

The Bottom-Intensification of Mixing Causes Large Abyssal Upwelling and Downwelling

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Abstract

We perform a buoyancy budget analysis of bottom-intensified mixing in the abyssal ocean and find that while the interior of the ocean exhibits diapycnal downwelling, strong diapycnal upwelling occurs in very thin continental bottom boundary layers. For a given amount of Antarctic Bottom Water which is upwelled through neutral density surfaces in the abyssal ocean (between 2000m and 5000m) up to *five* times this volume flux is upwelled in narrow turbulent sloping bottom boundary layers, while up to *four* times the net upward volume transport of Bottom Water flows *downward* across isopycnals in the near-boundary stratified ocean interior.

1 Introduction

The classic “abyssal recipes” paper of Munk (1966) achieves the diapycnal upwelling of Antarctic Bottom Water via a one-dimensional advection/diffusion balance which was consistent with a constant diapycnal diffusion coefficient of about $10^{-4} \text{m}^2 \text{s}^{-1}$ throughout the ocean interior. Since the buoyancy frequency increases with height, this one-dimensional advection/diffusion balance implies that the magnitude of the buoyancy flux and therefore the dissipation of turbulent kinetic energy *increases* with height; however observations and theory over the past twenty years have shown just the opposite, namely that diapycnal mixing activity increases towards the sea floor.

In the past twenty years, and particularly as a result of the Brazil basin experiment of WOCE, observations and theory have shown that most of the diapycnal mixing activity in the deep ocean occurs above rough bottom topography and is bottom intensified with an e-folding height above the bottom with a typical vertical e-folding length scale of $\sim 500\text{m}$ (Kunze et al (2006)). However, this bottom intensification of diapycnal mixing causes diapycnal downwelling, and so it is not possible to have the diapycnal upwelling of Antarctic Bottom Water in a one-dimensional situation. This points to the importance of the ocean topography, namely that the ocean does not have a flat bottom, nor vertical walls.

de Lavergne *et al.* (2016) have diagnosed the negative diapycnal transport in the ocean interior caused by near-boundary breaking internal waves and they have pointed towards the important role of the turbulent bottom layer (BBL) in order to upwell the AABW and to close the circulation. Ferrari *et al.* (2016) have studied the crucial role of these BBLs in allowing sufficiently strong upwelling across isopycnals therein to overcome the downwelling in the near-boundary stratified interior, while further away from the ocean boundaries there is no diapycnal motion. This view of the abyssal circulation contrasts sharply with our previous view of the diapycnal upwelling being distributed uniformly over the deep ocean basins. In the particular model studies performed by Ferrari et al (2016) the upwelling in the narrow turbulent boundary layers varied from two to three times the mean upwelling transport of AABW.

The feature that causes this rather dramatic change in where we expect diapycnal motion in the abyss is the bottom-intensification of the diapycnal buoyancy flux. In the present paper we examine the volume-integrated buoyancy budget between pairs of buoyancy surfaces in the abyss using the Walin framework for including the influence of diapycnal transports and the boundary flux of buoyancy (that is, the geothermal heat flux). By assuming that the bottom intensification occurs in an exponential fashion, we are able to relate the downwards diapycnal volume transport in the near-boundary ocean interior (called the Stratified Mixing Layer, SML) to the total diapycnal diffusive buoyancy flux across a buoyancy surface. Use of the steady-state volume-integrated buoyancy balance leads to very simple expressions (Eqns. (13) and (14)) for both the upwelling diapycnal volume flux in the BBL and the downwelling diapycnal volume flux in the SML, in terms of the net upwelling of AABW in the abyss. The application of the Walin budget framework with respect to density surfaces in the abyss, and the resulting Eqns. (13) and (14) are the main results of this work.

One of the main conclusions is that the magnitude of the area-integrated buoyancy flux F on a global buoyancy surface must be an increasing function of buoyancy in order to have net upwelling through a stably stratified ocean. There are two ways of ensuring that the magnitude of the area-integrated buoyancy flux increases with buoyancy (height). First, the magnitude of the buoyancy flux just above the turbulent boundary layer, \mathcal{B}_0 , can be an increasing function of buoyancy, and second, the area of the SML can increase with buoyancy. Neither of these ways of achieving the increase with buoyancy of the magnitude of the area-integrated buoyancy flux (i.e. $dF/db > 0$) were considered in the seminal boundary mixing descriptions of Thorpe (1987), Garrett (1990, 1991, 2001) or Garrett et al (1993) except perhaps in their reference to the “tertiary circulation” of Phillips et al. (1986) and McDougall (1989). These papers considered the mixing activity as arising right at the solid boundary and being mostly confined to the BBL, whereas in this study (as well as in Klocker and McDougall (2010), de Lavergne et al (2016) and Ferrari et al (2016)) we consider the diapycnal mixing activity to arise in the stratified ocean above the BBL.

Our focus is on the mixing in the stratified ocean interior, and this focus is crucial. Armi (1979) and Garrett (1990) both made the point that if near-boundary mixing were to make a significant contribution, then it would need to occur in the stratified near-boundary region. This is exactly the SML region in which the enhanced diapycnal mixing above rough topography is observed to occur.

2. Diapycnal volume transports expressed in terms of the turbulent buoyancy fluxes

In the present work we represent the boundary region in a particularly simple manner. We allow a turbulent boundary layer right against the sloping sea floor in which the isopycnals are assumed to be normal to the sea floor, and at the top of this turbulent boundary layer we have assumed that the stratification abruptly changes to have the isopycnals essentially flat.

The vertical profile of the magnitude of the diapycnal buoyancy flux \mathcal{B} in the deep ocean is taken to be zero at the sea floor and to increase with height in the BBL to a maximum value of \mathcal{B}_0 at the top of the BBL of thickness h , and then to decrease exponentially with height (with scale height d) in the SML (see Figure 1). The influence of the geothermal heat flux at the sea floor is secondary. The turbulent buoyancy flux can be written in terms of the turbulent diffusivity D acting on the vertical gradient of buoyancy b_z as the down-gradient flux $-Db_z$ (and note that $b_z = N^2$). We choose to frame the discussion in terms of the magnitude of the turbulent buoyancy flux per unit area which we give the symbol \mathcal{B} so that in the ocean interior we have $\mathcal{B} = Db_z$. Measurements of the dissipation of turbulent kinetic energy per unit mass, ε , are often used to estimate \mathcal{B} as $\mathcal{B} = \Gamma\varepsilon$ where Γ is the mixing efficiency following Osborn

(1980). In the BBL it is the strong variation of the mixing efficiency Γ with height that is responsible for the magnitude of the buoyancy flux per unit area going from \mathcal{B}_0 at the top of the boundary layer to zero at the sea floor (in the absence of the geothermal heat flux).

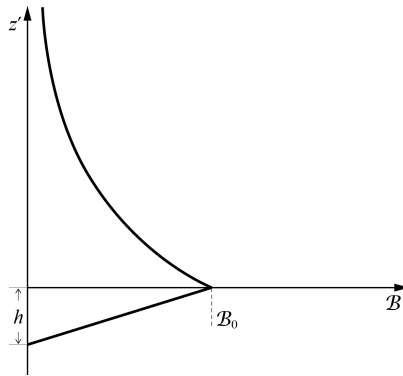


Figure 1. In the deep ocean each vertical cast is assumed to have the magnitude of the diffusive buoyancy flux \mathcal{B} start at zero at the sea floor and to increase with height in the turbulent bottom boundary layer (BBL) to a maximum value of \mathcal{B}_0 at the top of the BBL of thickness h , and then decrease exponentially towards zero as $\mathcal{B}_0 \exp(-z'/d)$ where z' is the height above the top of the turbulent boundary layer.

We examine the buoyancy budget for the volume between two closely-spaced buoyancy surfaces b and $b+\Delta b$, bounded by a sloping sea floor as shown in Figure 2, following the approach of the appendix of Klocker and McDougall (2010) and the volume-integrated buoyancy and volume conservation approach of Walin (1982). We ignore several subtleties of the equation of state of seawater and we take the vertical gradient of buoyancy b_z to be equal to the square of the buoyancy frequency, that is, $N^2 = b_z$, and we use subscripts to denote differentiation.

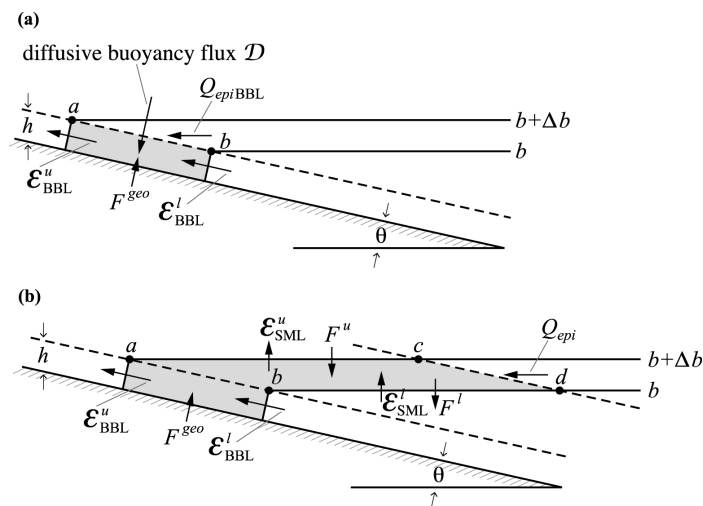


Figure 2. The geometry of the near-boundary mixing region, concentrating on the volume between two closely-spaced buoyancy surfaces. The turbulent bottom boundary layer (BBL) against the solid boundary has thickness h . The area integral of the diffusive flux of buoyancy, whose magnitude is F , is directed downwards while the diapycnal velocity e and the diapycnal volume fluxes \mathcal{E}_{SML} and \mathcal{E}_{BBL} are defined positive upwards. Panel (a) shows the fluxes required to establish the buoyancy budget for the turbulent bottom boundary layer (BBL) while panel (b) shows the corresponding terms needed for the buoyancy budget for the whole shaded near-boundary region that includes the BBL.

Because the mixing intensity decreases smoothly in the vertical, the shaded control volume of Figure 2(b) actually extends all the way to the right in the figure even though the shading is shown ending where the mixing intensity becomes sufficiently small. Along the upper $b+\Delta b$ surface the magnitude of the diffusive buoyancy flux is the maximum value \mathcal{B}_0 on that buoyancy surface at point a and decreases to the right, that is, away from the boundary along the buoyancy surface. Similarly, along the lower buoyancy surface, the magnitude of the diffusive buoyancy flux is the maximum value \mathcal{B}_0 on that buoyancy surface at point b and decreases to the right (the values of \mathcal{B}_0 at points a and b may be different).

We define the magnitude of the diffusive buoyancy flux across the whole interior area of an isopycnal as

$$F = \iint \mathcal{B}(b,x,y) dx dy , \quad (1)$$

where it is recognized that this integral only needs to be performed along the “near-boundary” stratified mixing layer (SML) where the dissipation is significantly non-zero. That is, because \mathcal{B} decreases rapidly with height it also decreases very strongly with horizontal distance from the sloping boundary (to the right) in Figure 2(b). The integral in Eqn. (1) is performed on a buoyancy surface so that F is a function only of buoyancy b .

When the volume and buoyancy budgets of the shaded fluid of Figures 2(a) and 2(b) are examined, the following results are found for the diapycnal volume transports in the turbulent bottom boundary layer (BBL), \mathcal{E}_{BBL} , and net diapycnal volume transport, \mathcal{E}_{net} , being the sum of \mathcal{E}_{BBL} and the diapycnal volume transport across the buoyancy surface in the SML, \mathcal{E}_{SML} ,

$$\mathcal{E}_{\text{BBL}} = \int \frac{G + \mathcal{B}_0}{b_z \tan \theta} \frac{1}{dc} dc , \quad (2)$$

and

$$\mathcal{E}_{\text{net}} \equiv \mathcal{E}_{\text{BBL}} + \mathcal{E}_{\text{SML}} = \frac{dF}{db} + \int \frac{G}{b_z \tan \theta} \frac{1}{dc} dc . \quad (3)$$

The difference between these two equations gives the following expression for \mathcal{E}_{SML}

$$\mathcal{E}_{\text{SML}} = \frac{dF}{db} - \int \frac{\mathcal{B}_0}{b_z \tan \theta} \frac{1}{dc} dc . \quad (4)$$

These statements for the various diapycnal volume transports apply locally to an area of diapycnal mixing near a boundary, and they apply even when the flow is not in a steady state and also when the near-boundary layer region receives (or exports) volume from/to the rest of the ocean. That is, a complete integration over the full area of a buoyancy surface is not needed to obtain these results; these three equations are applicable to a local area of mixing and also to the integral over a complete isopycnal, and they apply whether the ocean is stationary or non-stationary. The key assumptions we have made are that (i) the amplitude of turbulent diapycnal mixing decreases towards zero as one moves sufficiently far from the sloping boundary, and (ii) that a well-mixed turbulent boundary layer exists very close to the sloping solid boundary.

In these equations dF/db is the rate at which the magnitude of the isopycnally area-integrated turbulent buoyancy flux F varies with respect to the buoyancy label b of the isopycnals, G and \mathcal{B}_0 are the fluxes of buoyancy into the turbulent bottom boundary layer (BBL) per unit of exactly horizontal area due to the geothermal heat flux, G , and to the diffusive buoyancy flux at the top of the BBL, \mathcal{B}_0 , respectively, θ is the angle that the bottom topography makes with the horizontal, and dc is the element of spatial integration into the page of Figure 2.

Eqn. (2) shows that the sum of the geothermal heat flux per unit area at the seafloor, G , and the magnitude of the turbulent buoyancy flux per unit area at the top of the BBL, \mathcal{B}_0 , drive an upwelling volume transport along the BBL. This upwelling transport, \mathcal{E}_{BBL} , increases as the sea floor slope $\tan\theta$ decreases, and it increases in proportion to the “circumference” (or perimeter) of the edge of the isopycnal where it intersects the ocean boundary. Eqn. (3) confirms that the net diapycnal upwelling is proportional to the increase with buoyancy of the magnitude of the area-integrated turbulent buoyancy flux, plus the geothermal contribution coming into the BBL. Coming to grips with Eqn. (4) for the diapycnal sinking in the SML and its relationship to the BBL and net transports is a main focus of this work.

3 Relating the interior downwelling volume flux to the area-integrated buoyancy flux

The equation for the dianeutral velocity e in the stratified interior ocean can be found by taking the appropriate linear combination of the conservation equations for Absolute Salinity and Conservative Temperature (see McDougall (1984) or Eqn. (A.22.4) of IOC et al. (2010)). Ignoring various terms that arise from the non-linear nature of the equation of state of seawater, the dianeutral velocity can be expressed as (subscripts denote differentiation)

$$e b_z = \mathcal{B}_z, \quad \text{or} \quad e = \frac{\mathcal{B}_z}{b_z} = \frac{\partial \mathcal{B}}{\partial b} \Big|_{x,y}. \quad (6)$$

As explained in appendix A.22 of IOC et al. (2010), this equation is the evolution equation for the locally-referenced potential density; it is also the classic diapycnal “advection-diffusion” balance. In deriving this expression the curvature of the buoyancy surfaces in space has been neglected, so this expression is accurate when the buoyancy surfaces are relatively flat such as in the stratified ocean interior. Note that this expression for the diapycnal velocity applies even when the flow is unsteady, and it applies locally, on any individual water column. In Eqn. (6) both \mathcal{B}_z and b_z are evaluated on a vertical cast at constant x and y , so that the diapycnal velocity e is the exactly vertical component of the velocity that penetrates through the (possibly moving) buoyancy surface.

We now spatially integrate this expression for the dianeutral velocity over the buoyancy surface in the stratified mixing layer (SML), that is, over that part of the area of the buoyancy surface that excludes the BBL, to evaluate the diapycnal volume flux \mathcal{E}_{SML} (defined positive upwards, so that in the SML both e and \mathcal{E}_{SML} are negative) as

$$\mathcal{E}_{\text{SML}} = \iint e \, dx \, dy = \iint \frac{\mathcal{B}_z(b,x,y)}{b_z} \, dx \, dy. \quad (7)$$

It is now helpful to assume that the vertical shape of the turbulent buoyancy flux profile is exponential (see Figure 1), so that the variation of \mathcal{B} along the area of the buoyancy surface b in the stratified ocean interior is given by

$$\mathcal{B}(b,x,y) = \mathcal{B}_0(x,y) \exp\left(-\frac{z'}{d}\right), \quad (8)$$

where the magnitude of the diffusive buoyancy flux at the top of the BBL, \mathcal{B}_0 , is specified as a function of latitude and longitude, $\mathcal{B}_0(x,y)$, and z' is the height of the b buoyancy surface above the top of the turbulent bottom boundary layer (BBL) at a given latitude and longitude. From Eqns. (6) and (8) we see that the dianeutral velocity $e(b,x,y) = \mathcal{B}_z/b_z$ on buoyancy surface b at a general latitude and longitude is

$$e(b,x,y) = -\frac{\mathcal{B}_0(x,y)}{b_z d} \exp\left(-\frac{z'}{d}\right) = -\frac{\mathcal{B}(b,x,y)}{b_z d}, \quad (9)$$

whose integral over the buoyancy surface in the stratified mixing layer (SML) is

$$\mathcal{E}_{\text{SML}} = - \iint \frac{\mathcal{B}(b,x,y)}{b_z d} dx dy . \quad (10)$$

In the absence of knowledge of any spatial correlation between the variations of $\mathcal{B}(b,x,y)$ and $b_z d$ along the buoyancy surface in the SML, we take the vertical scale height d to be the fixed vertical scale $d = 500\text{m}$ and we approximate the right-hand side of Eqn. (10) as

$$\boxed{\mathcal{E}_{\text{SML}} \approx - \frac{F}{\langle b_z \rangle d}} , \quad (11)$$

where $\langle b_z \rangle$ is the average value of b_z along the whole area of the buoyancy surface (alternatively, this area average could be performed only in the SML). This approximation to Eqn. (10) is equivalent to ignoring any spatial correlation between the mixing intensity $\mathcal{B}(b,x,y)$ and the e-folding vertical buoyancy difference $\Delta b = b_z d$ over the SML on the buoyancy surface. If such a correlation exists it is probably in the sense of reducing the magnitude of the right-hand side of Eqn. (11) since we might expect that the largest values of $\mathcal{B}(b,x,y)$ on the SML would occur where the buoyancy surface is shallowest and b_z is probably also the largest. We note in passing that if we were justified in assuming that the vertical decrease in the magnitude of the buoyancy flux was an exponential function of *buoyancy* (rather than of *height* as in Eqn. (8) above) so that $\mathcal{B}(b,x,y) = \mathcal{B}_0(x,y) \exp(-(b - b_0)/\Delta b)$ where the e-folding buoyancy scale Δb is constant along the buoyancy surface, then \mathcal{E}_{SML} would be given by $\mathcal{E}_{\text{SML}} = -F/\Delta b$ so that \mathcal{E}_{SML} and F would simply be proportional to each other. But we are not aware of any observational support for the e-folding buoyancy scale Δb being spatially invariant, so we follow the conventional practice of adopting an e-folding scale in *height*, that is, we retain the form (8).

This rather direct relationship, Eqn. (11), between the downwelling volume transport \mathcal{E}_{SML} in the SML and the magnitude of the area-integrated interior buoyancy flux, F , is a direct result of the relationship between the diapycnal velocity and the diffusive buoyancy flux of Eqns. (6) and (8), namely $eb_z = \mathcal{B}_z = -\mathcal{B}/d$.

Just like our expressions Eqns. (2) and (3) for the net diapycnal volume flux in the BBL, the net diapycnal volume flux, Eqn. (11) for the SML near-boundary diapycnal volume flux applies to a local area integral along a buoyancy surface, and it applies even when the flow is unsteady and also when there is a mean epineutral transport between pairs of buoyancy surfaces.

4 Diapycnal upwelling in the BBL as a vertical integral of net global diapycnal upwelling

Recalling that we are ignoring the geothermal heat flux, the complete buoyancy budget, Eqn. (3), $\mathcal{E}_{\text{net}} = dF/db$, can be integrated with respect to buoyancy,

$$F = \int_{b_{\text{min}}}^b \mathcal{E}_{\text{net}} db' , \quad (12)$$

yielding a convenient expression for the area-integrated diffusive buoyancy budget F , where $\mathcal{E}_{\text{net}} = \mathcal{E}_{\text{BBL}} + \mathcal{E}_{\text{SML}}$ is the net diapycnal upwelling transport through both the BBL and the SML, and the definite integral is performed from the very densest water with buoyancy b_{min} .

Substituting this expression for F into Eqn. (11) gives

$$\boxed{\mathcal{E}_{\text{SML}} \approx - \frac{1}{\langle b_z \rangle d} \int_{b_{\text{min}}}^b \mathcal{E}_{\text{net}} db'} . \quad (13)$$

The lower limit of the integration here is the least buoyant (densest) water in the world ocean where F (and hence \mathcal{E}_{SML}) is zero since the area of this densest surface tends to zero.

Equation (13) is the key result of this work; it states that knowledge in the abyssal ocean of (i) the stratification $\langle b_z \rangle$, (ii) the vertical e-folding length scale of the diffusive buoyancy flux d , and (iii) the net upwelling of AABW as a function of buoyancy, $\mathcal{E}_{\text{net}}(b)$, yields an estimate of the sinking diapycnal volume flux \mathcal{E}_{SML} in the ocean interior.

The diapycnal volume flux in the BBL follows from Eqn. (13) and the volume conservation equation, $\mathcal{E}_{\text{net}} = \mathcal{E}_{\text{BBL}} + \mathcal{E}_{\text{SML}}$, so that

$$\mathcal{E}_{\text{BBL}} \approx \mathcal{E}_{\text{net}} + \frac{1}{\langle b_z \rangle d} \int_{b_{\text{min}}}^b \mathcal{E}_{\text{net}} db' . \quad (14)$$

As an initial demonstration of these equations, we will assume that the net upwelling volume flux \mathcal{E}_{net} is independent of height (buoyancy) in the abyss, and define buoyancy with respect to a Neutral Density value of 28.3 kg m^{-3} as

$$b / (\text{ms}^{-2}) = 0.01 (28.3 - \gamma / (\text{kg m}^{-3})) , \quad (15)$$

where γ is Neutral Density (Jackett and McDougall, 1997). We will assume that the buoyancy value $b_{\text{min}} = 0 \text{ ms}^{-2}$ characterizes the densest water in the world ocean. At a depth of 2500 m ocean atlases show that $\gamma \approx 28.05 \text{ kg m}^{-3}$, $b \approx 2.5 \times 10^{-3} \text{ ms}^{-2}$, $b_z \approx 10^{-6} \text{ s}^{-2}$, and taking d to be 500 m, Eqns. (13) and (14) yield $\mathcal{E}_{\text{SML}} \approx -5 \mathcal{E}_{\text{net}}$ and $\mathcal{E}_{\text{BBL}} \approx 6 \mathcal{E}_{\text{net}}$. In this way, if \mathcal{E}_{net} were say 18 Sv then the diapycnal transport in the BBL would be about 108 Sv while the downwelling in the interior SML would be 90 Sv.

If instead of assuming that \mathcal{E}_{net} is independent of height (buoyancy) in the abyss, we take it to be a linearly increasing function of buoyancy as suggested by the model studies of Ferrari et al (2016), then the above ratio of \mathcal{E}_{SML} to \mathcal{E}_{net} becomes $\mathcal{E}_{\text{SML}} \approx -2.5 \mathcal{E}_{\text{net}}$, closer to the values of approximately -1.5 seen in Figure 7 of Ferrari et al (2016). The remaining discrepancy could be due to the model runs having a larger stratification $\langle b_z \rangle$ than the observations or due to the correlation along isopycnals in the SML between the mixing intensity $\mathcal{B}(b, x, y)$ and the vertical stratification b_z in Eqn. (10). The ratio $|\mathcal{E}_{\text{SML}}|/\mathcal{E}_{\text{net}}$ in Figure 9 of Ferrari et al (2016) is based on applying the Nikurashin and Ferrari (2013) estimate of mixing induced by breaking topographic waves, and is slightly larger at about $|\mathcal{E}_{\text{SML}}|/\mathcal{E}_{\text{net}} \approx 2$ (and hence $\mathcal{E}_{\text{BBL}}/\mathcal{E}_{\text{net}} \approx 3$) in the abyss.

5 Conclusions

- The upward diapycnal volume transport in the turbulent bottom boundary layer (BBL) is typically several times as large as the net upwelling of AABW in the abyss.
- This implies that there is substantial cancellation between the large upwelling in the BBL and the (almost as large) downwelling in the stratified mixing layer (SML) that lies in the stratified ocean but is near the sea floor where the diapycnal mixing is significant.
- In order to upwell 100 Sv across isopycnals in the BBL, the turbulent diffusivity immediately above the BBL must be approximately $D_0 \approx 5 \times 10^{-3} \text{ m}^2 \text{ s}^{-1}$ on average along the incrop line of a buoyancy surface. Clearly, this is a large diapycnal diffusivity, and it remains to be seen if this will prove to be a realistic estimate.

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