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Theoretical Analysis of Laguerre-Gaussian Mode Generation for Microbunching Instability Suppression in Free-Electron Lasers.

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ECE 170A Principles of Photonics

Abstract

This paper reviews Tang et al.'s [1] application of Laguerre-Gaussian modes for suppressing microbunching instability in free-electron lasers. I further propose and model a spiral phase plate method to generate these modes while preserving linear polarization.

INTRODUCTION

Free-Electron Lasers (FELs), such as the Linac Coherent Light Sources (LCLS), are critical tools for probing matter at atomic scales using ultra-bright X-ray pulses [4]. LCLS uses incredibly fast X-ray pulses to essentially take pictures of atoms and molecules, which allows scientists to study processes that are fundamental across different fields [4]. However, the performance of these devices is fundamentally limited by collective electron beam effects. Of those effects, Microbunching Instability exists [1]. MBI occurs when small density modulations in the electron bunch are amplified during transport [1]. This, as a result, degrades the final X-ray beam quality. In order to get around this, this instability goes through Landau damping using an undulator [1]. This is a form of damping by which oscillations in a charged medium are damped with non-collisional interactions.

This review analyzes the recent experiment conducted by Tang et al. [1], which utilized a Laguerre-Gaussian (LG_{01}) donut mode to improve Microbunching Instability suppression. By shaping the transverse profile of the laser heater, the authors achieved a more uniform energy spread. Building upon their experimental results, we apply the theoretical framework from Principles of Photonics. More specifically we will be utilizing wave optics and polarization calculus to model an alternative method for generating these modes. Tan et al. used a specific mode converter. Instead this review will calculate the parameters required to implement a Spiral Phase Plate (SPP) system that satisfies the maintenance of fixed linear polarization in the interaction region.

METHODS

Experimental Review of Tang et al.

The experiment conducted at the LCLS injector utilized a laser heater to interact with a 135 MeV electron beam. The big innovation was the conversion of the heater laser from a standard Gaussian profile to a Laguerre-Gaussian (LG_{01}) profile. This mode features a null intensity at the center and a ring of high intensity, theoretically matching the transverse phase space of the electron beam more effectively as opposed to a Gaussian profile. The authors characterized the suppression of microbunching instability by measuring the spectral amplitude of the instability.

Theoretical Framework and Alternative Generation

To understand the propagation as well as the generation of these modes, we begin with the scalar Helmholtz equation. This governs the propagation of monochromatic optical fields [2]:

$$\nabla^2 U + k^2 U = 0$$

The solutions to this equation in cylindrical coordinates yield the Laguerre-Gaussian modes. The field distribution $U_{p,l}$ is characterized by a radial index p and an azimuthal index l . While the paper being reviewed used a specialized mode converter, this review paper proposes and analytically models a Spiral Phase Plate as an alternative for generating this mode while strictly preserving the linear polarization required for optimal laser-electron interactions.

Calculation 1: Spiral Phase Plate Design

An SPP creates an optical vortex by introducing an azimuthal phase delay. To convert a fundamental Gaussian beam into an (LG_{01}) donut mode, a topological charge of $l = 1$, meaning the phase must vary from 0 to 2π around the optic axis. The phase delay $\Delta\phi$ introduced by a plate of refractive index n and step height h is given by the modulation equation:

$$\Delta\phi_0(\theta) = k(n - n_{air})h(\theta)$$

For a full 2π phase shift at wavelength $\lambda = 760 \text{ nm}$, the required step height h_{step} is calculated assuming that the plate is made of fused silica ($n \approx 1.45$):

$$2\pi = \frac{2\pi}{\lambda}(n - 1)h_{step}$$

$$h_{step} = \frac{\lambda}{n-1} = \frac{760 \text{ nm}}{1.45-1} \approx 1688 \text{ nm}$$

This calculation confirms that a step height of about $1.69 \mu\text{m}$ is necessary to generate the necessary vortex.

Calculation 2: Polarization Analysis

This paper analyzes the SPP effect using Jones Calculus [2]. The input laser is horizontally linearly polarized:

$$J_{in} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

An ideal Spiral Phase Plate acts as a phase-only modulator. It does not exhibit birefringence. Jones matrix is an identity matrix multiplied by the spatial phase factor $e^{il\theta}$:

$$\mathbf{M}_{SPP} = e^{il\theta} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

The output field is then:

$$J_{out} = \mathbf{M}_{SPP} J_{in} = e^{il\theta} \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

The vector remains unchanged which proves theoretically that the proposed SPP method generates the donut intensity profile while maintaining the fixed linear polarization required for the magnetic undulator interaction in the laser heater.

Results and Interpretation

Suppression of Microbunching Instability

Tang et al. demonstrated that the LG_{01} mode significantly outperforms the Gaussian mode. As shown in their spectral measurements [1],

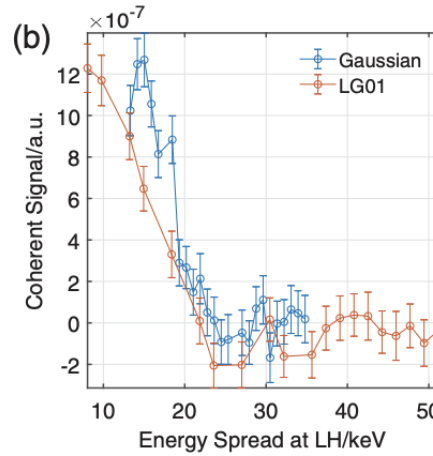


Figure 1: Energy spread vs Coherent Signal, adapted from Tang et al. [1]

the LG_{01} mode reduced the MBI gain across a broader range of laser energies which can be clearly seen in the 15-20 keV range. In a standard Gaussian heater, the electrons in the center of the bunch receive maximum heating. This heating often exceeds the cooling capacity, while outer electrons are under-heated. The LG_{01} mode allows for a heating where the energy is deposited on the periphery of the electron bunch. This produces a more uniform total energy spread which effectively suppresses instability. It does so without degrading the beam brightness.

Evaluation of Alternative Generation Method

The calculations in the methods section demonstrate that a spiral phase plate is a viable alternative to the complex mode converters used in the original experiment. The calculated step height of $1.69 \mu\text{m}$ is a component that can be easily manufactured. The Jones Calculus analysis confirms that the phase plate preserves the linear polarization state. This is a very important result because the laser heater relies on an undulator magnet. If the polarization were to become radial or azimuthal, the coupling efficiency between the laser and the electron beam would drop to zero. This confirms that the SPP approach is valid for specific scientific applications.

Conclusion

The work by Tang et al. establishes the LG_{01} mode as a superior tool for managing microbunching instability in next-generation FELs such as LCLS-II [4]. By redistributing the laser intensity to match the electron beam's phase space, the donut mode solves the overheating problem that prominently exists in Gaussian Beams. Through the theoretical analysis shown in this paper, it is determined that a spiral phase plate with a $1.69 \mu\text{m}$ step height offers a method for generating these modes. The application of Jones Calculus also verifies that this method maintains the fixed linear polarization necessary for the laser heat undulator. This leads to the idea that future FEL facilities could adopt passive phase optics to achieve these performance gains with simple optical beamlines.

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