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POSSIBLE BINDING OF A MAGNETIC MONOPOLE TO A
PARTICLE WITH ELECTRIC CHARGE AND A MAGNETIC DIPOLE MOMENT

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ABSTRACT

We argue that Dirac monopoles, if they exist, could be strongly bound to those naturally occurring free nuclei with magnetic dipole moments, and we discuss the effect this binding would have on the interpretation of experimental monopole searches.

I. INTRODUCTION

This subject was first discussed by Malkus,¹ who concluded that there cannot be bound states between Dirac monopoles and naturally occurring free nuclei. We disagree with Malkus's interpretation of his results, and believe that bound states of this type with binding energies up to the GeV range are a definite possibility if monopoles exist at all.

Because they involve assumptions about the properties of an unobserved particle, our arguments based on simple nonrelativistic calculations are not conclusive. We cannot be sure that we are not neglecting certain crucial features of the problem. What these calculations do indicate is that the possibility of magnetically charged nuclei must be considered in the design of magnetic monopole searches.²

The plan of this paper is as follows. In Sec. II we discuss the formulation of a nonrelativistic, quantum-mechanical Hamiltonian for the problem. In Sec. III we obtain the eigenvalues for the angular operator for a spin-0 monopole and a spin-1/2 nucleus. In Sec. IV we examine the radial equation in two separate cases in which binding is possible. The first case occurs when the monopole has an electric charge and the problem is similar to that of the hydrogen atom. In the second, the monopole is electrically neutral and there is a hard-core repulsion at small radius. In each case, we calculate typical values for the binding energy. Finally, in Sec. V we briefly discuss the validity of the calculations and the effect the results would have on the conclusions of various types of experimental monopole searches.

II. FORMULATION OF THE PROBLEM AND CHARGE QUANTIZATION

We work in units with $\hbar = c = 1$ and, since we are primarily interested in binding to nuclei, choose the basic unit of energy by taking the mass of the nucleon, $M_N = 1$. In these units the nuclear magneton is given by $e/2$. The properties of the charged particle and the monopole are summarized in Table I. Note the possibility that the monopole carries electric charge, as emphasized by Schwinger.³ We do not necessarily adopt Schwinger's suggestion that the monopole be given fractional electric charge and identified with the quark.

The magnetic field of a monopole of strength vg located at $\vec{r} = 0$ is given by

$$\vec{B} = (vg/r^2) \hat{i}_r. \quad (2.1)$$

In order to construct a nonrelativistic, quantum-mechanical Hamiltonian, we need to construct a suitable vector potential.⁴ This is awkward when $\vec{\nabla} \cdot \vec{B} \neq 0$, as in (2.1), since we cannot define \vec{A} by

$$\vec{B}_0 = \vec{\nabla} \times \vec{A} \quad (2.2)$$

together with the gauge condition

$$\vec{\nabla} \cdot \vec{A} = 0. \quad (2.3)$$

Dirac's solution to this problem was to find an expression for \vec{A} which satisfies (2.3), such that the \vec{B} field given by (2.2) agrees with (2.1) except on certain singularity lines. Two possibilities for the vector potential are

$$A_r^{(1)} = A_\theta^{(1)} = 0, \quad A_\phi^{(1)} = vg(1 - \cos \theta)/\sin \theta, \quad (2.4)$$

$$A_r^{(2)} = A_\theta^{(2)} = 0, \quad A_\phi^{(2)} = -vg \cos \theta/\sin \theta, \quad (2.5)$$

where $A^{(1)}$ is singular along the negative z axis ($r = -z$, $\theta = \pi$) and $A^{(2)}$ is singular along the entire z axis. The correspondence between (2.1) and (2.2) is then given by

$$(vg/r^2)\hat{i}_r = \nabla \times \tilde{A}^{(1,2)} - \tilde{B}_f^{(1,2)}, \quad (2.6)$$

where

$$\tilde{B}_f^{(1)} = \hat{i}_z \begin{cases} 4\pi vg \delta(x) \delta(y) & z < 0 \quad (\theta = \pi) \\ 0 & z > 0 \quad (\theta = 0) \end{cases}, \quad (2.7)$$

$$\tilde{B}_f^{(2)} = \hat{i}_z \begin{cases} 2\pi vg \delta(x) \delta(y) & z < 0 \quad (\theta = \pi) \\ -2\pi vg \delta(x) \delta(y) & z > 0 \quad (\theta = 0) \end{cases}. \quad (2.8)$$

It is, of course, also possible to take alternative orientations for the singularity line or to find expressions for the vector potential involving curved singularity lines. In order to have rotation invariance in the theory, all physical observables must be independent of the singularity line.

Within the approximations of nonrelativistic quantum mechanics, it is acceptable to use either vector potential, (2.4) or (2.5), directly in the Hamiltonian, although we must subtract the appropriate fictitious field, (2.7) or (2.8), if we intend to calculate the correct stress-energy

tensor.⁵ We therefore have the Hamiltonian⁶

$$H^{(1)} = -\frac{1}{2Tr^2} \left\{ \frac{\partial}{\partial r} r^2 \frac{\partial}{\partial r} + \frac{1}{\sin \theta} \frac{\partial}{\partial \theta} \sin \theta \frac{\partial}{\partial \theta} \right. \\ \left. + \frac{1}{\sin^2 \theta} \left[\frac{\partial}{\partial \phi} - iZveg(1 - \cos \theta) \right]^2 + v\mu eg T \vec{\Sigma} \cdot \hat{i}_r \right\} \\ + \frac{YZe^2}{r} + U(r), \quad (2.9)$$

where $T = M_Y M_Z / (M_Y + M_Z)$ and $U(r)$ is an undetermined potential which is assumed to be appreciable only at small distances. The necessity for including this undetermined potential is discussed later, but at this point it reflects our ignorance of hadronic and form-factor effects at small distances.

Schrödinger's equation is separable;

$$H^{(1)} R^{(1)}(r) \psi^{(1)}(\theta, \phi) = E R^{(1)}(r) \psi^{(1)}(\theta, \phi) \quad (2.10)$$

becomes

$$\left\{ \frac{1}{\sin^2 \theta} \left[\sin \theta \frac{\partial}{\partial \theta} \sin \theta \frac{\partial}{\partial \theta} + \left(\frac{\partial}{\partial \phi} - iZveg(1 - \cos \theta) \right)^2 \right] \right. \\ \left. + v\mu eg T \vec{\Sigma} \cdot \hat{i}_r \right\} \psi^{(1)}(\theta, \phi) = -\beta^{(1)} \psi^{(1)}(\theta, \phi) \quad (2.11)$$

and

$$\left\{ -\frac{1}{r^2} \left(\frac{\partial}{\partial r} \right) r^2 \left(\frac{\partial}{\partial r} \right) + \frac{\beta^{(1)}}{r^2} + 2TU(r) \right\} R^{(1)}(r) = 2TER^{(1)}(r). \quad (2.12)$$

There are completely analogous expressions for the Hamiltonian obtained by inserting the vector potential (2.5) instead of (2.4). As yet, we

have no way of knowing that the results depend on whether we choose a vector potential with a one-sided or a two-sided singularity line.

To investigate this point, we look at the solution of the angular eigenvalue equation for $\mu = 0$,

$$\Lambda^{(1)} \psi_{\ell m}^{(1)}(\theta) e^{im\phi} = -\alpha^{(1)}(\ell, m, Z\nu) \psi_{\ell m}^{(1)}(\theta) e^{im\phi}, \quad (2.13)$$

with

$$\Lambda^{(1)} = \frac{1}{\sin^2 \theta} \left[\sin \theta \frac{\partial}{\partial \theta} \sin \theta \frac{\partial}{\partial \theta} - \left(\frac{\partial}{\partial \phi} \left(\frac{1+\cos \theta}{2} \right) + \left(\frac{\partial}{\partial \phi} - 2Z\nu \right) \left(\frac{1-\cos \theta}{2} \right) \right)^2 \right]. \quad (2.14)$$

Equation (2.13) has the solution⁷

$$\psi_{\ell m}^{(1)}(\theta) = d_{m-Z\nu, Z\nu}^{\ell+s}(\theta) \quad (\ell = 0, 1, 2, \dots), \quad (2.15)$$

where $s = \max(|m - Z\nu|, |Z\nu|)$ and the $d_{mn}^J(\theta)$ are the familiar representations of the rotation group.⁸ In order to have rotation invariance, we must therefore have the quantization condition⁹

$$Z\nu = \frac{1}{2} n \quad (n = 1, 2, \dots). \quad (2.16)$$

Proceeding in the same way for the vector potential with the two-sided singularity line, we find the angular eigenfunctions

$$\psi_{\ell m}^{(2)} = d_{m, -Z\nu}^{\ell+s}(\theta), \quad (2.17)$$

and requiring rotation invariance in this instance gives the stronger quantization condition,⁹

$$Z\nu = 1 \cdot n \quad (n = 1, 2, \dots). \quad (2.18)$$

The familiar quantization conditions, (2.16) and (2.18), have been obtained in many independent ways.¹⁰ Here they are necessary in order that the angular eigenfunctions of the quantum mechanical Hamiltonian correspond to irreducible representations of the rotation group. The eigenvalues corresponding to (2.15) and (2.17) are respectively

$$\alpha^{(1)}(\ell, m, Z\nu) = \left\{ \ell + \frac{1}{2}(|m| + |m - 2Z\nu e g|) \right\} \left\{ \ell + 1 + \frac{1}{2}(|m| + |m - 2Z\nu e g|) \right\} - Z^2 \nu^2 e^2 g^2 \quad (2.19)$$

and

$$\alpha^{(2)}(\ell, m, Z\nu) = \left\{ \ell + \frac{1}{2}(|m + Z\nu e g| + |m - Z\nu e g|) \right\} \times \left\{ \ell + 1 + \frac{1}{2}(|m + Z\nu e g| + |m - Z\nu e g|) \right\} - Z^2 \nu^2 e^2 g^2. \quad (2.20)$$

From now on, we take

$$eg = 1/2, \quad (2.21)$$

and note that the stronger two-sided quantization condition, (2.18), is equivalent to requiring ν to be an even integer in (2.16). We then have

$$\alpha^{(2)}(\ell, m, Z\nu) = \alpha^{(1)}(\ell, m + \frac{Z\nu}{2}, Z\nu), \quad \nu = 0, \pm 2, \pm 4, \dots, \quad (2.22)$$

and all results can be obtained by using the $\psi_{\ell m}^{(1)}(\theta)$ and (2.22). Thus, from examining Schrödinger's equation with $\mu = 0$, the only difference between a two-sided and a one-sided vector potential are the different

quantization conditions, (2.16) and (2.18), and we cannot distinguish between the two possibilities unless we observe experimentally a magnetic monopole with $\nu = 1$. Note that the eigenvalues (2.19) and (2.20) are positive definite.

III. THE ANGULAR OPERATOR WHEN $\mu \neq 0$

Now, consider the solution of the angular eigenvalue equation, (2.11), for a spin- $\frac{1}{2}$ nucleus with $\mu \neq 0$. Let $\gamma = \mu T/2$ and $x = \cos \theta$, then (2.11) becomes

$$\begin{pmatrix} \Lambda^{(1)} + \nu\gamma x & \nu\gamma(1-x)^{1/2}(1+x)^{1/2}e^{-i\phi} \\ \nu\gamma(1-x)^{1/2}(1+x)^{1/2}e^{i\phi} & \Lambda^{(1)} - \nu\gamma x \end{pmatrix} \begin{pmatrix} \psi_A e^{i(m-1)\phi} \\ \psi_B e^{im\phi} \end{pmatrix} = -\beta \begin{pmatrix} \psi_A e^{i(m-1)\phi} \\ \psi_B e^{im\phi} \end{pmatrix}. \quad (3.1)$$

The polynomials $\psi_{\ell m}^{(1)}(\theta)$, (2.15), are orthogonal, and this suggests we make the expansion

$$\psi_A = \sum_{\ell=0}^{N_A} c_{\ell} \psi_{\ell, m-1}^{(1)}(x), \quad (3.2)$$

$$\psi_B = \sum_{j=0}^{N_B} d_j \psi_{j, m}^{(1)}(x). \quad (3.3)$$

We have to consider separately two cases of (3.1),

$$\text{Case 1: } m \geq 1, \quad (m - Z\nu) \geq 1, \quad (3.4a)$$

$$\text{Case 2: } m \geq 1, \quad (m - Z\nu) \leq 0. \quad (3.4b)$$

Other values of m can be reduced to one of these two case by using the identity⁸

$$d_{m,n}^J(x) = d_{-m,-n}^J(x). \quad (3.5)$$

In order to relate $\psi_{\ell,m}$ and $\psi_{\ell,m-1}$ it is convenient to express the d_{mn}^J in terms of Jacobi polynomials,⁷

$$\psi_{\ell m}(x) = (1-x)^{m/2} (1+x)^{|m-Z\nu|/2} P_{\ell}^{(m, |m-Z\nu|)}(x). \quad (3.6)$$

After the replacement, (3.6), we see that Eq. (3.1) has a different form in each of the two cases, (3.4).

Case 1. Absorbing the normalization of (3.6) into the coefficients of (3.2) and (3.3), we have the equations

$$\begin{aligned} \sum_{k=0}^{N_A} c_k \left[-\alpha(k, m-1, Z\nu) + \beta + \nu\gamma x \right] P_k^{(m-1, m-1-Z\nu)}(x) \\ + \sum_{\ell=0}^{N_B} \nu\gamma d_{\ell} (1-x^2) P_{\ell}^{(m, m-Z\nu)}(x) = 0, \end{aligned} \quad (3.7a)$$

$$\begin{aligned} \sum_{k=0}^{N_A} c_k \nu\gamma P_k^{(m-1, m-1-Z\nu)}(x) + \sum_{\ell=0}^{N_B} d_{\ell} \left[-\alpha(k, m, Z\nu) + \beta - \nu\gamma x \right] \\ \times P_{\ell}^{(m, m-Z\nu)}(x) = 0. \end{aligned} \quad (3.7b)$$

If the series terminates, then $N_A = N_B + 1$. Expanding the Jacobi polynomials in power series and matching powers of x , we find the eigenvalues

$$\beta_1^{\pm}(N_B, m, Zv, \gamma v) = \left(N_B + m - \frac{Zv}{2} \right)^2 - \frac{Z^2 v^2}{4} \pm \left[\left(N_B + m - \frac{Zv}{2} \right)^2 - \frac{Z^2 v^2}{4} + \left(\frac{Zv}{2} - \gamma v \right)^2 \right]^{1/2}. \quad (3.8)$$

These eigenvalues need not all be positive. The minimum occurs when $N_B = 0$, $m = Z + 1$, or $m = 1$,

$$\min_{(N, m)} (\beta_1^{\pm}) = 1 + |Zv| - \left[1 + |Zv| + \left(\frac{Zv}{2} - \gamma v \right)^2 \right]^{1/2}, \quad (3.9)$$

which can be negative when $(v^2/4)(Z - \mu T)^2$ is large enough. Inserting (3.5) into (3.7), we get the eigenvalues for the case when $m \leq 0$ and $(m - Zv) \leq 0$,

$$\beta_3^{\pm}(N_B, m, Zv, \gamma v) = \beta_1^{\pm}(N_B, -m+1, -Zv, -\gamma v), \quad m = 0, -1, -2, \dots \quad (3.10)$$

Case 2. In this instance, the eigenvalues (2.19) are independent of m and all equal to

$$\alpha^{(1)}(\ell, 0, Zv) = \ell(\ell + 1) + \frac{2\ell + 1}{2} |Zv|. \quad (3.11)$$

Equation (3.1) becomes

$$\sum_{k=0}^{N_A} c_k \left[-\alpha(k, 0, Z\nu) + \beta + \nu\gamma x \right] P_k^{(m-1, Z\nu-m+1)}(x) \\ + \sum_{j=0}^{N_B} d_j \nu\gamma(1-x) P_j^{(m, Z\nu-m)}(x), \quad (3.12a)$$

$$\sum_{k=0}^{N_A} \nu\gamma c_k (1+x) P_k^{(m-1, Z\nu-m+1)}(x) + \sum_{j=0}^{N_B} d_j \left[-\alpha(j, 0, Z\nu) \right. \\ \left. + \beta - \nu\gamma x \right] P_j^{(m, Z\nu-m)}(x), \quad (3.12b)$$

where $N_A = N_B$. Using the identity⁷

$$(1-x) \left[P_N^{(a,b)} - P_N^{(a-1,b+1)} \right] = \frac{(a+N-1)}{\left(\frac{a+b}{2} + N \right)} P_{N-1}^{(a-1,b+1)} \\ - \frac{N}{\left(\frac{a+b}{2} + N \right)} P_N^{(a-1,b+1)}, \quad (3.13)$$

we can combine the sums in (3.12) and obtain the eigenvalues¹¹

$$\beta_2(N_B, Z\nu, \gamma\nu) = N_B(N_B + 1) + (2N_B + 1) \frac{|Z\nu|}{2} - \gamma\nu \left(1 - \frac{N_B}{N_B + \frac{|Z\nu|}{2}} \right). \quad (3.14)$$

When $N_B = 0$ this reduces to

$$\beta_2(0, Z\nu, \gamma\nu) = \frac{Z\nu}{2} - \gamma\nu = \frac{\nu}{2} (Z - \mu T). \quad (3.15)$$

Inserting (3.5) into (3.12), we get the eigenvalues for the case when $m \leq 0$, $m - Z\nu \geq 1$,

$$\beta_4(N_B, Z\nu, \gamma\nu) = \beta_2(N_B, -Z\nu, -\gamma\nu). \quad (3.16)$$

Figure 1 shows a plot of $Z\nu$ vs m and indicates where the various expressions for the eigenvalues are valid. Tables II and III give a tabulation of β_{\min} for naturally occurring nuclei with various assumptions about the magnetic monopole mass and charge. As can be seen, negative eigenvalues are in abundance and, as is discussed in Sec. IV, this opens the possibility of bound states between an electrically neutral monopole and nuclei if there is a hard-core repulsion. Numerical calculations of eigenvalues for spin-1 and spin-3/2 nuclei also indicate the possibility of negative values for the angular eigenvalue. The methods used for the spin- $\frac{1}{2}$ case can lead to cubic or quartic equations involving β for spin 1 and spin 3/2 respectively, so no attempt was made to obtain the analytic form for the eigenvalues in these cases.

As can be seen in (3.8) and (3.14), the angular eigenvalues depend on the combination μT , where μ is the strength of the dipole moment in nuclear magnetons and T is the reduced mass. The value of β therefore depends on the monopole mass. In Table I we have therefore included the monopole mass that gives $\beta = -\frac{1}{4}$. The significance of this value is apparent in Sec. IV.

IV. THE RADIAL EQUATION

Hydrogen Atom Type of Solution

First consider the case when the Coulomb potential is attractive and the $1/r^2$ "potential" is repulsive. For this case the form of $U(r)$ is unimportant if it is appreciable only for small radii. We then have the familiar form of the radial equation for the hydrogen atom. Let $k^2 = -2TE$, $TYZe^2/k = p$, and $\rho = 2kr$; then (2.12) becomes

$$\frac{1}{\rho^2} \frac{\partial}{\partial \rho} \rho^2 \frac{\partial}{\partial \rho} R - \left(\frac{1}{4} + \frac{\beta}{\rho^2} - \frac{p}{\rho} \right) R = 0. \quad (4.1)$$

Equation (4.1) has the solution¹²

$$R(\rho) = e^{-\rho/2} \rho^{-(s+1)/2} {}_1F_1(-K; 1-s; \rho), \quad (4.2)$$

where

$$s = \pm(4\beta + 1)^{1/2}, \quad (4.3)$$

$$+2p = 2K - s + 1 = 2K + (4\beta + 1)^{1/2} + 1. \quad (4.4)$$

The positive root of (4.3) gives a divergence at the origin, and in order for s to be real we must have $\beta \geq -1/4$. The confluent hypergeometric function, ${}_1F_1(-K; c; z)$, reduces to a polynomial when K is a positive integer, and this gives the quantization condition

$$E_K = -2T \frac{Y^2 Z^2 e^4}{\left(2K + [4\beta + 1]^{1/2} + 1 \right)^2}. \quad (4.5)$$

Typical values of these binding energies are given in Table IV. As can be seen there, these binding energies are quite large for a wide range of values for the mass and the strength of the monopole. We therefore justify a posteriori the neglect of the atomic electrons in the problem.

Due to the factor $\exp(-kr)$ in (4.2), the wavefunction is small at the radius of a typical orbital electron. Including the effect of electrons in the calculation gives a perturbation of order M_e/M_Z . This type of binding can occur, of course, when $\mu = 0$ so that the nucleus can have spin 0.

Since Schwinger has emphasized the possibility of a magnetic monopole's having electric charge,³ this type of binding is potentially important.

Electrically Neutral Monopole with Repulsive Core

Let $Y = 0$, so that we have a $1/r^2$ "potential" and there is an infinite repulsive core of radius r_0 ,

$$U(r) = \begin{cases} \infty & 0 < r < r_0 \\ 0 & r_0 < r < \infty \end{cases} \quad (4.6)$$

Equation (2.12) then reduces to Bessel's equation subject to the boundary conditions

$$R(r_0) = 0, \quad (4.7)$$

$$\lim_{x \rightarrow \infty} R(x) = 0. \quad (4.8)$$

Let $k^2 = -2TE$, then the most general negative energy solution of (2.12) is given by

$$R(r) = r^{-1/2} \left[c_1 I_p(kr) + c_2 K_p(kr) \right], \quad (4.9)$$

where I_p and K_p are Bessel functions of order

$$p = \left(\frac{1}{4} + \beta \right)^{1/2}. \quad (4.10)$$

The asymptotic behavior of the Bessel functions is given by

$$K_p(x) \underset{x \rightarrow \infty}{\sim} (\pi/2x)^{1/2} e^{-x}, \quad (4.11)$$

$$I_p(x) \underset{x \rightarrow \infty}{\sim} (1/2\pi x)^{1/2} e^{+x}, \quad (4.12)$$

so that in order to satisfy the boundary condition, Eq. (4.8), we must have $c_1 = 0$. As for the boundary condition, (4.7), for p and x real, $K_p(x)$ has no zeros, as can be seen by examining the integral representation

$$K_p(x) = \int_0^\infty \exp \left\{ -x \cosh t \right\} \cosh(pt) dt. \quad (4.13)$$

Therefore, from (4.10), we cannot satisfy (4.7) when $\beta \geq -1/4$. When $\beta < -1/4$, p is purely imaginary and we have the possibility of zeros.

The location of the zeros can be approximately determined by the asymptotic expansion¹³

$$K_{ip}(x) \sim \frac{1}{3} \pi e^{-\pi p/2} \left[\frac{2(p-x)}{x} \right]^{1/2} \left[J_{1/3} + J_{-1/3} \right] \left(\frac{1}{2} \left[\frac{2(x-p)}{x^{1/2}} \right]^{3/2} \right). \quad (4.14)$$

The first zero on the rhs of (4.14) is at $p = x$, and this gives an approximate value for the binding energy

$$\frac{\frac{1}{4} + \beta}{2T r_0^2} \cong E_0 \quad \text{for} \quad \beta < -1/4. \quad (4.15)$$

Typical values of this binding energy for different nuclei are given in Table V. The importance of the repulsive core is now evident. Without

it, the wave function would "fall" to the origin and the energies (4.15) can become arbitrarily large in absolute value.¹⁴ We emphasize that the use of a repulsive core is not necessarily unphysical. Relativistic corrections to Schrödinger's equation can be approximated by a repulsive potential proportional to r^{-4} , and hadronic effects could give a repulsive Yukawa potential. In either of these cases, R would have to be small at some typical radius, r_0 , and this approximately leads to binding energies of the same order of magnitude as those given by (4.15), since the energy depends roughly on the smallest radius to which the wave function is allowed to fall. It seems reasonable that this radius would be near the nuclear Compton wavelength, which is the value we have used in Table V.

The use of the potential $U(r)$ attempts to circumvent the limitations of Schrödinger's equation at small radii. Once these limitations are recognized, the conclusion that bound states exist seems inescapable.

V. DISCUSSION AND CONCLUSIONS

The possibility that magnetic monopoles can be captured and bound by common nuclei must be considered in the evaluation of monopole searches. For a complete classification and review of the experimental situation we refer the reader to the paper by Amaldi.² Here we discuss only those experiments which search for monopoles trapped in geological materials that are potential collectors. Many of these experiments rely on extracting the monopoles from the host material by use of pulsed magnetic fields. If the monopole is loosely held with a binding energy of a few eV, the magnetic fields can easily extract it. But the binding

energies proposed in this paper are orders of magnitude greater than those envisioned by Malkus,¹ and cannot be overcome by conveniently available magnetic fields.

If the bound states discussed here exist, one must be prepared to show that the external field used in a particular experiment can remove a magnetically charged nucleus-monopole bound state from the surrounding chemical structure. If the magnetic charge is measured by ionization, one must consider the possibility that a large electric charge is also present. There is also the possibility that the total mass of the monopole and nucleus is considerably larger than the original monopole mass.¹⁵

Another consideration is the abundance of potential collecting material. Many experiments have searched for monopoles in ferromagnetic material that is scarce compared with the quantity of material represented by the nuclei in Table II. A rough figure of merit for experimental searches of this type is given by the area of the material exposed to cosmic rays multiplied by the exposure time. If common materials as well as ferromagnetic materials are potential monopole collectors, experiments involving a larger surface area of material may be considered.

We emphasize that those experiments which do not extract monopoles from potential collectors in order to detect them are cleaner than those which do, since their detection apparatus depends only on the classical, large-radius properties of monopoles. Their results are therefore less susceptible of possible modification from the quantum-mechanical properties of the monopole.

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FOOTNOTES AND REFERENCES

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 3. J. Schwinger, Phys. Rev. 144, 1087 (1966); 173, 1536 (1968). Giving the magnetic monopole an electric charge is permitted provided one uses a vector potential with a two-sided singularity line, and the appropriate quantization. We do not consider the extra factor of 2 that Schwinger obtains by considering surfaces which intersect a gauge line on their boundary.
 4. P. A. M. Dirac, Proc. Roy. Soc. (London) A133, 60 (1931); Phys. Rev. 74, 817 (1948). The use of the vector potential in quantum mechanics is obligatory because of the necessity of obtaining a gauge-independent translation operator--J. Zak, Phys. Rev. 134, A1602 (1964). It is easily shown that the correct generator of the translation group in

the presence of a magnetic field is

$$\underline{p} = -i \underline{\nabla} - e \underline{A} - \frac{1}{2} \underline{r} \times \underline{B}.$$

See the discussion of B. Zumino in 1966 International School of Physics "Ettore Majorana," edited by Z. Zichichi (Academic Press, New York, 1966), about the interpretation of the singularity lines and alternative forms for the vector potential.

5. E. Amaldi, op. cit., points out that in order to have the correct form for the stress-energy tensor we must subtract the fictitious field, (2.7) or (2.8). The stress-energy tensor is invariant under a rotation in "charge space" of the form

$$\begin{pmatrix} g' \\ e' \end{pmatrix} = \begin{pmatrix} \cos \delta & \sin \delta \\ -\sin \delta & \cos \delta \end{pmatrix} \begin{pmatrix} g \\ e \end{pmatrix}.$$

If a suitable rotation is chosen, our calculation can be valid for a spin-0 nucleus and a magnetic monopole with an electric dipole moment.

6. The Hamiltonian is in the Pauli approximation, neglecting the spin-orbit interaction. Relativistic corrections are assumed to be approximated by the undetermined potential, $U(r)$.
7. W. Magnus, F. Oberhettinger, and R. P. Soni, Formulas and Theorems for the Special Functions of Mathematical Physics (Springer-Verlag, New York, 1966), p. 209.
8. M. E. Rose, Elementary Theory of Angular Momentum (John Wiley and Sons, New York, 1957), p. 48.

9. E. P. Wigner, Gruppentheorie (Freidrichweinig und Sohn, Braunschwig, 1931) has shown the d_{mn}^J are the wave functions of a symmetric top. We can get a qualitative feel for why we have eigenfunctions of the form $d_{m, Zv/2}^J$ instead of $d_{m0}^J = Y_m^J$ by noting that applying a rotation to (2.10) also rotates the direction of the singularity line of the vector potential and we must apply a gauge transformation to restore the form, (2.4) or (2.5). The theory would therefore not be rotation-invariant without gauge transformations, that is, without massless photons. The quantization condition (2.16) comes from requiring m and n in d_{mn}^J to be half integral, (2.18) comes from requiring $m-n$ to be integral.
10. Perhaps the cleanest derivation of the quantization condition (2.16) is that of A. Goldhaber, Phys. Rev. 140, B1407 (1965), which does not depend on a singularity line.
11. The eigenvalues, (3.8), were obtained by Malkus in Ref. 1. The eigenvalues, (3.14), were omitted except for the special case, $N = 0$.
12. W. Magnus, et al., op. cit., p. 239.
13. Equation (4.15) is an asymptotic representation of the "Nicholsen" type valid for x/p near unity and $x-p$ large, and therefore gives only a rough estimate for the location of the zeros. See W. Magnus et al., op. cit., p. 142.
14. L. D. Landau and E. M. Lifschitz, Quantum Mechanics, 2nd Ed. (Addison-Wesley, Reading, Mass., 1965). See especially the discussion in Secs. 22 and 35.
15. As long as experiments do not detect any monopole signal, this criticism is academic. The problem is that these experiments could not determine monopole properties from a positive result.

Table I. The assumed properties of the magnetic monopole and the nucleus (Refs. 3, 5). The unit of magnetic charge is chosen to be $g = 1/2e$, from (2.16). The mass of the monopole is, of course, undetermined. In making sample calculations we have taken $M_Y = 10, 100$, but we have found no theoretical arguments to indicate that this is the right order of magnitude.

	<u>Magnetic monopole</u>	<u>Nucleus</u>
Magnetic charge	$vg, \quad v = 0, \pm 1, \pm 2, \dots$	0
Electric charge	$Ye, \quad Y = 0, \pm 1, \dots$	$Ze, \quad Z = 0, 1, 2, \dots$
Mass	M_Y	M_Z
Spin	0	$\Sigma = 0, \frac{1}{2}, 1, \dots$
Magnetic dipole mom.	0	$\mu e/2$
Electric dipole mom.	0	0

Table II. Typical values of β_2 for spin-1/2 nuclei. The value of monopole mass which gives $\beta_2 = -1/4$ is also shown.

Nucleus (% abundance)	Z	μ	Monopole Mass ($M_p = 1$)	ν	$\beta_2(0, Z, \nu, \mu T)$
n	0	-1.91	0.35	-1	- 0.25
			10	-1	- 0.87
			100	-1	- 0.95
			0.15	-2	- 0.25
			10	-2	- 1.74
p, H	1	2.79	100	-2	- 1.89
			1.16	1	- 0.25
			10	1	- 0.77
			100	1	- 0.88
			0.812	2	- 0.25
^{13}C (1.11×10^{-2})	6	0.702	10	2	- 1.54
			100	2	- 1.77
			10	1	1.01
			32.2	1	- 0.25
			100	1	- 1.04
			10	2	2.03
			23.3	2	- 0.25
			100	2	- 2.09

Continued

Table II. Cont.

Nucleus (% abundance)	Z	μ	Monopole Mass ($M_P = 1$)	ν	$\beta_2(0, Z, V, \mu T)$
F^{19} (100)	9	2.63	4.46	1	- 0.25
			10	1	- 4.12
			100	1	-16.6
			4.32	2	- 0.25
			10	2	- 8.24
			100	2	-33.1

P^{31} (100)	15	1.13	10	1	3.25
			24.6	1	- 0.25
			100	1	- 5.9
			10	2	6.50
			23.9	2	- 0.25
			100	2	-11.8

Table III. Typical values of β_1 for spin-1/2 nuclei.

Nucleus (% abundance)	Z	μ	Monopole Mass	ν	$\beta_1^{\text{Min}}(0, Z, \nu, \mu T)$
n	0	-1.91	10	-1	- 0.32
			100	-1	- 0.37
			10	-2	- 1.01
			100	-2	- 1.14

He ³ (1.3 × 10 ⁻⁶)	2	-2.12	10	1	- 0.86
			100	1	- 1.45
			10	2	- 2.20
			100	2	- 3.55

F ¹⁹ (100%)	9	2.63	10	1	4.81
			100	1	- 6.80
			10	2	9.70
			100	2	-14.32

Table IV. Typical values of the binding energy, E_0 , given by Eq. (4.5) for naturally occurring nuclei.

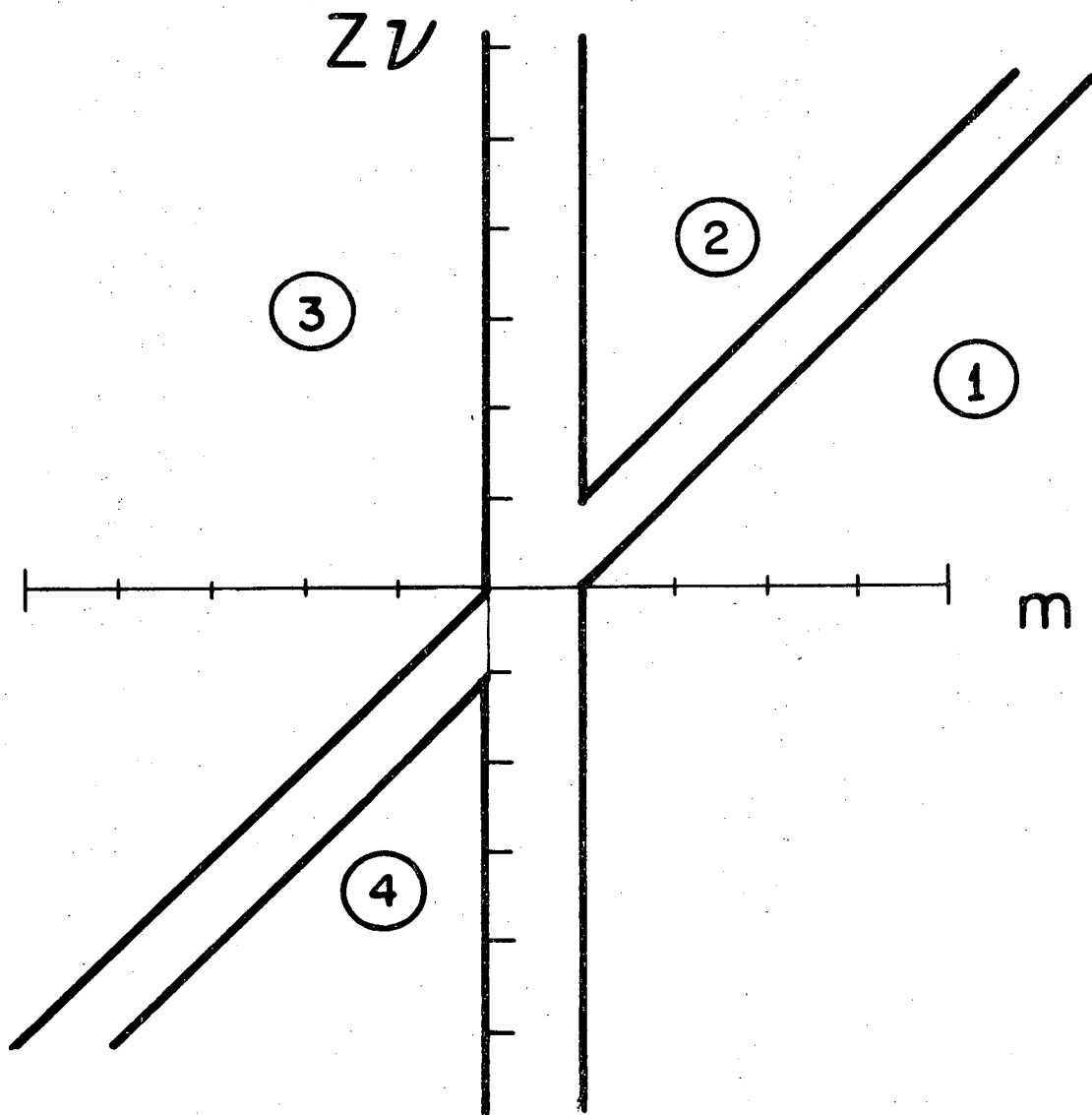
Nucleus	β	M_Y	Binding Energy (eV.)
p, H	0.52	100	1.9×10^4
C ¹²	6	100	5.8×10^5
N ¹⁴	7	100	9.3×10^5
P ³¹	12.8	100	8.4×10^6

Table V. Typical values of the binding energy, Eq. (4.15), for common nuclei.

Nucleus	M_Y	β	Binding energy (GeV)
n	100	-0.95	0.35
p, H	100	-0.88	0.32
C ¹³	100	-1.04	0.034
F ¹⁹	100	-16.6	0.51
P ³¹	100	-5.9	0.12

FIGURE CAPTION

Fig. 1. Plot of Zv vs m shows the regions where different expressions for the angular eigenvalues are obtained. In region 1, use Eq. (3.8); in 2, Eq. (3.14); in 3, Eq. (3.10); and in 4, Eq. (3.16).



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Fig. 1

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