

UNIVERSITY OF CALIFORNIA,
IRVINE

Intensionality and Intentionality: Phenomenology, Logic, and Mind

DISSERTATION

submitted in partial satisfaction of the requirements
for the degree of

DOCTOR OF PHILOSOPHY

in Philosophy

by

Kyle Banick

Dissertation Committee:
Professor David W. Smith, Co-Chair
Associate Professor Sean Walsh, Co-Chair
Professor Annalisa Coliva

2017

DEDICATION

To

Mutti and Pops, for everlasting support.

TABLE OF CONTENTS

	Page
ACKNOWLEDGMENTS	iv
CURRICULUM VITAE	v
ABSTRACT OF THE DISSERTATION	vi
CHAPTER 1: Epistemic Logic and the Halbach-Welch Rapprochement Strategy	1
The Halbach-Welch Approach	4
Epistemic Logic: Four Different Notions of Knowledge	9
Conclusion	24
CHAPTER 2: Hintikka’s Logic of Perception	31
Background Explication	34
Hintikka Semantics for “Sees”	52
The Validities and Invalidities of the Hintikka Semantics	60
Quantifiers and Direct Perception	67
Implications for Intentionality	79
Conclusion	87
CHAPTER 3: How to Be an Adverbialist about Phenomenal Intentionality	89
Introduction	89
Semantics of Adverbialism	92
The Nature of Phenomenally Conscious Intentional Events	106
Adverbial Realism and Phenomenal Intentionality	114
Appendix 1: Logic of Adverbs	124
Appendix 2: Proofs	127
BIBLIOGRAPHY	129

ACKNOWLEDGMENTS

I would like to thank my two co-advisors, David Woodruff Smith and Sean Walsh. David showed me the way into phenomenology, and Sean into logic. Without the support and knowledge of these two, this dissertation would not exist: David as sage helped me to understand how to combine my disparate interests into a coherent field of research, and Sean taught me how to turn my wandering thoughts into words that would actually end up on the dissertation page rather than in the back files of a hard drive.

Many thanks also to my other committee member, Annalisa Coliva, for her immediate willingness to serve on the committee even before settling into Irvine, and for her incisive perspective on issues in perception and self-knowledge that I would have otherwise overlooked. To Jeffrey Yoshimi for his sincere interest in the dissertation and for our telephone conversations relating to the modal logic of perception. To Aaron James, for an influential conversation in which he encouraged me to do more of the heavy lifting once the logic was done. To Martin Schwab, for early encouragement and for teaching me about Deleuze and Kierkegaard. To Ermanno Bencivenga, for being a transcendental idealist against the fashion. Thanks as well to the members of the California Phenomenology Circle—the fact that such a thing exists is reason enough to be optimistic.

Thanks to my fellow graduate students, in particular: Matt Dworkin, Phil Walsh, Dan Siakel, Ethan Galebach (for getting me through set theory), Adam Fox, Simona Capisani, Megan Zane, Valentina Ricci, Adam Sanders, Dylan Popowicz, Kino Zhao, Greg Lauro, and Will Stafford.

Last but not least, thanks to Alaska, who always kept me afloat, and to Katie, Caroline, Mark, and Melissa, for never even entertaining the thought that I wouldn't make it.

CURRICULUM VITAE

Kyle Banick

- 2011 B.A. in Philosophy, Chapman University
- 2013-17 Teaching Assistant, Philosophy Department,
University of California, Irvine
- 2015 M.A. in Philosophy, University of
California, Irvine
- 2015 Instructor, Philosophy Department,
University of California, Irvine
- 2016 Instructor, Philosophy Department,
University of California, Irvine
- 2016 Lecturer, Philosophy Department,
Chapman University
- 2017 Ph.D. in Philosophy,
University of California, Irvine

FIELD OF STUDY

Phenomenology, Philosophical Logic, Philosophy of Mind.

ABSTRACT OF THE DISSERTATION

Intensionality and Intentionality: Phenomenology, Logic, and Mind

By

Kyle Banick

Doctor of Philosophy in Philosophy

University of California, Irvine, 2017

Professor David W. Smith, Co-Chair
Associate Professor Sean Walsh, Co-Chair

Chapter 1 concerns issues in the construction of formal models of intensional notions.

Intensional notions may be treated as modal operators or as predicates. Halbach and Welch (2009) have proposed a new formal technique to reduce the necessity predicate to an operator, demonstrating that the two methods are ultimately compatible. I show that the monotonicity constraint in the Halbach-Welch technique fails for almost all possible-worlds theories of knowledge. Since the monotonicity constraint is an important element of the proof of the Halbach-Welch rapprochement strategy in the case of necessity, the present results show that the most obvious way of emulating this strategy in the epistemic setting fail.

In Chapter 2, I reconstruct and reconsider Hintikka's (1969) innovative operator approach to the logic of perception. I use a modal operator and a method of many-sorted substitutional quantification to formalize the notion of perceptual reference. I assimilate Hintikka's logic to present-day norms of formal logic and I explore Hintikka's treatment of quantification in the context of his overarching thesis that the possible-worlds theory of intensionality *eo ipso*

provides a theory of mental intentionality. My reconstruction makes us well-posed to consider what is true about Hintikka's philosophical claims, but also to see some of their decisive limitations.

In Chapter 3, I construct a framework for an adverbialist theory of phenomenal intentionality. I first pose the question: what is the *logic* of adverbial modification in the setting of phenomenal consciousness? I argue from tools in formal semantics that the logic is one of *events*, rather than a logic of *intensionality*. I then argue that the event adverbialist can offer elegant regimentations of crucial distinctions that give the event adverbialist purchase on phenomenal intentionality.

This chapter is understood as the other side of the coin from chapter 2 above. There, I reconstruct an account of intentional reference; here, I give a novel account of a structure that determines the ways in which acts are directed toward their (putative) reference.

Chapter 1

Epistemic Logic and the Halbach-Welch Rapprochement Strategy

This chapter concerns the formal representation of intentional notions in intensional logic. An intensional notion can be treated as a modal operator or as a specialized predicate. Traditionally, this choice has been seen as a substantive one: Kaplan and Montague (1960) demonstrate that standard axiomatic treatments of intensional predicates are inconsistent, a phenomenon referred to as the *knower's paradox*.¹ On the other hand, operator approaches have well-defined semantics in the form of possible-worlds models.

In a recent innovation, Halbach and Welch (2009) offer a formal procedure that reduces a necessity predicate to an operator, so that the choice between the two approaches is deflated: ‘necessity’, conceived as a predicate, can be reduced to ‘necessary truth’, conceived as an operator plus a truth predicate. This chapter concerns the question of whether this approach

¹It is well known also that Quine (1966) saw great philosophical import in the choice between predicates and operators, although his metaphysical arguments will not be the topic of the present paper.

can provide a uniform treatment of intensional notions generally.

Specifically, I argue that the scope of their reduction is likely to be restricted to necessity predicates, because my results in the *epistemic* setting indicate that a generalization of the technique to other intensional notions is not imminent. I thus provide evidence that operator and predicate approaches are genuinely incompatible when the formal treatment of knowledge is at issue. This is because the proof that works in the case of necessity fails for virtually all prominent possible-worlds semantics for knowledge. The problem is technical: the monotonicity constraint in Halbach and Welch’s construction guarantees the existence of a fixed point at which to interpret the necessity predicate in the possible-worlds structures. However, I find that most possible-worlds notions of knowledge lack the property of monotonicity. In the absence of monotonicity, it is an open question whether there will be any fixed points at which to interpret the knowledge predicate in the right way.

Their reduction is motivated by the task of finding a solution to the *quantification problem* for operator approaches: despite the now widespread adoption of the operator approach, predicates offer a simpler way of treating the interaction of intensional notions with quantifiers. For example, the operator’s syntactical resources do not provide nice representations of propositions such as, ‘All the theorems of arithmetic are necessary’, ‘There are necessary propositions that are not a priori,’ ‘Kurt knows something no one else knows,’ or ‘Something is known a priori.’ This problem arises because operators apply to formulas and not to objects. Hence, it is not well-formed to write, e.g., ‘ $\forall xThm(x) \rightarrow \Box x$ ’. By contrast, predicates handle these sentences straightforwardly (Halbach and Welch, 2009, pg. 71). Prima facie, the quantification problem bolsters the case for the incompatibility of operator and predicate approaches, since the predicate approach appears to have more expressive power; and such considerations would speak in favor of the predicate approach.

Prior proposals for strengthening the operator approach against the quantification problem have either applied only to a limited range of cases, or have not been worked out in sufficient

detail (Halbach and Welch, 2009, pp. 75-76). Indeed, some previous philosophers have had the intuition that the reduction of a predicate to an operator-plus-truth-predicate is the right approach; but Halbach and Welch are the first to give a rigorous specification of precisely what the reduction would consist in (Halbach and Welch, 2009, pg. 75, footnote 7). The first ingredient is a formal tool for interpreting a necessity predicate according to an ambient possible-worlds structure. The second ingredient is the construction of a translation procedure that preserves these truth-values relative to the fixed ambient possible-worlds structure (Halbach and Welch, 2009, Theorem 2, pg. 89ff). In this way, they demonstrate that predicate necessity can be reduced to operator necessity so long as one assumes a truth predicate in the operator language.

This can be seen as deflating the significance of the choice between the operator and the predicate: to the extent that their technique works, it turns out that the two approaches are ultimately complementary. The operator approach inherits the predicate's immunity to the quantification problem, and the predicate inherits the operator's possible-worlds models so that the intensional paradoxes no longer loom. The present results are thus significant in no small part because they place an undesirable restriction on Halbach and Welch's technique for demonstrating the required inter-translatability of intensional predicate languages and operator languages. This may serve to restore Montague's traditional arguments to the effect that there is philosophical gravity in the choice between the two approaches.

In §1.1 I give a reconstruction of Halbach and Welch's strategy. In §1.2 I then turn to the non-monotonicity results. These results constitute a series of counterexamples to the crucial monotonicity constraint in Halbach and Welch's proof. This will show that their proof does not carry over in any general way to epistemic operators, so that the results take on the force of an inductive argument: the structure of most of the important epistemic possible-worlds semantics exclude the technical possibility of Halbach and Welch's approach.

1.1 The Halbach-Welch Approach

In this section, I outline Halbach and Welch’s strategy in the case of necessity.

The first part of their proof involves constructing a translation function between a source predicate language and a target operator language. The operator language is restricted to languages with a truth predicate in addition to the modal operator.² Informally, then, the translation takes us from ‘ $2+2=4$ is necessary’ to ‘ $2+2=4$ is necessarily true’, and the thought is that such a transformation should not be troubling intuitively.³

The second part of their proof involves coupling the translation function with fixed-point model constructions that interpret the specialized predicates (necessity and truth). The notion of ‘fixed point’ here is the one familiar from Kripke’s theory of truth (Halbach (2011), Horsten (2011)). In essence, Halbach and Welch generalize Kripke’s truth construction to a structure with multiple worlds to treat a necessity predicate.

The two-part strategy thus shows that if we can obtain a semantics on the operator side, then there is one on the predicate side, since the translation function demonstrably preserves truth-values determined in a possible-worlds structure. Thus, the predicate will not be subject to the knower paradox, it will recover the nice possible-worlds semantics, and it will not require any syntactical wizardry to represent the interaction of the predicate with quantifiers.

²As Halbach and Welch note, Belnap and Gupta attempt a reduction in the absence of the auxiliary truth predicate on the modal operator language (Belnap and Gupta, 1993, Ch. 6E). This forces them to work with a predicate whose expressive power is constricted in accordance with the quantification problem. See §2.6 of this paper for more on this.

³For discussion, see (Halbach and Welch, 2009, pg. 72ff).

1.1.1 Semantics

To speak more precisely, we must introduce the technical machinery. We begin by defining the intensional predicate language.

The base language \mathcal{L} is unrestricted unless otherwise noted. For practical purposes we can think of \mathcal{L} as, for example, the language of arithmetic. To obtain the intensional language \mathcal{L}_N we introduce to the first-order language a primitive predicate N representing necessity. Thus ' $N\ulcorner\varphi\urcorner$ ' means ' φ ' is necessary.'

We define the semantics with a Strong Kleene version of a model-theoretic satisfaction relation.⁴ If w is a model for the base language \mathcal{L} , the pair (w, S) is a partial model for \mathcal{L}_N when S is a subset of sentences of \mathcal{L}_K . The satisfaction relation \models^{SK} is between partial models and formulae of \mathcal{L}_N . The necessity predicate N is given the extension $S^+ = S$. The extension of S^- is given by all the $\varphi \in \mathcal{L}_N$ such that $\ulcorner\neg\varphi\urcorner \in S^+$. Note that, since the Strong Kleene scheme allows for three-valued logic, it is not necessarily the case that for all $\varphi \in \mathcal{L}_N$ either $\ulcorner\varphi\urcorner \in S^+$ or $\ulcorner\varphi\urcorner \in S^-$. Hence, there may be some φ such that neither $N\ulcorner\varphi\urcorner$ nor $\neg N\ulcorner\varphi\urcorner$ obtain. In this case, $N\ulcorner\varphi\urcorner$ lacks a truth-value. Note, however, that the Strong Kleene scheme will assign the classical value to any formula whose atomic inputs are classical.

Definition 1. *Strong-Kleene Semantics*

- (i) $(w, S) \models^{SK} Pt_1\dots t_n$ iff the model w satisfies $Pt_1\dots t_n$ in the standard sense.
- (ii) $(w, S) \models^{SK} \neg Pt_1\dots t_n$ iff the model w does not satisfy $Pt_1\dots t_n$ in the standard sense.

⁴The standard Strong Kleene scheme is a three-valued logic which establishes an evaluation function V from sentences of the language to a set of values $\{0, i, 1\}$. 0 is understood as *false*, 1 as *true*, and i as *neither true nor false*. Atomic propositions get their value in the usual way. $\varphi \wedge \psi$ takes the minimum value of φ and ψ ; $\varphi \vee \psi$ takes the maximum value of φ and ψ . In our case (following the Halbach-Welch strategy), we will specify the Strong Kleene scheme in terms of the satisfaction relation instead of this more standard account. For the standard account of the Strong Kleene scheme, see Priest (2008), § 7.3, pg. 122

(iii) $(w, S) \models^{SK} N\ulcorner\varphi\urcorner$ iff $\ulcorner\varphi\urcorner \in S^+$

(iv) $(w, S) \models^{SK} \neg N\ulcorner\varphi\urcorner$ iff $\ulcorner\varphi\urcorner \in S^-$

(v) $(w, S) \models^{SK} \neg\neg\varphi$ iff $(w, S) \models^{SK} \varphi$

(vi) $(w, S) \models^{SK} (\varphi \& \psi)$ iff $(w, S) \models^{SK} \varphi$ and $(w, S) \models^{SK} \psi$

(vii) $(w, S) \models^{SK} \neg(\varphi \& \psi)$ iff $(w, S) \models^{SK} \neg\varphi$ or $(w, S) \models^{SK} \neg\psi$

(viii) $(w, S) \models^{SK} \forall x\varphi[a]$ iff for all variable assignments c differing from a only in the value of x : $(w, S) \models^{SK} \varphi[c]$

(ix) $(w, S) \models^{SK} \neg\forall x\varphi[a]$ iff there exists some variable assignment c differing from a only in the value of x : $(w, S) \models^{SK} \neg\varphi[c]$

We will define evaluation functions f to interpret the extension S of the necessity predicate. Each evaluation function f is from the total set of worlds W to a set $f(w) = (S^+, S^-) \subseteq W$. Intuitively, then, each f chooses an extension/anti-extension pair for S at some w , viz., each f chooses the extension of the N predicate at a given world. Each $f(w)$ has a positive subset $f(w)^+$, corresponding to S^+ ; as well as a negative part $f(w)^-$, corresponding to S^- .

1.1.2 Jump operator

With the semantics set up, we then build a *jump operator* Γ . The restriction for Halbach and Welch's proof of their theorem is that the Γ operator is monotonic. Formally:

Definition 2. Monotonicity

An operator $\Gamma : \mathcal{P}(C) \rightarrow \mathcal{P}(C)$ is *monotonic* if for any sets $A \subseteq C$ and $B \subseteq C$ we have: If $A \subseteq B$ then $\Gamma(A) \subseteq \Gamma(B)$.

Definition 3. Fixed Point

A set B is a *fixed point* of Γ if we have: $B = \Gamma(B)$

Intuitively, then, an operator is monotonic when it always preserves the ordering of the sets it operates on, and a fixed point is a point at which the iterations of the operation preserves the extension of the input set in the output. Importantly, if an operator is monotonic, it will produce a fixed point after iterative application.⁵

A famous application of these sorts of monotonic jump operators is in Kripke's theory of truth.⁶ Halbach and Welch generalize the Kripke jump to multiple worlds to obtain a model whose natural interpretation at the fixed point is necessity, rather than truth. In order to define this jump operator then, we need the notion of a frame from possible-worlds semantics:

Definition 4. A *frame* is a pair $\langle W, R \rangle$, where W is a non-empty set of *possible worlds* and R is a binary relation on W .

We can now define the Halbach-Welch jump operator Γ_N :

$$\Gamma_N(f)(w)^+ := \{\varphi \in \mathcal{L}_N : \forall v \in W \ wRv \Rightarrow (v, f(v)) \models^{SK} \varphi\}$$

$$\Gamma_N(f)(w)^- := \{\varphi \in \mathcal{L}_N : \exists v \in W \ wRv \wedge (v, f(v)) \models^{SK} \neg\varphi\}$$

⁵This is a key result in Kripke's approach to allowing a language to contain its own truth predicate while avoiding the truth-theoretic paradoxes. See Halbach (2011) and Horsten (2011).

⁶Kripke uses a jump operator to approximate the extension of the truth predicate of a given language. Intuitively, it begins by operating on the source set A of all true propositions of the language in which the truth predicate does not appear. The operator is applied in successive iterations to the source set until it reaches a fixed point. At the fixed point, then, all and only truths are in the jump, including truths with embedded truth predicates. Since sentences like the Liar have a non-reducible occurrence of the truth predicate, they never make it in the jump. And so, it is said, the jump operator approximates the extension of the truth predicate while avoiding the associated paradoxes.

At the least fixed-point f , then, S^+ is interpreted by $f(w)^+ = \Gamma_N(f)(w)^+ = \{\varphi \in L_K : \forall v \in W wRv \Rightarrow (v, f(v)) \models^{SK} \varphi\}$. Hence, for any fixed point f of Γ_N and for any $w \in W$ and any $\varphi \in L_N$, we have:⁷

$$(w, f(w)) \models^{SK} N^\top \varphi^\top \text{ iff } \forall v \in W wRv \Rightarrow (v, f(v)) \models^{SK} \varphi$$

$$(w, f(w)) \models^{SK} \neg N^\top \varphi^\top \text{ iff } \exists v \in W wRv \wedge (v, f(v)) \models^{SK} \neg \varphi$$

Note that the clause defining the jump operator here mirrors precisely a standard semantic clause for a necessity operator in a modal logic. Hence, the predicate approximates the truth-values specified by the ambient alethic theory.

Given the received wisdom on the Kripke jump and the natural use Halbach and Welch make of the device in their approach, it would be surprising if it was not generally applicable. Indeed, it is natural from a technical perspective to suggest that we simply use the same jump operator in the epistemic setting as Halbach and Welch use. But from a philosophical perspective this is unsatisfactory, for it would leave us with little reason to suppose that the intended interpretation of the predicate is knowledge instead of necessity. For one to know that φ would require that *all* non- φ circumstances have been completely ruled out.⁸

There is a considerable literature on contextual and counterfactual conditions on knowledge attribution in order to account for this, such as the safety and sensitivity conditions (Pritchard (2008)). In what follows I make use of the tradition in epistemic logic that uses modified possible-worlds structures to formalize these conditions. It turns out that most possible-worlds notions of knowledge produce non-monotonic jump operators, so that

⁷See Halbach and Welch's own demonstration, (Halbach and Welch, 2009, pg. 87)

⁸ In order for some φ to be known, it requires *certainty* of a very strong sort—namely, that *any* possible $\neg\varphi$ -world be eliminated by the evidence available to the agent in their context. Even the most far-fetched possibility of doubt forces us to reject the proposition. Obviously if we upheld this high standard of certainty for knowledge, the Halbach-Welch monotonicity argument would follow word for word.

the monotonicity constraint in Halbach and Welch’s proof is usually not satisfied in the epistemic case.⁹ There is therefore reason to think that predicates and operators remain genuinely competing approaches for intensional notions other than necessity.¹⁰

1.2 Epistemic Logic: Four Different Notions of Knowledge

1.2.1 Strong-Kleene Semantics for Epistemic Logic

We are investigating the epistemic analogue of the Halbach-Welch strategy for necessity. Hence, we share the same semantics given in Definition 1, above, with the exception of clauses (iii) and (iv) now being typographically altered to represent a knowledge predicate:

$$(iii') (w, S) \models^{SK} K\ulcorner\varphi\urcorner \text{ iff } \ulcorner\varphi\urcorner \in S^+$$

$$(iv') (w, S) \models^{SK} \neg K\ulcorner\varphi\urcorner \text{ iff } \ulcorner\varphi\urcorner \in S^-$$

In order to get our K predicate to behave like a knowledge predicate instead of a necessity predicate, we must turn to the topic of specifying the set S in such a way that our

⁹It is worth noting that Halbach and Welch themselves express doubt as to whether their method will extend to some propositional attitudes. They provide reason to doubt, for example, the applicability of their method to a doxic language with a belief notion, since their approach requires that we assume the operator language has a truth predicate. It then might be objected that it is possible to believe something without believing it to be true (Halbach and Welch, 2009, pg. 96). They hint at a solution to this problem. But for my purposes, it’s sufficient to note, first, that the worry they express is unrelated to the kinds of technical issues I am bringing up here, and, second, that Halbach and Welch hold a uniform approach to intensional logic to be a desideratum.

¹⁰It should be stressed that is that this is obviously an idealization. The operator we are constructing here is not tied to any specific agent and it doesn’t tell us anything about the quality or the source of the knowledge. Nor is it meant to be anything like an agent’s occurrent state of mind when he asserts that he knows something. However, such an idealization is informative both for epistemic logicians and epistemologists, given ubiquitous closure failures and other interesting epistemological phenomena that come up even in idealized settings. See Dretske (1970) and Holliday (2015).

specification corresponds to a possible-worlds concept of knowledge attribution. We will of course use evaluation functions f to interpret the S , and, in turn, we will use jump operators to approximate an ambient epistemic theory. So we must construct new epistemic jump operators.

1.2.2 Relevant Alternatives Models

What we need is to stipulate an epistemic jump operator Γ_K and define it in such a way that a hypothetical fixed point of the operator, if it has one, would correspond to some theory of knowledge attribution from the literature. I start with an important class of contextualist epistemic semantics called *Relevant Alternatives* (RA) models. We must first introduce the definition of an RA model on a possible-worlds frame.¹¹

Definition 5. A *relevant alternatives model* is a tuple of the form $\langle W, \rightarrow, \preceq, V \rangle$ with:

(1.1) a non-empty set W

(1.2) \rightarrow is a reflexive binary relation on W

(1.3) \preceq assigns to each $w \in W$ a binary relation \preceq_w on some $W_w \subseteq W$

a. \preceq_w is reflexive and transitive

b. $w \in W_w$ and for all $v \in W_w$ $w \preceq_w v$

(1.4) V assigns a set $V(P) \subseteq W$ to each atomic P in the language

As is familiar from standard modal logic, the elements of W are, intuitively, the ‘worlds’ or ‘possibilities.’ The epistemic analogue to the standard accessibility relation R of modal logic is \rightarrow . This relation is conceptualized in terms of *uneliminated possibilities*. So we write,

¹¹The formal definitions are borrowed from (Holliday, 2015, pg. 10)

' $w \rightarrow v$ ', to say that ' v is an uneliminated possibility from w .' Intuitively, we think of v as being some possible state of affairs that cannot be ruled out by the current evidence available to some subject in w . The \rightarrow relation must be reflexive, but for the sake of generality we will currently maintain reflexivity as the only restriction on \rightarrow .

The \preceq_w relation is understood as a relevance preorder on some W_w . So, for $u, v \in W_w$, ' $u \preceq_w v$ ' means that from the standpoint of w , u is either more relevant than v , or u is equally as relevant as v . Condition (1.3.a.) guarantees that the \preceq_w relation is a preorder. Other, stronger, restrictions can be placed upon \preceq_w to make \preceq_w , for example, a well-founded order, linear order, etc. For our purposes, we will refrain from taking these up so as to retain generality. Condition (1.3.b.) builds in Lewis' *Rule of Actuality*, which states that the actual world should always be maintained as a relevant alternative (Lewis, 1996, pg. 554). Finally, V should be understood as in a standard modal logic, such that $V(P)$ is the set of worlds in which P holds.

Definition 6. For $w, v, u \in W$ and $S \subseteq W$

$$(1.1) \quad u \prec_w v \text{ iff } u \preceq_w v \text{ and not } v \preceq_w u$$

$$(1.2) \quad \text{Min}_{\preceq_w}(S) = \{v \in S \cap W_w : \text{there is no } u \in S \text{ such that } u \prec_w v\}$$

We say ' $u \prec_w v$ ' to mean that u is strictly more relevant at w than v is. Intuitively, Min_{\preceq_w} is the set of most relevant possibilities of those ordered by \preceq_w .

The intuitive force of the kinds of epistemic models we will consider can be outlined with Holliday's medical exam example (Holliday, 2015, pg. 5). Suppose that two medical students are to take an exam in which they must diagnose a patient. In fact, the patient has a common condition c , which is easily differentiable from another related and equally common disease c' by means of a basic visual exam. There is a far rarer disease, x , and it requires extensive laboratory tests to differentiate x from c or c' . Suppose both medical students diagnose the

patient with c —they are both correct. However, the first medical student performed the laboratory tests for x and they came back negative, while the second student concluded c after only the visual exam, which only eliminates the c' possibility, not the x possibility. Did the second student know that the patient had c ? For, a world in which the patient had x after all remains uneliminated for him. Some may say that the rarity of x excludes it as a *relevant alternative*. In that case the verdict would be that attributing knowledge to the second student would be appropriate. The metrics of the RA models are meant to allow these sorts of analyses of closer and more distant worlds in the evaluation of knowledge attributions.

With these definitions in place, the next step will be to attempt to construct epistemic fixed-point models in the style of Halbach and Welch’s. The goal is to find operators that allow us to construct partial fixed-point models for the possible-worlds theories of knowledge that are expressible in the RA framework. First we will consider Lewis’s theory, followed by Dretske’s.

1.2.2.1 The Lewis Interpretation

We begin with Lewis’ epistemology. In the theory, the set of alternate scenarios considered by the agent is relativized to the proposition in question. In order to capture this idea in the RA model of Lewis’ theory, Holliday modifies the usual accessibility relation R of modal logic, so as to restrict the range of relevant uneliminated scenarios.¹² A formal statement of the model of Lewis’ theory might go, φ is known (at w) if and only if φ is true in all the *most relevant* uneliminated possibilities from the standpoint of w . Conversely, φ is definitely not known at w if there exists a $\neg\varphi$ world among the most relevant uneliminated possible worlds. Consider, then the following jump operator Λ_K :

¹²Holliday (2015)

$$\Lambda_K(f)(w)^+ := \{\varphi \in \mathcal{L}_K : \forall v \in W[(w \rightarrow v \wedge v \in \text{Min}_{\prec_w}(W)) \Rightarrow (v, f(v)) \models^{SK} \varphi]\}$$

$$\Lambda_K(f)(w)^- := \{\varphi \in \mathcal{L}_K : \exists v \in W[w \rightarrow v \wedge v \in \text{Min}_{\prec_w}(W) \wedge (v, f(v)) \models^{SK} \neg\varphi]\}$$

We can think of a world's membership in the $\text{Min}_{\prec_w}(W)$ as flagging that world as equally relevant to the actual world in the evidence it can present for a given proposition. Lewis' interpretation thus says that someone knows a proposition just in case that proposition holds true in all the maximally relevant scenarios that remain uneliminated by the evidence a person has. Hence, Lewis's verdict would be that the second medical student does indeed know the diagnosis: even though the x -world remains uneliminated (he has not done the lab tests to rule it out), the x -world is not in the $\text{Min}_{\prec_w}(W)$ set, and hence is not relevant to the knowledge attribution.

It can be shown that the Lewis operator does in fact return the extension of the Lewisian K predicate. Following the Halbach-Welch strategy, we define an ordering on the set of all evaluation functions: we have $f \leq g$ if and only if for all $w \in W$ we have both $f(w)^+ \subseteq g(w)^+$ and $f(w)^- \subseteq g(w)^-$. We know that Λ_K is monotonic with respect to this ordering, i.e., $f \leq g \Rightarrow \Lambda_K(f) \leq \Lambda_K(g)$. For, Halbach and Welch have proven a general result about accessibility relations R . Their proof can be carried over word for word to our setting, provided that we stipulate a new accessibility relation R' such $wR'v$ holds if and only if $w \rightarrow v$ and $v \in \text{Min}_{\prec_w}(W)$.

Hence, by standard results in set theory there exist fixed points of Λ_K , i.e., there exists some f with $\Lambda_K(f) = f$. Therefore, the argument we gave above for Γ_K carries over word for word to Λ_K . This, of course, will secure desired result for the Halbach-Welch strategy, indicating that we have the following:

$$(w, f(w)) \models^{SK} K^\top \varphi^\top \text{ iff } \forall v \in W[w \rightarrow v \wedge v \in \text{Min}_{\prec_w}(W)] \Rightarrow (v, f(v)) \models^{SK} \varphi$$

$$(w, f(w)) \models^{SK} \neg K \ulcorner \varphi \urcorner \text{ iff } \exists v \in W [w \rightarrow v \wedge v \in \text{Min}_{\preceq_w}(W)] \wedge (v, f(v)) \models^{SK} \neg \varphi$$

The Halbach-Welch recovery of possible-worlds semantics is thus successful for a Lewis style knowledge predicate. Regrettably, the success of the Lewis theory is an exception rather than the rule. Recall that one ingredient in our proof of the Lewis interpretation's success on the Halbach-Welch strategy was that the Lewis theory merely modified the R relation to pick out the set of most relevant uneliminated possibilities. We then just needed to stipulate a new R' so as to export to the epistemic setting Halbach and Welch's proof for any given R . The remaining theories will use metrics on the total set of worlds W in order to center the actual world w , so that we cannot formalize the theory as just a more nuanced R relation. So the same proof trick will no longer be available. Most epistemological theories, it will turn out, are not monotonic and thus may not be amenable to the Halbach-Welch reduction.

1.2.2.2 The Dretske Interpretation

The next jump operator, Δ_K , is inspired by the ideas of Dretske. The intuitive idea behind Dretske's theory is that, in order to know φ , one of two things must happen. First, in the best case, one has eliminated all relevant non- φ worlds. Short of that, one can know φ just in case there is an eliminated non- φ world in between the actual world and any uneliminated non- φ world.

The following operator is extracted from Holliday's exposition of the Dretske-semantics (Holliday, 2015, pg. 13):

$$\Delta_K(f)(w)^+ := \{\varphi \in \mathcal{L}_K : \forall v [(w \rightarrow v \wedge v \in W_w \wedge \forall u \prec_w v (u, f(u)) \models^{SK} \varphi) \Rightarrow (v, f(v)) \models^{SK} \varphi]\}$$

$$\Delta_K(f)(w)^- := \{\varphi \in \mathcal{L}_K : \exists v[w \rightarrow v \wedge v \in W_w \wedge \forall u \prec_w v (u, f(u)) \models^{SK} \varphi \wedge (v, f(v)) \models^{SK} \neg\varphi]\}$$

To illustrate the intuitive idea, let us return to the medical student case. A semantics based on the least fixed-point of this clause would rule that the second student indeed successfully knows the diagnosis. For, recall, the actual world is a c -world, and with the visual exam the student has eliminated the c' world that is between the actual world and the x -world. Hence, even though the x -world remains an uneliminated non- c world, the semantics predicts that the student has knowledge.

At best a claim of monotonicity for this Δ_K operator is highly problematic. We make this precise with the following proposition:

Proposition 7. *Suppose that (i) (W_w, \preceq_w) is a finite linear order. Suppose further that (ii) for $v, v' \in W_w$ we have $(v \cong_w v' \Rightarrow v = v')$. Finally, suppose (iii) $w \rightarrow v$ for all $v \in W_w$. Then*

$$f \leq g \Rightarrow \Delta_K(f)(w)^+ \subseteq \Delta_K(g)(w)^+ \wedge \Delta_K(f)(w)^- \subseteq \Delta_K(g)(w)^-$$

Proof. Enumerate W_w as $w = v_1 \prec_w v_2 \prec_w v_3 \prec_w \dots \prec_w v_n$. Then it suffices to show that $\Delta_K(f)(w)^+ = \Lambda_K(f)(w)^+$, since we know that $\Lambda_K(f)(w)^+$ is monotonic.¹³ Suppose $\ulcorner \varphi \urcorner \in \Delta_K(f)(w)^+$. For the first case, set $v = w$ in the definition of $\Delta_K(f)(w)^+$. Since $v = w = v_1$, there are no $u \in W_w$ with $u \prec_w v$. So we have, for all $u \prec_w v$, that $(u, f(u)) \models^{SK} \varphi$. Furthermore, since $v = w$, and we know that $\ulcorner \varphi \urcorner \in \Delta_K(f)(w)^+$, we know that $(v, f(v)) \models^{SK} \varphi$. In further cases where $v = v_n$ for $n > 1$, we have the same result, since we have proved it for the least element of W_w . So $\Delta_K(f)(w)^+ := \{\varphi \in L_K : \text{For all}$

¹³See §3.2.1 above.

v such that $w \rightarrow v$ ($v, f(v)$) $\models^{SK} \varphi$ $= \Delta_K(f)(w)^+$. The other direction follows easily from definitions, and similar reasoning can be used to establish the case for $\Delta_K(f)(w)^-$.

The problem with this proof lies with clauses (i)–(iii). We must presuppose (i)–(iii) in order to get monotonicity for $\Delta_K(f)(w)$, but these are very unnatural epistemic presuppositions. For, we have no prima facie reason to assume that two equally relevant worlds must be the same world. For example, consider a case of flipping a coin. It seems plausible to describe the case like so: there is one possible world in which the coin lands heads and one possible world in which the coin lands tails. Suppose we flip the coin but close our eyes for the landing. Now both possible worlds, the heads-world and the tails-world, are equally relevant and uneliminated scenarios for us. Worse yet, the presupposition that $w \rightarrow v$ for all $v \in W_w$ is much too strong. For, that entails that we can never eliminate any relevant possibility with the evidence that we have.

What happens when we do away with (i) – (iii)? Then monotonicity can fail for Δ_K .

Proposition 8. *There exist f, g and w such that $f \leq g$ and $\Delta_K(f)(w)^+ \not\subseteq \Delta_K(g)(w)^+$*

Proof. We construct the following countermodel. The set of worlds is a three-world set $W = \{w_1, w_2, w_3\}$. We have a relevance pre-order on W with $\preceq_{w_q} = \{\langle w_r, w_s \rangle : q < r < s\}$. Finally, the epistemic accessibility relation is defined as $\rightarrow_{w_1} = \{\langle w_1, w_1 \rangle, \langle w_1, w_3 \rangle, \langle w_2, w_2 \rangle, \langle w_3, w_3 \rangle\}$. It does not matter for present purposes what the interpretation of \rightarrow_{w_2} or \rightarrow_{w_3} is. Further, let the knowledge predicate be interpreted as follows:

		w_1	w_2	w_3
	f	$(\{\varphi\}, \emptyset)$	(\emptyset, \emptyset)	(\emptyset, \emptyset)
	g	$(\{\varphi\}, \emptyset)$	$(\{\varphi\}, \emptyset)$	(\emptyset, \emptyset)

We can look at the case of $\psi \equiv K^\top \varphi^\top$. We must demonstrate that $\psi \in \Delta_K(f)(w_1)^+$ and $\psi \notin \Delta_K(g)(w_1)^+$.

First, we can show that $\psi \in \Delta_K(f)(w_1)^+$. For, there are two cases. In the first case, $v = w_1$. We have $\varphi \in f(v)^+$. So $(v, f(v)) \models^{SK} \psi$. So the conditional is satisfied. In the second case, $v = w_3$. We do not have $(u, f(u)) \models^{SK} \psi$ for all $u \prec_{w_1} v$, since $\varphi \notin f(w_2)^+$. So the conditional is vacuously satisfied. So, $\psi \in \Delta_K(f)(w_1)^+$.

We can also show that $\psi \notin \Delta_K(g)(w_1)^+$. It suffices to consider the case of $v = w_3$. We know that $\varphi \notin g(v)^+$. So $(v, g(v)) \not\models^{SK} \psi$. But for all $u \prec_{w_1} v$, we have $\varphi \in g(u)^+$. So for all such u , we have $(u, g(u)) \models^{SK} \psi$. So $\psi \notin \Delta_K g(w)^+$.

This failure of monotonicity, as I've explained in earlier sections of this paper, suggests a corresponding failure for the generalization of the Halbach-Welch strategy. The strong conditions necessary for the Dretske operator to be monotonic make it implausible that the Dretske theory of knowledge can be amenable to the Halbach-Welch reduction. As we shall see, Dretske's theory is hardly alone.

1.2.3 Counterfactual Belief Models

So much for the RA models. The next class of theories we will consider is what Holliday refers to as *Counterfactual Belief* (CB) Theories. The basic idea behind CB theories is to extend the ideas of the RA models to a doxastic setting. Again we must introduce some new terminology in order to formalize these ideas.¹⁴

Definition 9. A *counterfactual belief model* is a tuple of the form $\langle W, D, \leq, V \rangle$. In this, W , \leq , and V are understood in the same way as their counterparts in Definition 5, and D is a serial binary relation on W .

We think of the serial relation D as a *doxastic accessibility relation* whereby ' wDv ' means 'everything that the agent believes in w is true in v .' A relation D ranging over some set A

¹⁴Once again, these definitions are taken directly from Holliday (2015), pg. 16-18

is *serial* just in case, for all $a \in A$, there exists some $b \in A$ such that aDb . Thus, instead of uneliminated scenarios, we have scenarios that are compatible with everything a subject believes in w , in the tradition of Hintikka’s alternativeness relation for doxic logic (Hintikka (1962)).

In order to incorporate the doxastic element of the CB models, Holliday extends the epistemic language to an epistemic-doxastic language with a modal belief operator B , such that:

$$(w, S) \models^{SK} B\varphi \text{ iff } \forall v wDv \Rightarrow (v, f(v)) \models^{SK} \varphi$$

I will not take the convenience of adding B to the language since it will just complicate matters for my purposes. Wherever the B operator would appear in Holliday’s formulations, I will just replace it with the clause on the right of the biconditional. Finally, we interpret \leq_w as *comparative similarity* in order to capture the idea behind the counterfactual conditions on a given agent’s belief.

1.2.3.1 The Safety Interpretation

One well known CB theory is the safety theory. According to this theory, I know φ just in case I believe φ , and also that I “play it safe” with my beliefs. I play it safe with my beliefs when all the close worlds in which I believe φ are in fact φ -worlds. For example, consider my current belief that I am sitting in front of this computer. The safety semantics would consider my belief safe, and therefore knowledge, just in case it would have been difficult for me to falsely believe that I am sitting in front of my computer.

We make this idea precise with the following jump operator. Note that the first conjunct in the jump clause is a translation of the semantics of the doxastic B operator. That is to say, as per Holliday’s account, we have $(v, f(v)) \models^{SK} B\varphi$ iff $\forall v wDv \Rightarrow (v, f(v)) \models^{SK} \varphi$.

$$\begin{aligned} \Sigma_K(f)(w)^+ &:= \{\varphi \in \mathcal{L}_K : \forall v \in W_w [wDv \Rightarrow (v, f(v)) \models^{SK} \varphi \wedge (*) ((**) \forall u \in W_w \\ u <_w v \Rightarrow (\exists t \in W_w uDt \wedge (t, f(t)) \not\models^{SK} \varphi) \wedge (***) (\forall t' \in W_w vDt' \Rightarrow (t', f(t')) \models^{SK} \\ \varphi)) \Rightarrow (v, f(v)) \models^{SK} \varphi]\} \end{aligned}$$

One can obviously present an analogous definition of $\Sigma_K(f)(w)^-$ as I have done for the previous operators. I leave it out here for simplicity.

As in the case of the Δ_K operator, we have a demonstrable failure of monotonicity for the Σ_K operator we have constructed here, and, hence, we know the the canonical method for showing a fixed point will not work. We make this precise with the following proposition:

Proposition 10. *There exist f, g and w such that $f \leq g$ and $\Sigma_K(f)(w)^+ \not\subseteq \Sigma_K(g)(w)^+$*

Proof. Suppose we had a five world model $W_{w_1} := \{w_1, w_2, w_3, w_4, w_5\}$ with

$D := \{\langle w_1, w_1 \rangle, \langle w_2, w_2 \rangle, \langle w_3, w_2 \rangle, \langle w_4, w_5 \rangle, \langle w_5, w_5 \rangle\}$. Suppose for all $w_x, w_y \in W_{w_1}$ $w_x \equiv_{w_1} w_y$. Further, interpret the knowledge predicate like so:

	w_1	w_2	w_3	w_4	w_5
f	$(\{\varphi\}, \emptyset)$	(\emptyset, \emptyset)	(\emptyset, \emptyset)	(\emptyset, \emptyset)	(\emptyset, \emptyset)
g	$(\{\varphi, \psi\}, \emptyset)$	$(\{\varphi\}, \emptyset)$	(\emptyset, \emptyset)	(\emptyset, \emptyset)	(ψ, \emptyset)

Let $\chi \equiv K\varphi \vee K\psi$. It suffices to show: $\chi \in \Sigma_K(f)(w_1)^+$ and $\chi \notin \Sigma_K(g)(w_1)^+$.

First we show that $\chi \in \Sigma_K(f)(w_1)^+$. For the first conjunct of the Σ_K clause, note that the only v such that w_1Dv is $v = w_1$. We have $\varphi \in f(w_1)^+$, so $(v, f(v)) \models^{SK} K\varphi$ for $v = w_1$. By the SK-Semantics, then, $(v, f(v)) \models^{SK} K\varphi \vee K\psi$. So $(v, f(v)) \models^{SK} \chi$.

We must now show that $(*)$ is satisfied for all $v \in W_{w_1}$. Note that the structure of $(*)$ is this: $\forall v \in W_{w_1} (**) \wedge (***) \Rightarrow (v, f(v)) \models^{SK} \chi$. So for each v' we must show either a failure of $(**)$, a failure of $(***)$, or the satisfaction of $(v, f(v)) \models^{SK} \chi$. As for $(**)$, note that in the

model all worlds are equally relevant. Hence, there does not exist any $u \in W_{w_1}$ such that $u <_{w_1} v$. This will hold true across the board, so we know that for all $v \in W_{w_1}$, condition (**) holds vacuously. So for each v' , we must show either that the consequent of (*) holds or a failure of (** *).

Suppose first $v' = w_1$. Then $\varphi \in f(w_1)^+$, so $(v, f(v)) \models^{SK} K\varphi$. By the SK-Semantics, then, $(v, f(v)) \models^{SK} K\varphi \vee K\psi$, that is, $(v, f(v)) \models^{SK} \chi$. So (*) holds.

Suppose $v = w_2$. The only t' such that w_2Dt' is $t' = w_2$. However, $f(w_2)^+ = \emptyset$, so neither $(w_2, f(w_2)) \models^{SK} K\varphi$ nor $(w_2, f(w_2)) \models^{SK} K\psi$. So $(w_2, f(w_2)) \not\models^{SK} \chi$. So (** *) fails. So (*) holds.

Suppose $v = w_3$. The only t' such that w_3Dt' is $t' = w_2$. However, $f(w_2)^+ = \emptyset$, so neither $(w_2, f(w_2)) \models^{SK} K\varphi$ nor $(w_2, f(w_2)) \models^{SK} K\psi$. So $(w_2, f(w_2)) \not\models^{SK} \chi$. So (** *) fails. So (*) holds.

Suppose $v = w_4$. The only t' such that w_4Dt' is $t' = w_5$. However, $f(w_5)^+ = \emptyset$, so neither $(w_5, f(w_5)) \models^{SK} K\varphi$ nor $(w_5, f(w_5)) \models^{SK} K\psi$. So $(w_5, f(w_5)) \not\models^{SK} \chi$. So (** *) fails. So (*) holds.

Suppose $v = w_5$. The only t' such that w_5Dt' is $t' = w_5$. However, $f(w_5)^+ = \emptyset$, so neither $(w_5, f(w_5)) \models^{SK} K\varphi$ nor $(w_5, f(w_5)) \models^{SK} K\psi$. So $(w_5, f(w_5)) \not\models^{SK} \chi$. So (** *) fails. So (*) holds.

So $\chi \in \Sigma_K(f)(w_1)^+$.

We must now show that $\chi \notin \Sigma_K(g)(w_1)^+$. The SK-semantics preserves the top truth-value under extensions. Hence, since $f \leq g$, the first conjunct of the Σ_K clause holds as before. We must then show one v such that (*) fails. By the equi-relevance of all the worlds, the case is the same as above for (**), that is, condition (**) holds vacuously throughout. So it remains to show some v such that condition (** *) holds while $(v, g(v)) \not\models^{SK} \chi$.

Consider $v = w_4$. We have $w_4 D w_5$ and $\psi \in g(w_5)^+$. So $(w_5, g(w_5)) \models^{SK} K\psi$. By the SK-Semantics, then, $(w_5, g(w_5)) \models^{SK} \chi$. So $(***)$ holds. However, $g(w_4) = \emptyset$, so $(w_4, g(w_4)) \not\models^{SK} K\varphi$ and $(w_4, g(w_4)) \not\models^{SK} K\psi$. So $(w_4, g(w_4)) \not\models^{SK} \chi$.

So $\chi \notin \Sigma_K(g)(w_1)^+$.

1.2.3.2 The Alternate Safety Interpretation

The inductive evidence is beginning to build against a generalization of the Halbach-Welch strategy. However, a safety theorist might worry that we have mischaracterized the safety theory in our operator. For, in the Σ_K operator above, we first gather up the set W_w of comparatively similar worlds as indexed to the actual world. And only thereafter do we determine which elements—*of those in W_w* —satisfy the safety conditions. However, one might say, we should take the entire set of possible worlds first, and then determine if a given proposition satisfies the safety conditions for a given possible world simultaneously to determining if the given possible world is in the set of comparatively similar worlds.

An alternate safety operator can be constructed to make this characterization of safety more precise:¹⁵

$$\Sigma'_K(f)(w)^+ := \{\varphi \in \mathcal{L}_K : \forall v (w D v \Rightarrow (v, f(v)) \models^{SK} \varphi) \wedge \forall v' \in W_w [(\neg \exists u (u <_w v') \wedge \forall t (v' D t \Rightarrow (t, f(t)) \models^{SK} \varphi)) \Rightarrow (v', f(v')) \models^{SK} \varphi]\}$$

Whether or not the hypothetical least fixed point of Σ'_K operator would be extensionally differentiated from the original interpretation in Σ_K is left open here; however, it doesn't matter for present purposes, since neither operator is monotonic.

¹⁵Note that this characterization corresponds more closely to Holliday's own formalization, (Holliday, 2015, pg. 18).

Proposition 11. *There exist f, g and w such that $f \leq g$ and $\Sigma'_K(f)(w)^+ \not\subseteq \Sigma'_K(g)(w)^+$*

Proof. By counter model. Consider a set of worlds $W := \{w_1, w_2, w_3\}$ with the epistemic-doxastic relation $D := \{\langle w_1, w_1 \rangle, \langle w_2, w_3 \rangle, \langle w_3, w_3 \rangle\}$ and the comparative similarly relation $\leq_{w_1} := \{\langle w_1, w_2 \rangle, \langle w_1, w_3 \rangle, \langle w_2, w_1 \rangle, \langle w_2, w_3 \rangle, \langle w_i, w_i \rangle \text{ for } i = 1, 2, 3\}$. Then fix the interpretation of the knowledge predicate like so:

	w_1	w_2	w_3
f	$(\{\varphi\}, \emptyset)$	(\emptyset, \emptyset)	(\emptyset, \emptyset)
g	$(\{\varphi\}, \emptyset)$	(\emptyset, \emptyset)	$(\{\varphi\}, \emptyset)$

Consider, again, $\psi \equiv K\varphi$. I claim that in the above model $\psi \in \Sigma'_K(f)(w_1)^+$ and $\psi \notin \Sigma'_K(g)(w_1)^+$.

We must first show that $\psi \in S_K(f)(w_1)^+$. First, we satisfy the conditional $w_1 D v \Rightarrow (v, f(v)) \models^{SK} \psi$ for all v , since there are no other v with $w_1 D v$. Hence, we know by construction that $(v, f(v)) \models^{SK} \psi$. It will now suffice to show that the second conjunct, the conditional with the long antecedent, holds for all v' . There are then three cases.

For [Case 1], set $v' = w_1$. We have $\varphi \in f(w_1)^+$, so $(v', f(v')) \models^{SK} \psi$. Hence, the consequent holds so the conditional holds. For [Case 2], set $v' = w_2$. Then the second conjunct of the antecedent fails, since $v' D w_3$ and $(w_3, f(w_3)) \not\models^{SK} \psi$. So the antecedent fails, and so the conditional holds vacuously. Finally, for [Case 3] we set $v' = w_3$. Then the first conjunct of the antecedent fails, since there does exist $u <_{w_1} v'$. Hence, the antecedent of the long conditional fails and so it holds. So $\psi \in \Sigma'_K(f)(w_1)^+$, as desired.

We must now show that $\psi \notin \Sigma'_K(g)(w_1)^+$. It will suffice to show one v' such that the conditional statement fails. Consider $v' = w_2$. We have $v' \in W_{w_1}$. Furthermore, $w_1 \leq_{w_1} w_2$ and $w_2 \leq_{w_1} w_1$, so $w_1 \simeq_{w_1} w_2$. Hence, there is no u with $u <_{w_1} v'$. So the first conjunct of the antecedent is satisfied. We also satisfy the second conjunct, since the only t such

that $v'Dt$ is w_3 , and $\varphi \in g(w_3)^+$. So $(w_3, g(w_3)) \models^{SK} \psi$. However, $\varphi \notin g(w_2)^+$, so in this case $(v', g(v')) \not\models^{SK} \psi$. Hence, the antecedent holds and the consequent does not, so the conditional fails. So, $\psi \notin \Sigma'_K(g)(w_1)^+$.

It seems, then, that no matter which way we prefer to characterize the safety theory, we will not end up with a monotonic jump operator.

1.2.3.3 The Sensitivity Interpretation

The next CB theory is known as the sensitivity theory. The intuitive idea here is that my beliefs are sensitive to falsehood: that is to say, I know some φ just in case I actually believe φ , and I would not believe φ in a situation in which it was actually false. Consider again my belief that I am currently sitting in front of the computer. My belief would be sensitive, and thus knowledge, just in case I would cease believing that I am sitting in front of a computer in a nearby world in which I was to stand up and go to the window over there.

Formally, this can be represented with the following operator:¹⁶

$$H_K(f)(w)^+ := \{\varphi \in \mathcal{L}_K : \forall v wDv \Rightarrow (v, f(v)) \models^{SK} \varphi \wedge \forall v \in \text{Min}_{\leq w}(\llbracket \bar{\varphi} \rrbracket) \exists v'(vDv' \wedge (v'f(v')) \not\models^{SK} \varphi)\}$$

And, once again, a variation on the same theme: there is a counter model demonstrating the failure of monotonicity for the H_K operator.

Proposition 12. *There exist f, g and w such that $f \leq g$ and $H_K(f)(w)^+ \not\subseteq H_K(g)(w)^+$.*

Proof. Again, by counter model. Consider a set of worlds $W := \{w_1, w_2, w_3, w_4\}$, with a doxastic accessibility relation $D := \{\langle w_1, w_3 \rangle, \langle w_2, w_4 \rangle, \langle w_3, w_4 \rangle, \langle w_1, w_2 \rangle\}$, and a comparative similarity relation $<_{w_1} := \{\langle w_q, w_r \rangle : q < r\}$. Then interpret the knowledge predicate like so:

¹⁶As above, one can define the analogous “minus” operator for the H_K operator.

		w_1	w_2	w_3	w_4
	f	$(\{\varphi\}, \emptyset)$	(\emptyset, \emptyset)	$(\{\varphi\}, \emptyset)$	(\emptyset, \emptyset)
	g	$(\{\varphi\}, \emptyset)$	(\emptyset, \emptyset)	$(\{\varphi\}, \emptyset)$	$(\{\varphi\}, \emptyset)$

Consider $\psi \equiv K\varphi$. I claim that the above model shows that $\psi \in H_K(f)(w_1)^+$ and $\psi \notin H_K(g)(w_1)^+$.

First, we can show that $\psi \in H_K(f)(w_1)^+$. The only v such that $w_1 Dv$ is $v = w_3$. We have $\varphi \in f(w_3)^+$. Hence, $(w_3, f(w_3)) \models^{SK} \psi$. Now we can show that for all the $v \in \text{Min}_{\leq w}(\llbracket \bar{\psi} \rrbracket)$ we have a v' with $v Dv'$ with $v', f(v') \not\models^{SK} \psi$. Note, w_2 is the only member in $\text{Min}_{\leq w}(\llbracket \bar{\psi} \rrbracket)$. So, set $v = w_2$ and $v' = w_4$. We have $w_2 \in \text{Min}_{\leq w}(\llbracket \bar{\psi} \rrbracket)$ and $w_2 D w_4$. We know that $\varphi \notin f(w_4)^+$, so $(w_4, f(w_4)) \not\models^{SK} \psi$. So $\psi \in H_K(f)(w_1)^+$.

Next, we can show that $\psi \notin H_K(g)(w_1)^+$. Set $v = w_2$. We have $w_2 \in \text{Min}_{\leq w}(\llbracket \bar{\psi} \rrbracket)$. However, there does not exist any v' with $w_2 Dv'$ and $(v', g(v')) \models^{SK} \psi$. For, the only world v' such that $w_2 Dv'$ is $v' = w_4$. However, $\varphi \in (g)(w_4)^+$. Hence, $(w_4, g(w_4)) \models^{SK} \psi$. So we have a $v \in \text{Min}_{\leq w}(\llbracket \bar{\psi} \rrbracket)$ such that for all v' , if $v Dv'$, then $(v', g(v')) \not\models^{SK} \psi$. So $\psi \notin H_K(g)(w_1)^+$.

1.3 Conclusion

I have in stayed in the predicate language and focused on the creation of jump operators to interpret the knowledge predicate. However, we can run the same counterexamples on the operator side. This can be easily verified. Consider, for example, the case of the Dretske operator above. Above we have considered only the jump operator on the predicate side of the translation. In that context, we were attempting to find an interpretation for a knowledge predicate. On the modal operator side, we will be using our jump operator to interpret the truth predicate, while our sentential modal operator will have a semantics defined in terms

of the Dretske-semantics we defined above. Consider, then, the target language \mathcal{L}_{\Box}^{Tr} . In this language we stipulate a primitive predicate Tr , whose intended interpretation is truth. Our truth jump operator $\Gamma_{Tr} := \{\varphi \in \mathcal{L}_{\Box}^{Tr} : (w, f) \models_{\Box}^{SK} \varphi\}$, which is an instance of “the Kripke jump parametrized by possible worlds” (Halbach and Welch, 2009, pg. 87). So for some f and some $w \in W$, we would have $\varphi \in \Gamma_{Tr}(f)(w)$ iff $(w, f(w)) \models_{\Box}^{SK} \varphi$. Take the case of $\psi \equiv \Box\varphi$. Then $\psi \in \Gamma_{Tr}(f)(w)$ iff $(w, f(w)) \models_{\Box}^{SK} \Box\varphi$ iff $\forall v[w \rightarrow v \wedge v \in W_w \wedge \forall u \prec_w v (u, f(u)) \models_{\Box}^{SK} \varphi] \Rightarrow (v, f(v)) \models_{\Box}^{SK} \varphi$. We can then run the exact same counter model as that in Proposition 8 above, in order to demonstrate a failure of monotonicity for Γ_{Tr} . Thus, in the absence of a fixed point, the theories in question fail to have the sorts of models definable by the Kripke construction.

These results are very suggestive, but they are not decisive. It remains to be seen whether fixed points can be produced in the absence of a monotone operator, or, alternatively, whether we can demonstrate that there are no fixed points for the operators in question. Another possibility is that the formal characterizations I’ve used here could be improved upon to produce perfectly adequate partial models of the suggested theories that yet have monotonic operators. Furthermore, other interesting counterfactual theories, such as Nozick’s tracking theory (Nozick (1981)) or Williamson’s margins of error theory (Williamson (2002)), remain to be investigated.¹⁷

There is, however, a pattern in the above results that calls for interpretation. I have suggested throughout that my results at least provisionally present a challenge to Halbach and Welch’s approach to intensionality. I suggest that these results begin to take on the weight of a kind of inductive argument against the technical feasibility of simply reducing a knowledge predicate to a knowledge operator. Lewis’s theory was an important exception, but given the pattern established here it seems to be a case of the exception proving the rule that possible-worlds theories of knowledge are non-monotonic. If this reading of the results is

¹⁷It’s natural also to wonder about received views on the range of application of the Kripke construction. Such considerations are outside the scope of this work.

correct, then we recover the philosophical gravity that Montague saw in the implications of choosing either the predicate or the operator approach ((Montague (1963)).

It might be objected that the problem is not with Halbach and Welch’s approach, but rather with the characterizations of knowledge that we have defined using the RA and CB models themselves. For, the non-monotonic epistemic jump operators we have considered here on the predicate side correspond to non-monotonic operators on the operator side.

Furthermore, it is worth exploring the assumption that the translation function itself is not an issue in the epistemic case. One might argue that the translation function actually is problematic from an epistemological perspective. For, in the translation we would transform, e.g., “Kurt knows something no one else knows” into “Kurt knows something to be true that no one else knows to be true.” Now, we might object to the equivalency of the translation on two basic grounds. First, we might want to say that an someone can know something without having any concept of truth at all, and therefore not know it to be true. Second, it might be charged that someone can know something to be true without knowing that a sentence that expresses this fact is itself true (since the truth predicate takes the names of sentences into its extension). It has been suggested to me that this problem is easily solved by switching the order of the operator and truth predicate in the target language, so that we get, e.g., “It is true that something is known.” But I leave this to future work.

We should differentiate the present objection to the Halbach-Welch strategy from other problems with the predicate approaches. In light of the inconsistency results associated with the knower paradox, it is natural to ask whether the intensional predicate approach fails independent of the success or failure of the Halbach-Welch construction. For, the following axiomatization of a knowledge predicate has been shown to be inconsistent Montague (1963, 1974):

$$(1) K\Gamma\varphi^{\neg} \rightarrow \varphi$$

$$(2) K^{\Gamma}K^{\Gamma}\varphi^{\neg} \rightarrow \varphi^{\neg}$$

$$(3) K^{\Gamma}\varphi^{\neg} \wedge K^{\Gamma}\varphi \rightarrow \psi^{\neg} \rightarrow K^{\Gamma}\psi^{\neg}$$

The worry is that these principles exact minimal demands on a knowledge predicate; and even so such an axiomatization is inconsistent. For, (1) is understood as representing the rather uncontroversial epistemic principle that what is known must be true, while (2) says that (1) is itself known. A moment's reflection on any instance (1), for any sophisticated epistemic subject, makes an instance of (2) evident, and so these two seem to come along with one another. (3), on the other hand, represents the highly controversial principle of epistemic closure, which says that the consequences of known implications of known truths are themselves known. It is controversial because, first, many epistemologists insist that actual agents are likely to miss or be mistaken about the implications of what they know. Second, it is argued that closure too easily defeats skeptical hypotheses, even in ideal reasoners.¹⁸ In any case, there is controversy surrounding the closure axiom in the extant literature, with some interpreting Kaplan and Montague's results as a decisive formal result in an argument against closure (Maitzen (1998)).

Given its infamy in epistemology, denying closure becomes more natural in the wake of these results. But Cross's work has presented a derivation of the paradox that does not rely on any closure axiom like (3).¹⁹ Cross's result will suggest that using a predicate for knowledge is simply unworkable, making the Halbach-Welch approach a non-starter and making the epistemic generalization of it moot.

However, consider the two axiom schemas (1) and (2) that Cross uses to derive his paradox. When we turn our eyes from Cross's set up to ours, we note that within the three-valued

¹⁸See Dretske (1970), (Steup and Sosa, 2005, pg. 13-43). See (Holliday, 2015, 5-6) for Holliday's account of this objection in terms of epistemic logic.

¹⁹The original discovery is in Cross (2001), though Cross later released an updated version with a small correction (see Cross (2012)) that didn't substantially affect the result. For secondary literature on the proof see Uzquiano (2004); Theorem 2.10 in Egge (2005); §3 of Schwarz (2013)

semantic setting we inherit here from Halbach and Welch, the answer is that we must reject both (1) and (2) as axioms for reasons independent of epistemological concerns. For, consider the fact that (1) and (2) are both appropriate for necessity as well. That is to say, there isn't anything controversial in holding that whatever is necessary is true, and that this very fact is itself necessary. So Halbach and Welch will have to answer to them. And they will have to do so by denying (1) and (2), since they work in a three-valued logic in which ungrounded sentences receive intermediary truth-value, as per the standard Kripke construction.²⁰ Hence, it may be that an instance of (1) is such that φ is a liar sentence, in which case φ will have intermediary truth-value, that is to say, will be neither true nor false. It's then possible for (1) to fail; hence (2) must fall with it. So Halbach and Welch have to reject (1) and (2) as axiom schemas. As it turns out, then, Cross' work doesn't present a problem for Halbach and Welch; and, since the present results pertain the Halbach-Welch approach, it is not a problem for me either.²¹

Egre has contended that a sufficient solution to the paradox of the knower has been dormant due to a lack of drawing certain crucial distinctions (Egre (2005)). He says that when Montague opposes predicate treatments to operator treatments of modalities, Montague is in fact conflating *self-referential treatments* with *metalinguistic* treatments, where there should be a distinction drawn between the two (Egre, 2005, pg. 37). In Egre's system of distinctions, what we have been referring to simply as predicate treatments are in fact self-referential meta-linguistic treatments. There are meta-linguistic options in which self-reference is not permitted, he says, as well as non-metalinguistic modal systems in which self-reference *is* permitted. The moral to draw from this, it seems, would be that framing the debate in a "predicate versus operator" way as I have been doing here will block options from view.

²⁰See Halbach (2011) and Horsten (2011).

²¹One might object that my discussion here does not accurately reflect what the arrow means in a three-valued semantics. For, we have not defined the arrow as a sequent arrow or as being defined by disjunction and negation. The matter is not crucial to my concerns here.

However, this objection doesn't really stick in our case. Recall that the basic setting of our logic here is inherited from Halbach and Welch. For them, the predicate language is clearly what Egge calls self-referential metalinguistic, since they permit diagonalization and their predicate applies to Gödel codes of sentences, so our predicate language really is just what Montague calls a syntactic treatment (or what we have been calling a predicate treatment). However, our three-valued semantic setting will avoid the semantic paradoxes by failing to assign liar-like sentences a stable truth-value. On the operator side of the translation, we are using a standard modal logic, which falls in Egge's categorization under non-self-referential non-metalinguistic. But recall also that on the operator language we also have a truth predicate, so there will be some self-reference available.

This very consideration may lead some to objections based on the requirement of having a truth predicate in the first place. First of all, modal operator languages need not contain a truth predicate, in which case, of course, the intended interpretation of $Tr\ulcorner\varphi\urcorner$ is entirely unclear. Second of all, inclusion of a truth predicate on the operator side generates the truth-theoretic paradoxes anyway, and so some might say that the Halbach-Welch strategy just pushes the problem back onto the truth predicate. As for this latter point, Halbach and Welch have replied to this objection in saying that at least the strategy has the advantage of reducing the paradoxes to one type of paradox (Halbach and Welch, 2009, pg. 79). The strategy thus allows logicians and philosophers to concentrate all efforts on the truth-theoretic paradoxes with some measure of confidence that the intensional paradoxes are reducible to the truth-theoretic paradoxes. In reference to the former objection, then, some logicians might be hesitant to subscribe to a translation which requires the stipulation of a truth predicate. But it seems this will be a matter of what considerations weigh more heavily in an individual logician's priorities. To some, like Halbach and Welch, the stipulation of a truth predicate in the operator language will be a very natural restriction, since such a reduction of the objects of necessity to the objects of truth will accommodate other deeper philosophical convictions. The above results give us reason to doubt that this move is available in the epistemic case.

Chapter 2

Hintikka's Operator Logic of Perception: Intensionality as Intentionality

Hintikka proposed that the possible-worlds theory of intensionality *eo ipso* provides a theory of intentionality (Hintikka (1969), Hintikka (1975)). What it is for a concept to be intentional is precisely for it to introduce the requirement of taking multiple possible worlds into account in the evaluation of sentences containing clauses embedded in an intentional context. His proposals in the logic of perception are thus part of his attempt to solve the traditional problems of intentionality associated with Brentano and Husserl. In this overarching project, he stresses the continuity of the *modality* involved in perceptual states with the modality involved in propositional attitudes such as knowledge and belief.

Hintikka's epistemic logic has been influential in the literature.¹ There is less to be found

¹Hintikka (1962) seems to be something of a watershed in the literature on epistemic operator logics, as opposed to the predicate approach of Montague (1963) and others. A citation search for the work reveals that it has thousands of citations, and pre-eminent epistemic logicians such as Holliday make reference to the Hintikka program in their work as being an important historical precursor.

in the literature on Hintikka’s perceptual logic, a distinctive branch of his modal system, perhaps due to the weakness of the perceptual logic in comparison to the epistemic logic.² Despite the empirical fact that Hintikka’s epistemic logic is more prevalent, the textual evidence indicates that Hintikka himself believed that his logic of perception was an indispensable element in understanding his larger project of using possible worlds to explicate the workings of mental intentionality. So understanding the logic of perception is crucial for understanding Hintikka’s possible-worlds approach to intentionality, and, by extension, crucial to understanding the general situation with respect to the possible-worlds approach to intentionality.³ For, the debates on intentionality have resurfaced in the literature, with possible-worlds approaches that were standard following Hintikka’s work being challenged by new trends, such as the phenomenal intentionality literature.⁴ It is thus more vital than ever that we understand Hintikka’s possible-worlds approach to intentionality. This chapter gives one of the first full reconstructions of Hintikka’s system of perceptual logic, formalizes it in a manner that is consistent with the present-day literature on intensional logic, and reflects on the results with respect to the philosophical problems in which the logic originated.

Prior work on Hintikka’s logic has been mostly literature on extending the spirit of Hintikka’s original ideas, or on criticizing important local issues in the logic that shed doubt on various of Hintikka’s claims. David Woodruff Smith and Ron McIntyre’s work—which departs from Hintikka’s substantially by using his ideas as a starting point for developing an Husserlian, phenomenological version of possible-worlds semantics—is an example of the former (Smith and McIntyre (1982)). Hintikka was quite influenced by Husserl in his discussions, but he distanced himself from many of Husserl’s claims, which he saw as implausible.⁵

²It is weak in the sense that few propositional modal axioms are valid in this system, see §2.3. Some literature on Hintikka’s logic of perception includes Smith-McIntyre Smith and McIntyre (1982); Shuger Shuger (1986); Mellema Mellema (1982); Bacon Bacon (1979); Saarinen Saarinen (1983).

³In addition, Hintikka used his logic as an apparatus for adjudicating problems in the philosophy of perception, such as the disputes between sense-datum and information-theoretic arguments.

⁴See Mendelovici Mendelovici and Bourget (2014) and Michelle Montague Montague (2010).

⁵See “The Intentions of Intentionality” in (Hintikka, 1975, pp. 192-222) for Hintikka’s take on where his theory compares and contrasts with Husserl’s.

Smith-McIntyre's work develops the idea that Hintikka's own solutions are closer in spirit to Husserl's than Hintikka made it seem; in fact, their work displays that Hintikka's possible-worlds theory can help provide plausible interpretations of some of Husserl's claims.⁶ Bacon has provided a useful proposal for formalizing an extension of Thomason's logic of perception, which itself comes on the heels of Hintikka's work (Bacon (1979), Thomason (1973)). So Bacon's formalization departs from Hintikka in a way that mine does not, and his work does not address the primary philosophical issues that occupied Hintikka in his original context.

Shuger's and Mellema's work are good examples of the second sort of literature mentioned. Mellema argues that the non-veridicality feature of Hintikka's logic restricts the sorts of sentences the semantics can handle in the way we'd like (Mellema (1982)). In particular, Mellema argues that sentences whose meaning requires that we join the actual world with the possible worlds encoding someone's visual information are less than illuminated by Hintikka's approach.⁷ Shuger addresses Hintikka's semantics with application to problems in the philosophy of visual perception such as visual Gettier cases and double vision cases (Shuger (1986)). However, there is yet to be a study such as the present one which attempts to give a full reconstruction of Hintikka's logic for the purposes of examining it within its philosophical origins in the problems of intentionality, which are again alive and active today.

Structure of the chapter. In §2.1, I set the stage intuitively for Hintikka's semantics by looking at the problems in modal logic that he is trying to solve and the underlying motivations for solving them in the way that he does. First, I introduce the problem of propositional attitudes in the setting of modal logic (§2.1.1). In §2.1.2, I discuss how Hintikka's problem of intentional individuation ties into the attitude problems, and in §2.1.3 I discuss how this line of thought gives rise to Hintikka's distinctive notion of quantification. After providing this intuitive gloss I begin formalizing the semantics in §2.2. Formalizing the logic allows

⁶In particular, the notion of explicating meaning in terms of possible worlds which Hintikka promotes accords in notable ways with Husserl's independent notion of explicating meaning in terms of *intentional horizon structure*. Cf. Smith-McIntyre (Smith and McIntyre, 1982, Ch.6, esp. §3.2).

⁷I address similar concerns from my own standpoint in remarks on equations (2.19) and (2.20) in §2.4.

us to get a clear grasp on what is distinctive and what is standard in Hintikka's approach. In addition, it makes it easy to test the fidelity of the reconstruction to Hintikka's original insofar as it at least obtains equivalent validities.

In §2.3, I look at the validities of the propositional fragment of Hintikka's semantics while couching and motivating the formal results in the setting of perception theory. In §§2.3.1 and 2.3.2, I review the results of the semantics with respect to standard modal axioms and the Boolean connectives, respectively. Readers interested in the distinctive aspects of Hintikka's system will want to hone in on §2.4, in which I discuss the more unfamiliar aspects of Hintikka's system. In §2.4.3 I explicate Hintikka's distinctive de re—de dicto reconstruction, and, following this, I employ his perceptual and physical quantifiers to clarify the ways in which these begin to answer the questions of intentionality (§2.4.4).

Following discussion of the logic itself, I discuss its implications for intentionality (§2.5). In §2.5.1 I point out some open philosophical problems that recommend a healthy caution in interpreting Hintikka's solution to intensionality as a solution to intentionality. Crucial to the intentional semantics is Hintikka's conception of perceptual meaning, and so in §2.5.2 I discuss the status of the notion of Fregean sense in Hintikka's proposal. In §2.6 I summarize what has been done.

2.1 Background Explication

2.1.1 Modal Logic and Propositional Attitudes

Hintikka's work arises in the intersection of quantified modal logic in the analysis of intensionality, on the one hand, and the theory of intentionality, on the other. Thus, as one works through his logic, one should have in the background the traditional questions of intention-

ality.⁸ It is, in fact, crucial that we keep this background in mind. For, the logic we develop here is a ‘*logic*’ only in a peculiar sense. I understand Hintikka’s logic of perception as a *device* for modelling the *intentional* features of perceptual concepts. It is thus a ‘logic’ only to the extent that it is developed within the resources of modal logic. This consideration distinguishes the sense of ‘logic’ intended here from other senses of ‘logic.’

First, this logic should not be understood as an attempt at a formal regimentation of natural language. I do not claim that my constructions here are in any sense what people *really mean* when they make perceptual reports or attribute perceptions to others, nor am I committed to the claim that the semantics here reveals “the correct” semantics of perceptual verbs in natural language. Secondly, the sense of logic of perception as a device for the analysis of actual perception is distinct from the project of providing a regimentation of the inferential relations between *judgments* that we make on the basis of our perceptions, or about the *beliefs* that we form thereby. This logic of perception is a device for modelling the concept of perception, insofar as it introduces the requirement that multiple possible worlds are taken into account when characterizing perceptual states. The validities of the resulting system thus reflect properties of the concept of perception, and do not necessarily provide the inferential relations between judgments that people make or ought to make on the basis of their perceptions.

To the extent that modal logic is a logic of *intensional* terms, then, it is natural to investigate to what extent we can extend these treatments of intensionality to treatments of intentionality. The formal parallels between intensional terms and intentional concepts are striking, but familiar to us today after Chisholm’s influential suggestion that the criterion for a concept’s being intentional is that the corresponding term behaves in an intensional fashion logically (Chisholm (1957)). Hintikka’s logic follows up on this suggestion with the further thesis that the correct theory of intensionality is given in the possible-worlds semantics of modal logic.

⁸See §2.5 below for discussion of these results in the context of intentionality.

The central features of intensionality are: the failure of inferences by substitution *salva veritate*, the failure of inferences by existential generalization, and the problem of the coherency of binding a free variable within the scope of an intensional term by a quantifier occurring outside.⁹ Let us address these briefly in turn, so as to set up the sense in which perception is an intentional concept insofar as perceptual verbs are subject intensional logical features.

Hintikka begins his logical reflections with the idea that moving quantifiers in and out of opaque intentional contexts occurs as a matter of course when using perceptual concepts, and in particular when reporting on one's visual perceptual situation. It is well known that there is a breakdown of classical inference in modal claims with respect to *substitution salva veritate*. For example, classically the following inference is valid (given the background information that Scott is the author of *Waverly*):

(2.1) The author of *Waverly* was born in 1771.

Therefore, Walter Scott was born in 1771.

However, the following is not valid (against the same background as above):

(2.2) Smith believes that the author of *Waverly* was born in 1771.

Therefore, Smith believes that Walter Scott was born in 1771.

For, it may be that Smith does not know or believe that Walter Scott is the author of *Waverly*. Next, in classical logic, we *generalize existentially* when we have an instance:

(2.3) The author of *Waverly* wrote *Waverly*

⁹Since Chisholm (1957), these problems of intensionality in logic have been correlated with problems of intentionality. On the side of intentionality, these problems are correlated respectively with the following problems: the conception-dependence of referential intentional contents; the existence-independence of directed intentional contents; and the indexicality of direct perception (Smith and McIntyre (1982)).

Therefore, there is someone who wrote *Waverly*.

In the attitude case, of course, the inference breaks down, i.e.:

(2.4) Smith believes that the author of *Waverly* wrote *Waverly*

Therefore, there is someone such that Smith believes it is he who wrote *Waverly*.

is invalid. For, it may be that Smith does not know the identity of the author of *Waverly*. Hence, there may be no one existing person of which Smith believes it is *he* who wrote *Waverly*.

Finally, the coherency of *quantifying-in and -out* has been disputed, with influential arguments originating in Quine's remarks on propositional attitudes (Quine (1956)).

Contra this line of thought, Hintikka maintains that it is difficult to make sense out of perfectly ordinary, coherent reports about one's attitudes without a method of quantifying-in. This case is particularly salient in the perceptual logic, since it is often held that there is an indexical or demonstrative structure underlying the semantics of direct-object perception.¹⁰

Compare:

(2.5) This is a jacaranda tree.

Therefore, there is something such that it is a jacaranda tree.

with:

(2.6) I see that this is a jacaranda tree.

¹⁰Cf., for example, Smith (1979), Miller (1984), Dickie (2015).

Therefore, there is something such that I see that it is a jacaranda tree.

In the attitude case, we must concede that the tree may be an hallucination, or that the perceiver has failed to actually see one particular tree, such that *it* is the jacaranda which the perception is of. It may be, for example, that the perceiver is looking at a forest and thinks he sees one particular tree which is, in fact, two trees.

We can account for these failures, and restore the basic intuition underlying the logical principles, Hintikka suggests, through recourse to his concept of *multiple reference* (Hintikka, 1969, pg. 90-93). This is the idea that any singular term falling in the scope of an attitude verb has references in multiple possible worlds, and it is precisely this feature that makes the corresponding attitude concept intentional.¹¹

The occurrence of an identity in the actual world need not carry over to other possible worlds that are compatible with an agent's background beliefs. For example, if Smith isn't sure which actual individual wrote *Waverly*, then this can be analyzed as saying that sentences characterizing Smith's beliefs are to be evaluated according to a spread of possible worlds wherein the reference of 'the author of *Waverly*' varies—Smith's beliefs are compatible with worlds in which Melville wrote *Waverly* and with worlds where Scott wrote *Waverly*. The method of multiple reference is thus a way of explicating Smith's attitudes and intentions, and the result is that the intensional phenomena are unsurprising: when multiple reference is countenanced in intentional contexts, we have no reason to suspect that an identity between two singular terms holding in the actual world should continue to hold once it is considered in Smith's belief-compatible worlds when Smith doesn't have a belief about the identity of the two terms.¹² Thus, the failure of substitution *salva veritate* should come as no surprise,

¹¹Hintikka hedges away from a metaphysically robust version of this notion and insists that he has something quite mundane in mind. That is, he wants us to understand the "reference in multiple possible worlds" in terms of different possibilities concerning our actual world, not as some worlds actually existing in a different mode of being. "In our sense," Hintikka says, "whoever has made preparations for more than one course of events has dealt with several 'possible courses of events' or 'possible worlds' " (Hintikka, 1969, pg.90).

¹²This point assumes, of course, that singular terms are not already defined as rigidly-designating.

and can be restored in cases where Smith’s beliefs are incompatible with such variation, i.e., when Smith comes to hold the belief that Scott wrote *Waverly*. Similar considerations account for the failure of existential generalization. These failures of inference are thus a direct consequence, Hintikka argues, of the fact that attitude contexts force us to consider individuals as members of different possible worlds, or as undergoing different courses of events, or as being perceivable from different perspectives, and, thus, of changing through any number of possible vicissitudes (Hintikka, 1969, pg. 159). Thus, on Hintikka’s thinking, any purported indispensability argument for something like Fregean senses in fact signals the phenomenon of “multiple reference” (in different possible worlds), rather than signalling a sharp distinction between meanings and references (Hintikka, 1969, pg. 106-108).

This line of thought is made quite intuitive when we entertain the idea that the meaning of anyone’s attitude report can be grasped when the listener is able to sift out the possible states of affairs that are consistent with the speaker’s attitudes from those that are not. Hence, for example, we can specify the belief-content of Smith’s doxic attitude at the actual world by specifying the sets of worlds that are consistent with all of Smith’s actual beliefs. Insofar as some p is true in all the worlds compatible with Smith’s background beliefs in the actual world, we can conclude that Smith believes that p . This basic line of analysis for belief generalizes to any given propositional attitude, among which Hintikka includes perception:¹³

(2.7) ‘ a believes that p ’ is true in w iff p is true in all possible worlds compatible with what a believes in w .

(2.8) ‘ a sees (that) p ’ is true in w iff p is true in all possible worlds compatible with what a sees in w .

¹³It may at first seem implausible to include perception as a *propositional* attitude, since a central case of perception seems to be direct-object perception that is best captured by direct-object language constructions such as ‘Smith sees John.’ However, Hintikka shows how to reduce direct-object constructions such as these in natural language to propositional attitude constructions. For more on his method for dealing with direct-object constructions, see §2.4 below.

In short, we can retrospectively interpret Hintikka's work here in light of results that came later on in intensional logic. Hintikka made an early version of the point that an important aspect of meaning—and perhaps an aspect of paramount importance—is what are now called *intensions*, or functions from worlds to extensions. Hintikka's notion of intensions in this sense is limited in scope to linguistic expressions whose extensions are objects. Putnam had a more general approach, applying the idea to all linguistic expressions, regardless of the type of extension they have (objects, classes, etc.) (Putnam (1975)). The point is familiar now from Montague grammar, in which the intension operator is defined as part of the semantics (Gamut, 1991, pg. 120). Of course, intensions have been important in many discussions of mind and semantics, for example in Stalnaker (1984) and Chalmers (2006).

However, what Hintikka takes to be the interesting problem is not the specification of intensional functions, but rather the question of intentionality—how and in what sense mental states represent things as being. In the case of *perception*, many important representations will intuitively be about ordinary objects, taken from different perspectives or seen under different aspects. An important part of Hintikka's idea of meaning, then, will be in tying together the appearance-adumbration of individuals in *world-lines* that determine a unique individual who is represented as changing through time and space (Hintikka, 1969, pg. 101-102). That is, perception will somehow track identical individuals through their manifestations in different possible worlds. We thus require a method of drawing these world-lines. In short, we need a theory of *trans-world identification*, i.e., a way of understanding how we hold fast to the identity of an individual under fluxing perceptual circumstance. Only if an object is genuinely individuated in this way as the same in different circumstances for a perceiver can a perception truly be said to be *of* or *about* that individual. The next section, thus, will address the problem of intentional individuation.

2.1.2 The Problem of Intentional Individuation

Up to this point, the discussion might sound familiar. For, Hintikka's logic, at the propositional level, is equivalent to today's traditional modal logic with standard possible-worlds semantics; the notion of an intensional function from worlds to extensions is now textbook; and the problems of trans-world identity might be considered old news. But Hintikka's interpretation of trans-world identity is distinctive, and his use philosophical use of the trans-world identity device is critical in his understanding of intentionality. In his theory the concept of trans-world identity is the semantic manifestation of a familiar ability that we all have to individuate objects for ourselves in perception and in other intentional modes. Thus, the foundation of trans-world identity for Hintikka lies in the notion of someone's being in the right position to *recognize* the identity in his attitudes, thus registering the identity across worlds. This picture is distinct from the discussion of trans-world identity in the literature surrounding the notion of metaphysical necessity, for example in Kripke (1980) and Lewis (1986), and it is within this distinctive line of thought that Hintikka develops his theory of quantification. Hintikka's shift of focus from *metaphysical* modalities to *intentional* modalities is therefore no trivial matter in the underlying theory of possible-worlds semantics. It is not hard to see, then, that the distinctive element of Hintikka's system, and the interest in studying it, is in his use of substitutional quantifiers to register the notion of intentional aboutness in the object-language in a way that the usual quantifiers cannot do, precisely by tying the sense of quantification to the notion of intentional individuation.¹⁴

His argument in this respect revolves around the idea that the use of quantifiers in perceptual contexts is understandable only relative to a rich perceptual-cognitive machinery. The basic idea is that we are only ever in perceptual or cognitive contact with a limited number of an object's modal presentations—yet this is sufficient to generate in our experience a total individuation of that object (Hintikka, 1969, pg. 99). In a way, this is the entire business of

¹⁴To see how this works in more detail, see §2.4 below.

intentionality on Hintikka’s possible-worlds understanding.

Quantifiers can only be said to bind individuals to the extent that such an individual is individuated, and, in the context of propositional attitudes, this just means that the subject has some means of so individuating. Since the use of quantifiers abounds in propositional attitude reports, he thus argues that we must have “nontrivial native conceptual capacities” that give us “ways of cross-identifying individuals, that is to say, ways of understanding questions as to whether an individual figuring in one possible world is or is not identical with an individual figuring in another world” (Hintikka, 1969, 99); i.e., perception has the special conceptual features built into visual processing that constitutes objects as the same through manifold differing partial perceptions of the object.

Individuation Functions. He thus adds to the semantic theory a stand-in for this conceptual capacity: he postulates a set of functions F , “each member f of which picks out a most one individual $f(\mu)$ from the domain of individuals $I(\mu)$ of each given model μ ” (Hintikka, 1969, pg. 100). He then gives a criterion for cross-world identity in terms of individuation functions, in that “the question whether $a \in I(\mu)$ is identical with $b \in I(\lambda)$ amounts to the question whether there is a function $f \in F$ such that $f(\mu) = a$, $f(\lambda) = b$ ” (Hintikka, 1969, pg. 101).¹⁵

Hintikka offers that we might think of the members of F as a distinguished kind of name or individual constant given to individuals that have a unique reference in all the relevant possible worlds. In this sense, we are to think of the things denoted by members of F not as separate individuals correlated by a function; rather, a function f is a name for some concrete individual being traced through or projected into different courses of events or states

¹⁵Note, first, that the I function here is distinct from the I function introduced in my formalization of the semantics in §2.2 below. Here, the I function simply takes us from worlds to the domain of individuals in that world. My own I function specified below is a standard interpretation function, which, e.g., assigns each constant, relation, and function symbol its interpretation at each world.

Note, second, that from a contemporary perspective, one can compare Hintikka’s idea of individuating functions here to the *roles* of Holliday and Perry in their epistemic logic (Holliday and Perry, 2014, pg. 605).

of affairs, different ways one and the same individual might have turned out in different circumstances (Hintikka, 1969, pg. 101). An individuation function thus constitutes the individual in a world-line, i.e., a mental projection of the individual constituted as the same in different possible states of affairs. The existence of some function $f \in F$ amounts to just the idea that there is some individuated thing for the perceiver.

Heuristically, at least, then one can think of individuation-functions-qua-individual-constants as denoting “senses” (Frege (1892), Husserl (1997)) or “individual concepts” (Carnap (1947), Hintikka (1969), (Smith and McIntyre, 1982, pg. 289)). For Hintikka, the crucial fact about individual concepts is that non-linguistic modes of presentation in fact *individuate* particular entities in the domain for subjects, by projecting the entity as the same in different situations. If no such constant existed for a perceiver, Hintikka argues, that perceiver would not be able to pick out the same entity under different possible vicissitudes, and there would not be any sense in which that person’s attitude is *about* that entity at all (Hintikka, 1969, 104). They thus play a role in the logic similar to an extended form of Fregean *Sinn*, or sense—i.e., they embody modes of presentation, but they also thereby individuate the entity so presented by tying together an entity’s distinct modes of presentation.¹⁶ We can thus refer to them as *individuating senses*. These will allow Hintikka to ascribe de re beliefs and perceptions where Frege had no straightforward means of doing so. The crucial insight, then, will be that even de re modalities have a mode of presentation—they involve an entity being presented in a particular way, but it is a special de re way of being presented.¹⁷

Physically- and Perceptually-Individuating Functions. As I noted in the introduction, studying the logic of perception in particular, as opposed to the epistemic modalities, for example, holds critical importance for Hintikka’s system. One particular reason for this is that the consideration of perception is crucial for the characterization of the individuating senses in

¹⁶Despite this overall parity between the idea of individuating functions and senses, one should be aware of their differences. See the discussion in §2.5.2 below.

¹⁷cf.(Smith, 1983, pg. 263).

the general possible-worlds theory. For, Hintikka argues, when we consider perception in its idiosyncrasies we find that there are at least two different sorts of individuating senses (or individual concepts)—physically-individuating senses, on the one hand, and perceptually-individuating senses, on the other.¹⁸

I will first address the *perceptually-individuating senses*. Senses that are perceptually-individuating are associated with a kind of geometrical fixing: the entity is individuated by virtue of its phenomenal-visual appearance being demarcated and fixed in a privileged position in a given subject’s field of vision. One can put the point in terms of the “cognitive machinery” required for this sort of individuating presentation.¹⁹ There is less cognitive machinery involved in perceptual individuation because this type of visual individuation is not belief-informed. By virtue of this non-doxastic character, perceptual individuation of objects maps to purely phenomenal modes of recognition, and thus plays into the intentional characteristics of a perceptual state in a distinctive way. For example, suppose I see someone in the distance walking toward me but cannot make out who it is. I can individuate the person walking towards me as one and the same person through a flux of different perceptions (first they are here, now there, first under these lighting conditions, then under those) due to the privileged position the colored shape maintains in my visual field, even when I have no further beliefs about who this person is.

On the other hand, *physically-individuating senses* are belief-informed and thus implicate us in a more nuanced cognitive machinery than pure phenomenal individuation. These senses are individuating by virtue of external, descriptive facts such as facts related to “bodily continuity, continuity of memory, certain obvious features of the behavior of material bodies,

¹⁸cf. Hintikka’s discussion following, “Now the great interest of perceptual concepts for a philosopher of logic is due precisely to the fact that in connection with them we all as a matter of fact use two different methods of individuation” (Hintikka, 1969, pg. 171).

¹⁹Where “cognitive” should be read loosely. We do not here intend to discuss “cognitive phenomenology”, or cognitive experiences of the sort that is involved in non-perceptual experiences such as thinking about mathematical theorems. Rather, these are innate capacities of perceivers by virtue of their having perceptual systems that deliver “information” of some sort that purports to be about the environment.

and many similar physical and psychological regularities” (Hintikka, 1969, pg. 170). Suppose, again, that I see the person off in the distance. Suddenly, I recognize the person’s face and see this *as* my father. In this new perception, I *recognize* my father. This recognition is plausibly informed by my beliefs, but not directly. That is, the recognition is enacted in a form of intentional *individuation* that *itself* is belief-informed. The very intentional individuation of this person in my experience *as* an object of my visual experience is now informed by my background beliefs with respect to my father’s bodily continuity, folk-psychological beliefs about my father, memories of his past actions, and so on.

There is thus a more robust set of beliefs and “background theories” that the perceiver brings to bear in physical individuation. Notably, these constants work much differently than *definite descriptions*. Hintikka makes an early version of the point, later made in a different context by Kripke (1980), that the definite description theory will not work here, since we usually don’t have enough descriptors. Again, it is the business of intentionality to individuate objects in a total way out of a potential poverty of descriptors.

We can further refine our notion of individuation in this sense by noting that this discussion brings out the point that the notion of perceptual and physical *individuation* is distinct from the notion of *recognition*, though the two are undoubtedly related. The conditions for recognition, it seems, are stronger than those for individuation. For example, an amnesiac would presumably individuate many objects that he/she does not recognize. However, it is difficult to imagine a case of recognizing an object that one has not individuated.

We have said that individuals are marked by functions $f \in F$. We cannot, Hintikka says, reify the members of F into ordinary individuals because we have at least two different methods of cross-identification in our native conceptual capacities. We account for this by stipulating (at least) two sets of individuation functions even though we are working with the same sets of possible worlds, i.e., the same individuals (Hintikka, 1969, pg. 102). Hence, an ordinary individual can have associated with it two different sorts of ways of being individually

presented.

We do not, however, want to say that each of these different sorts of ways of being individuated gives a different sort of object. On the contrary, they are different presentations *of* ordinary individual objects. Hence, reifying the members of F into ordinary unstructured individuals would violate this idea. Furthermore, we cannot simply postulate that one of these methods of individuation is more fundamental and thereby reify the members of that one into ordinary individuals. For, the individuation functions are associated with different methods for individuating objects in the world, but there is nothing that makes a given method of individuation the one that gives us “ordinary” individuals, and it is not a logical consideration that settles what an “ordinary” individual is *qua* ordinary. Only a deeply entrenched background social practice of generally using physical individuation makes it seem as though what we think of as ordinary individuals are the ordinary ones, according to Hintikka.

By extension, then, one can think of perceptual individuation as individuation in terms of the phenomenal appearances of some person’s visual situation considered independent of the physical veracity of that situation. Suppose, for example, I see a man in front of me, but I do not discern who he is visually—say his back is turned to me and I don’t recognize his gait. Hintikka says my visual information in this case is given by a set of *perceptually-possible* worlds. The perceptually-possible worlds are those that are “compatible with everything [I] see” (Hintikka, 1969, 171) at the time of the perception in question. In this case, since I cannot see who the man is, the variety of possible worlds compatible with what I see is such that a different man can be picked out in different worlds. After all, as far as my visual impressions are concerned, a phenomenal figure of such-and-such shape configuration is in front of me; but there are countless candidates for just who that might be as far as the pure visual information is concerned.

Already, though, we have lapsed into a potentially careless way of speaking on Hintikka’s

system. What I should say is—*according to physical methods of individuation* the man picked out will be different in different possible worlds. Different “physico-psychological individual[s]” will be compatible with my visual impressions, i.e., there are possible worlds in which Mr. Smith stands in front of me, and possible worlds in which Mr. Brown stands in front of me. But if we set these physical methods of individuation aside and consider only my appearances, only the phenomena at hand, then the situation is different. For, the only worlds compatible with what I see in the pure visual-information sense at w have fixed in the visual field one and the same entity—the entity in front of me now who, at w , occupies this privileged position in my visual field. These distinctions will help Hintikka draw formal distinctions between direct and indirect intentions, thus bringing into the object-language distinctions that hedge toward a theory of intentionality (cf. §2.4 below).

2.1.3 Quantification and Individuation

We have just seen how Hintikka motivates his distinctive theory of intentional individuation. We will now discuss how Hintikka motivates his method of quantification in terms of intentional individuation. The philosophical import of this move cannot be overstated. For, Hintikka’s strategy of defining special quantifiers in terms of modes of intentional individuation gives us a new way of thinking about the appearance of bound variables in perceptual reports. Variables, insofar as their sense is tied to modes of intentional individuation, are a *part* of the *meaning* of a perception. Bound variables are normally understood as standing in for bare pronouns anaphorically tied back to the appropriate quantifier. Here, they carry a distinctive sort of meaning; they inherently have a mode of presentation-qua-individuation-presentation. Furthermore, there are two sorts of senses any variable can carry—perceptual and physical senses. This can be seen as an extension of Frege’s idea that even singular terms have senses, extending the point even to variables standing in for singular terms. An earlier version of this point was made by Husserl, in which Husserl identifies two components

of meaning: the “predicates”, on the one hand, and the “determinable X,” abstracted from all possible predicates, on the other (Husserl, 1907, §113). Hintikka has, in effect, made this point more rigorous with his formulation of it in modal logic. Let’s see how it will work.

In Hintikka’s original formulation, a member of the substitution class F is thought of as a name standing in for a special kind of function which, given a possible world w assigns an individual from the universe who is constituted as “the same” individual in respect to either constitution within the visual field, or constitution as spatio-temporally continuous, etc.—as far as a given agent is concerned. Hintikka thus conceives of his substitution class F as individuating functions, a semantic stand-in for native conceptual capacities of intentional agents and defines his quantifiers in a (quasi-)substitutional fashion that ties the methods of individuation to the sense of the quantifiers.

This move adds plausibility to restrictions that may be put in place on the cardinality of the domain of a substitution class when using substitutional quantification. For, we are dealing here with entities as intentionally individuated, and since intentional individuation is accomplished by finite beings, it is natural to hold that the substitution class should be finite. It also makes sense of the requirement when defining truth in a model that truth of a formula at a world be relative to the availability of a constant whose substitution into the formula issues in a true sentence. For example, suppose someone has just been subjected to a short burst of an immensely powerful bright light, and his visual field becomes undifferentiated for a moment as his eyes adjust. In that case, no perception that could underwrite a determinate perceptual report as of being about any object in particular, since he has no individuated presentation of an object available in his perceptual field to pick out. This situation would be formally analyzed by saying there is no substitution instance of the right sort, and so first-order formulae will not be able to distinguish one object from the next.

To make more transparent the underlying logical structure of Hintikka’s system, we will have special domains of constants \mathcal{C}_i . This will be the substitution-class that stands in

for Hintikka’s substitution class of individuating functions, in terms of which we define our substitutional quantifier. The reason for this change from individuating functions to substitution constant symbols is that it allows a simplified standard presentation of the material while obtaining equivalent logical validities, and provides for a more natural and uniform formal presentation of the semantics than Hintikka himself provides. However, when addressing Hintikka’s solution to the identity puzzles of intensional logic and the relevance of his semantics for the question of intentionality, we cannot ignore that Hintikka’s special substitutional quantifier is of two different sorts, each corresponding to a different sort of individuation—one corresponding to perceptual individuation, the other to physical individuation. Hence, we will have two substitution classes, \mathcal{C}_{perc} and \mathcal{C}_{phys} , each with their own corresponding quantifier, $\Sigma_{\mathcal{C}_{perc}}$ and $\Sigma_{\mathcal{C}_{phys}}$.

These should not be understood as intrinsically typed entities, nor should these quantifiers be held to range over ontologically distinct sorts of entities.²⁰ Intuitively, the difference between $\Sigma_{\mathcal{C}_{perc}}$ and $\Sigma_{\mathcal{C}_{phys}}$ is that they are defined in terms of differing methods of individuation. It is tempting to think that $\Sigma_{\mathcal{C}_{perc}}$ thus must range over different sorts of individuals than does $\Sigma_{\mathcal{C}_{phys}}$ —perceptual individuals, on the one hand, and physical individuals on the other. But this would be inaccurate. For, Hintikka advocates for a realist ontology with respect to the kinds of individuals about which we report when making perception reports. A perceptual intention and its corresponding perceptual report is not *of* or *about* a perceptual object, like a mental presentation or a sense-datum. Rather, it is of or about the very object which the presentation, or the sense-data, presents. Allowing these different sorts of constants is as close as Hintikka wants to come to a pluralist, representationalist, or idealist ontology with respect to the sorts of individuals that serve as objects of a perceptual intention.²¹

Recall that this is a *device* for the analysis of perception and thus the point about perceptual

²⁰See the remarks on the \mathcal{C}_i sets from Definition 13 below.

²¹This point will be important in the consideration of intentional content in terms of possible worlds, cf. §2.5 below.

modes of presentation does not necessarily entail an ontological position about “perceptual objects.” The claim is not that there are perceptual objects in one domain, and, in a separate domain, physical objects. The difference between different modes of individuating presentation lie in the different sorts of singular terms used to denote, not in any difference with respect to the sort of objects denoted. In the Dummettian metaphor, they refer to different kinds of “routes to reference,” not to different objects of reference.²² One way of thinking of the distinction is that constants occurring in \mathcal{C}_{perc} do the work of denotation geometrically (i.e., by demarcating and fixing the coordinates of the phenomenon in the phenomenal field of the perceiver), while constants occurring in \mathcal{C}_{phys} do the work in a “descriptive” fashion. An example of a constant in \mathcal{C}_{perc} might be ‘the man currently in the right-hand side of a ’s visual field’, while a constant occurring in \mathcal{C}_{phys} might be something more like a proper name, e.g., ‘Mr. Smith’, or any sort of description that relies in some way on features of space-time bodily continuity, and so on. However, the constants should not be taken to *be* descriptions, since they are tied to *intentional*, and not necessarily linguistic, methods of individuation. Examples in §2.4 below will help flesh this notion out.

This leads to an important caveat with respect to the multiple quantifiers in Hintikka’s proposal. Hintikka pushes back against the idea of senses as entities that can serve as arguments in a function. Rather, for Hintikka, senses are better thought of as the very function that takes worlds as arguments and gives extensions. Still, Hintikka has invoked intensions as an important part of meaning, and in the Carnapian sense these just are senses or intensions. Again, Hintikka’s approach still commits us to functions that tie together world-lines of individuals in the same basic way as do constants standing in for senses. However, this notion should not be read ontologically. Rather, the import of the different quantifiers has to do with distinguishing between perceptual statements that invoke an individual *as* perceptually

²²Cf. the discussion of the notion of route to reference (Dummett, 1981, pp. 96, 102, 179 ff) and (Taschek, 2010, pg. 323). The comparison to Dummett’s notion should be taken loosely, however, since for Dummett routes to reference are couched in epistemic terms and can be conceived as a kind of algorithm. Hintikka’s notion of individuating senses is not an epistemic method that one uses to obtain reference. Rather, it is an immediate feature of our perceptual experience.

individuated for someone, and perceptual statements invoking an individual *as* physically individuated for someone.

Hence, the possible-worlds theory of intentionality that Hintikka provides still invokes the notion of a sense, regardless of the ontological status of senses as entities or as something else altogether. The possible-worlds theory of intentionality, which finds intentionality in a pattern of multiple reference, still invokes the notion of a sense or an intension. We should, however, not be led into thinking that the different sorts of senses (perceptual vs. physical, for example) implicate us in a pluralist ontology with respect to the sorts of individuals that exist in the world.²³ Furthermore, we should make sure we have a grasp on the philosophical point that I outlined at the very beginning of this subsection. For, this is a point that, as far as I can see, is unique to Hintikka's logic and thus will not be familiar from standard in modal logic. The point, again, is that the treatment of variables here implicates them in the meaning of a perception: variables inherently carry with them a sense or a meaning which is registered by the quantifier definitions.

What has been said so far, however, does not settle the matter as to what the real distinction is between physical and perceptual individuation, such that this real distinction is reflected in the formal distinction. This question, it seems to me, is a distinctly phenomenological one and requires phenomenological investigation over and above the modal logical model.

I have just set the stage for the semantics by looking at the philosophical approach Hintikka takes in tackling the problems of modal logic, leading to his theory of individuation and quantification. In the next section, §2.2, I will formalize the semantics. Following that, in §2.3, I will look at the validities and invalidities of the propositional level of Hintikka's semantics. These will be familiar from standard modal logic, and so the bulk of the discussion will be focused on Hintikka's quantifiers, in §2.4.

²³See §2.5.2 below.

2.2 Hintikka Semantics for “Sees”

Given that Hintikka developed his perceptual logic in an idiosyncratic way, it is notable that his proposal is a special instance of two well-studied logical mechanisms. It is easy for this observation to be obscured when looking at the special conceptual ideas that Hintikka uses to guide his presentation. In essence, Hintikka proposes a version of substitutional quantification combined with a standard semantics for a modal operator. In non-modal contexts, substitutional quantification is not usually used, but Kripke has shown that there is no formal barrier to extending a language L to a language L^Σ with a substitutional quantifier Σ over a class of expressions in L (Kripke (1976)). It follows that we should not expect to run into any technical barriers in implementing Hintikka’s ideas formally. Substitutional quantification is not as sophisticated as other theories of quantification, and it leads to some undesirable restrictions on the language—e.g., it may be that the language must contain a unique constant for each object in the domain, which is implausible for uncountable domains. In the modal setting, however, substitutional quantification comes apart from traditional quantifiers in important ways.²⁴

The formal presentation of Hintikka’s perceptual logic can be made quite similar to other sorts of more well-studied intensional logics, such as epistemic logic. Thus, in contrast to Hintikka’s non-standard presentation, I shall use more standard presentations from the literature to give a generalized formal account consistent with present day intensional logic. For example, he defines a special quantifier that only appears in front of particular sorts of formulas. In my presentation, we will avoid these idiosyncrasies. Hence, I will define the substitutional quantifier²⁵ and modal semantics²⁶ in a standard fashion from the literature.

²⁴See discussions in §2.2 and §2.4 below.

²⁵See Marcus (1961) and Kripke (1976), for influential discussions. For a survey of applications of substitutional quantification to problems in philosophy, see Hand (2007). Hintikka never offers a fully formalized presentation of his semantics in the way that I am doing here. However, compare my presentation of his perceptual semantics to that of David Woodruff Smith’s formal presentation of Hintikka’s semantics for the belief modality, (Smith, 1983, 262).

²⁶See Fitting and Mendelsohn (1998).

Definition 13. A *model* \mathcal{M} is a tuple $\langle W, D, I, R_{\mathbb{S}}, \mathcal{C}_1, \dots, \mathcal{C}_n \rangle$.

- (i) W is a non-empty set of possible worlds, and for each w from W one has that D_w is a non-empty set, and I is the interpretation function, which assigns, e.g., each constant, relation, and function symbol its interpretation at each world,
- (ii) $R_{\mathbb{S}}(a, w)$ is a *perceptual accessibility relation*, which, given an agent a in w selects the set of possible alternatives to w for a , and is thus a subset of W .
- (iii) Each \mathcal{C}_i is a distinguished class of constant symbols.

The set W is understood as a set of worlds, each of which constitutes a “complete novel” describing that world. Hintikka, in response to challenges from situation semantics, would later argue that the worlds in W need be understood as only partial descriptions of a world, and that the worlds associated with a given propositional attitude can just well be understood as “small worlds,” intuitively akin to the idea of a situation from situation semantics. These worlds, thus, can be understood as describing the various possible states of affairs or courses of events that genuinely existing individual entities might undergo. To the contrary, they should not be understood as describing the various possible entities or states of affairs that might exist, or that exist in some other possible world. Furthermore, in the perceptual case, it is natural to understand the possible worlds as different possible perspectives on an object or state of affairs.

The $R_{\mathbb{S}}$ relation is the perceptual accessibility (or “alternativeness”) relation, conceived of as the set of worlds (states of affairs, courses of events, visual perspectives) that are compatible at the time in question with everything the perceiver sees. We restrict $R_{\mathbb{S}}$ to non-reflexive relations only. For, on the semantics we will specify, reflexivity will force veridicality of perception. It will be beneficial to set up the formalism so that it is neutral with respect to veridicality. The force of non-reflexivity will be discussed later in §2.4. There is good reason

to think also that $R_{\mathbb{S}}$ is non-transitive, since a transitive relation would make it difficult to characterize misperception, as will become clearer later in an example in §2.3.2 below. It is natural to require also that $R_{\mathbb{S}}$ be serial, since the situation in which $R_{\mathbb{S}}(a, w)$ is empty for some a and w is one in which, intuitively, the perceiver is receiving no visual information at that world.²⁷

For the \mathcal{C}_i sets, it is often appropriate to constrain the constants to *rigid* constants of a certain sort, and some may choose to build in a rigidity constraint to the semantic definitions. That is to say, we must sometimes require that $c, d \in \mathcal{C}_i$ and if w_n is a world in W , then if $I(c, w_n) = I(d, w_n)$ then $I(c, w_m) = I(d, w_m)$ for all $w_m \in R_{\mathbb{S}}(a, w)$. This is a non-standard notion of rigidity, so I will often refer to it as *Hintikka-rigidity* to set it apart from standard notions.²⁸ The reasoning and significance behind this will be discussed later in §2.4. In short, Hintikka’s primary motivation in considering rigidity is in solving the problem of quantifying-out. Hence, his distinctive notion of rigidity is tailored to the considerations he makes with respect to the coherency of quantifying-out in attitude cases.

Finally, we assume a standard model-theoretic truth-definition for truth in a pointed model \mathcal{M}, w for $w \in W$.²⁹ We then extend the base language with two sorts of substitutional quantifiers, Σ and Π and a modal operator \mathbb{S}_a :

Definition 14.

- (i) For each \mathcal{C}_i , $\mathcal{M}, w \models \Sigma_{\mathcal{C}_i} x \psi(x)$ iff there is $c \in \mathcal{C}_i$ with $\mathcal{M}, w \models \psi(c)$.
- (ii) For each \mathcal{C}_i , $\mathcal{M}, w \models \Pi_{\mathcal{C}_i} x \psi(x)$ iff $\mathcal{M}, w \models \psi(c)$ for all $c \in \mathcal{C}_i$.
- (iii) $\mathcal{M}, w \models \mathbb{S}_a \varphi$ iff $\forall w' \in R_{\mathbb{S}}(a, w)$, $\mathcal{M}, w' \models \varphi$

²⁷A relation R ranging over a set W is *serial* just in case, for all $w \in W$, there exists some $v \in W$ such that wRv . For more on the seriality of $R_{\mathbb{S}}$, see the discussion of negation in §2.3.2 below.

²⁸Standardly, we would expect a definition of rigidity that says something like for all $w_n, w_m \in W$, we have $I(c, w_n) = I(c, w_m)$. See §2.4

²⁹See Fitting and Mendelsohn (1998) for standard treatments of truth in a pointed model.

- (iv) The other clauses are defined as is standard in modal predicate logic. In particular, the usual \exists and \forall clauses are defined as normal.

Note that the modal operator formalizes the perceptual verb “to see (that)...”, with the ‘ a ’ subscript standing in for some perceiving subject.³⁰ For example, ‘ $\mathbb{S}_a F(c)$ ’ translates intuitively to ‘ a sees that (the individual denoted by) c is F ’, or equivalently, ‘ a sees c as F .’ It should be borne in mind, however, that the formalism is neutral with respect to veridicality (i.e., it leaves it open as to whether c really is F , independent of a ’s present visual circumstance), and hence a more illuminating gloss may be, ‘ c appears F to a ’ or ‘ a seems to see c as F .’³¹

The quantifiers are obviously the distinctive element in this system. For ease of reading, I omit the subscript in $\Sigma_{\mathcal{C}_i} x$ and simply write Σx where the \mathcal{C}_i is clear from context, or in cases where nothing turns on its specification. As briefly noted in the introduction, and as manifest in the above definition, the $\Sigma_{\mathcal{C}_i}$ quantifier is a simple substitutional quantifier. Hintikka did not conceive of his quantifier as being a substitutional one per se, so, coupled with the fact that the quantifiers provide the lion’s share of the philosophical import here, it is important that I justify the simple substitutional definition. First, we can show that we obtain equivalent validities with a simplified presentation. More importantly, we have textual evidence that warrants such an interpretation. Hintikka intuitively approximates his quantifiers’ definition by specifying truth-conditions for it in terms of special sorts of substitution instances, and, he says, this shows “how close we can stick to the simple-minded idea that an existentially quantified sentence is true if and only if it has a true substitution instance” (Hintikka, 1969, 102-103). Here, then, we indeed stay as close as possible to that idea.

³⁰Other perceptual modalities (auditory, tactile, olfactory, proprioceptive, etc.) are of course not ruled out as relevant to the logic of perception. For the purposes of the present essay, I focus on the case of visual perception, as is standard in the literature. One should be cautious of assuming that the treatment of visual perception will easily generalize to the other perceptual modalities.

³¹In what follows, I use ‘ a sees...’ for shorthand, but a neutrality with respect to veridicality should be assumed unless otherwise noted.

In modal logic, a substitutional interpretation comes apart from the usual quantifier interpretation in important ways. For example, consider:

$$(2.9) \quad \forall x, y \ x = y \rightarrow \mathbb{S}_a x = y$$

$$(2.10) \quad \Pi x, y \ x = y \rightarrow \mathbb{S}_a x = y$$

Under the usual quantifier interpretation, as in (2.9), this is a valid inference, since variables are rigid with respect to a given variable assignment. Hence, the denotation of x and y relative to a variable assignment carries through from the actual world to the possible worlds. Under the substitutional interpretation, however, as in (2.10), the inference is invalid. Consider a model in which the actual world has only one object, say, a red ball. Suppose our perceiver has two constants, c and d , that pick out the red ball in the actual world. Then c and d stand witness to the antecedent since they pick out the same object in the actual world. However, our perceiver might be vastly disoriented, so that he sees two balls, one red, one blue, each occupying different portions of his visual field. Hence, in the alternative worlds, c and d could refer to two different objects. Intuitively, such a counter-model could describe a situation in which a perceiver has just woken up from fainting and has double-vision. Even though there is only one object before him, he sees two objects. Note, however, that (2.10) becomes valid if we assume Hintikka-rigidity. Indeed, the sentence is just the object-language statement of the Hintikka-rigidity condition.

Similarly, (2.11) is valid, while (2.12) is invalid:

$$(2.11) \quad \forall x, y \ x \neq y \rightarrow \mathbb{S}_a x \neq y$$

$$(2.12) \quad \Pi x, y \ x \neq y \rightarrow \mathbb{S}_a x \neq y$$

For, again, the (standard) rigidity of variable denotation relative to a given variable assignment makes (2.11) valid. On the other hand, the substitutional quantifier version says that

if we have no co-referring constants in the actual world then in all the worlds compatible with what *a* sees, there are no co-referring senses. But, again, there is nothing that says that a perceiver's visual information need be even remotely similar to the actual world, for example in a case in which the perceiver has lost his glasses and sees many different objects blurred into one big blob. This is a positive outcome in the case of Hintikka's aims, since it accords with intuition with respect to double- and blurred-vision cases. Finally, note that (2.12) is *not* made valid by assuming Hintikka-rigidity, as was the case with (2.10). For, the validity of (2.12) would amount to ruling out merging cases, but Hintikka-rigidity does not rule this out.

This consideration immediately leads to the worry that the many-sorted substitutional quantification leads to an ontological picture orthogonal to Hintikka's philosophical picture. For, Hintikka resists the an ontology of senses as objects. Here, it seems we are quantifying over senses and thus, by Quine's criterion, we are committed to the senses as pieces of our ontology. But Hintikka quantifies over senses as functions from worlds to extensions. Hintikka hedges against the ontological reading of this move by distinguishing between parts of our *ideology* and parts of our *ontology* (Hintikka, 1969, pg. 95). Thus, in a formal sense, the only objects we need have in our ontology are the individuals populating the domains of our worlds. Only what shows up in the domain of the model need be considered part of our ontology. The constant symbols standing in for senses are part of the semantic device for determining intentional characteristics of a perceptual state that individuate objects in the domain for perceivers. The constants are only objects in the *logician's* sense. Philosophically, the senses, as aspects of our semantic machinery, are aspects of our ideology, the machinations of our perceptual-cognitive capacities.

In my reconstruction there is admittedly a slight loss in this respect in the transition from functions to constant symbols. The idea of a function that takes us from worlds to extensions perhaps more intuitively fits the role of a "route" to reference than does a constant symbol,

which can have the “feel” of an object when dealing with the formalism. However, the constant symbols so defined play the exact same formal role as do Hintikka’s functions. And so there is no reason to suspect that the constant symbols should implicate us in any ontological difficulties any more than do Hintikka’s individuating functions.

The approach to quantifiers here may be held to be exceedingly unusual, so let us situate this idea in the literature on quantifiers. Defining two quantifiers has a precedent, for example, in modal logic debates on actualism and possibilism, in tense logic, and in free logic. Of course, in settings in which two different quantifiers have been defined, there is usually the additional question as to just which one should be used. However, there is a minority position in each of these traditions using both quantifiers. For example, in Cocchiarella’s free logic, we define two sets of quantifiers, one without existential import ranging over possible objects; the other with existential import ranging over actual objects ((Bencivenga, 1986, pg. 391), (Priest, 2008, pg. 295)).

Another method, distinct from Hintikka’s, is that of intrinsically typing variables corresponding to different sorts of intrinsically “typed objects,” rather than defining two different types of quantifiers ranging over the same sorts of objects. In their epistemic intensional logic, Holliday and Perry (2014) propose this. Similarly, in second-order logic, formulas are syntactically restricted by first- and second-order intrinsic typing. For example, the second order sentence, ‘ $\forall X \exists y (X(y))$ ’ is syntactically restricted to variables encoding second-order objects bound by the second-order quantifier, and variables encoding first-order objects bound by the first-order quantifier. Hintikka’s idea, on reflection, might be thought of as just another way of doing the same thing, and so one might thus think that Hintikka’s proposal as just a special weakened case of the Holliday-Perry system. Indeed, it is likely that these approaches converge when considering only fully quantified sentences. However, when looking at the larger class of formulas, there will be divergences, since each has a different treatment of singular terms and free variables.

For, there are many possible formulas that are well-formed for Hintikka that are not well formed on the Holliday-Perry set-up. Consider, for example, the following sort of formulas:

$$(2.13) \Sigma_{\mathcal{C}_{perc}} x \varphi(x) \rightarrow \Sigma_{\mathcal{C}_{phys}} x \mathbb{S}_a \varphi(x)$$

$$(2.14) \Sigma_{\mathcal{C}_{phys}} x \mathbb{S}_a \varphi(x) \rightarrow \mathbb{S}_a \Sigma_{\mathcal{C}_{perc}} x \varphi(x)$$

Holliday and Perry's typed objects rule out formulas in which mixed quantifiers are used in this way, since the object-position in the formula ' $\varphi()$ ' is encoded by intrinsic types. Thus, we have good reason to investigate the logic obtained from adding both quantifiers and allowing mixed quantifier formulas.

To further differentiate the Hintikka strategy from other approaches, consider a degenerate case in which Hintikka's semantics *is* definable in the usual possible-worlds way. Assume we can quantify over intensions,³² and assume we have a constant in the language for each function from worlds to individuals. Then we can redefine the truth-conditions of the quantifier as below:

$$(2.15) M, w \models \Sigma x \varphi(x) \text{ iff there is a function } s \text{ from worlds to individuals such that } \\ M, w \models \varphi(s(w)).$$

Using the extension operator, this happens iff $M, w \models \exists s \varphi(\check{s})$, where the quantifier is typed appropriately so that it ranges over functions from worlds to individuals.

However, Hintikka's semantics is not definable in this way in general. Consider the following representative example wherein the constants are more circumscribed than the functions from worlds to individuals. In particular, suppose that there is a finite number of worlds and that the domain is the real numbers, which is uncountable, and that the only relation in place

³²As in Gamut's construction of intensional type theory, (Gamut, 1991, Ch. 5).

on the real numbers is the ordering, and where there is a constant for every function from the worlds to the *rational* numbers (so that there are only countably many constants). Then consider the formula $\varphi(y) \equiv \exists x x = y$. At each world, this formula defines the rational numbers. But the real numbers with just the ordering are such that every set definable in the usual non-modal first-order semantics is a finite union of points and open intervals, i.e. sets of the form $(a, b) = \{c : a < c < b\}$ ((Marker, 2002, 99)). Since there are only finitely many worlds, any quantifier over functions from worlds to individuals can be defined in a first-order manner, and hence we won't be able to define the rationals in this way.

I will now move on to discussing the propositional validities in §2.3, below in order to demonstrate the faithfulness of the reconstruction and to understand the philosophical interpretation of the standard results of modal propositional logic for the modality of perception. After that, we will move on to discussion of Hintikka's substitutional quantifiers (§2.4).

2.3 The Validities and Invalidities of the Hintikka Semantics

This section deals with the validities of the propositional fragment of the logic. As I have said, the distinctive aspect of this semantics is the definition of an additional substitutional quantifier along with the usual quantifier, so it is natural to understand Hintikka's semantics at the propositional level as standard propositional modal logic with possible-worlds semantics. As a result, the primary interest here will be in couching the usual results from modal logic in the domain of perceptual concepts. For the discussion of quantification, see ahead §2.4.

2.3.1 Axioms

First, it will be helpful to review the status of some standard modal axioms:

$$\text{(K-axiom)} \quad \mathbb{S}_a(\varphi \rightarrow \psi) \rightarrow (\mathbb{S}_a\varphi \rightarrow \mathbb{S}_a\psi)$$

$$\text{(T-axiom)} \quad \mathbb{S}_a\varphi \rightarrow \varphi$$

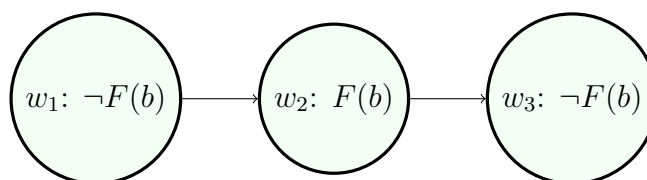
$$\text{(4-axiom)} \quad \mathbb{S}_a\varphi \rightarrow \mathbb{S}_a\mathbb{S}_a\varphi$$

It is clear that the K-axiom will be valid, as it is on any possible-worlds semantics. This is problematic for Hintikka's proposal, since the K-axiom in essence amounts to some sort of closure under implication, and it is implausible to suppose that the characterization of perceptual information is in any sense closed under logical implication. Other types of semantics, such as certain versions of topological semantics, allow the failure of this axiom, so it seems that the closure problems for Hintikka are an artifact of the possible-worlds semantics and not reflective of the concept of perception.³³

However, we can show that both the T-axiom and the 4-axiom will fail for Hintikka's perceptual operator. The sorts of counter-models that are available for this purpose are also quite well-motivated intuitively. We will find natural counter-models rooted in examples from perception. So it will be clear that perception resists axiomatization for reasons inherent in its way of working. It is immediate that the T-axiom will not hold, for, given the semantics, the T-axiom amounts to just the requirement that the $R_{\mathbb{S}}$ relation be reflexive. But, unlike most epistemic logics, for example, the perceptual case requires non-reflexivity in order to deal with non-veridicality and misperception. Any situation in which someone is seeing things incorrectly, either due to underdetermination of perceptual information, or incomplete perceptual identification, or cases of hallucination, and so on, will be a potential counter-example.

³³For example, see (Chellas, 1980, Ch. 7).

Furthermore, it is not hard to construct a counter-model to the 4-axiom. In particular, cases of “predicate explosion” provide counterexamples. Suppose I am walking through the streets of a city and in some store window I see a human standing. Upon closer inspection I come to find that the “human” I had seen was in fact a highly realistic wax mannequin. Suppose in this case that each accessible world is a different point of view on the object, denoted by $b \in \mathcal{C}_i$, and that $F(b)$ means ‘the figure, b , in the window is a human.’ We have $R_{\mathbb{S}}(a, w_1) = \{w_2\}$ and $R_{\mathbb{S}}(a, w_2) = \{w_3\}$.



Hence, $\mathcal{M}, w_1 \models \mathbb{S}_a F(b)$, since $F(b)$ is true at the only accessible world (namely, w_2). However, $\mathcal{M}, w_1 \not\models \mathbb{S}_a \mathbb{S}_a (F(b))$. For, suppose not. In that case, we must have $\mathcal{M}, w_n \models F(b)$ for all $w_n \in R_{\mathbb{S}}(a, w_2)$. But we have $\neg F(b)$ at w_3 , which is the only accessible world from w_2 . This situation also clarifies a philosophical foothold for the non-transitivity of $R_{\mathbb{S}}$.³⁴ For, transitivity would make it difficult to characterize situations in which an object is misperceived and later corrected from a more optimal point of view. Such cases are important in the perception literature, for example, in Husserl’s wax figure (Husserl (1970), Investigation V, §27, 137-138 and Husserl (1997), §15), in David Marr’s (1982) theory of visual information processing, and in van der Does and van Lambalgen (2000).

In addition to the formal results, there is the natural question of whether or not any axiomatization could be philosophically acceptable for a perception operator. With the exception of the T-axiom, it is problematic to give an intuitive interpretation of the axioms in the perceptual case. Consider the 4-axiom. It is unclear what it would be for one to “see that he sees” without equivocating on the meaning of “sees.” In the logic of perception, then, it

³⁴Recall in §2.2 above where non-transitivity was evoked for $R_{\mathbb{S}}$.

is far from clear whether iterated modalities are coherent.

With respect to the K-axiom one might ask: What is to “see” an implication? One plausible way of interpreting such sentences would be to think in terms of “geometrical information” present in a perceptual content. In the visual field, objects and states of affairs are individuated geometrically, and so you might make sense of a report such as, “I see that the upcoming doorway is too small for this man’s height” as a kind of seeing of an implication. To make sense of what it is to see an implication, then, we might invoke the notion of the *geometrical seeming* of the perception. We have said that perceptual individuation always occurs in a kind of “coordinate system.” So the idea of seeing of an implication could be couched in terms of comparisons between shapes in the visual field that can be made quite immediately. In this example, the visual comparison made would be between the shape of the person and the shape of the door, and in some sense the information—“*If* that person walks through that door, *then* they’ll hit their head”—is conveyed visually.³⁵ We must be careful here, however, to keep distinct the *perceptual judgments* someone might make, and the perceptual information implicit in geometrical seeming.³⁶

2.3.2 Boolean Connectives

We have seen the situation with respect to some of the standard axioms of modal logic, and we will now move on to the connectives. Again, the Hintikka semantics is standard in the propositional case. Hence, the treatment of the Boolean connectives is the same as in a standard possible-worlds semantics. I repeat briefly here the findings from the literature,

³⁵Widening the kinds of information that we conceive of perceptual contents in this way might even help to explain better how perceptual contents alone can motivate action. For example, one would be immediately motivated to duck out of the way, without recourse to beliefs about head pain and low-hanging ceilings. Indeed, Hintikka’s distinction between perceptual individuation and physical individuation offers a formalization of the early modern distinction between perception and apperception: the information-theoretic account of possible-worlds at various levels of visual processing fleshes out the idea of apperception.

³⁶For, again, this perceptual logic does not deal with perceptual judgments, but rather with the distinctive aspects of perception itself.

again with remarks about the interpretation of the results in the particular context of the logic of perception.

Conjunction and disjunction. With respect to conjunction and disjunction, the question in the semantics literature pertains to the transparency axioms. It is well known that conjunction is transparent with respect to a standard modal operator, but disjunction is not so. Thus, the same can be said for the Hintikka operator. This result for Hintikka is in agreement with the wider literature on the logic of perceptual terms, so in this context the result does not appear to be an artifact of possible-worlds semantics. For example, van Lambalgen’s model-theoretic approach agrees on this point ((van der Does and van Lambalgen, 2000, 17-19)). To use their example, we can infer from “*a* saw Sharon wink and Mary smile” to “*a* saw Sharon wink and *a* saw Mary smile.”

The potential counterexamples that have come up in the literature do not indicate that the inference is structurally pathological. Rather, some instances of the inference are ruled out at the level of the concept of perception. For example, perceptual claims are de facto restricted to those that require the perceiver to have one and only one unique visual field, and so a claim that would require of a subject that he have two fields of vision at the same time could constitute a counterexample to the transparency of conjunction ((van der Does and van Lambalgen, 2000, pg. 17), Barwise (1981)). Unfortunately, Hintikka’s logic gives us no way of making these distinctions, except insofar as his notion of perceptual individuation assumes the notion of a single field of vision with a unique origin point.³⁷

There are more pressing issues with respect to conjunction in perception, however. The literature on the semantics of perception has not fully appreciated the difficulty of understanding conjunction in the ascription of perceptual contents. In particular, *conjunction introduction* in perceptual contexts is problematic. There are several issues here, but the

³⁷See the remarks on physical and perceptual individuation in §2.4 below.

major complication is that we are trying to characterize the visual seemings of a person's mental state at a particular point in time. In connection with this, we must bear in mind that one's visual contents are also affected by one's conscious attention. One's conscious attention has a transtemporal element to it, and the attention may shift at a level that is immensely fine-grained with respect to our perception of time. Furthermore, it may be that both objects are in my visual field, but that I attend only to one with such focus that, in some sense, I do not even "see" the other object at all at the particular time in question. There is then a problem with introducing conjuncts in that we risk conjoining elements that are transtemporal in a perceptual ascription that is meant to capture a single point in time. In short, there is a problem here of how we attribute conjunctive contents to perceivers at all, much less do we know how to interpret transparency of conjunction in perceptual contexts when trying to characterize the way the perceptual state *seems* phenomenologically at a given moment in time.

So much for conjunction. On the other hand, in the case of disjunction, a counter-model to transparency is one which describes the intuitive situation in which someone cannot discern whether the object in his field is F or G , but can discern that it is one or the other. For example, suppose I see an object off in the distance, but cannot discern whether it is a wax mannequin or a real human. My visual information allows me to discern that it is one or the other, but it does not rule out the one or the other. For all I can see, both a human and a wax figure are compatible with everything I'm perceiving.

Negation. There has been some controversy in the literature as to the treatment of negative compliments in perceptual contexts. For in-depth discussion, see Higginbotham (1983). Here it suffices to note that the situation revolves around the treatment of inferences in which the negation is moved in and out of the scope of the perceptual term. The findings suggest that the direction from ' $\mathbb{S}_a\neg\varphi$ ' to ' $\neg\mathbb{S}_a\varphi$ ' is valid, and not vice-versa. The intuitive force of this result is seen when considering, for example, whether one can infer from "Jack saw Sharon

not cry” to “Jack did not see Sharon cry.” This direction seems obviously valid. For, in any model in which someone sees *that* not- F , this requires not- F obtaining in all accessible worlds. Hence, it’s easy to see that the inference is valid. Note, however, that in order to obtain this validity we must have a serial accessibility relation. For, in modal logic “blind worlds,” i.e., worlds with no accessible worlds associated with them, everything is necessary and nothing is possible. Hence, at such a blind world, for any φ , ‘ $\mathbb{S}_a\neg\varphi$ ’ is trivially true, but ‘ $\neg\mathbb{S}_a\varphi$ ’ is never true. If we assume seriality, this situation is blocked, and so the validity is obtained.³⁸

The other direction is patently invalid due to partiality of perceptual identification. For, it may be that Jack did not see Sharon cry, though Sharon did indeed cry— Jack simply saw the back of her head and did not see her face, and so could not visually discern she was crying. Or, perhaps, Jack saw her face and thought that she was sweating, rather than crying. Indeed, on the Hintikka semantics we have the same result, and again this is established by proofs familiar from propositional modal logic.

The results in the foregoing section show that Hintikka’s logic converges with standard modal logic in the propositional case, and also motivates Hintikka’s intentionality-via-intensionality thesis by giving an intuitive gloss on them in terms of perception concepts. Though there isn’t much in the way of big news in this section, it is promising as it is what we would expect from Hintikka’s own presentation, and it shows that his logic is amenable to present day treatments. Given that I have reconstructed Hintikka’s system in a way distinct from his own presentation, this is a good outcome.

We now move on to discussing Hintikka’s quantifiers, which will provide more philosophical interest. In §2.4–§2.4.2, I will distinguish further between the classical quantifiers and Hintikka’s versions and discuss the problems of quantified modal logic. In §2.4.3 I give an

³⁸In §2.2 above I noted that it is natural to apply a seriality condition to the accessibility relation, for reasons independent of the current consideration. Here, we add more evidence that a serial relation is reasonable to assume in this context, since it aligns the formalism with the intuitive situation.

analysis of Hintikka’s revisionary version of the de dicto—de re distinction, and in §2.4.4 I discuss the distinction between physical and perceptual quantifiers in the theory of intentionality.

2.4 Quantifiers and Direct Perception

So much for the propositional fragment of the logic. The distinctive aspect of Hintikka’s modal logic is his treatment of quantification, and so some extensive discussion of the quantifiers is warranted. First of all, his treatment of quantification is a critical component of his pluralist solution to the problem of direct-object constructions in perception. He is able to reduce such direct-object constructions to propositional constructions by means of distinguishing between different sorts of individuation, and further distinguishing between direct-object constructions that are de dicto from those that are de re.

Recall that, in addition to the usual quantifiers, we have defined a special substitutional quantifier $\Sigma_{\mathcal{C}_i}$. These additional quantifiers are geared toward two philosophical objectives. First, he gives formal shape to the idea that it is a subject’s *intentional situation* that determines the freedom we have to move quantifiers in and out of the scope of the perceptual term.³⁹ Second, he equips the logic with the ability to register intentional distinctions in the object language in a way that could not be achieved with the usual quantifiers. This is a unique feature of Hintikka’s system, since traditional versions of modal logic are motivated by metaphysical considerations, rather than by the theory of intentionality. The quantifiers are therefore critical to an understanding of Hintikka’s claim that the possible-worlds theory of intentionality eo ipso is a theory of intentionality. Let us see how his substitutional quantification works.

³⁹Van der Does and van Lambalgen make a similar point in their context (van der Does and van Lambalgen, 2000, 18).

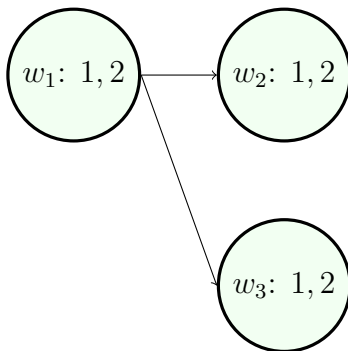
2.4.1 Existential Generalization and Morning Star—Evening Star

Existential Generalization. First, we can draw a distinction between the classical quantifier and the Hintikka quantifier in its treatment of existential generalization. For example, consider the following inference, noting that the quantifier here is the standard existential:

$$(2.16) \mathbb{S}_a F(c).$$

Therefore, $\exists x \mathbb{S}_a F(x)$.

As discussed in §2.1 above, the inference is invalid. However, the traditional counterexamples for the usual interpretation of the quantifier don't work on the Hintikka quantifiers. For simplicity, suppose we have the following possible-worlds structure with only two entities in each world:



In addition, we have an interpretation that flips the name and properties of each entity at the accessible worlds: $I(c, w_2) = 1$ and $I(F, w_2) = \{1\}$; $I(c, w_3) = 2$ and $I(F, w_3) = \{2\}$. The model illustrates the invalidity of the inference with respect to the usual quantifier, as is familiar from first-order modal logic. For, there is no entity such that it is the one that is F for all accessible worlds: in w_2 , 1 is witness, but in w_3 , 2 is witness.

However, switching gears to the perceptual quantifier, consider:

(2.17) $\mathbb{S}_a F(c)$.

Therefore, $\Sigma x \mathbb{S}_a F(x)$.

On the model, there is a substitution instance, namely c , at each accessible world, even though it is not the “same” individual that is designated by c at each world. So the inference holds. This shows that the usual counter-models do not extend to the substitutional quantifier.

The use of additional substitutional quantifiers to provide an account of quantification into the scope of an attitude verb from the outside is distinct from other well known accounts, such as Kaplan’s proposal in terms of *vivid names* (Kaplan (1968)). Roughly, Kaplan holds that quantifying in or out makes sense only when someone has a sufficiently vivid command over the causal history of a person’s name, such that no empirical vicissitudes could prevent the person from willingly bringing the right individual to mind.

Naturally, then, another way to restore the validity of (2.16) is to resort to the usual notion of rigidity. Suppose for all $w_n, w_m \in W$, one has $I(c, w_n) = I(c, w_m)$, i.e., one has standard rigidity. Then, since in standard modal logic variables are always rigid with respect to a given variable assignment, this sort of quantified inference is restored. Hintikka-rigidity, however, does not restore the validity of (2.16) in the way that standard rigidity does. The only way for Hintikka to obtain the validity of existential generalization is to use his special quantifier.

Morning Star—Evening Star. However, in the morning star—evening star case, Hintikka-rigidity ends up doing a lot of work. Consider:

(2.18) $(c = d) \wedge \mathbb{S}_a(F(c))$.

Therefore, $\mathbb{S}_a F(d)$.

Before going any further about rigidity, however, we must address the well known scope ambiguity of the string ‘ $\mathbb{S}_a F(c)$ ’. The first reading gives priority to the contingent designation of the constant, i.e., we first go to the actual world, pick out what c designates there, and say that in all possible worlds that entity is F . The second reading gives priority to the modal operator, i.e., we go to each alternative world, determine what c designates there, and evaluate for each world. The traditional semantics only gives the second reading. To distinguish between these readings and allow the language to handle the first reading, it is standard to introduce lambda terms, i.e., *predicate abstraction* into the formalism (Fitting and Mendelsohn, 1998, 189-200). Indeed, such a move could be made in the Hintikka semantics.

Hintikka’s approach is naturally suited to the second reading, i.e., the reading in which the move to possible worlds is primary, rather than the contingent designation of the constant in the actual world. After all, the purpose of Hintikka’s formalism is to model the perceiver’s visual information; the actual world is relevant only when we wish to quantify-out. It is no trivial question as to whether some constant is perceptually rigid, and a constant’s denotation in the actual world only holds fast in perceptually possible worlds based on the perceiver’s visual information, which may be vastly underdetermined or mistaken. For example, one important kind of constant will be ones that are perceptually individuated on the basis of a privileged position in the visual field, i.e., perceptually-individuating constants, and it is clear that the designation of these constants can change from world to world. So, e.g., the reference of the constant $d =$ ‘the man in front of a now’ will change from world to world, since, in the perceptual case, worlds stand in for different points of view.

Of course, there are cases in which the first reading would be preferred. Canonically, reports that attribute attitudes to others are best served with the first treatment, since it is ostensibly the case that one cannot have the sort of access needed to another’s perceptual information to ground the second reading. For example, supposing one heard on the news that “Michelle

Obama saw the President leaving the White House” the force of the statement is that Michelle, having a past relationship with the man Barack Obama, saw this figure before her *qua Barack Obama*. In this case, we would capture the meaning of the report only in the first reading. Contrast this with the case in which it is reported of a brand new immigrant to the United States, who, having no prior knowledge or beliefs of the man Barack Obama, that he saw the president leaving the White House. In that case, the man ostensibly sees this figure before him *qua the president*, and this could designate different individuals in different worlds.⁴⁰

Now, returning to the issue of rigidity. The Hintikka-rigidity of constants makes the morning star—evening star inference valid. For, insofar as $I(c, w_n) = I(d, w_n)$ for all $w_n \in R_S(a, w_1)$, any property that c has in those worlds, d will have as well. Similarly, with traditional rigidity, we obtain a validity because once a given entity is “baptized” with a certain name, that name doesn’t change through any possible world. Hence, since one and the same entity Venus is baptized with both the names ‘ c ’ and ‘ d ’ at the actual world w_n , we have $I(c, w_m) = I(c, w_n) = I(d, w_n) = I(d, w_m)$ for all w_m , and so c and d will have all the same properties in all the worlds. This result is a good outcome for Hintikka’s purposes, since he seeks for his theory of individuation to mimic the extensional inferences, but it is not without its intuitive problems. For, among many philosophers there is great sympathy for Frege’s point that the inference is not valid.

The problem from this perspective, then, is that of coordinating entities individuated perceptually with those individuated physically. This raises the question of how Hintikka will hook the actual world up to the perceptually-possible ones. Note that for the same reason that we cannot hold the perceptual alternativeness relation R_S to be reflexive (due to problems of non-veridical perception), we cannot use a similarity metric to solve the problem. For,

⁴⁰This example might be better-motivated in the context of belief attitudes than it is in the perceptual case. Still, the point is clear. The semantic meaning or information of a perception varies depending upon the background beliefs and attitudes the perceiver brings to bear on the perception, and so the formal disambiguation of sentences is sometimes a matter that cannot be settled a priori in the semantic theory.

due to the non-reflexivity of R_S , we cannot assume that the similarities between the worlds in someone's alternative worlds at w extend to w itself. The world as it appears to someone need not be similar to the actual world.

2.4.2 Non-veridicality and Causality

Due to this problem of non-veridicality, Hintikka must specify some method for weighing the actual world against the perceptual information, and thus of coordinating constants associated with perception and constants associated with extensional contexts or veridical modalities such as knowledge. This problem is relatively unique to Hintikka's perceptual logic, since the alternativeness relation of the epistemic logic is obviously reflexive. Thus, having a method of coordination of cross-identities in the actual and in the alternative worlds is crucial for the evaluation of such sentences as:

$$(2.19) \quad \Sigma x[(x = d) \wedge \mathbb{S}_a(x = c)]$$

in which a visual recognition in perceptual space is weighed against a descriptive recognition in the actual world. In order to specify the meaning of (2.19) in Hintikka's fashion, the truth-conditions must obviously require that the values of ' x ' here extend to the actual world, but there is the further matter of specifying how the comparison takes place. Another crucial sort of sentence in which this shows up as a problem are reports of "predicate explosion", in which someone sees something as having a property, only to come to see that it was a misperception. This case was invoked in the propositional fragment of the logic in §2.3 above, in the form of Husserl's favored wax mannequin example. Consider:

$$(2.20) \quad \Sigma x[F(x) \wedge \mathbb{S}_a G(x)]$$

as a formalization of someone reporting “this thing appeared to me to be a human, but it turns out it’s a wax mannequin.” This, indeed, is a hard problem for Hintikka, since the very purpose of requiring non-reflexivity for the alternativeness relation is to deal with non-veridical perception, and yet crucial cases of misperception of the “predicate explosion” sort require a strong account of how it is that the actual world is hooked up with the perceptually-possible worlds. In short, we need the objects in the worlds to be the same but we need the truths at the worlds to vary. This situation is easy to formalize, for it is just modal logic that does the job. But it is difficult to pin down philosophically the mechanics of perceptual concepts that assimilate the perceptual concepts to the variations in the modal logic.

Hintikka invokes the notion of causality to do the work, arguing that sentences such as (2.19) come out true just in case there is an appropriate, external causal relation obtaining between the perceiver and the actual object being presented. Cross-identification, thus individuation, by pure visual information, again, is much like fixing the coordinates of the individual in a geometrical field.⁴¹ It is natural to think that causality is intimately tied up with perception, and Hintikka wants to offer a way to understand this relation within the possible-worlds theory.

His theory thus tells us this much: causality is intimately tied up with perception in that causality is the means by which the geometry of the first-person visual field is brought into some sort of semantic consistency with the third-personal structure of space-time. Unfortunately, he has nothing more to say on the matter of determining precisely what the nature of the causal connection is, and thus of interpreting how exactly the comparison is supposed to occur. Hintikka relegates the project of specifying the concrete relation as being outside the scope of his inquiry, connecting the difficulty in specifying this connection with the difficulties of understanding causality in general, and not with any special interaction between

⁴¹This leaves a large role for the an ‘I’ to play in the perception, as such coordinate-fixing makes no sense without an ‘origin point’ or a ‘zero point’, a point made decisively in (Husserl, 1907, pg. 166). Similarly, in Husserl, the “interested” ‘I’ unifies phenomena as they appear in the disinterested flux with the structure of space-time continuity. Husserl’s theory of this is a priori in the notion of constitution.

the notions of perception and causality in particular. There is little he feels licensed to say, a priori, on the matter.

One might naturally argue that imposing success condition on perceptual statements fixes the problem a priori. However, as Hintikka argues, forcing a success condition is a matter of pragmatic conversational discourse, and not at all a matter of a priori necessity. For, in conversational discourse, we generally assume, perhaps in some cases by way of charity, that a speaker's perceptions have a certain level of veracity with respect to the actual world. However, the matter at hand is a matter of phenomenology and mental intentionality, not a matter of what people ordinarily mean when they use perception verbs. In perception theory we require the ability to separate out the matter of individuating a perceptual state's accuracy conditions from the evaluation of whether or not the world satisfies those conditions.⁴² If we assume a success condition, then we lose the idea of looking at the intentional situation of the perceiver independent of veridicality constraints. But this is not the goal of Hintikka's program in its full generality. There is obviously an exceedingly important class of statements, however, which require that we weigh the perceptually-possible worlds against space-time in its actuality, and so this important question remains with only a tenuous gesture at a solution.

2.4.3 De dicto—de re

Within his system Hintikka offers a distinctive reconstruction of the de dicto—de re distinction in the perceptual case (Hintikka, 1975, pg. 66). He argues that natural-language direct-object constructions can be read either de dicto or de re, depending upon the visual information that the perceiver has. For example, '*a* sees *b*' can be parsed in either of two ways. First, *a* might see *b* as someone (or something) with whom *a* is descriptively familiar,

⁴²Van der Does and van Lambalgen make a similar point in their work (van der Does and van Lambalgen, 2000, pg.3; §4).

thus, whom a recognizes on grounds other than pure visual information. For example, a might see Mr. Smith *as* Mr. Smith, the person a has met in the past who was born in Utah and who grew up in Arizona and who writes for the newspaper, and so on and so forth. This is a de dicto sort of visual recognition, and taken in this sense ‘ a sees b ’ entails that b finds a place among a ’s perceptually-individuated objects, i.e., b *as* b has a distinct and unique position in the visual field. This sense is captured with the following formula:

$$(2.21) \quad \Sigma_{C_{perc}} x \mathbb{S}_a(b = x)$$

Hence, we can conclude that, formally speaking, recognition de dicto involves a physically-individuating constant within the scope of the perception operator being equated with a perceptually-bound variable (a ‘this-here’ demonstratively before me is seen *as* a recognized object).

On the other hand, when no physically-individuating constant appears within the scope of the perception operator, as in (2.22) below, we have a de re construction. In such cases, a may be acquainted with b in a purely visual sense, so that ‘ a sees b ’ is to be read in the sense that a sees b *as this-here* i.e., in a de re sense:

$$(2.22) \quad \Sigma_{C_{phys}} x(b = x \wedge \mathbb{S}_a \Sigma_{C_{perc}} y(x = y))$$

Hence, in the logic of perception, de re constructions are associated with direct intentions that fail to recognize *who* or *what* something is in a sense beyond pure visual information. The disambiguation of these two readings of the direct-object construction ‘ a sees b ’ cannot be accomplished a priori; rather, it is based in an evaluation of the given perceiver’s background knowledge, beliefs, opinions, hopes, fears, and so on. Still, it is a priori *that* such information should come into play in the evaluation of perception reports.⁴³

⁴³Hintikka’s reconstruction of the de dicto—de re distinction is not standard. For standard treatments, see Fitting-Mendelsohn (Fitting and Mendelsohn, 1998, pg. 86-87) and McKay (McKay and Nelson, 2014, pg. 80 ff).

Understanding Hintikka's reading of the de dicto—de re distinction is crucial for understanding the manner in which his theory of possible-worlds semantics acts as a theory of intentionality. For, it is the business of intentionality to individuate objects as the same under varying senses for a perceiver. The combination of his special quantifier binding variables that are put into identity statements with the many-sorted constants is supposed to give us a schematic of intentionality's work in this regard.

2.4.4 Physical and Perceptual Quantifiers

We have just seen the situation with respect to some of the classical problems in modal logic as they pertain to objectual perception, and we have looked at Hintikka's solution to the problem of coordinating perceptual space with physical space. What remains is to discuss the distinctions drawn by the use of physical and perceptual quantifiers.

The theory of intentionality distinguishes between direct and indirect intentions, and, specifically, between direct and indirect perception. Direct perception is perception *of* an object, while indirect perception is perception *that* so-and-so. Since indirect perceptions are also intentional, Hintikka argues against the idea that intentionality is essentially directedness, or the feature of purporting to be *of* or *about* something in the world.

Of course, many important perceptual claims *do* regard perceptions that are directed in this way, and these are *direct perceptions*. However, the distinction between a direct intention and an indirect intention does not map cleanly onto the distinction between direct-object and propositional constructions in natural language. Hintikka is able to disambiguate this intentional ambiguity with respect to direct-object constructions with his multiple quantifiers.

The critical component of making this solution work out is in Hintikka's pluralism with

respect to methods of individuation, and his tying of the quantifiers' definitions to these methods. We have already seen one plurality with respect to direct-object constructions, in that they have both a de dicto and a de re way of being constructed. In addition, one can use mixed quantifiers to obtain a new distinction. For example, constructions such as 'a sees Mr. Smith' are to be distinguished from constructions such as 'a sees who b is', and the difference between these can be conceived in terms of the different quantifiers. Consider, for example:

$$(2.23) \quad \Sigma_{\mathcal{C}_{phys}} x \mathbb{S}_a(x = b)$$

$$(2.24) \quad \Sigma_{\mathcal{C}_{perc}} x \mathbb{S}_a(x = c)$$

where b = the man in front of a and c = Mr. Smith (Hintikka, 1969, 173). Notice that here b is drawn from the set of perceptually-individuating senses (i.e., the constant symbol is taken from \mathcal{C}_{perc}), since its reference relies only on fixing the coordinates of the figure in a 's visual field at w . On the other hand, c is a sort of proper name that picks out the entity on the basis of spatial-temporal, or psychological regularities, and hence the constant symbol ' c ' is drawn from the physically-individuating senses \mathcal{C}_{phys} . Recall, in general, that the two different sorts of quantifiers, physical and perceptual, are associated with the mode of presentation by virtue of which an individuating sense individuates an object for a perceiver.

We can thus paraphrase (2.23) and (2.24) in the following way:

(2.25) There is a physically-individuated person such that a sees that the man in front of him is this person.

(2.26) There is perceptually-individuated phenomenon such that a sees that the phenomenon is presenting Mr. Smith to him.

Thus, in (2.23) and its paraphrase (2.25), we have a physically-presented *this* recognized immediately as the perceptually-individuated phenomenon in one's visual field. More generally, then, (2.23) is tantamount to the natural-language report:

(2.27) *a* sees who *b* is

In (2.24) and (2.26), on the other hand, we have a physically-presented object identified with a perceptual presentation of an individual, i.e., the man in front of me now. So (2.24) is taken to be a proper translation of:

(2.28) *a* sees *b*.

The upshot is that the formal distinction between (2.23) and (2.24) maps onto a philosophical distinction between the *aboutness* or *intentionality* of a given report. Hintikka's quantifiers thus allow us to register differences intentionality in the object-language in a way that would be impossible with the usual quantifiers. Hintikka's solution here is not perfect, however, in that it might fail in certain unusual situations. For example, Shuger (1986) argues that Hintikka's theory is sometimes too strong and sometimes too weak in regard to its treatment of direct perception. Specifically, the theory's treatment of Gettier-style perception cases sometimes says there is a direct perception when there is not, making it too weak; and, on the other hand, in double vision cases, Hintikka's theory is too strong in that it says that there is no direct perception when our intuitive response is that there is. A natural response available to Hintikka is that, indeed, these unusual cases have artifactual residue on perception such that its usual workings are thrown asunder, and this should be expected. For example, he might argue that asserting a direct-object perception where there is none is akin to characterizing a visual illusion in which the perceiver takes there to be direct perception of an object when there is in fact none.

2.5 Implications for Intentionality

The purpose of the above reconstruction was to reconsider the results in light of Hintikka's central claims about intentionality, and so I will now turn the discussion to the theory of intentionality implicated by Hintikka's approach to possible-worlds semantics understood as an intentional semantics.

Hintikka's central claim in his intentional semantics is that his possible-worlds theory of intentionality solves the traditional problems of intentionality. The origin of the problem for modern purposes can be traced back to Brentano (1973). Brentano's doctrine of *intentional inexistence* was meant to be a theory about how it is that mental states have the contents they have, i.e., what it is for a mental phenomenon to have a "reference to a content" inherent in it. For many, Brentano's solution of intentional inexistence was only the name of the problem. Hintikka's theory reconstructs and tries to give more shape to Brentano's idea with recourse to possible-worlds structures. In short, then, the theory is that what it is for an object to "intentionally inexist" in a mental act is for the meaning of that act to be intensional in the sense of involving specification of extensions in multiple possible worlds, where the selection of which worlds to take into account is relativized to an agent in a situation (Hintikka, 1975, pg. 218-219).

Put this way, the claim may seem less than illuminating. For, our question is how it is that mental acts connect us to the unique actual world. It may seem perplexing, then, that Hintikka invokes the notion of our intentional states being compatible with *multiple worlds* to settle the matter. Despite the attractive elements of the theory that we saw above, it is natural to reflect on the limits of its achievements in the project of giving a *concrete* explication of the intentionality of perceptual (and other mental) acts.

2.5.1 Possible worlds, intensionality, and intentionality

From today's standpoint, the use of possible worlds in a theory of intentionality most naturally lends itself to *tracking* theories. Tracking theories of intentionality hold that the correct characterization of intentionality is to be given in terms of some sort of tracking relation between subjects and object formations. So possible worlds can be used in characterizing such relations in subjunctive or counterfactual ways. Hence, possible worlds have been central in offering an account of how such intentional tracking is supposed to work. Such theories are now challenged by a new tradition in the literature, rooted in the phenomenological tradition, of offering accounts of intentionality on the basis of phenomenality (Mendelovici and Bourget (2014), Montague (2010)).

When thinking about Hintikka's place in this debate, it is tempting to think of Hintikka as a tracking theorist with his use of possible worlds and his reduction of meaning to multiple reference. However, Hintikka's solution is in a tradition of intentional analysis which predates the strong distinction between intentional properties and phenomenal properties that we find in the recent literature. Hintikka, as his predecessors in Brentano and Husserl, saw phenomenological concepts as being logically dependent on intentional concepts, and vice-versa. There was not a sense, as there is in much of today's literature, that phenomenality and intentionality can come apart in any clean way. Importantly, most current innovative work on intentionality has been questioning this. Hence, it is of even more crucial importance to understand Hintikka's device in the analysis of intentionality, since it challenges the choices today's literature gives us and suggests that perhaps the choice between possible-worlds intentionality and phenomenal intentionality is a false dichotomy.

There is a sense in Hintikka's work that for him the notion of phenomenality is intimately tied up with information-theoretic accounts in terms of possible worlds (usually associated with

tracking theories), so that Hintikka seems to cross the divide.⁴⁴ This point is easily brought out when we see that the notion of phenomenal appearance extends even to Hintikka's theory of possible-worlds quantification. As is manifest from the above, cognitive capacities for discerning shapes in the visual-phenomenal field are a minimal requirement for perceptual individuation, and, by extension, perceptual quantification. It seems clear, then, that, insofar as Hintikka's possible-worlds theory can be understood as giving us a picture of a sort of causal-informational tracking, this tracking mechanism itself is at least partially understood in terms of the phenomenal concept of a visual field populated with colors and shapes whose edges can be detected in virtue of its place in the field. Hintikka, then, does not seem to fall cleanly onto either side of the current debate between tracking theorists and phenomenal intentionality theorists.

Regardless of the issue of retrospectively fitting Hintikka's position in the current debate, there is clearly a dissonance between Hintikka's claim that his possible worlds can do the work of intentionality, and the existence of a live and ongoing debate with respect to the problems of intentionality in the literature. There is some general consensus that possible-worlds semantics provides the correct analysis of modality and gives the best semantics for modal logic and the propositional attitudes. No similar consensus has been reached with respect to the debates on intentionality. In short, the asymmetry between the state of the possible-worlds literature in modal logic, on the one hand, and that with respect to intentionality and possible worlds, on the other, indicates that many take it that possible worlds cannot do the work of intentionality, at least not in the general sense which Hintikka argues for.

One might wonder, indeed, how it is that the invocation of manifolds of possible worlds solves the problem of intentionality—i.e., how is it that the notion of many different possible worlds

⁴⁴Cf. Hintikka's thesis in (Hintikka, 1975, pg. 59-75). The secondary literature on Hintikka's possible-worlds theory has pushed the basic principles of Hintikka's theory in a phenomenological direction. See Smith and McIntyre (1982), in which a Husserlian, and thus phenomenological, account of intentionalist semantics is offered in the possible-worlds style of Hintikka's approach.

is supposed to clarify how it is that my perceptions hook me up to this actual world? Hintikka does not offer as much guidance here as one would hope. But one response potentially available to Hintikka may be to push back against the worry by recourse to a revisionary interpretation of the underdetermination problem in perceptual psychology. The problem is that the same stimulations of a person's visual system are compatible with a variety of physical causes; thus a given visual field does not uniquely determine a physical world. This problem is particularly salient for Hintikka's account, since he argues that the geometrical forms of the phenomena in the visual field are coordinated with physical continuities through the notion of causality. That is to say, speaking illustratively, this object-like formation in my visual field which I am perceiving directly is *of* a physical object just in case there is a proper causal connection obtaining external to my attitude on the world, between the surface of the object and my retinal system. But the underdetermination problem says that different physical causes will be compatible with the same proximal stimulation, and, hence, with the same geometrical visual field.

Burge (2010) argues that this psychological problem of underdetermination is to be solved by setting veridicality conditions for perceptions. Hintikka's a priori theory of intentionality, however, cannot address these veridicality conditions because on his system they are decidedly empirical matters, i.e., matters of whether or not a given causal connection actually obtains. So one might argue that, in fact, Hintikka's theory of intentionality never gets us to the one, uniquely-determined world, and thus cannot tell us how it is that states of intentional perception uniquely determine an individual. This could lead critics to charge that we have no genuine individuation conditions for perceptual states. Rather, on Hintikka's proposal, any perception, veridical or not, is compatible with manifold possible worlds that can be distinct in various ways from the actual world. Hintikka might respond by saying this is precisely the point. Any given intentional state will have manifold possible worlds compatible with it. Hence, it is not the business of intentionality to connect us up to this one world but rather to constitute the actual world in an epistemological sense by virtue of

opening up a space of possibility within which we “move about.”

In this sense, note, the possible worlds cannot be thought of as existing apart from the intentional acts with which their possibility is correlated, since they are intimately tied to someone’s possible further acts with respect to the object. What it is for an object to be a physical object, in this sense, is to have correlated with it potentially infinite compatible manifolds of possible worlds. No matter how many angles from which we perceived the object, still we would never come to any one, self-contained determination of the object as thus-and-so—the object as physically real is, in the terms Hintikka picks up from Husserl, transcendent of any consciousness of it. Hintikka’s conviction seems to be, again, that it is the business of perceptual intentionality to individuate objects in a total way based on incomplete perceptual information, and to do so by way of tying together references in multiple possible worlds.

Reflections such as these might help clarify Hintikka’s claim that his intentional semantics is Kantian (Hintikka, 1969, pg. 108-109). For, Kant held that the conditions of possibility for being intentionally hooked up to a world underdetermine some class of physical truths about that world. In this sense we can understand Hintikka as providing a possible-worlds interpretation of the phenomena–noumena distinction. For Kant, phenomena appear to us in perception, but these phenomenal appearances originate, in some sense, in noumena which never appear but in some way “undergird” the phenomena. All of our knowledge is inscribed within phenomenal experience, so our knowledge cannot determine the noumena. In the possible-worlds semantics, if one understands the noumena as the actual world, it follows that our phenomenal appearances leave open any number of possibilities about how the noumena *really is* independent of our perceptions. Hence, the conditions for possibility of being intentionally hooked up to a world do not determine a unique world transcendent of our perceptions.⁴⁵

⁴⁵Hintikka’s Kantianism may also provide an interpretation of the antinomies. For example, Kant’s second antinomy he limits reason’s ability to decide the matter with respect to whether or not physical objects

To the extent that Hintikka embraces this Kantian reading of his picture, it seems open to him to argue on Kantian grounds that the problem of underdetermination in perception is an ingredient in the recipe of intentionality, not a defect in the theory. Indeed, the point might be this: intentionality hooks us up to a world precisely in that it does not uniquely determine any one world, but rather opens a space to move freely about in a world that is underdetermined with respect to at least some classes of truths. This might lead to a certain action-theoretic understanding of intentionality in the Merleau-Ponty sense: a perception is veridical just in case it invites an inexhaustible manifold of possible future actions that accord with the object in ways that I can anticipate in only a limited sense.⁴⁶

For example, my seeing my coffee cup on the table in front of me can be associated with a manifold of possibilities for future action: I can grab the cup, I can take a sip from it—even when it is at the peripheral of my attention, I must avoid hitting my elbow against it as I type or else it will spill. These routes for action would quickly dissolve if the perception were, for example, an hallucination, and they would be different with respect to a different sort of object, such as a piece of paper. Suppose I was hallucinating the cup on the table and went to grab for it. My hand would pass through the “cup”; hence, the availabilities for possible bodily action that my current perception offers up are quickly cancelled. On the other hand, if there is a cup there, then my routes to action with respect to the cup will persist through any manifold possibility of perceiving or handling the cup. Hence, the line of thought might go, the underdetermination of any given perceptual state leads us toward an account of bodily intentionality. Though this line of thought departs from Hintikka’s own substantially, there is textual evidence that something similar was on his mind when he ties his possible-worlds theory of intentionality to the Husserlian theory of the bodily orientation

decompose into basic constituents or are infinitely divisible. Since pure reason, in Kant’s philosophy, determines only conditions for the possibility of experience (of a world), it follows that we can be intentionally hooked up to a world, while intentionality still fails to determine a unique physical world—after all, both an infinitely and non-infinitely divisible physical reality are compatible with having the experiences we in fact have of this world.

⁴⁶This discussion, loosely based on Merleau-Ponty’s ideas, is indebted primarily to his account of bodily intentionality in the *Phenomenology of Perception* (Merleau-Ponty (2013)).

of the perceptual field (Hintikka, 1975, pg. 218), of which Merleau-Ponty's theory is a sort of amplification.⁴⁷

2.5.2 A life of Sinn: Fregean senses vindicated?

One of Hintikka's primary motivations is to shore up the notion of a Fregean sense. He thinks we can give a respectable interpretation to the notion, or at least that we can use possible-worlds to tease out a notion that plays most of the same explanatory roles in the theory of intentionality. It is natural, then, to ask whether or not he succeeds in providing such an interpretation that satisfies us with respect to the problems that moved the literature away from the idea in the first place.

It's not clear that he does. First, it is difficult to provide an account of what a Fregean sense *is*. Hintikka has resisted the idea of a sense as an entity, i.e., formally speaking, as an object that can act as an argument of a function. However, the explanatory work done by senses, he thinks, can be done without positing such objects. For him, the sense, if it is anything at all, just *is* the function that takes worlds as arguments and outputs extensions. In this sense, as I have gestured at, his idea of senses accords loosely with the Dummettian metaphor of a "route to reference." But has Hintikka done much to explicate exactly the notion of a route to reference, aside from positing it as a semantic function? Modal logic in itself can't give us a concrete explication of what a Fregean sense *is*, even if we did accept that Hintikka's modal logic can help us to see that positing it as an entity is a kind of category mistake with respect to the problem of intentional individuation.

⁴⁷Cf. Husserl's discussion of the lived body from the *Crisis of European Sciences*: "...In a quite unique way the living body is constantly in the perceptual field quite immediately, with a completely unique ontic meaning, precisely the meaning indicated by the word organ (here used in its most primitive sense), [namely, as] that through which I exist in a completely unique way and quite immediately as the ego of affection and actions, [as that] in which I hold sway quite immediately, kinesthetically—articulated into particular organs through which I hold sway, or potentially hold sway, in particular kinesthesia corresponding to them" (Husserl, 1936, pg. 106).

Second, there is a related worry that in invoking possible worlds we are trading in one ineffable idea for another. If we re-interpret Fregean senses in terms of possible worlds, we still must provide an account of possible worlds. Hintikka insists that the idea of considering possible worlds should be no more perplexing than the idea of considering the possible courses of action you will take tomorrow with respect to different possible ways the world might turn out. For example, someone who packs both a set of rain clothes and a set of dry clothes for a camping trip—taking into account the “possible world” in which it rains and also the “possible world” in which it doesn’t—is already effectively thinking about possible worlds. However, this stops quite short of allowing us to hold before us what it is we are doing when we make such considerations. Are we really mentally operating on possible worlds? And, if so, what would a metaphysical account of such look like?

Following this, it is natural to ask whether the constants of Hintikka’s system succeed in doing all the things we would want senses to do. There are many different explanatory roles that they are intended to play. A natural starting point, of course, is that senses are supposed to play a role in developing the truth-conditions of opaque contexts. In the traditional formulation, when we are in extensional contexts, Fregean senses determine the extension of the terms, and the terms are supposed to denote their customary extensions. However, when we switch to an opaque attitude context, the terms within the scope of the attitude term are taken as denoting their customary senses. Hintikka’s constants, of course, do not work in this way, but it is manifest from the above how it is that they deal with opaque contexts. Opaque contexts, for Hintikka, signal the phenomenon of “multiple reference.” The opacity of opaque contexts arises in virtue of the fact that we have to consider multiple possible ways the world might be or have been, over and above the actual. The constants tie together the extensions of a term in different possible worlds in world-lines that track an individual through any possible vicissitude. Hence, the constants help to determine the extension of a term within an opaque context in much the same way as do the terms in normal, non-opaque contexts. That is, they determine the reference. The only difference is

that, in the opaque context, the reference is relativized to each relevant possible world. And this, too, accounts for just what it is for an intentional act to have a content, and thus, for it to *be* intentional. Hence, having an individuating sense associated with it is the mark of an intentional act, but only if we understand senses in an ontologically deflated way.

2.6 Conclusion

In the above I have reconstructed Hintikka’s perceptual logic from the point of view of present-day intensional logic, and I have reconsidered the results in the setting of the problem of intentionality. First, we looked at the philosophical motivations for the path Hintikka takes in his theory of intensionality as theory of intentionality (§2.1). I then proposed a formalization of the semantics based on coupling a standard modal operator with the defining of additional substitutional quantifiers given in terms of special classes of substitution constant symbols \mathcal{C}_i (§2.2). The formal distinction between \mathcal{C}_{phys} and \mathcal{C}_{perc} marks the difference between two different ways that perceivers have of intentionally individuating objects in the domain with respect to a combination of pure visual information and the background intentions that the perceiver brings to bear on the visual information. In §2.3, I motivated the results of standard propositional logic in the setting of perception theory.

The quantifiers that were defined in terms of the \mathcal{C}_i substitution classes were studied in §2.4. There we looked at Hintikka’s distinctive theory of quantification, including his treatment of existential generalization (§2.4.1) and the de re—de dicto distinction (§2.4.3). In §2.4.2 we studied the way quantifiers work when the scope of an existential ranges over a variable occurring simultaneously both inside and outside the scope of a perception verb. Then §2.4.4 looked at how it is that Hintikka’s method of defining quantification in terms of physical and perceptual methods of intentional individuation registers important distinctions in the intentionality or aboutness of direct-object perceptions.

After the investigation of the formal framework, in §2.5 I examined the results in light of intentionality theory and suggested some ways of thinking about Hintikka's theory of intensionality as a theory of intentionality. In the process I reflected on some of the potential limits of Hintikka's theory. In §2.5.1, I suggested possible paths for giving a more concrete analysis of intentionality in terms of phenomenological concepts from Merleau-Ponty's work, and suggested a new way to think about the Kantian dimension of Hintikka's semantics. Finally, in §2.5.2 I discussed the status of the Fregean sense in Hintikka's proposal.

Chapter 3

How to Be an Adverbialist About Phenomenal Intentionality

3.1 Introduction

In philosophy of mind, it is increasingly common to hold that phenomenality is the ground of intentionality.¹ In a definitive part of this research, Kriegel (2011) proposes that intentionality be explicated as an intrinsic feature of experiences in virtue of their exhibiting the right kind of first-person phenomenal concepts, in contrast to first-order tracking theories that explain intentionality in third-personal terms of states that reliably track elements of the environment. To go beyond the old tracking theories, Kriegel presents a fork in the road for accounts of phenomenal intentionality: one must go to a higher-order tracking theory or to an adverbial theory.

In this article I examine the prospects for adverbialism in phenomenal intentionality theory.

¹See, e.g., Horgan (2013); Pitt (2004); Kriegel (2007) Kriegel (2011); Kriegel (2013). Literature surveys include Mendelovici and Bourget (2014) and Montague (2010).

At a first approximation, adverbialism is a view about the nature of conscious experience holding that consciousness is to be characterized in terms of the way in which experiences are modified. Thus, where tracking theorists hold the essence of intentionality to be a in a specific relational property, adverbialists hold the essence of intentionality to be in a non-relational characteristic of being directed somehow. This approach thus obviously resonates with the project of explicating phenomenal intentionality as an intrinsic, first-personal feature of experience.

However, there are forms of adverbialism that do not so resonate. In particular, much rides on the adverbialist's choice of a semantics for adverbs. As it turns out, one of the two main choices of semantics results in a form of adverbialism that is not a genuine alternative to tracking.

My approach in the present chapter is thus to ask, what is the *logic* of adverbial modification in this setting? I diagnose the application of the well-known Montagovian and Davidsonian semantics for adverbs to this problem in the philosophy of mind. I conclude that the most apposite semantics is one based on a logic of *events*. I use these results to elucidate a clear adverbial framework for thinking about phenomenal intentionality.

The semantic argument for the event view proceeds in two steps. First, in §3.2.1, I argue that the competing intensional semantics for adverbs—i.e., understanding conscious acts as involving appropriate kinds of intensions, formalized in the usual way as mappings from worlds or states to extensions—are ultimately not dissimilar from the tracking theory, which adverbialism is supposed to be an alternative to. I refer to Tye's (1984) formulation of intensional adverbialism for perception to show how the intensional form of adverbialism collapses into tracking. This shows that Tye's endorsement of the intensional route should not generalize to phenomenal intentionality.

In the second step of the semantic argument I regiment the event view with a modal type-

lowering principle that fine-grains intentional events according their phenomenal contents compared across possible worlds (§3.2.2). Capturing the fineness of grain of conscious content comes at the cost of potentially running into a form of Russell's paradox. I defend the event view against that objection by showing how to avert the paradox, and I argue that the move is consistent with the consensus view in philosophy of self-knowledge that we are often not privy to the deep inner workings of our own intentional apparatus.

Having argued for the event approach, I discuss how the choice of semantic framework matters for substantive concerns in the theory of intentionality (§3.2.3). In short, event adverbialism is the only regimentation of adverbialism that successfully follows up on a central tenet of adverbial thought—to firmly distinguish the notion of intentionality as *directedness* from intentionality as *reference*. While both senses of intentionality are important for adverbialists, neither tracking nor intensional adverbialism provides a cogent way of even conceiving of the distinction.

The second main topic of this chapter is then an argument as to the nature of conscious intentional events and the downstream implications for phenomenal intentionality (§§3.3-3.4). To ward off worries that the semantic argument is irrelevant to the concerns of phenomenal intentionality, I first argue that the semantic tools I use are structurally analogous to notions that have been independently motivated from within the phenomenological tradition (§3.3.1).

I then sketch out the idealized structure of conscious intentional events. I conceive of them as having the structure of *sense entertainings*: a conscious intentional experience is a structured whole characterized by a an entertainment relation extending from a subject to a sense at a time. Here senses are understood in a quasi-Fregean way. To explicate the notion, I argue that conscious events can be seen as structurally analogous to Jaegwon Kim's property-exemplification account of physical events (§3.3.2), while at the same time withholding judgment as to questions of mind-body dualism. The idea is to investigate the

meaningfulness of conscious states, their being directed in a particular way in virtue of their being phenomenal, and not to take a stance on the mind-body problem.

Finally, in §3.4, I give two results for phenomenal intentionality proffered by the semantic framework. In the prior sections, I've highlighted the importance to adverbial thought of the distinction between directedness and reference. Here I offer a regimentation of that distinction that gives us purchase on phenomenal intentionality. Directedness toward the world—the fundamental aspect of phenomenal intentionality—is understood in terms of *intersubjectivity*, i.e., making available a phenomenally-accessible *public possibility space*. I then ramify the notion of directedness by extending the semantic framework to include a logic of dynamic updating. This semantic structure is interpreted as giving an overriding norm for *successful* intentionality, which is found in the updating structure I call *intentional appositiveness and incongruity*.

3.2 Semantics of Adverbialism

3.2.1 Against Intensional Adverbialism

Formally, let us take *intensional adverbialism* to be the view that intentionality involves experiences being modified by an intension operating on the relevant property. For instance, representing redness involves the property of experience being modified by the intension of red.²

Let us clarify by considering a motivating example. Someone who knows that Melville wrote *Moby-Dick* but who is unaware that he wrote *Billy Budd* represents differently when he represents to himself author-of-*Moby-Dick*-wise than when he represents to himself author-

²This approach is obtained by applying Thomason and Stalnaker's adverb semantics to the setting of intentionality. See Thomason and Stalnaker (1973).

of-*Billy-Budd*-wise. The project in intensional semantics is to account for the failure of an inference like the following (assuming Smith does not know or believe the truth of (2)):

- (1) Smith is thinking about the author of *Moby-Dick*.
- (2) The author of *Moby-Dick* is the author of *Billy Budd*.
- (3) Therefore, Smith is thinking about the author of *Billy Budd*.

Intensional logics block this inference by assigning distinct intensions to the two denoting phrases in (2) and by arguing that it is intensions, and not the extensions, of the denoting phrases which contribute to the thought in (1) and (3). This move blocks free substitution of the two terms with one another, since it is the intension which is at issue in (1) and (3), while it is merely the extensions which are identified in (2).

Intensional adverbialism adapts this argument-structure to the philosophy of consciousness. To show this requires introducing the technical machinery in the underlying intensional type theory, which sets out the types of expressions we have. The first type are *propositions*, defined as functions from worlds to truth-values. We then have a type that can be thought of as *predicates*, more precisely a function which takes entities from the domain and makes a proposition, e.g., a function such that given input x we get ‘ x is large.’ Finally we specify our *adverbial modifiers* in two steps. First, we define a -wise operator, σ , which takes properties and outputs adverbial modifiers. For example, given an intension \hat{P} , we read $\sigma(\hat{P})$ as ‘P-wise’ or ‘P-ly’. Second, adverbial modifiers apply to predicates and yield new predicates, e.g., from Fx we obtain $\sigma(\hat{P})(Fx)$, i.e., ‘ x F’s P-wise.’

In this terminology, our previous example would be formalized as follows:

$$(1') \sigma(\hat{M})(Rx)(S)$$

(2') $M = B$

(3') $\sigma(\hat{B})(Rx)(S)$

In this, we use Rx for 'x is intending', Mx for 'x is an author of *Moby-Dick*', Bx for 'x is an author of *Billy Budd*', and S for intentional subject, as well as the intension operator, $\hat{}$, which given an expression φ returns its intension $\hat{\varphi}$ (Gamut 1991, pgs. 117 ff.; Dowty 1981, pgs. 154 ff). Thus, e.g., $\sigma(\hat{M})(Rx)(S)$ is read, "S intends M-wise", i.e., "someone intends author-of-*Moby-Dick*-wise." Since the adverb-making operator σ inputs the *intension* of the predicate following it, substitution *salva veritate* is outlawed for the predicates in its scope. So since propositions are maps from worlds to truth-values, a map sending person x to 'x represents author-of-*Moby-Dick*-wise will be different from a map that sends x to "x represents author-of *Billy-Budd*-wise."

This example illustrates the main idea behind intensional adverbialism. But it leaves much open about the nature of modification. Nonetheless it takes a clear stand on what it is about the world that is doing the modification: its the intension of properties like redness that are doing the modification.

To further see what the view portends, let us see how it might respond to the longstanding Jackson conjunction problem, also known as the many-properties problem (Jackson, 1977, pg. 64 ff). The idea is just that in some cases order of adverbs matters, and in some cases the adverbs must be paired together in certain ways with specific adverbs. For example, suppose I visually represent a white pelican and a black crow. The adverbial machinery will simply logically conjoin these adverbs like so: "I see whitely and pelicanly and blackly and crowly." But nothing about simple conjunction specifically pairs any two of these together, so that the theory cannot discriminate between my seeing a white pelican and a black crow and my seeing a black crow and a white pelican (or, for that matter, between my seeing a black white and a crow pelican).

We can follow Tye’s lead by adding a coincidence operator, Coin (Tye, 1984, pg. 218 ff.). This maps two input adverbs to a conjoined output adverb. For example, $\sigma(\hat{W})$ (i.e., whitely) and $\sigma(\hat{P})$ (i.e., pelicanly) and maps them to a new intensional function, $\text{Coin}(\sigma(\hat{W}))(\sigma(\hat{P}))$. This new predicate maps the property of intending onto the property of intending whitely-coincidental-with-pelicanly. So a solution to Jackson’s problem for our example looks like so:

$$(4) \text{Coin}(\sigma(\hat{W}))(\sigma(\hat{P}))(\text{Rx})(\text{me}) \wedge \text{Coin}(\sigma(\hat{B}))(\sigma(\hat{C}))(\text{Rx})(\text{me})$$

Such formalization clearly differentiates between the various possible pairings of adverbs.³

But advocates of this solution must answer the question: what is the nature of the coincidence function? Tye conceptualizes his function in relational terms (Tye 1984, pgs. 217-218). For example, the function $\text{Coin}(\sigma(\hat{W}))(\sigma(\hat{P}))$ stands in for that state which would typically be tokened in normal perceptual circumstances when a subject is viewing a real physical object having the properties of being white and being a pelican. It seems that this function, then, stands in for a kind of relation between the subject and a physical object.

The move that Tye makes here is standard in the Montagovian tradition. This tradition blocks inferences by operating on intensions rather than extensions, but compensates by adding meaning postulates. Hence, this tradition blocks the inference from *is a robotic pelican* to “*is a pelican*” by insisting that “*robotic*” modifies the intension of “*pelican*.” But then it compensates by adding a meaning postulate for the lexical entry for colors to ensure that “*is a white pelican*” entails “*is a pelican*” (cf. (Dowty et al., 1981, pg. 234)).

The general problem is that all this leaves the theory unavailable to play its prescribed

³Note that this method pairs adverbs in such a way that the resulting predicate has an internal structure. It is thus distinct from the approach of conjoining adverbs as simple, unstructured entities like ‘white-pelicanly.’ The detriment of the latter is that one loses compositionality and thereby loses crucial inferences (Cf. Kriegel (2007, pg. 316)), while the coincidence operation maintains compositionality as long as it is functional.

role as an alternative to tracking theories, and it thus cannot be a theory of phenomenal intentionality. If an indirect characterization like Tye's is to carry explanatory force, it can only offer the exact same explanation that tracking theories offer. For example, Fodor's tracking theory determines that I am conscious of a white pelican in my environment because I occupy an internal state that is reliably caused only by standing in a unique perceptual relation to a physical object that is white and is a pelican ((Fodor, 1987, pg. 126); cf. (Kriegel, 2011, pg. 72)). The meaning postulates that the intensional adverbialist will have to assume will be nothing more than the trace of this tracking theory: when one asks why this but not that meaning postulate is assumed, we will inevitably fall back into talk that some but not all parts of language track this or that part of the world.

In this way, the approach rises and falls with the very theories it is supposed to be an alternative to—it is a mere notational variant on the same intuitive picture. These considerations throw into question whether this framework constitutes an adverbial theory at all. But more importantly for the present concerns, the argument shows decisively that intensional adverbialism is not a viable approach to phenomenal intentionality, since it conceives of conscious intentionality in relational terms, rather than as an intrinsic feature of phenomenal experiences.

One might object that the coincidence function can be defined in more neutral terms that do in fact resonate with the adverbialist conception of phenomenal intentionality. For example, it might be said that the coincidence function at issue stands in for the experience that would typically obtain just when the experience is modified in a white-coincidental-with-pelican way. But that move makes the account plainly uninformative—for, what was meant to be explained was what it is for an experience to be so modified.

The problem is that this leaves the theory unavailable to play its prescribed role as an alternative to tracking theories. For, it ends up looking like a first-order tracking theory. First-order tracking theories are characterized in terms of relations defined with usually nat-

uralistic counterfactual tracking conditions. For example, as previously mentioned, Fodor's canonical proposal accounts for intentionality in terms of a pattern of causality that defines a representational relationship between mental states of F and actual instances of F (1987, pg. 126).⁴ There are other versions, all united by the specification of a unique relation as accounting for intentionality.⁵ It seems, then, that since intensional adverbialism appeals to a specific relation at the core of their account, intensional adverbialism rises and falls with the very theories it is supposed to be an alternative to.

3.2.2 Defending Event Adverbialism

The competing conception, which I seek to defend, views the nature of intentionality in terms of modifications of conscious *events*. This conception results from the application of Davidsonian event semantics for adverbs to the adverbial account of intentionality.⁶ However, my philosophical application of the event semantics is independent of Davidson's philosophical project, and it is worth pointing out here that the resulting conception of events turns out to be radically different than Davidson's.⁷ But before going further into the nature of intentions as conscious events, let us work out the semantics of the theory in some detail.

The contrast to the previous approach is brought into sharp relief by consideration of the underlying semantic structure. Here inferences are obtained by quantifying over events. Now, events involve change. In this context, then, typical intentionalities will eventually be understood as conscious, intentional events that involve a change of conscious awareness for someone. This involves two major devices: first, positing a map from second-order

⁴Cf. (Kriegel, 2011, pg. 72).

⁵For example, Dretske (1981), Dretske (1988) produces a version focusing on causal relations, while Millikan (1989) pushes in the direction of a biological teleological relation.

⁶See Davidson (1967) and Davidson (1985). Parsons (1990) has given canonical developments as well.

⁷Whatever the status of events in the philosophical literature today, it is worth noting that event semantics remains an active and ongoing project in formal semantics (Lasersohn (2006) and de Groote and Winter (2015) are representative examples). So novel applications of the semantics should not be seen as being inherently related to Davidson's concerns.

entities F to first-order concrete F-wise events e_F , and, second, adopting an assumption of an underlying mereology of events.

The map from second- to first-order entities is necessitated by the fact that events are specific, non-repeatable instances, while the second-order things that modify the events are repeatable and cover many different things. For example, we want to go from, say, green, which covers many different things, to the concrete, non-repeatable event of Peter’s seeing the green of this very tree in front of him now.

To illustrate the latter point about the mereology of events, let’s see how this helps solve the Jackson problem. In particular, we obtain the following disambiguation of the Jackson intention:⁸

$$(5) \exists e[\exists e' \leq e (\text{Subj}(e', I) \wedge (e'_W \wedge e'_P) \wedge \exists e'' \leq e (\text{Subj}(e'', I) \wedge (e''_B \wedge e''_C) \wedge (e' \neq e'')))]$$

This should be read as translating from the natural language report, “I visually represent a white pelican and a black crow” to the event-semantic structure: “There is an event, e , such that there are at least two distinct parts of the event, e' and e'' , such that I am the subject of both parts, and e is a visually conscious event white-wise and pelican-wise; while e'' is a visually conscious event black-wise and crow-wise.” The event mereology thus adds enough structure to solve Jackson’s many-properties problem, since it can be understood to articulate a part-whole structure on the subject’s visual field.

The event mereology has the additional benefit of relating the white-pelican-wise representation and the black-crow-wise representation as component parts of the very same intentional episode. This differentiates phenomenological reports where subjects take their experience to be one intentional episode with component parts, as opposed to a sequence of multiple distinct intentional episodes.

⁸Cf. (Schein, 1994, pg. 99 ff).

The primary challenge engendered by the conception of singular events being modified by generalities is to specify a method for transforming second-order concepts into first-order concrete events. A non-negotiable desiderata is that the mapping occur at a fine grain, since conscious contents are almost unanimously held to be at least as fine-grained as concepts, if not more so.⁹

This consideration makes it immediately natural to move to an ambient modal logic that can fine-grain phenomenal concepts according to their modal profiles. For, we can perceive, conceive, wish for, etc., merely possible states of affairs or objects. To use the famous Quinean example, in our world, the class of animals with hearts is coextensive with the class of animals with kidneys, while we clearly could conceive or hope for it to be otherwise. It is natural to interpret this in possible-worlds semantics: we can entertain the possibility of these two coming apart in some possible situation.

The corresponding modal principle would then be: two representation events e_F and e_G are equivalent with respect to F and G , just in case it is impossible to experience F as extensionally distinct from G , in whatever intentional modality is relevant. When displayed formally it is easy to see that this criterion can be formalized as the universal closure of the following:¹⁰

$$(6) \quad e_F = e_G \iff (\Box \forall x Fx \iff Gx)$$

Here, ‘ $\Box p$ ’ stands for ‘it is impossible to conceive not p ’. This principle is a natural regimentation of an adverbial theory of intentionality since it gives a map of concepts to representation events that is neither too strong nor too weak. As I said above, it predicts the distinction between thinking about renates and cordates, as well as any relevantly similar case.

⁹For example, Cf. (Peacocke, 1992, pg. 3).

¹⁰This principle is a modal variation on Basic Law V from Frege’s *Grundgesetze*; Cf. (Demopoulos and Clark, 2005, pg. 131).

Additionally, the modality introduced by principle (6) nicely regiments an intuitive aspect of conscious mental life. Thinking about mere possibilities is itself a richly structured activity, which principle (6) predicts. For example, principle (6) predicts a fundamental distinction in phenomenality between thinking about contingently non-existent entities like the Loch Ness Monster, on the one hand, and thinking about necessarily non-existent objects like four-sided triangles on the other. Nothing could be more intuitive to our normal, everyday experiences: what it is like to try and think of an impossible geometrical object is different than what it is like to try and think about a mythical creature that might have existed. Hence this theory clearly meets Kriegel’s necessary condition on adequacy with respect to intentional inexistence in its treatment of the intentionality of the non-actual (Kriegel, 2011, pp. 138ff).

But the major problem with (6) is that it may lead to a form of Russell’s paradox, depending on what we assume about the underlying modal logic. Specifically, if one assumes the T-axiom of modal logic, then principle (6) quickly leads to inconsistency. The T-axiom, recall, says that “if $\Box p$ then p .” That is, assuming that events associated to representation are sufficiently fine-grained to license (6), and assuming that the ambient modal logic validates a very minimal principle of modal logic, we quickly can derive an inconsistency.

What happens if we fine-tune the modal logic to permit failure of the T-axiom? Formally, it can be shown that permitting the failure of the T-axiom results in (6) being consistent. The details of this proof are set out in Appendix 2. Moreover, intuitively, permitting the failure of the T-axiom seems well-motivated. For, in most systems of doxastic logic (i.e., logic dealing with belief and conceivability) the axiom is permitted to fail (cf. Holliday 2013, 4). This is because belief is fallible; our beliefs need not reflect actuality, and the same goes for perception and most other intentional modes such as wishing.

However, the solution comes at a price: this forces us to deny the move from $(\Box\forall x(Fx \text{ iff } Gx))$ to $(\forall x(Fx \text{ iff } Gx))$. In effect, we are denying an instance of the idea that “if it is impossible

to conceive of F and G coming apart, then F and G are coextensive.” And one might think that one ought to be cautious about blithely rejecting the move from “it is impossible to conceive not p ” to “ p ”, since the question of whether and to what extent conceivability is a guide to possibility is a complicated topic in itself (cf. Gendler and Hawthorne (2002)).

But as the consistency proof in Appendix 2 indicates, the concepts that engender the paradox are complex in that they refer to the act of going from a concept to the associated mental event. It is then not implausible to suppose that we lack such knowledge of the intricate workings of our experiential apparatus. In the philosophy of self-knowledge, such views are now widely held (e.g., (Evans, 1982, pgs. 224-235); Moran (2001); (Burge, 2013, chp. 9)). Hence, since the modality is tied to our ability to conceive, it seems in keeping with recent work on self-knowledge to suggest that the T-axiom fails in certain rare contexts where the propositions whose modal profile we are attempting to assay concern the very workings of our representational apparatus.

In short, I have argued that once we have arrived at Kriegel’s fork, event adverbialism is the only genuine alternative to tracking. For, intensional adverbialism converges with tracking in its mechanisms, making it scarcely an alternative to tracking at all.

3.2.3 vive la différence

The differences between the two forms of adverbialism, and those between event adverbialism and tracking theories, run far deeper than a mere dispute about semantics and formalization. This choice between event and intensional semantics bears on deeper issues about intentionality. First, it is untenable to understand the event and intensional approaches as merely interchangeable methods of formalizing the same thing. Second, event adverbialism takes a stance on issues in the theory of intentionality that substantively distinguish it from tracking theories.

Regarding the first point, let us draw our attention to the fact that research in formal semantics has shown that the two programs present genuine alternatives to one another, in that their combination causes intuitively false interpretations. The incompatibility arises from the way event quantification combines with intensional requirements of compositionality. For example, under a combination of event and intensional semantics, a sentence like, ‘John went to every seminar on campus’, introduces the requirement that there be a single event involving John and all the seminars (de Groote and Winter, 2015, pg. 54). But that is a strange semantic requirement: one should be able to understand that as a series of distinct events, each one involving John and a single seminar.

It is, furthermore, widely held that the two approaches are motivated by distinct paradigm cases. Thus, when one surveys the solutions to these issues, it is evident that there will be no resulting philosophical equivalence established between the two programs. For, it is almost unanimously held that the core ideas behind event semantics are at odds with the treatment of the class of modal adverbs (de Groote and Winter, 2015, pg. 54). Traditional modal adverbs (such as ‘necessarily’ and ‘probably’) seem to be sentence modifiers and not event modifiers. This places the treatment of these cases in the jurisdiction of intensional semantics, for which modal adverbs are paradigm cases.

Setting the technical matters aside, event adverbialism allows us to introduce fruitful philosophical distinctions. A moral we can draw from the above is that, in order to be a genuine adverbialism, the theory must force a distinction between *intentionality* and *referentiality*. Tracking theories—as well as intensional adverbial theories, if the above is right—hold that a phenomenal experience of seeing red is intentional by virtue of there existing a specific relation of reference between a mental state and a red element of the environment. Proponents of such theories have usually specified a particular kind of content that does the work in establishing a referential relation. Tracking theories work by defining a relation that specifies why *F*-type mental states can be understood as indicating the presence of *F*s in

the world—because the *F*s are the thing causing them, ultimately. Again, in Fodor’s theory, what it is for an *F*-type mental state to be intentional is for it to be the kind of thing that reliably picks out instances of *F* by having a content tokened by a particular pattern of causality (Fodor, 1987, pg. 126). The core of the account of intentionality involves a mental state being related to an aspect of the world in virtue of the content that the mental state has.

Adverbialism, by contrast, traditionally recognizes acts and their modifications in a non-relational way, independent of any particular theory of reference or content. Adverbialism, thus, represents a move to an act-based understanding of consciousness, thus distinguishing between intentionality—characterized by having the right sort of acts with the right sorts of intrinsic features—and referentiality. Event adverbialism gives shape to this idea. Someone is conscious if they are undergoing a changing sequence of mental events, such that they can discern differences in one mental event to the next simply by virtue of undergoing the experience. In other words, intentionality minimally requires adverbial modification; but an adverbially modified sequence of mental events does not necessarily refer. In cases in which modification is hallucinatory, say, there is no relation of reference, but still an intentional directedness, since even someone who is severely hallucinating can discern differences in a sequence of mental events.

But intentionality is a complex phenomenon. It is not that reference is unrelated to a complete account of consciousness. I don’t claim that intentionality is *simply* adverbial directedness. In fact, philosophers who have conflated intentionality and reference are picking up on a crucial aspect of consciousness: namely, the *intersubjective normative elements of conscious experience*.¹¹ If reference is understood in this way—not as the essence of phenomenal consciousness, but as a normative constraint, perhaps imposed at a developmental stage as a child builds up its psychology in memories, anticipations, habits, preferences, and so

¹¹See §3.4, below, for discussion.

on—then yet more fruitful distinctions wait in the wings.

In particular, event adverbialism is in a unique position to further differentiate between reference and the weaker notion of successful directedness. Suppose I see a stick bent in water: in that case, even though the stick is not actually bent, I can still be successful, for example, by correcting my perception in a sequence of mental events. It would be natural to pull the stick out of the water and optimize my perception. This sequence of events, even if not referential, constitutes a successful directedness.

When these distinctions are countenanced, we find that reference, far from being a minimal element essential to conscious experience simpliciter, introduces the strongest possible normative requirement for intentionality: that I be directed, that my direction be successful, and that my direction ends up establishing a veridical, referential relationship of, say, correspondence, indication, or intersubjective agreement. On the other hand, successful directedness only requires adverbial modification plus some weaker, but still well-definable, norms of success. In §3.4.2, below, I extend the event semantics given so far to an account of how to conceive of these norms of success.

The long and short of it is that event adverbialism denies that intentionality is fundamentally a relational property. In this way, event adverbialism—just by virtue of adopting event semantics—automatically takes a stance on substantive issues in intentionality that distinguishes it firmly from intensional and tracking approaches. The latter theories do not draw these distinctions, and rather define consciousness by way of what are actually, in my view, certain norms of conscious experience. Adverbialists may hold that a successful intentionality can be referential in the best case, and that successful intentionalities thereby establish or ground an intentional relation (although they need not, for example, if the adverbialist seeks to minimize the ontology to acts and modifications). But in itself intentionality is not a relation, but an activity (a “verb”). The modifications of a mental event govern the directedness of a an experience, insofar as any conscious subject can discern that she is undergoing

an experience by being in a position to phenomenally differentiate one mental event from the next simply in virtue of undergoing the experience.

Of course, the choice of semantics leaves much unsettled about the nature of mental modification and intentionality. However, the view provides a clear framework that can regiment distinctions that are important for further understanding of the mental. The theory, insofar as it is a sort of adverbialism, is also subject to the consensus criticism that adverbialism in general is less amenable to naturalism than are tracking or higher-order tracking theories. But event adverbialism in particular bears no special burden in shouldering this objection. Kriegel thinks this tips the scale toward higher-order tracking, but I disagree. As Kriegel himself notes, naturalization is not necessarily an end-goal of theorizing, but rather one way of familiarizing a phenomenon and making it understandable (cf. (Kriegel, 2011, pgs. 166-171). Naturalization is a kind of third-person route to this end, but phenomenology is a kind of first-person route to the same. Providing a semantics that accords with the first-person experience of phenomenality seems to me to be a different, but equally valuable, project in understanding consciousness. In this sense, it seems to me that a theory that can countenance more distinctions is highly valuable.

By way of summary, I have so far argued that the way to be an adverbialist about phenomenal intentionality is to take on an event semantics for adverbs. For, tracking theories are incompatible with the basic tenets of phenomenal intentionality, and the competing intensional semantics results in a theory that is a mere notational variant on tracking theories. Furthermore, just by virtue of adopting event semantics, the adverbialist takes a stance on the distinction between reference and intentionality that resonates with the picture of phenomenal intentionality in a way that tracking theories and intensional adverbialism do not. Moreover, event adverbialism, in drawing a richer set of distinctions, is in a better position to characterize phenomenal experience.

3.3 The Nature of Phenomenally Conscious Intentional Events

I have shown that event adverbialism best fits the letter and spirit of adverbial tenets, and that it presents a different picture of intentionality than its primary competitors in the tracking approach. Insofar as we have pit tracking and adverbialism against one another, it follows that event adverbialism is the strongest alternative to the tracking approach. The question now is to look at event adverbialism in its own right, to fill out the event framework so that it gives us substantive purchase on the theory of phenomenal intentionality.

In thinking about this, one might wonder whether an answer to that question is forthcoming based on what has been said. After all, it may be cogently doubted whether the semantic tools I use are relevant to phenomenology and philosophy of mind at all. In this connection, I argue that my critical semantic idea (that of type-lowering event structures) has an exactly analogous idea independently motivated from within the phenomenological tradition (§3.3.1). After that, I appeal to Kim's property-exemplification account of physical events to sketch out the ideal structure of conscious intentional events as sense-entertainings (§3.3.2).

3.3.1 Type-Lowering in Phenomenology

The first step in showing that the logical properties I have highlighted are relevant to the theme of consciousness is to show the relevance of type-lowering principles in intentionality. After all, it would be entirely natural to ask what logical types have to do with intentionality. Above I have shown how the particular modal type-lowering principle I've proposed fits naturally in with traditional concerns about intentional inexistence. I will now further develop the connection in showing that type-lowering operations in general are well-motivated in the theory of intentionality, and in particular have been found in the phenomenological tradition

since Husserl at least.

I want to say, in short, that a key part of way the object is intended in an Husserlian approach is given by a semantic measurement on an experience that minimally generates in the theory a type of experience for that semantic content (e.g., an *event of consciously experiencing F-wise*). I will argue further that this measurement is something exactly structurally analogous to a type-lowering operation like the one I propose.

Before going into full detail, let us summarize the basic argument. Looking ahead, I want to say that a conscious intentional experience consists in a fusion of at least one second-order entity (usually many more) into a special kind of first-order event. To understand how this is explanatory at all, let us contrast this notion of ‘event’ with everyday events, like the event of a fire producing smoke. My point can be seen as an exceedingly natural point to make when we consider that ordinary events can be conceived of as fusions of second-order entities. Kim, in particular, conceives of events as *property exemplifications* (Kim (1976)). For example, the event of fire bringing about smoke is a fusion of the properties of fire and smoke. I will take up the phenomenological tradition in saying something analogous about experience events: in a structurally analogous way, first-person conscious experiences are not property exemplifications but *sense entertainings*, where senses are understood as fundamentally phenomenal ways of modifying experience. I will begin by showing that something structurally analysis is at work in large areas of the phenomenological tradition, and is thus well-motivated in this setting.

In his early *Logical Investigations*, Husserl delineated the most basic pattern of intentionality by making a distinction between the act, content, and object involved in a conscious experience (Husserl, 1970, Investigation V). The *act* of a conscious experience is described as a process or, indeed, a concrete event of a sort, one that takes place within a stream of consciousness, in a determinate spatio-temporal location, and in a particular way. The *content* of consciousness is the total way the object is given in the event of experience—including

the qualitative “semantic value” of the experience, which gives the object in a certain way as having these or those properties. The *object*, on the other hand, is different in kind from the content. It is the thing that the content of the experience ends up being about; the thing given as having the properties ascribed by the content, in the particular act. The content of my perception is “*included*” in my consciousness, while the object is somehow “*outside*” of my consciousness.¹² Indeed, suppose I see a tree in front of me and I begin walking around it. As I walk around the tree, seeing it from this angle and then from that, I will have different contents presenting the same tree. In this way, the content ‘tree’ is distinct from the physical tree.

Husserl characterizes the ‘act’ part of experiences with a technical term of his own invention, called ‘noesis.’ I will not use that term, but I will argue that Husserl’s conception of noesis involves a type-lowering idea:

Every intentional experience is noetic, thanks precisely to its inherent noetic aspects. It is its essence to contain in itself something like a “sense” (*Ideas I* §88¹³)

How does this characterization imply a type-lowering event? There is a longstanding precedence for interpreting Husserl’s use of the word ‘sense’ (*Sinn*) in quasi-Fregean terms, and I will take up this interpretation here.¹⁴ It is *Fregean* in that a sense has certain *logical* properties familiar to Fregean senses, and it is the sort of content that embodies a mode of presentation and helps determine reference (if there is one). It is only *quasi-Fregean* in that it

¹²Indeed, Husserl would describe the content of the experience as “immanent” to the experience. The object is “transcendent” of the experience, in that no experience can include every aspect of the object—there will always be further points of view I could take on the object, for example. The immanent content is not a thing in space and time, the way the tree is. As Husserl says, the physical thing tree is made of wood, could burn up, etc., while the content ‘tree’ is not made of wood, could not burn up, and is included immanently in my consciousness.

¹³Husserl (1907)

¹⁴Cf. Follesdal (1969) and Smith and McIntyre (1982) for representative examples.

involves a generalization of the notion of sense from linguistic meaning to cover non-linguistic intentional acts as well, such as perception.

What is crucial to point out when understanding the role of senses in intentionality is that the *same* sense can be present in different act-types:

Each perception, for example, has its noema [content], at the lowest level, its perceptual sense, i.e., the *perceived as perceived*. Similarly, the respective remembering has its *remembered as remembered* precisely as what is meant by it, what it is conscious of, exactly as it is “meant” in it, as that in it, of which it is “conscious.” Judging, in turn, has the *judged as judged*, the enjoying what is enjoyed as enjoyed, and so forth. In every case, the noematic correlate—that here means “sense” (*Sinn*) (in a very expanded meaning)—is to be taken *exactly in the way* that it lies “immanently” in the experience of the perception, of the judgement, the enjoying, and so forth, i.e., as it is presented to us by the experience, if *we inquire purely into this experience itself* (*Ideas I*, §88).

Note that Husserl points explicitly to the aforementioned generalization of the notion of “sense” from specifically linguistic to intentional meaning generally. More importantly, Husserl is pointing out that act-types have a modal character—the same sense can be present as the content in different act-types. I currently *perceive* under the sense ‘the tree in the yard across the way’; later on I *remember* under the sense ‘the tree in the yard across the way,’ now given to me under the mode of memory. The same sense appears as modifications in different modes of intentional experience.¹⁵

What is crucial to this characterization of an intentional sense is that it is *shareable*, not

¹⁵It is important to flag that the content of an act for Husserl *includes* the act-type. So the content of a perception with sense *F* is distinct from the content of a memory with sense *F*, precisely in that one presents *F*-as-perceived and the other presents *F*-as-remembered. But there is a sense “core” that remains invariant here, namely, the *F*.

only in different act-types but in different times, and between different people. Husserl will say that it has an “ideal” or “irreal” character, meaning it is not a singular thing located in time and space. Thus, it is natural to interpret it as a kind of second- or higher-order entity. For, again I can look at a tree from a particular angle. I can then invite my friend over so that she sees the tree at just the same angle. I can also return to the tree the next day and recreate my perspective. Thus, to the extent that the same sense can be shared between two different people (to an acceptable degree), or repeated at two different times, senses will be second-order entities in the theory. Hence, the shareability and repeatability of senses, in conjunction with the modal character of act-types allows us to interpret Husserl’s senses as making room for second-order entities in the theory of intentionality.

But more than that can be gleaned as well—Husserl’s idiosyncratic use of the technical term ‘noema’ in the above quotation is precisely meant to set the constituents of conscious contents apart from the other sorts of second-order entities like concepts or properties. Hence, the type-lowering operation in Husserlian phenomenology involves *senses*.

To further clarify the idea, let us sharpen the distinction between senses and other second-order entities. Husserl marshals a distinction between three different sorts of second- or higher-order entities: state of affair, property, and sense. The *state of affairs* of my perceiving the red apple—or the fact of the red apple’s being on the table—is distinct from the *property* of redness that is instantiated in the apple; and these are further distinct from the *sense* (or *meaning*) of my experience (the way the red visually appears to me from this perspective here, now). Consider, for example, that the red property of the apple holds static even under different lighting conditions. So properties are distinct from senses in that properties are illumination independent. However, senses are dependent on illumination and subject to the phenomena of inconstant illumination.

The distinction between property and sense can be cogently couched in terms of what these entities *do*. In short, objects *instantiate* properties, physical events *exemplify* properties,

while experiences *entertain* senses, or meanings.¹⁶ Only senses, and not properties, *mean* something about the world; properties are things that objects and events *have*; meanings are something conscious experiences of properties *entertain*.

On this heuristic, conscious intentional events are seen as the sort of event that entertains a sense by drawing that sense into the experience's phenomenology; the experience then *phenomenally entertains* that sense insofar as that sense's appearing determines the act's content, which in turn determines norms of success and reference (if there is one). One can thus think of phenomenal sense entertainment as the way in which an experience is adverbially modified by a type-lowering operation.

Two caveats. First, it doesn't follow that the type-lowering operation is the *only* way in which an act is modified; it appears here as a necessary but not sufficient condition for phenomenal intentional modification.

Second, it is important for what follows to point out that this characterization of senses conceives of senses as inherently phenomenal. In a way, it is precisely their phenomenality that sets them apart finally from other second-order entities like properties. The sense of the green of the tree is quite distinct from the green property of the tree, precisely in that there is a 'greenness-for-me' element to the sense which is not present in the property, which is greenness full-stop. The 'x-for-me' structure is present in any properly consciously intentional experience; and so there is uniquely something it is like for any given sense to be entertained in an experience. In short, the sense 'green' can be heuristically understood as the phenomenal presentation of the property green for someone in some circumstance.

What has been said in this section should suffice to show that the semantic notion of a type-lowering principle is no alien concept in the domain of phenomenology and philosophy of mind.

¹⁶Following the analogy proposed by David Woodruff Smith's exposition of Husserl (Smith, 2013, pg. 415).

3.3.2 Conscious Events as Sense Entertainings

Let us now say more about how to think about phenomenal sense entertainment.

First, in conceiving of sense-entertainment events, it is clear that I am departing greatly from Davidson's traditional program. Davidson thinks of events in terms of causation between state of affair and another distinct state of affairs. On such a view, my conscious representation events may even seem to be nothing like events at all.

But Kim has offered the alternative *property-exemplification account* of physical events (Kim (1976)). This serves as a more appropriate guide for developing the account of conscious intentional events. For Kim, an event is a change in a substance. So this leads to the idea of events as wholes with a structure of parts, consisting in a *substance* that has a *property* at a particular *time*. A physical event is then the concrete exemplification of that property by that substance at that time. Kim then applies an existence and identity condition for the individuation of events. An event with the structure of having x substance with P property at a time t exists just when that substance has that property at that time. And two events are identical when they have the same substance, property, and time.

There is a clear analogue of Kim's model available to the event adverbialist. The *sense-entertainment account* of conscious events, then, conceives of a conscious act as a change in consciousness. This leads to the idea of conscious events as wholes with a structure of parts consisting in a *consciousness* that has a *sense* at a particular *time*. The resulting conscious intentional event is then the concrete entertaining of that sense by that consciousness at that time.

We can also apply analogues of the existence and identity conditions for conscious events. A conscious event exists when there is a subject entertaining sense F , at a time t . In my account, we replace Kim's extensional identity condition with a modal identity condition to

account for the intensionality of intentionality.¹⁷ This is the modal type-lowering principle discussed earlier. An event is, then, identical (with respect to its sense) when the modal variations of the sense are aligned.

The sense-entertainment account of conscious events thus holds that intentional events are structurally analogous to property exemplifications, except that (i) the substrate of the event is a consciousness, and (ii) we appeal to a distinction between properties and senses to say that conscious acts harbor a sense, rather than a property. The word ‘entertaining’ is thus used to evoke the distinction between directedness and reference. One can *entertain* a sense in one’s experience even if that sense does not successfully refer. By contrast, a physical event cannot exemplify a property it does not have. Furthermore, the word ‘entertaining’ is more closely evocative of intentionality, in that the entertaining of something requires a fixing of regard on the thing as thematic. That is what is meant by saying that experiences entertain senses—they entertain them by fusing a second-order sense as a thematic regard into a first-order conscious experience. Experiences weld together senses.

It is important to understand, further, that the analogy to Kim’s physical events is to remain just that—a helpful tool in explicating the notion. It is not to be understood as, for example, entailing a metaphysical position on the issue of mind-body dualism. The question of the relationship between conscious events and physical events is an interesting question that is beyond the scope of this chapter.

One of the central objections to Kim’s property-exemplification account of physical events is that it multiplies events according to description or mode of presentation.¹⁸ For every distinct mode of presentation of an event, there is a separate event. For Kim’s purposes, this is troubling, since it will be a large bullet to bite to say that, e.g., the physical event of

¹⁷Of course, the general feature of intensional modality of intentional modes should not be confused for the use of intensional semantics in accounting for the meaningful behavior of adverbial modifiers.

¹⁸See, e.g., Rosenberg (1974). Kim reconstructs and responds to the objection himself in (Kim, 1976, pg. 42 ff.).

Brutus killing Caesar is distinct from that of Brutus stabbing Caesar.

But this problem for the property-exemplification account only speaks in favor of the sense-entertaining view of consciousness. For, the event of someone's *thinking* that Brutus killed Caesar *is* distinct from the event of her thinking that Brutus stabbed Caesar. Conscious contents are incorrigibly intensional, and every conscious event occurs under some mode of presentation that helps differentiate it from other conscious events; and so it is entirely natural to suppose that events are multiplied according to mode of presentation.

To be sure, we have not yet offered a positive account of conscious intentional events. But just as it would be a mistake to criticize Kim for failing to offer such an account of his property exemplifications, it would be a mistake to criticize my account in such a way. For, I will argue that taking up a realism about the semantics of this account offers significant purchase on the theme of phenomenal intentionality.

3.4 Adverbial Realism and Phenomenal Intentionality

Here I argue that being a realist about the event semantics has the advantage of giving an account of directedness that ramifies the distinction between directedness, successful directedness, and reference. In short, the notion of directedness is given by the accessibility relation of the ambient modal logic, and successful directedness is given as an extension on the same to a dynamic updating structure.¹⁹ Let us, then, investigate what a realism about the semantic structure of conscious events portends for phenomenal intentionality.

¹⁹Of course, appealing to the device of the modal accessibility relation in one's theory does not entail the ontological claim that intentionality is a relation. We are not identifying the accessibility relation and intentionality, but rather claiming that some important part of intentional structure involves modal variations of the sort that are given structure by something that has a function similar to that of an accessibility relation on possible worlds.

3.4.1 Intersubjectivity

The event picture of adverbialism understands intentional acts as changes in a consciousness at a time. Each actual change is individuated by virtue of a measurement of sameness of sense across possible worlds. Events are thus represented as changes in consciousness and individuated by virtue of an attitude to further possible changes in consciousness, and not, say, to possible states of the world, on the one hand, or to sets of propositions on the other. The accessibility relation in the semantics varies the possibility-spaces with what an agent takes to be *open* possibilities for future conscious events. When we combine the accessibility relation with the type-lowering principle, we get a picture of possibility-spaces that contract as one gets more information, so that less seems like an open possibility with respect to some object of experience. One might say that the directedness of consciousness a change that redistributes a person's *anticipations* of further possible changes of consciousness. But since the possibility-spaces are populated by potential consciousnesses, how does the event picture account for the directedness of consciousness toward the *world*?

Let us begin with the simplest possible example. Suppose I am visually presented with a vast white. Nothing populates my visual field but undifferentiable white. Am I having a conscious, intentional experience? The framework here says, “no.” This is because the vast swath of white carries no differentiations as to what is possible or impossible with respect to it. It does not make it possible for me to form an intentional attitude about future possible intentional attitudes. Heuristically speaking, we can say that the white carries no information that opens or closes some worlds for the perceiver. It may be objected that the vast white in fact does exclude the possibility of seeing black, or any other color. But I argue that it does no such thing. For, given what I have in front of me—namely, a vast, two-dimensional white swath—I have no grounds for ruling out that the swath might switch to black, or red, or blue, at any moment. It carries no information about my possible anticipations.

Let us add a bit more structure. Can an experience like that posited by sense-datum theorists, wherein my visual field is populated by differentiated patches of color, be intentional? On my view, I will say, “no.” Let us see why. Mere sense-datum by themselves do not have the potential to coarsen my sense of what I can do; they do not by themselves exclude enough possibilities. A sense datum can exclude only possibilities relating to the visual color of an object, and even then only under certain highly controlled conditions. For example, imagine you are walking through a hardware store when you come upon the painting section. You see an employee showing a red paint sample to a customer. Without further assumptions about the intentional situation, there is nothing about the patch of red of the paint sample that does the bulk of the representational work. After all, as Jackson among others has pointed out, it could be precisely that the employee is showing the customer what color the paint is *not*. Only the assumption that the employee is intentionally directed in a particular way cuts the ice. So a patch of color does not by itself do the representational work required; we need assumptions about intentional directedness to understand the patch of red as being representational at all.

Furthermore, the color of an object is scarcely the source of the important modal variations that present it as including some possibilities and excluding others. Rather, we require that the visual field be quite immediately populated with more structure: among other things, it is the spatiality of the cup, its being something I can grab and I drink out of, its being next to the fruit, on top of the table, in the room I’m in, etc. etc., that delineate possibilities as being included or excluded. In short, there has to be enough structure to dictate further possible conscious intentions in a way that excludes some and includes others with respect to adverbial parameters quite beyond mere color and shape.

The theorist interested in phenomenal intentionality may object that this diagnosis rules out my event approach as an approach to *phenomenal* intentionality. After all, the thesis of phenomenal intentionality is that the phenomenal determines the intentional. And what could

be more phenomenal than color qualia? My claim, then, in short, is that the phenomenal is more highly structured than the usual understanding of phenomenal qualia might suggest. The adverbial parameters mentioned above—the spatiality of the cup, the actions that are available on the basis of it, etc.—are themselves phenomenal senses.

So, the point of these examples has been to ask how much *phenomenal structure* is required to be directed at a world. We require more than that imposed by the sense-datum theorist. If I require structures of accessible possible worlds populated by possible further acts of consciousness, then this requires that my visual field be populated by more structure than *mere sense data*. And so to be intentional, an actual conscious state must encode possible further conscious states with adverbial parameters not only about color and shape but also about three-dimensional spatiality, potential for action, location, etc. But, again, how does this account for the traditional requirement of a theory of intentionality: that intentionality get us to the world?

Here, again, the adverbialist can appeal to their distinction between directedness and reference. For, one traditional goal of the theory of intentionality is that of accounting for the objective reference of an intention. If one takes intentionality to be tantamount to reference, then the only possible informative account of intentionality is one that specifies a strict relation between a given intention and the world. Here the project of specifying directedness by specifying an attitude toward further possible directednesses seems hopelessly circular.

But, of course, the adverbialist must account for more than reference—she must account for directedness. In this case, there is plenty informative to say that stops short of specifying a strict referential relation between intention and world. In particular, realism about the semantics here suggests that any interesting account of intentional reference must also—and perhaps first—account for the *intersubjectivity* of directedness.

The claim may seem anti-intuitive, but it is actually rather modest. Phenomenologically,

the experience of a world would not be experience of a *world* at all unless the world were populated with things that are not me, things that are other than me. Every object *I* see is available to any *other I* to see as well; any objective truth that I grasp is graspable for others as well, and so on. For, we take ourselves to live in a world with public facts and publicly observable states of affairs. And without this kind of taking, it would be difficult to see how it could be claimed that intentionality is directed at a world at all. So it's natural to think that intentionality is world-directed at least in part because it is capable of making the same future possibilities seem open to more than one person, hence, to produce the sense of a *public* space of possible directedness.

And this is exactly what the semantics predicts with its accessibility relation. For, the role of the accessibility relation is to vary with possibility spaces. Fix this as varying with different people's open possibility spaces. We now have a model of intersubjectivity that does not first go through objectivity. And this sheds a different light on the question of intentionality by shifting to view a different aspect of it—the active, ongoing, adverbial directing of consciousness rather than its referentiality. In the tradition of the act–content–object accounts of intentionality, then, one might say that what this adverbial framework does is switch the emphasis from the content and object—which emphasis naturally invites an account of reference—to the *act*, which invites an account of ways of an act's being directed.

It is an open question what account of objectivity this will supply. It may be the case that all we get is intersubjectivity. But this is not an entirely implausible result. The way we get a grip on whether we hold open the same possibilities is to talk with one another about it. Indeed, it is just as frequent in our ways of going about things that we establish the objectivity (to our own satisfaction at least) of our intentions by going through intersubjectivity; we establish whether the intention is shared by those who might have a comparable view on the matter, or we even defer to testimony from those who have a much better view on the

matter than we do. Suppose I take myself to see something off in the distance, but can't be sure what I'm seeing. I visually take it to be a boat, but perhaps it's a lighthouse. It will be exceedingly common to call a friend over and ask, "Is that a boat or a lighthouse?" Perhaps my friend, being more of an outdoorsman than I, even brought a pair of binoculars. So a big way that I get a grasp on what I'm directed toward is by reliance to a comparison—either one I enact explicitly or one passively done as a kind of modal operation native to my intentionality—that measures what I see against an intersubjective template. And this is precisely what is predicted by the semantics.

And this gives us a view quite distinct than what tracking theories offer. An intention is meaningful on the basis of its holding open some possibilities and closing off others in the same stroke, and doing so in a way that can match up with others in intersubjective agreement. But an intention's being referential is not constitutive of its being directed. The accessibility relation of the ambient modal logic, in combination with the type-lowering principle, plays the role of regimenting the notion of intersubjective directedness, since it includes information about the coarsening or fining of grain with respect to possibilities held open by different people. And so the view is that my being consciously directed or directable in a particular way gears into your being directed or directable in a particular way, by virtue of shared possibility for further conscious direction.

3.4.2 Intentional Appositeness and Incongruity

It remains to examine the question of *successful directedness*. To define intentional success, however, we must first distinguish two levels of success. First, there is the *intrinsic success* of a single intention as having accurate anticipations. A successful intention in this first sense is one that is *apposite* with respect to the other experiences in a given course of experience—the senses it anticipates end up being apposite with respect to actual senses in the course of

experience. An unsuccessful intention, by contrast, is one that is *incongruous* with respect to a given course of experience. At the second level of success, there is the *relative success* of a set of intentions that are related as together constituting a single, harmonious course of experience. A *successful course of experience* is one that instantiates a well-defined set of norms for *dynamic intentional updating*. Let us develop this further.

Above I have distinguished successful directedness from reference, in that reference is a stronger notion. While reference is most naturally characterized as a relation of correspondence, successful directedness can be understood by the weaker notion of the adherence to a set of norms. I claim that my account offers a natural account of the sorts of norms that can characterize successful directedness, even if these norms do not yet secure reference. Here we focus on the adverbial side of intentionality, i.e., the active ongoing conscious event whose modifications are meaningful in that they imply possibility for future conscious events.

The question of conscious reference is difficult, and I treated the matter of perceptual reference in prior chapters of this dissertation.²⁰ Perhaps that account can somehow be supplemented to the present one to account for the strong correspondence of reference. But there is also the question of the success of a person's intentional experience, and these seem to be appropriately independent of reference as to call for an isolated treatment. For, even if I were in a virtual reality, there are certain norms of success governing the course of my experience. Suppose I am playing a virtual reality game—even though the objects presented to me by the virtual reality do not refer to actual objects, my experience is modified in ways that imply intrinsic and relative success. I can mis-perceive an object in the game, and I can do things that make me successful or unsuccessful with respect to the course of experience, playing the game.

A primary consideration is that intentional anticipation will always be specific to varying degrees, but never up to maximal specificity. For example, as I become intimately familiar

²⁰See Chapter 2, above, on a reconstruction of Hintikka's perceptual logic.

with every property of the cup that I can see with the naked eye, the specificity of my anticipation increases. But I'll never be able to anticipate with maximal specificity what I would find if I took a microscope to it. And so the analysis of possible states of consciousness held open by actual states of consciousness suggest *norms for the closing and opening of possibilities*. Getting a grasp on an object in intentionality is an inexhaustible operation, but there are norms of being successful in our transactions with those objects, at least until further information tells us we had been wrong.

For example, when I see a tree from one side, I anticipate that it will have bark on the other, currently unseen side. My first intention is then apposite with respect to a second intention only if I actually or counter-factually have the sense of experiencing bark on the other side of the tree, when my body has been appropriately positioned to see the other side. My first intention is incongruous with the second intention if the second forces a cancellation of an anticipated meaning: I walk around to the other side and am surprised to find that the bark on this side has all been peeled off for a scientific experiment.

What exactly is happening in this example? In the first perception, I hold open that there is a conscious act of seeing bark on the other side. When I actually arrive to see the other side, having moved myself into a position to get a view of it, my experience is subject to certain norms. If I see the stripped bark, thus cancelling my anticipation, but I fail to close off the possibility of seeing bark, then something has gone “wrong” with my intentionality. There is thus an implicit norm involved in intentional appositeness and incongruity: when I come upon an incongruity, my intention ought to be updated to coarsen-out the cancelled possibilities. I should no longer anticipate bark there. And if I do this, then I am successful in that I have had an experience that is incongruous with an anticipation that I held open; but I have had a successful course of experience in that I have updated my anticipations to reflect the incongruity, thus restoring the appositeness of my ongoing intentional directedness.

So, while intentional directedness involves change in consciousness full-stop, the structure of

successful directedness involves *norms* for changing intentional directedness. But this paints a picture of success as always a sort of partial success. For, any given intention holds open manifold other possibilities for future intentions. And realism about the semantics supposes that there is an intersubjective variation on possibilities that is doing the work. Extending that theme, we may say that successful directedness occurs when the modifications of an experience fulfill norms of dynamic updating.

In order to obtain the distinction between directedness and successful directedness, then, we must thus regiment the structure of intentional appositeness and incongruity in the semantics. Such a structure can easily be represented in the semantics through a natural extension of the ambient modal logic. We go from a modal logic of intentional conceivability to one of *change* in intentional conceivability. In the modal part of the semantics, we have appealed to Kripke frames in combination with a type-lowering principle. This gave a picture of simple directedness as a kind of attitude-taking that intersubjectively coarsens the field of possibility for further attitude-taking. We must now add further structure to deal with norms for change of intentionality. Given our understanding of intentionality as an attitude on further possible intentionalities, there are some determinate things such update structure needs to do. In particular, the update structure should dynamically “filter” the set of accessible worlds so that, for example, dynamically cancelled possibilities are excluded from the modal variations in the accessibility relation.

There is a readily-available literature on the topic of extending modal logics to dynamic updating structures, so it will be formally unproblematic (Hans van Ditmarsch (2008)). Though the literature has focused on *belief* revision, these norms extend even to most acts of consciousness that have been primarily under consideration here, such as perceiving objects.²¹

²¹In previous chapters I have reconstructed one approach to the modal logic of perception originating in Hintikka’s work.

What is distinctive about the application of dynamic logic in this framework is that the updating structure gears into the type-lowering principle so crucial to earlier discussion of the event semantics. On the usual dynamic update structure, the semantics shifts the models around, but the combination of these shifts with the type-lowering principle gives a structure that dynamically fine- and coarse-grains the intersubjectively accessible possibility spaces, and this predicts the intuitive situation. For example, at one time I hold open that there could be renates that are not cordates, or vice-versa. But at some later time I learn that such state of affairs is not biologically possible. Intuitively, what I've done is stop holding open possibilities that I might experience a cordate that is not a renate.

The dynamic update structure gives us exactly that result. What occurs in the semantics is a course-graining of the possibility spaces at time t_0 , where worlds at which cordates were distinct from renates were held open, to the possibility space at time t_1 where these have been filtered out of the accessible worlds. We thus say that the original intention was not apposite with respect to the later discovery; and so successful intentionalities are ones that occur according to a structure that updates one's worlds according to one's new information.

And so we can say that a successful intentionality is one that is apposite with respect to the intentionalities that it anticipates as possible. And a successful *course* of intentionalities is one that instantiates the appositeness-incongruity structure of updating.

There are at least two ways in which the coarsening of possibilities occurs. First, there is a more or less "final" cancellation of a possibility. If I see what I take to be a human off in the distance, only to find as I approach closer that it had been a wax mannequin all along, then the possibility of there being a human right there is cancelled in a final manner.

But intentional coarsening is not always final in this way. Often, I coarsen-out possibilities only to re-open them later on. This is a fundamental kind of coarsening that intentionality does. For, in order for me to turn my intentional regard toward, say, the mug on the table

in front of me, I must turn my regard away from, say, the piece of fruit on the table. One can say that in this case my intention makes *thematic* the mug and shifts the fruit and table to the *background*. The fruit and the surface of the table are still in my experience, but only as offering a dull accessibility to their possibilities for further transaction. On the other hand, the possibilities with respect to the thematic cup are given finer grain. One can thus understand the intersubjective updating of coarsening to be one that contains information about how our regard is fixed with respect to a given experience, i.e., as picking out a subject matter.

We thus have the further result that the successful directedness of intentionality is *theme-sensitive*. I can successfully intend with respect to one theme, and then unsuccessfully intend with the exact same content, but fail to be successful, if my regard has shifted. That is, a successful partition of possibilities depends on how I take myself to be situated with respect to the world. For example, with my regard set on the mug, I can be successful in seeing it as grey. But if I shift regard to the red piece of fruit, then my intending while entertaining the sense ‘grey’ is no longer successful.

3.5 Appendix 1: Logic of Adverbs

As a first approximation, we can think of adverbs as the sorts of words or phrases that modify verbs, adjectives, or other adverbs. We are interested in the *logic* of adverbs, an inquiry centered around the question of entailment relations between sentences containing adverbs. Consider the following list of sentences:

- (a) Socrates ran swiftly across Athens.
- (b) Socrates ran swiftly.
- (c) Socrates ran.

Intuitively, any sentence in this list entails any sentence below, but not above.

To motivate the move to the more complicated Montagovian or Davidsonian approaches, let us briefly see why the most naive treatment will not work. In first-order predicate logic, it is natural to parse each sentence (a)-(c) as having a simple subject-predicate structure, so that we obtain a formal representation of (a)-(c) like so:

(a') $R''(s)$: Socrates *ran swiftly across Athens*.

(b') $R'(s)$: Socrates *ran swiftly*.

(c') $R(s)$: Socrates *ran*.

It is immediately apparent that this captures none of the entailment structure implicit in the original list of sentences, because in first-order logic no monadic predicate entails any other.

The Davidsonian *event* approach, further developed by Parsons, is under-girded by a sort of denial of the naive view of adverbs that they are modifying verbs. Rather, adverbs are properties attributed to events. So, for example, the event semanticist parses (c) as saying that there is an event that has Socrates as its agent and has the properties of being a running event. The event semanticist thus posits a more complex logical form underneath the surface grammar: there is an implicit quantification over events, and it is the event which the adverb modifies.

For philosophers, the point of deep interest here is the quantification over events. Indeed, much of the philosophical debate surrounding this approach has had to do with the metaphysics of event quantification. But for the logician, the quantification over events is unproblematic and solves the problem of entailment:

(a'') $\exists e[Subj(e, s) \wedge e_R \wedge e_W \wedge e_A]$

(b'') $\exists e[Subj(e, s) \wedge e_R \wedge e_W]$

(c'') $\exists e[Subj(e, s) \wedge e_R]$

It is easy to see that the entailments are given logical form as instances of conjunction elimination.

Setting aside the event approach, the Montagovian, or *intensional*, approach to this problem has been developed by Thomason and Stalnaker (1973). The underlying thoughts are, first, that it would be nice if we did not have to stray so far from the surface grammar of English adverbs as does the notion of event quantification. Second is the idea that it should be the *meaning* of particular adverbs, rather than the logical forms of adverbs simpliciter, that secures inferences. For, some adverbs should not entail in the way we have said in examples (a)-(c). Consider, for example, that, ‘Socrates allegedly ran’ should not entail, ‘Socrates ran,’ and this is due to a fact about the meaning of ‘allegedly.’

The formal ingredients behind intensional semantics for adverbs are the intension operator, $\hat{}$, and a predicate-modifying function, σ . The main idea is that the intension operator blocks certain inferences that can be re-instated with meaning postulates. This can be given a well-defined possible-worlds semantics, and the resulting functions can be correlated with English sentences without having to posit, as does event semantics, a hidden syntactic structure to secure the semantics.

Given an expression φ , the intension operator returns the intension of φ , i.e., $\hat{\varphi}$. Adverbs—i.e., the intensional predicate-modifying functions σ —can then be prefixed to the intensions of predicates to yield new intensional functions. For example, if R stands for runs, then we have \hat{R} , which is the intension of R . Suppose σ is a predicate-modifying function, e.g., *swiftly*, then prefixing σ to the intensional \hat{R} predicate yields the new intension $\sigma(\hat{R})$, i.e., *swiftly running*.

Let us apply this to the running examples. Note, first, that we will simplify matters since entailments here are more complicated than in the event approach. Still, let us consider the inference from (b) to (c), reproduced below in the form of the intensional theory:

(b'') $\sigma(\hat{R}(s))$

(c'') $\hat{R}(s)$.

On the Montague framework, this argument is not valid. This makes sense in cases like that mentioned above, when the adverb in question is supposedly or allegedly, since it is not the case that all alleged runners are runners. To obtain the validity for other sorts of adverbs, the intensional framework introduces a meaning postulate to ensure that, e.g., necessarily, if something runs swiftly, then it runs.

3.6 Appendix 2: Proofs

Recall the following from the earlier section:

$$(6) e_F = e_G \iff (\Box \forall x Fx \iff Gx)$$

Theorem 15. *Principle (6) is inconsistent with the T-axiom.*

Proof. Define a set $H = \{x : \exists F(x = e_F \wedge \neg Fx)\}$, and let $h = e_H$. Then consider whether h is a member of H . There are two possibilities. First, suppose h is in H . Then we know that there is some property F such that $h = e_F$ and h is not an F . But $h = e_H = e_F$, so by principle (6), of necessity F is co-extensive with H . By the T -axiom of modal logic, we can move from the necessity statement to the actual world.²² Hence, F is co-extensive with H .

²²The T -axiom states that whatever holds necessarily holds in the actual world, and is standard in most modal logics.

Then h is not in H , but that contradicts the case assumption. For the second case, suppose h is not in H . Then simply define F be co-extensive with H . Then, since e_H is not in H , e_H is not in F . But then by principle (6) e_F is e_H , so that h in fact meets the conditions for being in H , which contradicts the case assumption. \square

Theorem 16. *Principle (6) is consistent with the failure of the T-axiom.*

Proof. We must show that there is a model of principle (6) in which the T-axiom fails. Suppose we have a two-world model with worlds w_1 and w_2 . We have a non-reflexive accessibility relation such that w_1 sees w_2 and nothing else. Define D_1 as the domain of w_1 and D_2 as the domain of w_2 . Let the cardinality of D_2 be some arbitrary n . Then the power set of D_2 is 2^n . Then simply let the cardinality of D_1 be greater than or equal to 2^n . In that case, we guarantee the existence of an injection e from the power set of D_2 to D_1 . Further, as is not uncommon in variable domain semantics, we assume that Fa is true at a world iff F is a subset of the domain at that world and a is in F . So, $e_F = e_G$ at w_1 iff F and G are co-extensive at w_2 . Since w_1 sees w_2 and nothing else, this happens if and only if necessarily F and G are co-extensive at w_1 . \square

Bibliography

- Bacon, J. (1979). The logical form of perception sentences. *Synthese*, 41(2):271–308.
- Barwise, J. (1981). Scenes and other situations. *Journal of Philosophy*, LXXVIII:369–397.
- Belnap, N. and Gupta, A. (1993). *The Revision Theory of Truth*. MIT Press.
- Bencivenga, E. (1986). Free logics. *Handbook of Philosophical Logic*, Volume III: Alternatives in Classical Logic.
- Brentano, F. (1874/1911/1973). *Psychology from an Empirical Standpoint*. Routledge and Kegan Paul.
- Burge, T. (2010). *Origins of Objectivity*. Oxford University Press.
- Burge, T. (2013). *Cognition Through Understanding: Self-Knowledge, Interlocution, Reasoning, Reflection: Philosophical Essays, Volume 3*. Oxford University Press.
- Carnap, R. (1947). *Meaning and Necessity*. University of Chicago.
- Chalmers, D. (2006). Two-dimensional semantics. *Oxford Handbook of the Philosophy of Language*. Oxford University Press., pages 574–606.
- Chellas, B. F. (1980). *Modal Logic*. Cambridge University Press.
- Chisholm, R. (1957). *Perceiving: A Philosophical Study*. Cornell University Press.
- Cross, C. (2001). The paradox of the knower without epistemic closure. *Mind*, 110(438):319–333.
- Cross, C. (2012). The paradox of the knower without epistemic closure—corrected. *Mind*, 121(482):457–466.
- Davidson, D. (1967). Causal relations. *Journal of Philosophy*, 64(21):691–703.
- Davidson, D. (1985). Adverbs of action. In Vermazen, B. and Hintikka, M. B., editors, *Essays on Davidson: Actions and Events*, pages 230–241. Oxford University Press.
- de Groote, P. and Winter, Y. (2015). A type-logical account of quantification in event semantics. In *New frontiers in artificial intelligence*, volume 9067 of *Lecture Notes in Comput. Sci.*, pages 53–65. Springer, Heidelberg.

- Demopoulos, W. and Clark, P. (2005). The logicism of Frege, Dedekind, and Russell. In Shapiro, S., editor, *The Oxford Handbook of Philosophy of Mathematics and Logic*, pages 129–165. Oxford University Press.
- Dickie, I. (2015). Perception and demonstratives. *The Oxford Handbook of Philosophy of Perception*.
- Dowty, D. R., Wall, R. E., and Peters, S. (1981). *Introduction to Montague Semantics*. Springer.
- Dretske, F. (1970). Epistemic operators. *The Journal of Philosophy*, 67(24):1007–1023.
- Dretske, F. (1981). *Knowledge and the Flow of Information*. MIT Press.
- Dretske, F. (1988). *Explaining Behavior: Reasons in a World of Causes*. MIT Press.
- Dummett, M. (1981). *Frege: Philosophy of Language*. Harper & Row, New York, second edition.
- Egre, P. (2005). The knower paradox in the light of provability interpretations of modal logic. *Journal of Logic, Language and Information*, 14(1):13–48.
- Evans, G. (1982). *Varieties of Reference*. Oxford University Press.
- Fitting, M. and Mendelsohn, R. L. (1998). *First-order Modal Logic*. Kluwer Academic Publishers.
- Fodor, J. A. (1987). *Psychosemantics: The Problem of Meaning in the Philosophy of Mind*. MIT Press.
- Follesdal, D. (1969). Husserl’s notion of noema. *Journal of Philosophy*, 66(20):680–687.
- Frege, G. (1892). *Translations from the Philosophical Writings of Gottlob Frege*. Blackwell.
- Gamut, L. (1991). *Intensional Logic and Logical Grammar*, volume 2 of *Logic, Language, and Meaning*. University of Chicago Press, Chicago.
- Gendler, T. S. and Hawthorne, J. (2002). *Conceivability and Possibility*. Oxford University Press.
- Halbach, V. (2011). *Axiomatic Theories of Truth*. Cambridge University Press, Oxford.
- Halbach, V. and Welch, P. (2009). Necessities and Necessary Truths: A Prolegomenon to the Use of Modal Logic in the Analysis of Intensional Notions. *Mind*, 118(469):71–100.
- Hand, M. (2007). *Philosophy of Logic*. Elsevier.
- Hans van Ditmarsch, e. a. (2008). *Dynamic Epistemic Logic*. Springer.
- Higginbotham, J. (1983). The logic of perceptual reports: An extensional alternative to situation semantics. *Journal of Philosophy*, LXXX:100–127.

- Hintikka, J. (1962). *Knowledge and Belief: An Introduction to the Logic of the Two Notions*. Cornell University Press.
- Hintikka, J. (1969). *Models for Modality: Selected Essays*. D. Reidel Publishing Company.
- Hintikka, J. (1975). *The Intentions of Intentionality and Other New Models for Modality*.
- Holliday, W. H. (2015). Epistemic Closure and Epistemic Logic I: Relevant Alternatives and Subjunctivism. *Journal of Philosophical Logic*, 44(1):1–62.
- Holliday, W. H. and Perry, J. (2014). *Roles, Rigidity, and Quantification in Epistemic Logic*.
- Horgan, T. (2013). Original intentionality is phenomenal intentionality. *The Monist*, 96(2):232–251.
- Horsten, L. (2011). *The Tarskian Turn*. The MIT Press, Cambridge.
- Husserl, E. (1907). *Ideas for a Pure Phenomenology and Phenomenological Philosophy. First Book: General Introduction to a Pure Phenomenology*. Hackett, 2014.
- Husserl, E. (1936/). *The Crisis of the European Sciences and Transcendental Phenomenology*.
- Husserl, E. (1970). *Logical Investigations: Volume 2, 1901*. Routledge, 2001.
- Husserl, E. (1997). *Thing and Space: Lectures of 1907*. Springer.
- Jackson, F. (1977). *Perception: A Representative Theory*. Cambridge University Press.
- Kaplan, D. (1968). Quantifying in. *Synthese*, 19:178–214.
- Kaplan, D. and Montague, R. (1960). A paradox regained. *Notre Dame Journal of Formal Logic*, 1(3):79–90.
- Kim, J. (1976). Events as property exemplifications. In Brand, M. and Walton, D., editors, *Action Theory*, pages 310–326. D. Reidel.
- Kriegel, U. (2007). Intentional inexistence and phenomenal intentionality. *Philosophical Perspectives*, 21:307–340.
- Kriegel, U. (2011). *The Sources of Intentionality*. Oxford University Press.
- Kriegel, U. (2013). *Phenomenal Intentionality*. Oxford University Press.
- Kripke, S. (1976). *Is there a problem about substitutional quantification?*, pages 324–419. Oxford University Press.
- Kripke, S. (1980). *Naming and Necssity*. Harvard University Press.
- Lasersohn, P. N. (2006). Event-based semantics. *Encyclopedia of Language and Linguistics, 2nd edition*, ed. by K. Brown, 4(1):316–320.
- Lewis, D. (1986). *On the Plurality of Worlds*. Blackwell.

- Lewis, D. (1996). Elusive knowledge. *Australasian Journal of Philosophy*, 74(4):549–567.
- Maitzen, S. (1998). The knower paradox and epistemic closure. *Synthese*, 114(2):337–354.
- Marcus, R. (1961). *Modalities: Philosophical Essays*. Oxford University Press.
- Marker, D. (2002). *Model Theory: An Introduction*, volume 217 of *Graduate Texts in Mathematics*. Springer-Verlag, New York.
- McKay, T. and Nelson, M. (2014). Propositional attitude reports. In Zalta, E. N., editor, *The Stanford Encyclopedia of Philosophy*. Spring 2014 edition.
- Mellema, G. (1982). Multiple quantifiers and Hintikka’s logic of perception. *Philosophia*, 11(1-2):95–103.
- Mendelovici, A. and Bourget, D. (2014). Naturalizing intentionality: Tracking theories versus phenomenal intentionality theories. *Philosophy Compass*, 9(5):325–337.
- Merleau-Ponty, M. (1945/2013). *Phenomenology of Perception*. Routledge.
- Miller, I. (1984). Perceptual reference. *Synthese*, 61(1):35–59.
- Millikan, R. G. (1989). Biosemantics. *Journal of Philosophy*, 86(July):281–97.
- Montague, M. (2010). Recent work on intentionality. *Analysis*, 70(4):765–782.
- Montague, R. (1963). Syntactical treatments of modality, with corollaries on reflexion principles and finite axiomatizability. In Thomason, R. H., editor, *Formal Philosophy: Selected Papers of Richard Montague*, chapter 10, pages 286–302. Yale University Press.
- Montague, R. (1974). *Formal Philosophy: Selected Papers of Richard Montague*. Yale University Press.
- Moran, R. A. (2001). *Authority and Estrangement: An Essay on Self-Knowledge*. Princeton University Press.
- Nozick, R. (1981). *Philosophical Explanations*. Harvard University Press.
- Parsons, T. (1990). *Events in the Semantics of English: A Study in Subatomic Semantics*. Current Studies in Linguistics. MIT Press.
- Peacocke, C. (1992). *A Study of Concepts*. MIT Press.
- Pitt, D. (2004). The phenomenology of cognition, or, what is it like to think that *P*? *Philosophy and Phenomenological Research*, 69(1):1–36.
- Priest, G. (2008). *An Introduction to Non-Classical Logic*. Cambridge.
- Pritchard, D. (2008). Sensitivity, safety, and anti-luck epistemology. In Greco, J., editor, *The Oxford Handbook of Skepticism*. Oxford University Press.

- Putnam, H. (1975). The Meaning of 'Meaning'. In Gunderson, K., editor, *Language, Mind, and Knowledge*, pages 131–193. University of Minnesota Press.
- Quine, W. (1956). Quantifiers and propositional attitudes. *Journal of Philosophy*, 53(5):177–187.
- Quine, W. (1966). Three grades of modal involvement. In *The Ways of Paradox and Other Essays*. Random House.
- Rosenberg, A. (1974). On kim's account of events and event-identity. *Journal of Philosophy*, 71(11):327–336.
- Saarinen, E. (1983). On the logic of perception sentences. *Synthese*, 54(1):115–128.
- Schein, B. (1994). *Plurals and Events*. MIT Press.
- Schwarz, W. (2013). Variations on a montagovian theme. *Synthese*, 190(3377-3395).
- Shuger, S. (1986). Hintikka and the analysis of direct perception. *Philosophia*, 16(December):365–376.
- Smith, D. W. (1979). The case of exploding perception. *Synthese*, 41:239–269.
- Smith, D. W. (1983). Kantifying-in. *Synthese*, 54(2).
- Smith, D. W. (2013). *Husserl*. Routledge.
- Smith, D. W. and McIntyre, R. (1982). *Husserl and Intentionality: A Study of Mind, Meaning, and Language*. Kluwer Academic Publishers.
- Stalnaker, R. (1984). *Inquiry*. Bradford.
- Steup, M. and Sosa, E. (2005). *Contemporary Debates in Epistemology*. Blackwell Publishers Inc.
- Taschek, W. W. (2010). On Sense and Reference: A Critical Reception. In Potter, M. and Ricketts, T., editors, *The Cambridge Companion to Frege*, pages 293–341. Cambridge, Cambridge.
- Thomason, R. H. (Ed. Milton K. Munitz. New York University Press, 1973.). Perception and individuation. *Logic and Ontology*, pages 261–285.
- Thomason, R. H. and Stalnaker, R. C. (1973). A semantic theory of adverbs. *Linguistic Inquiry*, 4(2):195–220.
- Tye, M. (1984). The adverbial approach to visual experience. *Philosophical Review*, 93(April):195–226.
- Uzquiano, G. (2004). The paradox of the knower without epistemic closure? *Mind*, 113(449):95–107.

van der Does, J. and van Lambalgen, M. (2000). A logic of vision. *Linguistics and Philosophy*, 23(1).

Williamson, T. (2002). *Knowledge and Its Limits*. Oxford University Press, Oxford.