

UC San Diego

UC San Diego Electronic Theses and Dissertations

Title

Math is No Longer a Four-Letter Word: A Mixed Methods Study of Two Non-Traditional Fourth-Year Mathematics Classes

Permalink

<https://escholarship.org/uc/item/25d4s7vq>

Author

Heinzman, Erica

Publication Date

2020

Peer reviewed|Thesis/dissertation

UNIVERSITY OF CALIFORNIA SAN DIEGO
CALIFORNIA STATE UNIVERSITY SAN MARCOS

Math is No Longer a Four-Letter Word: A Mixed Methods Study of Two Non-Traditional
Fourth-Year Mathematics Classes

A dissertation submitted in partial satisfaction of the
requirements for the degree Doctor of Education

in

Educational Leadership

by

Erica Heinzman

Committee in charge:

University of California San Diego

Christopher Halter, Chair
Alan Daly
William Zahner

California State University San Marcos

Manuel Vargas

2020

Copyright

Erica Heinzman, 2020

All rights reserved

This Dissertation of Erica Heinzman is approved, and it is acceptable in quality and form for publication on microfilm and electronically:

Chair

University of California San Diego

2020

Dedication

This is dedicated to my students, past, present, and future, who every day make me a better human being. I am humbled and inspired by their tenacity, grace, and humanity.

Table of Contents

Signature Page	iii
Dedication	iv
Table of Contents	v
List of Figures	vii
List of Tables	viii
Acknowledgements	ix
Vita	xi
Abstract of Dissertation	xii
Introduction to the Reader	1
Special Note About the Pandemic	2
Chapter 1: Introduction	5
Organization of the Dissertation	5
Background	5
Theoretical Framework	10
Significance and Purpose of the Study	10
Research Questions	11
Methodology	12
Special Considerations to the Reader	13
Chapter 2: Literature Review	14
Overview	14
The Purpose of High School Mathematics	15
High School Mathematics Course-Taking Patterns and Outcomes	16
Increasing Access to High School Mathematics	19
The Content and Pedagogy of Secondary Mathematics	21
Leading Recommendations for High School Mathematics Pathways	26
Unique Fourth-Year High School Mathematics Courses	29
Theoretical Frame: Self-determination Theory	30
Gaps in the Research Literature	34
Summary	35
Chapter 3: Methodology	38
Research Design	38
Case Study Selection and Context	41
Data Collection	43
Data Analysis	46
Chapter 4: Findings	52
Overview	52

Introduction to the Three Findings	53
DMPC	54
Participants.....	55
Finding #1: Students Perceive Themselves as Connected within a Community of Learners	57
Finding #2: Students Experience Curiosity and Creative Freedom in DMPC	61
Finding #3: The Complex Use of Fun and Easy in DMPC	69
Summary	73
IDS	73
Participants.....	74
Finding #1: Students Perceive Themselves as Connected within a Community of Learners	76
Finding #2: Students Experience Curiosity and Creative Freedom in IDS	81
Finding #3: The Complex Use of Fun and Easy in IDS	90
Summary	94
Connections Across the DMPC and IDS Case Studies	95
Finding #1 Connections Across DMPC and IDS Cases	95
Finding #2 Connections Across DMPC and IDS Cases	96
Finding #3 Connections Across DPMC and IDS	97
Chapter 5: Conclusion	98
Overview.....	98
Overview of the Problem	98
Theoretical Framework.....	99
Methodology	99
Limitations	100
Summary of Findings.....	101
Implications for Policy, Practice, and Research	103
Appendices.....	107
Appendix A.....	107
Appendix B	110
Appendix C	112
Appendix D.....	114
Appendix E	115
References.....	117

List of Figures

Figure 2.1 Continuum of motivation from amotivation to intrinsic motivation	32
Figure 3.1. Two phases of the research design	40
Figure 4.1. Histograms of two relatedness Likert scale items from <i>strongly agree</i> (1) to <i>strongly disagree</i> (5)	60
Figure 4.2. Histograms of two competence Likert scale items from <i>strongly agree</i> (1) to <i>strongly disagree</i> (5)	67
Figure 4.3. Histograms of two autonomy Likert scale items from <i>strongly agree</i> (1) to <i>strongly disagree</i> (5)	68
Figure 4.4 Boxplots of the DMPC composite scores for competence, autonomy and relatedness	69
Figure 4.5 Histograms of two relatedness Likert scale items from <i>strongly agree</i> (1) to <i>strongly disagree</i> (5)	80
Figure 4.6. Histograms of two competence Likert scale items from <i>strongly agree</i> (1) to <i>strongly disagree</i> (5).	88
Figure 4.7. Histograms of two autonomy Likert scale items from <i>strongly agree</i> (1) to <i>strongly disagree</i> (5)	89
Figure 4.8 Boxplots of the IDS composite scores for competence, autonomy and relatedness. ..	90

List of Tables

Table 3.1 Methodology for Research Questions39

Table 3.2 Summary of Data Collection and Analysis49

Table 4.1 Demographics of DMPC Class and Student Focus Group Participants56

Table 4.2 Cumulative Unweighted GPA Range of DMPC Class and Student Focus Group Participants57

Table 4.3 Descriptive Statistics for DMPC Relatedness Composite Score59

Table 4.4 Inferential Statistical Tests Run on DMPC Data61

Table 4.5 Descriptive Statistics for DMPC Composite and Autonomy Composite Scores (N=30).....66

Table 4.6 Spearman’s Correlation of Competence, Autonomy, and Relatedness in DMPC69

Table 4.7 Demographics of IDS Class and Student Focus Group Participants75

Table 4.8 First Semester IDS Grades76

Table 4.9 Cumulative Unweighted GPA Range of IDS Class and Student Focus Group Participants76

Table 4.10 Descriptive Statistics for IDS Relatedness Composite Score79

Table 4.11 Inferential Statistical Tests Run on IDS Data80

Table 4.12 Descriptive Statistics for IDS Composite and Autonomy Composite Scores (n = 19)87

Table 4.13 Spearman’s Correlation of Competence, Autonomy, and Relatedness in IDS89

Acknowledgements

Though this dissertation has only one official author, myself, it is only possible with the expertise, passion, and sacrifice of many people.

My gratitude goes to the participants in this research, whose willingness, insights, and time were essential for creating this dissertation. Thank you to the directors of the Discrete Mathematics Project Collaborative, Dr. Osvaldo Soto, and Introduction to Data Science, Suyen Machado, for connecting me with the two research sites. A special thanks to the two teachers, Ms. North and Ms. Arcega (pseudonyms), who welcomed me into their classrooms, along with Sierra Madre's District Math Specialist. Also, I am grateful to the numerous students who consented to participate and candidly share their experiences in the two courses.

A wise soul once advised the best people are those who see more in you than you see yourself, which is also an apt description of my dissertation committee. They have had an unwavering faith in my work and expanded my thinking in new and productive ways. I value Dr. Manuel Vargas's deep regard for the power of writing and participants' voices, which helped shape my narrative; Dr. Alan Daly's infectious optimism which gave me renewed enthusiasm throughout this journey; and Dr. Bill Zahner's thought-provoking and critical questions, which strengthened my research. I am especially grateful to Dr. Chris Halter, my chair, who has been a mentor since my intern year of teaching and saw limitless potential in me from the very beginning.

I would be remiss if I failed to mention UC San Diego's Writing Hub, in particular the Director, Matt Nelson, and Associate Director, Dr. Erica Bender, who share an admiration for the craft of writing as a generative and creative practice. They nurture a healthy and reflective relationship with writing, which I hope will endure for me far after this dissertation. The

numerous writing retreats I attended, the consultations, and the talented team of graduate writing consultants they lead, all played an essential role in my finished dissertation.

I am deeply grateful for my cohort, who are inspiring leaders, which actively seek to transform lives and education. I could never have imagined better companions for this journey. I am especially grateful to Martha and Cheers, for the carpool rides and for the countless writing sessions. While writing is mostly a solitary activity, it is beneficial to do it in community.

Of course, I am indebted to my family, friends, and colleagues, who would kindly ask about my progress, offer endless support, and celebrate the milestones, both small and large, along the way. Thank you to my spouse, Dave, who has seen me through three degrees; my parents, Michael and Stephanie, who instilled in me an appreciation for knowledge and the wider world long before I could read; and to Pamela, Aly, John, Audrey, Lor, and Mary & Glenn, who gave me renewed hope throughout. Finally, a special thank you to the community of #MTBoS, who have shaped my thinking on mathematics, teaching, equity, and anti-racism.

Vita

1997	Bachelor of Arts, University of San Diego
2003	Master of Education, University of California San Diego
2020	Doctor of Education, University of California San Diego and California State University, San Marcos
2002-2016	Mathematics/AVID/English Teacher, Kearny High School
2012-2017	Noyce Master Teacher Fellow, University of California San Diego
2016-2020	Lecturer and Supervisor of Secondary Education, University of California, San Diego

Field of Study

Major field of study: Education

Abstract of Dissertation

Math is No Longer a Four-Letter Word: A Mixed Methods Study of Two Non-Traditional
Fourth-Year Mathematics Classes

by

Erica Heinzman

Doctor of Education in Educational Leadership

University of California San Diego, 2020

California State University, San Marcos, 2020

Christopher Halter, Chair

Educators and policymakers envision high school mathematics as vital in the pursuit of Science, Technology, Engineering, and Mathematics (STEM) professions and a way to cultivate a deep appreciation of mathematics in society. With two such significant aspirations, there is advocacy for expanding the traditional course offerings in high school mathematics to include courses such as Discrete Mathematics Project Collaborative (DMPC) and Introduction to Data

Science (IDS). Research on non-traditional high school mathematics courses has mostly focused on pathways, content, and pedagogy. This mixed methods case study expands our understandings by examining the perceptions and experiences of students enrolled in the DMPC and IDS course at two separate California high schools. Classroom observations, student focus groups, one-on-one teacher interviews, and a student survey were collected and analyzed using the analytical framework of self-determination theory, which posits competence (self-efficacy), autonomy (agency), and relatedness (a sense of belonging) are essential for positive motivation and meaningful learning. Three significant findings unite the DMPC and IDS case studies. Students in these two courses (a) perceive themselves as connected within a community of learners; (b) experience curiosity and creative freedom, unlike previous mathematics courses they may have completed; (c) use the words *fun* and *easy* in complex ways to describe their experiences. These findings have important implications as interest greatly increases to expand high school mathematics pathways and to implement the DMPC and IDS courses in more high schools statewide and nationally.

Introduction to the Reader

I owe this dissertation in part to Thomas Bayes (1701 - 1761), known for his posthumous contributions to the field of probability. I was vacationing in Hawaii and felt listless in my career as a public health contact tracer, two decades before the pandemic made it a trendy profession. Reading about Cold War submarine espionage while lounging on the sand, I became intrigued by a description of Bayes Theorem of Subjective Probability and Operational Search Theory in finding sunken and missing nuclear weapons and submarines. I glimpsed a new perspective of mathematics far from the mathematics of my childhood and young adulthood. Bayes's theory had nothing to do with speed or repetitive formulaic procedures. Taking weeks to solve, calculations were not abstract and rational but dependent on human intuition and emotion. They were messy, unpredictable, and evolving.

I put the book down and vowed to become a high school mathematics teacher. In a matter of months, I quit my job and enrolled in the University of California, San Diego's credential and master's program. As with any epiphany, there is a mystery to how the dots connected together, though here I am writing a dissertation on mathematics education.

Unfortunately, it would take me many more years before I would begin to translate what captivated me about Bayes's theorem into my teaching of mathematics. As a secondary mathematics teacher, I admittedly perpetuated an inaccurate and stifling view of mathematics for far too long. I have watched students' eyes glaze over after the tedium of rationalizing the denominator and had students slog through factoring dozens of trinomials. I have agonized over 'not covering' standards, reduced rich concepts into formulaic procedures, and assigned mindless worksheets.

Quite literally it was Thomas Bayes and his theorem who helped transform my practice. The opportunity to teach probability and statistics reconnected with me once again with that vision of mathematics as transcendent and reflective of our humanity. I rediscovered what I had appreciated about Bayes's theorem all those years ago on the sand in Hawaii. The discipline was alive with nuance, passion, and relevance. Lived experience and intuition mattered in interpreting data. Differing conjectures and conclusions resulted in lively discussions. I scoured news stories for data sets I could use for tasks and spark engaging conversations.

Thus, it is ironic that in my dissertation research, I have found myself thinking again of Thomas Bayes, his theorem, and the glimpse of mathematics I had on that beach in Hawaii. Researching student experiences and perceptions in the Discrete Math Project Collaborative (DMPC) and Introduction to Data Science (IDS) courses, part of the California Mathematics Readiness Challenge Initiative (California Department of Education [CDE], 2018a), I got to examine an effort that reimagined what high school mathematics could be. As I sat in the DMPC and IDS classrooms and chatted with the students, I once again glimpsed the wondrous, multidimensional, and meaningful nature of mathematics.

Special Note About the Pandemic

Like nearly all research happening in 2020, the pandemic touched this project. Early in my data collection, the specter of COVID-19 felt distant and remote. My experiences in the IDS and DMPC classrooms bookend the rapidly unfolding reality of the coronavirus pandemic from a happenstance far from Southern California to the entire world upended.

When I observed in the IDS course in late January, a student wore a black cloth face mask. At the time, I wondered if the student was genuinely frightened of the virus or simply mimicking the images he saw of the epidemic in other parts of the world. In retrospect, the

student was prescient of the future when we would all don face coverings to venture outside our homes. Kobe Bryant's untimely passing the preceding Sunday heightened the somber mood that day. Southern California was awash in tributes to the Laker's basketball legend from the scrolling "Kobe RIP" on the transit buses to the car dealership marquees memorializing him in purple and gold lettering.

During my first observation in the DMPC classroom, the Sequence & Series module opened with the "Congraduation!" problem (misspelling intentional by the curriculum authors), where students explore a pattern of folding chairs for the graduates in which a set number of seats are added to each row. On that day, no one could have imagined those seniors would be the first graduating class in the school's history to forgo a traditional ceremony. The seniors in the DMPC class would never march out to a football field and sit in folding chairs while they awaited their ceremonial walk across the stage.

As the weeks flew by, more ominous signs of the coming pandemic appeared and a general apprehension settled over school and life. A Costco-sized bottle of hand sanitizer materialized in the classroom where I conducted my DMPC focus groups, which the students and I liberally applied, and the DMPC teacher's lingering cough required reassurance it was just the common cold. In January the IDS teacher had shared with the students her eagerness to run the Los Angeles marathon the first weekend of March, only to cancel the Friday before the race, no longer believing it wise to gather in large crowds. The world had dramatically changed.

I am deeply grateful for the school personnel at both research sites who remained committed to my research, especially as the possibility of school closures loomed on the horizon. Their generous accommodations and flexibility ensured I have the dissertation before you. The IDS teacher and district mathematics specialist made herculean efforts in the final hours before

the school closures. After the school district barred outside visitors to campus, they hastily crafted an alternate plan for students to huddle around a laptop in a cramped office as I conducted the focus group via Zoom. At the end of the interview, the district math specialist handed passes to each of the students and squeezed hand sanitizer into their palms before they departed for what would be their final day on campus.

Though I had the good fortune to have collected my data before the school closures, my research was nevertheless impacted by the pandemic. When the schools transitioned to remote learning in spring 2020, it precluded me from any additional member checking with the students and presenting my initial findings to both classes before the end of the school year to gain insights from the students themselves. It also situates the findings of this dissertation in the world before coronavirus.

Chapter 1: Introduction

Organization of the Dissertation

Compelling narratives often follow a familiar progression - setting the stage, the climax, and the resolution. Following this storytelling framework, my dissertation begins by setting the context with a literature review and introducing the two case studies (Chapters 1-3). The heart of the story is next in my findings (Chapter 4), followed by the denouement or discussion (Chapter 5). The first chapter introduces the reader to the context of high school mathematics in the United States. The second chapter reviews the literature on high school mathematics course-taking patterns and outcomes, the ubiquitous debate over content and pedagogy of K-12 mathematics, and the leading recommendations for designing high school pathways before concluding with research on the self-determination theory. The methodology chapter highlights the plan for a mixed methods collective case study of two non-traditional high school mathematics courses. Chapter 4 reveals the findings and Chapter 5 looks forward to implications and areas of future research.

Background

In the past few decades, economists, policymakers, and educators have advocated simultaneously to grow and to diversify the pathways to Science, Technology, Engineering, and Mathematics (STEM) careers and to establish a more mathematically literate society. Secondary mathematics education serves a crucial role in meeting these aspirations by attracting young adults to STEM-related careers, providing a foundation for studying STEM disciplines in college, and fostering a life-long appreciation and understanding of mathematics (Burdman, 2018; Carnevale & Desrochers, 2003; Committee on STEM, 2018; NCTM, 2018; President's Council of Advisors on Science and Technology [PCAST], 2012).

As a nexus on the pathway between elementary schooling and post-secondary education, researchers have examined secondary mathematics course-taking outcomes, access, content, and pedagogy, which are central to how students experience the discipline. Numerous studies have documented the benefits of enrolling in additional years of high school mathematics including greater odds of persisting in and graduating from college (Adelman, 1999; Byun, Irvin, & Bell, 2015; Gaertner, Kim, DesJardins, & McClarty, 2014), higher earnings (Rose & Betts, 2004), and a greater likelihood of majoring in STEM fields (Wang, 2013). Not surprisingly, organizations such as the National Council of Teachers of Mathematics (NCTM) and the Charles A. Dana Center at the University of Texas, Austin (Dana Center) recommend students take mathematics all four years of high school (Fitzpatrick & Sovde, 2019; NCTM, 2018). Based on the recommendations of a California State University (CSU) system task force to increase mathematics course-taking among freshmen applicants (Academic Senate, 2016), the CSU Trustees approved requiring an additional year of quantitative reasoning to the already required three years of mathematics for freshmen applicants. The proposal broadly defines quantitative reasoning as courses that include mathematics in the curricula and fall under the mathematics, laboratory science, or elective course categories (CSU, 2020a, 2020b; Taylor, 2019).

Unfortunately, even with such desirable outcomes and strong recommendations from equity-minded institutions, far too many students lack access to additional years of high school mathematics, especially Black, Indigenous, Students of Color. U.S. Department of Education Office of Civil Rights (2018) noted significant discrepancies in access to mathematics courses beyond Algebra II, especially in high schools with at least 75 percent Black and Latino enrollment. According to a national survey of public and private high schools, only a third of underrepresented minority students enroll in a mathematics course beyond algebra II (Banilower,

Smith, Malzahn, Plumley, Gordon & Hayes, 2018). These disheartening data highlight structural impediments to diversifying the STEM pipeline.

Offering advanced mathematics to significantly more high school students would be a logical remedy. However, the algebra-for-all movement of the past two decades suggests a cautionary warning about focusing solely on access and failing to consider student experiences within mathematics classrooms. While the movement succeeded in increasing enrollment in algebra nationally, studies show concerning findings including a decline in mathematics achievement on standardized tests (Clotfelter, Ladd, & Vigdor, 2012; Domina, Penner, Penner, & Conley, 2104; Domina, Hanselman, Hwang, & McEachin, 2016; Loveless, 2013). These negative outcomes belie the original intent of the movement to open access to more students. Failing to consider factors such as content, pedagogy, and the learning experiences of students, increasing enrollment in algebra was insufficient to improve achievement in mathematics (Domina, McEachin, Penner, & Penner, 2015).

Although much of the general public views K-12 mathematics as immutable and static (e.g., $2 + 2 = 4$), the discipline has been a lightning rod of controversy for more than a half-century, especially at the high school level. A contentious and polarizing debate has centered on what mathematics is considered rigorous and worthy of study and how mathematics should be taught, which directly impacts how students experience the discipline (Burdman, 2018; Gutiérrez 2012; Schoenfeld, 2002, 2004, 2007).

The content of high school mathematics promotes vastly different experiences for students. Since the late 19th century, high school mathematics has focused on the algebra and geometry pathway in preparation for calculus (Burdman, 2015; NCTM, 2018). Traditionalists contend any change to the orthodoxy of high school mathematics is diluting the rigor so more

diverse learners can succeed in mathematics (Klein, 2007; Schoenfeld, 2004), while reformers question the utility of this pathway for career preparation in the 21st Century and to instill an inherent appreciation of mathematics in students (Burdman, 2015; Carnevale & Desrochers, 2003; Steen, 2001). Leading recommendations for high school mathematics pathways strike a compromise by encouraging both traditional and non-traditional content based on the intrinsic and career interests of students (Daro & Asturias, in press; Fitzgerald & Sovde, 2019; NCTM, 2018).

Traditionally, high school mathematics students experience the discipline as centered on rote and arbitrary procedures, which represents a dissonance from how mathematicians engage in the discipline. Many high school students find mathematics repetitive, limited, and unrelated to life; in contrast, mathematicians view the discipline as creative, vast, and meaningful (Burton, 1998; Boaler, 2015; Boaler & Greeno, 2000; NCTM, 2018). Attempting to bridge the chasm between the contrived mathematics experiences of K-12 students and the creative endeavor experienced by mathematicians, leading frameworks of K-12 mathematics pedagogy, such as NCTM's (2014) *Principles to Action* and Schoenfeld's (2014) *Teaching for Robust Understanding*, position the student as a doer of mathematics; emphasize reasoning; highlight the multi-dimensional nature of mathematics; facilitate the social construction of knowledge; and affirm the essential need for quality tasks. Despite decades of research evidence about the effectiveness of these pedagogical practices, some students continue to experience a narrow focus on procedures instead of a more balanced approach, which includes conceptual understanding and applications alongside procedural fluency (Boaler & Staples, 2008; NCTM, 2014; National Governors Association [NGA], 2010; Schoenfeld, 2002, 2004, 2007).

Emerging policy recommendations envision uniting these strands of research on course-taking patterns, student outcomes, access, content, and pedagogy to improve the high school student experiences in mathematics pathways. Instead of one singular pathway for high school mathematics, high school students would experience mathematics relevant to their interests and future career. The decision point would happen after completing a shared set of mathematics through sophomore or junior year. Theoretically, these pathways would allow for flexible movement if students' pursuits change (Daro & Asturias, in press; Fitzpatrick & Sovde, 2019; NCTM, 2018).

Beginning in 2017, the California Mathematics Readiness Challenge Initiative (CMRCI) funded the implementation of five pilot senior year high school mathematics courses, which are designed for students, who have already completed three years of mathematics and may need to strengthen their quantitative reasoning ability before college (CDE, 2018a, 2019). The CMRCI courses offer the rare opportunity to study the student experience in newly created courses, which incorporate the leading recommendations on high school mathematics pathways. Two of the five courses, Discrete Math Project Collaborative (DMPC) and Introduction to Data Science (IDS), deviate from the calculus pathway and feature non-traditional branches of mathematics (CDE, 2018a). The DMPC curriculum includes units in game theory, cryptography, and graph theory, while the IDS course introduces the students to descriptive and inferential statistics using large data sets and the statistical software R (DMPC, n.d.; Gould et al., 2016; IDS, n.d.). Examining student experiences in these two CMRCI courses will provide a uniquely rich opportunity to study how pedagogical practices and non-traditional branches of mathematics may influence students' perceptions of the discipline and their motivation to engage in mathematics.

Theoretical Framework

Self-determination theory provides a framework for studying student experiences in the DMPC and IDS courses through the lens of motivation. Through years of research, self-determination theory has identified three psychological needs essential for encouraging extrinsic and intrinsic motivation. These three elements - competence, autonomy, and relatedness - provide a useful tool for categorizing and analyzing student experiences in the DMPC and IDS courses. Competence captures a student's confidence and efficacy in mathematics. Autonomy describes the agency and free will the student has when doing mathematics. Relatedness is the sense of belonging or community a student feels in the classroom (Ryan & Deci, 2000a, 2000b).

Significance and Purpose of the Study

In the past few years, much of the literature has focused on the structural elements of redesigning high school mathematics pathways (e.g., sequence, content, and pedagogy) instead of the student experiences in newly created courses for these pathways. In California, the CSU Trustee proposal for a year of quantitative reasoning has renewed interest in high school mathematics pathways, especially multiple options for fourth-year courses (CSU, 2020a, 2020b; Taylor, 2019). Understanding the student experiences in non-traditional fourth-year mathematics courses has the potential to inform and enhance the implementation of these recommended pathways and changes to the CSU admission policy. Without research on the student perspective, it is unknown to what extent these courses achieve their aim of providing more meaningful and relevant high school mathematics experiences.

The DMPC and IDS courses were designed to expand access to rigorous mathematics by students typically marginalized and excluded from enrolling in advanced mathematics. Featuring non-traditional branches of mathematics, such as game theory and data science, both courses

employ student-centered instructional practices that emphasize reasoning, quality tasks, and collaboration (DMPC, n.d.; Gould et al., 2016; IDS, n.d.). Examining the student experiences in these courses will provide valuable insight into their perceptions and motivation to engage in mathematics. To date, the DMPC course has yet to publish research findings and the little published research on the IDS course focuses on instructional practices and content (e.g., Gould et al., 2016; Gould, Bargagliotti, & Johnson, 2017), not on the student experience in the course. Studying the student experiences in DMPC and IDS courses will contribute to the nascent literature on non-traditional fourth-year mathematics courses and the findings, and in turn, may provide valuable insights, especially as the leading recommendations for high school mathematics pathways encourage the proliferation of these types of courses (CSU 2019a, 2019b; Daro & Asturias, in press; Fitzpatrick & Sovde, 2019; NCTM, 2018). The purpose of this study is to examine student perceptions and experiences in the DMPC and IDS courses through the lens of self-determination theory, which identifies the three psychological needs of autonomy, relatedness, and competence (Ryan & Deci, 2000a, 2000b).

Research Questions

The literature on high school mathematics course-taking, outcomes, access, content, pedagogy, and pathways combined with the research on self-determination theory has informed these research questions:

- 1) Within the Discrete Math Project Collaborative or Introduction to Data Science course, how do students experience competence, autonomy, and relatedness?
 - 1a) How might competence, autonomy, and relatedness differ based on student demographics and academics within the same course?

2) What are students' perceptions of and experiences with mathematics in the Discrete Math Project Collaborative or Introduction to Data Science courses?

Methodology

This research study will use a mixed methods collective case study design to examine students' perceptions of mathematics and their experiences in DMPC and IDS fourth-year high school mathematics courses. The mixed methods approach draws upon the strengths of both qualitative and quantitative methodologies to explore student perceptions and experiences in two non-traditional fourth-year mathematics courses (Creswell, 2012; Teddlie & Tashakkori, 2009). Quantitative methods capture responses from a large number of participants, whereas qualitative methods reveal a nuanced and profound perspective of what is happening from a smaller sample of participants (Creswell, 2012; Emerson, Fretz, & Shaw, 2011; Seidman, 2006; Teddlie & Tashakkori, 2009). The purpose of a case study is not generalizability, but to deepen understanding about a bounded system. The bounding refers to the criteria used to select the case and situates the research in a specific context. A collective case study allows for the expansion beyond one case (Creswell, 2012; Stake, 2005). The research study will focus on two cases, one DMPC classroom and one IDS classroom at two separate high schools. The collective case study of the DMPC and IDS courses will shed light on students' perceptions and experiences in mathematics courses purposefully designed as a rigorous alternative to the calculus pathway.

This research study will unfold in two phases in both the DMPC and IDS classrooms. After an initial set of observations in both classrooms, students in the two courses will respond to an electronic survey. The initial analysis of the survey will guide the second phase: semi-structured student focus groups in both courses and semi-structured one-on-one interviews with the DMPC and IDS teachers.

Special Considerations to the Reader

In this dissertation, I made deliberate use of language. I prefer *mathematics* to *math* since the former is suggestive of the many branches which fall under the discipline. After all, this dissertation examines two courses that feature aspects of the discipline rarely ever introduced to K-12 students. The discipline is not singular but composed of a multitude of ways to explore and contextualize patterns, hence mathematics. I want to acknowledge educational research is foremost about humans. Therefore, I have attempted to never lose sight of the human actors at the forefront of this study, including the researcher (myself). When quoting the student and teacher participants in Chapter 4, I only edit the quotes for length and minimally add clarification (e.g., vague pronouns), so their words remain front and center. I also break from tradition and deliberately use the first-person periodically, since it is I who enacted the methodology and interpreted the data, not a nameless researcher. Similarly, I use Black, Indigenous, and People of Color (BIPOC) when discussing ethnic and racially marginalized populations in my commentary and analysis to highlight the ongoing struggle for racial justice (The BIPOC Project, n.d.). When I cite research and data, I use the terminology of the authors and publishers of the data. The student participants in my study classified themselves under broad racial/ethnic categories in the survey, such as Latinx. In my findings, I use the category selected by the student, yet I recognize this may not fully capture how they personally identify racially and ethnically.

Chapter 2: Literature Review

Overview

This literature review explores how the purpose of K-12 mathematics, course-taking patterns, access, content, and pedagogy influence (1) student experiences in high school mathematics and (2) leading recommendations for high school mathematics pathways. Self-determination theory adds a layer of understanding to those student experiences through the framework of three critical elements needed to foster motivation for studying high school mathematics (Ryan & Deci, 2000a, 2000b).

The literature documents the pivotal role of high school mathematics in achieving the goals of growing and diversifying the STEM pipeline and fostering a deep appreciation for mathematics (Burdman, 2018; Committee on STEM, 2018; NCTM, 2018; PCAST, 2012). Unfortunately, far too many students, especially BIPOC, fail to take four years of high school mathematics, a vital milestone in achieving these goals (Asim, Kurlaender, & Reed, 2019; Finkelstein, Fong, Tiffany-Morales, Shields, & Huang, 2012; U.S. Department of Education, 2012, 2018). Opening access to higher-level mathematics would appear an obvious remedy, yet as the algebra-for-all movement demonstrated, access without careful consideration of the experiences of the newly recruited students, may have the opposite results by exacerbating and entrenching inequitable experiences and outcomes (Loveless, 2013; Domina et al., 2016).

Core tensions exist within secondary mathematics pedagogy and content. Despite decades of research on effective pedagogical practices, there is resistance to implementing these practices in secondary mathematics classrooms (Schoenfeld, 2007). While the field of mathematics is vast, the current high school curriculum is predominantly a narrow pathway from algebra to calculus (Burdman, 2015; Carnevale & Desrochers, 2003; Daro & Asturias, 2019).

Critics of innovative approaches often resist changes to instructional practices or mathematics content, arguing such modifications would dilute the rigor, yet empirical research suggests otherwise (Boaler & Greeno, 2000; Boaler & Staples, 2008; Klein, 2007; NCTM 2014, 2018; Schoenfeld, 2002).

The Purpose of High School Mathematics

An irony exists between the imperative to grow and diversify the STEM pipeline and the reality that falls drastically short, categorized by an ever-increasing demand for STEM professionals, and a persistent lack of diversity. Economic and labor forecasts show STEM job creation is outpacing non-STEM job growth (Noonan, 2017) and suggest an additional one million more STEM professionals are needed than are currently projected to enter the job market by 2024 (PCAST, 2012). Of concern, is not only educating more individuals to fill STEM positions but also ensuring the STEM workforce is representative of this country's diversity (Committee on STEM, 2018). According to recent figures from the National Science Foundation (NSF, 2019), underrepresented minorities earned 21.5 percent of the bachelor's degrees and 8.8 percent of the doctoral degrees awarded in science and engineering fields in 2016. While women earn nearly half the bachelor's degrees in science and engineering fields, a pernicious and wide gap continues in physical sciences, computer science, and engineering, with women earning less than a quarter of the doctorates in these three fields (NSF, 2019). To address the twin aspirations of growing and diversifying STEM professions, policymakers and researchers focus on high school mathematics, as a vital foundation for STEM careers, which makes it crucial to examine the experiences students have traveling the pipeline.

There is an additional purpose of secondary mathematics. Beyond preparing future STEM professions, some scholars envision high school mathematics as necessary for an

informed democratic society, regardless of career pursuits. NCTM's (2018) bold definition embraces a threefold purpose: "High school mathematics empowers students to - (1) expand professional opportunity; (2) understand and critique the world; and (3) experience wonder, joy, and beauty" (p. 9). Policymakers, industry, and schools tend to focus on the first reason, especially when promoting mathematics as integral to a STEM pathway (Burdman, 2015; Raymond, 2018). The latter two reasons suggest a more transcendent view of mathematics with personal significance. The verb *empowers* conceives of the student as the doer of mathematics rather than a passive learner. This expanded view of mathematics has crucial implications in terms of content and pedagogy in high school mathematics courses, by emphasizing the learning experiences of students.

High School Mathematics Course-Taking Patterns and Outcomes

Within high school mathematics, there are positive outcomes of taking additional years of high school mathematics, as well as enduring inequities, which act as a barrier to higher-level mathematics, especially for Black, Indigenous, and Students of Color. These persistent inequities in course-taking have influenced the design of the leading recommendations for high school mathematics pathways (Daro & Asturias, 2019; Fitzpatrick & Sovde, 2019; NCTM, 2018). Using national longitudinal cohort samples, researchers have analyzed the role of high school mathematics in predicting academic outcomes and majors in college (Byun et al., 2015; Gaertner et al., 2014; Wang, 2013); the odds of earning a college degree (Adelman, 1999, 2006; Trusty & Niles, 2003); and future career earnings (Levine & Zimmerman, 1995; Rose & Betts, 2004).

Empirical studies imply a positive correlation between taking at least three years of high school mathematics (equivalent to Algebra II or beyond) and a variety of post-secondary outcomes. For example, taking additional high school mathematics increases the likelihood of

attending college (Byun et al., 2015) and has influenced students to major in STEM fields (Levine & Zimmerman, 1995; Wang, 2013). Each semester of high school mathematics increases the likelihood of female students majoring in a STEM discipline (Levine & Zimmerman, 1995). Using two separate cohorts a decade apart, researchers demonstrated taking Algebra II is associated with higher college GPAs and the likelihood of graduating college (Gaertner et al., 2014). Similarly, other studies found the most important predictive factor in college degree attainment is the highest level of high school mathematics taken. In fact, completing at least four years of high school mathematics more than doubles the odds a student will graduate from college (Adelman, 1999, 2006). Narrowing the cohort to high school students whose goal was to earn a college degree expanded on these findings; students who took at least three years of high school mathematics more than double their odds of completing a college degree (Trusty & Niles, 2003). What is uncertain is if a causal relationship exists between taking additional years of high school mathematics and these positive post-secondary outcomes (Asim et al., 2019). Presumably, if students take three or more years of high school mathematics, they have other factors in their life that may be influencing their later post-secondary success (Byun et al., 2015). The question remains if more students enrolled in additional years of high school mathematics would they experience the same post-secondary benefits.

Economic scholars have shown enrolling in additional years of high school mathematics is correlated with higher incomes (Levine & Zimmerman, 1995; Rose & Betts, 2004). A decade after high school, a quarter of the wage gap between low-income and middle-income participants can be explained by the number of high school mathematics courses taken (Rose & Betts, 2004). These seminal studies suggest a powerful connection between high school mathematics course-taking and future earnings, though new studies should explore if these economic benefits persist

with more recent cohorts of students. Participants in these often-cited studies attended high school during the last century (Levine & Zimmerman, 1995; Rose & Betts, 2004).

While empirical studies show the positive effect of taking additional years of high school mathematics, there is an opportunity gap with many high school students never enrolling in Algebra II or beyond, which requires the construction of equitable mathematics pathways. Nationwide, only 38 percent of high schools, which enroll at least three-quarters of Black or African American and Hispanic or Latino students, offer calculus a course long considered a gateway to STEM majors in college (U.S. Department of Education, 2018) and females are underrepresented in AP mathematics courses (U.S. Department of Education, 2012).

In California, about a quarter of high school seniors fail to enroll in a mathematics class their last year of high school (Asim et al., 2019; Finkelstein et al., 2012) and approximately one-third of the high-performing high school mathematics students, who took Algebra I or equivalent in middle school, never fulfill the a-g requirements for admission to the California public universities by completing at least three years of high school mathematics (Gao & Johnson, 2017). Analysis of the California public school senior year course-taking highlights the opportunity gap in secondary mathematics pathways based on race. Thirty-four percent of African American seniors and 39 percent of Latinx seniors in 2017-2018 enrolled in a mathematics course beyond Algebra II, falling short of the overall rate of 46 percent (Asim et al., 2019). Seniors who failed to take mathematics or science in twelfth grade are less likely to persist in college (Gao & Johnson, 2017), which complement previous findings on the positive association between high school mathematics course-taking and college completion (Adelman, 1999, 2006; Gaertner et al., 2014).

The limited access to higher-level mathematics classes in California may be partially a result of the state's paltry high school graduation requirements. The state only requires two years of high school mathematics to earn a high school diploma. Many large districts have aligned their graduation requirements with a-g requirements, -- the minimum high school coursework required to apply for freshman admission to the California public university systems -- which require three years of high school mathematics through Algebra II or equivalent, but highly recommend four years (Asim et al., 2019; Burdman, 2018; CDE, 2018b; University of California, n.d.). Wanting to improve four-year college graduation rates, the CSU Trustees voted on a proposal in January 2020 to require a year of quantitative reasoning in addition to three years of high school mathematics, which would be phased in over the next seven years (CSU, 2020a, 2020b). The CSU Trustees refer to the additional year as quantitative reasoning, since a wide variety of courses may meet the requirement such as a laboratory science, computer science course, or a Career and Technical Education class (CSU, 2020a, 2020b). In addition, there are practical roadblocks denying access to additional years of high school mathematics courses. If students fail lower-level high school mathematics courses, enrolling in Algebra II or beyond may be precluded since mathematics courses are often sequential (Finkelstein et al., 2012)

Increasing Access to High School Mathematics

A tension festers between wanting to rapidly enroll more high school students in additional years of mathematics, thereby potentially expanding the positive outcomes to a new group of students, and taking a more measured and systematic approach to expanding access. The results of the earlier algebra-for-all movement portend following a methodical approach to redesigning pathways. Like the CSU Trustee proposal to require a year of quantitative reasoning, which seeks to have freshmen enter college better prepared, the algebra-for-all movement was

born of noble intent (CSU, 2020a, 2020b; Domina et al., 2014; Loveless, 2013). Beginning in the 1990s, many states experimented with algebra-for-all initiatives, incentivizing and in some cases mandating, all students to be enrolled in algebra courses in eighth grade. The rationale behind such efforts focused on the entry-level high school course as a gateway to further mathematics learning by leading to a key destination on the STEM pipeline: calculus. By successfully completing algebra in freshman year of high school or earlier, students could finish at least four years of high school mathematics (Burdman, 2015; Domina et al., 2014; Loveless, 2013). Regrettably, the outcome of the movement fell far short of the lofty aspirations.

While the percentage of eighth-graders enrolling in algebra increased threefold nationwide, a decline in mathematics achievement was observed on the National Assessment for Educational Progress exam (Loveless, 2013). Regional case studies show a similar inverse relationship between increased enrollment in algebra and subsequent decline in student achievement (Allensworth, Nomi, Montgomery, & Lee, 2009; Clotfelter et al., 2012; Domina et al., 2014; Domina et al., 2015; Domina et al., 2016). Leveraging a high-stakes accountability system, California increased the percentage of eighth-graders enrolled in algebra from 32 to 84 percent in three academic years (Domina et al., 2014). Researchers found eighth-grade enrollment in Algebra I increased, yet mathematics achievement decreased as measured by the California High School Exit Exam in multiple school districts (Domina et al., 2014; Domina et al., 2015). In an ironic twist, California's effort to improve access to challenging mathematics resulted in increased tracking at well-resourced middle schools with the creation of additional advanced eighth-grade mathematics pathways isolating students who struggled in algebra (Domina et al., 2016).

Examining North Carolina's ten largest school districts, researchers found similar negative findings from the state's push to require eighth-graders to enroll in algebra: students in the lower quintile of mathematics achievement were less likely to enroll and pass geometry by junior year of high school when they took algebra prior to high school (Clotfelter et al., 2012). With similar ambitions in mind to open access to high-level mathematics courses, Chicago public schools mandated high school freshmen to enroll in algebra or a higher mathematics course in the late 1990s. During the initiative, more Chicago high students earned algebra credits. However, the district test scores and the likelihood of district graduates entering college showed no significant increase while the academic failure rate increased (Allensworth et al, 2009).

The algebra-for-all policies viewed access as the catalyst to higher mathematics achievement. These mandates neglected to address the learning environment and student experiences in these courses despite the fact the composition of the students in the Algebra I course dramatically changed. Resisting a deficit interpretation, scholars attribute the concerning results to the myopic approach which favored access at the expense of other essential factors, such as content and pedagogy (Allensworth et al., 2009; Clotfelter et al., 2012; Domina et al., 2014; Domina et al., 2015; Domina et al., 2016).

The Content and Pedagogy of Secondary Mathematics

Tension over the *what* and *how* of teaching secondary mathematics stems from contrasting visions of the discipline and has manifested in a decades-long scholarly and public debate over content and pedagogy (Burdman, 2015; Schoenfeld, 2002, 2007). Pervasive inertia fuels resistance to transforming the content and pedagogy of mathematics classrooms despite decades of compelling research. Examining the literature of the struggle and related studies

exposes differing values about learning mathematics, which translate into vastly different experiences for students. These experiences shape students' perceptions of the discipline and affect their persistence in STEM disciplines (Boaler, 2015; Boaler & Greeno, 2000). The content and pedagogy of K-12 mathematics are key to understanding the leading recommendations for high school mathematics pathways.

Content

Tension over the featured mathematics of K-12, especially at the high school level, center on the changing needs of society. Differing views on the content of high school mathematics shape students' experiences and perceptions of the subject. Since the late 19th Century, the content of secondary mathematics has remained relatively static focusing on algebra to calculus pathway (Burdman, 2015; NCTM, 2018). The world has dramatically changed in the last 100 years, sparking a debate about revisions to the content in secondary mathematics. Views differ on how to respond best to changing realities from leaving the current content mostly intact, yet fusing it with innovative research-based instructional approaches, to de-emphasizing the traditional algebra and geometry strand in favor of other branches of mathematics that may have more relevance and engagement for adolescents (Burdman, 2015, 2018; Carnevale & Desrochers, 2003; Daro & Asturias, 2019; Tucker et al., 2013). The leading recommendations for high school pathways endorse both viewpoints (Daro & Asturias, 2019; Fitzgerald & Sovde, 2019; NCTM, 2018).

The imperative to grow and to diversify the STEM pipeline sustains this tension with a continued emphasis on the algebra to calculus pathway, though the labor market prefers a broader range of mathematical understanding including knowledge in mathematical modeling, statistics, and computer science (Burdman 2015; Carnevale & Desrochers, 2003; Tucker et al.,

2013). The dominant pathway from algebra to calculus may fail to serve the needs of some students when many careers may never require knowledge of calculus, leaving many high school students struggling to find relevance in studying mathematics (Burdman, 2015; Carnevale & Desrochers, 2003; Daro & Asturias, 2019). If the content could be presented as more relevant to students pursuing a diverse range of professions, they may form positive perceptions of mathematics and achieve a higher level of quantitative reasoning, benefiting society, the knowledge economy, and the overall standard of living (Burdman, 2015; Carnevale & Desrochers, 2003; Daro & Asturias, 2019).

The Common Core State Standards for Mathematics (CCSS-M), adopted by 43 states including California, largely preserves traditional K-12 content and inserts a set of eight Standards for Mathematical Practice (SMPs), which describe active learning experiences students should have when doing mathematics (NGA, 2010). The CCSS-M defines *rigor* as “conceptual understanding, procedural fluency and skills, and application with equal intensity” (NGA, 2010). Along with the SMPs, these three aspects of CCSS-M rigor facilitate a multi-dimensional view of mathematics, though recent findings suggest the balance has yet to happen in many secondary classrooms. Theoretically, in the CCSS-M era, students should experience traditional content through reasoning, though an Education Trust study of nearly 2,000 secondary mathematics assignments during the 2015-16 school year found less than 40 percent of assignments include conceptual understanding and applications (Dysarz, 2018).

Pedagogy

The pedagogy of teaching mathematics has sparked a fierce debate for decades between two competing visions of how students should experience the content. In the 1990s this contentious struggle earned the nickname *the math wars*, suggestive of the vitriolic and bitter

debate, which endures in the CCSS-M era (NCTM 2014, 2018; Schoenfeld, 2002, 2004, 2007). The two starkly different views of teaching mathematics have profound effects on how students experience and perceive the discipline. The traditional teaching of mathematics is teacher-centered and categorized by individual learning, rote memorization, and disconnected topics; whereas the inquiry-based approach to teaching mathematics is student-centered and favors the inclusion of all students in collaboration, sense-making, and meaningful tasks (Boaler, 2015; Boaler & Greeno, 2000; Gutiérrez, 2012, 2013; NCTM 2014).

For purposes of this dissertation, I will identify the inquiry-based approach as reform-oriented pedagogy in mathematics, signifying how this pedagogy contrasts with traditional teaching. Multiple statements and frameworks layout a vision for reform-oriented K-12 mathematics teaching: NCTM's (2014) Eight Effective Mathematics Teaching Practices; Schoenfeld's (2014) five dimensions of Teaching for Robust Understanding Framework; National Council of Supervisors of Mathematics (NCSM) and TODOS's (2015) social justice statement; and Krall's (2018) *Necessary Conditions* to name a few. Although each one has distinctive features, the reform-oriented frameworks share common themes in teaching K-12 mathematics by emphasizing the social nature of doing mathematics, reasoning, coherence, and an equity mindset, which are suggestive of the experiences students should have in the mathematics classroom.

Reform-oriented mathematics teaching facilitates student experiences that mirror how mathematicians encounter the discipline. Mathematicians describe their craft as a creative, multi-dimensional, and social endeavor (Boaler, 2015; Burton, 1998). Reform-oriented pedagogy seeks to honor these descriptions in the classroom by encouraging multiple approaches to problem-solving; emphasizing conceptual understanding, procedural fluency, and applications; and

fostering collaboration and discourse (NCTM, 2014; NGA, 2010). Like mathematicians, students are solving meaningful and intellectually engaging problems with their peers (Boaler, 2015; Krall, 2018; Schoenfeld, 2014).

When high school teachers employ a reform-oriented pedagogy, students are more likely to develop a positive relationship with the discipline. Students' descriptions of reform-oriented mathematics classes reflect how mathematicians practice their craft (Boaler & Greeno, 2000; Boaler & Sengupta-Irving, 2016). Students depict mathematics as purposeful and they value the co-construction of knowledge with their peers and teachers. As a result, students in reform-oriented mathematics classrooms are more likely to enroll in additional mathematics classes and to pursue future studies in mathematics-based professions (Boaler & Greeno, 2000; Boaler & Staples, 2008; Boaler & Sengupta-Irving, 2016). In contrast, students who experience a traditional mathematics classroom tend to form an inaccurate perception of the discipline. In traditional high school classrooms, students characterize mathematics as following a set of procedures and view it as a meaningless answering gathering exercise (Boaler, 2000; Boaler & Greeno, 2000; Boaler & Sengupta-Irving, 2016; Boaler & Staples, 2008; PCAST, 2012). In Boaler and Greeno's (2000) study of traditional AP Calculus classes, 94 percent of the students expressed a dislike for the subject and were less likely to pursue further studies in mathematics.

Not surprisingly, student experiences with contrasting mathematics pedagogies produce different outcomes. K-12 students in reform-oriented mathematics classrooms test significantly higher on conceptual understanding than students enrolled in traditional classrooms (Schoenfeld, 2002; Swafford, 2003). Researchers have found that traditional mathematics teaching constrains high school and community college students from developing conceptual understanding, making

connections between topics, and flexibly applying procedures to novel problems (Schoenfeld, 1988; Stigler, Givvin, & Thompson, 2010).

The intense debate between the two styles of teaching continues today and affects the experiences students have in secondary mathematics classrooms. The previously mentioned Education Trust study of nearly 2,000 assignments implies many teachers persist with traditional pedagogy, as evidenced by the coursework samples. Only nine percent of assignments have a cognitive demand of level three or four according to Webb's Depth of Knowledge and less than third required students to communicate their thinking. While nearly 90 percent of assignments in the study focused on procedural fluency, less than half include conceptual understanding (Dysarz, 2018). Scholars express frustration traditional pedagogy persists, marginalizing students, especially Black, Indigenous, and Students of Color, from mathematics (Bartell, Wagner, Edwards, Battey, Foote, & Spencer, 2017; Gutiérrez, 2012, 2013; NCTM, 2014, 2018;

Leading Recommendations for High School Mathematics Pathways

NCTM (2018), the Dana Center at the University of Texas, Austin (Fitzpatrick & Sovde, 2019), and Just Equations (Daro & Asturias, 2019) have issued recommendations on designing high school mathematics pathways, which build on years of research about high school mathematics. Although each statement differs slightly, all three place equity and experiences of students at the forefront. Recognizing one-size-fits-all is inherently inequitable, the recommendations seek to create multiple rigorous high school mathematics pathways, which address the interests and future pursuits of students (Daro & Asturias, 2019; Fitzpatrick & Sovde, 2019; NCTM, 2018). Too often, the algebra to calculus pathway acts as a gatekeeper instead of a gateway to opportunity, excluding students from pursuing post-secondary education (Burdman, 2018; Daro & Asturias, 2019; Tucker et al., 2013).

The three leading recommendations incorporate findings from years of research on high school mathematics purpose, course-taking patterns, student outcomes, access, content, and pedagogy to create equitable pathways and improve student experiences. The core tenets of the leading recommendations include (1) four years of high school mathematics are encouraged, (2) purposeful pathways offer traditional and non-traditional branches of mathematics aligned to student interests, and (3) teachers use reform-oriented pedagogy (Daro & Asturias, 2019; Fitzpatrick & Sovde, 2019; NCTM, 2018).

Nuance exists in the language of the recommendation for four years of high school mathematics. The Dana Center uses the strongest language, recommending states *require* four years of high school mathematics (Fitzpatrick & Sovde, 2019). NCTM (2018) and Just Equations (Daro & Asturias, 2019) use more muted language such as *should* and *encouraged* respectively. Just Equations offers the least strong language to discourage policymakers from rushing to issue mandates without strengthening the first three years of high school mathematics and having a menu of meaningful fourth-year mathematics courses in place (Daro & Asturias, 2019).

High school students would experience a shared pathway when they first enter high school and then splinter off in junior or senior year to mathematics reflective of their personal and future career interests. NCTM (2018) spends nearly half of *Catalyzing Change* describing the shared content, which it terms, *Essential Concepts* (see pp. 37-79). At the decision point, the students would have a choice of rigorous mathematics courses aligned to their interests. The Dana Center and Just Equations have clever terms to describe the courses after the decision point. The Dana Center refers to these courses as *launch years* and Just Equations refers to them as *branch* courses, describing how the students will *launch* or *branch* off into mathematics courses that match their post-secondary field of study (Daro & Asturias, 2019; Fitzpatrick &

Sovde, 2019). The three organizations encourage a broad range of options in addition to the traditional pathway to calculus. NCTM (2018) defines the necessary elements to ensure rigor and mathematics are paramount in these launch or branch courses. Concerned with *de facto* tracking and flexibility between the pathways, all three organizations emphasize students select their junior or senior mathematics courses based on their interests and aspirations, but can move between pathways as their interests and career plans may change (Daro & Asturias, 2019; Fitzpatrick & Sovde, 2019; NCTM, 2018). As an example, Oregon Department of Education (2020) is proposing a *2+1 course model*, in which high school students would take an initial two years focused on algebra, geometry, and statistics and then select from one of three pathways ending in calculus, data science, or quantitative mathematics.

The three sets of recommendations endorse both traditional and non-traditional content with reform-oriented pedagogy. At the decision point, high school students would select between the traditional pathway to calculus and non-traditional options based on a variety of factors including future career plans (Daro & Asturias, 2019; Fitzpatrick & Sovde, 2019; NCTM, 2018). Unanimously, all three sets of recommendations advocate for reform-oriented pedagogy. NCTM (2018) draws on the Eight Effective Mathematics Teaching Practices (NCTM, 2014) and specifically addresses equity within each one. The Dana Center and Just Equations mention reform-oriented instructional practices that foster sense-making, collaboration, and a positive mathematics identity in students (Daro & Asturias, 2019; Fitzpatrick & Sovde, 2019). Ostensibly, high school students will form a more positive perception of mathematics if they find personal relevance in what they study and are engaged in reasoning and constructing meaning from the mathematics, making it compelling to examine student experiences in these *launch year* or *branch* courses.

Unique Fourth-Year High School Mathematics Courses

The California Math Readiness Challenge Initiative (CMRCI), which generously funded the Discrete Math Project Collaborative (DMPC) and Introduction to Data Science (IDS) courses, along with three other pilot courses in the state, focused on increasing access to four years of rigorous mathematics and providing extensive professional learning on reformed-oriented pedagogy for teachers implementing the courses. These goals directly address core tenets of three leading recommendations for high school mathematics pathways (CDE, 2018a; DMPC, n.d.; Gould et al., 2016; IDS, n.d.). CMRCI funding financed the creation of the DMPC course; the IDS course existed prior to CMRCI and received NSF funding (DMPC, n.d.; IDS, n.d.; NSF, n.d.; O. Soto, personal communication, April 30, 2019;). Although the creation of DMPC and IDS courses predates the recommendations set forth by NCTM, Dana Center, and Just Equations by a few years, the two classes follow the guidelines, which make them intriguing courses to research (Daro & Asturias, 2019; Fitzpatrick & Sovde, 2019; NCTM, 2018). Just Equations even lists both DMPC and IDS courses as examples of non-traditional branch courses (Daro & Asturias, 2019).

IDS and DMPC were specifically designed to prepare high school seniors for post-secondary mathematics and expand access to four years of rigorous high school mathematics and post-secondary options (CDE, 2018a). Both courses feature non-traditional branches of mathematics and are intended to provide a rigorous quantitative reasoning experience for students, who may never pursue a STEM-related career, but could do so when they enter higher education. DMPC emphasizes branches of mathematics rarely seen in high school curricula such as game theory and cryptography (DMPC, n.d.). IDS focuses on data science using R statistical software (Gould et al., 2016; IDS, n.d.). The CMRCI grant funded extensive professional

learning of the DMPC and IDS content and reform-oriented pedagogy for teaching the course (CDE, 2018a). Student sense-making, reasoning, and agency are at the heart of both courses (DMPC, n.d.; Gould et al., 2016; IDS, n.d.).

The body of literature on high school mathematics clearly leads to the recommendations for high school pathways made by NCTM, the Dana Center, and Just Equations (Daro & Asturias, 2019; Fitzpatrick & Sovde, 2019; NCTM, 2018). What appears to be less well understood in the literature is the student experiences within courses, which follow the high school mathematics pathway recommendations. Given the recent creation of DMPC and IDS, there is no published research on the perceptions and experiences of students in either course. There are a few research publications on the content of the IDS course and instructional challenges, but none focused on the student experience (e.g., Gould et al., 2016 and Gould et al., 2017). As the CSU Trustees plan to require a year of quantitative reasoning of freshmen applicants, they are touting the CMRCI courses as an option for high schools to implement, making it all the more urgent to examine how students perceive and experience these courses (CSU 2020a, 2020b; Taylor, 2019).

Theoretical Frame: Self-determination Theory

Self-determination theory provides a useful frame for examining the experiences of students enrolled in high school mathematics courses, such as DMPC and IDS, by exploring the origins of motivation and psychological factors that can foster or suppress it. Studying student experiences through the lens of motivation affords insights into how high school mathematics courses may support the STEM imperative and NCTM's (2018) three-fold purpose of learning mathematics (Committee on STEM, 2018; Deci & Ryan, 2008; Ryan & Deci, 2000a, 2000b). In high school mathematics courses, student experiences may motivate them to pursue STEM

careers, to make sense of the world, and to find personal satisfaction in doing mathematics. Self-determination theory views motivation as the reason behind an action. Behavior can result from compliance or a choice (Ryan & Deci, 2000a). For example, a student may do a homework assignment out of fear of being scolded or out of intellectual curiosity. In either case, the result is the same - a completed homework task - but for far different reasons.

Self-determination theory rests on decades of psychology and educational research into the inner workings and influences of motivation, which is viewed as mutable and varying in degree. The social context and the individual work in concert to shape motivation (Deci & Ryan, 2008; Deci, Vallerand, Pelletier, & Ryan, 1991; Guay, Ratelle, & Chanel, 2008; Ryan & Deci, 2000a, 2000b). Self-determination theory conceives of motivation along a continuum from amotivation (disengagement) to intrinsic motivation (self-directed) with extrinsic motivation situated in the center (see Figure 2.1). The source of the motivation determines whether it is extrinsic (external), such as a job promotion, or intrinsic (internal), such as enjoyment of playing a musical instrument. Extrinsic motivation is subdivided into four types based on varying degrees of self-agency from external regulation to integrated regulation. The desire for a reward or avoidance of unpleasant consequences (e.g., punishment, feelings of shame, etc.) characterizes external regulation and introjected regulation. The furthest right category of extrinsic motivation, integrated regulation, aligns most closely with intrinsic motivation. In this classification, the individual is self-directed and finds meaning in the external outcome. In the example of homework, a student would feel an external purpose in completing the assignment regardless of a reward or punishment. Understanding extrinsic motivation is critical in education. Over the course of 180 school days, an average school year in California, it is unrealistic that every task will engender intrinsic motivation; therefore, fostering integrated regulation is crucial for

students to find meaning in the learning (Deci & Ryan, 2008; NCES, 2018; Ratelle, Guay, Vallerand, Larose & Senécal, 2007; Ryan & Deci, 2000a, 2000b).

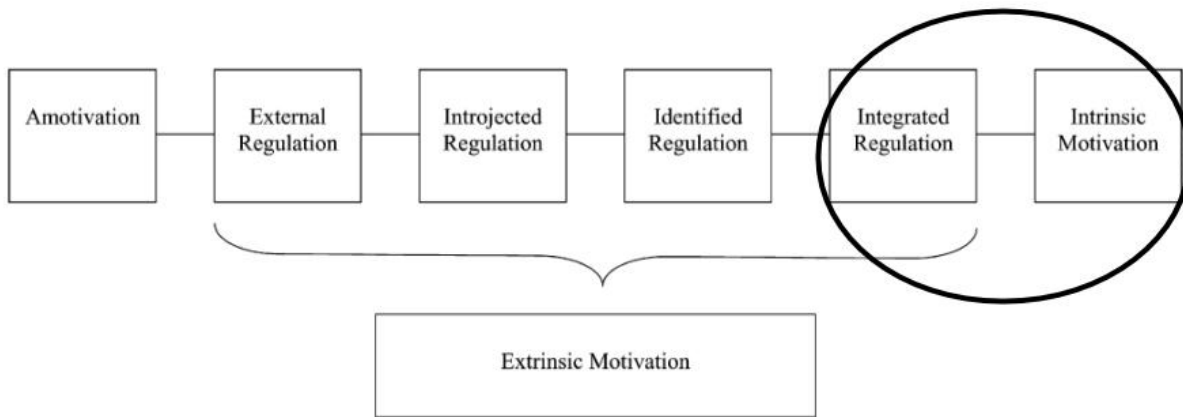


Figure 2.1 Continuum of motivation from amotivation to intrinsic motivation. Fairchild, A. J., Horst, S. J., Finney, S. J., & Barron, K. E. (2005). Evaluating existing and new validity evidence for the Academic Motivation Scale. *Contemporary Educational Psychology*, 30(3), 331-358.

Sub theories within self-determination theory, organismic integration theory and cognitive evaluation theory, specialize in extrinsic and intrinsic motivation respectively and how factors (e.g., deadlines, feedback) can either enhance or diminish both types of motivation. Of particular interest is how the lack of or abundance of three psychological needs, competence, autonomy, and relatedness, can influence integrated regulation and intrinsic motivation. For integrated regulation (extrinsic) or intrinsic motivation, all three psychological needs must be met. Competence is characterized by feelings of confidence and efficacy to engage in challenging tasks and to find purpose in doing so. Autonomy describes a sense of agency, a person acting freely regardless of any external pressures. Finally, a sense of belonging and being part of the community define relatedness. All three psychological needs touch upon vital aspects of a high school mathematics classroom. Ideally, these students experience empowerment,

agency, and a sense of community when engaging in mathematics (Deci & Ryan, 2008; Ryan & Deci, 2000a, 2000b).

Spanning decades, several self-determination studies show supporting the three psychological needs of competence, autonomy, and relatedness has a profound impact on education. Students who feel a stronger sense of competence, autonomy, and relatedness demonstrate more persistence, have higher achievement, prefer intellectual challenges, and find more enjoyment in academic pursuits (Deci et al., 1991; Guay et al., 2008; Ratelle et al., 2007; Ryan & Deci, 2000a, 2000b). Studies on mathematics education outside of the self-determination field also identify these three psychological needs as essential, citing how the lack of competence, autonomy, and/or relatedness marginalizes and drives students from the discipline. Ladson-Billings (1997) described how K-12 mathematics classrooms focused on rote formulas impeded African-American students' sense of autonomy or free will. Bressoud (2015) found rapport, or a sense of relatedness, between the teacher and students significantly influenced whether students persisted in undergraduate calculus. In Good, Rattan, and Dweck's (2012) study of the gender gap in mathematics, the authors cited a sense of community and perceptions of self-confidence, as crucial for women who intend to study the discipline. Boersma and Savina (2019) recommend mathematics faculty foster students' sense of belonging and self-efficacy to improve academic outcomes in college.

Two prominent frameworks for teaching K-12 mathematics speak to the student experience in the classroom and allude to the three psychological needs of competence, autonomy, and relatedness from self-determination theory (Ryan & Deci, 2000a, 2000b). The frameworks imply effective mathematics teaching supports the three psychological needs. Aguirre, Mayfield-Ingram, and Martin's (2013) equity-focused teaching framework list the

practices of “leveraging multiple mathematical competencies, affirming mathematics learners’ identities, challenging spaces of marginality, and drawing on multiple resources of knowledge” (pp. 43-48). These four practices infer the need for the teacher to facilitate competence, autonomy, and relatedness in students by valuing students’ strengths, encouraging multiple approaches to problem-solving, and building an inclusive community. Likewise, Schoenfeld’s (2014) Teaching for Robust Understanding framework implies the three needs in the fourth dimension titled “agency, authority, and identity” (p. 407). Agency entails autonomy. Authority infers competence and identity suggests the students viewing themselves as members of the mathematics community.

Gaps in the Research Literature

The leading recommendations for high school mathematics pathways rest on a robust literature of K-12 mathematics and in particular, high school mathematics. This body of research supplies the *why* for the recommendations from NCTM, the Dana Center at the University of Texas, Austin, and Just Equations, yet there is little to no research on *how* students experience and perceive courses incorporating these recommendations into their design and practice (Daro & Asturias, 2019; Fitzpatrick & Sovde, 2019; Gould et al., 2016; NCTM, 2018). The intention of these leading recommendations is for high school students to have a more engaging and meaningful experience in high school mathematics. Without research into student experiences in the non-traditional pathway courses, it is unknown if the intentions are being achieved. As the CSU Trustees move forward to require a year of quantitative reasoning, they promote the CMRCI courses as an option to fulfill the requirement, making it even more compelling to examine and understand the nature of student experiences in the DMPC and IDS courses (CSU, 2020a, 2020b; Taylor, 2019).

Summary

There exists a through line in the literature on high school mathematics purpose, course-taking patterns, student outcomes, access, content, and pedagogy to the leading recommendations on high school mathematics pathways. Student experiences and perceptions frame the strands of high school mathematics research literature. These recommendations purport to create equitable pathways through high school mathematics and to support students having meaningful and relevant experiences in the discipline based on their personal and career interests (Daro & Asturias, 2019; Fitzpatrick & Sovde, 2019; NCTM, 2018). Self-determination theory provides a useful lens for studying student experiences in specially designed fourth-year high school mathematics courses (Ryan & Deci, 2000a, 2000b).

High school mathematics sits at the pinnacle of K-12 mathematics and provides the foundation for post-secondary pursuits. Therefore, policymakers, industry leaders, and educators have defined a broad purpose for the role of high school mathematics. Most commonly, they cite the integral role mathematics serves in growing and diversifying the STEM pipeline (Burdman, 2018; Committee on STEM, 2018; NCTM, 2018; PCAST, 2012). NCTM (2018) expands beyond this imperative to view high school mathematics as inspirational and necessary for contextualizing the world. These goals imply student experiences in mathematics should be both relevant and possibly transcendent.

Research studies on high school mathematics course-taking patterns and subsequent outcomes affirm the crucial role of the subject. Findings from national longitudinal studies of high school students associate taking additional years of high school mathematics, especially at least four years, with a greater likelihood of majoring in a STEM field, higher odds of graduating college, and higher incomes years after high school (Adelman, 1999, 2006; Byun et al., 2015;

Gaertner et al., 2014; Rose & Betts, 2004; Wang, 2013). Tragically, far too many students, especially BIPOC, never have the opportunity to enroll in four years of high school mathematics (Asim et al., 2019; Finkelstein et al., 2012; Gao & Johnson, 2017; U.S. Department of Education Office of Civil Rights, 2018). Opening access to additional years of high school mathematics seems to be a clear way to redress the inequity, yet historical efforts suggest access without consideration of student experiences with the content and the pedagogy will be insufficient. The algebra-for-all movement dramatically increased enrollment in eighth-grade algebra, yet student achievement decreased because little attention was given to the experiences these new students would have in the course (Allensworth et al., 2009; Clotfelter et al., 2012; Domina et al., 2014; Domina et al., 2015; Loveless, 2013).

The content and pedagogy of high school mathematics have been contentious for decades. Within content, there is a debate between the dominant pathway from algebra and geometry to calculus and non-traditional branches of mathematics, such as statistics and computational thinking, which may hold more relevance for a wide variety of professions (Burdman, 2015; Carnevale & Desrochers, 2003; Daro & Astarius, 2019). The debate over high school pedagogy stems from two contrasting views of mathematics. Traditional teaching views mathematics as isolated topics and sets of procedures to memorize, whereas reform-oriented teaching regards the subject as sense-making, collaborative, and empowering (Aguirre et al., 2013; Boaler, 2015; Schoenfeld, 2004, 2007, 2014).

The research on the high school mathematics purpose, course-taking, student outcomes, access, content, and pedagogy all informed the three leading recommendations on constructing high school mathematics pathways for equity and enhanced student experiences. NCTM, the Dana Center, and Just Equations advocate for (1) four years of high school mathematics, (2)

multiple pathways of traditional and non-traditional content based on student interests and career goals, and (3) reform-oriented teaching (Daro & Asturias, 2019; Fitzgerald & Sovde, 2018; NCTM, 2018). While these recommendations are designed to improve opportunity and student experiences, there is minimal literature on students' experiences and perceptions in courses that put these guidelines into practice. Two such courses, DMPC and IDS, offer a unique opportunity to examine how students experience and perceive courses designed to open access, feature non-traditional branches of mathematics, and embed reform-oriented pedagogy (DMPC, n.d.; Gould et al., 2016, IDS, n.d.).

Self-determination theory is a helpful frame for studying students' experiences and perceptions in the DMPC and IDS courses. The theory explores extrinsic and intrinsic motivation through the presence of competence, autonomy, and relatedness, known as the three psychological needs. Competence is feeling confident in being able to engage in mathematics. Autonomy describes self-directed actions free of external pressure or threats. Relatedness is the sense of belonging to the class community (Ryan & Deci, 2000a, 2000b). Examining student experiences through the lens of competence, autonomy, and relatedness provides insights into how the DMPC or IDS course influences student perceptions of mathematics.

Chapter 3: Methodology

Research Design

Overview

The literature on high school mathematics course-taking patterns, pedagogy, and pathways influenced the research questions and design of this research study on student experiences in the non-traditional high school mathematics courses. The DMPC and IDS courses incorporate the leading recommendations on high school mathematics pathways, by offering students who are ill-served by the calculus pathway, an opportunity to engage in rigorous reasoning through non-traditional branches of mathematics. In order to expand our knowledge of students' experiences within these unique courses, one DMPC classroom and one IDS classroom were selected for a collective case study. Observations, a survey, semi-structured student focus groups, and semi-structured one-on-one interviews generated data for analysis and triangulation, adding depth and credibility to the findings.

Self-determination theory provides a framework for studying student experiences within the DMPC and IDS courses. Self-determination theory defines both extrinsic and intrinsic motivation by the presence of three essential psychological needs: competence, autonomy, and relatedness (Ryan & Deci, 2000a, 2000b). Examining competence, autonomy, and relatedness of students through the qualitative and quantitative data collected provided insights into how students experience and perceive the DMPC and IDS courses. Table 3.1 relates the research methods to the two research questions.

- 1) Within the Discrete Math Project Collaborative or Introduction to Data Science course, how do students experience competence, autonomy, and relatedness?
 - 1a) How might competence, autonomy, and relatedness differ based on student

demographics within the same course?

2) What are students' perceptions of and experiences with mathematics in the Discrete Math Project Collaborative or Introduction to Data Science courses?

Table 3.1 Methodology for Research Questions

Research Questions	O	S	FG	I	DR
1. Within the Discrete Math Project Collaborative or Introduction to Data Science course, how do students experience competence, autonomy, and relatedness?	X	X	X	X	X
1a. How might competence, autonomy, and relatedness differ based on student demographics and academics within the same course?		X			
2. What are students' perceptions of and experiences with mathematics in the Discrete Math Project Collaborative or Introduction to Data Science courses?	X	X	X	X	X

Note: (O) Observation; (S) Survey; (FG) Semi-Structured Student Focus Group; (I) Semi Structured One-on-One Teacher Interview; (DR) Document Review

This study utilized a mixed methods collective case study design. Mixed methods build on the individual strengths of quantitative and qualitative methods. Quantitative methods capture the perceptions of a large number of participants, while the qualitative methods flesh out the *how* and *why* of those perceptions from a carefully selected subset of participants. Together, the two methods form a more robust understanding of the phenomena (Creswell, 2012; Teddlie & Tashakkori, 2009). The design of this research study had two distinct phases (see Figure 3.1). Data collected in the first phase informed the data collected in the second phase (Creswell, 2012). Phase 1 was observations in the DMPC and IDS classrooms followed by a student survey. The second phase built on the areas of interest from phase 1 and featured semi-structured student focus groups and one-on-one interviews of the DMPC and IDS teachers. Within each case, the teacher interviews were sandwiched between an initial and a final student focus group. This way, the DMPC and IDS teachers shed light on emergent themes from the analysis of previous student

focus groups, yet the final student focus group had the last word so to speak, expounding on themes that emerged from previous interviews with their peers and teacher. Thus, the student perspective remained paramount. The arrows in Figure 3.1 show the direction of influence for the data collected. Phase 1 data partially influenced which students were invited to participate in the focus groups and the questions asked in the focus groups and teacher interviews. The bidirectional arrow in phase 2 indicates how initial student focus groups influenced the interview of their teacher, which in turn, influenced the final student focus group.

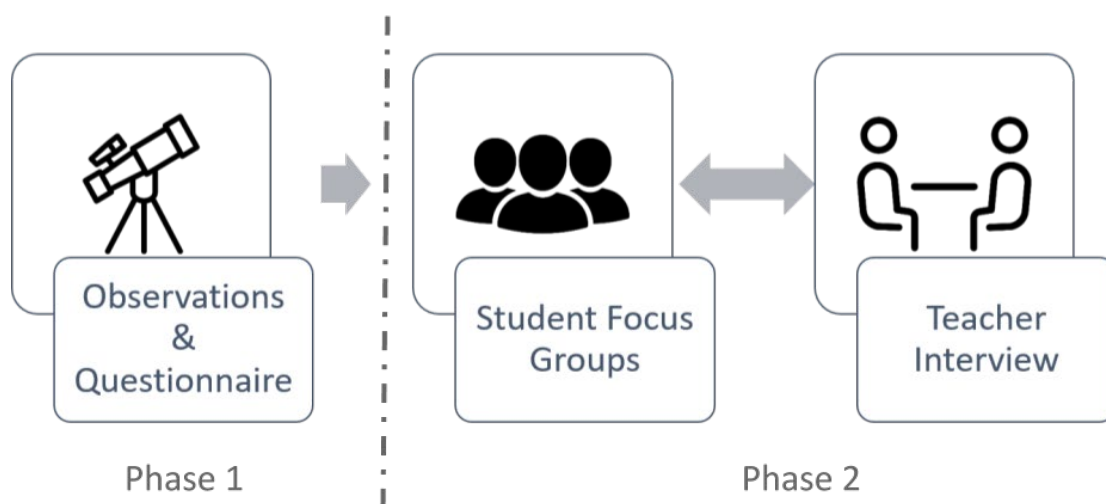


Figure 3.1. Two phases of the research design.

According to Stake (2005), “Case study is both a process of inquiry about the case and a product of that inquiry” (p. 444). A case study is a methodology and results in a deep understanding of a contextualized phenomenon (Stake, 2005). For this research study, the experiences and perceptions of students were bounded by one DPMC class and one IDS class at two different high schools (Creswell, 2012; Stake, 2005).

Maxwell (2012) highlighted *selection, data collection, and data analysis* as necessary pillars to designing the methodology of a study. Therefore, these three pillars frame the organization of this chapter.

Case Study Selection and Context

Selection

The DMPC and IDS classrooms were selected in consultation with the Director of the Discrete Math Project Collaborative at San Diego State University and the Director of Introduction to Data Science Project at UCLA Center X. To minimize confounding variables, such as a teacher new to the curriculum, the collective case study is bound to one teacher of the DMPC course, Ms. North, and one of the IDS course, Ms. Arcega. Pseudonyms were given to the two schools, two teachers, and all students in this study. Both Ms. North and Ms. Arcega serve as mentors and trainers for new teachers of the DMPC and IDS courses respectively and were strongly recommended by their program directors. They have multiple years of experience teaching the respective course along with traditional high school mathematics courses (S. Machado, personal communication, October 16, 2019; O. Soto, personal communication, April 30, 2019).

Context

DMPC. Ms. North teaches at Mediterranean High School in a large urban school district, which serves nearly 40,000 secondary students. Sixty percent of the students are socioeconomically disadvantaged, and 22 percent are English learners. Nestled among pre-planned housing developments -- bearing names suggestive of a French or Italian Riviera resort - - Mediterranean High School is located in a more affluent corner of the district with over 2,400 students; 27 percent of the students are socioeconomically disadvantaged and 6.5 percent are

English learners. A six-lane parkway is the main artery through the community with the housing developments dotting the hillsides and the newly constructed strip malls of national chain stores and casual eateries below. A majority of Mediterranean High's student population is BIPOC: Hispanic (55 percent), Filipino (20 percent), two or more races (9 percent), African-American (6 percent), and White (7 percent) (CDE, n.d.). Built in the last two decades to accommodate the expanding housing developments, Mediterranean High School reflects the classic Southern California school architecture of sprawling single-story buildings painted in neutral tones. Drought-resistant plants landscape the outdoor pathways between the buildings and solar panels shade a large parking lot at the entrance of the school.

IDS. Ms. Arcega teaches at Sierra Madre High School. The high school of 2,000 students is located in one of the most populous regions in Southern California and is part of a mid-size district that serves less than 20,000 students K-12. Sixty-four percent of the district's students are socioeconomically disadvantaged and nearly a quarter are English learners. Among the handful of high schools in the district, Sierra Madre has the highest percentage of socioeconomically disadvantaged students (80.1 percent) and a lower percentage of English learners (20 percent) than the district overall (CDE, n.d.). The school is located in an older residential neighborhood; wide streets are lined with decades-old tall trees. Many of the charming bungalow homes were built in the 1930s and 1940s, reminiscent of old Hollywood films. Some of the homes have fallen into disrepair and others have been razed to make way for larger homes and two-story apartment complexes. A short distance from the school is a historic commercial area. The ground level businesses reflect diverse communities, with many restaurants serving regional cuisines from Asia and Latin America alongside familiar national chains. These communities are reflected in the demography of the school district and at Sierra Madre High. Asian (50.4 percent) and

Hispanic (40.8 percent) students comprise over 90 percent of the student population in the school district. Similarly, at Sierra Madre High 94 percent of the student population is Asian (59.5 percent) and Hispanic (33.9 percent) (CDE, n.d.). Sierra Madre High School appears to have been built originally as the Baby Boom generation entered high school. Over the intervening decades, the nondescript one and two-story buildings have been remodeled. Taking advantage of the temperate California weather, visitors enter through an outdoor kiosk and there are wide courtyards between buildings.

Data Collection

Data Collection Instruments

Students completed more than half the DMPC and IDS courses when data collection began in January 2020, which was sufficient time for them to form a perspective on their experiences in the class. Data collection concluded prior to the school closures for the pandemic. The classroom observations, semi-structured focus groups, semi-structured interviews, and the curriculum generated a qualitative data set including field notes, interview transcripts, and analytical memos. A student survey was used to collect the quantitative data and one additional piece of qualitative data, a written prompt (see Appendix A). A preliminary analysis of the survey and classroom observations informed the purposeful sampling of students for the focus groups and some of the questions asked in the interviews.

Observations. Observations happened three times each in the DMPC and IDS courses during phase 1. The observations were essential to understanding the learning environment and context of each classroom (Emerson et al., 2011), which DMPC and IDS students and teachers later referred to in the semi-structured student focus groups and one-on-one teacher interviews. In the DMPC course, many of the problem sets unfold over multiple school days (O. Soto,

personal communication, April 30, 2019). The IDS course distinguishes between *classroom days* and *lab days*, with lab days focusing on programming in statistical software R (Gould et al., 2016). Observations were scheduled over multiple days to ensure that the unique nature of each class was revealed. I took great care to mitigate any disruption to the learning environment when I observed, by minimally interacting with the teacher and students during my observations. My intent was to be the proverbial fly-on-the-wall, except for rarely asking the teacher or the students for clarification of the unfolding events in the classroom (Emerson et al., 2011; Maxwell, 2012).

Survey. After three observations each in the DMPC and the IDS courses, the course teacher invited students, who submitted the necessary permissions, to complete an electronic survey via Qualtrics. The survey (see Appendix A) has common demographic questions and adapted items from three different instruments designed to address the three psychological needs of self-determination theory: competence, autonomy, and relevance (Ryan & Deci, 2000a, 2000b). The items addressing the psychological needs of competence and autonomy are adapted from two scales on mathematics attitude, Shortened Form of the Fennema-Sherman Mathematics Attitude Scales (Mulhern & Rae, 1998) and Attitudes Toward Mathematics Inventory (Tapia & Marsh, 2004). The two instruments were created for Irish school children (Mulhern & Rae, 1998) and secondary students in the United States (Tapia & Marsh, 2004). The relatedness items are adapted from Good, Rattan, and Dweck's (2012) survey on a sense of belonging in mathematics. I updated the formal language of the established instruments for current high school students and narrowed the focus to their experiences in either the DMPC or IDS course. Students responded to these items using a 5-point Likert scale ranging from *strongly disagree* to *strongly agree*. Neither instrument had items addressing specific aspects of autonomy and

relatedness, such as self-direction on starting problems (autonomy) nor forming connections with their peers in the course (relatedness); therefore, I included additional items to capture these subtleties. Finally, the survey had one open-ended item, which prompted students to describe their perceptions of mathematics in either the DMPC or IDS course. The student responses to the written prompt informed the purposeful sampling of students for the focus groups and were analyzed with the other qualitative data (Creswell, 2012).

Semi-structured focus groups and interviews. Semi-structured focus groups and interviews provide structure and flexibility simultaneously to the interviewer (see Appendix B and C for protocols). There was a predetermined set of questions to gather information on student experiences and perceptions, paying particular attention to the three components of self-determination theory. The semi-structured nature provided flexibility to ask additional follow-up questions based on unanticipated responses of interest during the interviews and emergent themes from prior data collected (Creswell, 2012; Maxwell, 2012; Siedman, 2006). I recorded the student focus groups and interviews for transcription. The heart of the corpus of data came from the transcripts of the student focus groups. For the DMPC course, I conducted four semi-structured focus groups with 12 students over a three-week period. For the IDS course, I conducted two semi-structured focus groups, totaling eight students over a two-week period. I situated the one-on-one DMPC and IDS teacher interviews in the middle of conducting the student focus groups. The two teachers commented and elaborated on emerging themes from data already collected, but the insights and perspectives of the students remained at the forefront by ending on a student focus group.

The structure of a focus group has benefits and limitations. The student focus group participants deepened their responses by building on what their peers shared. Being with friendly

peers may have alleviated some anxiety students felt about participating. However, students may have felt reluctant to share in front of their peers or may have limited time to share (Creswell, 2012). To partially mitigate these drawbacks, the focus groups were limited most often to three students and careful consideration went into the composition of each focus group. Every effort was made to capture a diversity of perspectives among the students who submitted consent forms (see Chapter 4 and Appendix D for a more detailed description of the participating students). I relied on my previous observations, preliminary survey analysis, teacher recommendations, and demographics to purposefully sample the pool of consenting students (Creswell, 2012; Maxwell, 2012).

Document review. Both DMPC and IDS feature non-traditional branches of mathematics and in the case of IDS, there is specific statistical software. Therefore, a contemporaneous review of the curricula happened during the data collection process, providing a richer understanding of the observations and the student and teacher responses that cited specific tasks (Creswell, 2012). Prior to each observation, I reviewed the curricula material to deepen my understanding of the featured mathematical content.

Data Analysis

Preliminary data analysis occurred iteratively during the two phases to inform subsequent data collection. After the conclusion of the second phase, I analyzed the corpus of data in its entirety. I describe the analysis process below for the qualitative and quantitative data collected in this research study (see Table 3.2).

Qualitative Data Analysis

The two phases of data collection resulted in a rich trove of field notes, memos, and interview transcripts for analysis. During the observations of the DMPC and IDS classes, I

attended to student experiences in each course. Later, I expanded these jottings of descriptions, events, and dialogue into detailed field notes and memos for later coding (Emerson et al., 2011). I also wrote memos after each focus group and teacher interview focusing on persistent and intriguing themes from the sessions (Maxwell, 2012).

Thematic coding of the qualitative data – field notes, open-ended survey responses, interview transcripts, and memos - was both open and focused for etic and emic themes (Emerson et al., 2011). Coding is a heuristic, a crucial tool for discovering underlying and enduring themes in the qualitative data. Codes function as noticings which are then connected together into overarching ideas or themes (Saldaña, 2013). I utilized predetermined, descriptive, emotion, values, versus, and in vivo coding strategies (Miles, Huberman, & Saldaña, 2014; Saldaña, 2013). Self-determination theory framed the predetermined or etic coding of the qualitative data for the presence of competence, autonomy, and relatedness (Miles et al., 2014; Ryan & Deci, 2000a, 2000b). Emic codes emerged beyond the self-determination framework that enriched my interpretation and analysis, in particular how the students conceptualized mathematics in these unique courses (namely through descriptive, emotion, values, and versus coding). The versus coding approach was especially helpful in highlighting the difference between students' experiences in previous mathematics classes and the DMPC or IDS course. I noticed patterns in the use of common terms that held deeper meanings, in particular the repeated use of *fun* and *easy* by the students. I explored these nuances in the interviews and in vivo coding of the transcripts (Miles et al., 2014; Saldaña, 2013).

The coding of the data was an iterative process done using MAXQDA software. I recorded emergent ideas, impressions, themes, and patterns in memos, which informed future iterations of coding and the next direction of the inquiry (Emerson et al., 2011; Miles et al.,

2014; Saldaña, 2013). To reduce the potential for bias or myopic focus when coding, I engaged in peer debriefing with colleagues of de-identified data to uncover any emergent themes or interpretations that I may have missed (Lincoln & Guba, 1985).

Quantitative Data Analysis

I designed the electronic survey to collect demographic information and student self-ratings on items regarding competence, autonomy, and relatedness in the DMPC or IDS mathematics course (see Appendix A). The students responded to the items about their mathematics experiences using a 5-point Likert scale from *strongly disagree* to *strongly agree*. I analyzed the ordinal and categorical data from the survey using JASP statistical software, developed at the University of Amsterdam. I used a principal component analysis (PCA) to confirm the items that cluster around the three pre-planned themes of competence, autonomy, and relatedness. All statements below the component coefficient threshold of .5 were disregarded. After the PCA, I generated a composite score for each of the psychological needs of competence, autonomy, and relatedness from the remaining statements (see Appendix E). I investigated the descriptive statistics of competence, autonomy, and relatedness composite scores by analyzing means, medians, modes, standard deviations, and interquartile ranges (IQR), and ranges. Prior to running any independent t-tests or one-way ANOVA tests, I checked for normality. For nonparametric data, I used Mann-Whitney for independent t-tests and Kruskal-Wallis for one-way ANOVA. The appropriate parametric or nonparametric measures calculated if there were differences between a variety of demographic factors (e.g., gender, race/ethnicity, first-semester mathematics grades, etc.) and the three psychological needs of self-determination theory in the DMPC and IDS courses (Price et al., 2015; Ryan & Deci, 2000a, 2000b). Finally, Spearman's Rho correlation coefficient calculated the strength and direction of the relationship

between the three elements of self-determination theory: competence, autonomy, and relatedness (Price et al., 2015). Throughout the process, I captured my interpretations and perceptions of the statistical results in analytical memos which in turn informed the qualitative data coding and analysis (Emerson et al., 2011).

Table 3.2 Summary of Data Collection and Analysis

	Data Collected	Data Analysis
Observations	<ul style="list-style-type: none"> • Field notes • Analytical memos 	Open and focused coding for etic and emic themes
Survey	<ul style="list-style-type: none"> • Student self-reported demographics • 5-point Likert scale items related to competence, autonomy, and relatedness • Open-ended item 	<ul style="list-style-type: none"> • Principal component factor analysis (Likert scale items) • Mean, median, mode, standard deviation, IQR, range (Likert scale items) • Non-parametric Mann-Whitney for independent t-tests (Demographic & Likert scale items) • Non-parametric Kruskal-Wallis for ANOVA analysis (Demographic & Likert scale items) • Open and focused coding for etic and emic themes (open-ended item)
Semi-Structured Student Focus Groups	<ul style="list-style-type: none"> • Interview Transcripts • Analytical Memos 	Open and focused coding for etic and emic themes
Semi-Structured One-on-One Teacher Interview	<ul style="list-style-type: none"> • Interview Transcripts • Analytical Memos 	Open and focused coding for etic and emic themes
Document Review of Curricula	Memos	N/A collected to inform the analysis of qualitative data

Triangulation

A mixed methods approach lends itself well to triangulation, defined as multiple types of data supporting a set of coherent and logical conclusions (see Table 3.2). As a process, triangulation builds on the strengths and mitigates the limitations of each type of data to advance

well-supported findings (Creswell, 2012; Maxwell, 2012; Teddlie & Tashakkori, 2009). In this research study, triangulation connected the analysis of the observations, the student survey, student focus groups, and teacher interviews, leading to a set of findings. The two phases of this study facilitated triangulation. The observations and survey informed the subsequent semi-structured focus groups and interviews. In turn, each interview informed the next one and interpretations of prior ones.

Limitations

While the mixed methods collective case study design will address the research questions, there are limitations to the findings. In general, case studies are not intended to be generalizable (Creswell, 2012; Stake, 2005). The results of this study will be limited by size (only two classrooms), time (only a few months), participation (must consent to be interviewed), and context (two high schools in Southern California), and by no means should be interrupted as representative of all DMPC and IDS classrooms, or by that matter all non-traditional fourth-year mathematics classes. Instead, the findings of this study are intended to be a starting point for further research and a deepening of understanding about the phenomena investigated: student experiences and perceptions in two specific non-traditional fourth-year mathematics courses, DMPC and IDS (Stake, 2005).

Positionality and Validity

As a researcher, I must acknowledge my subjectivity and beliefs about secondary mathematics. Since exorcizing them from me is impossible, I instead interrogated them throughout the process to avoid being blinded by contrary results, finding myself distracted by an agenda, or myopically analyzing my evidence (Peshkin, 1988). My goal is to mitigate my subjectivity, not to extinguish it (Emerson et al., 2011; Peshkin, 1988). My interest in fourth-year

mathematics stems from my nearly two decades in the education profession. I began as a high school mathematics teacher and I now teach and supervise pre-service secondary mathematics teachers. Through my position, I am invested in the conversations about equity in secondary mathematics and how to expand and diversify the STEM professions. I have never wavered in my belief of education as a means to social justice and equity, though my perceptions of mathematics have dramatically evolved. The advent of common core along with teaching statistics transformed my thinking and expanded my view of mathematics as multi-dimensional and essential to our humanity.

Maxwell (2012) defines validity as ensuring the data analysis has credibility and actively mitigates potential erroneous interpretations. Inevitably, researchers are prone to faulty interpretations (Emerson et al., 2011; Maxwell, 2012). Geertz's (1973) cautionary story of a *wink* illustrates how a researcher may mistake a twitch for a mocking gesture. To mitigate validity threats to the data, peer debriefing with researchers inside and outside of secondary mathematics tested my assumptions and interpretations of the field notes, survey, memos, and interview transcripts. I met with the DMPC and IDS program leadership to present my initial data analysis. Their historical perspective and expertise deepened my understanding of the data and interpretation of the initial findings I generated (Lincoln & Guba, 1985; Price et al., 2015). I did member checking with the participating students and teachers during the interview to verify their thoughts were accurately captured (Maxwell, 2012; Seidman, 2012). Finally, looking for discrepant pieces of data guarded against possible underlying bias and strengthened the eventual findings (Maxwell, 2012).

Chapter 4: Findings

Overview

Three significant findings emerged from the analysis of quantitative and qualitative data of the DMPC and the IDS cases: (a) students perceive themselves as connected within a community of learners; (b) students experience curiosity and creative freedom in the two courses unlike previous mathematics courses they may have completed; and (c) students use the words *fun* and *easy* in complex ways to describe their experiences in the two courses.

The first two findings directly address the first research question and the third finding relates to the second research question (see below). The first research question is based on self-determination theory (Ryan & Deci, 2000a, 2000b) and the second question is intentionally broader to capture any findings of interest that may extend beyond the self-determination analytical framework. Through decades of studies, Ryan and Deci (2000a, 2000b) have defined the three psychological needs of competence, autonomy, and relatedness underlying positive motivation. The more contemporary terms for competence, autonomy, and relatedness would be self-efficacy, agency, and sense of community and belonging, respectively (Ryan & Deci, 2000a, 2000b). Self-determination framework is a lens for analyzing student experiences and perceptions within the DMPC and IDS course. When students possess competence, autonomy, and relatedness, they find meaning and purpose in the course. The two research questions are:

1) Within the Discrete Math Project Collaborative or Introduction to Data Science course, how do students experience competence, autonomy, and relatedness?

1a) How might competence, autonomy, and relatedness differ based on student demographics and academics within the same course?

2) What are students' perceptions of and experiences with mathematics in the Discrete Math Project Collaborative or Introduction to Data Science courses?

I begin this chapter by briefly introducing each finding. Due to the unique nature of each case, I consider how the nuances and context of these three findings present themselves within each case individually before concluding with commonalities and differences of the findings across the two cases.

Introduction to the Three Findings

The first two findings align with how the DMPC and IDS students' experience the three psychological factors of self-determination theory: competence, autonomy, and relatedness (Ryan & Deci, 2000a, 2000b). The first finding highlights students' feelings of being part of a community of learners and their perceptions of relatedness. And the second finding illuminates how students experience curiosity and creative freedom through their conceptualization of the course content, leading toward feelings of competence and autonomy. Finally, the third finding examines the specific and prevalent word usage of *fun* and *easy* to frame the DMPC and IDS students' experiences and perceptions in the two courses.

In a standard high school mathematics class, the teacher is positioned as the expert in the content. Harel and Rabin (2010) aptly describe this phenomenon as the teacher being *the sole arbiter of correctness* (p. 18), resulting in the students engaging in mathematics only after being shown a procedure and rarely questioning the purpose or reasoning of explanations. The novel content of the DMPC and IDS courses disrupts this traditional paradigm. Instead, the students learn *from* and *with* each other in the DMPC and IDS courses, fostering a community of learners among the teacher and the students. The novel nature of mathematics, along with free-flowing collaboration, nurtures a sense of belonging and relatedness within the students.

The DMPC and IDS students frequently portray their previous mathematics courses as marginalizing and isolating. Being part of the DMPC or IDS class community, the students become active doers of mathematics, feeling a sense of competence and autonomy. The DMPC and IDS students portray the two courses as full of curiosity and creative freedom, similar to how professional mathematicians and statisticians describe their work (Burton, 1998; Boaler, 2015; Boaler & Greeno, 2000; NCTM, 2018).

Finally, during the student focus group interviews, students used two words, *fun* and *easy*, in specific ways to describe the mathematics they were experiencing. In general, *fun* alluded to the joy and satisfaction the DMPC and IDS students find engaging in the mathematics and *easy* implies the accessibility and accomplishment they feel in the courses. Although the two words are deceptively simple, the students employ them to convey multiple and profound ideas.

DMPC

Ms. North's DMPC classroom has lively energy. Being the last period of the day, the DMPC students animatedly file into Ms. North's classroom. At the door, Ms. North greets her students with radiant eyes and an infectious smile, inquiring about their school day, an upcoming driving test, basketball season, or cello recital. She sustains the enthusiasm in the room by grinning as students engage in problem-solving half-conversations across the room ("Wait, why do we use the 9? ...Tell me again what does N represent...Oh, I get it!"); express their frustrations with good-natured and dramatic protests of mock anger trying to discern a recursive pattern ("THIS makes me so mad"); and admonish each other to avoid revealing the answer in the midst of discovery ("Don't be a ruiner.").

Glossy posters dot the four walls. By Ms. North's desk, there is a collection of former students' senior portraits on the wall, and in the opposite corner, a shoe rack hangs filled with

calculators. Piles of papers of varying heights sit on her desk and along the side counter. The one empty spot remains under the document camera, ready for students to share their thinking with the class. Formally trained as a professional dancer, Ms. North nearly glides or *chassés* between groups as she makes sense of students' reasoning and representations with her joyous smile. Ms. North deliberately unfolds the learning, building student anticipation. She strategically selects students to share their approaches under the document camera at specific moments, reminding students of the class refrain, "Don't be a ruiner."

Participants

Ms. North is a gregarious and energetic White teacher in her early 40s. Ms. North switched careers from the performing arts to secondary education a decade ago, uniting her fondness for creativity and mathematics. She has taught nearly the continuum of high school mathematics courses with the exception of pre-calculus and calculus. She began teaching the DPMC course four years ago, and for nearly as long, has mentored new teachers of the course, facilitated DPMC professional learning, and advised administrators throughout the district on effective implementation of the course.

The self-reported student demographics of the DMPC course mostly reflect the overall student population at Mediterranean High School (see Table 4.1). Thirty-one high school seniors were enrolled in Ms. North's DMPC course during the spring semester and 30 seniors consented to participate in the research. A nearly representative subset of 12 students participated in the focus groups with two notable exceptions (see Appendix D and Table 4.1). While the two genders were near parity in the DMPC class, females were overrepresented in the student focus group participants (75 percent). Black students also were overrepresented in the student focus group participants, comprising 10 percent of the DMPC class and 25 percent of the student focus

group participants. There is a higher percentage of students in the DMPC class qualifying for the federal lunch program (40 percent) than would be suggested by 27 percent of the students at Mediterranean High classified as socioeconomically disadvantaged (CDE, n.d.).

Table 4.1 Demographics of DMPC Class and Student Focus Group Participants

	DMPC Class (n = 30)	DMPC Focus Group (n = 12)
Gender		
Male	14	4
Female	16	8
Ethnicity/Race		
Latinx	16	5
White	2	1
Asian	5	2
Multiracial	2	1
Black/African American	3	3
Other	1	0
American Indian/Native American	1	0
Federal Lunch Status		
Qualify for Free or Reduced Lunch	14	4

From the self-reported data on the student survey, the 30 DMPC seniors completed one of three mathematics courses just prior to enrolling in DMPC: Integrated Mathematics III (26 students), Pre-Calculus (1 student), and Other (3 students). The twelve DMPC focus group students closely matched the overall class, with ten having taken Integrated Mathematics III. The other two students took pre-calculus and an unspecified mathematics course, *other*. In the focus group interview, the student who selected *other* on her survey shared she took a compacted mathematics course her junior year which consisted of a semester of Integrated Mathematics III and another semester of Pre-Calculus content. I am uncertain what mathematics course the other two students who marked *other* previously completed since they were never interviewed. Academically, 93 percent of the DMPC students report having a cumulative unweighted grade

point average (GPA) between 2.5 and 4.0, with 70 percent having at least a cumulative unweighted GPA of 3.0 or higher (see Table 4.2). The students participating in the focus groups almost precisely mirror the academic achievement of the class as a whole.

Table 4.2 Cumulative Unweighted GPA Range of DMPC Class and Student Focus Group Participants

GPA Range	DMPC Class (n = 30)	DMPC Focus Group (n=12)
Below 2.0	1	1
2.0 to 2.5	1	0
2.5 to 3.0	7	3
3.0 to 3.5	14	5
3.5 to 4.0	7	3

Note: Students selected a range that best matched their cumulative unweighted GPA.

Finding #1: Students Perceive Themselves as Connected within a Community of Learners

The uniqueness of discrete mathematics disrupts the paradigm of the teacher as the only authority of mathematical knowledge, by lending itself to explorations and reasoning by the students (Goldin, 2018; Harel & Rabin, 2010). Students engage in problems that feature familiar contexts (e.g., folding chairs at a graduation ceremony, Towers of Hanoi game, etc.) and have an easily accessible entry point to begin solving (e.g., students can play with manipulatives, draw a visual, make a table, etc.) (Goldin, 2018). Ms. North capitalizes on this unique nature of discrete mathematics to foster a sense of belonging in the DMPC class. She deliberately begins the school year with the Games unit, which simultaneously challenges her students’ notions what it means to do mathematics and builds community for the first six weeks of the school year:

So a lot of these mathematical habits that we want our students to do, they learn through playing games, which again doesn't seem like they're doing math to them.... The games are at least two-player games or two-team games...[which are] highly collaborative.

Though the DMPC students may be unaware of Ms. North's strategy, the impact is appreciated and felt by them. Ms. North's attention to community building coupled with the novel mathematical content results in the DMPC students perceiving their peers as capable doers of mathematics. The comments of DMPC students Ava, Brittany, Savannah, and Jayden, illustrate this sentiment. Ava, a Latinx female, described the classroom as an affirming and supportive space:

Even if your answer may be incorrect, you don't feel like, 'Oh, I did it wrong. It's all wrong, I can't share.' But here, everyone wants to share their ideas and you want to see how other people are thinking. So it's more of a positive space.

With an emphasis on reasoning, Ava values the sense-making of her peers and views making and sharing mistakes as beneficial to the learning process. Brittany, an Asian female, describes Ms. North's efforts to make the DMPC class feel egalitarian:

She [Ms. North] makes you feel like you're an equal.... It's really refreshing because normally, in other classrooms, it's just the teacher talking to the students, but our teacher really tries to make connections with her students to help...them understand what we're doing.

Likewise, Savannah, a White female, describes the DMPC class as a level playing field, "We're all in the same slate. We don't understand it. So we're trying different ways, and we're trying to figure out together. It's not just an individual thing. When we figure out things, we tell them."

Savannah reveals a camaraderie with her peers as she engages in problem-solving, emphasizing how the effort is communal. Building on this sentiment, Jayden, a Black male, seeks validation from his DMPC peers for his sense-making:

Because other years, I've just been, okay, I got the answer.... But now, I'm actually like, I want to get the answer before other students, and I want to put it on a projector and be like, this is my work. I did it like this. Is there anybody else that relates to me?... I genuinely get happy...because I have an urge to push myself and show it to other people. Whereas, I didn't like math before.

Thus, Ava, Brittany, Savannah, and Jayden feel a sense of community and relatedness that arises from doing challenging and unique mathematics together. Unlike many mathematics classrooms, the DMPC students have ownership of the sense-making. Brittany uses the word ‘equal’ to describe Ms. North and the students. Savannah notes the DPMC students are ‘in the same slate.’ And for Jayden, joy comes from his peers affirming his explanation.

The sense of community in Ms. North’s class is evident in the quantitative analysis of the survey data from the 30 DPMC students. The survey had 14 Likert scale items devoted to students’ sense of relatedness. Students ranked each item from *strongly agree* (1) to *strongly disagree* (5). Seven items exceeded the component coefficient threshold of .5 (see appendix E) in the principal component analysis. From these seven items a composite score was generated. The mean of the relatedness composite score is 1.548, the median is 1.286 (see Table 4.3). The corresponding measures of spread, standard deviation and IQR, suggest many of the student responses cluster near *agree* and *strongly agree*. In fact, the mode for all seven items was 1.0. Together, these results imply the strong sense of relatedness DMPC students feel.

Table 4.3 Descriptive Statistics for DMPC Relatedness Composite Score

	Mean	Median	Std. Deviation	IQR	Range
Relatedness Composite Score (n = 30)	1.548	1.286	0.611	.964	1.857

Visual representations of two example relatedness Likert scale items further highlight the connections DMPC students express and give insights on the measures of central tendency and spread discussed above (see Figure 4.1). These two statements had a mode of 1.0 (*strongly agree*) and are among the seven items that comprise the relatedness composite score (see Appendix E). The student responses overwhelmingly cluster around *strongly agree* (1) and *agree* (2), with only one *disagree* (4) response for each item. The first item focuses on the social

cohesion of the DMPC class in general. The histogram for the statement, “I feel like I am part of a community in this math class,” is skewed to the right with 28 of 30 students agreeing or strongly agreeing they feel part of the classroom community. The first bin is slightly taller when the Likert scale item focuses on collaboration when problem-solving. Two-thirds of the DMPC students *strongly agree* they appreciate working together with their peers when solving problems.

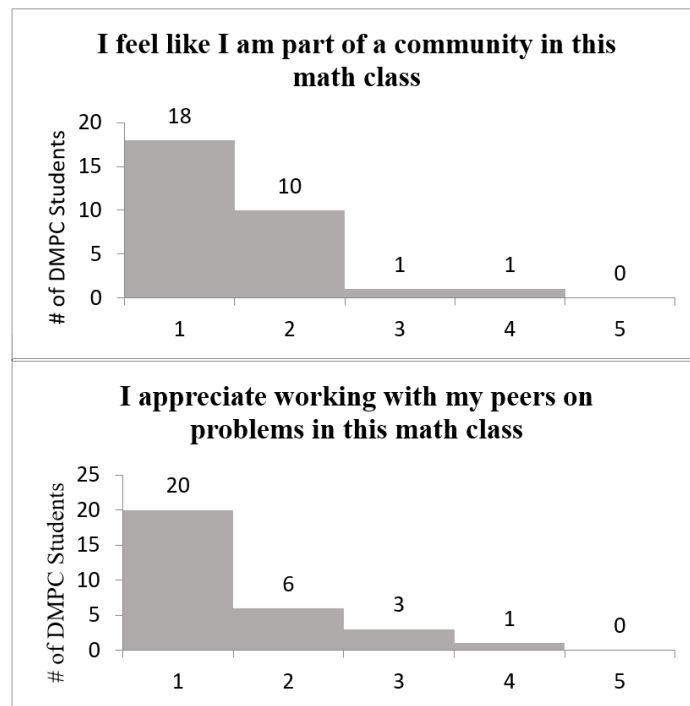


Figure 4.1. Histograms of two relatedness Likert scale items from *strongly agree* (1) to *strongly disagree* (5).

The results of the statistical tests on the survey data suggest the DMPC students feel similarly about competence, autonomy, and relatedness, regardless of various factors (e.g., gender, GPA, etc.). With the exception of one statistics test, none showed a statistically significant difference between a number of demographic and academic factors (see Table 4.4 for a complete list of factors) and the three psychological needs of competence, autonomy, and relatedness. Only a marginal statistically significant difference was detected between the first-

semester grade earned in the DMPC course and the composite score for competence (Kruskal-Wallis, p -value = .045). This result was considered negligible since the p -value was close to the 0.05 threshold and the limited power of the small sample size ($n = 30$; 13 A's, 12 B's, and 5 C's). Amongst several demographic and academic factors, the lack of any statistically significant differences lends further validity to the finding in which the DMPC students describe viewing the contributions of their peers as valuable.

Table 4.4 Inferential Statistical Tests Run on DMPC Data

ANOVA Tests	Independent T-Tests
Cumulative Unweighted GPA Range	Gender (M F)
First-semester Grade in DMPC	Native English Speaker (Y N)
Ethnicity/Race	Qualify for Free or Reduced Lunch (Y N)
Previous Mathematics Course (IM III, Pre-calculus or Other)	

Note: IM (integrated mathematics), M (male), F (female), Y(yes), N (no)

Finding #2: Students Experience Curiosity and Creative Freedom in DMPC

Beyond feelings of relatedness, the uniqueness of discrete mathematics engenders feelings of competence and autonomy in the students through sense-making. Goldin (2018) posits that discrete mathematics differs from more traditional instruction in algebra and geometry with a “central emphasis on reasoning, strategy use, decision-making, and open-ended problem solving” (p. 54). These attributes set the stage for students to conceptualize discrete mathematics in terms of pattern-seeking, which invites curiosity and creative freedom into problem-solving.

Differs from previous mathematics. The students prefer to juxtapose their previous experiences in high school mathematics with their current experiences in the DMPC course to emphasize the uniqueness of they feel in the course. Teddy, Audrey, and Darius paint a picture of their prior mathematics courses as rote and stifling, devoid of competence and autonomy.

Teddy, a Latinx male, depicts his prior experiences in mathematics as dull and detached, “One

thing that kind of sticks out to why I wasn't the biggest fan of math [previous to DMPC] was...just bland.... It's learning the same thing over and over again, but it's just equation over equation over equation.” A fixation on correct equations left Audrey, a Latinx female, feeling marginalized in her previous mathematics classes:

I usually would just be the one getting it wrong or having to have extra help.... Then coming into high school, I feel like it just got worse. The whole you have an equation. You don't get it, then you're wrong. That whole process made me feel belittled and not encouraged to keep trying.

Darius, a multiracial male, contrasts his previous reluctance to engage in mathematics with his new perspective in the DMPC course:

Because I know when I was in regular IM2, IM3 [IM is Integrated Mathematics] and all that, whenever I would get stuck on an equation or I didn't know what to do, I'd just sit there and wait until the teacher explained it. Because I already knew that I didn't know how to finish the problem so I had nothing else to try. But with this class [DMPC], it makes me look for different options, different ways to actually explore a problem.

Teddy, Audrey, and Darius lacked feelings of competence and autonomy in their prior mathematics coursework. The overemphasis on equations left Teddy with little choice or autonomy. Audrey's mistakes in mathematics made her feel fearful and incompetent to engage in mathematics. Similarly, Darius disengaged in his prior mathematics class, feeling unconfident to problem solve, yet he expresses feelings of competence when doing discrete mathematics.

Discrete mathematics is pattern-seeking. The DMPC students liken problem-solving in discrete mathematics to solving puzzles or seeking patterns, which supports them in feeling competence and autonomy. Akino, Audrey, and Aira illustrate how approaching problem-solving in this way sparks curiosity and sense-making. Akino, an Asian female, explains:

It's like we're not only restricted to one way [in DPMC]. We're open to any other ways. To me, discrete [DMPC] is more patterns and finding different ways to solve the problem....We look at patterns to solve problems and we have to find solutions to it.

Akino's analogy between solving problems in DMPC and finding patterns emphasizes the focus on student sense-making and inquiry in the course. Akino has embraced this cognitively demanding approach to problem-solving, which encourages feelings of competence and autonomy. Audrey provides a specific example from the Sequence & Series unit about finding a recursive pattern for Rapunzel's growing hair:

So that's when you look at it, it's a problem, but then once you break it down, it's like, 'Okay. Well, we know this. Her [Rapunzel] hair's going to grow this much every day.' You don't start seeing as a problem anymore. You start seeing it as kind of like a puzzle. You have the pieces and you're just putting them together. Then once you have your answer, you're like, 'Okay. There. That all makes sense.'

By reframing the problem as a puzzle, Audrey becomes curious and invested in discovering the recursive expression, highlighting the competence and autonomy she feels. Aira, a Latinx female, takes it a step further, making connections between finding patterns in discrete mathematics and solving life's dilemmas:

It shows how you can get to a solution in different patterns.... It doesn't have to be one way. You always have many other options that it leads you to the same answer. I think it relates to real life, as I said because I think when you present a problem or a situation in your family or friends or at school, there's always your solution, but if you talk to someone then someone can help you to solve that problem but in a different way.

Aira compares the multiple ways to solve a mathematical problem (finding a pattern) to resolving issues in life. She finds personal significance and purpose problem solving in the DMPC course, which is a characteristic of competence. However, one DMPC student expressed concern about the utility of focusing on patterns in the course. Miguel, a Latinx male, questions the emphasis on pattern-finding:

I want to pursue biology. So if you had to take lecture in math classes [in college], ... I want it to be prepared.... We don't really get any equations or solving circles, circumferences or stuff like that, I think that is a downside to it. Because for the whole college thing, I think, we need that sometimes. Because this class just

really offers just finding patterns and just getting it to keep you motivated. So I think that's the downside.

Miguel admits his own increased motivation and curiosity for finding patterns in DMPC. But he worries this emphasis is misguided. He wonders if he will be less prepared for college-level mathematics as a potential STEM major than if he had taken a more standard high school mathematics class focused on memorizing formulas.

Discrete mathematics is liberating. In the DMPC class, the focus on sense-making or pattern-seeking encourages students to engage freely in problem-solving and to value a diversity of approaches. Ava, Nia, and Aria draw a stark contrast between their previous experiences in high school mathematics courses and their current experiences in the DMPC course. Bemoaning how previous mathematics classes focus on rote procedures, Ava, a Latinx female, appreciates the freedom to solve problems multiple ways:

Before [DMPC], it was always like there's the one way to do it, and it sucks if that one way doesn't work for you.... And then it's like, 'Oh well, moving on to the next thing.' But here [DMPC], we always go over two or three different ways to do each problem. And it really helps because everyone's different...I feel like everyone can understand better this way.

Nia, a Black female, shared a theory on why DMPC is less constrained than her previous mathematics courses:

With regular math, it's the set equation and then you find your numbers. But with discrete [DPMC], you have your numbers and then you find the equation....It doesn't have to be a set equation that you have to memorize. So one can be easier than the other [way]. And that's why I like discrete a lot more than I like regular math.

She astutely observes her previous courses began with the formula and then she would solve for the value of the variable using a prescribed method. In the DPMC course, students are given the number and then have to construct the equation to express the pattern, providing the freedom for creativity and autonomy. Similarly, Aira, a Latinx female, laments the focus on isolated

procedures in her previous mathematics courses, yet traces her increased feelings of competence and autonomy to the focus on reasoning in DPMC:

Yeah. I don't like math, just memorizing the formulas or the equation. But I like when I understand the problem or get very involved in it. So I think discrete math helped me...because it gives you more options to get a solution and to help you thinking and just understand why.... Not just memorizing and pass the class as a requirement.

Aira is dismissive of the extrinsic reward of a grade in favor of gaining satisfaction from understanding a problem in discrete mathematics.

Brittany, Jayden, and even Ms. North all credit discrete mathematics with reshaping their view of the discipline. The emphasis on creative freedom in problem-solving nurtures a deep appreciation for diversity in perspective. Brittany, an Asian female, revealed how she is genuinely curious about how her peers approach a problem in the DMPC course:

I feel like you have more of a free creative outlet in finding your answers because...there's different methods we've used in finding our equations.... In normal math classes, you're taught to think one way, and this is the only way you could do this problem, but in discrete math, you could have multiple ways. There's more than one way to find the answer to your problem. It's a lot of communication with other people to see what they found versus what you found and how your way is different from others.

Jayden, a Black male, finds this freedom and creativity in problem-solving to be revealing of his peers' personalities:

I think that's the best part. Not everybody in our class will get the same equation.... Your equation is well thought out, but mine is using tricks.... I like explaining to other people how my brain works.... I see things in a different pattern so I feel like I'm able to express myself through the math a little bit more.... It definitely shows what kind of person and what kind of character. How do you learn? And what clicks inside of people.

Like the students, Ms. North credits teaching the DMPC course with a deepened and renewed appreciation for mathematics:

I was definitely a kid who was brought up on algorithms and not allowed to find my own way into math.... And so I think, seeing the creativity and the multidimensional aspect of a lot of the problems that we do in [DMPC], I've really begun to see math as this kind of beautiful, magical thing.

The curiosity and creative freedom nurtured in the DMPC course heightened students' sense of competence and autonomy. Free from the burden of only one approach to problem-solving, students can confidently experiment in their approaches and have renewed appreciation for multiple perspectives from their peers.

The quantitative data from the survey complement the qualitative data from the focus group interviews around students' perceptions of competence and autonomy. Seven of the Likert scale items on competence and eight of the Likert scale items on autonomy exceeded the component coefficient threshold 0.5 from the principal component analysis (see Appendix E). The means and medians of the resulting competence and autonomy composite scores lie between *strongly agree* (1) and *agree* (2) (see Table 4.5). The standard deviations and IQRs of competence and autonomy are all under 0.7, indicating the students' responses clustered near the measures of central tendency. Fourteen of the 15 competence and autonomy Likert scale items have a mode of 1.0 or 2.0. Thus, the quantitative findings closely parallel the qualitative findings: DMPC students feel a sense of competence and autonomy in the course.

Table 4.5 Descriptive Statistics for DMPC Composite and Autonomy Composite Scores (n = 30)

	Mean	Median	Std. Deviation	IQR	Range
Competence Composite Score	1.810	1.786	0.567	0.679	2.143
Autonomy Composite Score	1.767	1.750	0.552	0.625	1.750

In self-determination theory, competence relates to students' feelings of confidence and self-efficacy. Histograms of two selected Likert scale items capture the high degree of competence expressed by the DMPC students from among the seven items that form the

composite score of competence (see Figure 4.2 and Appendix E). The first item echoes the students' assuredness with sharing their thinking in the DMPC class. Eighty-three percent of the students reported feeling confident to share their approaches to difficult problems. The second Likert scale item suggests there is a greater sense of competence when focused on their individual ability. Overwhelmingly, 96 percent of the DMPC students express their sense of self-efficacy when engaging in discrete mathematics; only one student reported feeling *neutral* (3).

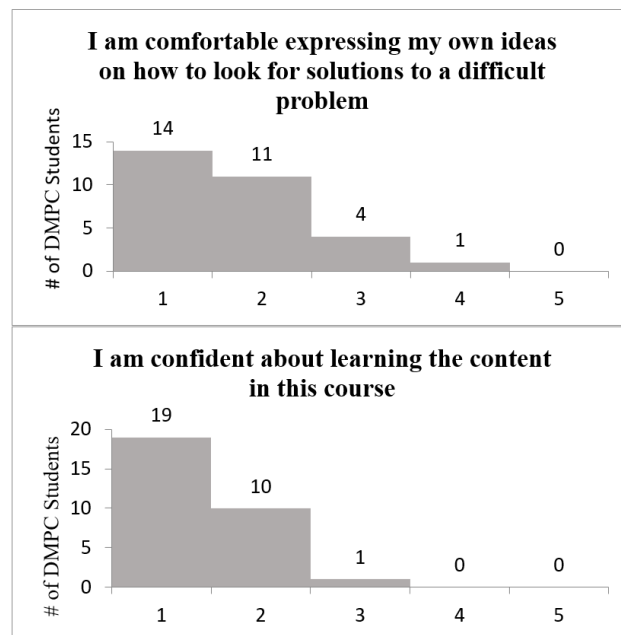


Figure 4.2. Histograms of two competence Likert scale items from *strongly agree* (1) to *strongly disagree* (5).

The autonomy Likert scale items follow a similar pattern with student responses skewing to the right since most DMPC students selected *agree* or *strongly agree*. The autonomy items in Figure 4.3 are representative of the pattern in the eight items making up the autonomy composite score (see Appendix E). The first item reflects the DMPC students' overall enthusiasm. The second item highlights a reason for their enjoyment, alluding to the agency the students experience in the DMPC class. In the student focus groups, many depicted a sense of creative

freedom they felt in selecting how to approach to problems and to represent their thinking in a variety of forms. Of note, on these two autonomy items, none of the DMCP students selected (4) *disagree* or (5) *strongly disagree*.

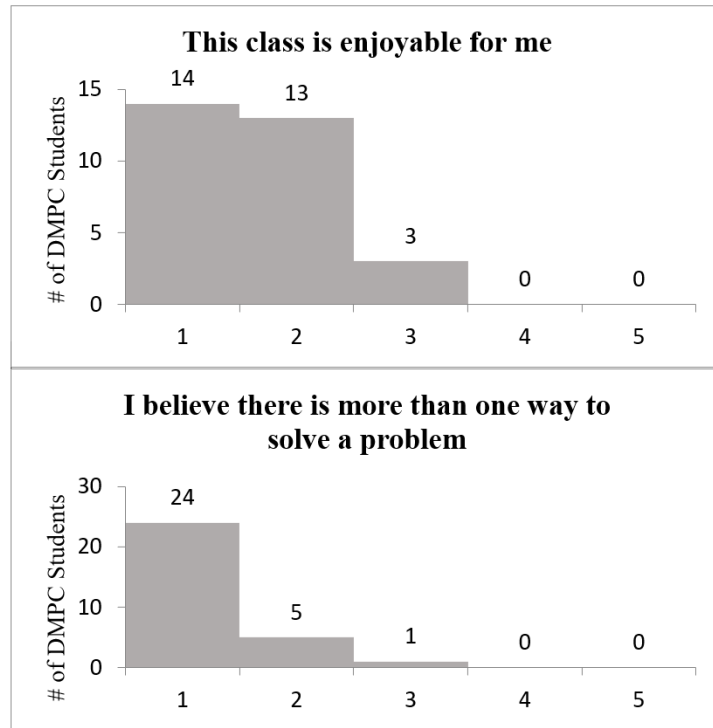


Figure 4.3. Histograms of two autonomy Likert scale items from *strongly agree* (1) to *strongly disagree* (5).

Correlations between competence, autonomy, and relatedness suggest DMPC students feel similarly across all three psychological factors. There is a moderate to strong positive Spearman’s correlation between the composite scores for competence, autonomy, and relatedness (see Table 4.6), providing additional support to students’ perception that the class is a “positive space” for doing mathematics.

Table 4.6 Spearman’s Correlation of Competence, Autonomy, and Relatedness in DMPC

	Competence	Autonomy
Competence Spearman’s rho	--	--
Autonomy Spearman’s rho	0.551	--
Relatedness Spearman’s rho	0.455	0.727

The students’ similar composite scores for competence, autonomy, and relatedness are also reflected visually in side-by-side boxplots (see Figure 4.4). The composite scores represent the seven competence items, eight autonomy items, and seven relatedness items that exceed the component coefficient threshold of 0.5 (see Appendix E). Notably, the IQRs for all three composite scores fall between *strongly agree* (1) and *agree* (2). The boxplots show how tightly clustered the data is above *neutral* (3) and slightly skewed to the right.

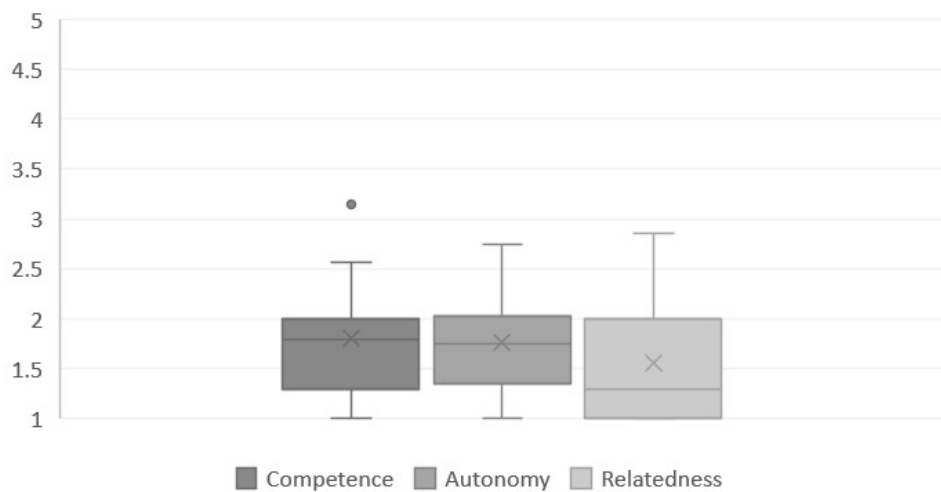


Figure 4.4 Boxplots of the DMPC composite scores for competence, autonomy and relatedness.

Finding #3: The Complex Use of Fun and Easy in DMPC

The DMPC students use the terms *fun* and *easy* in unique ways to describe their experiences and perceptions of the course. Nine out of 30 students referred to the DMPC course

as *fun* in the free-response item on the survey, heightening my awareness for when students used the word 27 times during the focus groups. DMPC students uttered the word *easy* 35 times and *easier* 28 times during the focus group discussions, highlighting its significance. Taken together, the DMPC students assign specific meanings to the vague descriptors of *fun* and *easy*, conveying their experiences and perceptions of engagement and accessibility in the course.

The meanings of *fun*. DMPC students, Nyah and Audrey, employ the word *fun* to express their engagement with mathematics and the freedom and enjoyment they perceive when problem-solving. Discrete mathematics relies on relatable contexts but poses intriguing twists to pique students' curiosity (Goldin, 2018). Not surprisingly, many DMPC students fondly recall the contexts of specific tasks and units including the Games unit. For Nyah, a Black female, the Games unit surpassed her expectations:

Alright, games. That's going to be chill, playing games. But it turned out to be actually fun. And even though at times it did get annoying because it'd be every single day...same game or just a slightly different variation of it, but it was still fun... Even now, we're all like, 'Can we go back to the games unit?' We all miss that unit. It was so much fun.

Audrey, a Mexican/Latinx female, recalled the enjoyment she had discovering and applying the winning strategy in the games unit:

That was fun because then we got to play with each other. Then it would be like, 'Okay. I'll let you win this time,' or, 'I'll lose this time.' You could control that because we got to understand why it was. Who won was that person. Just the method and finding that. Then once you found that, you were just like, 'Okay. Let's play. I'm going to win. I'm going to win.' So that was fun.

Repeatedly using the word *fun* in their descriptions, Nyah and Audrey capture the excitement they had exploring two-player games and conjecturing about the guaranteed winning strategy.

Additionally, Nia's and Savannah's descriptions of *fun* highlight their engagement with the process of problem-solving. Nia, a Black female, uses *fun* to describe the autonomy and

agency she perceives when problem-solving in DMPC, “It’s [DMPC] funner. Math is funner than it was before. And I like to just be able to do my own thing and figure it out myself without having a set equation.” Savannah, a White female, associates the *fun* with making mistakes:

Sometimes when you're trying to figure out something in math, it's kind of fun because it's like, ‘Oh, I got that wrong. Let me try this.’ I don't know. I just think working through problems is just fun to me, especially when you know what you're doing and when you know what you're doing, and you're getting it wrong, but you know you're still going to get it right eventually because you're not worried about it.

Savannah's use of *fun* shows her self-efficacy in problem solving (“you’re still going to get it right”) and the creative freedom she has in revising her approach (“let me try this”).

Nyah and Savannah both use *fun* to capture how they perceive the supportive DMPC class community. Nyah depicts the lively nature of Ms. North’s classroom, “So we're all jumping around but also getting work done. So it's a fun class but also a very productive class. And that's the first time I've ever had that in a math class, and I'm really loving there right now.” Nyah marvels over how joy and productivity can coexist in a mathematics class. Savannah equates *fun* with being in a community of learners, “I think it's really fun when you understand it, and you're up on the screen, and you're teaching people how to do it. They're like, ‘Oh, that's how you do it.’ It's like we're coming to this understanding together.”

The meanings of *easy*. The DMPC students invoke *easy* to contrast DMPC with the rigidity of their previous mathematics courses and to describe the focus on reasoning. Interestingly, while DMPC students freely use *easy* to describe their experiences in the course, they bristle when their peers not enrolled in the course refer to it as *easy*.

Nia uses *easier* to distinguish DMPC from her prior perspective mathematics classes, appreciating the freedom she experiences when problem-solving:

And math [DMPC] for me, it's made...it's easier for me. And it feels like I can do what I want when it comes to figuring out the equation. So I can start from where I want to start and still get the same answer if someone started differently. And you don't have to use that same set equation as regular math. So with discrete, I can start the way I want to start and not the way someone told me I should start.

The agency, or autonomy, Nia perceives allows her to engage in the DPMC content since she is not confined to one approach and one starting point. Ava, a Latinx female, suggests having to justify her reasoning makes the course *easier* than her previous honors class:

It's [DMPC] different than the other math classes. Because last year I was in pre-calc honors. And it's there just... You just do the equation, you plug in the numbers, and then just get your answer. And then here, well, you do the same thing a little bit, but the math is easier.... You're focused on why the equation or whatever works, and how you got your answer.

Although DMPC students may ascribe the label *easy* to different aspects of the course, they reject the perception of the class itself as *easy* by outsiders. Akino, an Asian female, admitted she enrolled in the course because of the reputation of the course as *easy*, yet no longer shares that belief:

Yeah. I'm going to be honest with you. I thought it was an easy A class, but the first day we walked in it was already patterns and just everything at us. Because I remember it was the connect computers and we had to draw lines, and I was like what is this. But then, as I was solving, I was like, whoa this is so cool. We could see different patterns connecting all the computers to each other. And it is not an easy A class. You actually have to put in work into it.

Ava feels slighted by how her peers in AP mathematics classes dismiss the rigor and cognitive challenge of DPMC:

Oh, some of my friends are in AP stats. Then you have some in AP calc. And then I'm like, 'Yeah, I'm in Discrete.' And they're like, 'Oh, why are you in that class? It's so easy. You guys don't do anything.' But it's not that easy. You actually have to think more. I mean, I don't know if think more but think in a different way. And everyone thinks it's easy, but you still do the same amount of work.

Akino and Ava both reject insinuations of the course as undemanding or *easy*. Ms. North accurately predicted how the DMPC students perceive the course, “I think they'd say it's fun,....I think fun and challenging is probably the top two.” The students routinely use *fun* and *easy* to describe their positive learning experiences in DPMC, though they reject others' perceptions of the course as less rigorous.

Summary

The three findings for the DMPC case study piece together a picture of the students' experiences and perceptions in discrete mathematics. The three psychological needs of competence, autonomy, and relatedness create a framework for examining those experiences and perceptions. The contrast between their prior experiences in high school mathematics and the DMPC course sharpen the clarity of the findings. Previously, the DMPC students considered high school mathematics to be isolating, judgmental, and uninspiring. However, discrete mathematics dramatically differs by dismantling traditional hierarchies and fostering a vibrant community of doers of mathematics. DMPC students feel a sense of belonging and relatedness within the course, by being active participants in the sense-making process. In the DMPC course, students conceive of discrete mathematics as pattern-seeking alluding to the curiosity and creative freedom they have in exploring the content. These two nouns underlie their sense of competence and autonomy in the course. Finally, the DMPC students use *fun* and *easy* or *easier* in clever ways to accentuate the engagement and accessibility of the course.

IDS

Being the first period of the day, a few IDS students enter clutching the school-issued breakfast to munch on during class. Ms. Arcega stands near the door, wearing her ballet flats. A wavy bob frames her friendly face. She greets the students as they trickle into the room. The

students affectionately refer to Ms. Arcega as “Miss” dropping off her last name. “Miss, did you watch more episodes of *You* on Netflix last night?” “Miss, did you remember your water bottle today?” “Miss, I need help with ___.” After the final bell rings, Ms. Arcega counters the subdued mood of the early class by generating enthusiasm for the context of the lesson. One day, students eagerly listed what made them feel stressed or relaxed, perking up as they shouted out ideas to their group mates. Another day, Ms. Arcega played the classic Celine Dion song from the *Titanic* soundtrack, before launching into an exploration of the probability of living or dying on the doomed ocean liner.

Ms. Arcega decorated her classroom walls with an eclectic mix of charts of lines of code, statistical concepts, and inspiring social justice messages. The We the Future illustrated posters - - done in the style of the iconic Women’s March posters -- feature young BIPOC individuals with empowering sentiments: “The tyrant fears the poet” and “Write your own liberation.” The single-person desks are grouped in pods of four throughout the room. The structure lends itself to camaraderie as the students program in R-studio and glance at each other’s screens debugging lines of code and exchanging light-hearted banter.

Participants

Ms. Arcega, a Russian-Mexican American in her early 40s, originally wanted to be an Algebra II teacher. Dissuaded by her high school counselor, Ms. Arcega, majored in business instead. After a series of unsatisfying jobs, Ms. Arcega returned to her original aspiration and earned her single-subject mathematics credential. Ms. Arcega now teaches a full line of high school statistics in all its variations -- statistics, AP Statistics, and IDS. She has taught the IDS course for the past four years. The program director routinely invites representatives from other

districts, considering adopting the IDS course, to observe Ms. Arcega’s class. She facilitates workshops for new IDS teachers and participates in the monthly IDS teacher network.

Ms. Arcega’s IDS class is small and predominantly male. Nineteen IDS students out of 23 consented to participate in the research study, self-reporting demographic and academic data (see Tables 4.7, 4.8, 4.9 and Appendix D). Similar to the overall school population, 84 percent of the surveyed IDS students self-identified as either Latinx or Asian. However, unlike Sierra Madre High School as a whole, there was a higher percentage of Latinx (47 percent) than Asian (37 percent) students. Seventy-nine percent of participating students qualify for the federal lunch program closely reflective of the 80 percent of the school population identified as socioeconomically disadvantaged (CDE, n.d.). The eight IDS focus group students (see Table 4.7 and Appendix D) reflect the gender imbalance and federal lunch program status of the IDS class.

Table 4.7 Demographics of IDS Class and Student Focus Group Participants

	IDS Class (n = 19)	IDS Focus Group (n = 8)
Gender		
Male	14	6
Female	5	2
Ethnicity/Race		
Latinx	9	5
White	1	1
Asian	7	2
Multiracial	2	0
Federal Lunch Status		
Qualify for Free or Reduced Lunch	15	6
Grade Level		
Juniors	6	4
Seniors	13	4

The IDS students surveyed performed better academically in the course than in high school generally. Fifty-eight percent of the IDS students surveyed earned at least a B in the IDS

course for their fall semester grade, while only 26 percent reported a cumulative unweighted GPA of at least 3.0 (See Tables 4.8 and 4.9). Most of the surveyed IDS students (79 percent) enrolled in the course after finishing either Integrated Mathematics II or III. The other 21 percent completed statistics before enrolling in the IDS course. The eight IDS students who participated in the focus groups represented all three pathways into the course and closely mirrored the cumulative unweighted GPA ranges of the surveyed class (see Table 4.9).

Table 4.8 First Semester IDS Grades

Fall IDS Grade	A	B	C	D	F
IDS Students (n = 19)	7	4	4	3	1

Table 4.9 Cumulative Unweighted GPA Range of IDS Class and Student Focus Group Participants

GPA Range	IDS Class (n = 19)	IDS Focus Group (n = 8)
Below 2.0	3	1
2.0 to 2.5	6	2
2.5 to 3.0	5	3
3.0 to 3.5	3	1
3.5 to 4.0	2	1

Note: Students selected a range that best matched their cumulative unweighted GPA.

Finding #1: Students Perceive Themselves as Connected within a Community of Learners

The paradigm of the teacher as the holder of all knowledge is unsustainable in data science since the students engage in “investigations in which learners pose questions, obtain data, and communicate findings” (Wilkerson & Polman, 2020, p.5). In other words, the students perceive themselves as active learners alongside the teacher, translating into a *we are in this together* community feeling. This sense of community or relatedness in the IDS course manifests itself in three meaningful ways - pride in being one of the first to enroll in the course, an appreciation for the collective wisdom of their peers, and an expansion of their social circle.

The students feel they are participating in a groundbreaking experience by taking the IDS course. The IDS students Nolan and Conner are aware of the course's beginnings in a nearby school district and count themselves fortunate to be among the first to enroll in the course. Nolan, a Latinx junior, senses his enrollment in the course is historical, "This class [IDS] is just starting and we're going to be part of something very big. And I'm really grateful for the opportunity of being something that's going to be going big later on." Conner, a Latinx senior, notes how the IDS course began in Los Angeles and values being part of a course that may expand far beyond the state:

IDS class just started here in Los Angeles...What I've experienced. I feel as though this class will continue to grow exponentially. So I think that's like the coolest part about IDS right now, too. That we're at the start, right when it's just starting up. So we're learning as we go. It's the best part for me.

The student's sense of being part of something larger than themselves translates into a feeling of unity, belonging, and relatedness in the IDS classroom.

The IDS students frequently comment on the togetherness they feel in the IDS classroom.

Ms. Arcega attributes this sense of community to her efforts early in the school year:

We do a lot of group work and getting to know each other and I rotate seats often.... Since it is a small class, we gotten to know each other fairly well. They... know who needs more help...and they help each other out.

As a result, Ms. Arcega notes how the students willingly and mutually rely on each other. Nolan and Catherine illustrate how they view their peers as capable doers of mathematics. Nolan values learning from his peers, even forming new friendships:

I love the aspect of working together in groups of two or more. And sometimes I don't understand it, I'll be like, 'Hey Will [another IDS student], will you help me out?' And I really like the aspect that we can help each other. And also it gives us an opportunity to talk to people that we never hung around with in our life. And so I really like that because you get to conversate with someone new, make new friends.

Since the data science content is new to the students, Catherine, a Latinx junior appreciates the *we are in this together* feeling and trusts the collective wisdom of her peers:

It's good because everyone in that class could help you because we're all at the same level. We're all beginners into IDS, so we're at the same level and we could really help each other. And I feel like, if I'm ever stuck, I have multiple people to go to...and everybody gets along, and there's never really any drama or nothing like that.

Two IDS students, Grace and Connor, use words such as “kind” and “worry” to capture the sense of community among the IDS students in the course. Grace, an Asian senior, clarifies the sense of relatedness she feels, “For us, we're all kind to each other, we help one another and we give ideas to each other as well, and we come up with conclusions to solve the question.” Taking a step further, Connor shares how the sense of community extends beyond collaboration, “So yeah, we do get to collaborate a lot in this class, and it does give us a chance to kind of worry about each other and help each other out.” Connor’s feelings of relatedness include concern for his fellow IDS classmates.

The quantitative data from the survey of IDS students echoes the sense of community and relatedness the students describe in the focus groups. Ten of the 17 Likert scale items on relatedness passed the principal component analysis coefficient threshold of 0.5 (see Appendix E). The mean and median composite score for relatedness is close to 2.0 or *agree* on the scale (see Table 4.10). Both the standard deviations and IQRs are less than 1.0, indicating many of the student responses are above 3.0 or *neutral* on the scale. Sixty percent of the Likert scale relatedness items have modes of 1.0 or 2.0. In sum, the descriptive data imply the positive sense of relatedness reported by the students.

Table 4.10

Descriptive Statistics for IDS Relatedness Composite Score

	Mean	Median	Std. Deviation	IQR	Range
Relatedness Composite Score (n = 19)	2.189	2.200	0.706	0.800	2.300

The histograms of two relatedness Likert scale items give additional insights into trends observed with the 10 statements comprising the relatedness composite score (see figure 4.5 and Appendix E). The first item focuses on the students’ comfort level with the class. The histogram skews to the right and is bimodal with an equal number of students selecting *agree* (2) and *strongly agree* (1) as their response to the statement, “When I am in this math class, I feel comfortable.” Thus, 84 percent of the students feel at ease in the course. The second item examines student perceptions of collaboration with their peers. With the exception of one student, who selected *strongly disagree* (5), all the IDS respondents either welcome (14 students) or feel impartial (4 students) to working with their peers to solve data science problems. Combined, these two histograms mirror the patterns observed with the relatedness composite scores.

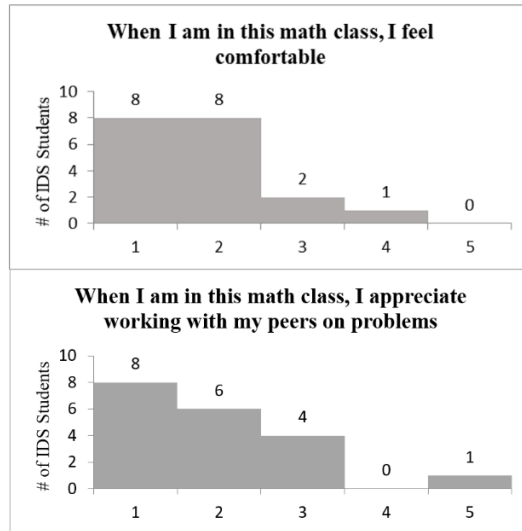


Figure 4.5 Histograms of two relatedness Likert scale items from *strongly agree* (1) to *strongly disagree* (5).

No statistical differences were found between multiple academic and demographic factors of the IDS students and their composite ratings of competence, autonomy, and relatedness (see Table 4.11 for a complete list of factors). The lack of any statistically significant differences complements the students' perceptions of relatedness in the course and the *we are in this together* feeling the students described, though it is worth noting that only 19 of the 23 IDS students participated in the survey. There is less power in such a limited sample, making it harder to detect statistically significant differences.

Table 4.11 Inferential Statistical Tests Run on IDS Data

ANOVA Tests	Independent T-Tests
Cumulative Unweighted GPA Range	Gender (M F)
First-semester Grade in IDS	Native English Speaker (Y N)
Ethnicity/Race	Qualify for Free or Reduced Lunch (Y N)
Previous Mathematics Course (IM II, IM III, Statistics)	

Note: IM (integrated mathematics), M (male), F (female), Y(yes), N (no)

Finding #2: Students Experience Curiosity and Creative Freedom in IDS

We are in this together sense of community or relatedness originates in part from the IDS students conceptualizing their experiences and perceptions of the course through the lens of coding. In their descriptions of the course, IDS students conflate coding with data science, admiring coding's inherent purpose, precision, and efficiency. The students make sense of the IDS course, by comparing it to their previous high school mathematics courses including statistics. The IDS students feel connected to the curriculum and are empowered problem solvers, which is diametrically opposed to what they previously experienced, namely detachment and failure.

Catherine and Connor both exemplify this sentiment. Catherine, a Latinx junior, strongly favors the IDS course over her previous mathematics courses (Integrated Mathematics I and II), "I like the math in IDS but I don't like the math out of IDS." Connor, a Latinx senior who previously took statistics, shares Catherine's view, "I love math only if it's coding math now. If I am being honest." By conceptualizing data science as coding, the IDS students feel a sense of competence and autonomy. They confidently engage in coding and value the freedom they have to explore in R Studio.

Coding is purposeful. The IDS students found memorization tedious and largely onerous in previous mathematics courses. Joseph, a Latinx junior, laments the futility of memorizing formulas, "One week we would be learning this formula [in a previous mathematics course] and after that then we go to another one. I could never...memorize it fully." Grace, an Asian senior, concurs complaining, "I would say math, in general, is quite hard for all the formulas you had to remember like from elementary till now," musing, "some of the stuff you don't really need in the future." But Grace draws a sharp contrast to knowing codes in R Studio, "For IDS everything is

on the computer and you just to take down the code.... You just got to know what it means and it's done.” Grace finds a purpose in knowing codes for the IDS course. Catherine further asserts the convenience and utility of memorizing these codes:

I feel it [previous mathematics] was too much things that I feel I don't need in life. Like a whole bunch of formulas, you had to memorize. And with IDS...you just memorize codes, and the codes aren't that hard to memorize because it's all the same codes throughout the whole year.... You don't really have to memorize that many codes. You learn new codes on the way, but I still use codes that I learned from the first day of school.

Unlike abstract formulas of previous mathematics courses, Catherine finds the codes she learned early in the school year for R Studio continue to be useful. Connor finds purpose and meaning in the precision required for coding:

If you miss a comma...you can't have a proper graph. If you miss a parenthesis, you can't have [a] proper graph. If you misspell something, it won't work. You have to be right 100% of the time for it to work. That's what I like, you have to be 100 percent with everything.

Any omitted or incorrect character will make a code inoperable. A characteristic of competence is finding meaning and purpose when engaging in cognitively rich tasks. IDS students perceive coding, a tool of data science, as useful and necessary for the challenging tasks of the curriculum.

Coding is efficient. The IDS students view coding in R Studio as far more sophisticated than hand calculations on paper, derisively describing their previous courses as *paper and pencil* mathematics. Caleb, a Latinx junior, extols the benefits of coding for large data sets, “Coding...work for like big sets of data. Where you can like it would take you forever to do on just paper and pencil.” Additionally, Nolan marvels at using technology to solve problems in the IDS course, “For me, I wasn't the best at math...I thought it [the IDS class] will be the same thing. Until after the first day, and then I was like, wow, this is very interesting. I get to use

computers, to solve my problems.” Coding in R Studio provides Nolan with a sense of competence and autonomy in the IDS course. Unlike calculations by hand or typing in a calculator as in previous mathematics courses, Catherine surmises the efficiency of coding makes it seem effortless, especially when she compares IDS work with her peers in other mathematics courses

‘Oh, I have so much homework.’ [her peers complain] And I look at it, and I do nothing like that. [IDS work] It's all just coding. It's like... all you do is just code, and it [R Studio] does everything for you. And I feel that's pretty cool.

Connor also admires the power of a line of code, “Hit a couple of keys and you get your answer... [made a clicking sound]. That's it, that simple.” Such sentiments seemingly dismiss and obscure the intellectual challenge, competence, and autonomy necessary to code accurately. Although Catherine and Connor possess skill and knowledge to write the code, they ascribe the hard work to the code itself in R Studio. In other words, the efficiency of coding is appreciated, especially in comparison to the tedium of former mathematics classes, yet it can also conceal the intellectual accomplishment required to code correctly.

Coding is empowering. Learning codes in R Studio supports the IDS students’ feelings of competence and autonomy, through students’ interpretations of the data and the choices they make to represent the data. For example, Liam, a White senior, appreciates how his point of view in the IDS course matters in solving tasks:

Some days we talk about different stuff like for example, we recently did something about the Titanic. About like how many people like survived and died. It [any task in the IDS curriculum] involves like our perspective and what we think in our opinion.

He feels empowered and qualified to give his thoughts, unlike his previous mathematics courses. In Integrated Mathematics I through III, Liam had minimal feelings of competence remarking, “most of the time...I didn't know what I was presenting.” Although Connor previously failed

statistics (by his own admission), his sense of competence and confidence in coding empowers him to learn more advanced concepts:

I ask Miss, our teacher, about like hard graphs if there were any. And she gave me a paper, and it was about GG plots. So I started messing around, and I got all the possible plots somehow. Honestly, I didn't know I was going to do it, but I did it.

Coding also empowers IDS students' freedom of expression and autonomy. Nolan values the agency he has in making representations of the data:

I like that.... I don't have to follow one simple rule, one way, I don't have to follow one way to get this answer, I like how I can just change it up a bit. I like it how I can use a histogram or maybe a bar graph or like tally or a XY plot [for] my answer.

Similarly, Catherine finds empowerment in executing the codes to create visualizations of the data:

One interesting thing that I like with the coding, I like how you could change the colors of the graphs.... I like how there are these things called arguments...they could do certain things. So, I find it cool how every little thing can do something, like every piece of code can do a big thing with the graph.

Through coding in R Studio, the IDS students experience competence and autonomy. They feel self-efficacy, challenging themselves to go beyond the curriculum. And they feel empowered and autonomy to explore and represent the data using the codes they have learned.

Ms. Arcega shares how a former IDS student felt empowered by what he learned in the course to enact social change at Sierra Madre High. As a junior he took the IDS course and then senior year, "he created a documentary on... grab-and-go breakfast. He...noticed that they [cafeteria personnel] require the kids take all the food, whether they want to eat it or not." The student was concerned about the resulting food waste. He approached Ms. Arcega about how he could apply what he learned in the IDS course the previous year. She organized her IDS class at the time to collect data on the discarded food each morning, "They logged it how much each

there was in each food and then we created a CSV file and uploaded into R studio. Boom, made some graphs and... now we have a food recycling program.”

Context is engaging. Underlying the interest in coding is the intriguing context of the IDS data sets, which spark curiosity and engagement in the IDS students. Students can be both a *consumer* and *producer* of data, bringing unique relevancy to a data science course (Gould, 2010, p. 310). In the course, the IDS students explore pre-existing data sets (e.g., Titanic passenger logs) and create their own (e.g., Stress and Chill lab). The featured contexts provide authentic opportunities for IDS students to experience competence and autonomy. The IDS students express genuine interest in the context-driven, challenging labs (competence) and have the freedom to explore these data sets using coding (autonomy). Grace can easily rattle off some of the most intriguing labs:

I thought some of the labs that we did were really cool. We did one with songs and another with horror movies. For the songs, we see what people like, either like rap, hip hop, jazz. And noting carefully how much people liked which kind of genre of the music that they liked, which was really nice to know. And for horror movies, we see which specific gender tends to survive more, and we found out that females survived more than males. And we were also thinking why did the female survive more, and we would try to think of ways, why and what did they do.

Grace fondly recalls both the context and the crux of the lab investigation, continuing on to speculate on why males may perish more frequently in horror movies. Joseph perks up as he describes the participatory sensing lab about his and his classmates' stress levels:

Stress and chill...It was one of the labs that we had to go home and at a specific time, we would record what we think our stress level was. If we were watching TV at 7, we would write down a 1 or a 0 because that's pretty chill or if we were maybe doing some pretty difficult homework and at a specific time then we would record [a higher number]. Then after everything was done, every one put in their stress levels then we would make it into a data set. Then we would work with the data that we all put in.

He was fascinated by a data set generated from his peers' stress levels and what he could learn using the skills he gained in the IDS course. Both Grace and Joseph feel competence, by confidently engaging in the IDS labs and genuinely interested in the exploration. Connor experienced competence and autonomy when Ms. Arcega directed the class to select a data set from the internet and upload it into R Studio:

Oh, there was a time where...we picked up our own data. We had to explore the web. We uploaded our own data onto R Studio. We were asking questions that we want to know. Like, for me, I think of weight training, powerlifting. I was wondering like if age had anything to do with how much weight they pick up and specific workouts, how much would they pick up? So yeah, we actually do a lot of freedom, a lot of choice.

Connor chose a database on weightlifting to explore his hobby using the techniques he learned in the IDS course and what interested him most (e.g., how age may relate to weight lifting limit). The given and curated contexts intrigued the students and let them meaningfully engage in data science.

Like the ratings for relatedness, the Likert scale items for competence and autonomy closely mirror the qualitative findings. IDS students' descriptions of curiosity and creative freedom parallel the positive ratings of the competency and autonomy on the survey. Six Likert scale items for competence and 10 of the Likert scale items for autonomy exceeded the 0.5 threshold on the principal component analysis (see Appendix E). The means and medians for competence and autonomy composite scores are close to 2.0 or *agree* on the rankings (see Table 4.12). Not surprisingly, nearly all the mode rankings for competence and autonomy items are either 1.0 (*strongly agree*) or 2.0 (*agree*), with the exception of one competence item that had a mode of 3.0 (*neutral*). With the small sample size of 19 students more variation is expected in the data. The competence rankings show slightly greater variation than autonomy, but standard

deviations and IQRs for both factors are 1.0 or less, indicating many students' rankings cluster between 1.0 *strongly agree* and 3.0 *neutral*.

Table 4.12 Descriptive Statistics for IDS Composite and Autonomy Composite Scores (n = 19)

	Mean	Median	Std. Deviation	IQR	Range
Competence Composite Score	2.079	2.000	0.802	1.000	3.000
Autonomy Composite Score	2.053	1.900	0.654	0.800	2.600

Reviewing the six Likert scale items focused on competence, the IDS students' responses imply their curiosity and confidence in the course and parallel the pattern already observed in the competence composite scores. Most of the IDS student responses cluster between *agree* (2) and *strongly agree* (1) as seen visually in the pronounced rightward skew in the histograms for the two selected competence items (see Figure 4.6). These two items address characteristics of competence – finding relevance (“The content is very interesting”) and possessing confidence in one’s ability (“I am confident about learning the content in this course”). Nearly 90 percent of the IDS students find the content of the IDS course interesting and confidently engage in data science.

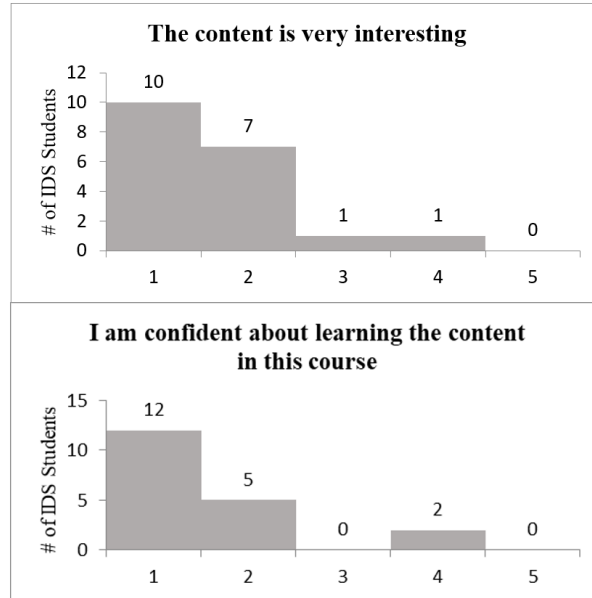


Figure 4.6. Histograms of two competence Likert scale items from *strongly agree* (1) to *strongly disagree* (5).

Similarly, an analysis of two of 10 autonomy Likert scale items show the tell-tale histogram skewed to the right, with the majority of IDS students choosing *agree* (2) or *strongly agree* (1) (see Figure 4.7). The two statements explore aspects of autonomy, namely the IDS students' positive engagement and the freedom they feel in analyzing the data and choosing how to display their findings. These sentiments are mirrored in the 79 percent of students who agree the IDS course is enjoyable and the 89 percent of students who agree there are multiple ways to solve a problem.

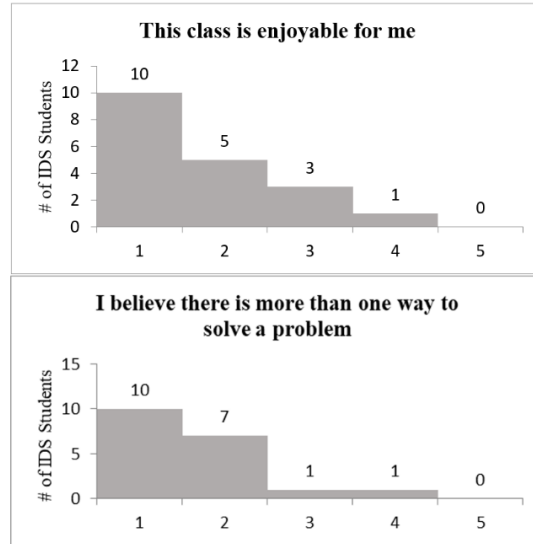


Figure 4.7. Histograms of two autonomy Likert scale items from *strongly agree* (1) to *strongly disagree* (5).

IDS students' composite scores for competence, autonomy, and relatedness are remarkably similar with a moderate to strong positive Spearman's correlations (See Table 4.13), implying the three factors are associated together. Thus, if students feel a strong sense of relatedness, they also likely felt a similar level of competence and autonomy.

Table 4.13 Spearman's Correlation of Competence, Autonomy, and Relatedness in IDS

	Competence	Autonomy
Competence Spearman's rho	--	
Autonomy Spearman's rho	0.545	--
Relatedness Spearman's rho	0.640	0.466

The boxplots show visually the similarity between the composite scores for competence, autonomy, and relatedness in the IDS course. Six of the 12 competence items, 10 of the 17 autonomy items, and 10 of the 14 relatedness items exceeded the component coefficient threshold of 0.5. For all three psychological factors, most of the composite score data lie on the

agreement side of *neutral* (3.0), with the three IQRs tightly clustered between approximately 1.5 and 2.5. The median of competence and autonomy composite scores are close to 2.0 (*agree*) and the median of relatedness is slightly beyond 2.0. Competence has the most variation, extending to 4.0 (*disagree*).

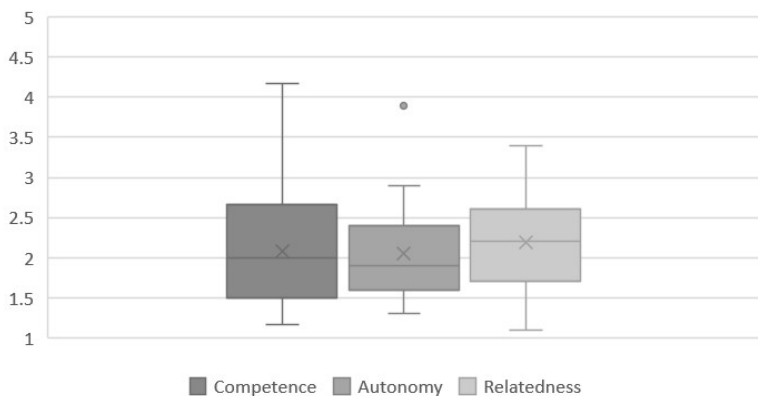


Figure 4.8 Boxplots of the IDS composite scores for competence, autonomy and relatedness.

Finding #3: The Complex Use of Fun and Easy in IDS

When the IDS students describe their experiences and perceptions in the course, they frequently rely on the generic words *fun* and *easy* in their responses. In the IDS student focus group transcripts, *fun* appears 20 times, *easy* appears 22 times, and its variant, *easier*, appears another 25 times. These popular yet vague descriptors fail to adequately capture the more nuanced perceptions the IDS students intend to convey. During the focus group interviews and the subsequent transcript analysis, I sought to clarify what the IDS students meant by *fun* and *easy*. The IDS students often employ these two words as foils to draw contrasts with their previous negative experiences in and perceptions of high school mathematics.

The meaning of *fun*. The IDS students use *fun* as a proxy to capture the novel or revolutionary nature of data science, in particular, coding. The IDS course is a dramatic

departure from their previous high school mathematics content and experiences. The word *fun* appears in close proximity with hallmarks of the IDS course such as computers, coding, and graphical displays. Liam notes his initial uncertainty about the IDS course, “When I first came to IDS, I was a little bit nervous...But then afterwards..., it was..., kind of fun because like, it's not like the typical math classes. It's...math on the computers. So that was a kind of interesting to me.” Caleb compares coding to solving puzzles using the adjective *fun*, “I like puzzles. I like putting things together and coming up with something. So like, coding has been really fun for me.” Likewise, Will associates *fun* with coding, “solve problems with different graphs and all that, and all these different codes. And it's fun to learn, really fun to learn.” Catherine is even more blunt declaring, “So, I took IDS and I don't regret it at all. It's been a cool experience and it's nothing compared to IM3 [Integrated Math III] ... [IDS] It's all coding and it's really interesting and fun.” For the IDS students, the use of the word *fun* is closely associated with the positive and novel experiences students are having in the IDS course, especially centered on coding.

The meanings of *easy*. IDS students use the word *easy* to describe the accessibility of the course, the efficiency of coding, improved academic standing, and a less strenuous workload. Ms. Arcega anticipated one of the definitions for the word *easy*, “They'll [IDS students] say it's easy, and when asking them what makes it easy, and they say, ‘Well, it makes sense. I understand. The problems make sense.’” The IDS students associate the word *easy* with being able to understand the content. Grace viewed her previous mathematics courses, including statistics, as nearly impenetrable:

Since I wasn't great at math, I took the class and it was a bit different than IM1 and IM2 [Integrated Mathematics I and II] or stats...I get to learn quicker and in a different way from math. For regular math, it's a harder way to learn so it's more

difficult for me, and I can't really understand, it's hard to understand. While IDS it's really easy to understand.

Grace attributes this accessibility to the coding in the IDS course:

It was much easier, all I had to do was remember all the codes and what they meant. I didn't have to remember any formulas or anything. I didn't have to put all the numbers down in a calculator. I didn't have to write anything down, I just had to type it down and the computer does it for me, which was way easier than all those other stuff.

Dismissing the trappings of her previous courses -- memorizing formulas, entering data in the calculator, and writing notes -- Grace praises the efficiency of coding, declaring it *easier*.

Ironically, she acknowledges having to 'remember all the codes,' but unlike the abstract formulas, codes have relevance. She understands the meaning behind the codes.

Will and Caleb also admire the efficiency of coding, using the word *easier* to capture how quickly an answer is obtained and their newfound academic success. Will asserts coding results in a faster answer, "Being able to code to solve your problems and you get a faster answer is easier." Caleb appreciates how the IDS course relies on technology (R Studio) making problem-solving efficient, "And now it's [technology] finally in math.... Like using technology to solve big problems. And it's easier, more convenient, it's fast." Like Grace, Will and Caleb relate *easier* with the quick answer from coding. The use of *easier* implies the result is nearly effortless, obscuring the cognitive skills Will and Caleb gained to code accurately. Additionally, they associate the word *easy* with improved grades. Will admits his previous disengagement from mathematics and low academic performance:

So my past math experiences were kind of bad. I hated math. You know, I was the type of person who would slack off on homework, fail my test. And but I feel like IDS for me, I started using coding and that was a lot easier for me to understand.

Similarly, Caleb explains that he previously failed a high school mathematics course but feels confident in the IDS course:

I failed it [Integrated Mathematics I] Math has never been my favorite. I'm not very good at it. I sometimes I don't really pay attention. But somehow, like for coding, it's been explained to me in a way where it's easy for me to understand and for me to do.

Easier is associated with coding and efficiency in Will and Caleb's responses. They imply understanding and enjoyment are the drivers of their new perspective in the IDS course. Unlike their previous negative experiences in high school mathematics, Will and Caleb describe feeling confidence and competence in the IDS course, which stems from their understanding of and proficiency with coding.

Finally, the IDS students use the word *easy* to describe a less onerous workload than their previous mathematics classes. Joseph bemoans the expectations of his prior classes:

Whenever we would have a test we would turn it [a thick packet] in.... By the end when we had the test, it would be this thick [motions with his hands a half dozen inch wide gap]. Yeah, pretty much a whole packet of answering questions that he [former teacher] gave us. So it was a lot.

Joseph recounts how he previously would complete thick packets of worksheets. When Joseph notes he was 'answering questions that he [the teacher] gave us,' he implies a level of detachment from the work, finding little meaning in the endless stream of worksheets. Not surprisingly, Joseph compliments the IDS course, "I think easy as in the workload isn't that much as like a normal math class." Having the work on the computers and mostly confined to the class period is refreshing to Joseph. Catherine also shares how less stressful the IDS course is in comparison to Integrated Mathematics I and II:

For me, I feel that it has helped me personally. The past two years when I would have math, I would be really stressed about math and then I'd have to go home and then do the math homework and be stressed at home. IDS is way less stressful.... We rarely ever get homework. It's easy and it's not stressful at all.

Catherine ascribes the word *easy* to the seemingly lighter workload in the IDS course. She goes on to clarify, "I feel like that's cool because they're not really putting so much into your brain and

it doesn't get all cluttered in your head. And since you already have codes memorized it's easier to solve problems and do your work.” Catherine perceives the work in IDS as related and purposeful, such as knowing codes, which is far less onerous or *easier* than previous courses.

IDS students use *fun* and *easy* to capture their experiences and perceptions in the course. *Fun* conveys the IDS students’ enjoyment and engagement with data science. They use the word to describe the revelatory experiences they are having coding and exploring intriguing data sets. Many of the IDS students previously had little academic success in high school mathematics, so they employ the words *easy* to express their newfound feelings of efficacy and competence in the IDS course. However, the frequent and consistent use of the word *easy* minimizes the cognitive demand required of their IDS labs. IDS students associate the word *easy* with the computer quickly spitting out an answer and a lack of homework assignments, which renders the students and their intellectual prowess invisible in the process.

Summary

The self-determination framework provides a useful lens for making sense of how the IDS students experience and perceive the course (Ryan & Deci, 2000a, 2000b). The psychological factors of competence, autonomy, and relatedness organize the students’ experiences and perceptions into three significant findings. The first finding explores IDS students’ sense of relatedness. The nature of data science, an inherently participatory subject (Wilkerson & Polman, 2020), fosters a sense of community among the IDS students. They also derive a sense of belonging from being part of a movement larger than themselves and envision the IDS course growing nationwide. For IDS students, data science is synonymous with coding. Thus, understanding the multiple ways the students portray coding -- purposeful, efficient, empowering, and engaging -- reveals their feelings of competence and autonomy in the course.

The IDS students' use of the words *fun* and *easy* extends an understanding of how they experience and perceive the course beyond the self-determination framework. *Fun* captures their excitement and enthusiasm for engaging in the course and the word *easy* or *easier* conveys their newfound sense of achievement. No longer is mathematics, in this case, data science, viewed as arcane and esoteric.

Connections Across the DMPC and IDS Case Studies

The DMPC and IDS courses are distinctive and unique, yet it is meaningful to consider the findings from both case studies collectively. The three significant findings are: (a) students feel connected to an intellectual and social community which fosters a sense of relatedness, (b) the students conceptualize of discrete mathematics as pattern-seeking and data science as coding, highlighting their feelings of competence and autonomy, and (c) the students frequently use the words *fun* and *easy* to convey their experiences and perceptions in the two courses.

Finding #1 Connections Across DMPC and IDS Cases

The uniqueness of discrete mathematics and data science builds community in the DMPC and IDS course and dispels students' prior perceptions of what it means to do mathematics or statistics. The hallmarks of both courses invite collaboration and foster feelings of relatedness. The DMPC course emphasizes reasoning (Goldin, 2018), which makes the DMPC students naturally curious to discover their peers' understanding of the tasks. In the IDS course, the newness of coding nurtures a supportive classroom environment, where students forge friendships over programming and exploring intriguing data sets. The lack of any significant difference in relatedness composite scores among the DMPC and IDS students imply they feel similarly about the community in their respective course regardless of various demographic and academic factors. Further evidence of relatedness and community are found in the quantitative

data analysis of both case studies. The measures of center for the relatedness composite score in the DMPC and IDS courses correspond to the students' descriptions of feeling connected to a community. For both courses, the item – “I appreciate working with my peers on problems in this math class” – had a mode of 1.0 (*strongly agree*). Eighty-seven percent of DMPC students and 73 percent of IDS students either *agreed* or *strongly agreed* with the statement.

Finding #2 Connections Across DMPC and IDS Cases

In describing what is discrete mathematics or data science, the students from the respective courses begin by distinguishing the current course from their prior experiences in high school mathematics and statistics. DMPC students lament an overemphasis on a singular procedure to solve problems and favoring correct answers over understanding whereas, IDS students dreaded memorizing abstract formulas and spending hours on rote exercises for their previous mathematics and statistics courses. These disempowering and marginalizing experiences contrast with the competence and autonomy the DMPC and IDS students perceive in their respective courses. Discrete mathematics shifted the focus from product to process, by valuing multiple representations, ways of seeing, and solving problems. In data science meaning is integral to the content and consequently, IDS students depict their experiences in the course as purposeful, empowering, and engaging. Analysis of the quantitative data further bolster these findings. DMPC and IDS students rank Likert scale items for competence and autonomy favorably as evidenced by the measures of center and spread of these composite scores. In the DMPC and IDS courses, some of the same competence and autonomy items shared a mode of 1.0 (*strongly agree*). In the case of competence, 63 percent of DMPC and IDS students, express a strong confidence in their ability to learn discrete math and data science respectively. With autonomy, two of the same items had a mode of 1.0 (*strongly agree*) in both courses. These two

items underscore characteristics of the psychological need. One statement captures the positive engagement in the DMPC or IDS course: “This class is enjoyable for me.” The other item affirms the creative freedom the DMPC and IDS students repeatedly referred to in the focus groups: “I believe there is more than one way to solve a problem.” In previous mathematics courses, both sets of students described feeling constrained in their thinking and approach to the content, yet in DMPC and IDS students cherished the freedom they had in sense-making and communicating their reasoning.

Finding #3 Connections Across DPMC and IDS

The DMPC and IDS students use the words *fun* and *easy* in remarkably similar ways. The use of *fun* and *easy* underscores the difference between their prior negative experiences high school mathematics and statistics and their positive experiences in the DMPC or IDS course. DMPC students use the word *fun*, namely in two ways - to convey their engagement and pleasure in the problem-solving process, and their excitement for the freedom to explore multiple approaches to any task. *Fun* for the IDS students captures their enthusiasm for coding. They marvel over its power and efficiency. The use of the word *easy* or *easier* centers on the concept of accessibility in both courses. DMPC students use the word *easy* when describing the non-prescriptive approaches to problem solving in the course. IDS students use the word *easy* in a more nuanced manner. *Easy* represents the efficiency of coding, greater academic success (e.g., higher grades earned in IDS), and less stressful workload. While DMPC students often use the word *easy*, they fully acknowledge the course is cognitively demanding and reject when others use the word to imply DMPC is a less rigorous course. On the other hand, the IDS students never acknowledge the irony in using the word *easy*. Their knowledge and skill at programming in R Studio is intellectually challenging.

Chapter 5: Conclusion

Overview

In this chapter, I summarize the importance of this dissertation study, by reviewing the statement of the problem, theoretical framework, methodology, limitations and findings, which logically lead to a discussion of implications for policy, practice, and research. In essence, I review what this dissertation adds to the literature on secondary mathematics education and how the limitations and implications lay the foundation for future advocacy and research.

Overview of the Problem

The leading recommendations for the design of equitable high school mathematics pathways argue for the creation of multiple, flexible, and rigorous pathways for students based on their current interests and future aspirations. Students may elect to follow a traditional pathway (e.g., algebra to calculus) or an alternative one which may feature non-traditional branches of mathematics (e.g., data science, discrete mathematics) and more closely align with professional goals beyond STEM (Daro & Asturias, 2019; Fitzpatrick & Sovde, 2019; NCTM, 2018). These policy recommendations draw on decades of research including (a) secondary mathematics pedagogy and content (Aguirre et al., 2013; Boaler, 2015; Burdman, 2015; Schoenfeld, 2004, 2007, 2014); (b) the need to grow and diversify STEM professions (Burdman, 2018; Committee on STEM, 2018; NCTM, 2018; PCAST, 2012); and (c) the positive outcomes associated with taking additional years of high school mathematics (Adelman, 1999, 2006; Byun et al., 2015; Gaertner et al., 2014; Rose & Betts, 2004; Wang, 2013). However, missing from the research is an understanding of student experiences and perceptions in non-traditional high school mathematics courses. This research is especially significant as policy makers consider

implementing these recommendations (Oregon Department of Education, 2020) and promote expanding access to non-traditional high school mathematics courses (CSU, 2020a, 2020b).

Thus, this mixed methods collective case study sought to examine the students' experiences and perceptions in two non-traditional high school mathematics courses -- DMPC and IDS -- related to self-efficacy, agency, and a sense of belonging.

Theoretical Framework

Self-determination theory provides a suitable framework for examining student experiences in the DMPC and IDS courses. Self-determination theory explores intrinsic and extrinsic motivation through three psychological needs: competence, autonomy, and relatedness. Competence is best understood as self-efficacy, autonomy as agency, and relatedness as a feeling of belonging or community. When all three needs are met, a person finds meaning and purpose in their actions (Deci & Ryan, 2008; Ryan & Deci, 2000a, 2000b). Therefore, competence, autonomy, and relatedness supply a useful lens for studying student experiences and making sense of their perceptions of the DMPC and IDS courses.

Methodology

The design of this research study - mixed methods collective case study - complements the goal of understanding student experiences and perceptions in the DMPC and IDS courses. A mixed methods design builds on the individual strengths of quantitative and qualitative methodology, enriching the findings more than either methodology could do alone (Creswell, 2012; Teddlie & Tashakkori, 2009). The collective case study was bounded by one DMPC section at Mediterranean High School and one IDS section at Sierra Madre High School. The DMPC and IDS course directors recommended these two research sites based on the expertise

and leadership of the two teachers (Creswell, 2012; S. Machado, personal communication, October 16, 2019; O. Soto, personal communication, April 30, 2019; Stake, 2005).

The first phase of the research design was focused on becoming familiar with the context of the school, the course itself, the two teachers, and the students. During this phase, I conducted three classroom observations in each classroom and administered a survey to consenting students in both the DMPC and IDS classes. The survey gathered information on the demographics of the students and their Likert scale ratings on 43 statements related to competence, autonomy, and relatedness (Creswell, 2012; Emerson et al., 2011; Maxwell, 2012).

The second phase consisted of student focus groups and one-on-one interviews with the DMPC and IDS teachers. My initial analysis of the data collected in the first phase partially guided decision making in the second phase. During the second phase, I conducted an initial set of small student focus groups in both courses, followed by one-on-one teacher interviews. I concluded the second phase with the student perspective by conducting a final student focus group at each research site (Creswell, 2012; Maxwell, 2012; Siedman, 2006).

Once the second phase ended, I analyzed the quantitative and qualitative data using the lens of self-determination theory, by coding, memoing, and running statistical tests (Emerson et al., 2011; Miles et al., 2014; Price et al., 2015; Ryan & Deci, 2000a, 2000b; Saldaña, 2013). Significant themes emerged from the integration of quantitative and qualitative data from both research sites, adding credibility and depth to the findings about student experience in both courses.

Limitations

The limitation section precedes a summary of the findings and implications to avoid overstatements and to seed directions for future practice and scholarship.

The goal of a case study -- in this instance two cases (DMPC and IDS) -- is to closely examine a particular, well-defined phenomenon, making it impossible to generalize the findings beyond this case study (Creswell, 2012; Stake, 2005). Thus, the above findings are constrained by context, time, and participants. I only studied one section of DMPC at Mediterranean High School and one section of IDS at Sierra Madre High School. Therefore, I am unable to extrapolate the findings to all DMPC or IDS courses currently running. I collected the data before the pandemic school closures, preventing any findings related to student experiences and perceptions during distance learning. Ms. North, the DMPC teacher, and Ms. Arcega, the IDS teacher, graciously consented to participate along with 30 DMPC students and 19 IDS students. Ms. North and Ms. Arcega are highly qualified teachers and not necessarily indicative of all DMPC or IDS teachers. And the two small student cohorts limit the power of the statistical tests making it harder to detect significant differences.

The survey was created from three separate validated instruments (Good et al., 2012; Mulhern & Rae, 1998; Tapia & Marsh, 2004) and had 43 Likert scale items (12 on competence, 17 on autonomy, and 14 on relatedness). Being an amalgamation of surveys, conducting cognitive interviews with high school students would have been beneficial and may have shed light on the length of the survey and the clarity of the demographic and Likert scale items (Snijkers, 2002). For example, when I ran the principal component analysis, many of the reverse coded items failed to pass the correlation threshold of 0.5 (Salkind, 2007), which may be a result of the wording and students' interpretations.

Summary of Findings

Three meaningful findings emerged from the data analysis of the DPMC and IDS courses which I relate to the literature review and theoretical framework, setting the stage for implications.

The first finding focuses on the DMPC and IDS students' sense of belonging, known as relatedness in the self-determination theory (Ryan & Deci, 2000a, 2000b). The DMPC and IDS students attribute this sense of relatedness to the uniqueness of the content and pedagogy, which invites collective problem-solving. The content of DMPC and IDS courses differs substantially from the students' previous high school mathematics courses (DMPC, n.d.; Gould et al., 2016; IDS, n.d.). Discrete mathematics is centered in familiar contexts with an emphasis on reasoning (Goldin, 2018) and data science sparks curiosity through exploring meaningful data sets (Gould, 2010; Wilkerson & Polman, 2020). This finding relates to research on the importance of students finding relevance in the content (Burdman, 2015; Carnevale & Desrochers, 2003; Daro & Astarius, 2019; NCTM, 2018) and how students are more likely to persist and achieve academically when they feel a sense of connection or relatedness (Boersma & Savina, 2019; Bressoud, 2015; Good et al., 2012).

The second finding explores the DMPC and IDS students' feelings of efficacy and agency, known as competence and autonomy under self-determination theory (Ryan & Deci, 2000a, 2000b). The unique way the DMPC and IDS students conceptualize mathematics or statistics nurtures curiosity and creative freedom to explore. The DMPC students view discrete mathematics as a pattern to discern, whereas the IDS students view data science through the lens of coding. These conceptualizations contribute to DMPC and IDS students taking an active role in sense-making and reasoning in the two courses, which is recommended by empirical studies on how student student-centered practices can enhance mathematical learning (Boaler & Greeno, 2000; Boaler & Staples, 2008; Schoenfeld, 1988, 2002; Stigler et al., 2010; Swafford, 2003). DMPC and IDS students describe their confidence and genuine curiosity in problem-solving,

which alludes to their feelings of competence; they appreciate the creative freedom to explore multiple approaches and representations, which represents their feelings of autonomy.

The third finding adds depth and nuance to the previous two findings. Throughout the focus group interviews, the students compared their experiences in the DMPC or the IDS course with prior experiences in high school mathematics and statistics. These students utilize the word *fun* and *easy* to set apart their experiences in the DMPC and the IDS courses. The word *fun* captures their excitement and engagement with the two courses, and the word *easy* describes the accessibility of the two courses, which disrupts the view of mathematics or statistics as impenetrable. Through the use of these two words, the DMPC and IDS students depict experiences aligned to major frameworks on mathematics pedagogy, which provide guidance on facilitating engagement, meaningful problem-solving, and productive struggle (Krall, 2018; NCSM & TODOS, 2016; NCTM, 2014; Schoenfeld, 2014).

Implications for Policy, Practice, and Research

While a limitation of case studies is generalizability, the methodology is generative and illuminating of implications for policy, practice, and research. Policymakers, educators, and the general public tend to view mathematics as a conduit to growing and diversifying the STEM professions (Burdman, 2018; Committee on STEM, 2018; NCTM, 2018; PCAST, 2012). Some posit mathematics as integral to our humanity beyond lucrative careers, urging individuals to find personal meaning and relevance in the discipline (NCTM, 2018 Su & Jackson, 2020). My dissertation findings address both of these aspirations.

The DMPC and IDS courses foster an empowering view of mathematics in students. During the student focus groups, students from both courses expressed a new sense of confidence, agency, and belonging. They drew a clear line between their prior negative and

alienating experiences and the current motivating and affirming experiences in either the IDS or DMPC course. This positive view is especially relevant and important given 90 percent of the interviewed students identify as BIPOC. If policy makers and the public are committed to growing and diversifying STEM professions, these courses may be helpful in attracting more BIPOC students to STEM, and in the very least giving them a perspective of mathematics as meaningful in their lives, regardless of their future careers. It would be valuable to conduct a longitudinal study and examine the duration, malleability, and evolution of these beliefs. For example, do these beliefs endure when the DMPC or IDS students enroll in post-secondary mathematics? Do students who previously eschewed the STEM career pathway, now join it?

There is a laudable movement to expand high school mathematics offerings, beyond the traditional algebra to calculus pathway (Daro & Asturias, 2019; Fitzpatrick & Sovde, 2019; NCTM, 2018). My findings of student experiences in the DMPC and IDS courses are especially relevant as policymakers recommend the expansion of these courses (CDE 2000a, 2000b; Oregon Department of Education, 2020; Taylor, 2019). Mathematics has a tragic history of tracking and elitism. As consequence, some branches of mathematics have more prestige and esteem (Burdman, 2015, 2018; NCTM, 2014, 2018). In the interviews, the students and teachers noted others' misperceptions of the DMPC and IDS courses as less rigorous. Miguel, a DMPC senior, questioned whether the course was adequately preparing him for pursuing a STEM major in college. For multiple pathways in mathematics to thrive and these courses in particular to flourish, the public must perceive the intellectual rigor and merit of these non-traditional courses and pathways. As part of this effort, we must address the public paradigm of what it means to do mathematics. Too often, that paradigm is filled with false notions of the discipline as

disconnected procedures to memorize, instead of a meaningful sense-making tool to deepen humanity's understanding of the world.

Although I chose the unit of analysis to be students, it was apparent when collecting and analyzing the data that a teacher's content knowledge and pedagogy are inextricably intertwined with the experiences the students have (CDE, 2018a; DMPC, n.d.; Gould et al., 2016; IDS, n.d.). In the rush to implement these courses in more high schools, it is vital to consider the preparedness of the teachers to teach non-traditional high school mathematics courses and the crucial role professional learning may play. Therefore, educational leaders and elected officials must prioritize adequately funding professional learning, especially at time of unprecedented budget shortfalls during the pandemic. Expanding these courses without properly supporting teachers in high quality implementation will malign the efforts for more students to "experience wonder, joy, and beauty" (NCTM, 2018, p. 9) inherent in these two courses. A worthwhile future study would examine the role of teacher professional learning and instructional practice in concert with the experiences of the students to inform the successful expansion of these courses to more high schools nationwide.

The current pandemic provides a unique research opportunity to explore the enduring elements of student experiences in these two courses. Undoubtedly, the 2020-21 school year will be unprecedented given the continued spread of coronavirus. My research study documents findings prior to the school closures and the dramatic shift to remote learning. It would be intriguing to examine student experiences and perceptions in the two courses during remote or hybrid learning. Mediterranean and Sierra Madre High Schools have started the school year with distance learning and plan to continue for at least a few months according to their district websites. Both course directors have undertaken the process of developing online modules from

the respective curricula for teachers to use regardless of the learning configuration (S. Machado, personal communication, September 3, 2020; O. Soto, personal communication, July 24, 2020). The experiences lost and gained in distance learning may have influential implications after the pandemic ends for the continued refinement of these profound courses.

Appendices

Appendix A

DMPC / IDS Student Survey

What is your gender?

1. Female
2. Male
3. Non-binary
4. Decline to State

Which ethnicity/race do you identify with most?

1. Mexican or Latina or Latino
2. Black or African American
3. American Indian or Alaska Native
4. Native Hawaiian or Pacific Islander
5. White
6. Asian
7. Other
8. 2 or more
9. Decline to State

What is your current grade level?

1. 9th Grade
2. 10th Grade
3. 11th Grade
4. 12th Grade

What is your current mathematics class?

1. Discrete Math
2. Introduction to Data Science (IDS)

What prior high school math courses have you taken? (select all that apply)

1. Integrated Math I / Algebra I
2. Integrated Math II / Geometry
3. Integrated Math III / Intermediate Algebra
4. Pre-calculus
5. Statistics
6. Other

What is your overall unweighted GPA?

1. Below 2.0 - 1
2. 2.0 to 2.5 -2
3. 2.5 to 3.0 -3
4. 3.0 to 3.5 - 4

5. 3.5 to 4.0 - 5
6. Decline to State

Last semester, my grade in this math course was:

1. A
2. B
3. C
4. D
5. F

Was your first language English?

1. Yes
2. No
3. Decline to State

Do you qualify for free or reduced lunch?

1. Yes
2. No
3. Decline to State

Competence Statements

In this math class:

1. I feel secure about attempting problems
2. I am confident about learning the content in this course
3. I think I can handle more challenging problems
4. I feel lost, though I do okay in my other classes
5. I look forward to taking more math classes
6. The content makes me feel nervous
7. I have a sinking feeling when I try problems
8. I have self-confidence
9. My teacher has encouraged me to study more math
10. I am comfortable expressing my own ideas on how to look for solutions to a difficult problem.
11. The content is very interesting.
12. I am comfortable answering questions in math class.

Autonomy Statements

1. I believe I will need math in my future work
2. I am in this course because it is useful
3. I see math as a subject I will rarely use in daily life
4. I believe this math class is worthwhile
5. This class is enjoyable for me
6. When I am confused by a problem in this math class, I stick with it
7. I'd rather have someone give me the answer to a difficult math problem.
8. Knowing math will help me earn a living.

9. When I start working on a problem in this course, I find it hard to stop.
10. This class is dull and boring.
11. I like to solve new problems in this class
12. I would like to avoid using math in college.
13. This class would be very helpful no matter what I decide to study.
14. The challenge of this class appeals to me.
15. My teacher values only one method and answer
16. In this class, I can choose my way to solve a problem
17. I believe there is more than one way to solve a problem

Relatedness Statements

When I am in this math class:

1. I feel that I belong
2. I feel like I am part of this community
3. I feel like an outsider
4. I feel accepted
5. I feel valued
6. I feel excluded
7. I feel comfortable
8. I feel inadequate
9. I wish I could fade into the background and not be noticed
10. I try to say as little as possible
11. I enjoy being an active participant
12. Even when I struggle, I trust my teacher to have faith in my potential
13. I appreciate working with my peers on problems
14. I feel supported by peers

In this class, I would describe math [write what comes to mind]:

Appendix B

Student Focus Group Semi-Structured Interview Guide

Script: Hi. I'm honored to talk to you about Discrete/Introduction to Data Science Math Class, which privilege untraditional branches of mathematics. For this interview, I am curious to hear your thoughts on the course. In the moment, I want to concentrate on your responses. Therefore, I will take a few notes and I will audio record the interview so I can listen again and ensure I accurately capture your responses. There are no wrong and right answers, just your honest opinion.

For this conversation to be successful, we should make a few agreements.

- (1) To maintain anonymity, when referring to people you know, please use their role in your life instead of names (e.g., my teacher, a counselor, a peer, my friend, etc).*
- (2) To maintain confidentiality, do not repeat what is said in the focus group after you leave this room.*
- (3) Please be mindful to speak one at a time.*
- (4) These recordings will be transcribed and then destroyed. All quotes will be anonymized.*
- (5) If you don't understand my question, please let me know and I will re-word the question.*
- (6) You are free to decline to answer any questions or leave at any point*

Are you willing to participate?

1. Please tell me a little about yourself in high school. What do you take pride in? What have you appreciated in high school? [Follow up: What do you like about this high school?]
2. What is Discrete/Intro to Data Science class like for you?
3. Take me through an average few days in the Discrete/Intro to Data Science. [Follow up: What type of assignments do you have? What do you find challenging?]
4. How did you decide to take Discrete/Intro to Data Science math class? [Follow up: What had you heard about the class, if anything? What impressions did you have about the course? What interested you, if at all? Who, if anyone, helped you decide to take the class?]
5. What are some of the experiences you found **most impactful** in Discrete/Intro to Data Science math class? What are some of the experiences you found **least meaningful** in Discrete/Intro to Data Science math class? [Follow up: What unit(s) from Discrete/Intro to Data Science math class stand out to you? What makes you say that? How would you describe the classroom community?]
6. How do you feel about math? [Follow up: What do you love? What do you NOT love about math?]
7. How do you compare your experiences in Discrete/Intro to Data Science math class with previous mathematics classes you have taken? [Follow up: What are some examples of ____?]
8. How has your opinion of mathematics changed by being in the Discrete/Intro to Data Science math course, if at all?

9. In this moment, what are your future aspirations? [Follow up: College plans? Future major? Potential careers? Areas of interest?]
10. What else do you believe is important for me to know about your experiences in Discrete/Intro to Data Science?

Legend:

Theme	Question #'s
Competence	2, 3, 4, 5, 7, 8, 10
Autonomy	2, 3, 4, 5, 7, 8, 10
Relatedness	2, 5, 7, 8, 9, 10
Experiences & Perceptions in general	2 - 10

Appendix C

Teacher Semi-Structured Individual Interview Guide

Hi. I'm honored to talk to you about Discrete/Intro to Data Science Math Class. As you know the State of California is piloting courses, like Discrete/Intro Data Science Math Class, which privilege untraditional branches of mathematics. For this interview, I am curious to hear your thoughts on teaching the course and experiences students have in the course. To maintain anonymity, when referring to people you know, please use their role in your life instead of names (e.g., a colleague, a counselor, an administrator, etc). In the moment, I want to concentrate on your responses. Therefore, I will take a few notes and I will audio record the interview so I can listen again and ensure I accurately capture your responses. There are no wrong and right answers, just your honest opinion. You are free stop at any time or refuse to answer any question. Are you willing to participate?

1. What drew you to teach high school mathematics? [Follow-up: What do you appreciate about teaching mathematics?]
2. What other math courses have you taught?
3. What are some of the highlights and challenges of teaching the Discrete/Intro to Data Science Math Class? Challenges in teaching Discrete/Intro to Data Science Math Class? [Follow up: How do these highlights and challenges compare with teaching other math classes?]
4. Over a week, how does the learning unfold in Discrete/Intro to Data Science Math Class?
5. Describe the typical students who enroll in Discrete/Intro to Data Science Math Class. [Follow up: What attracts students to enroll in Discrete/Intro to Data Science Math Class?]
6. What student experiences do you feel are **most impactful** and **least meaningful** in Discrete/Intro to Data Science Math Class?
7. How do the students view math in Discrete/Intro to Data Science Math Class? How does this compare with students in other math classes you may teach? [Follow-up: How do student experiences in Discrete/Intro to Data Science Math Class compare with other math classes?]
8. How are choice and freedom offered in Discrete/Intro to Data Science Math Class?
9. What do you do to create a community in Discrete/Intro to Data Science Math Class?
10. What else do you believe is important for me to know about Discrete/Intro to Data Science Math Class?

Legend:

Theme	Question #'s
Competence	3, 4, 5, 6, 7,

Autonomy	3, 4, 6, 7, 8,
Relatedness	3, 5, 6, 7, 9,
Experiences & Perceptions in general	3 -10

Appendix D

DMPC Student Focus Group Participants

Pseudonym	Gender	Ethnicity/ Race	Grade	Previous Math Class	1 st Semester Grade	Free or Reduced Lunch Status	Native English Speaker	Unweighted Cumulative GPA range
Teddy	Male	Latinx	12	IM III	B	No	Yes	2.5 to 3.0
Aira	Female	Latinx	12	Other	A	Yes	No	3.5 to 4.0
Akino	Female	Asian	12	IM III	C	Yes	Yes	3.0 to 3.5
Darius	Male	Multiracial	12	IM III	C	No	Yes	2.5 to 3.0
Nia	Female	Black	12	IM III	B	No	Yes	3.0 to 3.5
Miguel	Male	Latinx	12	IM III	A	No	No	3.5 to 4.0
Jayden	Male	Black	12	IM III	A	No	Yes	3.0 to 3.5
Nyah	Female	Black	12	IM III	A	No	Yes	2.5 to 3.0
Ava	Female	Latinx	12	Pre-Calc	A	Yes	No	3.5 to 4.0
Savannah	Female	White	12	IM III	A	No	Yes	3.0 to 3.5
Audrey	Female	Latinx	12	IM III	B	Yes	Yes	Below 2.0
Brittany	Female	Asian	12	IM III	A	Decline	Yes	3.0 to 3.5

Note: IM (integrated mathematics)

IDS Student Focus Group Participants

Pseudonym	Gender	Ethnicity/ Race	Grade	Previous Math Class	1 st Semester Grade	Free or Reduced Lunch Status	Native English Speaker	Unweighted Cumulative GPA range
Grace	Female	Asian	12	Statistics	A	Yes	Yes	3.5 to 4.0
Joseph	Male	Latinx	11	IM II	A	Yes	Yes	2.5 to 3.0
Catherine	Female	Latinx	11	IM II	A	Yes	Yes	3.0 to 3.5
Nolan	Male	Latinx	11	IM II	C	Yes	Yes	Below 2.0
Will	Male	Asian	12	Statistics	B	No	Yes	2.0 to 2.5
Liam	Male	White	12	IM III	B	No	Yes	2.5 to 3.0
Caleb	Male	Latinx	11	IM II	B	Yes	No	2.5 to 3.0
Connor	Male	Latinx	12	Statistics	B	Yes	No	2.0 to 2.5

Note: IM (integrated mathematics)

Appendix E

The following are the Likert scale items from the original survey which exceeded the component coefficient threshold of 0.5 (PCA) and comprise the composite score for competence, autonomy, and relatedness of each course.

DMPC Course

Competence Statements

In this math class:

I feel secure about attempting problems

I am confident about learning the content in this course

I think I can handle more challenging problems

I feel lost, though I do okay in my other classes

I have self-confidence

I am comfortable expressing my own ideas on how to look for solutions to a difficult problem

I am comfortable answering questions in math class

Autonomy Statements

This class is enjoyable for me

When I am confused by a problem in this math class, I stick with it

This class is dull and boring

This class would be very helpful no matter what I decide to study

The challenge of this class appeals to me

My teacher values only one method and answer

In this class, I can choose my way to solve a problem

I believe there is more than one way to solve a problem

Relatedness Statements

When I am in this math class:

I feel that I belong

I feel like I am part of this community

I feel accepted

I feel valued

I feel comfortable

I appreciate working with my peers on problems

I feel supported by peers

IDS Course

Competence Statements

In this math class:

I feel secure about attempting problems
I am confident about learning the content in this course
My teacher has encouraged me to study more math
I am comfortable expressing my own ideas on how to look for solutions to a difficult problem
The content is very interesting
I am comfortable answering questions in math class

Autonomy Statements

I believe I will need math in my future work
I am in this course because it is useful
I believe this math class is worthwhile
This class is enjoyable for me
Knowing math will help me earn a living
When I start working on a problem in this course, I find it hard to stop
I like to solve new problems in this class
This class would be very helpful no matter what I decide to study
The challenge of this class appeals to me
I believe there is more than one way to solve a problem

Relatedness Statements

When I am in this math class:

I feel like an outsider
I feel accepted
I feel valued
I feel excluded
I feel comfortable
I wish I could fade into the background and not be noticed
I enjoy being an active participant
Even when I struggle, I trust my teacher to have faith in my potential
I appreciate working with my peers on problems
I feel supported by peers

References

- Academic Senate of the California State University Quantitative Reasoning Task Force* (Rep.). (2016, August 1). Retrieved from <https://www.asccc.org/sites/default/files/V. G. QRTF Final Report 08-01-2016.pdf>
- Adelman, C. (1999). *Answers in the tool box: Academic intensity, attendance patterns, and bachelor's degree attainment*. Washington, DC: U.S. Department of Education.
- Adelman, C. (2006). *The toolbox revisited: Paths to degree completion from high school through college*. US Department of Education.
- Aguirre, J., Mayfield-Ingram, K., & Martin, D. (2013). *The impact of identity in K-8 mathematics: Rethinking equity-based practices*. The National Council of Teachers of Mathematics.
- Allensworth, E., Nomi, T., Montgomery, N., & Lee, V. E. (2009). College preparatory curriculum for all: Academic consequences of requiring algebra and English I for ninth graders in Chicago. *Educational Evaluation and Policy Analysis*, 31(4), 367-391.
- Asim, M., Kurlaender, M., & Reed, S. (2019, August 01). 12th Grade Course-taking and the Distribution of Opportunity for College Readiness in Mathematics. *Policy Analysis for California Education*. Retrieved from <https://www.edpolicyinca.org/publications/12th-grade-course-taking-and-distribution-opportunity-college-readiness-mathematics>
- Banilower, E. R., Smith, P. S., Malzahn, K. A., Plumley, C. L., Gordon, E. M., & Hayes, M. L. (2018). *Report of the 2018 NSSME+*. Chapel Hill, NC: Horizon Research, Inc.
- Bartell, T., Wager, A., Edwards, A., Battey, D., Foote, M., & Spencer, J. (2017). Toward a framework for research linking equitable teaching with the standards for mathematical practice. *Journal for Research in Mathematics Education*, 48(1), 7-21.
- The BIPOC Project. (n.d.). Retrieved September 3, 2020, from <https://www.thebipocproject.org/>
- Boaler, J. (2015). *Mathematical mindsets: Unleashing students' potential through creative math, inspiring messages and innovative teaching*. John Wiley & Sons.
- Boaler, J., & Greeno, J. G. (2000). Identity, agency, and knowing in mathematics worlds. *Multiple perspectives on mathematics teaching and learning*, 171-200.
- Boaler, J., & Sengupta-Irving, T. (2016). The many colors of algebra: The impact of equity focused teaching upon student learning and engagement. *The Journal of Mathematical Behavior*, 41, 179-190.
- Boaler, J., & Staples, M. (2008). Creating mathematical futures through an equitable teaching approach: The case of Railside School. *Teachers College Record*, 110(3), 608-645.

- Boersma, S., & Savina, F. (2019). Re-envisioning the pathway to calculus: Supporting all students. In Rebecca Hartzler & Richelle Blair (Eds.), *Emerging issues in mathematics pathways* (pp. 13-22). Austin, TX: The University of Texas at Austin, Charles A Dana Center.
- Bressoud, D. (2015). Insights from the MAA national study of college calculus. *Mathematics Teacher*, 109(3), 178-185.
- Burdman, P. (2015). Degrees of freedom: Diversifying math requirements for college readiness and graduation. *Oakland CA: Learning Works and Policy Analysis for California Education*.
- Burdman, P. (2018). *The Mathematics of Opportunity: Rethinking the Role of Math in Educational Equity* (Rep.). Retrieved from <https://justequations.org/resource/the-mathematics-of-opportunity-report/>.
- Burton, L. (1998). The practices of mathematicians: What do they tell us about coming to know mathematics?. *Educational Studies in Mathematics*, 37(2), 121.
- Byun, S. Y., Irvin, M. J., & Bell, B. A. (2015). Advanced math course taking: Effects on math achievement and college enrollment. *The Journal of Experimental Education*, 83(4), 439-468.
- California Department of Education. (2018a). CA Math Readiness Grants. Retrieved from <https://www.cde.ca.gov/pd/ps/cmrcigrants.asp>
- California Department of Education. (n.d.). California School Dashboard (CA Dept of Education). Retrieved from <https://www.caschooldashboard.org/>
- California Department of Education. (2018b). High school graduation requirements. Retrieved from <https://www.cde.ca.gov/ci/gs/hs/cefhsgradreq.asp>
- California Department of Education. (2019). Request for Applications. Retrieved September 11, 2020, from <https://www.cde.ca.gov/fg/fo/r12/cmrci16rfa.asp>
- California State University Board of Trustees. (2020a, January). *Agenda January 28-29, 2020*. Retrieved from: <https://www2.calstate.edu/csu-system/board-of-trustees/past-meetings/2020/Documents/jan-28-29-full-agenda.pdf>
- California State University Board of Trustees. (2020b, March). *Agenda March 24, 2020*. Retrieved from: <https://www2.calstate.edu/csu-system/board-of-trustees/past-meetings/2020/Documents/march-22-24-full-agenda.pdf>
- Carnevale, A. P., & Desrochers, D. M. (2003). The democratization of mathematics. *Quantitative literacy: Why numeracy matters for schools and colleges*, 21-31.

- Clotfelter, C. T., Ladd, H. F., & Vigdor, J. L. (2012). *Algebra for 8th graders: Evidence on its effects from 10 North Carolina districts* (No. w18649). National Bureau of Economic Research.
- Creswell, J. W. (2012). *Educational research: planning, conducting, and evaluating quantitative and qualitative research* (4th ed.). New York, NY: Pearson.
- Committee on STEM Education of the National Science & Technology Council. (2018). *Charting a course for success: America's strategy for STEM education* (Report). (ERIC Document Reproduction Service No. ED590474)
- Daro, P. & Asturias, H. (In press). *Branching out: Designing high school mathematics pathways with equity in mind*. (Rep).
- Deci, E. L., & Ryan, R. M. (2008). Self-determination theory: A macrotheory of human motivation, development, and health. *Canadian psychology/Psychologie canadienne*, 49(3), 182-185.
- Deci, E. L., Vallerand, R. J., Pelletier, L. G., & Ryan, R. M. (1991). Motivation and education: The self-determination perspective. *Educational psychologist*, 26(3-4), 325-346.
- Discrete Math Project Collaborative. (n.d.). Discrete Math Project Collaborative – A Collaborative Project between San Diego State University and Sweetwater Union High School District. Retrieved from <https://dmopc.sdsu.edu/>
- Domina, T., Hanselman, P., Hwang, N., & McEachin, A. (2016). Detracking and tracking up: Mathematics course placements in California middle schools, 2003–2013. *American Educational Research Journal*, 53(4), 1229-1266.
- Domina, T., McEachin, A., Penner, A., & Penner, E. (2015). Aiming high and falling short: California's eighth-grade algebra-for-all effort. *Educational Evaluation and Policy Analysis*, 37(3), 275-29.
- Domina, T., Penner, A. M., Penner, E. K., & Conley, A. (2014). Algebra for All: California's eighth-grade Algebra initiative as constrained curricula. *Teachers College record* (1970), 116(8), 1.
- Dysarz, K. (2018). Checking In: Are Math Assignments Measuring Up? Equity in Motion. *Education Trust*.
- Emerson, R. M., Fretz, R. I., & Shaw, L. L. (2011). *Writing ethnographic fieldnotes*. University of Chicago Press.

- Fairchild, A. J., Horst, S. J., Finney, S. J., & Barron, K. E. (2005). Evaluating existing and new validity evidence for the Academic Motivation Scale. *Contemporary Educational Psychology*, 30(3), 331-358.
- Finkelstein, N., Fong, A., Tiffany-Morales, J., Shields, P., & Huang, M. (2012). College Bound in Middle School & High School? How Math Course Sequences Matter. *Center for the Future of Teaching and Learning at WestEd*.
- Fitzpatrick, L.P., & Sovde, D. (2019). The case for mathematics pathways from the launch years in high school through postsecondary education. In Rebecca Hartzler & Richelle Blair (Eds.), *Emerging issues in mathematics pathways* (pp. 97-104). Austin, TX: The University of Texas at Austin, Charles A Dana Center.
- Gaertner, M. N., Kim, J., DesJardins, S. L., & McClarty, K. L. (2014). Preparing students for college and careers: The causal role of algebra II. *Research in Higher Education*, 55(2), 143-165.
- Gao, N. & Johnson, H. (2017). Improving College Pathways in California. *Public Policy Institute of California*. Retrieved from <https://www.ppic.org/publication/improving-college-pathways-in-california/>
- Geertz, C. (1973). Chapter 1, Thick Description: Toward an Interpretive Theory of Culture. In *The interpretation of cultures*. London: Fontana Press.
- Goldin, G. A. (2018). Discrete mathematics and the affective dimension of mathematical learning and engagement. In *Teaching and Learning Discrete Mathematics Worldwide: Curriculum and Research* (pp. 53-65). Springer, Cham.
- Good, C., Rattan, A., & Dweck, C. S. (2012). Why do women opt out? Sense of belonging and women's representation in mathematics. *Journal of personality and social psychology*, 102(4), 700-717.
- Gould, R. (2010). Statistics and the modern student. *International Statistical Review*, 78(2), 297-315.
- Gould, R., Bargagliotti, A., & Johnson, T. (2017). An analysis of secondary teachers' reasoning with participatory sensing data. *Statistics Education Research Journal*, 16(2), 305-334.
- Gould, R., Machado, S., Ong, C., Johnson, T., Molyneux, J., Nolen, S., ... & Zanontian, L. (2016). Teaching data science to secondary students: The mobilize introduction to data science curriculum. Iase-Web. Org.
- Guay, F., Ratelle, C. F., & Chanal, J. (2008). Optimal learning in optimal contexts: The role of self-determination in education. *Canadian Psychology/Psychologie canadienne*, 49(3), 233-252.

- Gutiérrez, R. (2012). Context matters: How should we conceptualize equity in mathematics education?. *Equity in discourse for mathematics education*, 17-33.
- Gutiérrez, R. (2013). Why (Urban) Mathematics Teachers Need Political Knowledge. *Journal of Urban Mathematics Education*, 6(2), 7–19.
- Harel, G., & Rabin, J. M. (2010). Teaching practices associated with the authoritative proof scheme. *Journal for research in mathematics education*, 14-19.
- Introduction to Data Science. (n.d.). Retrieved from <https://www.mobilizingcs.org/>
- Klein, D. (2007). A quarter century of US ‘math wars’ and political partisanship. *BSHM Bulletin*, 22(1), 22-33.
- Krall, G. (2018). *Necessary Conditions: A Self Study Guide for Teachers and Coaches on Improving Math Discussions*. Stenhouse Publishers.
- Ladson-Billings, G. (1997). It Doesn't Add up: African American Students Mathematics Achievement. *Journal for Research in Mathematics Education*, 28(6), 697. doi:10.2307/749638
- Levine, P. B., & Zimmerman, D. J. (1995). The benefit of additional high-school math and science classes for young men and women. *Journal of Business & Economic Statistics*, 13(2), 137-149.
- Lincoln, Y., & Guba, E. (1985). *Naturalistic inquiry* (1st edition). Beverly Hills, Calif: Sage Publications.
- Loveless, T. (2013). The 2013 Brown Center Report on American Education: How Well are American students learning? Washington, DC: The Brown Center on Education Policy. The Brookings Institution.
- Maxwell, J. A. (2012). *Qualitative research design: An interactive approach* (3rd ed.). Sage publications.
- Miles, M. B., Huberman, A. M., & Saldana, J. (2014). *Qualitative data analysis*. Sage publications.
- Mulhern, F., & Rae, G. (1998). Development of a shortened form of the Fennema-Sherman Mathematics Attitudes Scales. *Educational and psychological measurement*, 58(2), 295-306.

- National Council of Supervisors of Mathematics and TODOS Mathematics for ALL (NCSM and TODOS). (2016). *Mathematics Education Through the Lens of Social Justice: Acknowledgment, Actions, and Accountability a Joint Position Statement*. Retrieved from <https://www.mathedleadership.org/docs/resources/positionpapers/NCSMPositionPaper16.pdf>
- National Council of Teachers of Mathematics. (2018). *Catalyzing change in high school mathematics: Initiating critical conversations*. Author. Washington, DC.
- National Council of Teachers of Mathematics. (2014). *Principles to action: Ensuring mathematical success for all*. Author. Washington, DC.
- National Governors Association Center for Best Practices, Council of Chief State School Officers. (2010). *Common Core State Standards Math*. Retrieved from <http://www.corestandards.org>
- National Science Foundation, National Center for Science and Engineering Statistics. (2019). *Women, minorities, and persons with disabilities in science and engineering*. (Special Report NSF 19-304). Alexandria, VA: Author. Retrieved from <https://www.nsf.gov/statistics/wmpd>.
- Noonan, R. (2017). *STEM Jobs: 2017 update*. (ESA Issue Brief #02-1) Washington, DC: Office of the Chief Economist, Economics and Statistics Administration, U.S. Department of Commerce. Retrieved from <http://www.esa.doc.gov/sites/default/files/stem-jobs-2017-update.pdf>
- NSF Award Search: Award#0962919 - MOBILIZE: Mobilizing for Innovative Computer Science Teaching and Learning. (n.d.). Retrieved September 20, 2020, from https://www.nsf.gov/awardsearch/showAward?AWD_ID=0962919
- Oregon Department of Education. (2020, August). *High School Core Math Guidance – Publication Version 4*. Retrieved September 12, 2020, from <https://www.oregon.gov/ode/educator-resources/standards/mathematics/Documents/High%20School%20Core%20Mathematics%20Guidance.pdf>
- Peshkin, A. (1988). In search of subjectivity—one's own. *Educational researcher*, 17(7), 17-21.
- President's Council of Advisors on Science and Technology (PCAST). (2012). *Engage to excel: Producing one million additional college graduates with degrees in science, technology, engineering, and mathematics*. Washington, DC. Retrieved from http://www.whitehouse.gov/sites/default/files/microsites/ostp/pcast-engage-to-excel-final_2-25-12.pdf

- Price, P. C., Jhangiani, R., & Chiang, I. C. A. (2015). *Research methods in psychology*. BCCampus.
- Ratelle, C. F., Guay, F., Vallerand, R. J., Larose, S., & Senécal, C. (2007). Autonomous, controlled, and amotivated types of academic motivation: A person-oriented analysis. *Journal of educational psychology*, 99(4), 734-746.
- Rose, H., & Betts, J. R. (2004). The effect of high school courses on earnings. *Review of Economics and Statistics*, 86(2), 497-513.
- Ryan, R. M., & Deci, E. L. (2000a). Intrinsic and extrinsic motivations: Classic definitions and new directions. *Contemporary educational psychology*, 25(1), 54-67.
- Ryan, R. M., & Deci, E. L. (2000b). Self-determination theory and the facilitation of intrinsic motivation, social development, and well-being. *American psychologist*, 55(1), 68 - 78.
- Saldaña, J. (2013). *The coding manual for qualitative researchers* (2nd ed.). London: SAGE.
- Salkind, N. J. (2006). *Encyclopedia of measurement and statistics*. SAGE publications.
- Schoenfeld, A. H. (1988). When good teaching leads to bad results: The disasters of 'well-taught' mathematics courses. *Educational psychologist*, 23(2), 145-166.
- Schoenfeld, A. H. (2002). Making mathematics work for all children: Issues of standards, testing, and equity. *Educational researcher*, 31(1), 13-25.
- Schoenfeld, A. H. (2004). The math wars. *Educational policy*, 18(1), 253-286.
- Schoenfeld, A. H. (2007). Problem solving in the United States, 1970–2008: research and theory, practice and politics. *ZDM*, 39(5-6), 537-551.
- Schoenfeld, A. H. (2014). What makes for powerful classrooms, and how can we support teachers in creating them? A story of research and practice, productively intertwined. *Educational researcher*, 43(8), 404-412.
- Seidman, I. (2006). *Interviewing as Qualitative Research: A Guide for Researchers in Education and the Social Sciences, Fourth Edition* (3 edition). New York: Teachers College Press
- Snijkers, G. J. M. E. (2002). *Cognitive Laboratory Experiences: On Pre-testing Computerised Questionnaires*. Utrecht University.
- Stake, R.E. (2005). Qualitative case studies. In N.K. Denzin & Y.S. Lincoln (Eds.), *The SAGE handbook of qualitative research* (2nd ed., pp. 435-454). Thousand Oaks, CA: Sage

- Steen, L. A. (Ed.). (2001). *Mathematics and democracy: The case for quantitative literacy*. The National Council on Education and the Disciplines.
- Stigler, J. W., Givvin, K. B., & Thompson, B. J. (2010). What community college developmental mathematics students understand about mathematics. *MathAMATYC Educator*, 1(3), 4-16.
- Swafford, J. (2003). Reaction to high school curriculum projects' research. *Standards-oriented school mathematics curricula: What does the research say about student outcomes*, 457-468.
- Su, F. E., & Jackson, C. (2020). *Mathematics for human flourishing*. New Haven; London: Yale University Press.
- Tapia, M. & Marsh, G. E. (2004). An Instrument to Measure Mathematics Attitudes. *Academic Exchange Quarterly*, 8(2), 16-21.
- Teddlie, C., & Tashakkori, A. (2009). *Foundations of mixed methods research: Integrating quantitative and qualitative approaches in the social and behavioral sciences*. Sage.
- Taylor, P. (August 19, 2019). *Notice of meeting – Thursday, August 29, 2019*. [Memorandum]. Long Beach, CA: The California State University. Retrieved from <https://www2.calstate.edu/csu-system/board-of-trustees/PastNotices/Documents/20190819-10-Day-Meeting-Notice-Ed-Policy-Open-Forum-August-29-2019.pdf>
- Tucker, M., Daro, P., Snow, C., Pellegrino, J., Everson, H., Glasper, R., ... & Fain, P. (2013). What Does It Really Mean to Be College and Work Ready? The Mathematics and English Literacy Required of First Year Community College Students.
- Trusty, J., & Niles, S. G. (2003). High-school math courses and completion of the bachelor's degree. *Professional School Counseling*, 99-107.
- U.S. Department of Education Office for Civil Rights. (2018, April). 2015–16 Civil Rights Data Collection STEM Course Taking. Retrieved from <https://www2.ed.gov/about/offices/list/ocr/docs/stem-course-taking.pdf>
- U.S. Department of Education Office of Civil Rights. (2012, June). Gender Equity in Education: A Data Snapshot. Retrieved from <https://www2.ed.gov/about/offices/list/ocr/docs/gender-equity-in-education.pdf>
- University of California. (n.d.). A-G Subject Requirements. Retrieved from <https://www.ucop.edu/agguide/a-g-requirements/>
- Wang, X. (2013). Why students choose STEM majors: Motivation, high school learning, and

postsecondary context of support. *American Educational Research Journal*, 50(5), 1081-1121.

Wilkerson, M. H., & Polman, J. L. (2020). Situating data science: Exploring how relationships to data shape learning. *Journal of the Learning Sciences*, 29(1), 1-10.