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# Engineered jumpers overcome biological limits via work multiplication

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8 For centuries, scientists have explored the limits of biological jump height<sup>1,2</sup>, and for decades, engineers have designed jumping machines<sup>3-18</sup> that often mimicked or took inspiration from 9 10 biology. Despite these efforts, general analyses are missing that compare the energetics of biological 11 and engineered jumpers across scale. Here we show how biological versus engineered jumpers have 12 key differences in jump energetics. The jump height of a biological jumper is limited by the work its 13 linear motor (muscle) can produce in a single stroke. In contrast, the jump height of an engineered 14 device can be far greater because its ratcheted or rotary motor can "multiply work" during 15 repeated strokes or rotations. As a consequence of these differences in energy production, biological 16 versus engineered jumpers should have divergent designs for maximizing jump height. Following 17 these insights, we created a device that can jump over 30 m high, far higher than previous 18 engineered jumpers and over an order of magnitude higher than the best biological jumpers. Our 19 work advances the understanding of jumping, shows a new level of performance, and underscores 20 the importance of considering differences between engineered and biological systems.

# 2223 Introduction

24 "Jumping [is] a peculiarly attractive subject for investigations," noted preeminent biomechanist R.M. 25 Alexander<sup>19</sup>. It is found across diverse species and size scales, yet is performed in strikingly similar 26 manners and has clear, quantifiable metrics by which ultimate capabilities can be compared: jump height 27 and distance. Indeed, the seemingly simple act of jumping has intrigued thinkers for centuries. Aristotle 28 pondered how humans could increase jump height with halteres<sup>1</sup>, while a Renaissance model 29 approximates that all animals, regardless of size, jump roughly the same height of one metre<sup>2</sup>. More 30 recent biological jumper models have examined performance limits across scale in more detail<sup>20</sup>, 31 incorporating effects of leg length<sup>19</sup>, jumper height<sup>21,22</sup>, muscle dynamics<sup>23-25</sup> as well as considering the use 32 of springs<sup>26,27</sup> and latches<sup>28,29</sup> for power-limited jumpers, and air drag for small and light jumpers<sup>11,30</sup>. The 33 performance limits of jumping across scale are thus well-studied, within the domain of biology. 34

These studies have informed the design of many bio-inspired engineered jumpers, dating back to at least 1967<sup>3</sup>. However, a general modelling framework to capture and quantify inherent differences in biological and engineered jumpers across scale is missing from the literature. Most engineering works focus on specific designs<sup>3-18,31</sup>, draw conclusions based on previous biological models<sup>10</sup>, or present models that only describe single-stroke linear motors, as found in biology<sup>14,32</sup>.

## 41 Model

42 Here we present a model of jumping that compares the energetics of both biological and engineered

- jumpers. We define a jump as a movement created by forces applied to the ground by the jumper, whilemaintaining a constant mass (Fig. 1a). Thus, a rocket does not jump, nor does an arrow shot from a bow.
- 44 maintaining a constant mass (Fig. 1a). Thus, a focket does not jump, not does an arrow shot from a bow. 45 We examine two aspects of a jump: specific energy production limits (the maximal energy that could be
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46 created for a single jump per unit mass of a jumper) and specific energy utilisation (the efficiency of 47 converting this specific energy into jump height). We concentrate the following discussion on specific 48 energy production limits, as in previous biological studies<sup>26</sup>, because we are interested in the upper 49 bounds of jumping, and specific energy directly corresponds to maximum (lossless) jump height in a 49 given gravitational field (e=gh). (See Methods: Energy Utilisation Model for discussion of energy 49 utilisation non-idealities such as non-vertical motions<sup>19</sup>, distributed mass in the spring<sup>32</sup>, and air drag<sup>30</sup>.)

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For our analysis, we consider the following components of a jumper: a motor, an optional spring, a 54 linkage, and a payload (Fig. 1b). For the motor, we consider two types: biological and engineered (we 55 focus on electromagnetic, though others could be substituted). Both motor types can have one of two 56 transmission types—direct-drive (no spring) or spring-actuated (with spring)—resulting in four jumper 57 configurations. For direct-drive transmissions, the motor directly connects via a stiff, light tendon to a 58 linkage, the structure necessary to transmit forces to the ground. For spring-actuated (also termed power-59 amplified) transmissions<sup>32–34</sup>, the motor may slowly pre-stretch a spring before the spring rapidly releases the energy into the linkage<sup>28</sup>; this can be done without a latch<sup>14,29</sup>, but here we focus on the latched case. 60 61 Finally, the payload comprises all remaining parts of a jumper, including the energy supply (assumed to 62 be sufficient for multiple jumps) and non-energetic items, and we assume the payload does not directly 63 limit or affect the single-jump energy production. We thus consider payload in the Methods: Energy 64 Utilisation Model.

66 Of the energy-production components, we find the motor to have the most important differences between 67 biological and engineered jumpers. A biological motor is a linear muscle with a finite single stroke 68 bounding its work capacity. An engineered motor, in contrast, can overcome this single-stroke work limit. 69 A linear engineered motor may use ratchets to combine multiple strokes (e.g., in jumping microrobots<sup>35-</sup> 70  $^{38}$ ), and a rotary motor may turn repeatedly to combine multiple rotations (e.g., in centimetre-scale 71 robots<sup>13,15</sup>). We term this "work multiplication." The number of strokes or rotations can be raised by 72 increasing the gear reduction between the motor stroke and the jumper's overall motion (see Extended 73 Data Fig. 1). For a direct-drive transmission, work multiplication occurs during the acceleration phase, 74 and for a spring-actuated transmission, it primarily occurs during the pre-stretch phase. Work 75 multiplication is available only to engineered jumpers as ratchets and rotary motors have not been found 76 above the cellular scale in biology<sup>39</sup>.

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78 To describe the upper limits of specific energy production, the model considers three primary limiters. 79 The first limiter is the motor's single-stroke specific work<sup>27</sup>, or the integral of specific force over stroke 80 length (termed "motor work limiter"). This limiter is not present for engineered jumpers, due to work 81 multiplication. The second limiter is the motor's specific power-time, or the product of specific power and available acceleration time<sup>26</sup> (termed "motor power limiter"). The third limiter is the spring specific 82 83 energy, or energy that can be stored and released per unit mass of the spring, and it is only present for 84 spring-actuated transmissions (termed "spring energy limiter"). We assume sufficient time between jumps 85 to fully pre-stretch the spring regardless of the motor's power as well as sufficient spring power to 86 discharge the energy during the acceleration time. Additionally, we consider the linkage mass necessary 87 to transfer and apply the energy. Thus, the per-unit-mass specific jumper energy will approach but never 88 reach its bounding limiter, especially at the high specific energies of the best engineered jumpers that 89 require significant linkages (for scaling of linkage mass, see Methods: Energy Production Model).

#### 91 Model Results and Insights

92 The results of our model (Fig. 2) show that for biological jumpers, specific energy production can never93 surpass the motor work limiter, yet for engineered jumpers, the upper bound can be far greater.

94 Specifically, biological direct-drive jumpers at a large scale (e.g., a leopard) can produce specific energy
 95 approaching the motor work limiter; at a small scale (e.g., a lizard), specific energy will be lower due to
 96 power limitations<sup>26,27,32</sup>. Biological spring-actuated jumpers at a small scale (e.g., a flea) have sufficient

- 97 power, but again specific energy is capped by the motor work limiter; at a large scale, springs are
- 98 unnecessary and actually decrease specific energy due to added mass and muscle-spring force-
- 99 displacement characteristics<sup>26,27,32</sup>. These trends align with previous models in the literature and biological
- 100 jump data (Extended Data Fig. 2).
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For engineered jumpers, work multiplication eliminates the motor work limiter. At a small scale, springactuated transmissions result in higher jumps than direct-drive transmissions, with an upper bound set by the spring energy limiter and the linkage mass. Theoretically, at very large scales, direct-drive transmissions are superior, with an upper bound set by the motor power limiter.

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107 These differences in energetics lead us to find that biological versus engineered jumpers should have 108 divergent designs for maximizing specific energy production-and thus jump height. We present three 109 key insights into these design differences. First, for biological jumpers, the crossover scale below which 110 spring-actuated jumpers produce more specific energy and above which direct-drive jumpers produce 111 more specific energy is approximately 1 m (0.6 s acceleration time). In contrast, for engineered jumpers, 112 this crossover scale is nearly two orders of magnitude larger, at approximately 100 m (3 s acceleration 113 time). (For crossover times, see Fig. 2; for conversion to scale, see Methods: State-space Model and 114 Extended Data Fig. 3-4.)

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116 Second, engineered jumpers should use a ratio of spring mass to motor mass (termed "spring-motor mass 117 ratio") that is much larger than that of biological jumpers (Fig. 2e). In biology, the motor work is the 118 limiting factor; therefore, to maximize specific energy production, the spring energy capacity (i.e., the 119 product of specific energy and mass) should equal but not exceed motor work (See Methods: Spring-120 motor mass ratio). Since spring specific energy is much larger than motor specific work, only a spring 121 mass much smaller than the motor mass is needed. We find an optimal ratio of 0.029 for biological 122 jumpers, in line with morphological data (0.025-0.06<sup>26</sup>). In contrast, for engineered jumpers with 123 sufficient work multiplication, large amounts of energy can be accumulated in the spring. Thus, we find 124 that the spring-motor mass ratio should be much larger than the ideal ratio for biology. Essentially, work 125 multiplication allows engineering to better utilize the high specific energy of springs. 126

127 Third, for spring-actuated transmissions, biological jumpers should maximize the specific work of the 128 motor, but engineered jumpers should maximize the combined specific energy of the spring plus linkage 129 (spring-linkage specific energy). This is because each system should maximize that which primarily limits 130 its specific energy.

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# 132133 From Insights to Jumper Design

134 We followed these three insights to push the limits of specific energy production, and consequently jump 135 height, for engineered devices with electromagnetic motors. First, we chose a spring-actuated 136 transmission, given the selected scale of 0.3 m. Second, we set a high spring-motor mass ratio by 137 selecting a small rotary motor (10 g) with a large gear reduction (1000:1). This enables the motor to 138 compress a relatively large spring with 150 N of tension in a line wrapped around its spindle (See 139 Methods: Jumper Design). Third, with this peak force constraint, we designed a high-specific-energy 140 hybrid spring-linkage via a custom non-linear simulation framework (See Methods: Jumper Design). We 141 simulated two spring-linkage designs from the literature: a tension linkage<sup>13</sup> (passive carbon fibre linkage

142 with rubber in tension) and a compression bow<sup>10</sup> (carbon fibre only) (Fig. 3a). Our simulation found the 143 tension linkage has only slightly higher specific energy (1,638 J/kg versus 1,313 J/kg), despite the high 144 specific energy of the rubber (7,000 J/kg<sup>40</sup>). To improve, we designed a hybrid-tension-compression 145 spring-linkage, supporting rubber in tension with a compression-bow (1,922 J/kg). The improvement can 146 be thought of in two ways: compared to the tension linkage, we enable the linkage to store energy so it is 147 no longer passive; compared to the compression bow, we add high-specific-energy rubber in tension. Our 148 spring-linkage also has a nearly constant force-displacement curve, which helps it store a large amount of 149 energy given the force constraint. It provides a spring-motor mass ratio (considering the whole spring-150 linkage mass, 12.4 g) of 1.2 (versus 0.025-0.06<sup>26</sup> in biology).

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Using this hybrid spring-linkage and motor design, we created a jumper (Fig. 3b, Extended Data Fig. 5) with minimal losses in six identified stages of energy utilisation (See Methods: Energy Utilisation). For instance, we minimized the mass of the "foot" (components of the jumper that are stationary during acceleration) to make energy transfer losses small, and created a shape-changing morphology that becomes streamlined after take-off to minimize air drag. We measured a payload-free specific energy of 1075 J/kg (24.2 J per 22.5 g, see Methods: Jumper Design), and observed our 30 g jumper accelerating from 0 to over 28 m/s in 9 ms (> 3000 m/s<sup>2</sup>) and reaching a height of 32.9 +/- 0.7 m (n = 3) (Fig. 3c).

160 For comparison, we calculated a payload-free specific energy production for the best biological jumpers 161 of ~170 J/kg<sup>26</sup>, and for the best engineered jumpers with electromagnetic motors of ~100 J/kg<sup>15</sup> and ~115 162 J/kg<sup>13</sup>. Considering payload and other utilisation non-idealities, these specific energies result in jump 163 heights for a galago of 2.25 m<sup>41</sup> and for the engineered jumpers of 3.7 m<sup>15</sup> and 3.8 m<sup>13</sup>.

### 165 Conclusion

166 In this work, we presented modelling, insights, and a demonstration. Via modelling, we showed that 167 specific energy production of biological jumpers cannot exceed the motor specific work, yet through work 168 multiplication, engineered jumpers can overcome this limit, resulting in the potential to jump much 169 higher. As a consequence, biological and engineered jumpers have different designs for maximizing 170 specific jump energy—and lossless jump height—which we described in three design insights. According 171 to these, we designed a jumper that demonstrated a jump over 30 m high.

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173 Our model suggests that this is near the feasible limit of jump height with currently available materials. 174 Within specific energy production, assuming that the spring specific energy is near the limit of available 175 materials for solid elastic springs, the primary potential improvement is in the spring-motor mass ratio. 176 However, even increasing the ratio from 1.2 to infinite would only increase jump height by approximately 177 17% (See Methods: State-space Model, Extended Data Fig. 6). Within specific energy utilisation, the 178 primary improvement comes from minimizing drag effects by increasing scaling; we see less room for 179 improvement in other losses (See Methods: Utilisation Model). However, isometrically scaling the 180 presented jumper by 10x (the predicted optimum which is large enough to eliminate drag but not too large 181 to incur other losses) would result in only a 19% increase in jump height.

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Finally, we note that our specialized design trades off adaptability, as found in biology, for high performance. Nevertheless, our results change the implications of jumping as a means of locomotion, changing how and where jumping could be used (see Supplementary Video S4). On Earth, jumping robots could overcome obstacles previously only navigated by flying robots while collecting vision-based data of the ground below (see Supplementary Video S5), and on the Moon, the leaps of the presented jumper would be even loftier: 125 m high while covering half a kilometre in a single bound. Our work

fundamentally advances the understanding of the "peculiarly attractive subject" of jumping and underlines the importance of considering the differences between biological and engineered systems.

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293 294 Fig. 1 | Graphical overview of the modelling framework. a, Temporally, we consider a jump to include an 295 optional pre-stretch phase (for jumpers with latched springs), an acceleration phase during which a force is applied 296 to the ground to accelerate the jumper upward, and a flight phase (See SI for details of simulation shown). 297 Energetically, we consider two aspects of a jump. First is the specific energy production limit (red dashed), giving 298 the upper bound on the energy per unit mass that can be produced. Second is the specific energy utilisation (grey 299 curve), considering losses, for example due to non-idealities in energy transfers and air drag (see Methods: Energy 300 Utilisation). Note: only the latched-spring case is shown. b, We categorize jumpers according to transmission type 301 (direct-drive versus spring-actuated) and motor type (biological versus engineered). For direct-drive transmissions, 302 the motor connects directly to the linkage, and for spring-actuated transmissions, the motor stretches a spring, which 303 drives the jump. For biological motors (muscles), the output work can never exceed the work of a single stroke. For 304 engineered motors, the output work is the product of the single stroke work multiplied by the number of strokes, 305 termed "work multiplication." Here a ratcheted linear motor is shown; rotary motors perform similarly (Extended 306 Data Fig. 1).



308 309 310 Fig. 2 | Trends of specific energy upper bounds for biological versus engineered, and direct-drive versus spring-actuated jumpers. a-d, The upper bound of a jumper's specific energy (black line) will approach limiters 311 (broken lines), but remain below due to required linkage mass (blue shading). a, Biological, direct-drive: two 312 limiters are present, (i) the motor's single-stroke specific work limiter (integral of specific force over stroke length; 313 muscle: ~200 J/kg<sup>26</sup>, dash-dot pink), and (ii) the motor's specific power-time limiter (product of specific power 314 (muscle: ~200 W/kg<sup>42</sup>) and acceleration time, dotted red). **b**, Engineered, direct-drive: because work multiplication 315 removes the motor work limiter, only the motor power limiter is present (electromagnetic:  $\sim 2000 \text{ W/kg}^{43}$ ). c, 316 Biological, spring-actuated: the addition of a latched spring helps overcome the motor power limiter, but adds a new 317 limiter: spring specific energy (tendon/apodeme in pure tension: ~7000 J/kg<sup>26</sup>, dashed purple). However, the 318 jumper's specific energy can never surpass the motor work limiter and thus never approaches the spring energy 319 limiter. d, Engineered, spring-actuated: work multiplication again removes the motor work limiter, allowing the 320 jumper's specific energy to rise closer to the spring energy limiter (latex in pure tension:  $\sim 7000 \text{ J/kg}^{40}$ ). e, These 321 differences result in different ideal spring-motor mass ratios (ratio of spring mass to motor mass) in spring-actuated 322 jumpers: ~0.025 for biological and much larger for engineered. Dots in (c-e) mark ratio at 0.01 s acceleration time, 323 corresponding to presented jumper. Note: the x-axis for (a-d) shows acceleration time, to easily relate power and 324 energy; acceleration time relates monotonically to length scale for an isometrically scaled jumper (see Methods: 325 State-space Model).



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Fig. 3 | From insights to an engineered jumper that exceeds 30 m. a, Simulation results of specific energy (area under curve) for three different spring configurations: two from the literature (tension-linkage<sup>13</sup> and compression $bow^{10}$ ) and one that we designed (hybrid tension-compression). **b**, The presented jumper with the hybrid spring in a stable pre-jump configuration. Scale bar 10 cm. c, Image of the device jumping with lines added over the position of the jumper every 200 ms (see Supplementary Video S2). Human is 1.83 m. d, Jump height of the presented and other jumpers (reference in bracket) shown as a function of the mechanism specific energy (where mechanism is the motor, linkage, and optional spring). Biological jumpers tend to have lower utilisation efficiency (lower jump height for a given mechanism specific energy) due to higher payloads. Two points are shown for jumpers not using electromagnetic motors, but instead propane<sup>44</sup> and CO<sub>2</sub><sup>45</sup> (specific energy of propane is 10,000 kJ/kg<sup>44</sup> and compressed  $CO_2$  in a composite tank is 250 kJ/kg<sup>46</sup>; mechanism specific energy is greatly reduced due to required structure). Grey lines represent energy utilisation. See Extended Data Tables 1 and 2 for details of data and calculations. e, Frames from Supplementary Video S1 with acceleration phase occurring in 9 ms. Scale bar: 10 cm. f, Frames from Supplementary Video S3 of self-righting using the legs as a roll-cage<sup>10,17</sup>, enabled by adding four 341 tapered legs (see Supplementary Video S4 and Methods: Jumper Design for details). Scale bar: 10 cm.

## 342 Methods:

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# 344 Limits of Energy Production Model

345 We first consider the maximum energy that could be available to the jumper, dependent on its 346 components. We consider the payload, which we assume does not directly limit or affect the single-jump 347 energy production in the Energy Utilisation Model, and segment the remainder of the jumper as follows: 348 (i) a motor, providing the mechanical energy, (ii) optional elastic energy storage or springs, temporarily 349 accumulating mechanical energy, and (iii) inelastic linkage or other elements, applying the energy via 350 ground reaction forces. In the extremes, the jumper may contain no springs (purely inelastic) or it may 351 utilise the springs for structural support and require little to no inelastic linkage elements. Note we include 352 linkage mass in the energy production rather than utilisation, as neither motors nor springs can operate in 353 isolation. Indeed, to fairly evaluate the energy production, we must consider how much linkage mass a 354 design requires to function. We begin with direct-drive transmissions before considering spring actuation. 355 Also note that specific energy per unit mass is denoted by a lowercase e, versus absolute energy is 356 denoted by an uppercase E

357

358 *Direct-drive Transmission:* We determine the maximum specific jump energy,  $e_{jump}^{\text{direct}}$ , assuming the jumper 359 contains only a motor of mass  $m_m$  and a linkage of mass  $m_l$ . In biology, the muscle's specific energy is 360 limited by the maximum specific work of a full stroke

$$361 \quad e_m^{bio} = \frac{1}{m_m} \int_0^d F_{max} dx$$

defined by integrating the maximum force,  $F_{max}$ , over the entire stroke, d. In engineering, a linear motor with the addition of a ratchet could complete multiple strokes to overcome such a limit. Similarly, a rotary motor has an unlimited stroke and hence an unlimited energy (limited ultimately only by the energy supply; because battery specific energy is orders of magnitude larger than those considered in this analysis, approximately 500 kJ/kg<sup>46</sup>, we assume it is nearly infinite). We generally apply  $e_m^{eng} = \infty$ .

Interestingly, biological muscle ratchets at a microscopic scale, but its macroscopic structure loses thefeature and limits its stroke.

370

Both biological and engineered motors are also limited by their maximum specific power  $p_m = \frac{P_m}{m_m}$ , available during the acceleration time  $t_0$ . The specific jump energy is thus limited by both as

$$374 \quad e_{jump}^{\text{direct}} = \frac{E_{jump}^{\text{direct}}}{m_m + m_l} = \frac{\operatorname{Min}(E_m, P_m t_0)}{m_m + m_l} = \frac{\operatorname{Min}(m_m e_m, m_m p_m t_0)}{m_m + m_l} = \frac{m_m}{m_m + m_l} \operatorname{Min}(e_m, p_m t_0)$$

375

376 *Spring-actuated Transmission:* We further include a spring of mass  $m_s$ , with a maximum specific energy 377 capacity of  $e_s = \frac{E_s}{m_s}$ . If we allow the motor a pre-stretch time  $t_p$ , the spring may store energy up to

378

$$\begin{array}{ll} \mathbf{379} \quad E_{store} = \mathrm{Min}\left(m_{s}e_{s}, m_{m}e_{m}, m_{m}p_{m}t_{p}\right) \\ \mathbf{380} \end{array}$$

381 If we allow the motor to continue providing energy during the acceleration phase (not possible for many jumper designs, but represents the upper limit) and assume the spring can deliver specific power up to

383  $p_s = \frac{P_s}{m_s}$ , we have a maximum specific jump energy of 12

384

385 
$$e_{jump}^{\text{spring}} = \frac{E_{jump}^{\text{spring}}}{m_m + m_s + m_l} = \frac{\text{Min}[m_m e_m, E_{store} + m_m p_m t_0, m_s p_s t_0]}{m_m + m_s + m_l}$$

386

**387** Finally, if we assume the spring's output specific power  $p_s$  far exceeds the requirements  $p_s \gg \frac{e_s}{t_0}$ , we find

388 
$$e_{jump}^{\text{spring}} = \frac{E_{jump}^{\text{spring}}}{m_m + m_s + m_l} = \frac{\text{Min}[m_m e_m, m_m p_m(t_p + t_0), m_s e_s + m_m p_m t_0]}{m_m + m_s + m_l}$$

389

390 Spring-Motor Mass Ratio: The mass ratio is used in Fig. 2. We see that increasing the spring mass  $m_s$ 391 only helps when the system is neither limited by motor energy nor by motor power 392

393 
$$e_{jump}^{\text{spring}} = \frac{m_m}{m_m + m_s + m_l} \operatorname{Min}\left[e_m, p_m(t_p + t_0), \frac{m_s}{m_m}e_s + p_m t_0\right]$$

394

395 Thus, assuming sufficient pre-stretch time  $t_p$ , biological jumpers are helped up to an optimal mass ratio 396 where  $e_m = \frac{m_s}{m} e_s + p_m t_0$ , or

397 
$$\left(\frac{m_s}{m_m}\right)_{optimal} = \frac{e_m - p_m t_0}{e_s} \approx \frac{e_m}{e_s},$$

398 which equates to ~0.029 when values of  $e_m$  and  $e_s$  from Fig. 2 are used (~200 J/kg and ~7000 J/kg, 399 respectively). In contrast, adding spring mass to engineered jumpers helps indefinitely, approaching the 400 maximal specific energy 401

402 
$$\left(e_{jump}^{\text{spring}}\right)_{limit} = \frac{m_s e_s + m_m p_m t_0}{m_m + m_s + m_l} \rightarrow \frac{m_s}{m_s + m_l} e_s,$$

403

405

404 assuming the linkage mass  $m_l$  also needs to increase to support the higher energies.

406 Size/Length Scaling: Beyond mass ratios, the jump energy limit only depends on the motor/spring specific 407 energy/power properties, as well as the pre-stretch time (which we assume can be freely selected) and the 408 acceleration time,  $t_0$ . The specific energies and powers are assumed to be scale-invariant: in biology<sup>47,48</sup>, muscle forces scale with area,  $F_{max} \propto L^2$ , while distance (stroke) and velocity scale with length, 409 d  $\propto$  L,  $v_{max} \propto L$ , and mass scales with volume,  $m \propto L^3$ . In engineering, with electromagnetic rotary 410 motors, the torque scales with the 4<sup>th</sup> power of length,  $\tau \propto L^4$ , while angular speed scales inverse 411 linearly,  $\omega \propto L^{-1}$ , so again the power remains scale invariant<sup>47,49,50</sup>. Similar to biological muscle, all 412 spring forces scale with area,  $F_{max} \propto L^2$ , while distances (stroke) scale with length, d  $\propto$  L, such that 413 414 spring specific energies are scale independent.

415

416 We also note that assume the required linkage mass  $m_l$  simply scales with jump energy: to maintain a 417 constant stress across scale, the cross-sectional area of the linkage will scale with force  $F_{max}$ , while the 418 length scales with stroke d. As such, we can define a scale-invariant specific energy transfer capacity

$$420 \quad e_l = \frac{E_{jump}}{m_l}.$$

 $\mathbf{r}$ 

422 Equivalently, and again assuming sufficient pre-stretch time  $t_p$ , we write the required linkage mass as  $\frac{E_{jump}}{e_{l}} = \frac{\operatorname{Min}[\tilde{m}_{m}e_{m}, m_{s}e_{s} + m_{m}p_{m}t_{0}]}{e_{l}}$ 172

423 
$$m_l = \frac{1}{e_l} = \frac{1}{e_l}$$

to obtain 424

$$e_{jump} = \frac{E_{jump}}{m_m + m_s + m_l} = \frac{1}{\frac{1 + \frac{m_s}{m_m}}{\min\left[e_m, \frac{m_s}{m_m}e_s + p_m t_0\right]} + \frac{1}{e_l}}.$$

425

426

427 For a spring-actuated engineered jumper, this brings the maximal specific energy for an infinite spring-428 motor mass ratio to

429 
$$(e_{jump})_{eng-limit} = \frac{1}{\frac{1}{e_s} + \frac{1}{e_l}} = \frac{e_s e_l}{e_s + e_l}.$$

430

431 We note that this final expression is similar to how stiffnesses of springs add in series. In Fig. 2, we approximate for engineering  $e_l$  as 2650 J/kg, based on the values from our jumper (where  $e_s = 7000 \frac{J}{kg}$ 432 and  $\left(e_{jump}^{\text{spring}}\right)_{limit} = 1922 \frac{J}{kg}$ ). For a motor-work-limited biological jumper with minimal spring mass, we 433 434 calculate

 $(e_l)_{bio} = \frac{e_m m_m}{m_l},$ 435

436

437 as 1130 J/kg, based on the values for muscle specific energy (200 J/kg) and percent of mass of the 438 skeleton (~14%<sup>51</sup>).

439

440 Only the acceleration time  $t_0$  is scale dependent (for both direct-drive and spring-actuated transmission 441 types). For isometric scaling assumptions, acceleration time monotonically increases with scale. More specifically, for all but the largest direct-drive jumpers, configured to operate at peak power during the 442 entire acceleration phase, time scales with a 2/3 power,  $t_0 \propto L^{2/3}$  (see State-space Model below and 443 previous biological model<sup>26</sup>). Meanwhile, the acceleration time for spring-actuated jumpers increases 444 445 linearly with scale,  $t_0 \propto L$  (also see State-space Model below).

446

447 We finally note that our model showing that spring-powered jumpers are scale-invariant in specific 448 energy production contrasts with the conclusions of previous work<sup>32</sup>, which stated that specific energy 449 production decreases at small scales for spring-powered jumpers. The discrepancy arises from differing 450 model assumptions: the previous work, in an effort to model not just jumpers but also many other high-451 power movements, considered a catapult launching a projectile, where only the projectile, but not the 452 catapult, changed size during scaling. This led to the conclusion that the catapult's spring would meet 453 material limits; however, during scaling of a jumper, spring and all, this effect is not present, and spring-454 powered biological jumpers should be scale-invariant, as shown in more recent work<sup>27</sup>.

455 456

#### 457 **Energy Utilisation Model**

458

459 An energy utilisation model is a helpful design tool, as it describes the effects of different losses on the 460 achievable jump height, h, (defined as the change in vertical COM position from standing to apex), given 461 a maximum payload-free specific jump energy,  $e_{jump}$ . Using an energy flow perspective, we lump all 462 losses and reductions into the following six types or stages. We note that many of the individual 463 components have been discussed in separate papers, as referenced below, and that this general framework 464 that assembles disparate models is helpful for design and analysis. Further, we realize the numerical 465 computation of each reduction may require assumptions or approximations (see the Supplementary 466 Information for derivations). The six stages are:

Specific Energy

Losses

1	Produced Energy:	e <sub>∏ii</sub>	j	$e^{\Box}_{_{jump}}\eta_{\prod {}_{\dot{\iota}\dot{\iota}}}$	(production inefficiency)
2	Available Energy:	$e_0$	i	$e_{\prod i \left(1-rac{m_{payload}}{m}\right)i}$	(payload apportionment)
3	KE, Total:	$e_{\rm KE}$	j	$e_0 - Lg \frac{m_{body}}{m}$	(less energy-to-stand)
4	KE, Vertical:	e <sub>vert</sub>	j	$\boldsymbol{e}_{K\!E} \big[ 1 - \boldsymbol{\beta}_{xy} - \boldsymbol{\beta}_{\theta} \big]$	(less non-vertical)
5	KE, Centre of Mass:	e <sub>COM</sub>	j	$e_{vert}\left[1-\frac{m_{foot}}{m}\right]$	(less energy transfer losses)
6	PE, Centre of Mass:	$e_{apex}$	j	$e_{COM}\left[1 - \frac{D_s e_{COM}}{2}\right]$	(less aerodynamic drag)

<sup>467</sup> 

468 Where  $\eta_{\prod i,i}$  is production efficiency,  $m_{payload}$  is the mass of the payload, *m* is the total mass,  $m_{body}$  is the 469 lumped mass that is moving during acceleration (see SI),  $m_{foot}$  is the lumped mass that is static,  $\beta_{xy}$  and 470  $\beta_{\theta}$  are the fraction of the kinetic energy due to movements in any horizontal direction and due to 471 rotations, respectively, and  $D_s$  is a drag constant (see SI). Overall, we write the model as

$$472 \quad h = \frac{1}{g} e_{apex} = \frac{1}{g} i$$

- 473 Further details of each stage:
- Produced specific energy, e<sub>∏ id</sub>, considering the production efficiencies: Any impedance mismatches between components or viscous losses will reduce the available energy. Practically, the force-displacement profiles of biological muscle and tendons limits jumpers to obtain 30-50% of the potential muscle energy<sup>26,27</sup>. In contrast, our nearly constant force spring matches the nearly constant force output of our motor, mitigating this loss. Further, a very small damping ratio (0.02) was experimentally determined for the carbon fibre experimentally measured using a clamped beam oscillation method<sup>52</sup>.
- 481 2) Initial specific energy before movement,  $e_0$ , that can be released in a single jump: Any payload 482 requires apportionment across the entire mass<sup>26</sup>. Our jumper has a payload-free mass of 22.5 g and 483 payload mass of only 7.9 g. Thus the 1075 J/kg payload-free specific energy is reduced to 796 J/kg in 484 this step. However, we see little room for improvement here. Our battery is a lithium polymer battery, 485 which is the lightest commercially available option. Our release mechanism has a mass of 1.23 g 486 (made from 7075 aluminium) and our nose cone 1.2 g. All other components are less than a gram.
- 487 3) Total specific kinetic energy,  $e_{KE}$ , after the full stroke has occurred: This deducts the potential energy 488 surrendered to raise the centre of mass from crouch to stand<sup>21</sup>. This delivers the jump height as the

- change in height of the centre of mass above its position when the jumper is fully standing. Our jumper has negligible loss here, due to its small size and high jump.
- 491 4) Vertical specific kinetic energy,  $e_{vert}$ , due to movements in the vertical (Z) direction only<sup>19</sup>. This 492 deducts the fraction of the kinetic energy due to movements in any horizontal direction ( $\beta_{xy}$ ) and due 493 to rotations ( $\beta_{\theta}$ ). This also includes potential losses due to sliding on the ground surface or frictional 494 losses in joints. Along with (5), below, our jumper has roughly 50% efficiency for these stages. 495 Interestingly, a pure compression spring of a solid material with only vertical motion mitigates this 496 non-vertical loss, but is only 50% for the energy transfer loss. Alternatively, a realistic compression 497 spring, such as a coil or bow spring, will have substantial non-vertical motion, meaning its efficiency 498 will be below 50%. Our device exceeds 50% because of its payload, which is placed at the top of the 499 jumper, while the "foot" mass is minimized (only the bottom of the spring).
- 5) Vertical Centre-Of-Mass (COM) specific kinetic energy,  $e_{COM}$ , after launch: This deducts the transfer losses shifting energy from individual masses to the COM motion<sup>18,19,32</sup>. It effectively removes energy of internal relative motions, accelerating the foot and the portion of the spring that was stationary prior to launch.
- **6)** Potential energy at the jump apex,  $e_{apex}$ : This deducts the aerodynamic drag losses occurring during the jump<sup>11,30</sup>. Our jumper loses about 25% of its energy due to air drag, even with its highly streamlined body. This loss is possible to mitigate by scaling 10x (Extended Data Fig. 6).
- 507
- Regarding isometric scaling, we note the energy-to-stand losses dominate at large scales. For any jumperwith a scale-invariant maximum jump energy, we find a maximum standing height

510 
$$L_{stand} = \frac{1}{g} \frac{m - m_{payload}}{m - m_{foot}} \eta_{\prod i e_{jump}^{\Box} i}$$

beyond which jumping is no longer possible. Meanwhile, at small scales the aerodynamic losses
dominate. See State-space Model and Supplementary Information, Energy Utilisation Model for further
details.

514 515

## 516 State-space Model: Adding Jumper Specifics

517

518 We also simulate jumpers using a simple  $2^{nd}$ -order model. Assume a single moving lumped body mass, 519  $m_b$ , with vertical position z, velocity v, acceleration a, consistent with previous jumping models<sup>32</sup>. The 520 jumper has a length scale, L, which we define as the leg stroke, or difference between the body height 521 when the jumper is fully crouched (z=0) and when standing ( $z=L\dot{c}$ ; the jump height is measured above 522 z=L. This model neglects the effects of geometric linkages, motor internal inertias, multiple masses, etc., 523 but captures the fundamental power and energy production and predicts acceleration times relative to the 524 jumper scale. We consider direct-drive and spring-actuated transmissions.

526 *Direct-drive Transmission:* Assume the body is driven, via a reduction G, by an inertia-free motor with 527 linear viscous losses:

528 
$$m_b a = G F_m \left( 1 - \frac{Gv}{v_m} \right) - m_b g$$

529

530 where  $F_m$  and  $v_m$  are the motor's maximum force and velocity respectively. We consider both fixed 531 reductions and variable reductions, where the motor continually operates at maximum power. The latter 532 case is modelled by

533 
$$m_b a = \frac{F_m v_m}{4v} - m_b g = \frac{m_m p_m}{v} - m_b g$$

- 534
- 535 We again note biological and engineered motor specific power is scale invariant<sup>47,49,50</sup>
- 536  $p_m = \frac{F_m v_m}{4 m_m} = \text{constant}.$
- 537

538 For biological jumpers, assume G is upper-bound by a value of one, in the case when the muscle 539 completes a full stroke during the leg stroke. Consequently, the scale-invariant motor specific energy is F I

540  $e_m^{bio} = \frac{F_m L}{m_m} = \text{constant}$ . 541

We simulate a payload-free system using the same motor parameters as described in Fig 2. The body mass is composed of the motor and linkage mass such that  $m_b = m_s + m_l$ . Extended Data Fig. 3 graphs the results.

Not surprisingly, operating at maximum power, when possible, delivers the most energy. It also provides acceleration times scaled with a 2/3 power of size. We further note that biological jumper's finite motor stroke limits both maximum power operation and the reduction, such that large scale animals have limited energy and see a drop off in their kinetic energy due to increasing energy to stand, ultimately limiting the size of animals that can jump. In contrast, linkage-less engineered jumpers theoretically can produce more energy the larger they are (Extended Data Fig. 3a); when the energy to stand is considered, the kinetic energy eventually plateaus (Extended Data Fig. 3b).

553 554

555 Spring-actuated Transmission: Alternatively, assume the motor pre-stretches a latched linear spring 556 linkage assembly of additional mass  $m_s$ . In turn, when fully stretched and released, the spring propels the 557 body upward. If the spring shows uniformly distributed mass and uniform strain rate, then effectively 559  $\frac{1}{1}$ 

558  $\frac{1}{2}m_s$  contributes to potential energy, while  $\frac{1}{3}m_s$  contributes to kinetic energy. We can thus model

559

560 
$$\left(m_b + \frac{1}{3}m_s\right)a = k(L-z) - \left(m_b + \frac{1}{2}m_s\right)g$$
  
561

562 where the stiffness, k, relates to the effective spring specific energy

563  
564 
$$e_s = \frac{\frac{1}{2}kL^2}{m_s}$$

565

This implicitly maps the appropriate portion of the spring to the foot and body. We again simulate the system using the spring specific energy from Fig. 2 and assume the body mass consists of only a motor mass. We then vary the effective spring-motor mass ratio.

570 Extended Data Fig. 4 shows the results. Note, for such spring-actuated jumpers, the acceleration time
571 increases linearly with scale. We also see that smaller springs impose a limit on the size of jumper –
572 smaller spring forces cannot overcome larger weights. And naturally, smaller springs provide less energy,
573 specific to the total jumper mass.

- 574
- 575 A more detailed description of state space model is found in the Supplementary Information.
- 576 577

#### 578 Jumper Design: 579

- 580 Spring Material Selection: We search a material database to maximize the "material factor," or the ratio581 of the elastic stored energy during axial extension to mass:
- 582

 $\kappa = \frac{\sigma_y^2}{E\rho},$ 

where  $\sigma_y$  is the yield stress, *E* is the modulus of elasticity, and  $\rho$  is the density (Extended Data Fig. 5a). The largest material factor occurs among two main groups of materials: elastomers at lower values of  $E/\rho$  (we choose latex rubber), and fibre-reinforced composites at higher values of  $E/\rho$  (we choose carbon-fibre composite). See SI for further details.

- 588 Spring Design: To explore the design space of springs, we built a non-linear quasistatic simulation 589 framework, comparing the three designs outlined in the main text. The simulation provides a guideline for 590 selecting spring parameters for designing a hybrid spring, suggesting ratios of rubber cross-section to 591 length for a given carbon-fibre cross section to length, such that peak strain in the carbon-fibre is reduced 592 compared to the no-rubber case. See SI for details of the simulation and comparison. 593
- Jumper Design: A small highly geared motor reels in an ultra-high-molecular-weight polyethylene line (Spectra) to compress the spring, storing ~24.2 J of energy at a stress of >90% of the bow ultimate strength. A lightweight release mechanism unlatches to relieve the tension in the line and initiate a jump (see Extended Data Fig. 5). We mount the motor, this release mechanism, the batteries and the nose cone at the top of the bows. Placing as much of the necessary mass on the moving body helps reduce the foot mass ratio and improves the energy transfer in stage 4 of the energy utilisation model.
- We power the motor using a small lithium polymer cell battery (enough energy for roughly 10 jumps). The battery is packaged in a model rocket nose cone which helps reduce the drag of the jumper. To further reduce drag, the robot shape-changes during the acceleration phase, from a wide, stable configuration to a streamlined, rocket-like shape (See Supplementary Videos S6-S7). Cyanoacrylate adhesive is used throughout for bonding. The combination of a lightweight construction and high-strength materials means the jumper can survive landing on even concrete surfaces from its apex height of over 30 m. All of the components are shown in Extended Data Fig. 5d and listed in Extended Data Table 3.
- 609 *Release Mechanism:* The goals of the release mechanism are to quickly release tension in the string that 610 compresses the bow spring (extension of the spring occurs in less than 9 ms), enable resetting for another 611 jump, manage the high forces (~130 N), and be as light as possible. This is achieved with a hinged arm 612 that supports a roller, which turns on bearings, and over which the string passes (Extended Data Fig. 5c). 613 A latch opens to release the tension from the string, after which a rubber band resets the arm, allowing the 614 motor to begin winding for another jump without ever stopping. Given the small size of the motor, reset 615 time is roughly 2 minutes. This could be decreased by increasing the motor size (e.g., doubling the motor 616 mass (and power) would approximately halve reset time).
- 617
- 618 *Self-Righting Mechanism:* To right between jumps, a simple modification to the jumper can be made: 619 adding four bows, one between each set of the main bows, that are tapered and split such that they have

- 620 an asymmetric shape when compressed (Extended Data Fig. 5e, Fig. 3f). The concept of a roll-cage has
- been employed previously for self-righting<sup>10,17</sup>. In the presented design, the tapered and split bows contact the answer d and deform during compression to much the immer unright (See Supplementary Video S2)
- 622 the ground and deform during compression to push the jumper upright. (See Supplementary Video S3.)
- 623

624 Determining Jumper's Payload-free Specific Energy: This value can be determined as the energy the

- 625 motor is able to store in the spring-linkage per mass of the motor and spring-linkage. A force-626 displacement curve was measured for the hybrid compression-tension spring, using the displacement that
- 627 the motor is able to create (20.3 cm) (Extended Data Fig. 5b). The stored energy is measured as 24.2 J,
- 628 the hybrid spring-linkage mass as 12.4 g, and motor mass as 10.1 g; thus, we find an overall payload-free
- 629 specific energy of 1075 J/kg.
- 630
- 631
- 632 Code availability. MATLAB code for energy production and utilization models and state-space model,
- 633 as well as spring simulation, are available upon request.
- **634 Data availability.** All data is available in Extended Data Tables 1-3.

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688

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modeling and simulations; C.K., R.P., and E.H. built the jumpers; E.H., G.N., and C.X. wrote the paper;
E.H., M.P., and G.N. supervised the project.

- 693
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- 696 Additional information
- 697 Extended data is available for this paper at
- 698 Supplementary information is available for this paper at
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704 Extended Data Fig. 1 | Work multiplication in more detail. a, Similar to a ratcheted motor in Fig. 1, a rotary 705 motor can accomplish work multiplication through multiple rotations instead of multiple strokes. **b**, The output work 706 of a biological jumper is determined by fixed parameters (motor stroke, leg stroke, and motor force), but work 707 multiplication overcomes this for engineered jumpers: For biological jumpers, motor stroke and leg stroke 708 determine an effective gear ratio, if the entire stroke of both is to be used (in animals, the gear ratio varies around 709 this value slightly throughout the jump<sup>19</sup>). With this determined gear ratio and a fixed motor force (assuming a size 710 of motor), the leg force is determined. Finally, with the fixed leg stroke and determined leg force, the output work is 711 determined. In contrast, for engineered jumpers, although the leg stroke is roughly fixed (assuming a size of 712 jumper), the motor can make multiple strokes or rotations, allowing the gear ratio to be designed (higher gear ratio 713 will result in more strokes, at the cost of more time). With this designed gear ratio and a fixed motor force (assuming 714 a size of motor), the leg force is also multiplied with respect to the leg force in the single-stroke case. Finally, with 715 the fixed leg stroke and the multiplied leg force, the output work is also multiplied. 716

717 Extended Data Fig. 2 | Biological mechanism specific energy data. The model (Fig. 2a,c) predicts an upper limit 718 to specific energy for all biological jumping mechanisms, regardless of transmission type, at approximately 200 J/kg 719 (dash-dot green). Across scales found in nature, this limit holds. Note that the energy utilisation was estimated at 720 15%, similar to previous biological work<sup>26,27</sup>. However, variation likely occurs, with jumpers with higher take-off 721 velocities likely having more mass dedicated to jumping muscles, and thus having a higher energy utilisation 722 efficiency. A higher utilisation efficiency, e.g. 30%, would result in a lower mechanism specific energy than shown 723 here. The model also predicts a limit due to motor specific power. Direct-drive jumpers fall on or below this limit 724 (dashed blue). Non-latched spring-actuated jumpers can exceed this limit, and latched spring-actuated jumpers can 725 exceed it by even greater amounts (distance from blue dashed line). However, all still fall below. See Extended Data 726 Table 1 for data.

727 728 Extended Data Fig. 3 | Direct-actuated jumper simulations. a, The produced energy specific to the jumper mass. 729 b, The centre-of-mass kinetic energy, specific to the jumper mass. c, The acceleration time. d, The optimal fixed 730 reduction, G, producing the highest acceleration velocity for each jumper scale. The simulations are performed (i) 731 for biological jumpers with fixed reductions of 0.01, 0.1, and 1 (dotted lines), and (ii) for biological jumpers (blue 732 solid) and engineered jumpers (red solid: no linkage; red dotted: with linkage) using variable reduction to operate at 733 maximum power. Each fixed reduction is only possible up to a limiting scale, where the motor force balances the 734 body weight. Biological jumpers operating at full power are also limited in scale, as the motor runs out of stroke. 735 Consequently, biological energy production is always limited by the motor energy (black dashed line). Finally, when 736 operating at the optimal fixed or full-power variable reduction, the acceleration time scales with a 2/3 power of size, 737 reflected in the same scaling in energy and gear reduction.

Fig. 4 | Spring-actuated engineered jumper simulations. a, The produced energy specific to the jumper mass. b, The centre-of-mass kinetic energy, specific to the jumper mass. c, The acceleration time. The simulations are performed for spring mass ratios of ranging from 0.001-10. A lower mass ratio lowers the produced energy specific to the total mass and also imposes an upper bound on size, as smaller springs cannot match larger weight forces. The acceleration time scales nearly linearly with the size, and bigger springs create faster jumps.

744 745 Extended Data Fig. 5 | Jumper design details. a, Ashby plot of materials with the largest material factor, or the 746 square of yield strength over density. At low elastic moduli are elastomers, but these require a passive linkage to 747 load in tension. At high elastic moduli are fibre-reinforced composites, which can act as stand-alone compression 748 bow springs, but have lower specific energies than elastomers in tension. We therefore design a hybrid spring with 749 elastomer in tension and carbon fibre in bending, replacing the passive linkage. b. Force-displacement plot of our 750 hybrid linkage-spring, with total area under the curve (energy) shown. c, Schematic and pictures of the minimalistic 751 release mechanism for unlatching. During winding of the string, the motor shaft turns, pulling the string over a shaft 752 supported by bearings in the arm and compressing the hybrid spring-linkage. With further winding, a lever on the 753 string eventually hits the latch, prying it open. The arm swings open, allowing the string to unspool from the 754 shaft. d, Components of the jumper before assembly. e, Self-righting mechanism. Without a self-righting 755 mechanism, the top-heavy jumper will roll nose-down during compression of the bow springs, given its mass 756 distribution. However, if tapered and split bows are added between each pair of the main, non-tapered bow springs,

757 the behaviour can be reversed. The taper in the bow near the nose creates a high radius of curvature during compression, contacting the ground and forcing the nose to roll upward. The split section continues this as the jumper nears completion of the righting behaviour.

**Extended Data Fig. 6** [Simulating presented jumper across spring-motor mass ratio and scale. Using the statespace model modified with the specifics of the presented jumper, we simulated jump height. We included both energy production and energy utilisation. When the spring-motor mass ratio is increased to infinite, we see only a 17% increase in jump height (from 32.9 to 38.6 m). When the scale is increased by 10x, we find an increase of only 19% in jump height (from 32.9 to 39.1 m). The star denotes the presented jumper (0.3 m scale, 32.9 m jump height).

Extended Data Fig. 7 |Schematic of simplified jumper. a, Schematic of jumper used in the Fig. 1a. b-d, Free body diagrams of the body, top linkage, and bottom linkage, respectively.

#### Extended Data Table 1: Biological jump data.

Extended Data Table 2: Engineered jumper data. Mechanism specific energy is calculated as the energy production divided by the mass of the mechanism (motor, spring, and linkage). The "~" represents numbers estimated from source.

**Extended Data Table 3: Jumper specifications.**