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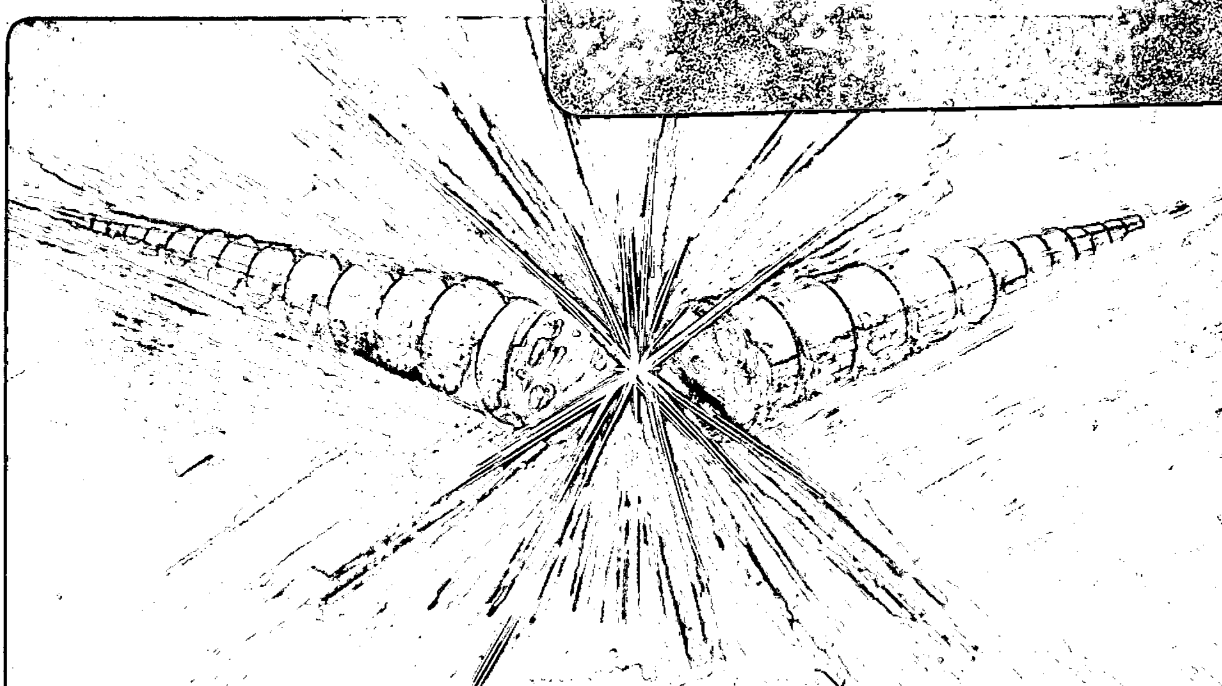

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The evolution of the radiation-beam system in long undulators

Kwang-Je Kim

Abstract

The process by which the radiation intensity grows and correlation in the electron distribution develops, as the beam passes through a long undulator, is described.

I. Introduction

In undulators, the interaction of one electron with another by means of the emitted radiation is usually negligible. However, this interaction does exist and that is why a coherent signal develops from noise in a free electron laser oscillator. For certain cases involving long undulators and high density electron beams, the cumulative effects of the electron-beam interaction can lead to a significant enhancement in the radiation intensity of the spontaneously emitted undulator radiation. The purpose of this paper is to describe a recent theoretical analysis[1] on this subject, which will be called the self-amplified spontaneous emission (SASE). The results indicate that a long undulator placed in a special by-pass of a low-emittance storage ring could become an important source of high power radiation at short wavelengths.[2]

Section II outlines the theoretical analysis. Section III gives a general discussion of the evolution of the radiation-beam system. In Section IV, the results of the analysis are compared with the recent microwave experiment at Livermore.[3] Section V considers the performance of a high-gain, single-pass free electron laser in the short wavelength region. Section VI gives concluding remarks.

II. Outline of the analysis

To analyze SASE, it is necessary to generalize the usual free electron laser (FEL) analysis in two respects. First, a continuous range of frequency about the resonant frequency must be explored since the spectral characteristics change as the system evolves. Second, the discreteness of the electron distribution must be taken into account, since otherwise spontaneous emission is not possible. This is accomplished by working with the Klimontovich distribution function,[4] rather than the Vlasov distribution. The coupled Klimontovich-Maxwell equations are solved by perturbation theory, in which the deviation of the fine-grained distribution from the smooth average is regarded as being a first-order quantity. One finds that the radiation field is composed of two terms: the first term is proportional to the input coherent signal and describes the well-known FEL gain process, and the second term is proportional to the sum of random phase factors and represents the SASE process.

The radiation intensity corresponding to the SASE term reduces to the well-known result for spontaneous emission in the limit of small interaction. In the regime of exponential growth, one obtains an explicit formula for the power and the spectral characteristics of the SASE radiation, as well as insight into the correlation properties of the electron beam distribution. The exponential growth eventually saturates because of nonlinear effects, which can be analyzed in a quasi-linear approximation.[5]

III. SASE in the exponential growth regime

The average energy of the electron beam will be expressed as $mc^2\gamma_0$ (m = electron mass, c = velocity of light). The beam travels along the z-direction through a planar undulator of period length λ_u with a peak magnetic field B_0. At the beginning of the undulator, the radiation spectrum, which has been well-studied in the theory of undulator radiation, is peaked at the frequency

$$\omega_1 = \frac{2\pi c}{\lambda_1} = k_u c \, \frac{2\gamma^2}{1 + K^2/2} \quad , \tag{1}$$

where $K = eB_0/mck_u$, e = electron charge, $k_u = 2\pi/\lambda_u$ and λ_1 is the radiation wavelength. (MKS units are used throughout this paper).

After some distance, the intensity starts go grow exponentially. An important dimensionless parameter which characterizes the behavior in this regime is[6]

$$\rho = \left(\frac{eK[JJ]Z_0 j}{32\ \gamma_0^2\ mc^2\ k_u^2}\right)^{1/3} \quad . \tag{2}$$

where j is the current density, Z_0 = 377 ohms, and

$$[JJ] = [J_0(\xi) - J_1(\xi)], \quad \xi = \frac{K^2}{4(1 + K^2/2)} \quad , \tag{3}$$

The growth rate at a given frequency $\omega = \nu\omega_1$ is determined by the following dispersion relation:

$$D(x,\nu) = x + \frac{\Delta\nu}{2} + \rho^3 \int d\eta \, \frac{dV/d\eta}{x + \eta\nu} = 0 \quad , \tag{4}$$

where $\Delta\nu = \nu - 1 = \omega/\omega_1 - 1$, i.e., the detuning parameter, and $V(\eta)$ is the energy distribution of the electron beam as a function of

$$\eta = \frac{\gamma - \gamma_0}{\gamma_0} \quad , \tag{5}$$

where $mc^2\gamma$ is the electron energy. The function $V(\eta)$ is normalized so that $\int V d\eta = 1$. Equation (4) is essentially the cubic equation known in the literature,[7] but generalized to include the effects of frequency detuning and finite energy spread of the electron beam.

In general, the dispersion relation has a solution $x = \rho\mu$, with a positive imaginary part x_I that gives rise to an exponentially growing intensity term proportional to $\exp(4k_u x_I z)$. The spectral property is determined by the behavior of x_I as a function of detuning $\Delta\nu$. For a given momentum distribution $V(\eta)$, let x_I^m be the maximum value of x_I and $\Delta\nu_m$ the value of $\Delta\nu$ at which this maximum occurs. This means that the growth is strongest at frequency $\omega = \omega_m = \omega_1(1 + \Delta\nu_m)$, and the spectral shape is obtained by studying the behavior of x_I near $\Delta\nu = \Delta\nu_m$. One obtains

$$\frac{dP}{d\omega} = \rho \, \frac{mc^2\gamma_0}{2\pi} \, G \, e^{\tau} \, e^{-\frac{(\omega - \omega_m)^2}{2(\omega_1\sigma_\nu)^2}} \quad , \tag{6}$$

where

$$\tau = 4k_u \, x_I^{\,m} \, z \quad , \tag{7}$$

$$G = \frac{\int d\eta \; V(\eta)/|\mu + \eta/\rho|^2}{|dD/dx|^2_{x = \rho\mu}} \quad . \tag{8}$$

In Eq. (6) σ_ν is the rms value of the relative bandwidth. For the zero energy spread case, where $V(\eta) = \delta(\eta)$, one obtains[8]

$$G = \frac{1}{9} \, , \quad x_I^{\,m} = \rho \, \frac{\sqrt{3}}{2} \quad \text{and} \quad \sigma_\nu = \sqrt{\frac{9\rho}{2\pi\sqrt{3}(z/\lambda_u)}} \quad . \tag{9}$$

The exponential growth of SASE saturates when $\rho z/\lambda_u$ becomes of order unity, as will be seen later. The bandwidth of SASE in the exponentially growing region is smaller by a factor $(\rho z/\lambda_u)^{1/2}$ than that of the spontaneous radiation, which is about λ_u/z. However, the bandwidths of SASE and the spontaneous radiation are comparable at saturation.

For a more realistic $V(\eta)$, the dispersion relation must be solved numerically to obtain x_I^m, etc. The results for a rectangular distribution are summarized in Fig. (1) and Fig. (2). One notices that the growth rate becomes negligible when the width of $V(\eta)$ is much larger than ρ.

The total power is obtained by integration, whereby one obtains

$$P_T = \int \frac{dP}{d\omega} \, d\omega = \rho \, P_{beam} \, \frac{\sqrt{2\pi} \, \sigma_\nu}{N_\lambda} \, G \, e^{\tau} \quad . \tag{10}$$

Here $P_{beam} = mc^2\gamma_0 \, I/e$ is the power contained in the electron beam.

As the radiation intensity grows from the initial random noise, the electron distribution develops periodic variation. It turns out that the variation occurs in the two-particle correlation function, whereas the single particle distribution function remains spatially uniform. Thus, there develop regions of periodic density variation, which are scattered randomly throughout the beam. To express this quantitatively, one introduces the two-particle correlation function $C(\theta,\theta',z)$, defined as the excess probability of finding one electron at θ and one at θ'. Here θ is a variable which represents the longitudinal position of an electron inside the bunch in unit of $\lambda_1/2\pi$. In the exponential growth regime, one finds

$$C(\theta,\theta',z) = \frac{2\sqrt{2\pi}\,\sigma_\nu}{9N_\lambda}\, e^{\tau}\, e^{-\sigma_\nu^2(\theta-\theta')^2/2} \cos(\theta-\theta') \quad . \tag{11}$$

Thus, the density variation has period equal to the radiation wavelength λ_1. The correlation is negligible for electrons separated by more than the distance λ_1/σ_ν.

The exponential growth cannot continue indefinitely. The saturation can be studied by extending the linear theory to a quasi-linear theory by means of a procedure familiar from plasma physics.[5] It is found that the average value of beam energy decreases so as to conserve the total energy of the radiation-beam system. It is also found that the rms spread σ_η of η increases as[9]

$$\sigma_\eta^{\,2} \sim \rho^2 \left(\frac{\sqrt{2\pi}\,\sigma_\nu}{9N_\lambda}\, e^{\tau} \right) \quad . \tag{12}$$

On the other hand, the growth rate becomes negligible when $\sigma_\eta \gg \rho$. Thus, SASE saturates when the factor in brackets in Eq. (12) becomes of order unity. In view of Eq. (10), the power at saturation is

$$P_{sat} \sim \rho\, P_{beam} \quad . \tag{13}$$

This relation was previously derived by an intuitive argument.[6]

IV. Comparison with the Livermore experiment

A high-gain FEL experiment in the microwave region has been carried out at Livermore.[3] Using the parameters of the experiment ($\gamma_0 = 7$, $\lambda_u = 9.8$ cm, $\lambda = 8.67$ mm, $I = 850$ A, $K = 2.5\sqrt{2}$, $\Delta\eta$ = full width of a rectangular momentum distribution = 6.4%, $\sigma = 3 \times 10/2$ cm^2), one obtains $\rho = 5.66 \times 10^{-2}$ and the corresponding growth rate of 42.1 dB/m. The observed growth rate, 35 dB/m, is smaller than that predicted because of space charge effects. Taking the observed growth rate and computing the coefficient in Eq. (10), one finds

$$P_T(\text{Watts}) = 2.8 \times 10^{-5} \times 10^{3.5z(m)}/\sqrt{z(m)} \quad . \tag{14}$$

Figure (3) compares this formula with the experimental result. For small z, the theoretical values are less than the experimental points, possibly because of the presence of other modes, i.e., higher harmonics. For $z \geq 2$m the theoretical value is higher. This could be due to saturation effects, not included in the quasi-linear theory.

V. A high-gain FEL at 400 Å

A high-gain FEL operating in a special by-pass of an optimized storage ring is a promising way to achieve high-power radiation at short wavelengths.[2] A design of such a system for a 400 Å FEL is described in Ref. 10, where $\rho \sim 1.5 \times 10^{-3}$ for an electron beam with $I \sim 200$ A and an rms momentum spread of 0.2%, which translates to a full width of 0.7% for a rectangular distribution. Such a system would generate about 100 MW of peak power, saturated at about 1000 undulator periods. The relative band width of the spectrum would be about 10^{-3}.

VI. Conclusion

The theory presented here is a consistent classical treatment of the development of a coherent signal from noise.[11] The theory is one-dimensional and, strictly speaking, only applicable to the situation in which the radiation is guided by an external structure such as a wave guide. Even in the absence of such external structure, it is still possible that the radiation would be self-guided by high-gain effects.[12,13] The discussion in this paper would then be of general validity. These and other extension of the theory will be reported elsewhere.

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References

1. K-J. Kim, An Analysis of Self-Amplified Spontaneous Emission, Preprint LBL-19991, 1985, Proceedings of the 7th Int. Free Electron Laser Conf (to be published).
2. J. Murphy and C. Pellegrini, J. Opt. Soc. Am., Vol. 1B, p. 530. 1984.
3. T.J. Orzechowski et al., High Gain and High Extraction Efficiency from a Free Electron Laser Amplifier Operating in the Millimeter Wave Regime, these proceedings; Phys. Rev. Lett., Vol. 54, p. 889. 1985.
4. Y.L. Klimontovich, Soviet Phys. JETP, Vol. 6, p. 753. 1958. See also S. Ichimaru, Basic Principles of Plasma Physics (W.A. Benjamin, Inc., 1973) Appendix A.
5. See, for example, G. Schmidt, Physics of High Temperature Plasmas, Academic Press, New York, 1979. Section 9-2.
6. B. Bonifacio, C. Pellegrini and N. Narducci, in Free Electron Generation of Extreme Ultraviolet Coherent Radiation, J.M.J. Madey and C. Pellegrini, eds., Am. Inst. Phys., New York, 1984, 236.
7. N.M. Kroll and W.A. McMullin, Phys. Rev., Vol. A17, p. 300. 1978; A. Gover and Z. Livni, Opt. Comm., Vol. 26, p. 375. 1978.
8. This result has been also obtained by J.M. Wang and L.H. Yu (private communication) using a Green's function method.
9. S. Krinsky, Proceedings of 1985 SLAC Summer Schools on High Energy Particle Accelerators, Stanford Linear Accelerator Center (to be published).
10. J.J. Bisognano et al., Feasibility Study of a Storage Ring for a High Power XUV Free Electron Laser, preprint LBL-19771, submitted to Particle Acc.; K-J. Kim, J.J. Bisognano, A.A. Garren, K. Halbach and J.M. Peterson, Nucl. Inst. and Methods, Vol. A239, p. 54. 1985. See also A. Jackson et al., these proceedings.
11. For previous analysis on the related subject, see P. Sprangle, C.M. Tang and L. Bernstein, Phys. Rev. Lett., Vol. 50, p. 1775. 1983, A.T. Georges, Free-Electron Generation of Coherent Radiation, SPIE Proc., Vol. 453, p. 297. 1983.
12. G.T. Moore, Optics Comm., Vol. 54, p. 121. 1985.
13. E.T. Scharlemann, A.M. Sessler and J.S. Wurtele, Phys. Rev. Lett., Vol. 54, p. 1925. 1985.

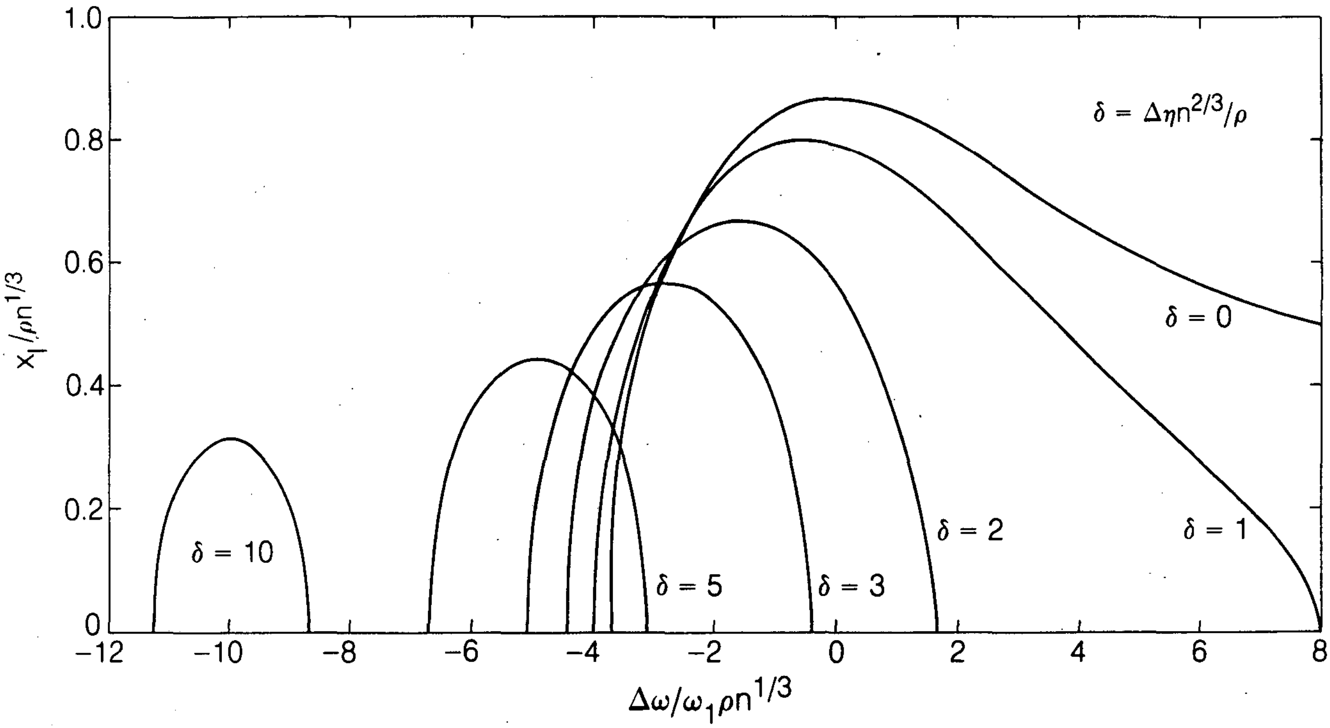

Fig. 1. The solution of the dispersion relation for arbitrary harmonic number n (n = 1 in the rest of this paper). The curves show the values of $x_I/\rho n^{1/3}$ as functions of $\Delta\omega/(\omega_1 \rho n^{1/3})$ for various values of $\delta = \Delta\eta\, n^{2/3}/\rho$. The momentum distribution is assumed to be rectangular; $V(\eta) = 1/\Delta\eta$ for $|\eta| \le \Delta\eta/2$ and $V(\eta) = 0$ for $|\eta| \ge \Delta\eta/2$.

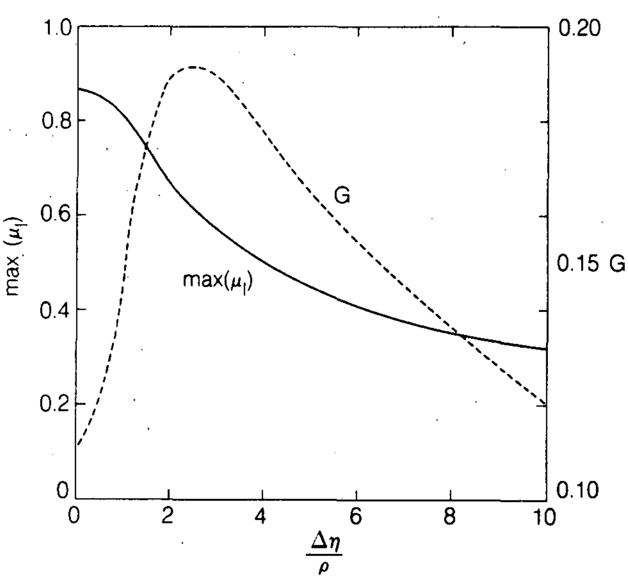

Fig. 2. The behavior of $\mathrm{Max}(\mu_I) = X_I^m/\rho$ and G (Eq. (8)) as functions of $\Delta\eta/\rho$.

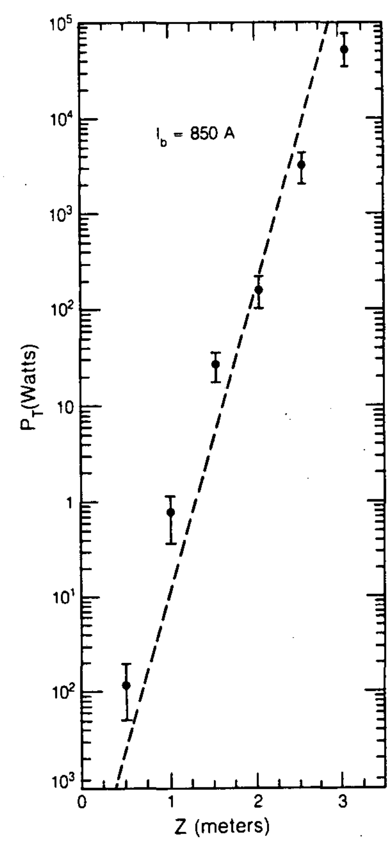

Fig. 3. A comparison of the experimental results (solid dots) and the theoretical prediction (dotted line) corresponding to the Livermore experiment (Ref. 3).

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