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S.-J. Wang

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Non-Adiabatic Effect On Berry's Phase For Light Propagating In An Optical Fibre

Shun-Jin Wang

Nuclear Science Division, Lawrence Berkeley Laboratory
University of California, Berkeley, CA 94720
and
Center of Theoretical Physics, CCAST(World Lab.) Beijing and
Department of Modern Physics, Lanzhou University, Lanzhou 730001, PR China

Abstract: A Schrödinger equation for a photon propagating in a helix has been derived from Maxwell's equations and their quantization. The exact solutions of this equation are employed to study the non-adiabatic effect on Berry's phase. It is shown that non-adiabaticity alters the time evolution ray and in turn changes its Berry's phase. A new expression for Berry's phase is given, which indicates a close link between Berry's phase and spin expectation value. For SU(2) dynamical group, non-adiabatic effect on Berry's phase manifests itself as spin-alignment (familiar in nuclear physics). Two kinds of experiments are suggested to measure non-adiabatic effect on Berry's phase.

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Manifestations of Berry's phase[1] for the photon were predicted by Chiao and Wu[2] and soon after observed by Tomita and Chiao.[3] Chiao and Wu's description of a photon propagating in an optical fibre is based on a postulated photon Hamiltonian. The related equation of motion is,in this sense, not derived rigorously from quantum electrodynamics or Maxwell's equations. To obtain the solution of the proposed equation of motion, an analogy between a photon propagating in a helical optical fibre and a particle with spin moving in a rotating magnetic field was employed by Chiao and Wu. The adiabatic approximation was used to describe the phenomena, with the background Hamiltonian unspecified. Since in the adiabatic limit Berry's phase of the photon depends only on its path in k–space, the background Hamiltonian is irrelavent.

On the countrary, the effect of non–adiabaticity on Berry's phase depends on the whole specific dynamics,[4] and hence the detailed knowledge of the Hamiltonian is crucial. Thus, to study the effect of non–adiabaticity on Berry's phase, one needs to derive the photon Hamiltonian from QED or from its classical correspondence according to Maxwell's equations. This is the goal of the present work. Unlike the previous papers[5] which obtained a Schrödinger–like equation of motion for the electromagnetic fields, we shall derive a Schrödinger–like equation of motiom for the electromagnetic potential. This makes it easier to find analytical solutions. For light propagating in a helical optical fibre, the resulting exact solutions are used to study the effect of non–adiabaticity on Berry's phase.

We start from Maxwell's equations in vacuum or in a uniform medium

$$\frac{1}{C^2}\frac{\partial^2 \overrightarrow{A}(\vec{r},t)}{\partial t^2} = \nabla^2 \overrightarrow{A}(\vec{r},t) \tag{1a}$$

with the transverse wave condition,

$$\overrightarrow{\nabla}\cdot\overrightarrow{A} = 0 \quad . \tag{1b}$$

Two independent solutions of eqs. (1a–b) for monochromatic waves are circularly polarized plane waves

$$\vec{A}(\vec{r},t) = \vec{e}_{\pm} \exp[i\vec{k}\cdot\vec{r} - i\Omega t\} \tag{2}$$

The polarization vectors $\vec{e}_{\pm}$ are eigenvectors of the helicity operator

$$\hat{h} = \vec{k}\cdot\vec{S}/k \quad , \tag{3a}$$

$$k = \Omega/c \quad , \tag{3b}$$

$$\hat{h}\vec{e}_{\pm} = \pm e^{\pm} \quad , \tag{4}$$

where $\vec{S}$ are spin operators for spin−1 particles, $\vec{k}$ and Ω are wave vector and frequency of the (laser) light respectively. To study Berry's phase, one needs a Schrödinger−like equation of motion, i.e., Maxwell's equation should be "linearized" to obtain an equation of motion with only the first order derivative in time. The task becomes easier if one inverts the problem: Since one knows the exact solutions of eqs. (1a−b), the solution (2), what is the implied Schrödinger−like equation of motion? To simplify the problem, let us rewrite $\vec{A}$ as

$$\vec{A}(\vec{r},t) = \vec{\psi}(t) \exp\{i\vec{k}\cdot\vec{r}\} \quad . \tag{5}$$

The Schrödinger−like equation of motion for $\vec{\psi}(t)$ is

$$i\frac{\partial\vec{\psi}(t)}{\partial t} = \hat{H}_o\vec{\psi}(t) \quad . \tag{6}$$

Where

$$\hat{H}_o = [(\vec{\Omega}\cdot\vec{S})^2]^{1/2} \tag{7a}$$

$$\vec{\Omega} = c\vec{k} \tag{7b}$$

The solutions of eq. (6) are

$$\vec{\psi}_{\pm}(t) = \vec{e}_{\pm} \exp\{-i\Omega t\} \tag{8}$$

Combining eqs. (8) and (5), we recover the solutions (2). Therefore equation (6) is the Schrödinger−like equation of motion for polarized electromagnetic plane waves. The salient feature of eq. (6) is that the transverse wave condition is incorporated in the Hamiltonian, and therefore satisfied automatically. To see this, let us examine the third eigen−solution of the helicity operator and the third solution of eq. (6)

$$\hat{h}\vec{e}_0 = o\vec{e}_0 \tag{9a}$$

$$\hat{H}_0\vec{e}_0 = o\vec{e}_0 \tag{9b}$$

$$\vec{\psi}_0(t) = \vec{e}_0\eta(t) \quad , \tag{9c}$$

$$i\frac{\partial\vec{\psi}_0(t)}{\partial t} = \hat{H}_0\vec{\psi}_0(t) = 0 \quad . \tag{9d}$$

Eqs. (9a–d) mean that, although the third solution $\vec{\psi}_0$ exists formally, its physical effect is zero, i.e., it is non–observable.

Now let us deform the boundary of the medium which guides the electromagnetic waves. Suppose the undeformed wave propagates in the $\vec{k}$–direction

$$\vec{k} = k(\sin\theta\,, 0\,, \cos\theta) \quad . \tag{10}$$

We deform the waveguide into a helix, which can be realized by the following unitary transformation

$$\hat{H}(t) = \exp\{-i\vec{S}_z\omega t\}\,\hat{H}_0\exp\{i\vec{S}_z\omega t\} = [(\vec{\Omega}(t)\cdot\vec{S})^2]^{1/2} \quad , \tag{11}$$

$$\vec{\Omega}(t) = \Omega(\sin\theta\cos\omega t\,, \sin\theta\sin\omega t\,, \cos\theta) \quad . \tag{12}$$

Laser light propagating in the deformed medium (the helical optical fibre) obeys the following equation of motion

$$i\frac{\partial\vec{\psi}_0(t)}{\partial t} = \hat{H}(t)\,\vec{\psi}(t) \quad . \tag{13}$$

We shall solve the above equation by using the cranking method developed in nuclear physics. Let

$$\vec{\psi}(t) = \exp\{-i\hat{S}_z\omega t\}\,\vec{\eta}(t) \quad , \tag{14}$$

then $\vec{\eta}(t)$ satisfies

$$i\frac{\partial\vec{\eta}(t)}{\partial t} = \hat{H}(\omega)\,\vec{\eta}(t) \quad . \tag{15}$$

where

$$H(\omega) = \hat{H}_0 - \omega\hat{S}_z \quad . \tag{16}$$

The solution is

$$\vec{\eta}(t) = \exp\{-i\hat{H}(\omega)t\}\,\vec{\eta}(0) \quad , \tag{17}$$

$$\vec{\psi}(t) = \hat{U}(t)\,\vec{\eta}(0) = \hat{U}(t)\,\vec{\psi}(0) \quad , \tag{18}$$

$$\hat{U}(t) = \exp\{-i\hat{S}_z\omega t\}\cdot\exp\{-i\hat{H}(\omega)t\} \quad . \tag{19}$$

Consider the eigensolutions of $\hat{H}(\omega)$. Firstly consider eigensolutions of $\hat{H}_o$

$$\hat{H}_o\vartheta_m = \varepsilon_m\vartheta_m \quad , \tag{20}$$

$$\vartheta_m(\theta) = \sum_{m'} D^1_{m'm}(0,\theta,0)\,|m'\rangle \quad , \tag{21}$$

$$\varepsilon_m = |m|\Omega \quad , \tag{22}$$

where $|m\rangle$ are eigenstates of $\hat{S}_z$

$$\hat{S}_z|m\rangle = m|m\rangle \quad . \tag{23}$$

Now consider eigensolutions of $\hat{H}(\omega)$

$$\hat{H}(\omega)\,\eta_m(\theta,\omega) = E_m(\theta,\omega)\,\eta_m(\theta,\omega) \quad . \tag{24}$$

Since $H(\omega = 0) = \hat{H}_o$, there is a one–one correspondence between $\vartheta_m(\theta)$ and $\eta_m(\theta,\omega)$.

For $m = +1$

$$\hat{H}_o = \vec{\Omega}\cdot\vec{S} \quad , \quad \hat{H}(\omega) = \vec{\Omega}\cdot\vec{S} - \omega\hat{S}_z \quad ; \tag{25a}$$

while for $m = -1$

$$\hat{H}_o = -\vec{\Omega}\cdot\vec{S} \quad , \quad \hat{H}(\omega) = -\vec{\Omega}\cdot\vec{S} - \omega\hat{S}_z \quad .$$

Since the frequency, i.e.,the eigen value of H( ), is always positive, from eqs. (25a–b) we have

$$\hat{H}(\omega) = [(\vec{\Omega}'\cdot\vec{S})^2]^{1/2} \quad , \tag{26}$$

where

$$\vec{\Omega}' = \overline{\Omega}(\sin\overline{\theta} \quad , \quad 0 \quad , \quad \cos\overline{\theta}) \tag{27a}$$

$$\overline{\Omega} = \Omega[1 \mp 2(\omega/\Omega)\cos\theta + (\omega/\Omega)^2]^{1/2} \tag{27b}$$

$$\sin\overline{\theta} = \sin\theta/[1 \mp 2(\omega/\Omega)\cos\theta + (\omega/\Omega)^2]^{1/2} \tag{27c}$$

$$\cos\overline{\theta} = (\cos\theta \mp \omega/\Omega)/[1 \mp 2(\omega/\Omega)\cos\theta + (\omega/\Omega)^2]^{1/2} \quad . \tag{27d}$$

In analogy with eqs. (20)–(22), we obtain eigensolutions of $\hat{H}(\omega)$

$$\eta_m(\overline{\theta},\omega) = \sum_{m'} D^1_{m'm}(0\,,\overline{\theta}\,,0)\,|m'\rangle \tag{28}$$

$$E_m(\overline{\theta},\omega) = |m|\,\Omega[1 \mp 2(\omega/\Omega)\cos\theta + (\omega/\Omega)^2]^{1/2} \quad , \tag{29}$$

where $D^1_{m'm}$ are the rank−1 Wigner functions.

With the eigensolutions of $\widehat{H}(\omega)$ in hand, we begin to discuss the time−dependent solution $\psi(t)$. Most interesting is the time evolution of $\psi(t)$ after n periods, i.e. $\psi(nT)$. The evolution operator in one period is

$$\widehat{U}(T) = \exp[-2\pi i\widehat{S}_z\}\cdot\exp\{-i\widehat{H}(\omega)T\} \quad , \quad (T = 2\pi/\omega) \quad . \tag{30}$$

Since

$$\exp\{-i2\pi\widehat{S}_z\}\,\widehat{H}(\omega)\,\exp\{+i2\pi\widehat{S}_z\} = \widehat{H}(\omega) \quad , \tag{31}$$

we have

$$[\widehat{U}(T) \quad , \quad \widehat{H}(\omega)] = 0 \quad . \tag{32}$$

This means that $\widehat{U}(T)$ and $\widehat{H}(\omega)$ have common eigenstates, i.e.,

$$\widehat{U}(T)\,\eta_m = \exp\{-i\phi_m\}\,H_m \quad , \tag{33}$$

$$\widehat{H}(\omega)\,\eta_m = E_m\eta_m \quad . \tag{24}$$

A classification of the time−dependent solutions

$$\psi(nT) = [\widehat{U}(T)]^n\,\psi(0) \tag{34}$$

can be made according to the initial states $\psi(0)$:

(i) cyclic or recurrent solutions require

$$\psi(0) = \eta_m \quad , \tag{35}$$

and lead to

$$\psi(nT) = \exp\{-in\phi_m\}\,\psi(0) \quad . \tag{36}$$

This case is related to the problem of Berry's phase.

(ii) non−cyclic or non−recurrent solutions require

$$\psi(0) \neq \eta_m \quad , \tag{37}$$

and lead to

$$\psi(nT) \neq c\psi(0) \quad . \tag{38}$$

In short, when the initial state is an eigenstate of $\hat{H}(\omega)$ (or $\hat{U}(T)$), we have cyclic (recurrent) solutions; while if the initial state is not an eigenstate of $\hat{H}(\omega)$, we have non-cyclic (non-recurrent) solutions. In a forthcoming paper we will show that the non-cyclic solutions are related to the dissipation in a quantum system if its dynamical symmetry is seriously destroyed by time-dependent external fields. In this letter, we concentrate on the cyclic evolution and the related Berry's phase.

It is worth noting that, like eq. (6), eq. (13) desribes a physical photon (laser light) propagating in the "cranked" optical medium (a helical fibre). This can be made clear as follows. Firstly, as is the case for $\hat{H}_0$, $\hat{H}(t)$ and $\hat{H}(\omega)$ have only two physically observable solutions, the right circular mode (m = +1) and the left circular mode (m = −1) (or their linear combinations), with the longitudinal mode (m = 0) projected out by the Hamiltonian automatically. Secondly, if the initial state is a physical state, i.e., if

$$\psi(0) = a\eta_+ + b\eta_- \quad , \tag{39a}$$

from eq. (18), we have

$$\psi(t) = a\exp\{-iE_+t\}\exp\{-i\hat{S}_z\omega t\}\,\eta_+ + b\exp\{-iE_-t\}\exp\{-i\hat{S}_z\omega t\}\,\eta_- \quad . \tag{39b}$$

Since $\exp\{-i\hat{S}_z\omega t\}\,\eta_\pm$ are the right and left circular modes propagating along $\vec{k}'_\pm = \overline{\Omega}/c(\sin\overline{\theta}_\pm\cos\omega t, \sin\overline{\theta}_\pm\sin\omega t, \cos\overline{\theta}_\pm)$ and physically observable, their combination $\psi(t)$ is therefore also physically observable. Thus we conclude that eq. (13) guarantees physical solutions. It should be pointed out that the physical modes η_+ and η_- are not orthogonal, although independent of each other. The eigenstates of $\hat{H}_0$, $\vartheta_\pm$, can be expanded in terms of $\eta_\pm$

$$\vartheta_\pm = a_\pm\eta_+ + b_\pm\eta_- \quad , \tag{40}$$

since the vectors $\vec{k}(\theta)$, $\vec{k}'(\overline{\theta}_+)$ and $\vec{k}'(\overline{\theta}_-)$ are in the same x–z plane.

We proceed to calculate Berry's phase for the solution

$$\psi_m(t) = \exp\{-i\hat{S}_z\omega t\}\cdot\exp\{-i\hat{H}(\omega)t\}\,\eta_m$$

$$= \exp\{-iE_m t\} \sum_{m'} D^1_{m'm}(0\,,\bar{\theta}_m\,,0)\exp\{-im'\omega t\}\,|m'\rangle \quad . \tag{41}$$

After one period, the state returns to its starting point up to a phase factor,

$$\psi_m(t) = \exp\{-iE_m T\} \sum_{m'} D^1_{m'm}(0,\bar{\theta}_m,0)\exp\{-i2\pi m'\}\,|m'\rangle = \exp\{-\phi_m\}\,\psi(0)\;, \tag{42}$$

where then total phase is

$$\phi_m = E_m T + 2\pi m \quad . \tag{43}$$

To calculate the dynamical phase ϕ^d_m, we have first to calculate the expection value of $\hat{H}(t)$ over $\psi_m(t)$,

$$\varepsilon_m(t) = \langle\psi_m(t)|\hat{H}(\omega)|\psi_m(t)\rangle = \langle H_m|\hat{H}_o|\eta_m\rangle = E_m + \omega\bar{S}^m_z \tag{44a}$$

where

$$\bar{S}^m_z = \langle H_m|\hat{S}_z|\eta_m\rangle \quad . \tag{44b}$$

Since

$$\eta_\pm = \frac{1}{2}\,(1 \pm \cos\bar{\theta}_\pm)\,|+\rangle \mp \sqrt{1/2}\,\sin\bar{\theta}_\pm|0\rangle + \frac{1}{2}\,(1 \mp \cos\bar{\theta}_\pm)\,|-\rangle \quad , \tag{45}$$

we have

$$\bar{S}^m_z = \pm\cos\bar{\theta}_\pm \quad , \tag{46}$$

and

$$\varepsilon_\pm(t) = E_\pm \pm \omega\cos\bar{\theta}_\pm \quad . \tag{47}$$

The dynamical phase in one period is

$$\phi^d_\pm = \int_o^T \varepsilon_\pm dt = E_\pm T \pm 2\pi\cos\bar{\theta}_\pm \quad . \tag{48}$$

Finally, we find for Berry's phase the expression

$$\phi^b_\pm = -[\phi_\pm - \phi^d_\pm] = -[\pm 2\pi(1 - \cos\bar{\theta}_\pm)] \quad , \tag{49}$$

or

$$\phi_m^b = -2\pi m(1 - \cos\overline{\theta}_m) \tag{50a}$$

$$= -2\pi m\{1 - (\cos\theta - m(\omega/\Omega))\,[1-2m(\omega/\Omega)\cos\theta + (\omega/\Omega)^2]^{-1/2}\} \tag{50b}$$

$$= \phi_m^{adia} + \Delta\phi_m^{dia} \tag{50c}$$

The non-adiabaticity contributes the term $\Delta\phi_m^{dia}$ (proportional to ω/Ω for small ω/Ω) to Berry's phase. In the adiabatic limit, $\omega/\Omega \to 0$ and $\Delta\phi_m^{dia} \to 0$, we recover the adiabatic Berry's phase and Chiao–Wu's results.

The expressions for Berry's phase, eqs. (50) and (46), provide us with new physical insight into the problem. Let us combine expressions (50) and (46),

$$\phi_m^b = -2\pi m(1 - \langle\eta_m|\hat{S}_z|\eta_m\rangle/m) = -2\pi m(1 - \langle\psi_m(t)|\hat{S}_z|\psi_m(t)\rangle/m) \quad . \tag{51}$$

Expression (51) indicates that Berry's phase for the SU(2) dynamical group is closely related to the expectation value of the spin z–component, which in turn determines the geometrical solid angle subtended by the closed path traced by the state η_m (the ray m) in its time evolution. As the dynamical group and the time–evolution ray (a vector in some irreducible representation space of the dynamical group) are fixed, Berry's phase is fixed. Therefore Berry's phase depends only on the dynamical group, its dictated geometry and the specific geometric object — the ray. In this sense, Berry's phase is a geometric object in the group representation space.

However, the non–adiabatic effect will change the evolution vector η_m. For the adiabatic case, $\psi(0) = \vartheta_m$ are eigenfunctions of $\hat{H}(\omega)$; while for the non–adiabatic case, $\psi(0) = \eta_m$ are eigenfunctions of $\hat{H}(\omega)$ and depend on the frequency ω. Through its effect on the evolution state η_m, the non–adiabatic effect also changes Berry's phase. In this case the non–adiabatic Berry's phase is a dynamical gauge geometrical phase, while the adiabatic Berry's phase can be considered as a static gauge geometrical phase.[6–9]

To exploit the physical meaning of the non–adiabatic effect on Berry's phase, we consider the case where $\theta = \pi/2$, which means that the helical optical fibre becomes a circular coil in the x–y plane. Suppose laser light in the right helical mode is injected into the fibre coil. From eqs. (51) and (46) we have

$$\phi_+^b = -2\pi\left[1 + (\omega/\Omega)/[1 + (\omega/\Omega)^2]^{1/2}\right] \tag{52}$$

and

$$\langle\eta_+|\hat{S}_z|\eta_+\rangle = -(\omega/\Omega)/[1 + (\omega/\Omega)^2]^{1/2} \tag{53}$$

This means that a laser initially with $\langle\hat{S}_z\rangle = 0$, after propagating in a circular fibre, will acquire a non–zero value $\langle\hat{S}_z\rangle$. In nuclear physics,[10] especially in high spin nuclear physics , the above phenomenon is called spin–alignment. Therefore the non–adiabatic effect on Berry's phase for the SU(2) dynamical group is related to spin–alignment. This problem will be investigated in detail in a separate paper.

Before turning to experimental aspects of non–adiabatic Berry's phase for laser light, we consider the problem of quantization of the laser equation of motion. It is simple and straightforward. According to the de Broglie relations

$$E = \hbar\Omega \tag{54a}$$

$$\vec{P} = \hbar\vec{k} \ . \tag{54b}$$

After multiplying both sides of equations (6) and (4) with $\hbar$, we obtain the quantal equation of motion for the photon

$$i\hbar\frac{\partial\vec{\psi}(t)}{\partial t} = \hat{H}_o^q\vec{\psi}(t) \ , \tag{55}$$

$$\hat{h}^q\vec{e}_\pm = \pm\hbar\vec{e}_\pm \ , \tag{56}$$

where

$$\hat{H}_o^q = [(\vec{\Omega}\cdot\vec{S}^q)^2]^{1/2} \ , \tag{57}$$

$$\hat{h}^q = \vec{k}\cdot\vec{S}^q/k \ , \tag{58}$$

$$\vec{S}^q = \hbar\vec{S} \quad . \tag{59}$$

For the cranked case, we have

$$i\hbar\frac{\partial\vec{\psi}(t)}{\partial t} = \widehat{H}^q(t)\,\vec{\psi}(t) \quad , \tag{60}$$

$$\widehat{H}^q(t) = \hbar\widehat{H}(t) \quad . \tag{61}$$

Eqs.(55) and (60) constitute the Schrödinger equation for the photon.

Now we consider experimental aspects of non–adiabatic Berry's phase. We suggest two kinds of experiments to measure non–adiabatic effect on Berry's phase for laser light. (i) Linearly polarized laser light in helical optical fibre—the Tomita–Chiao like experiments. Since

$$\omega/\Omega = \lambda\!\!\!^{-}/[(2\pi a)^2 + d^2]^{1/2} \quad , \tag{62}$$

where $\lambda\!\!\!^{-}$ is the wavelength of the laser, a the radius of the helix , d the pitch of the helix, ω/Ω is very small in practice. One should increase the winding number N of the helix to accumulate the non–adiabatic effect on Berry's phase. For Tomita–Chiao's experiment, the non–adiabaticity contributes an extra rotation angle

$$\Delta\gamma = -2N\pi\,\cos\theta[1 - (1 + (\frac{\omega}{\Omega})^2)^{-3/2}] \tag{63}$$

which is calculated from eq. (50b) for linearly polarized laser light and might be measurable in Tomita–Chiao–like experiments for sufficiently large N.

(ii) Circularly polarized laser light in a circular fibre. Inject a circularly polarized laser light into a circular fibre. If initially the laser is in the right circular mode of $\widehat{H}_0$

$$\psi(0) = \vartheta_+ \quad , \tag{64}$$

after the time evolution it will acquire a left component. Since

$$\vartheta_+ = \sum_m D^1_{m+1}(0\,,\theta\,,0)\,|m\rangle \quad , \tag{65}$$

$$\eta_- = \sum_m D^1_{m+1}(0\,,\overline{\theta}_-\,,0)\,|m\rangle \quad , \tag{66}$$

and

$$\langle\eta_-|\vartheta_+\rangle| = [1 - \cos(\theta - \overline{\theta}_-)]/2 = (\omega/\Omega)^2/4 \qquad (67)$$

the intensity of the left component is

$$I_- = |\langle\eta_-|\vartheta_+\rangle|^2 = (\omega/\Omega)^4/16 \ . \qquad (68)$$

Moreover, since η_+ and η_- are not othogonal, their transition probability can be calculated similarly

$$P_\pm = |\langle\eta_+|\eta_-\rangle|^2 = [1 - \cos(\overline{\theta}_+ - \overline{\theta}_-)]^2/4 = (\omega/\Omega)^4 \sin^4\theta \ . \qquad (69)$$

Therefore, in the case of spiral propagation of a photon, the right mode and the left mode will oscillate.

Before concluding this paper, we would like to discuss an interesting and also fundamental question:Is Berry's phase a classical or a quantal phenomenon? From the above results, we see that Berry's phase for laser light can be described by either the classical Maxwell's equations (the deformed version,Schrödinger–like equation) or by the quantal Schrödinger equation for a photon. Thus Berry's phase for laser light is both a classical and a quantal phenomenon, since both are characterized by their wave nature. Hence one could say that Berry's phase is a phenomenon of waves, irrespective of whether the waves are classical or quantal.

We conclude with a summary of information presented in this paper. A Schrödinger–like equation of motion for laser light propagating in a helix ("cranked" propagation) has been derived from Maxwell's equations. By applying the de Broglie quantization rule, its quantum correspondence—the Schrödinger equation for the corresponding photon is obtained. This equation of motion has been solved analytically and the exact solutions are employed to study the non–adiabatic effect on Berry's phase. It is shown that although Berry's phase is completely determined by the path traced by the time evolving ray in projective Hilbert space, non–adiabaticity will alter the time evolution ray itself, and in turn change its path and the related Berry's phase.

The non–adiabaticity contributes an extra term to Berry's phase depending on the rate of time evolution. A new expression for Berry's phase is presented, which indicates a close link between Berry's phase and spin expectation value along the rotated axis, and gives Berry's phase a physical explanation besides its gauge geometrical interpretation. From the new expression, it is easy to see that the non–adiabatic effect on Berry's phase manifests itself as spin–alignment for a system with SU(2) dynamical group, which is familiar in nuclear physics. Finally, we have suggested two kinds of experiments to measure the non–adiabatic effect on Berry's phase.

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