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Publication Date

1981-12-01

Lawrence Berkeley Laboratory
UNIVERSITY OF CALIFORNIA

Accelerator & Fusion Research Division

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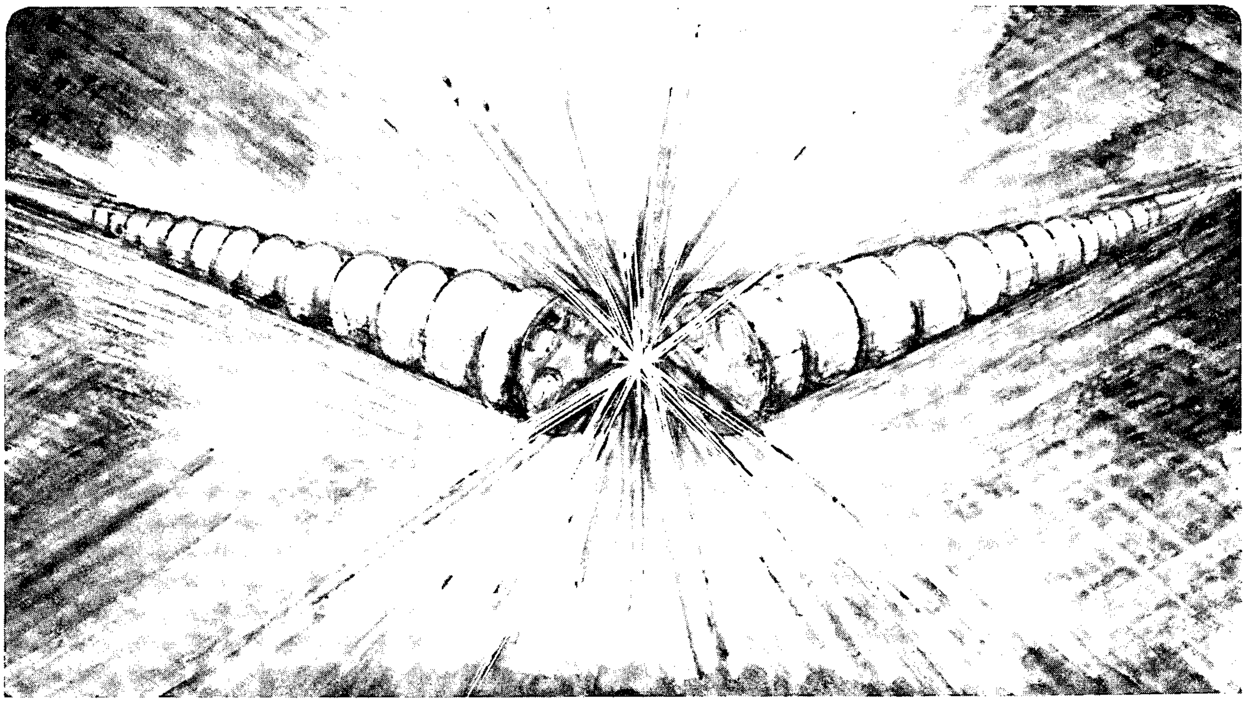

Prepared for the U.S. Department of Energy under Contract DE-AC03-76SF00098

LBID-536
BETA-18

Lawrence Berkeley Laboratory

Technical Report of the Betatron Design Study

THE AZIMUTHAL COUPLING IMPEDANCE OF A RING BEAM SITUATED MIDWAY BETWEEN INFINITE PARALLEL CONDUCTING PLANES*

Victor Brady, Andris Faltens, and L. Jackson Laslett

December 1981

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*Sponsored by Defense Advance Research Projects Agency (DoD)
ARPA Order No. 3718, Amend. 37
Monitored by NSWC under Contract No. N60921-81-LT-W0031

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER BETA-18	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) The Azimuthal Coupling Impedance of a Ring Beam Situated Midway between Infinite Parallel Conducting Planes		5. TYPE OF REPORT & PERIOD COVERED 6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) Victor Brady, Andris Faltens, and L. Jackson Laslett		8. CONTRACT OR GRANT NUMBER(s) N60921-81-LT-W0031
9. PERFORMING ORGANIZATION NAME AND ADDRESS Lawrence Berkeley Laboratory University of California Berkeley, California 94720		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS 61101E; 0; 0; OR40AA
11. CONTROLLING OFFICE NAME AND ADDRESS Defense Advanced Research Projects Agency 1400 Wilson Boulevard, Arlington, Virginia 22209 Attn: Program Management/MIS		12. REPORT DATE December 1981 13. NUMBER OF PAGES 7
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office) Naval Surface Weapons Center White Oak, Silver Spring, Maryland 20910 Attn: Code R401		15. SECURITY CLASS. (of this report) UNCLASSIFIED 15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

azimuthal coupling impedance
circular accelerators
cyclotrons
betatron
longitudinal stability

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

The azimuthal coupling impedance, which pertains to the longitudinal stability of intense particle beams in circular accelerators, is calculated for a ring beam situated midway between infinite, parallel, conducting planes as a function of frequency. The peak value is consistent with the approximation $Z_n/n = 300\ h/R$.

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SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

BETA-18
December 11, 1981

The Azimuthal Coupling Impedance of a Ring Beam Situated Midway between Infinite Parallel Conducting Planes*

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The azimuthal or longitudinal coupling impedance, Z_n, usually enters into the analysis of the stability of intense particle beams in circular machines as

$$\frac{Z_n}{n} \equiv - \frac{2\pi R E_n}{n I_n} ,$$

where R is the beam major radius, E_n is the ac electric field amplitude, and I_n is the ac current at the n^{th} harmonic of the beam revolution frequency. In a few machine types such as cyclotrons, betatrons, and the electron ring compressors, the beam-surrounding geometry is well approximated by conducting sideplates. The effect of the sideplates, or other similar conductors, is to suppress the electromagnetic radiation of the ring at the lowest few harmonics, that otherwise would be the major contributor to the coupling impedance of a relativistic ring, as well as to modify the self-field distribution and its minor contribution to the impedance.

The subject of coupling impedance is closely related to electromagnetic radiation by a charge moving in a circular orbit.[1,2] Starting with Eq. 7 of Nodvick and Saxon,[2] converted to MKSA units, the power P_n radiated at

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the n^{th} harmonic is,

$$P_n^{(\infty)} = \frac{1}{4\pi\epsilon_o} \frac{n\omega e^2}{R} \frac{4\pi R}{2h} \mathrm{Re}\left\{\sum_{j=1,3,\ldots}^{\infty} - H_n^{(1)} J_n + \frac{\beta^2}{2}\left(H_{n-1}^{(1)} J_{n-1} + H_{n+1}^{(1)} J_{n+1}\right)\right\} \tag{1}$$

where the argument of the Bessel functions is

$$\gamma_{nj} R = \left[(n\beta)^2 - \left(\frac{j\pi R}{2h}\right)^2\right]^{1/2},$$

the plate separation is $2h$, the charge of the particle is e, and the radian frequency is ω. The power may be related to an impedance Z_n as

$$P_n^{(\infty)} = \frac{1}{2} \mathrm{Re}\left\{I_n^2 Z_n\right\}. \tag{2}$$

For a δ-function charge, $I_n = 2I_o = \omega e/\pi$; therefore

$$\mathrm{Re}\left\{\frac{Z_n}{n}\right\} = 2\pi^2 \sqrt{\frac{\mu_o}{\epsilon_o}} \left(\frac{R}{2h}\right) \mathrm{R_e}\left\{\sum_{j=1,3,\ldots} - H_n^{(1)} J_n + \frac{\beta^2}{2}\left(H_{n-1}^{(1)} J_{n-1} + H_{n+1}^{(1)} J_{n+1}\right)\right\} \tag{3}$$

The results of evaluating this expression for $\beta = 1$ and several values of the ratio of beam radius to plate separation are shown in Fig. 1. Also shown is the free-space radiation asymptote, and the values predicted from the approximate formula for the peak of the impedance function,

$$\left(\frac{Z_n}{n}\right)_{max} \approx 300 \frac{h}{R} \text{ ohms}, \tag{4}$$

obtained in Ref. 3, for a beam situated between coaxial conducting cylinders. The close agreement of the approximate formula to the computed values is not surprosing for the present geometry in view of the results of Ref. 3, where the same approximation held for the essentially resonant geometry of a beam within a conducting cylinder as well as the nonresonant geometry of a beam outside of a conducting cylinder.

The impedance values shown in Fig. 1 are for a beam of vanishingly small minor dimensions and $\beta = 1$. The effect of β approaching 1 is to increase the cutoff of the synchrotron radiation spectrum to higher harmonics ($n_{crit} \propto \gamma^3$), but, because Z_n/n decreases with n at high harmonics, the detailed behavior there is not important for stability analyses. The effects of finite beam size, such as caused by the transverse and longitudinal emittances, are favorable for both the self-field and the radiation contributions to the impedance. The self-fields give a reactive term of the form

$$\frac{Z_n}{n} \approx - \frac{i\sqrt{\frac{\mu_o}{\epsilon_o}}}{\beta r^2} \left(\frac{1}{4} + \ln \frac{h}{a}\right), \tag{5}$$

at low harmonics, where a is the beam minor radius. The radiation from the ring near the maximum of the Z_n/n function for typical geometries of interest is largely due to the lowest axial harmonic, $j = 1$, and the peak occurs far enough above the cutoff for the radial wavelength to be comparable to the free-space wavelength. For an extended beam with a spatial current density $\vec{J}$ a factor

$$F \approx \frac{\int \vec{J}_{beam} \cdot \vec{E}_{mode} \, dA}{I \, E_{max}} \tag{6}$$

enters in the way the beam drives a given mode and in the way a mode drives the beam (which is completely analogous to the transit time factor in accelerating gaps), therefore the results of the impedance for a line beam should be multiplied by

$$F_n{}^2 \simeq \left(\frac{\sin\frac{\pi a}{2h}}{\frac{\pi a}{2h}}\right)^2 \left(\frac{\sin\frac{2\pi b}{\lambda_n}}{\frac{2\pi b}{\lambda_n}}\right)^2 \tag{7}$$

where a and b are the axial and radial minor radii of the beam. For an example of current interest, let $a = b = 5$ cm, $h = 17.5$ cm, $R = 2$m, $n_{max} \simeq 60$, and $\lambda_n = \frac{2\pi R}{n} \simeq 21$ cm, these factors give 0.41, or approximately a halving of the effective coupling impedance. In addition to these extended beam effects, the formulation of the stability criteria for such beams will be re-examined by members of the theory group.

To make the infinite plane results applicable to a finite ring geometry, it is necessary to provide an rf absorber at the outer radius of the vacuum chamber. In the ERA compressor this consisted of a few centimeter deep layer of loosely woven absorptive cloth cut from 100 Ω/□ material. With such an absorber, the wave impedance does not differ greatly from the free space value and there are no abrupt geometric discontinuities, resulting in very broad band absorption of propagating waves. A termination other than an absorptive outer wall will lead to undesirable reflections of radiation, and higher peaks in the impedance curve.

References

1. J. Schwinger, Phys. Rev. 75, 1912 (1949).
2. J. S. Nodvick and D. S. Saxon, Phys. Rev. 96, 180 (1954).
3. A. Faltens and L. J. Laslett, Particle Accelerators 4, 151 (1973) and ERAN 195.

APPENDIX

The program PINIF calculates the sum

$$S = \sum_{j=1,3,\ldots} \left[-H_n^{(1)} J_n + \frac{1}{2} \beta^2 \left(H_{n-1}^{(1)} J_{n-1} + H_{n+1}^{(1)} J_{n+1} \right) \right]$$

where $H_n^{(1)}$ is the Hankel function

$$H_n^{(1)} = J_n + i Y_n .$$

The argument of the Bessel functions is

$$\left[(n\beta)^2 - (j\pi R/H)^2 \right]^{1/2}$$

and the summation is carried out for all odd j such that

$$j \leqslant n\beta H/\pi R .$$

The program is set to calculate the sum for $n = 1,2,\ldots,400$ but seldom reaches $n = 400$ due to overflow in Y_n. The output consists of the argument value for the largest j, the real part of S, the imaginary part of S, and the magnitude of S. This output is printed for each value of n.

The program is stored as subset PINIF in PSS library COILS and may be accessed by the command

```
LIBCOPY,COILS,PINIF,PINIF.
```

The values of the parameters BETA, R, and H may be changed, and the program may then be submitted to the 7600 computer. The output is disposed to the printer with the hold-out option, and it may then be claimed from a terminal. A listing of the program follows.

```
 A2<J B22                                                                                        5
0                         XXXX  XXXX   XXX  XXXX  X   X
                          X   X X   X X   X X   X  X X
                          XXXX  XXXX  X   X X   X   X
                          X   X X  X  XXXXX X   X   X
                          XXXX  X   X X   X XXXX    X
0               4590 03 . . PINIF . 16 APR 82 . 14.52 . 4590 03
                .                                                   .
  X      XXX    .                                                   .
 X     X    X   .                                                   .
 X    X     X   .                                                   .
 X   X      X   .                                                   .
  XXXX      X   .                                                   .
                .                                                   .
  XXXXXXXXXX    .                                                   .
 X          X   .                                                   .
 X          X   .                                                   .
 X          X   .                                                   .
  XX       XX   .                                                   .
                .                                                   .
       XXXXXX   .                                                   .    XXXX XXXXX
    XXX         .                                                   .   X    X     X
 XXX            .                                                   .   X    X     X
    XXX         .                                                   .   X    X     X
       XXXXXX   .                                                   .   XXXXXXXXXXXX
                .                                                   .
 XXXXXXXXXXXX   .                                                   .   XXXXXXXXXXXX
 X          X   .                                                   .   X          X
 X          X   .                                                   .   X          X
 X          X   .                                                   .   X          X
 XXXXXXXXXXXX   .                                                   .   XXXXXXXXXXXX
                .                                                   .
 XXXXXXXXXXXX   .                                                   .   XXXXXX
 X     X     X  .                                                   .         XXX
 X     X     X  .                                                   .            XXX
 X     X     X  .                                                   .         XXX
  XXXXX XXXX    .                                                   .   XXXXXX
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                .                                                   .   X          X
                .                                                   .    XXXXXXXXXX
                .                                                   .
                .                                                   .    X      XXXX
                .                                                   .   X     X    X
                .                                                   .   X    X     X
                .                                                   .   X   X      X
                .                                                   .    XXX       X
                .                                                   .
0               4590 03 . .FRIDAY . 16 APR 82 . 14.55 . 4590 03
                   XXXXX X     XXXXX XXXXX XXXX       XXXXX
                   X     X     X   X X   X X   X      X
                   XXX   X     X   X X   X XXXX       XXXX
                   X     X     X   X X   X X  X           X
                   X     XXXXX XXXXX XXXXX X   X       XXX
```

```
PINIF,1,100,170030,XXXXXX,BRADY
FLOOR(5)
FTN4,L=LIST,ROUND.
MATHLIB.
LINK,X,L=LIST,PF.
DISPOSE,TAPE1=PR,PA=1F,HO,R=[FLOOR 5].
EXIT.
DOB.
COPY,DAYFILE,LIST.
DISPOSE,LIST=PR,HO,R=[FLOOR 5].
EXIT.
DUMP,0,L=LIST.
DOB.
COPY,DAYFILE,LIST.
DISPOSE,LIST=PR,HO,R=[FLOOR 5].
      PROGRAM PINIF(INPUT,OUTPUT,TAPE1)
*
*  THIS PROGRAM CALCULATES THE PART OF FORMULA 7 CONTAINED
*  IN CURLY BRACKETS FROM THE PAPER "SUPPRESSION OF
*  COHERENT RADIATION BY ELECTRONS IN A SYNCHROTRON" BY JOHN S.
*  NODVICK AND DAVID S. SAXON PUBLISHED IN PHYISCAL REVIEW VOLUME
*  95, NUMBER 1. IN THIS PROGRAM THE DISTANCE OF SEPARATION IS CALLED H
*  INSTEAD OF A. THE CALCULATION IS DONE FOR N = 1,...,400.
*  THIS PROGRAM IS STORED AS SUBSET PINIF IN PSS LIBRARY COILS.
*
      DIMENSION Y(3)
      REAL JAY(3)
      COMPLEX S,HAN(3)
      CALL DATE(L) $ WRITE(1,120) L
      PI=2.*ACOS(0.)
      H=.35
      R=2.
      BETA=1.
      BETSQ=BETA**2
      A1=PI*R/H
      A3=BETA/A1
      WRITE(1,100) BETA,R,H
      DO 40 N=1,400
      JMAX=A3*FLOAT(N)
      IF(2*(JMAX/2).EQ.JMAX) JMAX=JMAX-1
      IF(JMAX.GT.0) GO TO 10
      WRITE(1,110) N
      GO TO 40
10    S=CMPLX(0.,0.)
      DO 30 J=1,JMAX,2
      ARG=SQRT((BETA*FLOAT(N))**2-(A1*FLOAT(J))**2)
      NORD=N-1
      CALL BESYN(ARG,NORD,3,Y)
      ALPHA=FLOAT(NORD)
      CALL BESJ(ARG,ALPHA,3,JAY,NZ)
      DO 20 I=1,3
      HAN(I)=CMPLX(JAY(I),Y(I))
      A2=.5*BETSQ
      IF(I.EQ.2) A2=-1.
      S=S+A2*JAY(I)*HAN(I)
20    CONTINUE
30    CONTINUE
      P=REAL(S)
      AIS=AIMAG(S)
      SMAG=SQRT(P**2+AIS**2)
      WRITE(1,110) N,ARG,P,AIS,SMAG,JMAX
40    CONTINUE
```

```
      STOP
100   FORMAT(* BETA =*,F6.3/*      R =*,F6.3/*      H =*,F6.3//
     * 3X,1HN,4X,9HARG(JMAX),3X,4HREAL,8X,4HIMAG,8X,3HMAG,6X,4HJMAX/)
110   FORMAT(I5,1PE12.4,3E12.4,I5)
120   FORMAT(28X,A10//)
      END
```

```
555555555.555555555.555555555.555555555.555555555.555555555.555555555.55555555
555555555.555555555.555555555.555555555.555555555.555555555.555555555.55555555
```

```
555555555,555555555.555555555.555555555,555555555.555555555.555555555.55555555
555555555,555555555.555555555.555555555.555555555.555555555.555555555.55555555
555555555,555555555.555555555.555555555,555555555,555555555.555555555,55555555
555555555.555555555.555555555.555555555.555555555.555555555.555555555.55555555
        1         2         3         4         5         6         7

  A2<J822

  **3<Y74J*C 16 APR 82 14.55.11.

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Fig 1

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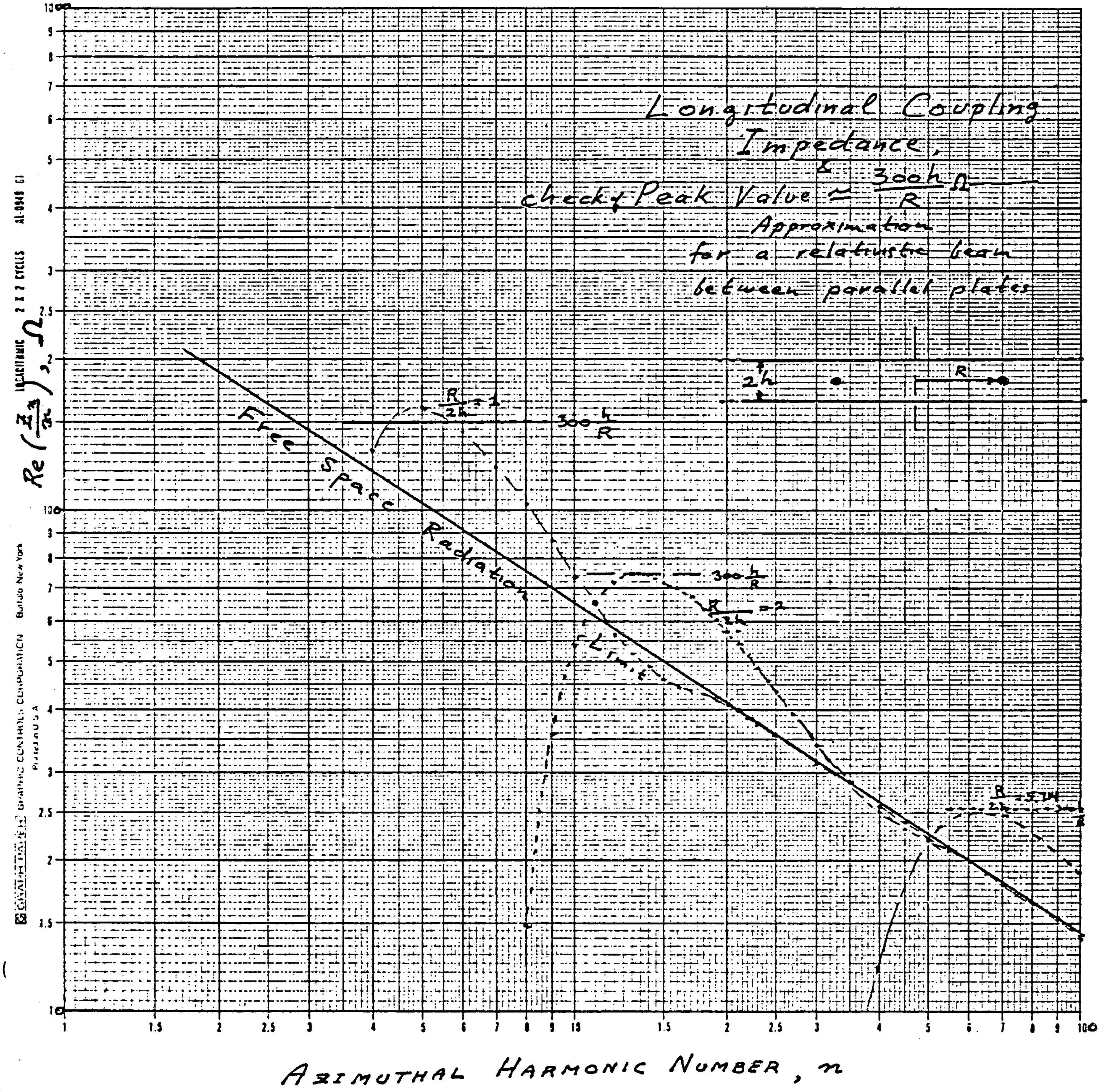

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