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Abstract

Schwinger's result for the rate of pair creation in a uniform external field has been transcribed previously into QCD. We reexamine this problem. Schwinger's implicit neglect of the mutual interaction of the pair and strict energy conservation, while generally of negligible importance in QED, are of major importance in QCD. In the picture of strict color confinement in a tube we derive a new result which is free of these defects.

In a classic paper on quantum electrodynamics, Schwinger derived, among other things, the persistence of the vacuum against e^+e^- creation in a uniform external electric field [1]. It is implicit in his derivation that the mutual interaction of the pair is neglected and that the energy stored in the external field is large compared to $2m_e$. His explicit formula for the rate per unit volume that a e^+e^- pair will be created in the uniform field of strength ε is,

$$p = \frac{(e\varepsilon)^2}{4\pi^3} \sum_{n=1}^{\infty} \frac{1}{n^2} \exp\left(-\pi m^2 n/e\varepsilon\right) \tag{1}$$

where e is the electron charge and m its mass. Recently a number of authors have applied this result to hadronic production in high-energy e^+e^-

annihilation [2,3]. In the annihilation event, a tube of color flux is supposed to be formed connecting a rapidly receding quark and antiquark. In this uniform color field a virtual $q\bar{q}$ pair may tunnel to a real state causing the tube to fission and thus creating mesons. The mass of the created quark appears explicitly in (1), thus giving different production rates for K and π.

However, there are essential differences between the QED and QCD processes of pair creation. In QED the flux lines between opposite charges spread out in space. This causes essential difficulties in accounting for the mutual interaction of a virtual pair immersed in an external field. Nevertheless, it is generally true in QED that the mutual interaction is negligible compared to the force exerted on each member of the pair by the external field. In contrast, the lines of color flux in QCD are believed to be confined to a narrow tube connecting opposite color charges. We adopt this picture quite literally. A virtual pair in such a field will generate a color field of the same strength as that in which they were born. Thus the mutual interaction is not negligible. Indeed, if it were negligible, as has been tacitly assumed in earlier work [2,3], it would be inconsistent to assume that pair creation causes the fission of a color tube. It is precisely because the field is equal and oppositely directed to the original field that the region between the created pair is devoid of color flux and causes fission. Unlike QED, the confinement of color allows us to account precisely for the mutual interaction (in the idealization of a uniform field in the tube). This we now do.

Suppose that a virtual pair appears spontaneously at some point in the flux tube. The quark and antiquark in this pair can have any combination of momenta, spins, flavors, and colors if they carry as a whole the same quantum numbers as the vacuum from which they emerge. This means that the spin, flavor, color charge, and momenta of each component of the pair should be opposite. Let the magnitude of the transverse momentum be denoted by p_T. (We shall use longitudinal to denote the orientation of the tube and transverse to denote an orthogonal direction.) First we calculate the probability that each component will tunnel from the virtual state to a real state having the same energy as the original state. The longitudinal momentum of each component at the point where the virtual pair first appears must satisfy

$$p_L^2 + p_T^2 + m^2 = 0 \qquad (2)$$

or

$$p_L = iE_T \equiv i(p_T^2 + m^2)^{1/2} \qquad (3)$$

As they move apart in the field of the tube their mutual interaction produces a field equal in magnitude but opposite in direction to the field in the tube, thus destroying the field between. After they have each moved a distance r in opposite directions from the point of first appearance, the energy balance reads

$$2\left(p_L^2(r) + p_T^2 + m^2\right)^{1/2} = 2\sigma r \qquad (4)$$

The right side is the energy of the field of the tube that is destroyed when the quark and antiquark each moves a distance r. The string constant, σ, which is the energy per unit length stored in the field is given by

$$\sigma = \frac{1}{2} A\epsilon^2 \qquad (5)$$

where A is the cross-sectional area of the tube and ϵ is the color field strength. Gauss' law gives the relation between the flux and the quark color charge $q/2 = \epsilon A$, so that

$$\sigma = \frac{1}{4} g\epsilon \tag{6}$$

Hence

$$p_L(r) = i\left(E_T^2 - (g\epsilon r/4)^2\right)^{1/2} \quad , \quad E_T = \left(p_T^2 + m^2\right)^{1/2} \tag{7}$$

The action of both quarks integrated from the initial point to the point where they materialize given by $p_L(r) = 0$ is

$$\text{Action} = 2 \int_0^{4E_T/g\epsilon} |p_L| dr = 2 \frac{\pi E_T^2}{g\epsilon} \tag{8}$$

The probability that a virtual pair can tunnel to a real state in the field of the tube, with each component having transverse momentum, p_T, is therefore

$$P(p_T) = \left|e^{-\text{Action}}\right|^2 = \exp\left(-4\pi E_T^2/g\epsilon\right) \tag{9}$$

Our action, and hence the exponent in $P(p_T)$, differs by precisely a factor of 2 from the result of Casher at al. [2]. Their result can be derived by saying that the quark (or antiquark) acquires an energy from the applied field $(g/2)\epsilon r$ when it moves a distance r, but does not feel the field of its partner. One obtains instead

$$P'(p_T) = \exp\left(-2\pi\, E_T^2/g\epsilon\right) \tag{10}$$

Knowing the correct tunneling probability from a virtual to a real state we can now calculate the probability that a pair will actually be created. Following Casher et al., we compute the vacuum persistence probability, which is the probability that no such tunneling event for any spin, flavor, or transverse momentum has occurred at any point r in the tube at any time t during the existence of the tube,

$$\langle 0_+|0_-\rangle^2 = \prod_{\text{flavor}} \prod_{\text{spin}} \prod_{\tilde{p}_T} \prod_{r} \prod_{t} [1 - P(p_T)]$$

$$= \exp\left\{\sum\sum\sum\sum\sum \ell n\, [1 - P(p_T)]\right\} \tag{11}$$

Let $L_xL_yL_zT$ be the space-time region of the tube in which no such event is supposed to have occurred. Let z be the longitudinal direction. Divide it into cells of length equal to that required for the materialization of a pair, which is according to (7),

$$\Delta z = 8\, E_T/g\epsilon \tag{12}$$

The time interval T is divided into cells according to the frequency with which such tunneling attempts can occur in accordance with the uncertainty principle,

$$\Delta t = \frac{2\pi}{\omega} = \frac{2\pi}{2E_T} = \frac{\pi}{E_T} \tag{13}$$

Since $P(p_T)$ is independent of r, t, and spin, we obtain

$$|\langle 0_+|0_-\rangle|^2 = \exp\left\{\gamma \frac{L_z}{\Delta z} \frac{T}{\Delta t} \int \frac{p_T dp_T d\phi}{\left(\frac{2\pi}{L_x}\right)\left(\frac{2\pi}{L_y}\right)} \ell n\, [1 - P(p_T)]\right\}$$

$$= \exp\,(-L_xL_yL_z\, T\, p) \tag{14}$$

where

$$p = -\gamma \frac{g\epsilon}{32\pi^2} \sum_{\text{flavor}} \int_{m_f^2}^{\infty} dE_T^2\, \ell n\, [1 - P(p_T)]$$

$$= \frac{(g\epsilon)^2}{64\pi^3} \sum_{\text{flavor}} \sum_{n=1}^{\infty} \frac{1}{n^2} \exp\left(-4\pi m_f^2 n/g\epsilon\right) \tag{15}$$

In the above $\gamma = 2$ is the spin degeneracy. Taking into account that the quark color charge is g/2 we see that this differs from Schwinger's result simply by replacing the charge g/2 by g/4.

The above p may be interpreted as the rate at which a $q\bar{q}$ pair having opposite color charges g/2 are created per unit volume inside the tube containing the color field of strength ε. Of course, it is independent of position in the tube and of time.

Equation (15) was derived with reference to pair creation in a flux tube having a field strength ε equal to that connecting oppositely charged quark and antiquark. However, if quark configurations are possible that give rise to a larger uniform field strength ε', a review of our derivation readily reveals that (15) remains true, where ε is the field strength connecting the created pair. The only restriction is that the color of the created pair must be the same as the color of the quark-antiquark giving rise to the flux tube, for otherwise the pair cannot absorb energy from the external field.

In summary, we have derived the probability per unit four volume for creation of a quark-antiquark pair in a chromoelectric flux tube. Our result differs from Schwinger's because we are able to take into account the mutual interaction of the pair and strict energy conservation. While neglect of these in QED are justified for macroscopic electric fields, they are not justified in QCD. It is remarkable that within the assumed confinement of the color field it is possible to solve a problem in QCD that has remained intractable in QED. It is of course precisely the confinement of color that allows an easy solution.

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References

[1] J. Schwinger, Phys. Rev. 82 (1951) 664.

E. Brezin and C. Itzykson, Phys. Rev. D2 (1970) 1191.

[2] A. Casher, H. Neuberger, and S. Nussinov, Phys. Rev. D20 (1979) 179.

[3] E.G. Gurvich, Phys. Lett. 87B (1979) 386.

B. Andersson, G. Gustafson, T. Sjostrand, Z. Phys. C6 (1980) 235.

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