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### Journal

Journal of Geotechnical and Geoenvironmental Engineering, 139(8)

### ISSN

1090-0241

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### Publication Date

2013-08-01

### DOI

10.1061/(asce)gt.1943-5606.0000847

Peer reviewed

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[http://dx.doi.org/10.1061/\(ASCE\)GT.1943-5606.0000847](http://dx.doi.org/10.1061/(ASCE)GT.1943-5606.0000847)

1                    **p-y Plasticity Model for Nonlinear Dynamic Analysis of Piles in Liquefiable Soil**  
2   **by, Scott J. Brandenburg, M.ASCE<sup>1</sup>, Minxing Zhao<sup>2</sup>, Ross W. Boulanger, M.ASCE<sup>3</sup>, and Daniel W. Wilson,**  
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4   **Abstract**

5   Liquefiable soil-structure interaction material models are developed and implemented in the open-  
6   source finite element modeling platform, OpenSees. Inputs to the free-end of the p-y materials include  
7   the ground motion and mean effective stress time series from a free-field soil column. Example  
8   simulations using a single p-y element attached to a soil element demonstrate key features. The models  
9   are then used to analyze centrifuge experiments of a single pile in a level liquefiable profile, and a six-  
10   pile group in a sloping liquefiable profile that resulted in lateral spreading. Measured displacements and  
11   mean effective stress time series are utilized as inputs to isolate the response of the material models  
12   from predictive uncertainties in free-field ground motion and excess pore pressure. The predicted pile  
13   response agrees reasonably well with measurements. The cyclic mobility behavior of sand in undrained  
14   loading is shown to be an important mechanism affecting bending moments in the piles; neglecting the  
15   dilatancy component of the sand's response (i.e., ignoring the cyclic mobility behavior) can result in  
16   under-prediction of the demands imposed on the piles.

17   **CE Database subject headings:** Soil liquefaction; Pile lateral loads; Plasticity; Centrifuge models;  
18   Dynamic analysis.

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## 20 **Introduction**

21 Liquefaction has damaged many pile foundations in past earthquakes, resulting in significant  
22 research into the fundamental loading mechanisms. Research studies include centrifuge modeling (e.g.,  
23 Abdoun et al. 2003, Wilson et al. 2000, Brandenberg et al. 2005, Haigh and Madabhushi 2011), 1-g shake  
24 table testing (e.g., Tokimatsu and Suzuki 2004), full-scale field testing using blast-induced liquefaction  
25 (e.g., Ashford et al. 2004), and numerical simulations (e.g., lai 2002). Among the important findings from  
26 these studies are: (1) liquefied sand provides some non-zero lateral resistance to piles, and the p-y  
27 behavior often exhibits a concave-upward trend that is similar to the undrained stress-strain cyclic  
28 mobility response of sand due to dilatancy (Wilson et al. 2000, Rollins et al. 2005), (2) loads from a  
29 nonliquefiable laterally spreading crust layer often dominate pile foundation response (e.g., Dobry et al.  
30 2003) and significantly larger deformations are required to mobilize passive resistance compared with  
31 nonliquefied soil profiles (Brandenberg et al. 2007a), (3) kinematic demands induced by lateral  
32 spreading ground deformation can act simultaneously with inertia demands imposed by a  
33 superstructure and pile cap (Boulangier et al. 2007), and (4) static beam on nonlinear Winkler foundation  
34 (BNWF) analyses can provide reasonable predictions of bending moments and pile deformations  
35 provided that the inputs are carefully selected (e.g., Juirnarongrit and Ashford 2006, Brandenberg et al.  
36 2007b).

37 The primary benefits of static BNWF simulations are that they can capture many of the salient  
38 features of the loading mechanisms, and can be easily performed using commercially available software  
39 (e.g., LPile, Reese et al. 2004). The disadvantages are that assumptions must be made regarding the  
40 appropriate combination of kinematic and inertia demands, and static simulations cannot reasonably  
41 capture the evolution of pile demands as the soil transitions from non-liquefied to liquefied, nor the  
42 cyclic mobility behavior following liquefaction. Dynamic simulations have become routine for structures  
43 founded on nonliquefiable soils, yet dynamic simulations are quite rare for liquefiable sites simply

44 because well-vetted tools for performing such simulations are not readily available, and numerical  
45 approaches can be computationally expensive. There is a clear need for development and  
46 documentation of relatively simple computational tools that permit dynamic analysis of structures at  
47 liquefiable sites.

48 This paper formulates dynamic liquefiable soil-structure interaction materials (i.e., p-y and t-z) that  
49 are implemented in a BNWF framework and compared with results from two dynamic centrifuge model  
50 tests of pile systems in liquefiable soil profiles, with and without lateral spreading. While the material  
51 models described herein have been implemented in OpenSees and used in a number of dynamic  
52 numerical studies, their basic formulation and initial examination of their performance have not been  
53 previously presented in the literature. This paper therefore presents the mathematical formulation of  
54 the material models, followed by a description of the centrifuge models and the analyses of the pile  
55 responses using the BNWF method.

## 56 **PySimple1 Material**

57 Formulation of the PySimple1 material was first presented by Boulanger et al. (1999) and compared  
58 with centrifuge model results for piles in soft clay. This material forms the basis for the liquefiable p-y  
59 material, PyLiq1, and the PySimple1 equations are presented first. The equations used to describe p-y  
60 behavior were chosen as a versatile means of approximating established p-y relations, and are  
61 structured for implementation in a displacement-based finite element code. The nonlinear p-y behavior  
62 is conceptualized as consisting of elastic ( $p-y^e$ ), plastic ( $p-y^p$ ), and gap ( $p-y^g$ ) components in series (Fig. 1).  
63 A dashpot is placed in parallel with the elastic component to model radiation damping. This formulation  
64 is consistent with the observation that radiation damping consists largely of elastic wave propagation in

65 the far-field, whereas hysteretic damping dominates the near-field response. The gap component  
 66 consists of a drag ( $p^d \cdot \dot{y}^g$ ) and closure ( $p^c \cdot \dot{y}^g$ ) element in parallel. Note that  $p = p^c + p^d$ , and  $y = y^e + y^p + y^g$ .

67 Elastic and-Plastic Components

68 The elastic component consists of an elastic material with stiffness  $K^e$  in parallel with a dashpot to  
 69 model radiation damping. Force in the elastic component is  $p = K^e y^e$ , where  $y^e$  is the elastic component  
 70 of displacement. The elastic component is placed in series with a plastic component such that the force,  
 71  $p$ , in these components is equal. The force in the plastic component is defined on the right side of Eq. 1,  
 72 where  $y^p$  is the plastic component of displacement,  $C$  and  $n$  are model constants that control the shape  
 73 of the plastic component,  $y_{50}$  is the displacement where  $p = 0.5p_{ult}$ , and  $p_o$  and  $y_o^p$  are the values of  $p$   
 74 and plastic displacement, respectively, at the start of the current plastic loading cycle.

$$p = K^e y^e = p_{ult} - (p_{ult} - p_o) \left( \frac{C \cdot y_{50}}{C \cdot y_{50} + |y^p - y_o^p|} \right)^n \quad (1)$$

75  
 76 The yield function is defined in Eq. 2, where  $p_{ult}$  is the ultimate strength,  $C_r \cdot p_{ult}$  is the yield stress, and  $p_\alpha$   
 77 is the back stress (i.e., the value of  $p$  at the center of the elastic region). A kinematic hardening law  
 78 defines evolution of the back stress such that  $\dot{p}_\alpha = \dot{p}$  for a plastic loading increment, and  $\dot{p}_\alpha = 0$  for an  
 79 elastic loading increment. The plastic modulus is defined in Eq. 3.

$$f = |p - p_\alpha| - (C_r \cdot p_{ult}) \leq 0 \quad (2)$$

$$K^p = \frac{\partial p}{\partial y^p} = \frac{n \cdot \text{sign}(\dot{y}) \cdot (p_{ult} - p_o)}{|y^p - y_o^p| + C \cdot y_{50}} \left( \left[ \frac{C \cdot y_{50}}{|y^p - y_o^p| + C \cdot y_{50}} \right]^n \right) \quad (3)$$

80

81 Material constants  $C$ ,  $n$ , and  $C_r$  define the shape of the backbone curve of the PySimple1 material, and  
 82 have been adjusted to fit the functional form suggested by Matlock (1970) for piles in clay ( $C=10$ ,  $n=5$ ,  
 83  $C_r=0.35$ ), and API (1993) for piles in sand ( $C=0.5$ ,  $n=2$ ,  $C_r=0.2$ ).

84 Gap Component

85 The gap component consists of a nonlinear drag element in parallel with a nonlinear closure  
 86 element such that  $p^d + p^c = p$ , and the displacement across the gap element is  $y^g$ . Force in the drag  
 87 component,  $p^d$ , and closure component,  $p^c$ , are defined by Eqs. 4 and 5, respectively, where  $C_d$  is a  
 88 material constant, and  $p_o^d$  and  $y_o^g$  are the force and plastic gap displacement in the component at the  
 89 start of the current plastic loading cycle. Evolution of the gap follows logic similar to that of Matlock et  
 90 al. (1978) with  $y_o^+$  equal to the maximum past value of  $y^e + 1.5y_{50}$  and  $y_o^-$  equal to the maximum past  
 91 value of  $y^e - 1.5y_{50}$ , where  $1.5y_{50}$  represents some rebounding of the gap. The tangent modulus for the  
 92 gap component,  $K^g$ , is defined in Eq. 6.

$$p^d = C_d \cdot p_{ult} - (C_d \cdot p_{ult} - p_o^d) \left[ \frac{y_{50}}{y_{50} + 2|y^g - y_o^g|} \right]^n \quad (4)$$

$$p^c = 1.8 \cdot p_{ult} \left[ \frac{y_{50}}{y_{50} + 50(y_o^+ - y^g)} - \frac{y_{50}}{y_{50} + 50(y_o^- - y^g)} \right] \quad (5)$$

$$K^g = \frac{\partial p}{\partial y^g} = \frac{2n(p_o^d - C_d p_{ult})}{y_{50} + 2|y^g - y_o^g|} \left( \frac{y_{50}}{y_{50} + 2|y^g - y_o^g|} \right)^{n-1} + \frac{1.8 p_{ult} \frac{y_{50}}{50}}{\left( \frac{y_{50}}{50} - y^g + y_o^+ \right)^2} - \frac{1.8 p_{ult} \frac{y_{50}}{50}}{\left( \frac{y_{50}}{50} - y^g + y_o^- \right)^2} \quad (6)$$

93

94 Combined Material

95 Example behavior for the combined material is shown in Fig. 1 for the second cycle of sinusoidal  
96 displacement-controlled loading with amplitude equal to  $10y_{50}$ . Values of  $C_d = 0.1, 1.0,$  and  $10.0$  are  
97 shown in the figure and radiation damping is zero for all cases. The material with  $C_d=0.1$  is pinched in the  
98 middle, clearly exhibiting behavior that is consistent with a pile moving through an open gap (e.g.,  
99 Matlock 1970). Resistance in the open gap arises from friction along the sides of the pile. The force  
100 amplitude abruptly increases when the gap closes. The material with  $C_d=10.0$  essentially removes the  
101 gap component from the material by making it rigid (notice the essentially rigid response in Fig. 1e).

102 The tangent modulus for the combined material,  $K$ , is defined as  $K = (1/K^e + 1/K^p + 1/K^g)^{-1}$ . The  
103 consistent tangent operator is equal to the elasto-plastic tangent modulus for one dimensional  
104 problems, and is important to preserve the quadratic rate of asymptotic convergence for iteration  
105 schemes commonly used in nonlinear finite element problems (e.g., Simo and Hughes 1998).

106 **PyLiq1 Material**

107 The PyLiq1 material follows the same logic as the PySimple1 material with the only difference being  
108 that the capacity of the p-y material,  $p_{ult\_liq}$ , is treated as a variable that depends on the mean effective  
109 stress in the free-field,  $\sigma'$ , rather than being specified as a material constant. The value of  $p_{ult\_liq}$  is  
110 degraded as pore pressure develops in the free field, eventually reaching a residual value  $p_{res}$  when  $\sigma'=0$   
111 according to Eq. 7, where  $\sigma'_o$  is the initial free-field effective stress.

$$p_{ult\_liq} = p_{res} + (p_{ult} - p_{res}) \frac{\sigma'}{\sigma'_o} \tag{7}$$

112 This formulation is intended to incorporate the influence of ground shaking and liquefaction on p-y  
113 behavior, while retaining some small p-y capacity for the fully-liquefied condition that has been



114 observed in many model studies (e.g., Wilson et al. 2000, Dobry et al. 2003). The ground motion and  
115 mean effective stress are input to the free-ends of the PyLiq1 elements as demonstrated in Fig. 2. These  
116 quantities can be obtained from an effective stress site response analysis, though measured quantities  
117 are also used as inputs in this paper. The site response simulation can be run separately from the  
118 structural analysis, with the recorded outputs written to file and subsequently input to the free-ends of  
119 the p-y elements, or it can be run concurrently with the p-y elements and pile part of the same domain  
120 as the soil mesh. If run concurrently, the out-of-plane thickness of the soil mesh should be made very  
121 large so that an essentially free-field site response condition is achieved (i.e., so that the pile and p-y  
122 elements do not affect the site response). The motivation for utilizing free-field motions is that the p-y  
123 materials are intended to capture all of the soil-structure-interaction effects, and none of it is modeled  
124 by a soil continuum. A three-dimensional continuum with appropriately sized elements near the pile  
125 would be required to properly model SSI effects, and such approaches are computationally very  
126 expensive for dynamic problems with liquefaction.

127 In addition to modeling degradation of the p-y behavior as excess pore pressure develops in the  
128 free-field, the material is also capable of modeling the transient stiffening associated with the cyclic  
129 mobility behavior of sands in cyclic undrained loading. Cyclic mobility behavior is defined as the  
130 transition from incrementally contractive to incrementally dilative behavior that is associated with an  
131 increase in the tangent stiffness and inverted s-shaped stress-strain behavior. Cyclic mobility  
132 significantly influences free-field site response behavior, and this influence is captured as an input to the  
133 PyLiq1 material. However, a limitation of the model is that the dilatancy induced by local strains  
134 imposed on the soil by the pile can only indirectly be incorporated by specifying an appropriate value for  
135  $p_{res}$ . The concave-upward p-y behavior that has been observed in the absence of shaking-induced free-  
136 field dilatancy during blast-induced liquefaction studies (Rollins et al. 2005) and in numerical simulations

137 (e.g., lai 2002) is not captured by the PyLiq1 formulation. Furthermore, the inverted cone-shaped  
138 negative pore pressure region around the pile that was observed by Gonzalez et al. (2009) is not  
139 captured by the PyLiq1 material. Assimaki and Varun (2009) formulated a p-y macroelement that links a  
140 Bouc-Wen type backbone curve with a pore pressure function that combines free-field pore pressure  
141 response with near-field response related to plastic work in the p-y element. This added feature of  
142 material behavior requires specification of additional input parameters for the macro-elements.  
143 Development of multiple independent models is important for quantifying the effects of epistemic  
144 uncertainty.

145 An illustration of the PyLiq1 material behavior in level ground conditions (i.e., without static shear  
146 stress and lateral spreading) is shown in Fig. 3 for a case where a PyLiq1 material with  $p_{res}=0.1p_{ult}$   
147 attaches a soil element to a rigid pile. The soil element is modeled as a PressureDependMultiYield02  
148 material using the default input parameters suggested by Yang et al. (2003) for medium dense sand with  
149  $D_R=50\%$ , and it is subjected to simple shear loading with a cyclic stress ratio of  $CSR=0.3$ . The harmonic  
150 simple shear stress path is applied at a low enough frequency to render essentially zero inertial stresses.  
151 The simulation was performed in OpenSees, with the soil response computed first and the  
152 displacements and mean effective stresses from the soil response subsequently imposed on the free  
153 end of the p-y element in a separate analysis. This approach ensures that the soil behavior is a free-field  
154 input. The excess pore pressure builds up and reaches 1.0 after approximately 6 cycles, and the material  
155 behavior is characterized by transient reductions in pore pressure associated with dilatancy. The dilatant  
156 tendency at large strains causes sharp increases in the shear stress when the shear strain exceeds the  
157 maximum past strain, resulting in the inverted s-shaped stress-strain behavior that characterizes  
158 undrained loading of sands. The p-y behavior in the model mimics the stress-strain behavior of the sand

159 in this case and also exhibits an inverted s-shape that is similar to trends observed in centrifuge tests  
160 (e.g., Wilson et al. 2000).

161 An illustration of the PyLiq1 material behavior in the presence of lateral spreading is shown in Fig. 4  
162 for the same configuration as in Fig. 3, but with a static shear stress imposed on the soil in addition to  
163 the cyclic shear stress. Shear strains accumulate in the direction of static shear stress, resulting in  
164 permanent deformation of the soil element in a manner that is consistent with lateral spreading. The  
165 free-field soil response was input to the same PyLiq1 material as in Fig. 3, but this time the analysis was  
166 performed for a rigid pile, and for a flexible pile whose stiffness was adjusted so that the peak pile  
167 displacement is equal to  $10y_{50}$ . The rigid pile attracted large loads during each cycle as the soil spreads  
168 past, whereas the flexible pile attracted much smaller loads. The flexible pile deformation is  
169 characterized by alternating episodes in which (1) the down-slope movement of the sand is slowed as  
170 the sand becomes incrementally dilative, the excess pore pressures reduce, and the sand stiffens, (2) the  
171 temporarily stiffened sand exerts greater loads on the pile and displaces it down-slope, (3) the dynamic  
172 shear stress on the sand reverses direction, such that the sand becomes incrementally contractive, the  
173 excess pore pressures increase again, and the sand softens, and (4) the temporarily softened sand exerts  
174 lesser loads on the pile, which allows the pile to rebound in the upslope direction. On the other hand,  
175 the rigid pile does not displace downslope during dilatancy cycles, and a much larger load is transmitted  
176 to the pile due to the cyclic mobility behavior of the soil. This observation is consistent with centrifuge  
177 testing by Haigh (2002) that showed that stiff piles attract much larger lateral spreading loads than  
178 flexible piles.

179 In addition to PySimple1 and PyLiq1 for lateral soil-pile interaction, TzSimple1 and TzLiq1 materials  
180 were formulated for axial shaft friction, and QzSimple1 was formulated for end bearing resistance.  
181 TzSimple1 follows the same logic as PySimple1, except the gapping component is removed, and TzLiq1

182 follows the same logic as PyLiq1, except the residual capacity of the material is set to zero. The  
183 backbone of the TzSimple1 and TzLiq1 materials can be selected to match the relation by Mosher (1984)  
184 for axially loaded piles in sand, or by Reese and O'Neill (1987) for drilled shafts. QzSimple1 exhibits a  
185 direction-dependent response in which a small uplift capacity can be included to model suction stresses  
186 in undrained loading. The backbone of the QzSimple1 material can be selected to match the relation by  
187 Reese and O'Neill (1987) for drilled shafts in clay or Vijayvergia (1977) for piles in sand. Including axial  
188 interaction can be important for pile groups that rotate in response to lateral loading.

### 189 **Description of Centrifuge Models**

190 Simulations incorporating the PyLiq1 material are compared with experimental data from centrifuge  
191 model CSP2 (Wilson et al. 1997) for single piles in nearly level liquefiable sand, and from model SJB03  
192 (Brandenberg et al. 2005) for pile groups in laterally spreading ground. Model sketches are shown in Fig.  
193 5. Properties of the sand utilized in the studies are summarized in Table 1 for CSP2 and Table 2 for  
194 SJB03. Peak friction angles reported in Tables 1 and 2 are estimated based on relative density. Pile  
195 properties are summarized in Table 3. Results are presented in prototype units unless otherwise  
196 specifically noted.

197 Model CSP2 consisted of a 0.67m diameter single extended pile shaft embedded 16.8m into a  
198 horizontally-layered soil profile consisting of liquefiable loose Nevada sand ( $D_R=35\%$ ) over dense Nevada  
199 sand ( $D_R=75\%$ ). Pile groups were also embedded in the model, but only the single pile is studied herein.  
200 The single pile was at least 15 diameters from the nearest pile group. A 49 MN mass was attached to the  
201 top of the pile at a height of 3.81m above the ground surface. The model was saturated with a pore fluid  
202 consisting of water mixed with methylcellulose with a viscosity equal to ten times that of water. Testing

203 was performed at a centrifugal acceleration of 30g. Scaling effects related to pore fluid viscosity are  
204 discussed by Wilson et al. (2000).

205 Model SJB03 consisted of a six-pile group of 1.17m diameter piles embedded into a sloping soil  
206 profile consisting of a nonliquefiable overconsolidated crust of San Francisco bay mud over loose  
207 Nevada sand ( $D_R=35\%$ ) over dense Nevada sand ( $D_R=75\%$ ). A thin layer of Monterey sand was placed on  
208 top of the bay mud to prevent desiccation due to wind currents during spinning. The piles were tied  
209 together by an embedded pile cap with length x width x height of 14.2m x 9.2m x 2.2m and mass of  
210 726Mg. A channel was carved in the downslope end of the model to simulate a common geologic  
211 condition that results in lateral spreading. The model was saturated with water rather than a viscous  
212 pore fluid because some water was squeezed out of the clay into the sand during consolidation on the  
213 hydraulic press prior to shaking and this water could not be replaced by viscous pore fluid during  
214 saturation. Since the viscosity of the pore fluid was not scaled, the prototype hydraulic conductivity of  
215 the sand was 57.2 times higher than for the same water-saturated sand at 1-g. The hydraulic  
216 conductivity is nevertheless low enough to fully liquefy the sand during shaking. Testing was performed  
217 at a centrifugal acceleration of 57.2g.

218 A sequence of ground motions was imposed on each model, and seven of the ground motions  
219 imposed on CSP2 and four of the ground motions imposed on SJB03 are analyzed in this paper. The  
220 analyzed motions were scaled versions of the UCSC/Lick Lab, Ch. 1 - 90° strong motion record from the  
221 1989 Loma Prieta earthquake, and the downhole record (depth = 83m) from Port Island during the 1995  
222 Kobe earthquake. The ground motion profile was recorded at discrete points using vertical arrays of  
223 horizontal accelerometers, and the acceleration records were double-integrated in time to obtain free-  
224 field displacement records. The free-field pore pressure profile was recorded using vertical arrays of  
225 piezometers. Test CSP2 consisted of a level ground profile and the permanent component of the soil

226 displacement was negligible, hence time series of ground displacement could be obtained from double  
227 integration of acceleration records. On the other hand, the low frequency component of lateral  
228 spreading displacement from SJB03 could not be obtained by integration of acceleration records, but  
229 was measured using displacement sensors attached to the nonliquefied crust. The complete ground  
230 motion time series, including low frequency and high frequency components of the crust displacement,  
231 were computed using complementary filters applied to the accelerometer and displacement sensor  
232 records. The low frequency components of the soil displacements below the ground surface were  
233 assumed to be proportional to those at the ground surface, and the final displaced shape of the soil  
234 profile (as determined from post-test profiles of vertical markers embedded in the model) was used to  
235 determine the coefficients of proportionality for those low frequency components. Displacement time  
236 series were then computed by combining the low and high frequency components. Validation of this  
237 procedure and the selection of appropriate filters for the equipment used in these centrifuge tests are  
238 discussed in Kutter and Balakrishnan (1998).

### 239 **Material Properties for p-y, t-z, and Q-z elements**

240 The capacity of the p-y materials,  $p_{ult}$ , was estimated using the API (1993) equations for piles in sand.  
241 A p-multiplier approach was to define the residual capacity as  $p_{res} = m_p p_{ult}$ , where  $m_p$  was defined based  
242 on Brandenburg (2005). The modulus of subgrade reaction in sand deposits is often assumed to vary  
243 linearly with depth, however the elastic modulus for clean sands is known to vary approximately with  
244 the square root of confining stress (e.g., Yamada et al. 2008). Hence, a parabolic relation was used to  
245 define the elastic stiffness with depth. The crust load acting on the embedded pile cap for test SJB03  
246 was estimated to be 6940kN based on the sum of passive earth pressure and side and base friction  
247 summarized by Brandenburg et al. (2005). Pile group effects are considered for the mobilized crust load  
248 because clay may become trapped between the piles, thereby causing the pile group to act as an

249 equivalent block (Brandenberg et al. 2005). However, group effects are not included for the sand layers  
250 based on tests that show group interaction effects to be negligible in liquefied soils (Rollins et al. 2005).  
251 Lateral load transfer between pile groups and nonliquefied crusts was observed to be significantly softer  
252 for crusts spreading over liquefied soil compared with nonliquefied soil profiles (Brandenberg et al.  
253 2007a). The p-y materials in the nonliquefied crust were therefore adjusted so that the ultimate capacity  
254 was mobilized at a relative displacement of 40% of the crust thickness rather than a more typical value  
255 of 1 to 7% of the crust thickness observed in load tests in nonliquefied soil profiles.

256 The single-pile for CSP2 required only p-y elements to model lateral load transfer, but the pile group  
257 in SJB03 required t-z elements to model shaft friction and Q-z elements to model end bearing since the  
258 pile group forms a frame that can rock during lateral loading. The t-z elements were modeled using  
259 TzLiq1 materials with  $t_{ult} = K_o B \sigma_{vo}' \tan(2/3\phi')$ , where  $K_o = 1 - \sin\phi'$ , and  $\sigma_{vo}'$  is the initial vertical effective  
260 stress prior to shaking. Horizontal stresses at the soil-pile interface typically increase when closed-ended  
261 pipe piles are driven into the soil. However, in this case the piles were driven into the models at 1g,  
262 therefore significant changes in horizontal pressure are not anticipated. The bearing capacity at the tip  
263 of the piles was estimated to be 10MN using bearing factors by Meyerhof (1976), and end bearing  
264 resistance was modeled using QzSimple1 materials since there was little excess pore pressure generated  
265 in the end bearing stratum in each case.

## 266 **Numerical Modeling Approach**

267 Numerical models of the pile were constructed in OpenSees (McKenna and Fenves 2001) using 50  
268 elements along the length of the pile with p-y elements attached at each node below the ground  
269 surface. For CSP2 the piles did not yield during testing, and were therefore modeled as elastic beam  
270 column elements with properties summarized in Table 3. A mass was assigned to the top node. For

271 SJB03, the piles did slightly yield during testing, and were therefore modeled using nonlinear beam  
272 column elements. The piles were tied together at their head by a pile cap composed of very stiff  
273 (essentially rigid) beam column elements. An elastic rotational element attached the pile head to the  
274 pile cap to model the measured rotational stiffness of 1300 MN-m/rad at the connection. Masses were  
275 distributed among the pile cap nodes and PyLiq1 materials were attached to the top and bottom of the  
276 pile cap. For convenience, PyLiq1 materials were utilized for the nonliquefied crust layer as well as in the  
277 liquefiable layers but the recorded mean effective stress time series input to the free-ends of the crust  
278 were essentially constant since the clay did not generate significant excess pore pressure during the  
279 tests. Hence, the PyLiq1 material response was nearly identical to what the PySimple1 material response  
280 would have been. TzLiq1 materials were distributed along the length of the piles and QzSimple1  
281 materials were attached to the pile tips. This configuration permits the pile group to rotate during  
282 lateral loading, which is an important feature of the response of laterally loaded pile groups. The free-  
283 ends of the t-z and Q-z elements were fixed.

284 Time series of displacement and mean effective stress were linearly interpolated from the recorded  
285 data, and imposed on the free-ends of the p-y elements. Recordings of acceleration and pore pressure  
286 were from sensors at least 10 pile diameters from the nearest foundation. In forward predictions,  
287 displacements and pore pressures would need to be estimated from a site response simulation.  
288 However, in this study the measured inputs are utilized to isolate the response of the PyLiq1 materials  
289 so that errors in the p-y elements could be separated from errors in site response simulations. Small-  
290 strain damping of approximately 2% in the frequency range of interest was achieved using Rayleigh  
291 damping. The convergence tolerance on the norm of the displacement residuals was set to  $10^{-6}$  (using  
292 the normDispIncr command), and displacement constraints were enforced using the transformation  
293 method (using the Transformation command). The equation of motion was integrated using the Hilbert-



294 Hughes-Taylor integrator (using the HHT command) with  $\alpha=0.7$ . The time step was adjusted as needed  
295 to facilitate convergence (using the VariableTransient command).

## 296 **Numerical Results for CSP2**

297 Example analysis results for CSP2 are shown in Fig. 6 for a Santa Cruz motion with a peak base  
298 acceleration of 0.42g. The excess pore pressure ratio near the center of the loose sand layer reaches 1.0  
299 approximately 5s after the start of strong shaking, and subsequently exhibits modest dilation-induced  
300 drops in pore pressure. Excess pore pressure ratios are plotted in the free-field, far away from the pile  
301 groups, and also at a location immediately adjacent to the pile. Dilation-induced spikes are apparent in  
302 both records, but are slightly more pronounced in the record near the pile, presumably due to the  
303 additional increment of dilatancy caused by strains imposed on the near field soil by the pile. The peak  
304 bending moment (measured and predicted) is mobilized during one such dilation-induced drop in pore  
305 pressure at approximately time = 26s. The bending moment and superstructure acceleration records are  
306 predicted quite well. The bending moment record was taken at a depth of approximately 3B, where the  
307 peak bending moments were measured. Computed values of subgrade reaction near the center of the  
308 loose sand do not agree with measurements as well as computed values of bending moment and  
309 superstructure acceleration, but nevertheless, the peak responses are predicted well during critical  
310 cycles.

311 The same records from Fig. 6 are plotted in Fig. 7 for a Kobe motion with a peak base acceleration of  
312 0.61g. The frequency content of the Kobe motion is significantly lower than the Santa Cruz motion, and  
313 the dilatancy response in the liquefied sand is much more pronounced as a result. Once again, the peak  
314 bending moment and peak superstructure acceleration occurred during a transient drop in excess pore  
315 pressure. The superstructure acceleration reached a peak near 1.5g that was under-predicted by the

316 analyses by about 0.5g, but the computed values track the measured response quite well other than for  
317 the one cycle that produced the peak value. The bending moment mobilized in the pile is also predicted  
318 reasonably well.

319 Acceleration response spectra were computed for the superstructure motion for three Santa Cruz  
320 motions and four Kobe motions of various intensities. The shapes of the spectra tend to agree quite well  
321 with measurements. However, for the Santa Cruz motions, the computed values tend to be too high for  
322 the high-intensity motions and too low for the low-intensity motions. For the Kobe motions, the  
323 disagreements could not be so simply characterized based on input motion intensity. Better agreement  
324 could be achieved by adjusting the stiffness of the p-y materials on a motion-specific basis by increasing  
325  $K_{ref}$  for the small motions and decreasing  $K_{ref}$  for the large motions. This may partly reflect the effect of  
326 loading history on p-y behavior, which is not included in the analyses. Another factor may be that the  
327 functional form of the API (1993) sand curve is very linear at small values of  $\gamma$ , hence there is very little  
328 small-strain nonlinearity in the PyLiq1 materials. Varun (2010) also demonstrated that the API curve is  
329 too linear, and suggested an alternative form that resulted in better agreement with measurements.

### 330 **Numerical Results for SJB03**

331 Computed results for SJB03 are compared with measurements for the medium intensity Santa Cruz  
332 motion with peak base acceleration of 0.35g in Fig. 9. Some residual loads were present in the pile  
333 groups at the end of each motion for SJB03 due to lateral spreading deformations and the motions were  
334 applied in sequence in the numerical simulations, which explains the non-zero initial values of some  
335 quantities in Fig. 9. The excess pore pressure ratio in the loose sand does not reach 1.0 for this motion  
336 even though surface evidence of liquefaction was observed in the form of 0.4m of lateral spreading  
337 ground displacement. This may be attributed to the effect that sustained downslope shear stresses has

338 on limiting values of excess pore pressure ratio (e.g., Ishihara and Nagase 1980). Computed values are  
339 reasonably consistent with the measurements for bending moment, crust load, pile cap inertia, and pile  
340 cap displacement, though the residual loads on the pile group are larger than predicted.

341 Computed values for the large Kobe motion with peak base acceleration of 0.67g are shown in Fig.  
342 10. This motion fully liquefied the loose sand in the first cycle of strong shaking, and the response is  
343 characterized by very pronounced dilatancy spikes. Each downward spike in the pore pressure record is  
344 associated with a local peak in the crust load, a local maximum bending moment amplitude (the largest  
345 amplitude bending moments were negative in this case), and a local maximum in pile cap inertia. The  
346 measured crust load sometimes exceeded the predicted maximum crust load, and as a result the peak  
347 bending moments were slightly under-predicted in the analysis. Nevertheless, agreement between the  
348 measured and predicted responses is quite reasonable.

349 Response spectra for the pile cap motion are shown in Fig. 11. The shape of the spectra was  
350 reasonably predicted in each case, though the amplitude was not predicted perfectly. Once again, the  
351 small-strain stiffness of the p-y materials could be adjusted to provide a better prediction of pile cap  
352 motion (results not shown for brevity), which may either represent the effects of loading history not  
353 being accounted for in the analyses or indicate a need for a p-y material functional form that more  
354 correctly captures small-strain nonlinearity.

## 355 **Discussion**

356 A key feature of the PyLiq1 material is that it incorporates not only the development of excess pore  
357 pressure during cyclic loading, but also transient reductions in pore pressure caused by dilatancy.  
358 Dilatancy was shown to be an important driver of peak bending moments in the pile foundations from  
359 centrifuge studies by Wilson et al. (2000) and Brandenberg et al. (2005). This paper input the measured

360 pore pressures; hence the dilatancy response was included to the extent it was measured in the free-  
361 field during a particular motion. However, the pore pressure response would need to be numerically  
362 simulated in a forward analysis. Advanced plasticity models are capable of capturing the dilatancy  
363 response (e.g., Dafalias and Manzari 2004; Yang et al. 2003, Boulanger and Ziotopoulou, 2012) whereas  
364 other models can capture the development of excess pore pressure but not the transient reductions  
365 caused by dilatancy (e.g., Martin and Qiu 2001, Hashash 2011). To explore the influence of dilatancy on  
366 pile response, simulations for the single pile from CSP2 were repeated with the same displacement  
367 records input to the free-ends of the p-y materials, but with the measured pore pressure response  
368 adjusted so that it only increased (Fig. 12). The motions in Fig. 12 were selected because in both cases  
369 the loose sand fully liquefied, but the extent of the post-liquefaction dilatancy-induced drops in pore  
370 pressure were quite different. The Santa Cruz motion exhibited very small pore pressure drops whereas  
371 the Kobe motion exhibited very pronounced drops.

372 For the Santa Cruz motion, the simulation with the always-increasing pore pressure record is very  
373 similar to the simulation that utilized the measured pore pressure input. On the other hand, significant  
374 differences arise for the Kobe motion, where the always-increasing pore pressure response resulted in a  
375 significantly smaller prediction of peak bending moments and superstructure acceleration (Table 4).  
376 These cases demonstrate that dilatancy can be a significant factor that drives the critical loading cycles  
377 for piles in liquefied ground. Similar conclusions were reached in previous studies that utilized an always  
378 increasing pore pressure response to model the single pile by Wilson (2000). Liyanapathirana and Poulos  
379 (2005) analyzed the Kobe motion, and found that bending moments were under-predicted following  
380 liquefaction. Finn et al. (2000) analyzed the Santa Cruz motion, and found that bending moments were  
381 reasonably predicted. Similarly, Kramer et al. (2011) and Ziotopoulou et al. (2011) illustrate that

382 dilatancy is an important factor in obtaining reasonable ground motion simulations of the seismic  
383 responses of liquefying soil profiles.

384 The cases studied in this paper involved liquefaction of loose sands. Wilson et al. (2000) studied  
385 liquefaction potential of medium dense sands and found them to be more dilatant and exhibit more  
386 pronounced cyclic mobility behavior compared with the looser sands. Analysis of a medium dense sand  
387 case is presented by Boulanger et al. (2004).

### 388 **Conclusions**

389 Static methods for analyzing piles in liquefied ground are appropriate for many structures for which  
390 dynamic simulations are too complex and costly, and uncertainties inherent to static analysis  
391 approaches can be accommodated by adequate conservatism. However, dynamic simulations may be  
392 warranted for important structures, and may be required for complex structures for which liquefaction-  
393 compatible inertia demands are difficult to quantify without performing a dynamic simulation.  
394 Advancing beyond equivalent static analysis of piles in liquefied ground requires development of robust,  
395 validated numerical tools for performing dynamic simulations. This paper addresses this need by  
396 formulating liquefiable soil-structure interaction elements that utilize free-field site response quantities  
397 as inputs.

398 Comparisons with centrifuge test data show that the materials can reasonably capture key features  
399 of dynamic response when measured displacements and excess pore pressures are utilized as inputs.  
400 Forward predictions would require a site response simulation to obtain ground motion and effective  
401 stress time series to input to the p-y model, which introduces additional uncertainty to the predictions.  
402 The measured inputs were utilized instead of a site response prediction in this study to isolate the  
403 behavior of the p-y materials.

404 Cyclic mobility behavior that is associated with inverted s-shaped stress strain behavior during  
405 undrained loading of sands also causes inverted s-shaped p-y behavior for piles embedded in liquefied  
406 sand. Cyclic mobility behavior was found to be the mechanism responsible for mobilization of the peak  
407 bending moments in the piles presented in this study. Simulations that neglected cyclic mobility  
408 behavior (i.e., by inputting an always-increasing pore pressure response) under-predicted bending  
409 moments and superstructure accelerations compared with measurements, and compared with  
410 simulations that included cyclic mobility behavior. Therefore, neglecting the influence of cyclic mobility  
411 on pile response could result in unconservative predictions and unforeseen damage or failure.

#### 412 **Acknowledgments**

413 Funding for this work was provided by Caltrans and the National Science Foundation through the  
414 Pacific Earthquake Engineering Research Center. The contents of this paper do not necessarily represent  
415 a policy of either funding agency or endorsement by the state or federal government. The authors  
416 would like to thank Christina Curras for doing the initial model development work on the p-y material  
417 models prior to their implementation in OpenSees. Tom Shantz provided valuable technical comments  
418 and suggestions. The centrifuge shaker was designed and constructed with support from the National  
419 Science Foundation (NSF), Obayashi Corp., Caltrans, and the University of California. Upgrades were  
420 funded by NSF award CMS-0086566 through the George E. Brown, Jr. Network for Earthquake  
421 Engineering Simulation (NEES).

#### 422 **Notation**

423  $B$  = pile diameter

424  $C$  = modeling constant that contributes to shape of p-y backbone curve

425  $C_r$  = modeling constant that controls size of elastic region

426  $C_d$  = modeling constant that controls subgrade reaction load in open gap

427  $CSR$  = cyclic stress ratio

- 428  $D_R$  = relative density
- 429  $K_o$  = coefficient of at-rest earth pressure
- 430  $K$  = tangent stiffness of p-y element
- 431  $K^e$  = tangent stiffness of elastic component
- 432  $K^p$  = tangent stiffness of plastic component
- 433  $K^g$  = tangent stiffness of gap component
- 434  $n$  = modeling constant that contributes to shape of p-y backbone curve
- 435  $p$  = subgrade reaction due to relative displacement between soil and pile
- 436  $p^d$  = component of subgrade reaction in drag element
- 437  $p^c$  = component of subgrade reaction in closure element
- 438  $p_\alpha$  = subgrade reaction value at center of elastic region
- 439  $p_{res}$  = ultimate resistance of p-y element for fully-liquefied condition (i.e., with  $r_u=1$ )
- 440  $p_{ult}$  = ultimate resistance of p-y element for non-liquefied condition (i.e., with  $r_u=0$ )
- 441  $p_{ult\_liq}$  = ultimate resistance of p-y element corresponding to  $0 < r_u < 1$
- 442  $p_o$  = value of subgrade reaction at start of current plastic loading cycle
- 443  $p_o^d$  = component of subgrade reaction in drag element at start of current plastic loading cycle
- 444  $r_u$  = excess pore pressure ratio
- 445  $t_{ult}$  = ultimate shaft friction load per unit pile length
- 446  $y$  = relative displacement between soil and pile
- 447  $y_{50}$  = relative displacement between soil and pile when half of ultimate load is mobilized in p-y element
- 448  $y^e$  = elastic component of relative displacement between soil and pile
- 449  $y^p$  = plastic component of relative displacement between soil and pile
- 450  $y^g$  = gap component of displacement between soil and pile
- 451  $y_o^g$  = value of gap component of relative displacement between soil and pile at start of current plastic
- 452 loading cycle

453  $y_o^p$  = value of plastic component of relative displacement between soil and pile at start of current plastic  
454 loading cycle

455  $y_o^+$  = gap evolution term equal to maximum past value of  $y^e + 1.5y_{50}$

456  $y_o^-$  = gap evolution term equal to maximum past value of  $y^e - 1.5y_{50}$

457  $\phi'$  = peak friction angle

458  $\sigma'$  = current effective stress in free-field soil

459  $\sigma'_o$  = initial effective stress in free-field soil

460  $\sigma_{vo}'$  = vertical effective stress

461

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## 20 **Introduction**

21 Liquefaction has damaged many pile foundations in past earthquakes, resulting in significant  
22 research into the fundamental loading mechanisms. Research studies include centrifuge modeling (e.g.,  
23 Abdoun et al. 2003, Wilson et al. 2000, Brandenberg et al. 2005, Haigh and Madabhushi 2011), 1-g shake  
24 table testing (e.g., Tokimatsu and Suzuki 2004), full-scale field testing using blast-induced liquefaction  
25 (e.g., Ashford et al. 2004), and numerical simulations (e.g., lai 2002). Among the important findings from  
26 these studies are: (1) liquefied sand provides some non-zero lateral resistance to piles, and the p-y  
27 behavior often exhibits a concave-upward trend that is similar to the undrained stress-strain cyclic  
28 mobility response of sand due to dilatancy (Wilson et al. 2000, Rollins et al. 2005), (2) loads from a  
29 nonliquefiable laterally spreading crust layer often dominate pile foundation response (e.g., Dobry et al.  
30 2003) and significantly larger deformations are required to mobilize passive resistance compared with  
31 nonliquefied soil profiles (Brandenberg et al. 2007a), (3) kinematic demands induced by lateral  
32 spreading ground deformation can act simultaneously with inertia demands imposed by a  
33 superstructure and pile cap (Boulangier et al. 2007), and (4) static beam on nonlinear Winkler foundation  
34 (BNWF) analyses can provide reasonable predictions of bending moments and pile deformations  
35 provided that the inputs are carefully selected (e.g., Juirnarongrit and Ashford 2006, Brandenberg et al.  
36 2007b).

37 The primary benefits of static BNWF simulations are that they can capture many of the salient  
38 features of the loading mechanisms, and can be easily performed using commercially available software  
39 (e.g., LPile, Reese et al. 2004). The disadvantages are that assumptions must be made regarding the  
40 appropriate combination of kinematic and inertia demands, and static simulations cannot reasonably  
41 capture the evolution of pile demands as the soil transitions from non-liquefied to liquefied, nor the  
42 cyclic mobility behavior following liquefaction. Dynamic simulations have become routine for structures  
43 founded on nonliquefiable soils, yet dynamic simulations are quite rare for liquefiable sites simply

44 because well-vetted tools for performing such simulations are not readily available, and numerical  
45 approaches can be computationally expensive. There is a clear need for development and  
46 documentation of relatively simple computational tools that permit dynamic analysis of structures at  
47 liquefiable sites.

48 This paper formulates dynamic liquefiable soil-structure interaction materials (i.e., p-y and t-z) that  
49 are implemented in a BNWF framework and compared with results from two dynamic centrifuge model  
50 tests of pile systems in liquefiable soil profiles, with and without lateral spreading. While the material  
51 models described herein have been implemented in OpenSees and used in a number of dynamic  
52 numerical studies, their basic formulation and initial examination of their performance have not been  
53 previously presented in the literature. This paper therefore presents the mathematical formulation of  
54 the material models, followed by a description of the centrifuge models and the analyses of the pile  
55 responses using the BNWF method.

## 56 **PySimple1 Material**

57 Formulation of the PySimple1 material was first presented by Boulanger et al. (1999) and compared  
58 with centrifuge model results for piles in soft clay. This material forms the basis for the liquefiable p-y  
59 material, PyLiq1, and the PySimple1 equations are presented first. The equations used to describe p-y  
60 behavior were chosen as a versatile means of approximating established p-y relations, and are  
61 structured for implementation in a displacement-based finite element code. The nonlinear p-y behavior  
62 is conceptualized as consisting of elastic ( $p-y^e$ ), plastic ( $p-y^p$ ), and gap ( $p-y^g$ ) components in series (Fig. 1).  
63 A dashpot is placed in parallel with the elastic component to model radiation damping. This formulation  
64 is consistent with the observation that radiation damping consists largely of elastic wave propagation in

65 the far-field, whereas hysteretic damping dominates the near-field response. The gap component  
 66 consists of a drag ( $p^d \cdot \dot{y}^g$ ) and closure ( $p^c \cdot \dot{y}^g$ ) element in parallel. Note that  $p = p^c + p^d$ , and  $y = y^e + y^p + y^g$ .

67 Elastic and-Plastic Components

68 The elastic component consists of an elastic material with stiffness  $K^e$  in parallel with a dashpot to  
 69 model radiation damping. Force in the elastic component is  $p = K^e y^e$ , where  $y^e$  is the elastic component  
 70 of displacement. The elastic component is placed in series with a plastic component such that the force,  
 71  $p$ , in these components is equal. The force in the plastic component is defined on the right side of Eq. 1,  
 72 where  $y^p$  is the plastic component of displacement,  $C$  and  $n$  are model constants that control the shape  
 73 of the plastic component,  $y_{50}$  is the displacement where  $p = 0.5p_{ult}$ , and  $p_o$  and  $y_o^p$  are the values of  $p$   
 74 and plastic displacement, respectively, at the start of the current plastic loading cycle.

$$p = K^e y^e = p_{ult} - (p_{ult} - p_o) \left( \frac{C \cdot y_{50}}{C \cdot y_{50} + |y^p - y_o^p|} \right)^n \quad (1)$$

75  
 76 The yield function is defined in Eq. 2, where  $p_{ult}$  is the ultimate strength,  $C_r \cdot p_{ult}$  is the yield stress, and  $p_\alpha$   
 77 is the back stress (i.e., the value of  $p$  at the center of the elastic region). A kinematic hardening law  
 78 defines evolution of the back stress such that  $\dot{p}_\alpha = \dot{p}$  for a plastic loading increment, and  $\dot{p}_\alpha = 0$  for an  
 79 elastic loading increment. The plastic modulus is defined in Eq. 3.

$$f = |p - p_\alpha| - (C_r \cdot p_{ult}) \leq 0 \quad (2)$$

$$K^p = \frac{\partial p}{\partial y^p} = \frac{n \cdot \text{sign}(\dot{y}) \cdot (p_{ult} - p_o)}{|y^p - y_o^p| + c \cdot y_{50}} \left( \left[ \frac{c \cdot y_{50}}{|y^p - y_o^p| + c \cdot y_{50}} \right]^n \right) \quad (3)$$

80



81 Material constants  $C$ ,  $n$ , and  $C_r$  define the shape of the backbone curve of the PySimple1 material, and  
 82 have been adjusted to fit the functional form suggested by Matlock (1970) for piles in clay ( $C=10$ ,  $n=5$ ,  
 83  $C_r=0.35$ ), and API (1993) for piles in sand ( $C=0.5$ ,  $n=2$ ,  $C_r=0.2$ ).

#### 84 Gap Component

85 The gap component consists of a nonlinear drag element in parallel with a nonlinear closure  
 86 element such that  $p^d + p^c = p$ , and the displacement across the gap element is  $y^g$ . Force in the drag  
 87 component,  $p^d$ , and closure component,  $p^c$ , are defined by Eqs. 4 and 5, respectively, where  $C_d$  is a  
 88 material constant, and  $p_o^d$  and  $y_o^g$  are the force and plastic gap displacement in the component at the  
 89 start of the current plastic loading cycle. Evolution of the gap follows logic similar to that of Matlock et  
 90 al. (1978) with  $y_o^+$  equal to the maximum past value of  $y^e + 1.5y_{50}$  and  $y_o^-$  equal to the maximum past  
 91 value of  $y^e - 1.5y_{50}$ , where  $1.5y_{50}$  represents some rebounding of the gap. The tangent modulus for the  
 92 gap component,  $K^g$ , is defined in Eq. 6.

$$p^d = C_d \cdot p_{ult} - (C_d \cdot p_{ult} - p_o^d) \left[ \frac{y_{50}}{y_{50} + 2|y^g - y_o^g|} \right]^n \quad (4)$$

$$p^c = 1.8 \cdot p_{ult} \left[ \frac{y_{50}}{y_{50} + 50(y_o^+ - y^g)} - \frac{y_{50}}{y_{50} + 50(y_o^- - y^g)} \right] \quad (5)$$

$$K^g = \frac{\partial p}{\partial y^g} = \frac{2n(p_o^d - C_d p_{ult})}{y_{50} + 2|y^g - y_o^g|} \left( \frac{y_{50}}{y_{50} + 2|y^g - y_o^g|} \right)^{n-1} + \frac{1.8 p_{ult} \frac{y_{50}}{50}}{\left( \frac{y_{50}}{50} - y^g + y_o^+ \right)^2} - \frac{1.8 p_{ult} \frac{y_{50}}{50}}{\left( \frac{y_{50}}{50} - y^g + y_o^- \right)^2} \quad (6)$$

93

94 Combined Material

95 Example behavior for the combined material is shown in Fig. 1 for the second cycle of sinusoidal  
96 displacement-controlled loading with amplitude equal to  $10y_{50}$ . Values of  $C_d = 0.1, 1.0,$  and  $10.0$  are  
97 shown in the figure and radiation damping is zero for all cases. The material with  $C_d=0.1$  is pinched in the  
98 middle, clearly exhibiting behavior that is consistent with a pile moving through an open gap (e.g.,  
99 Matlock 1970). Resistance in the open gap arises from friction along the sides of the pile. The force  
100 amplitude abruptly increases when the gap closes. The material with  $C_d=10.0$  essentially removes the  
101 gap component from the material by making it rigid (notice the essentially rigid response in Fig. 1e).

102 The tangent modulus for the combined material,  $K$ , is defined as  $K = (1/K^e + 1/K^p + 1/K^g)^{-1}$ . The  
103 consistent tangent operator is equal to the elasto-plastic tangent modulus for one dimensional  
104 problems, and is important to preserve the quadratic rate of asymptotic convergence for iteration  
105 schemes commonly used in nonlinear finite element problems (e.g., Simo and Hughes 1998).

106 **PyLiq1 Material**

107 The PyLiq1 material follows the same logic as the PySimple1 material with the only difference being  
108 that the capacity of the p-y material,  $p_{ult\_liq}$ , is treated as a variable that depends on the mean effective  
109 stress in the free-field,  $\sigma'$ , rather than being specified as a material constant. The value of  $p_{ult\_liq}$  is  
110 degraded as pore pressure develops in the free field, eventually reaching a residual value  $p_{res}$  when  $\sigma'=0$   
111 according to Eq. 7, where  $\sigma'_o$  is the initial free-field effective stress.

$$p_{ult\_liq} = p_{res} + (p_{ult} - p_{res}) \frac{\sigma'}{\sigma'_o} \tag{7}$$

112 This formulation is intended to incorporate the influence of ground shaking and liquefaction on p-y  
113 behavior, while retaining some small p-y capacity for the fully-liquefied condition that has been

114 observed in many model studies (e.g., Wilson et al. 2000, Dobry et al. 2003). The ground motion and  
115 mean effective stress are input to the free-ends of the PyLiq1 elements as demonstrated in Fig. 2. These  
116 quantities can be obtained from an effective stress site response analysis, though measured quantities  
117 are also used as inputs in this paper. The site response simulation can be run separately from the  
118 structural analysis, with the recorded outputs written to file and subsequently input to the free-ends of  
119 the p-y elements, or it can be run concurrently with the p-y elements and pile part of the same domain  
120 as the soil mesh. If run concurrently, the out-of-plane thickness of the soil mesh should be made very  
121 large so that an essentially free-field site response condition is achieved (i.e., so that the pile and p-y  
122 elements do not affect the site response). The motivation for utilizing free-field motions is that the p-y  
123 materials are intended to capture all of the soil-structure-interaction effects, and none of it is modeled  
124 by a soil continuum. A three-dimensional continuum with appropriately sized elements near the pile  
125 would be required to properly model SSI effects, and such approaches are computationally very  
126 expensive for dynamic problems with liquefaction.

127 In addition to modeling degradation of the p-y behavior as excess pore pressure develops in the  
128 free-field, the material is also capable of modeling the transient stiffening associated with the cyclic  
129 mobility behavior of sands in cyclic undrained loading. Cyclic mobility behavior is defined as the  
130 transition from incrementally contractive to incrementally dilative behavior that is associated with an  
131 increase in the tangent stiffness and inverted s-shaped stress-strain behavior. Cyclic mobility behavior  
132 significantly influences free-field site response behavior, and this influence is captured as an input to the  
133 PyLiq1 material. However, a limitation of the model is that the dilatancy induced by local strains  
134 imposed on the soil by the pile can only indirectly be incorporated by specifying an appropriate value for  
135  $p_{res}$ . The concave-upward p-y behavior that has been observed in the absence of shaking-induced free-  
136 field dilatancy during blast-induced liquefaction studies (Rollins et al. 2005) and in numerical simulations

137 (e.g., lai 2002) is not captured by the PyLiq1 formulation. Furthermore, the inverted cone-shaped  
138 negative pore pressure region around the pile that was observed by Gonzalez et al. (2009) is not  
139 captured by the PyLiq1 material. Assimaki and Varun (2009) formulated a p-y macroelement that links a  
140 Bouc-Wen type backbone curve with a pore pressure function that combines free-field pore pressure  
141 response with near-field response related to plastic work in the p-y element. This added feature of  
142 material behavior requires specification of additional input parameters for the macro-elements.  
143 Development of multiple independent models is important for quantifying the effects of epistemic  
144 uncertainty.

145 An illustration of the PyLiq1 material behavior in level ground conditions (i.e., without static shear  
146 stress and lateral spreading) is shown in Fig. 3 for a case where a PyLiq1 material with  $p_{res}=0.1p_{ult}$   
147 attaches a soil element to a rigid pile. The soil element is modeled as a PressureDependMultiYield02  
148 material using the default input parameters suggested by Yang et al. (2003) for medium dense sand with  
149  $D_R=50\%$ , and it is subjected to simple shear loading with a cyclic stress ratio of  $CSR=0.3$ . The harmonic  
150 simple shear stress path is applied at a low enough frequency to render essentially zero inertial stresses.  
151 The simulation was performed in OpenSees, with the soil response computed first and the  
152 displacements and mean effective stresses from the soil response subsequently imposed on the free  
153 end of the p-y element in a separate analysis. This approach ensures that the soil behavior is a free-field  
154 input. The excess pore pressure builds up and reaches 1.0 after approximately 6 cycles, and the material  
155 behavior is characterized by transient reductions in pore pressure associated with dilatancy. The dilatant  
156 tendency at large strains causes sharp increases in the shear stress when the shear strain exceeds the  
157 maximum past strain, resulting in the inverted s-shaped stress-strain behavior that characterizes  
158 undrained loading of sands. The p-y behavior in the model mimics the stress-strain behavior of the sand

159 in this case and also exhibits an inverted s-shape that is similar to trends observed in centrifuge tests  
160 (e.g., Wilson et al. 2000).

161 An illustration of the PyLiq1 material behavior in the presence of lateral spreading is shown in Fig. 4  
162 for the same configuration as in Fig. 3, but with a static shear stress imposed on the soil in addition to  
163 the cyclic shear stress. Shear strains accumulate in the direction of static shear stress, resulting in  
164 permanent deformation of the soil element in a manner that is consistent with lateral spreading. The  
165 free-field soil response was input to the same PyLiq1 material as in Fig. 3, , but this time the analysis was  
166 performed for both a rigid pile and for a flexible pile whose stiffness was adjusted so that the peak pile  
167 displacement is equal to  $10y_{50}$ . The rigid pile attracted large loads during each cycle as the soil spreads  
168 past, whereas the flexible pile attracted much smaller loads. The flexible pile deformation is  
169 characterized by alternating episodes in which (1) the down-slope movement of the sand is slowed as  
170 the sand becomes incrementally dilative, the excess pore pressures reduce, and the sand stiffens, (2) the  
171 temporarily stiffened sand exerts greater loads on the pile and displaces it down-slope, (3) the dynamic  
172 shear stress on the sand reverses direction, such that the sand becomes incrementally contractive, the  
173 excess pore pressures increase again, and the sand softens, and (4) the temporarily softened sand exerts  
174 lesser loads on the pile, which allows the pile to rebound in the upslope direction. On the other hand,  
175 the rigid pile does not displace downslope during dilatancy cycles, and a much larger load is transmitted  
176 to the pile due to the cyclic mobility behavior of the soil. This observation is consistent with centrifuge  
177 testing by Haigh (2002) that showed that stiff piles attract much larger lateral spreading loads than  
178 flexible piles.

179 In addition to PySimple1 and PyLiq1 for lateral soil-pile interaction, TzSimple1 and TzLiq1 materials  
180 were formulated for axial shaft friction, and QzSimple1 was formulated for end bearing resistance.  
181 TzSimple1 follows the same logic as PySimple1, except the gapping component is removed, and TzLiq1

182 follows the same logic as PyLiq1, except the residual capacity of the material is set to zero. The  
183 backbone of the TzSimple1 and TzLiq1 materials can be selected to match the relation by Mosher (1984)  
184 for axially loaded piles in sand, or by Reese and O'Neill (1987) for drilled shafts. QzSimple1 exhibits a  
185 direction-dependent response in which a small uplift capacity can be included to model suction stresses  
186 in undrained loading. The backbone of the QzSimple1 material can be selected to match the relation by  
187 Reese and O'Neill (1987) for drilled shafts in clay or Vijayvergia (1977) for piles in sand. Including axial  
188 interaction can be important for pile groups that rotate in response to lateral loading.

### 189 **Description of Centrifuge Models**

190 Simulations incorporating the PyLiq1 material are compared with experimental data from centrifuge  
191 model CSP2 (Wilson et al. 1997) for single piles in nearly level liquefiable sand, and from model SJB03  
192 (Brandenberg et al. 2005) for pile groups in laterally spreading ground. Model sketches are shown in Fig.  
193 5. Properties of the sand utilized in the studies are summarized in Table 1 for CSP2 and Table 2 for  
194 SJB03. Peak friction angles reported in Tables 1 and 2 are estimated based on relative density. Pile  
195 properties are summarized in Table 3. Results are presented in prototype units unless otherwise  
196 specifically noted.

197 Model CSP2 consisted of a 0.67m diameter single extended pile shaft embedded 16.8m into a  
198 horizontally-layered soil profile consisting of liquefiable loose Nevada sand ( $D_R=35\%$ ) over dense Nevada  
199 sand ( $D_R=75\%$ ). Pile groups were also embedded in the model, but only the single pile is studied herein.  
200 The single pile was at least 15 diameters from the nearest pile group. A 49 MN mass was attached to the  
201 top of the pile at a height of 3.81m above the ground surface. The model was saturated with a pore fluid  
202 consisting of water mixed with methylcellulose with a viscosity equal to ten times that of water. Testing

203 was performed at a centrifugal acceleration of 30g. Scaling effects related to pore fluid viscosity are  
204 discussed by Wilson et al. (2000).

205 Model SJB03 consisted of a six-pile group of 1.17m diameter piles embedded into a sloping soil  
206 profile consisting of a nonliquefiable overconsolidated crust of San Francisco bay mud over loose  
207 Nevada sand ( $D_R=35\%$ ) over dense Nevada sand ( $D_R=75\%$ ). A thin layer of Monterey sand was placed on  
208 top of the bay mud to prevent desiccation due to wind currents during spinning. The piles were tied  
209 together by an embedded pile cap with length x width x height of 14.2m x 9.2m x 2.2m and mass of  
210 726Mg. A channel was carved in the downslope end of the model to simulate a common geologic  
211 condition that results in lateral spreading. The model was saturated with water rather than a viscous  
212 pore fluid because some water was squeezed out of the clay into the sand during consolidation on the  
213 hydraulic press prior to shaking and this water could not be replaced by viscous pore fluid during  
214 saturation. Since the viscosity of the pore fluid was not scaled, the prototype hydraulic conductivity of  
215 the sand was 57.2 times higher than for the same water-saturated sand at 1-g. The hydraulic  
216 conductivity is nevertheless low enough to fully liquefy the sand during shaking. Testing was performed  
217 at a centrifugal acceleration of 57.2g.

218 A sequence of ground motions was imposed on each model, and seven of the ground motions  
219 imposed on CSP2 and four of the ground motions imposed on SJB03 are analyzed in this paper. The  
220 analyzed motions were scaled versions of the UCSC/Lick Lab, Ch. 1 - 90° strong motion record from the  
221 1989 Loma Prieta earthquake, and the downhole record (depth = 83m) from Port Island during the 1995  
222 Kobe earthquake. The ground motion profile was recorded at discrete points using vertical arrays of  
223 horizontal accelerometers, and the acceleration records were double-integrated in time to obtain free-  
224 field displacement records. The free-field pore pressure profile was recorded using vertical arrays of  
225 piezometers. Test CSP2 consisted of a level ground profile and the permanent component of the soil

226 displacement was negligible, hence time series of ground displacement could be obtained from double  
227 integration of acceleration records. On the other hand, the low frequency component of lateral  
228 spreading displacement from SJB03 could not be obtained by integration of acceleration records, but  
229 was measured using displacement sensors attached to the nonliquefied crust. The complete ground  
230 motion time series, including low frequency and high frequency components of the crust displacement,  
231 were computed using compatible complementary filters applied to the accelerometer and displacement  
232 sensor records. The low frequency components of the soil displacements below the ground surface were  
233 assumed to be proportional to those at the ground surface, and the final displaced shape of the soil  
234 profile (as determined from post-test profiles of vertical markers embedded in the model) was used to  
235 determine the coefficients of proportionality for those low frequency components. Displacement time  
236 series were then computed by combining the low and high frequency components. Validation of this  
237 procedure and the selection of appropriate filters for the equipment used in these centrifuge tests are  
238 discussed in Kutter and Balakrishnan (1998).

### 239 **Material Properties for p-y, t-z, and Q-z elements**

240 The capacity of the p-y materials,  $p_{ult}$ , was estimated using the API (1993) equations for piles in sand.  
241 A p-multiplier approach was to define the residual capacity as  $p_{res} = m_p p_{ult}$ , where  $m_p$  was defined based  
242 on Brandenburg (2005). The modulus of subgrade reaction in sand deposits is often assumed to vary  
243 linearly with depth, however the elastic modulus for clean sands is known to vary approximately with  
244 the square root of confining stress (e.g., Yamada et al. 2008). Hence, a parabolic relation was used to  
245 define the elastic stiffness with depth. The crust load acting on the embedded pile cap for test SJB03  
246 was estimated to be 6940kN based on the sum of passive earth pressure and side and base friction  
247 summarized by Brandenburg et al. (2005). Pile group effects are considered for the mobilized crust load  
248 because clay may become trapped between the piles, thereby causing the pile group to act as an



249 equivalent block (Brandenberg et al. 2005). However, group effects are not included for the sand layers  
250 based on tests that show group interaction effects to be negligible in liquefied soils (Rollins et al. 2005).  
251 Lateral load transfer between pile groups and nonliquefied crusts was observed to be significantly softer  
252 for crusts spreading over liquefied soil compared with nonliquefied soil profiles (Brandenberg et al.  
253 2007a). The p-y materials in the nonliquefied crust were therefore adjusted so that the ultimate capacity  
254 was mobilized at a relative displacement of 40% of the crust thickness rather than a more typical value  
255 of 1 to 7% of the crust thickness observed in load tests in nonliquefied soil profiles.

256 The single-pile for CSP2 required only p-y elements to model lateral load transfer, but the pile group  
257 in SJB03 required t-z elements to model shaft friction and Q-z elements to model end bearing since the  
258 pile group forms a frame that can rock during lateral loading. The t-z elements were modeled using  
259 TzLiq1 materials with  $t_{ult} = K_o B \sigma_{vo}' \tan(2/3\phi')$ , where  $K_o = 1 - \sin\phi'$ , and  $\sigma_{vo}'$  is the initial vertical effective  
260 stress prior to shaking. Horizontal stresses at the soil-pile interface typically increase when closed-ended  
261 pipe piles are driven into the soil. However, in this case the piles were driven into the models at 1g,  
262 therefore significant changes in horizontal pressure are not anticipated. The bearing capacity at the tip  
263 of the piles was estimated to be 10MN using bearing factors by Meyerhof (1976), and end bearing  
264 resistance was modeled using QzSimple1 materials since there was little excess pore pressures  
265 generated in the end bearing stratum in each case.

## 266 Numerical Modeling Approach

267 Numerical models of the pile were constructed in OpenSees (McKenna and Fenves 2001) using 50  
268 elements along the length of the pile with p-y elements attached at each node below the ground  
269 surface. For CSP2 the piles did not yield during testing, and were therefore modeled as  
270 elasticBeamColumn elements with properties summarized in Table 3. A mass was assigned to the top

271 node. For SJB03, the piles did slightly yield during testing, and were therefore modeled using nonlinear  
272 beam column elements. The piles were tied together at their head by a pile cap composed of very stiff  
273 (essentially rigid) beam column elements. An elastic rotational element attached the pile head to the  
274 pile cap to model the measured rotational stiffness of 1300 MN-m/rad at the connection. Masses were  
275 distributed among the pile cap nodes and PyLiq1 materials were attached to the top and bottom of the  
276 pile cap. For convenience, PyLiq1 materials were utilized for the nonliquefied crust layer as well as in the  
277 liquefiable layers but the recorded mean effective stress time series input to the free-ends of the crust  
278 were essentially constant since the clay did not generate significant excess pore pressure during the  
279 tests. Hence, the PyLiq1 material response was nearly identical to what the PySimple1 material response  
280 would have been. TzLiq1 materials were distributed along the length of the piles and QzSimple1  
281 materials were attached to the pile tips. This configuration permits the pile group to rotate during  
282 lateral loading, which is an important feature of the response of laterally loaded pile groups. The free-  
283 ends of the t-z and Q-z elements were fixed.

284 Time series of displacement and mean effective stress were linearly interpolated from the recorded  
285 data, and imposed on the free-ends of the p-y elements. Recordings of acceleration and pore pressure  
286 were from sensors at least 10 pile diameters from the nearest foundation. In forward predictions,  
287 displacements and pore pressures would need to be estimated from a site response simulation.  
288 However, in this study the measured inputs are utilized to isolate the response of the PyLiq1 materials  
289 so that errors in the p-y elements could be separated from errors in site response simulations. Small-  
290 strain damping of approximately 2% in the frequency range of interest was achieved using Rayleigh  
291 damping. The convergence tolerance on the norm of the displacement residuals was set to  $10^{-6}$  (using  
292 the normDispIncr command), and displacement constraints were enforced using the transformation  
293 method (using the Transformation command). The equation of motion was integrated using the Hilbert-

294 Hughes-Taylor integrator (using the HHT command) with  $\alpha=0.7$ . The time step was adjusted as needed  
295 to facilitate convergence (using the VariableTransient command).

## 296 **Numerical Results for CSP2**

297 Example analysis results for CSP2 are shown in Fig. 6 for a Santa Cruz motion with a peak base  
298 acceleration of 0.42g. The excess pore pressure ratio near the center of the loose sand layer reaches 1.0  
299 approximately 5s after the start of strong shaking, and subsequently exhibits modest dilation-induced  
300 drops in pore pressure. Excess pore pressure ratios are plotted in the free-field, far away from the pile  
301 groups, and also at a location immediately adjacent to the pile. Dilation-induced spikes are apparent in  
302 both records, but are slightly more pronounced in the record near the pile, presumably due to the  
303 additional increment of dilatancy caused by strains imposed on the near field soil by the pile. The peak  
304 bending moment (measured and predicted) is mobilized during one such dilation-induced drop in pore  
305 pressure at approximately time = 26s. The bending moment and superstructure acceleration records are  
306 predicted quite well. The bending moment record was taken at a depth of approximately 3B, where the  
307 peak bending moments were measured. Computed values of subgrade reaction near the center of the  
308 loose sand do not agree with measurements as well as computed values of bending moment and  
309 superstructure acceleration, but nevertheless, the peak responses are predicted well during critical  
310 cycles. Furthermore, the "measured" subgrade reaction values were obtained by double-differentiation  
311 of recorded bending moments, and are prone to more significant measurement error than the  
312 measured bending moment and superstructure acceleration (particularly at high frequencies).

313 The same records from Fig. 6 are plotted in Fig. 7 for a Kobe motion with a peak base acceleration of  
314 0.61g. The frequency content of the Kobe motion is significantly lower than the Santa Cruz motion, and  
315 the dilatancy response in the liquefied sand is much more pronounced as a result. Once again, the peak

316 bending moment and peak superstructure acceleration occurred during a transient drop in excess pore  
317 pressure. The superstructure acceleration reached a peak near 1.5g that was under-predicted by the  
318 analyses by about 0.5g, but the computed values track the measured response quite well other than for  
319 the one cycle that produced the peak value. The bending moment mobilized in the pile is also predicted  
320 reasonably well.

321 Acceleration response spectra were computed for the superstructure motion for three Santa Cruz  
322 motions and four Kobe motions of various intensities. The shapes of the spectra tend to agree quite well  
323 with measurements. However, for the Santa Cruz motions, the computed values tend to be too high for  
324 the high-intensity motions and too low for the low-intensity motions. For the Kobe motions, the  
325 disagreements could not be so simply characterized based on input motion intensity. Better agreement  
326 could be achieved by adjusting the stiffness of the p-y materials on a motion-specific basis by increasing  
327  $K_{ref}$  for the small motions and decreasing  $K_{ref}$  for the large motions. This may partly reflect the effect of  
328 loading history on p-y behavior, which is not included in the analyses. Another factor may be that the  
329 functional form of the API (1993) sand curve is very linear at small values of  $y$ , hence there is very little  
330 small-strain nonlinearity in the PyLiq1 materials. Recent research by Varun (2010) also demonstrated  
331 that the API curve is too linear, and suggested an alternative form that resulted in better agreement  
332 with measurements.

### 333 **Numerical Results for SJB03**

334 Computed results for SJB03 are compared with measurements for the medium intensity Santa Cruz  
335 motion with peak base acceleration of 0.35g in Fig. 9. Some residual loads were present in the pile  
336 groups at the end of each motion for SJB03 due to lateral spreading deformations and the motions were  
337 applied in sequence in the numerical simulations, which explains the non-zero initial values of some

338 quantities in Fig. 9. The excess pore pressure ratio in the loose sand does not reach 1.0 for this motion  
339 even though surface evidence of liquefaction was observed in the form of 0.4m of lateral spreading  
340 ground displacement. This may be attributed to the effect that sustained downslope shear stresses has  
341 on limiting values of excess pore pressure ratio (e.g., Ishihara and Nagase 1980). Computed values are  
342 reasonably consistent with the measurements for bending moment, crust load, pile cap inertia, and pile  
343 cap displacement, though the residual loads on the pile group are larger than predicted.

344 Computed values for the large Kobe motion with peak base acceleration of 0.67g are shown in Fig.  
345 10. This motion fully liquefied the loose sand in the first cycle of strong shaking, and the response is  
346 characterized by very pronounced dilatancy spikes. Each downward spike in the pore pressure record is  
347 associated with a local peak in the crust load, a local minimum bending moment (the largest amplitude  
348 bending moments were negative in this case), and a local maximum in pile cap inertia. The crust load  
349 sometimes exceeded the predicted maximum crust load, and as a result the peak bending moments  
350 were slightly under-predicted in the analysis. Nevertheless, agreement between the measured and  
351 predicted responses is quite reasonable.

352 Response spectra for the pile cap motion are shown in Fig. 11. The shape of the spectra was  
353 reasonably predicted in each case, though the amplitude was not predicted perfectly. Once again, the  
354 small-strain stiffness of the p-y materials could be adjusted to provide a better prediction of pile cap  
355 motion (results not shown for brevity), which may either represent the effects of loading history not  
356 being accounted for in the analyses or indicate a need for a p-y material functional form that more  
357 correctly captures small-strain nonlinearity.

## 358 **Discussion**

359 A key feature of the PyLiq1 material is that it incorporates not only the development of excess pore  
360 pressure during cyclic loading, but also transient reductions in pore pressure caused by dilatancy.  
361 Dilatancy was shown to be an important driver of peak bending moments in the pile foundations from  
362 centrifuge studies by Wilson et al. (2000) and Brandenberg et al. (2005). This paper input the measured  
363 pore pressure response, hence the dilatancy response was included to the extent it was measured in the  
364 free-field during a particular motion. However, the pore pressure response would need to be  
365 numerically simulated in a forward analysis. Advanced plasticity models are capable of capturing the  
366 dilatancy response (e.g., Dafalias and Manzari 2004; Yang et al. 2003, Boulanger 2010) whereas other  
367 models can capture the development of excess pore pressure but not the transient reductions caused by  
368 dilatancy (e.g., Martin and Qiu 2001). To explore the influence of dilatancy on pile response, simulations  
369 for the single pile from CSP2 were repeated with the same displacement records input to the free-ends  
370 of the p-y materials, but with the measured pore pressure response adjusted so that it only increased  
371 (Fig. 12). The motions in Fig. 12 were selected because in both cases the loose sand fully liquefied, but  
372 the extent of the post-liquefaction dilatancy-induced drops in pore pressure were quite different. The  
373 Santa Cruz motion exhibited very small pore pressure drops whereas the Kobe motion exhibited very  
374 pronounced drops.

375 For the Santa Cruz motion, the simulation with the always-increasing pore pressure record is very  
376 similar to the simulation that utilized the measured pore pressure input. On the other hand, significant  
377 differences arise for the Kobe motion, where the always-increasing pore pressure response resulted in a  
378 significantly smaller prediction of peak bending moments and superstructure acceleration (Table 4).  
379 These cases demonstrate that dilatancy can be a significant factor that drives the critical loading cycles  
380 for piles in liquefied ground. Similarly, Kramer et al. (2011) and Ziotopoulou et al. (2011) illustrate that

381 dilatancy is an important factor in obtaining reasonable simulations of the seismic responses of  
382 liquefying soil profiles.

383 The cases studied in this paper involved liquefaction of loose sands. Wilson et al. (2000) studied  
384 liquefaction potential of medium dense sands and found them to be more dilatant and exhibit more  
385 pronounced cyclic mobility behavior compared with the looser sands. Analysis of a medium dense sand  
386 case is presented by Boulanger et al. (2004).

### 387 **Conclusions**

388 Static methods for analyzing piles in liquefied ground are appropriate for many structures for which  
389 dynamic simulations are too complex and costly, and uncertainties inherent to static analysis  
390 approaches can be accommodated by adequate conservatism. However, dynamic simulations may be  
391 warranted for important structures, and may be required for complex structures for which liquefaction-  
392 compatible inertia demands are difficult to quantify without performing a dynamic simulation.  
393 Advancing beyond equivalent static analysis of piles in liquefied ground requires development of robust,  
394 validated numerical tools for performing dynamic simulations. This paper addresses this need by  
395 formulating liquefiable soil-structure interaction elements that utilize free-field site response quantities  
396 as inputs.

397 Comparisons with centrifuge test data show that the materials can reasonably capture key features  
398 of dynamic response when measured displacements and excess pore pressures are utilized as inputs.  
399 Forward predictions would require a site response simulation to obtain ground motion and effective  
400 stress time series to input to the p-y model, which introduces additional uncertainty to the predictions.  
401 The measured inputs were utilized instead of a site response prediction in this study to isolate the  
402 behavior of the p-y materials.

403 Cyclic mobility behavior that is associated with inverted s-shaped stress strain behavior during  
404 undrained loading of sands also causes inverted s-shaped p-y behavior for piles embedded in liquefied  
405 sand. Cyclic mobility behavior was found to be the mechanism responsible for mobilization of the peak  
406 bending moments in the piles presented in this study. Simulations that neglected cyclic mobility  
407 behavior (i.e., by inputting an always-increasing pore pressure response) under-predicted bending  
408 moments and superstructure accelerations compared with measurements, and compared with  
409 simulations that included cyclic mobility behavior. Therefore, neglecting the influence of cyclic mobility  
410 on pile response could result in unconservative predictions and unforeseen damage or failure.

#### 411 **Acknowledgments**

412 Funding for this work was provided by Caltrans and the National Science Foundation through the  
413 Pacific Earthquake Engineering Research Center. The contents of this paper do not necessarily represent  
414 a policy of either funding agency or endorsement by the state or federal government. The authors  
415 would like to thank Christina Curras for doing the initial model development work on the p-y material  
416 models prior to their implementation in OpenSees. Tom Shantz provided valuable technical guidance.  
417 The centrifuge shaker was designed and constructed with support from the National Science Foundation  
418 (NSF), Obayashi Corp., Caltrans, and the University of California. Upgrades were funded by NSF award  
419 CMS-0086566 through the George E. Brown, Jr. Network for Earthquake Engineering Simulation (NEES).

#### 420 **Notation**

421  $B$  = pile diameter

422  $C$  = modeling constant that contributes to shape of p-y backbone curve

423  $C_r$  = modeling constant that controls size of elastic region

424  $C_d$  = modeling constant that controls subgrade reaction load in open gap

425  $CSR$  = cyclic stress ratio

426  $D_R$  = relative density



- 427  $K_o$  = coefficient of at-rest earth pressure
- 428  $K$  = tangent stiffness of p-y element
- 429  $K^e$  = tangent stiffness of elastic component
- 430  $K^p$  = tangent stiffness of plastic component
- 431  $K^g$  = tangent stiffness of gap component
- 432  $n$  = modeling constant that contributes to shape of p-y backbone curve
- 433  $p$  = subgrade reaction due to relative displacement between soil and pile
- 434  $p^d$  = component of subgrade reaction in drag element
- 435  $p^c$  = component of subgrade reaction in closure element
- 436  $p_\alpha$  = subgrade reaction value at center of elastic region
- 437  $p_{res}$  = ultimate resistance of p-y element for fully-liquefied condition (i.e., with  $r_u=1$ )
- 438  $p_{ult}$  = ultimate resistance of p-y element for non-liquefied condition (i.e., with  $r_u=0$ )
- 439  $p_{ult\_liq}$  = ultimate resistance of p-y element corresponding to  $0 < r_u < 1$
- 440  $p_o$  = value of subgrade reaction at start of current plastic loading cycle
- 441  $p_o^d$  = component of subgrade reaction in drag element at start of current plastic loading cycle
- 442  $r_u$  = excess pore pressure ratio
- 443  $t_{ult}$  = ultimate shaft friction load per unit pile length
- 444  $y$  = relative displacement between soil and pile
- 445  $y_{50}$  = relative displacement between soil and pile when half of ultimate load is mobilized in p-y element
- 446  $y^e$  = elastic component of relative displacement between soil and pile
- 447  $y^p$  = plastic component of relative displacement between soil and pile
- 448  $y^g$  = gap component of displacement between soil and pile
- 449  $y_o^g$  = value of gap component of relative displacement between soil and pile at start of current plastic
- 450 loading cycle

451  $y_o^p$  = value of plastic component of relative displacement between soil and pile at start of current plastic  
452 loading cycle

453  $y_o^+$  = gap evolution term equal to maximum past value of  $y^e + 1.5y_{50}$

454  $y_o^-$  = gap evolution term equal to maximum past value of  $y^e - 1.5y_{50}$

455  $\phi'$  = peak friction angle

456  $\sigma'$  = current effective stress in free-field soil

457  $\sigma'_o$  = initial effective stress in free-field soil

458  $\sigma_{vo}'$  = vertical effective stress

459

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- 547
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**Table 1. Soil properties for centrifuge model CSP2.**

Soil Layer	Depth to Top of Layer (m)	$\gamma$ (kN/m <sup>3</sup> )	$D_R$ (%)	$\phi'_{pk}$ (deg)	$V_s$ (m/s) <sup>a</sup>	$K_{ref}$	$m_p^c$
						(kN/m <sup>3</sup> ) <sup>b</sup>	
Loose Nevada Sand	0	19	35	32°	170	12500	0.05
Dense Nevada Sand	9.1	20	75	38°	230	55500	0.3

<sup>a</sup> Shear wave velocity based on measurements from SJB03 for sand with same relative density.

<sup>b</sup> Modulus of subgrade reaction,  $K = K_{ref}(\sigma_v'/50\text{kPa})^{0.5}$ .

<sup>c</sup> p-multipliers based on Brandenberg (2005).

**Table 2. Soil properties for centrifuge model SJB03.**

Soil Layer	Depth to Top of Layer (m)	$\gamma$ (kN/m <sup>3</sup> )	$D_R$ (%)	$\phi'_{pk}$ (deg)	$s_u$ (kPa) <sup>a</sup>	$V_s$ (m/s) <sup>b</sup>	$K_{ref}$	$m_p^d$
							(kN/m <sup>3</sup> ) <sup>c</sup>	
Monterey Sand	0	17	--	36°	--	--	--	--
Bay Mud	1.2	16	--	--	44	160	--	--
Loose Nevada Sand	3.9	19	35	32°	--	170	12500	0.05
Dense Nevada Sand	9.4	20	75	38°	--	230	55500	0.3

<sup>a</sup> Average value for thickness of clay layer measured using T-bar.

<sup>b</sup> Shear wave velocity measured in-flight using mini air hammers.

<sup>c</sup> Modulus of subgrade reaction,  $K = K_{ref}(\sigma_v'/50\text{kPa})^{0.5}$ .

<sup>d</sup> p-multipliers based on Brandenberg (2005).

**Table 3. Pile properties.**

Test	b (m)	E (GPa)	$I$ (m <sup>4</sup> )	$A$ (m <sup>4</sup> )	$M_y$ (kPa) <sup>a</sup>
CSP02	0.67	68.9	$6.06 \times 10^{-3}$	0.135	7522
SJB03	1.17	68.9	$22.0 \times 10^{-3}$	0.166	17050



**Table 4. Peak superstructure acceleration and pile bending moment predicted with and without dilatancy effects compared with measured quantities. Percent error is indicated in parenthesis.**

	Santa Cruz			Kobe		
	Measured	PyLiq1	No Dilatancy	Measured	PyLiq1	No Dilatancy
Superstructure acceleration (g)	0.24	0.35 (+46%)	0.37 (+54%)	1.54	1.25 (-18%)	0.78 (-97%)
Pile bending moment (kN·m)	490	896 (+83%)	971 (+98%)	3464	3189 (-9%)	2144 (-38%)

Figure 1

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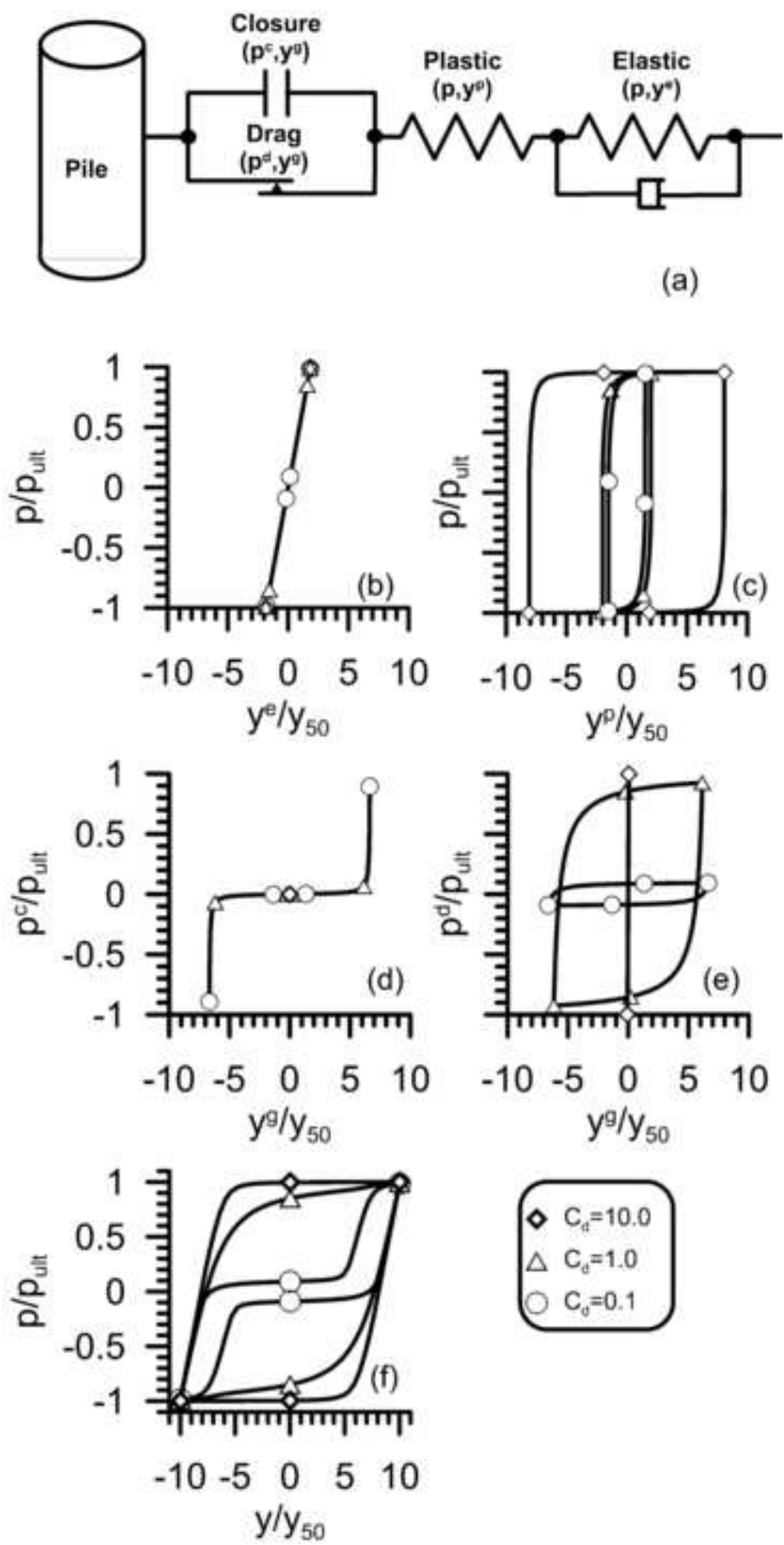


Figure2  
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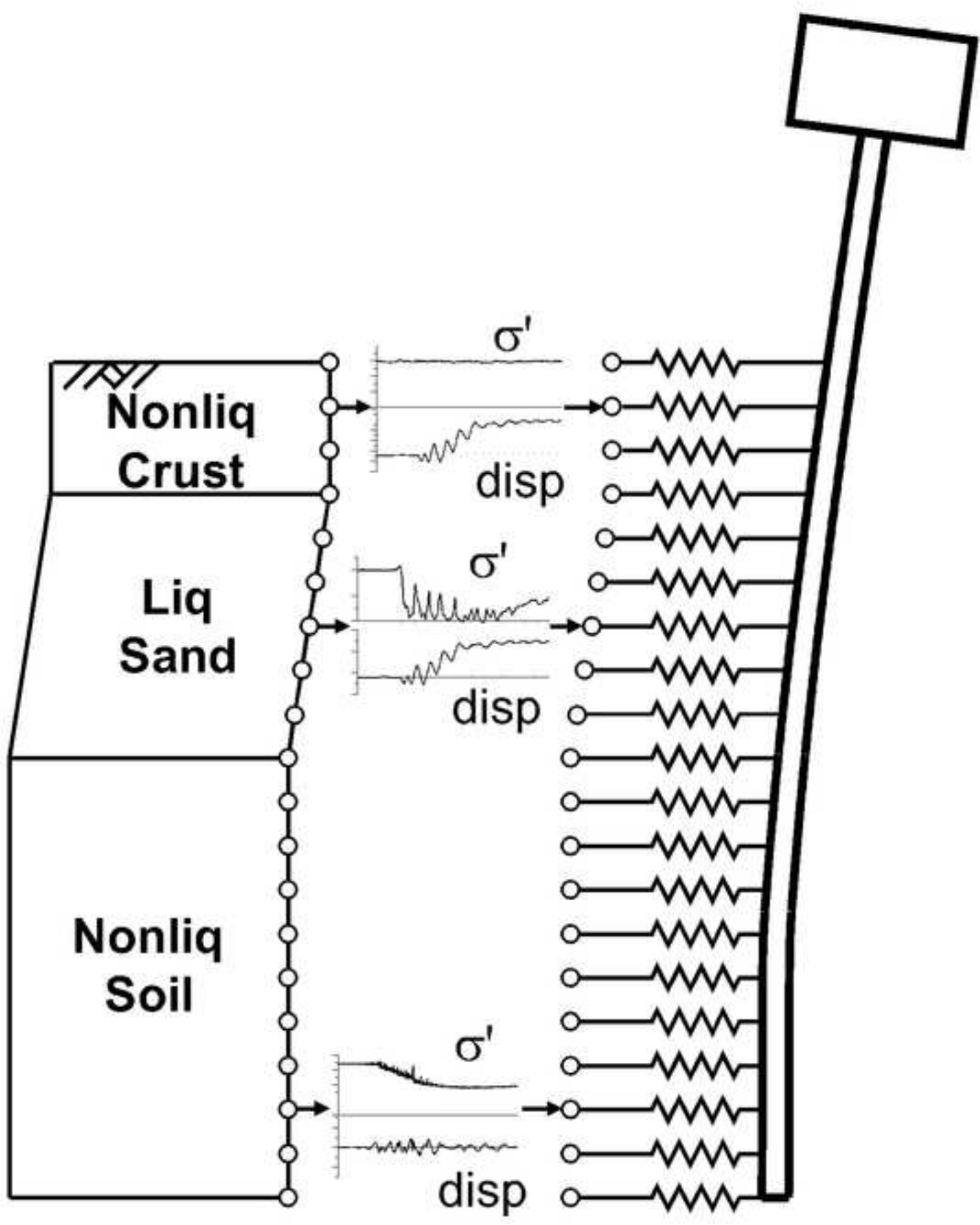


Figure3  
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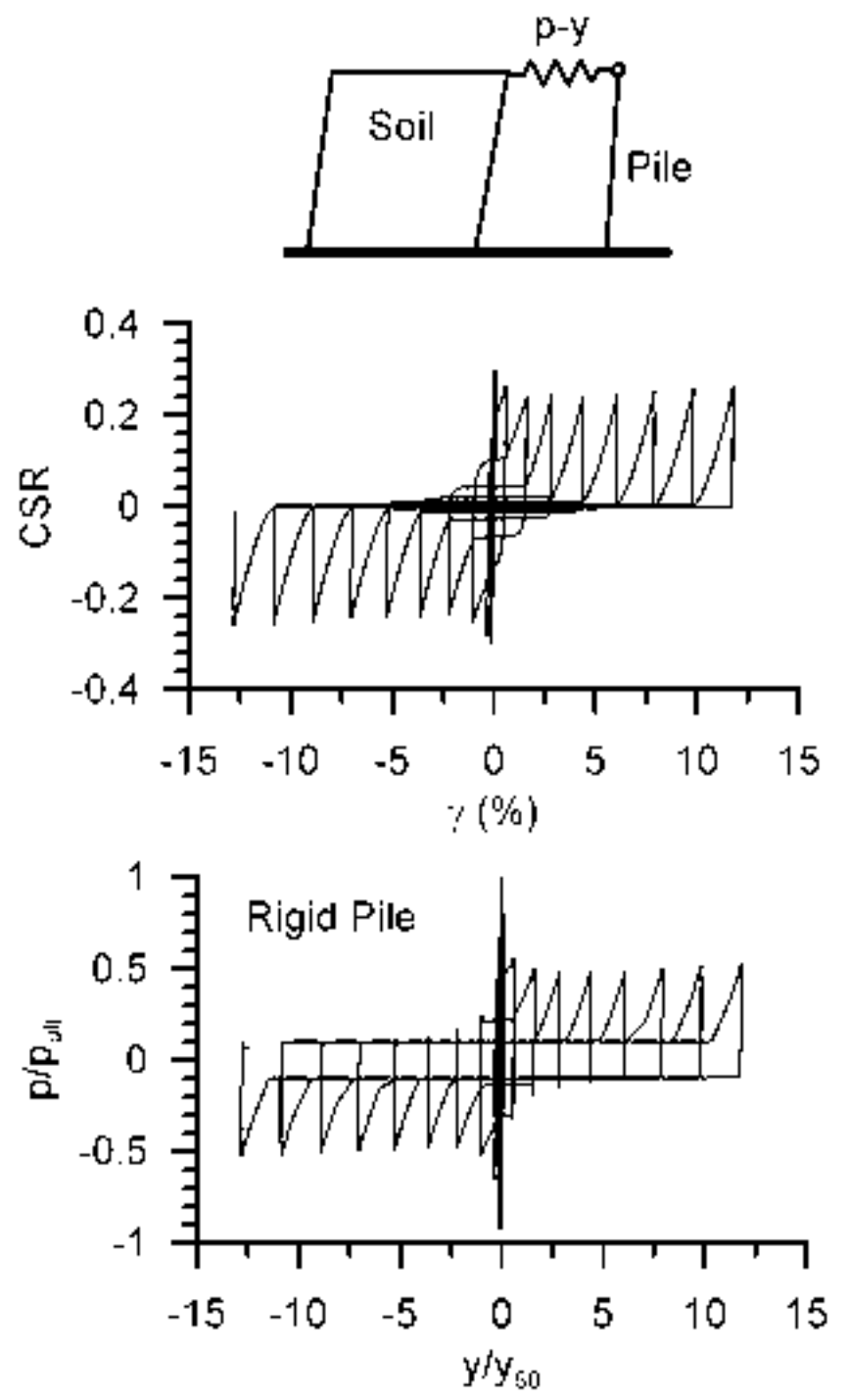
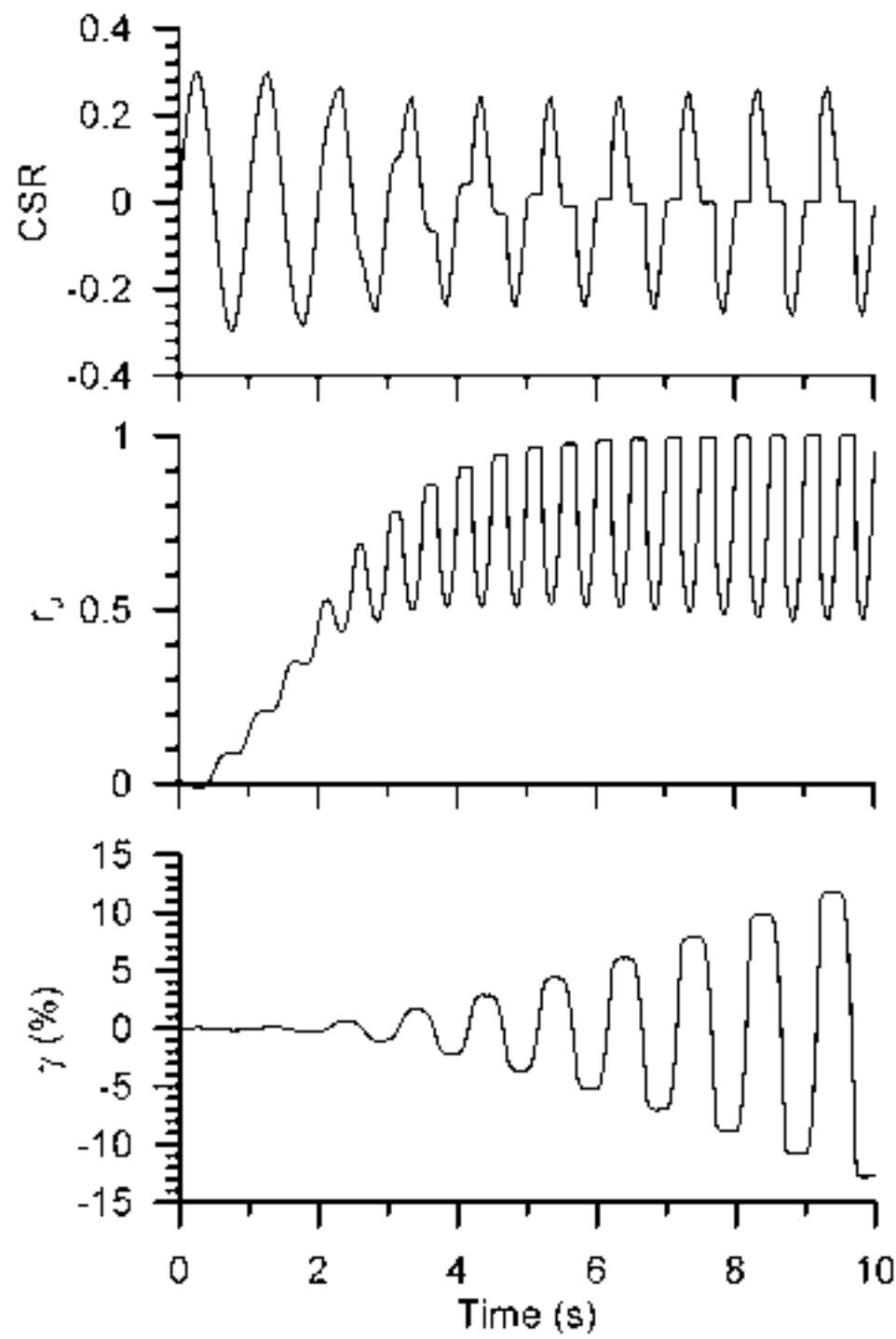


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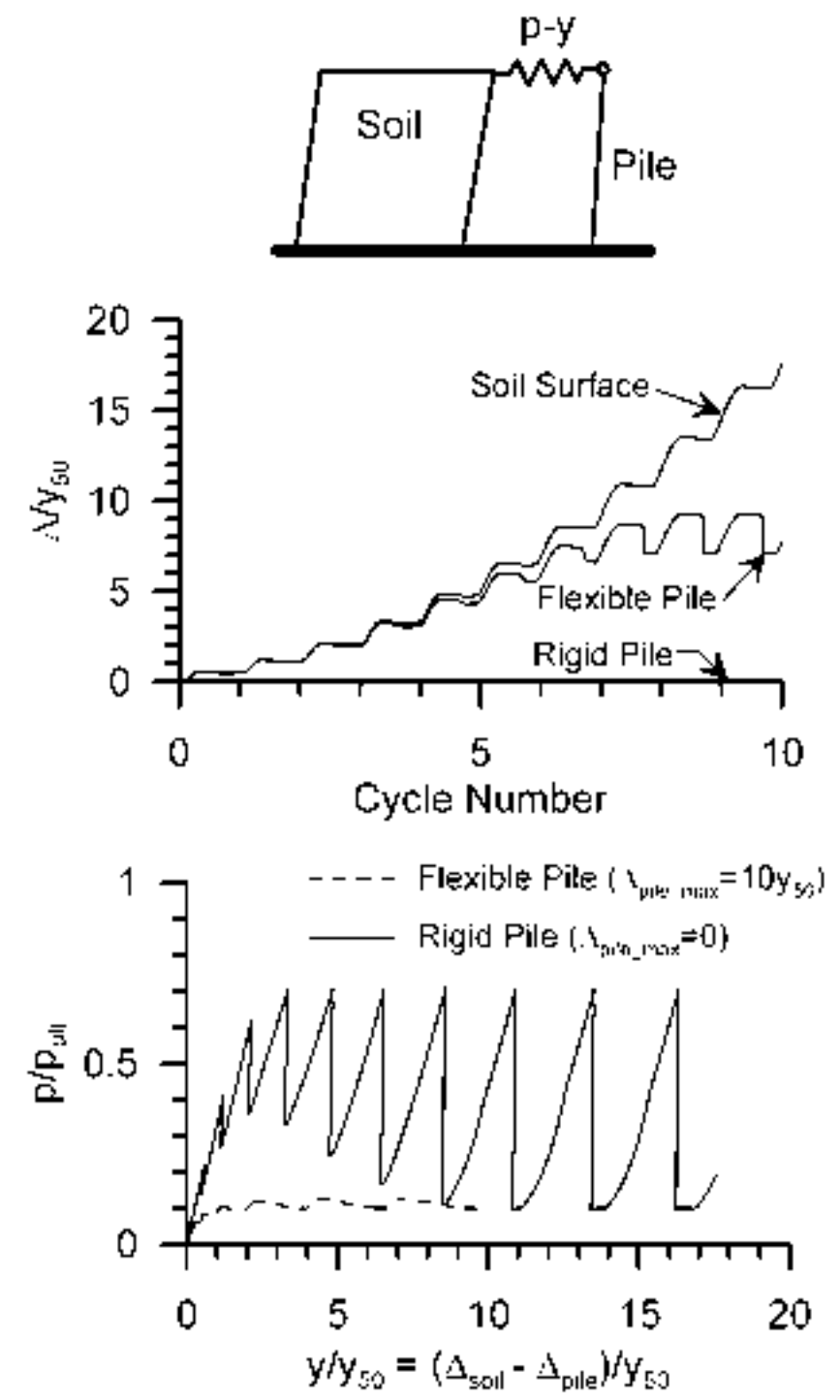
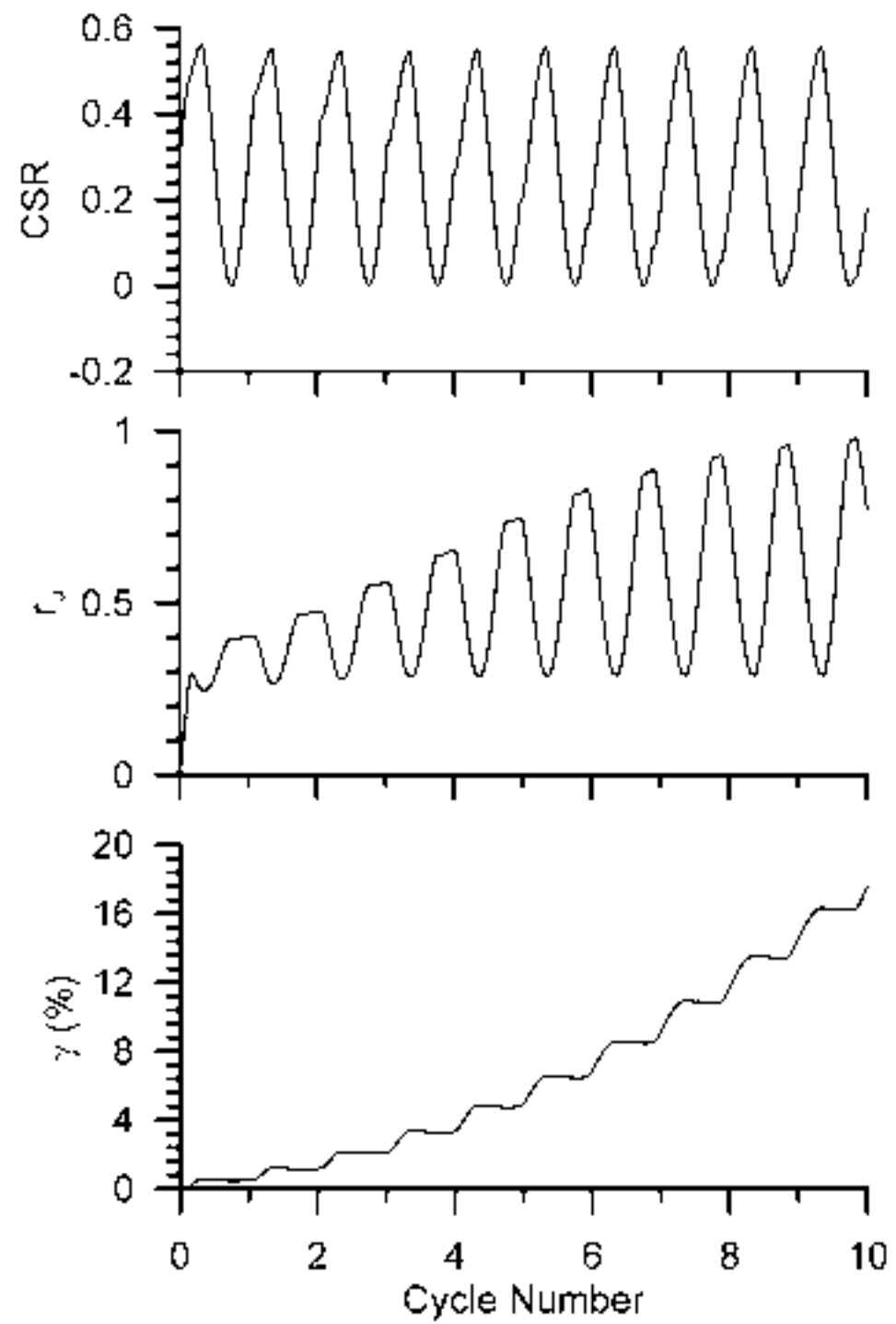
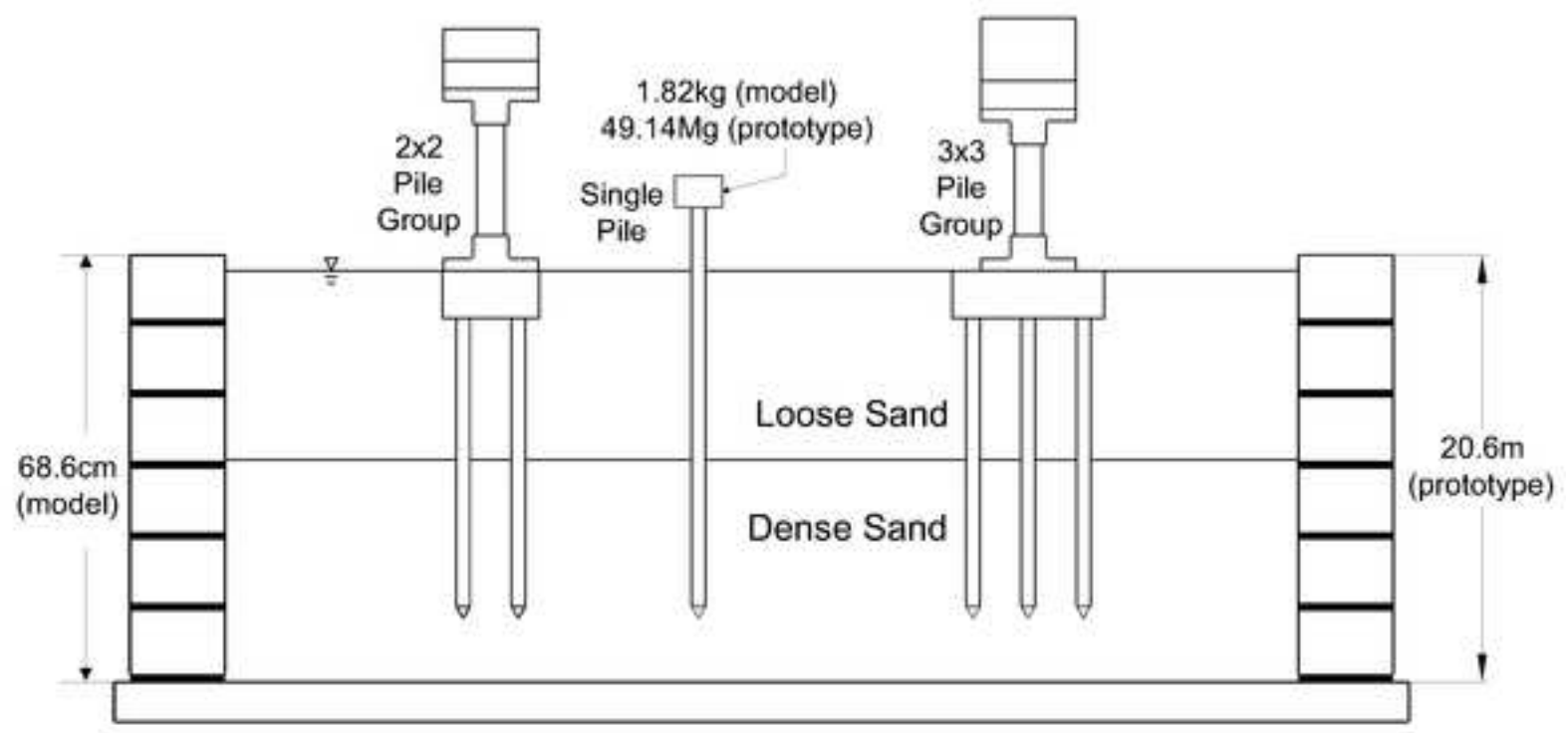
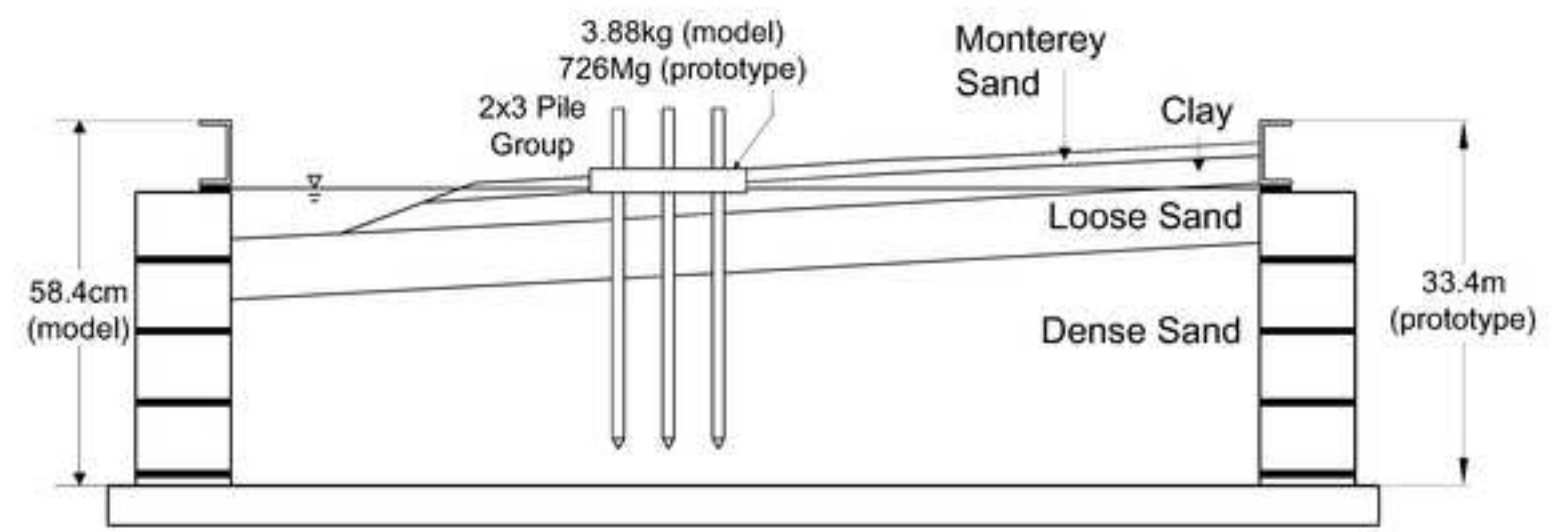


Figure5  
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(a) CSP2



(b) SJB03

Figure6

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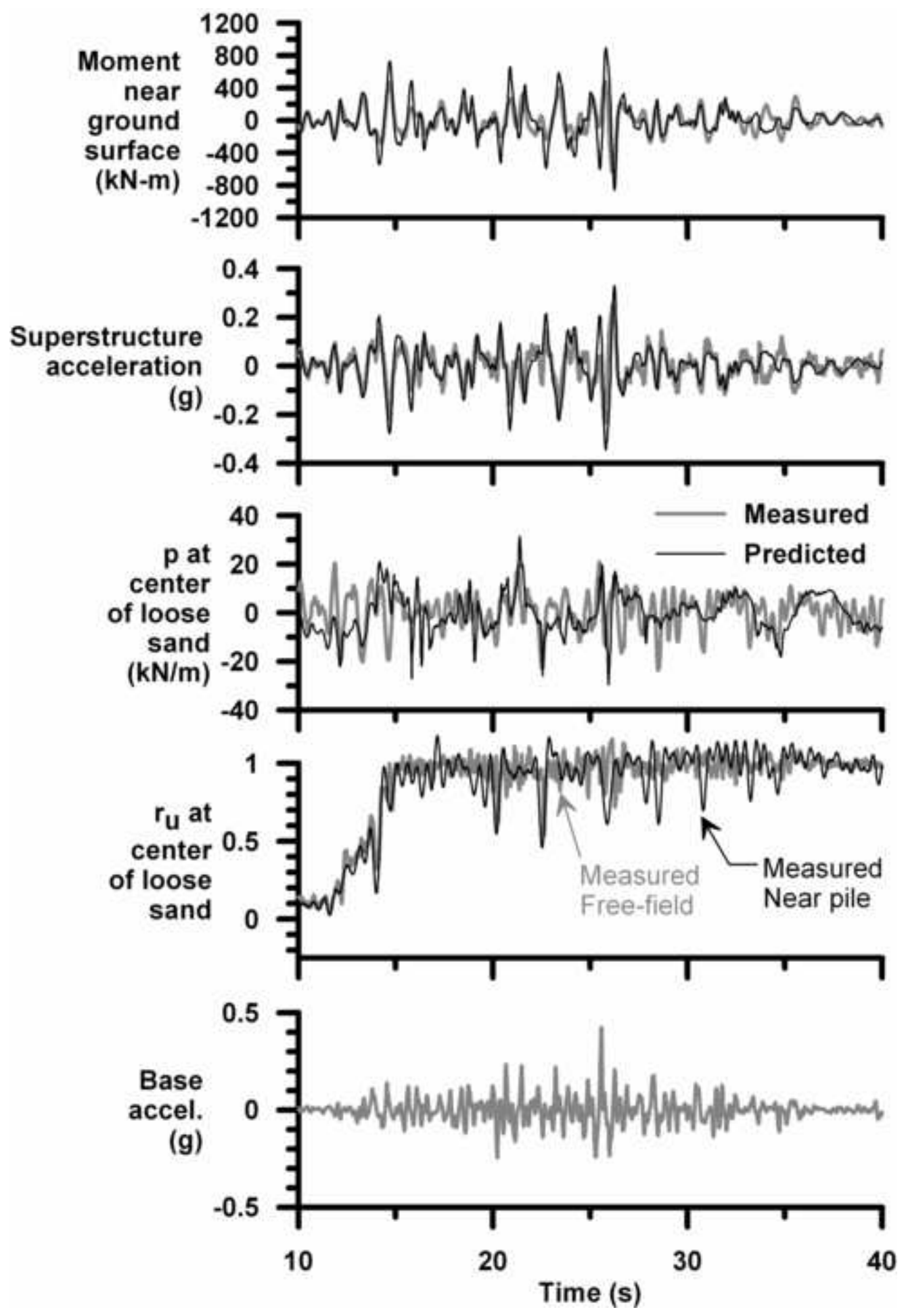


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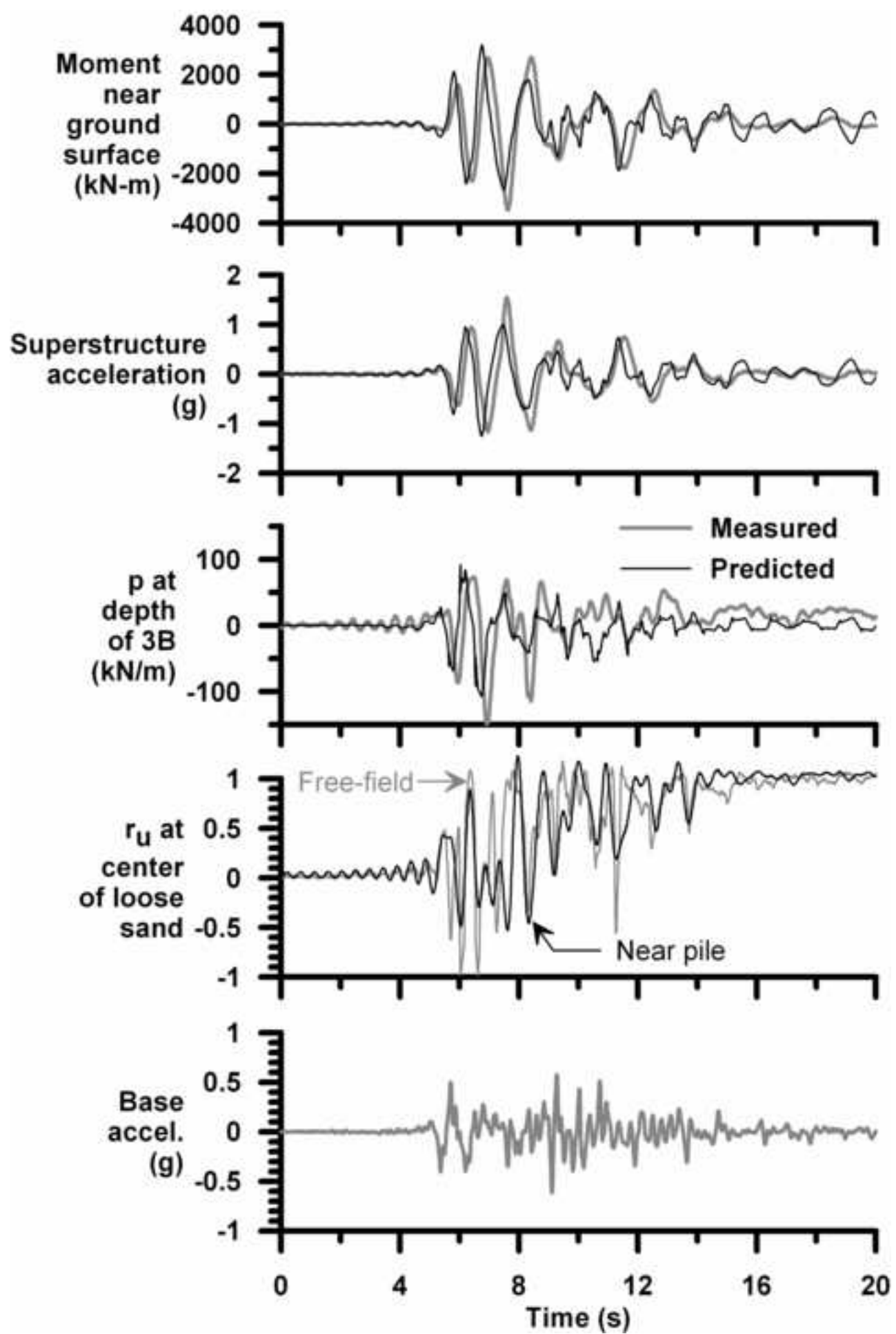




Figure8

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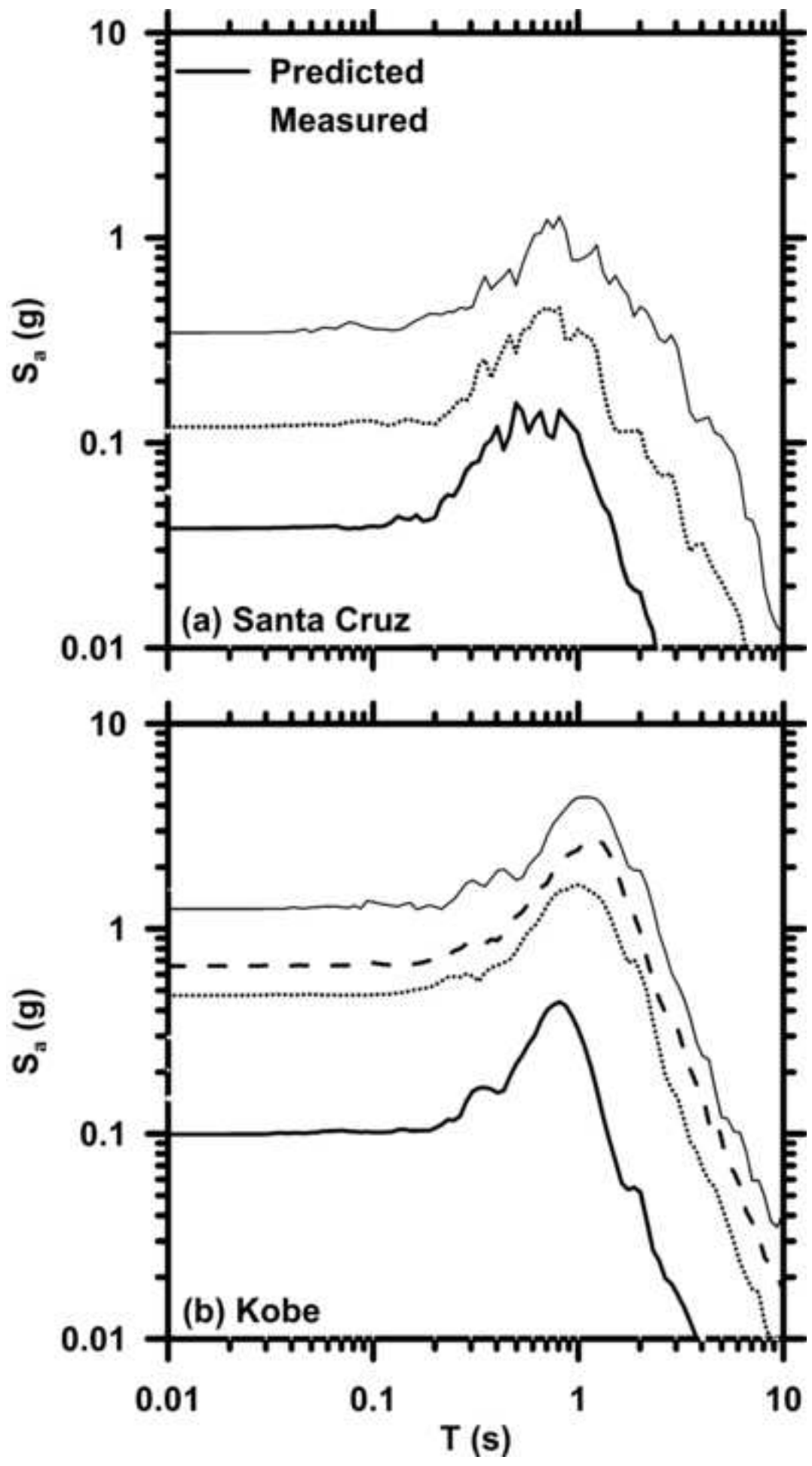


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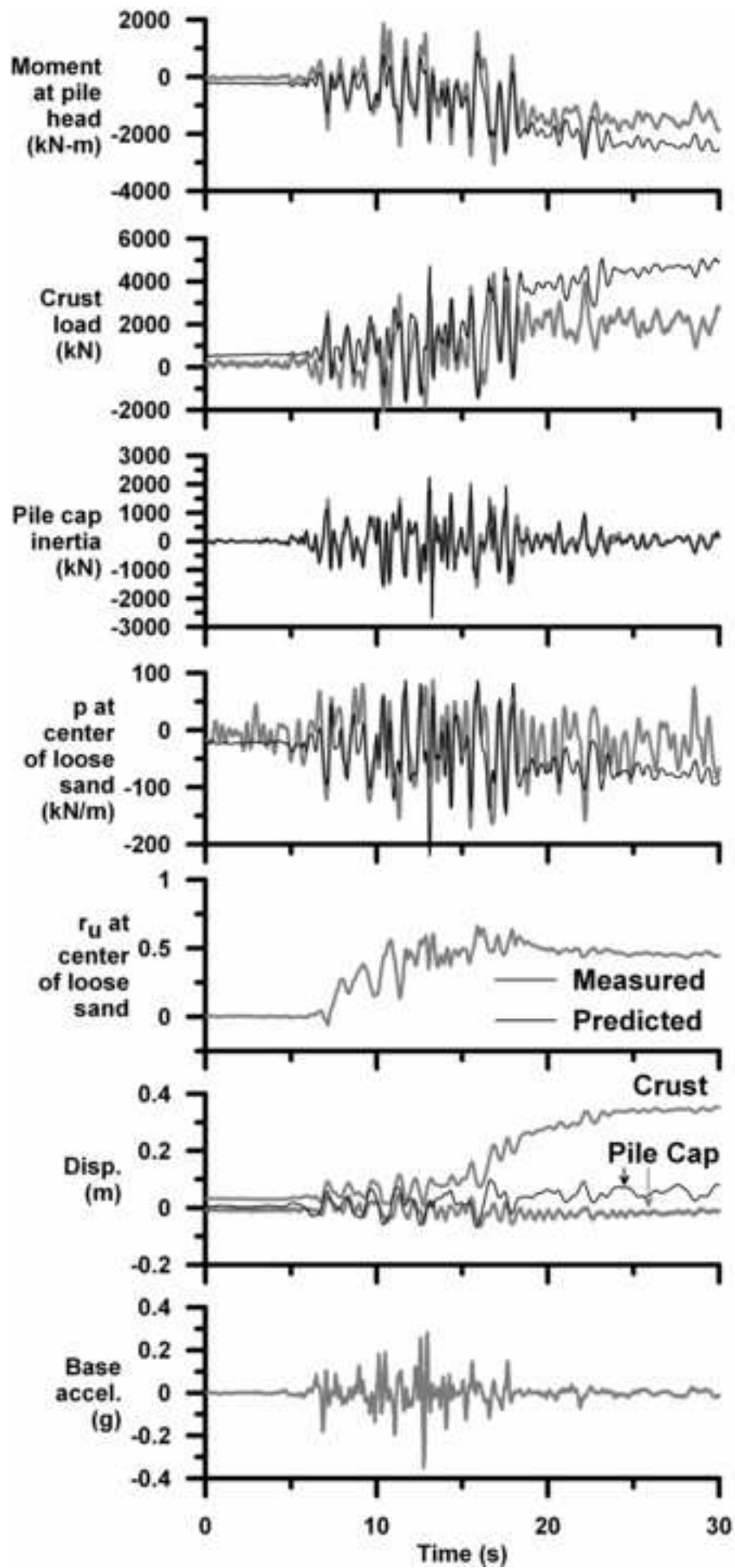


Figure10

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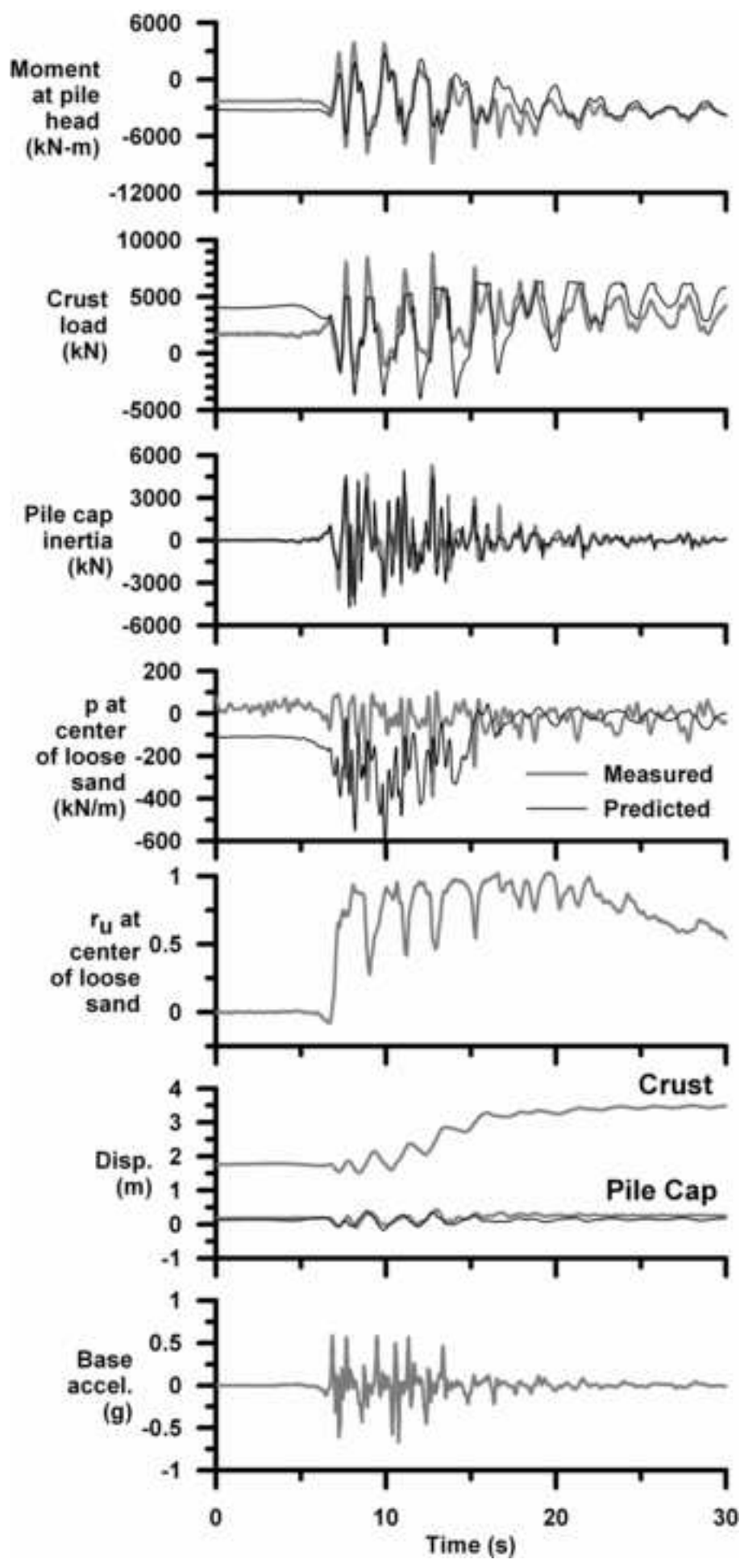


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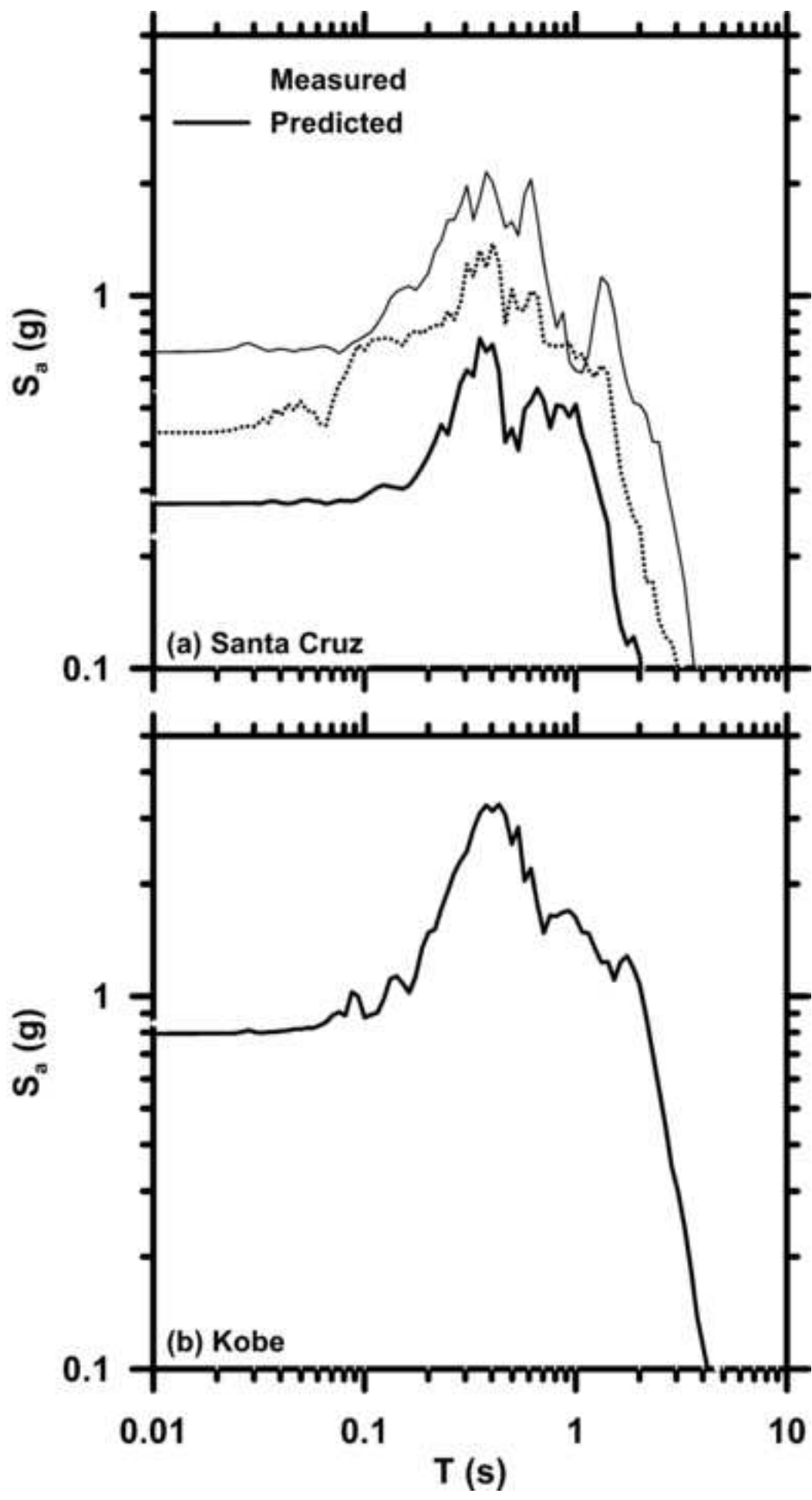
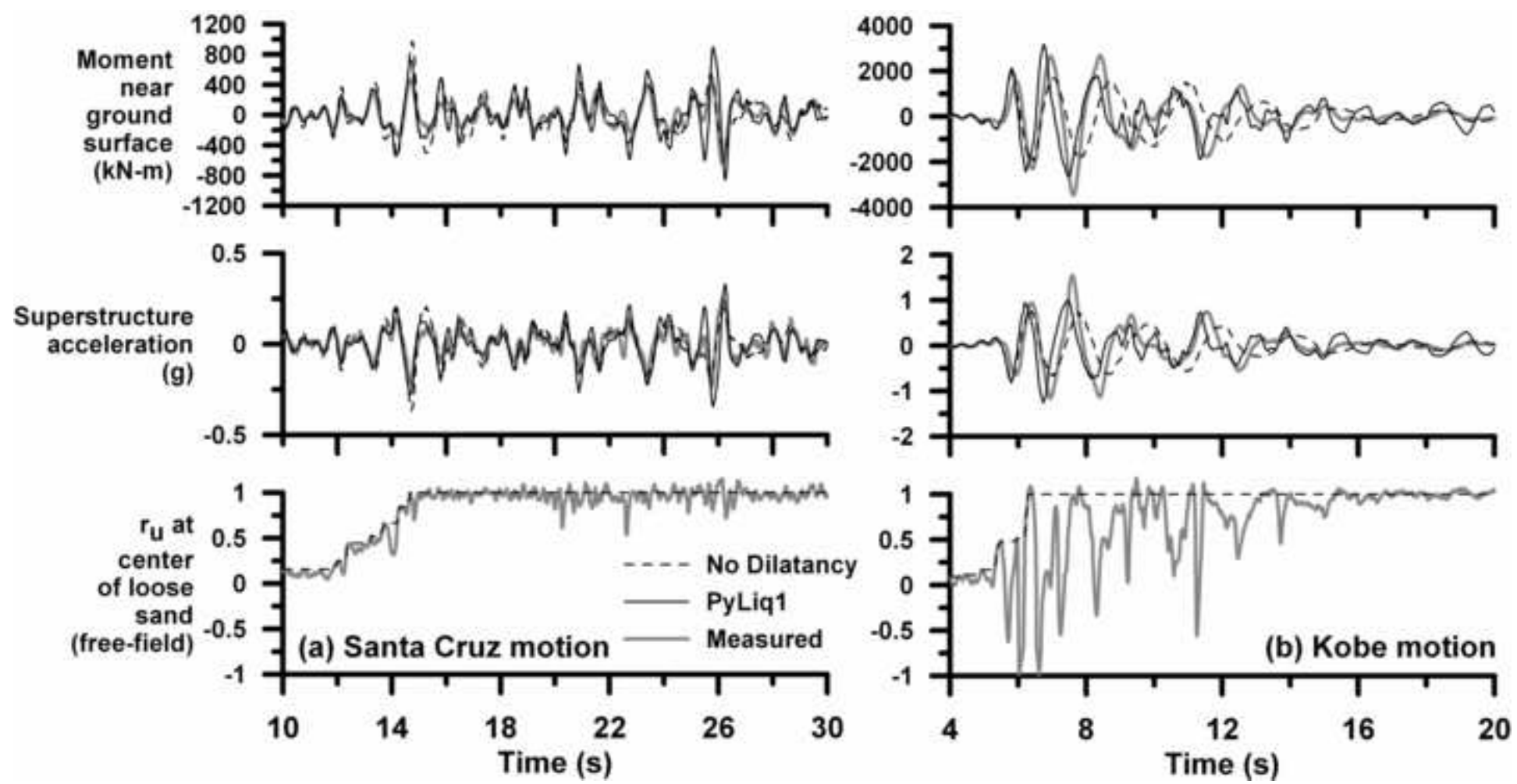


Figure12  
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