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Journal
Proceedings of the Annual Meeting of the Cognitive Science Society, 22(22)

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Publication Date
2000

Peer reviewed

# Visual and Spatial Representations in Relational Reasoning

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**Abstract**

Psychologists have argued that visual imagery plays a vital role in human reasoning. If so, then reasoning with materials that are easy to visualize should be better than reasoning with materials that are hard to visualize. The literature, however, reports inconsistent results. Our starting point was that the inconsistencies arise from confounding imageability with the spatial nature of the materials. Hence, we manipulated the ease of envisaging the materials as visual images and also as spatial layouts. An experiment showed that materials that are easy to visualize impair reasoning unless they are also easy to envisage spatially.

## Introduction

"I am by the sea and I have a picture. This is a picture of a picture. I am – " She screwed up her face and scowled – "thinking." . . . She paused, frustrated by the vivid detail of her picture, not knowing how to extract from it the significance she felt was there.

– *The Inheritors,* William Golding, 1955, p. 62

Speculations about the role of visual imagery in human reasoning have a long history, and have recently surfaced again in the claims of computer scientists that reasoning based on diagrams has advantages from a computational point of view (Glasgow, Narayanan, & Chandrasekaran, 1995). Yet, the situation is not so clear as it should be from either a psychological or computational standpoint. In psychology, Kosslyn (e.g. 1994) and his colleagues have no doubt that visual imagery plays a key role in reasoning. The origins of this idea are the pioneering studies of DeSoto, London, & Handel (1965) and Huttenlocher (1968), who investigated so-called three-term series problems, such as:

Ann is taller than Beth.
Cath is shorter than Beth.
Who is tallest?

DeSoto et al. argued that reasoners imagine the three individuals on the vertical axis of a visual image, and then read off the answer by inspecting the image. Various sorts of evidence support this hypothesis, including the well-known effects of mental rotation (Shepard & Cooper, 1982) and mental scanning (Kosslyn, 1980). Indeed, metrical information, which is often posited as the main characteristic of mental images, affects reasoning performance (Kelter & Kaup, 1995; Rinck, Hähnel, Bower, & Glowalla, 1997). Likewise, Pearson, Logie, & Gillhooly (1999) studied mental synthesis tasks, which elicit reasoning, and detected interference from visual secondary tasks.

In contrast, several studies have failed to find any effect of imageability on reasoning (Mynatt & Smith, 1977; Sternberg, 1980; Newstead, Pollard, & Griggs, 1986; Richardson, 1987; Johnson-Laird, Byrne, & Tabossi, 1989). Furthermore, Sternberg (1980) did not find a reliable correlation between reasoning ability and scores on imageability items of IQ-tests (Sternberg, 1980). Knauff and his colleagues found interference between relational reasoning and spatial secondary tasks but no such effects of visual secondary tasks (Knauff, Rauh, Schlieder, & Jola, 1999; Knauff, Jola, Strube, Rauh, & Schlieder, 2000).

From a computational point of view, the situation is similar. Researchers into diagrammatic reasoning have argued that diagrams are useful in solving problems, ranging from the analysis of molecular structure (Glasgow & Papadias, 1992) to the navigation of robots (Stein, 1995). Reasoning based on such analog representations can be more powerful than traditional propositionally based reasoning (Glasgow et al., 1995). This approach, however, appears to conflict with theories of qualitative spatial reasoning. Their proponents argue that abstract representations of spatial relations together with an appropriate reasoning engine are a better way to enable computers to make predictions, diagnoses, and plans, when quantitative knowledge is unavailable or leads to computationally intractable inferences (Hernández, 1994; Cohn, 1997).

The aim of our research was to clarify the role of mental images in human reasoning. Our basic assumption is that the inconsistent psychological effects of imageability arise from a failure to distinguish between visual characteristics and spatial characteristics of mental representations. On the one hand, if reasoning relies on mental images, then the easier it is to visualize the information in the premises, the better performance should be. On the other hand, if reasoning relies on spatial models, then the easier it is to envisage a spatial array, the better performance should be. We carried out a preliminary study of various relational terms to assess the ease of imagining assertions based on them as visual images and as spatial arrays. We then carried out an experiment to investigate the effects of both these factors on relational reasoning.

## A preliminary study

In order to generate the materials for our main experiment, 10 students at Princeton University, who were native speakers of English, filled out a questionnaire about the ease of forming visual images and spatial arrays for a set of thirty relational assertions, such as:

The cat was above the dog.

The assertions were based on such relations as *cleaner-dirtier*, *uglier-prettier*, *heavier-lighter*, *smarter-dumber*, and *above-below*. The participants rated the ease of forming visual images and of forming spatial arrays of the assertions on separate seven-point scales ranging from “very easy” to “very difficult”. The frequencies of usage of the relational terms were controlled word frequencies were controlled (Francis & Kucera, 1982), and the order of the assertions was counter-balanced across the participants.

Table 1: Three sorts of relational terms from the preliminary study and their mean ratings for ease of forming a visual image and a spatial array. The scales ranged from 7 (very easy) to 1 (very difficult)

| | Visual image ratings | Spatial ratings |
|---|---|---|
| Spatio-visual relations | | |
| above-below | 5.3 | 5.4 |
| front-back | 5.2 | 5.3 |
| Visual relations | | |
| cleaner-dirtier | 5.1 | 1.6 |
| fatter-thinner | 4.8 | 2.0 |
| Control relations | | |
| better-worse | 2.1 | 1.1 |
| smarter-dumber | 2.8 | 1.2 |

The ratings of assertions based on a relation and its converse did not differ reliably, and so we pooled the results. The ratings enabled us to select three sorts of pairs of relations from the set as a whole. These pairs and their mean ratings are shown in Table 1. The three sorts of relations are: 1. relations such as *above-below* that were easy to envisage spatially and visually, which we henceforth refer to as spatio-visual relations; 2. relations such as *cleaner-dirtier* that were hard to envisage spatially but easy to envisage visually, which we henceforth refer to as visual relations; and pairs such as *better-worse* that were hard to envisage either spatially or visually, which we henceforth refer to as control relations.

The differences between the three groups were statistically reliable, whereas there were no significant differences within the groups. None of the relations in the preliminary study were easy to envisage spatially but difficult to envisage visually.

## The Experiment

**Design.** The aim of the experiment was to investigate the effects of the three sorts of relational terms (visuo-spatial, visual, and controls) on relational reasoning. The participants acted as their own controls and evaluated inferences of all three sorts in 12 three-term series problems and 12 four-term series problems. The relations in these problems were those in Table 1. There were two valid and two invalid problems of each of the three sorts in both the three-term and four-term series problems, making a total of 24 problems. The problems were presented in a counterbalanced order over the set of participants.

**Participants.** We tested 22 undergraduate students of Princeton University (mean age 19.5; 12 female, 10 male), who received a course credit for their participation.

**Materials.** The three-term and four-term series problems all concerned the same terms (*dog*, *cat*, *ape* and *bird*). Here is an example of a problem with a valid conclusion:

The dog is cleaner than the cat.
The ape is dirtier than the cat.
Does it follow:
The dog is cleaner than the ape?

**Procedure.** The participants were tested individually in a quiet room, and they sat at a laptop computer that administered the experiment in separate stages (Potts & Scholz, 1977). The premises were presented one at a time on a sequence of screens (in black letters) followed by a putative conclusion (in red letters). The participants were told to evaluate whether or not the conclusion followed necessarily from the premises. They made their response by pressing the appropriate key on the keyboard, and the computer recorded their response and latency. Prior to the experiment, there were eight practice trials.

**Results.** The problems were easy, and 89 percent of the responses were correct. Furthermore, there were no significant differences in error rates for the three sorts of problems. Figure 1 shows the mean latencies for the correct responses to the three sorts of relational problems. As there was no reliable difference between the three-term and four-term series, we have pooled the results. The participants responded faster to the visuo-spatial problems (2200 ms) than to the control problems (2384 ms), though this difference was not significant, but slower to the visual problems (2654 ms) than to the control problems (Wilcoxon test $z = 3.07$; $p < .002$). Overall, the difference over the three groups was reliable (Friedman analysis of variance, $F = 8.08$; $p < .02$).

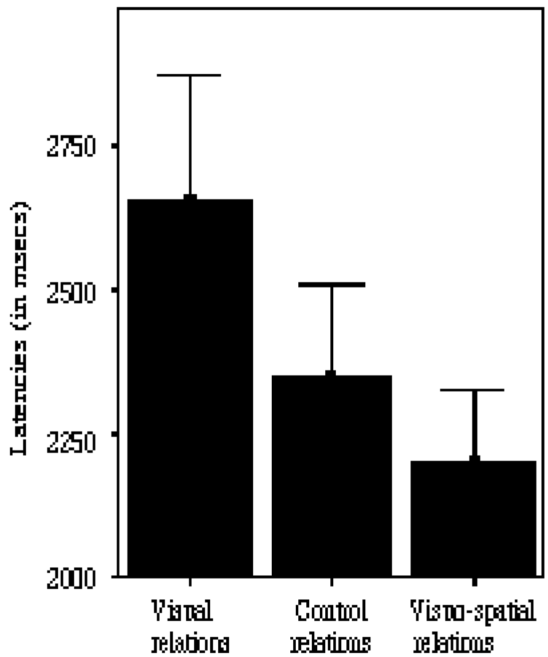


Figure 1: Mean reaction latencies [in milliseconds] and standard errors in the relational reasoning with the three sorts of relation: visual relations, control relations and visuo-spatial relations.

The differences are also reflected in the premise reading times. Because for all three premises we obtained a similar pattern of results, we pooled together all three premises. The mean reading times were 6.6s for the visual premises, 6.2s for the control premises, and 6.0s for the visuo-spatial tasks. The trend over the three groups was reliable (Page's $L = 284$; $p < .05$). Likewise, the difference between the visual and visuo-spatial premises was reliable (Wilcoxon test $z = 2.07$; $p < .05$).

## General Discussion

Our starting point was the conjecture that the conflicting results in the literature on imagery and relational reasoning arose from a failure to distinguish between visual images and spatial representations. Our preliminary study enabled us to identify (a) visuo-spatial relations, such as *above-below*, which are easy to envisage both visually and spatially, (b) visual relations, such as *cleaner-dirtier*, which are easy to envisage visually but hard to envisage spatially, and (c) control relations, such as *better-worse*, which are hard to envisage both visually and spatially. Unfortunately, we were unable to identify relations that were easy to envisage spatially but hard to imagine visually; and some colleagues doubt the existence of such relations. Nevertheless, the results of our experiment established the importance of distinguishing between visual and spatial representations. Visual relations such as *fatter* and *thinner* significantly impede the process of reasoning in comparison with control relations such as *smarter* and *dumber*. In contrast, visuo-spatial relations, such as *front* and *back*, which are easy to envisage visually and spatially, speed up the process of reasoning in comparison with control relations (though the difference did not reach significance).

What causes the trend in our results? One possible explanation is suggested by the theory of mental models (Johnson-Laird, 1983; Johnson-Laird & Byrne, 1991). It postulates that people make transitive inferences by constructing models of the situations that the premises describe. They possess neither axioms nor rules of inference for transitivity, but merely construct an appropriate model. For example, given the premises:

The cat is above the ape.
The dog is below the ape.

they construct a spatial model representing the relative positions of the three individuals:

cat
ape
dog

They evaluate a putative conclusion by checking whether it holds in the model. If it does, they search for a counterexample, i.e., a model that satisfies the premises but refutes the conclusion. Given that no such counterexample exists, the conclusion is valid (see Byrne and Johnson-Laird, 1989). Perhaps the ability to envisage spatial models is a precursor to many forms of abstract reasoning (Johnson-Laird, 1996). Likewise, relational terms that lead naturally to spatial models should speed up the process of reasoning. In contrast, a visual relation, such as *dirtier*, may elicit irrelevant visual detail. One imagines, say, a cat caked with mud, but such a representation is irrelevant to the transitive inference. It takes additional time to replace this vivid image with one in which dirtiness is represented in degrees. In other words, the visual relations, which are hard to envisage spatially, lead to a mental picture. But, the vivid details in this picture interfere with the process of thinking – much as they did for the character in our epigraph from William Golding's novel.

This interpretation is consistent with Logie's (1995) distinction between the visual and spatial subsystems in Baddeley's conception of working memory (Baddeley & Hitch, 1974; Baddeley, 1986). One subsystem (the visual cache) is linked to visual perception and the "visual buffer" (Kosslyn, 1994), and the other subsystem (the inner scribe) is amodal and handles spatial information for use by different cognitive and motor systems (Logie, 1995). Knauff and his colleagues have carried out a series of experiments in which the participants evaluated three-term series inferences as primary tasks together with visual and spatial secondary tasks (Knauff et al., 1999, 2000). The results showed that the spatial tasks interfered with reasoning, whereas the visual tasks did not interfere with reasoning.

A theoretical argument corroborating our hypothesis comes from a comparison of computational accounts of spatial reasoning. Schlieder (1999) compared two computational models of empirical data from Knauff and his colleagues (Knauff et al., 1995, 1998). One model was based on visual images with metrical information (Berendt, 1996), and the other model was based on diagrams that represent only the characteristic points of objects with no metrical information (Schleider's own model). This second spatial account yielded a better account of the empirical results.

An alternative account of our results, however, makes no appeal to the nature of mental representations. It is conceivable that the critical difference between the three sorts of relations is that they differ in the extent to which they suggest transitive relations over the individuals in our problems. Spatial relations among them are unequivocal, whereas the visual relations are more dubious. Given, say, the following premises:

The cat is fatter than the ape.
The ape is fatter than the dog.

reasoners may wonder whether the *fatness*of cats, apes, and dogs, is commensurate. Thus, when one asserts than an elephant is thin, the claim is relative, and so it is perfectly sensible to assert that a thin elephant is fatter than a fat dog. Hence, the criteria of fatness shift from one animal to another. This factor might have confused reasoners in our experiment momentarily, and accordingly lead to longer latencies with the visual relations. One strong argument against this account, however, is that the reading times for the individual premises also showed an advantage for visuospatial relations over visual relations. There remains one other possibility: the visuospatial relations were expressed by prepositions whereas the other relations in our experiment were expressed by comparative adjectives. It is conceivable that this factor, or some other unknown confound, might be responsible for our experimental results. Our next task is to examine in more detail our explanation in terms of irrelevant visual data.

## Acknowledgments

The first author is a visiting research fellow at the Department of Psychology, Princeton University, and the holder of a post-doctoral scholarship from the German National Research Foundation (DFG; Kn465/2-1). Both authors are grateful for the helpful comments of Uri Hasson, Juan Garcia Madruga, Vladimir Sloutsky, Yingrui Yang, and Lauren Ziskind on an earlier draft of this paper.

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