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Essays on Keynesian Models of Closed and Open Economies

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Essays on Keynesian Models
of Closed and Open Economies

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Konstantin Platonov

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ABSTRACT OF THE DISSERTATION

Essays on Keynesian Models
of Closed and Open Economies

by

Konstantin Platonov
Doctor of Philosophy in Economics
University of California, Los Angeles, 2019
Professor Roger E. Farmer, Chair

My dissertation contributes to the macroeconomics of self-fulfilling prophecies. It demonstrates the importance of shocks to beliefs in accounting for aggregate fluctuations. The dissertation consists of three chapters.

The first chapter is a paper joint with Roger E.A. Farmer. We integrate Keynesian economics with general equilibrium theory in a new way. We develop a simple graphical apparatus, the IS-LM-NAC framework, that can be used by policy makers to understand how policy affects the economy. A new element, the No-Arbitrage-Condition (NAC) curve, connects the interest rate to current and expected future values of the stock market and it explains how 'animal spirits' influence economic activity. Our framework provides a rich new approach to policy analysis that explains the short-run and long-run effects of policy.

The second chapter studies implications of self-fulfilling beliefs in open economies. Uncovered interest parity states that the carry trade should deliver zero profit, on average. The data robustly reject this hypothesis. In a large sample of countries, high interest rate currencies earn excess returns at short horizons and negative excess returns at longer horizons. I rationalize this observation in a two-country overlapping generations model with complete markets that features multiple dy-

namic equilibria. Because newborns cannot make decisions about consumption and savings before they are born, there is a set of self-fulfilling beliefs of the currently alive generations about the decisions of the future newborns. I utilize the multiplicity of dynamic equilibria by imposing a structure on the formation of beliefs. Beliefs are self-fulfilling, and shocks to these beliefs generate a large and volatile risk premium that is correlated with the interest rate differential. Changes in uncertainty about beliefs cause a reversal of expected excess returns associated with the current interest differential, similar to the reversal found in the data. I provide empirical evidence in favor of my mechanism and show that persistence of past expectations can account for most of the observed deviation from uncovered interest parity.

The third chapter extends the methodology developed in the first chapter to the open economy framework. I build a model of the eurozone crisis. I study a two-country model in which agents form self-fulfilling beliefs about asset prices. Using the labor market search and matching frictions and abandoning Nash bargaining over wage, I create multiple equilibria on the labor market where any unemployment rate can be sustained as a steady state. Self-fulfilling beliefs about the future value of assets select the equilibrium. I show that sudden downward revisions of beliefs ('animal spirits') cause stagnation in real economic activity, international financial contagion, and, in the absence of recovery in confidence, permanently high rates of unemployment. High unemployment is viewed as a new steady state, not a temporary deviation from the natural rate of unemployment. Policy aimed at recovering the eurozone needs to trigger optimistic beliefs about the value of assets.

The dissertation of Konstantin Platonov is approved.

Francois Geerolf

Steven A. Lippman

Aaron Tornell

Roger E. Farmer, Committee Chair

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2019

To my family

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CHAPTER 1

Animal Spirits in a Monetary Model

This paper was jointly written with Roger E.A. Farmer and published as ”Animal Spirits in a Monetary Economy,” Roger E.A. Farmer, Konstantin Platonov, European Economic Review, 115, 2019, pages 60-77

1.1 Introduction

In the lead-up to the 2008 financial crisis, a consensus developed among academic macroeconomists that the problem of macroeconomic stability had been solved. According to that consensus, the New Keynesian dynamic stochastic general equilibrium (DSGE) model provides a good first approximation to the way that monetary policy influences output, inflation and unemployment. In its simplest form, the New Keynesian model has three equations; a dynamic IS curve, a policy equation that describes how the central bank sets the interest rate, and a New-Keynesian Phillips curve. In its more elaborate form, the New-Keynesian DSGE model is reflected in work that builds on the medium scale DSGE model of Frank Smets and Raf Wouters (2007).

The New Keynesian model evolved from post-war economic theory in which the Keynesian economics of the *General Theory*, (Keynes, 1936), was grafted onto the microeconomics of Walrasian general equilibrium theory (Walras, 1899). Paul Samuelson, in the third edition of his undergraduate textbook, (Samuelson, 1955), referred to this hybrid theory as the ‘neoclassical synthesis’. According to the neoclassical synthesis, the economy is Keynesian in the short-run, when not all wages and prices have adjusted to clear markets; it is classical in the long-run, when all wages and prices have adjusted to clear markets and the demands and supplies for all goods and for labor are equal.[1]

The neoclassical synthesis is still the main framework taught in economics textbooks, and, in the form of 'dynamic IS-LM analysis', it is used by policy makers to frame the way they think about the influence of changes in fiscal and monetary policy on economic activity.[2] This paper proposes an alternative framework. Building on work by Roger Farmer (2010a) we integrate Keynesian economics with general equilibrium theory in a new way to demonstrate that low-income high-unemployment inefficient equilibria may be sustained in the long run. Our work displays two main differences from the New Keynesian model.

First, the steady state equilibria of our model display dynamic indeterminacy. For every steady state equilibrium, there are multiple dynamic paths, all of which converge to the same steady state. We use that property to explain how changes in the money supply may be associated with immediate changes in real economic activity without invoking artificial barriers to price change. Prices in our model are set one period in advance, but there are no explicit costs of price adjustment.[3]

Second, our model displays steady state indeterminacy. We adopt a labor search model in which the presence of externalities generates multiple steady state equilibria. Unlike classical search models we do not close the model by assuming that firms and workers bargain over the wage.[4] Instead, as in Farmer (2010a;

[1]This characterization of the history of thought is drawn from Farmer (2010b) and elaborated on in Farmer (2016a).

[2]See, for example, Mankiw (2014).

[3]For earlier papers that invoke that idea see Farmer and Woodford (1997), Farmer (1991, 1999, 2000, 2002), Matheny (1998), and Benhabib and Farmer (2000). Although we do not explicitly adopt the assumptions of menu costs (Mankiw, 1985) or price rigidity (Christiano et al., 2005; Smets and Wouters, 2007), these are both possible explanations for agents in our model to select an equilibrium in which prices are predetermined.

[4]By classical search models, we mean the literature that builds on work by Peter Diamond, (1982), Dale Mortensen, (1970), and Chris Pissarides (1976).

2012b; 2016a), firms and workers take wages and prices as given and employment is determined by aggregate demand. We use that feature to explain why unemployment is highly persistent in the data. Persistent unemployment, in our model, represents potentially permanent deviations of the market equilibrium from the social optimum.[5]

To close our model, we assume that equilibrium is selected by 'animal spirits' and we model that idea with a *belief function* as in Farmer (1999, 2002, 2012a). We treat the belief function as a fundamental with the same methodological status as preferences and endowments and we study the implications of that assumption for the ability of monetary policy to influence inflation, output and unemployment. Although we use a stylized calibration of our model to generate impulse response graphs, this paper is not a serious piece of data analysis: Our contribution is to introduce a new pedagogical tool, the IS-LM-NAC model, and to illustrate the use of that tool with a series of policy exercises. We refer the reader to Farmer and Nicolò (2018), Keynesian economics without the Phillips curve, for an empirical application of our framework.

There have been many attempts to build micro-foundations to the IS-LM model. The most popular is the dominant New-Keynesian model that appears in modern graduate textbooks (Galí, 2008; Woodford, 2003). Bilbiie (2008, 2019) and Dong et al. (2016) use credit market imperfections to generate Keynesian results from micro foundations and a micro-founded approach that stresses finan-

[5] King et al. (1991); Beyer and Farmer (2007b) and Farmer (2012c, 2015) find evidence of a unit root in the U.S. unemployment rate. Sticky-price models with a unique determinate steady-state equilibrium have difficulty generating enough persistence to understand this fact, as do unique-equilibrium models of the monetary transmission mechanism that assume sticky information (Mankiw and Reis, 2007) or rational inattention, (Sims, 2003). Our approach generates *permanent* equilibrium movements in the unemployment rate that are consistent with a unit root, or near unit root, in U.S. unemployment data and is complimentary to theories that explicitly model small costs of price change. Olivier Blanchard and Lawrence Summers (1986; 1987) attribute persistent unemployment to models that display hysteresis. Our model has that feature, but for different reasons than the explanation given by Blanchard and Summers. For a recent survey that explains the evolution of models of dynamic and steady state indeterminacy, see Farmer (2016b).

cial market imperfections has found its way into the undergraduate curriculum in the UK with the influential textbook by Carlin and Soskice (2014). Our main difference from these approaches is the ability of our model to generate deviations of the unemployment rate from the social optimum that can persist forever in the absence of monetary or fiscal policy intervention.

The supply side of our model was developed in Farmer (2012b, 2013) and in Farmer (2016a) where Farmer refers to a model closed by beliefs, as a "Keynesian search model" to distinguish it from the classical approach to search theory (Diamond, 1982; Mortensen, 1970; Pissarides, 1984). Keynesian search theory replaces the classical assumption that the bargaining weight is a parameter with the alternative assumption that the unemployment rate is demand determined.

The current paper builds on the Keynesian search approach by including the real value of money balances in the utility function to capture the function of money as a means of exchange. The addition of money leads to genuinely new results from the real model in Farmer's previous work. For example, we show that, under some specifications of beliefs, money may be non-neutral. An unanticipated shock to the money supply may have a permanent effect on the unemployment rate through its influence on beliefs about the real value of future wealth.

1.2 The Model

We construct a two-period overlapping generations model. In every period there are two generations of representative households; the young and the old.[6] The young inelastically supply one unit of labor, but, due to search frictions, a fraction of young individuals remain unemployed in any given period. We assume that there is perfect insurance within the household and that labor income is split

[6]The restriction to two-period lives is made for expository purposes only. In section 1.12 we develop a long-lived version of our approach using the Blanchard (1985) perpetual youth model.

between current consumption, interest bearing assets, and money balances.

Households hold money, physical capital and financial assets in the form of government bonds. Money is dominated in rate-of-return and is held for transaction purposes. We model this by assuming that real money balances yield utility as in Patinkin (1956). The old generation receives interest on capital and bonds and they sell assets to the young generation. We close the markets for physical capital and labor by assuming that there is one unit of non-reproducible capital and that the labor-force participation rate is constant and equal to one. We also assume that government bonds are in zero net supply.

There is a single good produced by a continuum of competitive firms. Firms rent capital from old generation individuals and hire young generation individuals. Hiring labor is subject to search frictions. Firms take prices and wages as given and they allocate a fraction of labor to recruiting. We assume that every worker allocated to recruiting can hire q new workers, where q is taken as given by firms but determined in equilibrium by the search technology. Every worker allocated to recruiting is one less worker allocated to production.

Search in the labor market generates multiple equilibria. To select equilibrium, we assume that economic agents form beliefs about the real value of their financial wealth using a belief function that is a primitive of our model. Our Keynesian search approach differs from the more usual assumption in the classical labor search literature where the equilibrium is pinned down by Nash bargaining over the real wage.[7]

To make our model transparent, we consider only permanent unanticipated shocks to beliefs. There is no uncertainty regarding other fundamentals of the economy.

[7] Farmer (2016a, Chapter 7) distinguishes Keynesian search models, where employment is determined by aggregate demand, from classical search models, where employment is determined by Nash bargaining.

Our model provides a microfoundation for the textbook Keynesian cross, in which the equilibrium level of output is determined by aggregate demand. Our labor market structure explains why firms are willing to produce any quantity of goods demanded, and our assumption that beliefs are fundamental determines aggregate demand. In our model, beliefs select an equilibrium and in that equilibrium, the unemployment rate may differ permanently from the social planning optimum.

1.3 Aggregate Supply

There is a unit continuum of competitive firms. We represent the capital and labor employed and output produced by each individual firm with the symbols K_t, L_t, and Y_t.[8] To refer to aggregate labor and aggregate output we use the symbols $\bar{L}_t$ and $\bar{Y}_t$. The variables K_t, L_t, and Y_t are indexed by $j \in [0,1]$ where

$$\bar{K}_t = \int_j K_t(j)dj, \quad \bar{L}_t = \int_j L_t(j)dj, \quad \bar{Y}_t = \int_j Y_t(j)dj.$$

Since all firms face the same prices and will make the same decisions, it will always be true that $K_t(j) = K_t$, $L_t(j) = L_t$ and $Y_t(j) = Y_t$, hence, we will dispense with the subscript j in the remainder of our exposition.

All workers work only in the first period of their life. A firm puts forward a production plan in which it proposes to allocate X_t workers to production and V_t workers to recruiting where

$$L_t = X_t + V_t.$$

Output is given by the expression

$$Y_t = K_t^{\alpha} X_t^{1-\alpha},$$

[8]The model developed in this section is drawn from Farmer (2012b).

and the total number of workers employed at the firm is equal to

$$L_t = q_t V_t, \tag{1.1}$$

where the firm takes q_t as given. Putting these pieces together, we may express the output of the firm as

$$Y_t = K_t^{\alpha} \left[\left(1 - \frac{1}{q_t} \right) L_t \right]^{1-\alpha}. \tag{1.2}$$

Firms maximize profit,

$$P_t Y_t - R_t K_t - W_t L_t,$$

by choosing how much capital and labor to hire. Here P_t is the money price of goods, R_t is the money rental rate of capital and W_t is the money wage. Perfect competition implies that factors earn their marginal products and profit is equal to zero.

$$(1-\alpha)\frac{Y_t}{L_t} = \frac{W_t}{P_t} \quad \text{and} \tag{1.3}$$

Notice that equation (1.2) looks like a classical production function with one exception. The variable, q_t, which represents labor market tightness, influences total factor productivity. One may show that q_t is greater than 1 in equilibrium. A low value of q_t corresponds to a tight labor market in which firms must devote a large amount of resources to recruiting and in which productivity is low. A high value of q_t corresponds to a loose labor market in which firms may devote a small amount of resources to recruiting and in which productivity is high.

At the aggregate level, we assume the existence of a matching technology that determines aggregate employment $\bar{L}_t$ as a function of aggregate resources devoted to recruiting, $\bar{V}_t$, and the aggregate number of unemployed searching workers, $\bar{U}_t$. This function is given by,

$$\bar{L}_t = m\left(\bar{V}_t, \bar{U}_t\right) \equiv \left(\Gamma \bar{V}_t\right)^{1/2}, \tag{1.4}$$

where $\bar{U}_t = 1$ because workers are fired every period and the number of searching workers is equal to 1 at the beginning of every period.[9] Because the economy is endowed with one unit of labor, the end-of-period unemployment rate can be defined as

$$U_t = 1 - \bar{L}_t.$$

The parameter Γ in the matching function determines the efficiency of the matching technology. In a symmetric equilibrium where $L_t = \bar{L}_t$, we may combine equations (1.1), (1.2) and (1.4) to find an expression for Y_t in terms of L_t and $\bar{L}_t$

$$Y_t = K_t^{\alpha}\left[L_t\left(1 - \frac{\bar{L}_t}{\Gamma}\right)\right]^{1-\alpha}, \tag{1.5}$$

where $\bar{L}_t/\Gamma = 1/q_t$.

Equation (1.5) is the private production function. This function represents the connection between the output of an individual firm, Y_t, the capital and labor inputs at the level of the firm, K_t and L_t, and the labor input of all other firms, $\bar{L}_t$. The private production function is distinct from the social production function, equation (1.6),

$$\bar{Y}_t = \bar{K}_t^{\alpha}\left[\bar{L}_t\left(1 - \frac{\bar{L}_t}{\Gamma}\right)\right]^{1-\alpha}, \tag{1.6}$$

which represents the connection between aggregate output $\bar{Y}_t$ and aggregate capital and labor inputs, $\bar{K}_t$ and $\bar{L}_t$. We illustrate the properties of the social pro-

[9] This simplification requires the assumption that workers can, in effect, recruit themselves. Farmer (2012b) discusses the assumption further and Farmer (2013) drops the assumption and treats employment as an additional state variable. In the complete dynamic model, the fraction of workers assigned to recruiting is a small fraction of the workforce, as opposed to the current formulation where, at the social optimum, 50% of the firm's workers are engaged in recruiting. Because nothing of substance is added to the model by studying the full dynamics of the labor market we have chosen, in this paper, to use the simpler model, where labor is not a state variable, for expositional purposes.

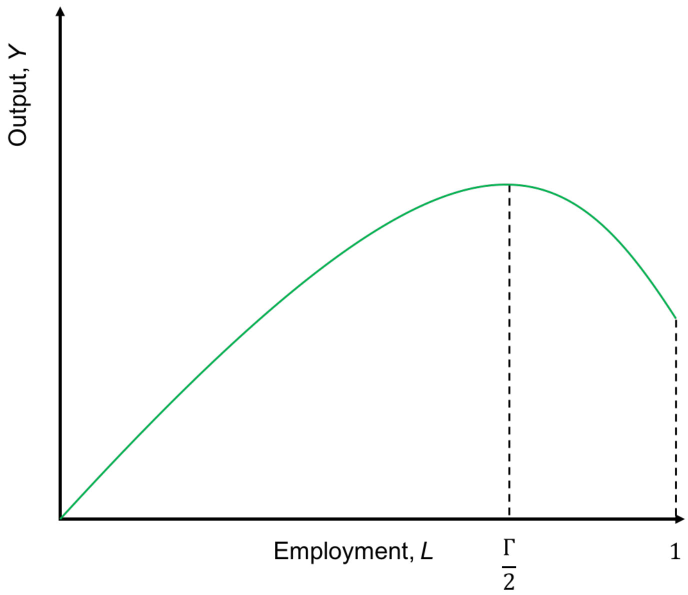

Figure 1.1: The Social Production Function

duction function on Figure 1.1. On this figure, we see that output is increasing in employment up to a maximum that occurs at $\Gamma/2$.

The social production function exhibits search externalities. For large values of aggregate employment, $\bar{L}_t$, the labor market becomes very tight and further reduction of unemployment is costly. As firms allocate more workers to the recruiting activity, those workers are withdrawn from production. If employment increases beyond $\Gamma/2$, additional increases in aggregate employment become counter-productive.[10] The value of unemployment at the social optimum,

$$U = 1 - \frac{\Gamma}{2}, \tag{1.7}$$

is our definition of the *natural rate of unemployment*.[11]

[10] In the special case when $\Gamma = 1$, output is maximized when $\bar{L} = 1/2$ and, when $\bar{L} = 1$, aggregate output falls to zero.

[11] Friedman (1968) defined the natural rate of unemployment to be the *equilibrium* rate. That definition only makes sense when equilibrium is unique. In our model, there is a continuum of steady state equilibria and in this framework it makes more sense to define the natural rate of unemployment to be the unemployment rate at the social planning optimum.

1.4 Aggregate Demand

There is a continuum of households. Each household lives for two periods and derives utility from consumption when young C_t^y, consumption when old C_{t+1}^o, and real money balances accumulated in the first period of their life M_{t+1}/P_t. Labor does not deliver disutility, and therefore the participation rate is always equal to 1.[12]

Preferences are given by a logarithmic utility function. Households maximize expected utility,

$$u_t = \log\left(C_t^y\right) + \beta \mathbb{E}_t\left[\log\left(C_{t+1}^o\right)\right] + \delta \log\left(\frac{M_{t+1}}{P_t}\right) \tag{1.8}$$

where the mathematical expectation is taken with respect to the future realization of the stock market. We describe determination of the value of the stock market in the next section.

In the first period of their life, households earn labor income $W_t L_t$. They use their income to purchase current consumption $P_t C_t^y$, capital goods $P_{K,t} K_{t+1}$ and government bonds B_{t+1}. All prices are in terms of money.

In the second period of life, households rent capital to firms and earn the rental payment $R_{t+1} K_{t+1}$ and interest accrued on their loan to the government $(1 + i_t) B_{t+1}$. In addition, at the end of the period they sell capital and money to the new young generation. The first and second period budget constraints are given by the following equations:

$$P_t C_t^y + M_{t+1} + B_{t+1} + P_{K,t} K_{t+1} = W_t L_t, \tag{1.9}$$

[12] Allowing for disutility from participation in the labor market would make the participation rate endogenous. We do not pursue that modification here because, in the U.S. data, the participation rate appears to be driven mostly by demography and does not exhibit a pronounced co-movement with unemployment at business cycle frequencies.

$$P_{t+1}C^o_{t+1} = (R_{t+1} + P_{K,t+1})K_{t+1} + (1 + i_t)B_{t+1} + M_{t+1}. \tag{1.10}$$

The no-arbitrage condition (NAC) implies that the return to government bonds must be equal to the return on physical capital, when evaluated in terms of utility from consumption in the second period,

$$\mathbb{E}_t \left[\frac{\beta}{C^o_{t+1}} \left(\frac{1 + i_t}{P_{t+1}/P_t} - \frac{(P_{K,t+1} + R_{t+1})/P_{t+1}}{P_{K,t}/P_t} \right) \right] = 0. \tag{1.11}$$

Here the first term in round parentheses is the expected real interest rate payed on government bonds. The second term in the round parentheses is the real return to physical capital. In words, this equation states that the young are indifferent between investing in bonds and capital. Using this condition, and defining real savings of the young in interest-bearing non-monetary assets as follows,

$$S^y_t = (B_{t+1} + P_{K,t}K_{t+1})/P_t, \tag{1.12}$$

we can write the young's consumption function C^y_t, the demand for real money balances M_{t+1}/P_t and the young's real savings function S^y_t that solve the utility maximization problem:

$$C^y_t = \frac{1}{1 + \beta + \delta} \frac{W_t L_t}{P_t}, \tag{1.13}$$

$$\frac{M_{t+1}}{P_t} = \frac{\delta}{1 + \beta + \delta} \left(\frac{1 + i_t}{i_t} \right) \frac{W_t L_t}{P_t}. \tag{1.14}$$

$$S^y_t = \frac{W_t L_t}{P_t} - C^y_t - \frac{M_{t+1}}{P_t}. \tag{1.15}$$

Substituting for consumption and money balances in (1.15) gives the following alternative expression for saving

$$S^y_t = \frac{1}{1 + \beta + \delta} \left(\beta - \frac{\delta}{i_t} \right) \frac{W_t L_t}{P_t}. \tag{1.16}$$

The saving of the young is an increasing function of the money interest rate because money and consumption are substitutes in utility and the money interest rate is the opportunity cost of holding money. In the traditional IS-LM model, saving is sometimes written as an increasing function of the real interest rate. That channel for the interest rate to influence saving is missing from our model because of our simplifying assumptions that utility is logarithmic and that labor supply occurs only in youth.[13]

To simplify the exposition of our model, we assume that government bonds are in zero net supply and we concentrate on the role of monetary policy. We study a policy in which the central bank keeps the money supply M_t^* constant, and where that policy is expected to continue forever. In that environment we study the effect of an unanticipated change in M_t^* that we implement through an unanticipated cash transfer to the old generation. In future work we plan to study the role of fiscal interventions.

1.5 The Role of Beliefs

Although our work is superficially similar to the IS-LM model and its modern New Keynesian variants; there are significant differences. By grounding the aggregate supply function in the theory of search and, more importantly, by dropping the Nash bargaining assumption, we arrive at a theory where preferences, technology and endowments are not sufficient to uniquely select an equilibrium.

Following Farmer (2012b) we close our model by making beliefs fundamental. Farmer studies that assumption in the context of a purely real representative agent model. In the current paper we explore the implications of multiple steady

[13]Relaxing the unitary elasticity of intertemporal substitution by considering a utility function of the form $U(C^y, C^o, M/P) = \log(C^y) + \beta \log\left(C^o + \bar{C}\right) + \delta \log(M/P)$ would add the real interest rate as an argument of the savings function. When $\bar{C} > 0$, the intertemporal substitution effect dominates the income effect, making the savings function increasing in both money interest rate as the price of money and the real interest rate as the relative price of consumption when old. In this model, we adopt $\bar{C} = 0$ for expository purposes.

state equilibria in a model where money is used as a means of exchange and where the representative agent assumption is replaced by a model of overlapping generations.[14]

The assumption that beliefs are fundamental is not sufficient to explain *how* they are fundamental and the belief function could take different forms. In our view, beliefs are most likely learned and we see the work of George Evans and Seppo Honkapohja (Evans and Honkapohja, 2001) as a promising avenue in describing how a particular belief function may arise. In this respect beliefs are similar to preferences.[15]

Economists assume that a human being is described by a preference ordering and that by the time a person achieves adulthood he or she is able to make choices over any given commodity bundle. But those choices are learned during childhood; they are not inherited. At the age of twenty one, an Italian is likely to choose a glass of wine with a meal; a German is more likely to choose a beer. But a German child, adopted into an Italian family at birth, will grow up with the preferences of his adoptive parents, not with those of his biological parents. Beliefs, in our view, are similar.

During a period of stable economic activity, people learn to make forecasts about future variables by projecting observations of variables of interest on their information from the recent past. When there is a change in the environment, caused by a policy shift or a large shock to fundamentals, they continue to use the beliefs that they learned from the past. That argument suggests that we should treat the parameters of the belief function in the same way that we treat the parameters of the utility function. They are objects that we would expect to remain stable over the medium term and that should be estimated using econometric

[14] Plotnikov (2013, 2019) explores a similar idea in a version of a real business cycle model, closed with Farmer's (2012b) Keynesian search model of the labor market. Plotnikov closes his model with the assumption that beliefs about future human wealth are adaptive and he shows that a model, closed in this way, generates jobless recoveries.

[15] The discussion in this section closely follows the presentation in Farmer (2016a).

methods.

In this paper we investigate one plausible assumption about the belief function and we study its role as a way of closing our model. We assume that beliefs are determined by the equation

$$\mathbb{E}_t^*\left[\frac{P_{K,t+1}}{P_{t+1}}\right] = \Theta_t, \tag{1.17}$$

where the expectations operator in equation (1.17) is subjective and reflects the beliefs of a representative person of the probabilities of future events. To impose discipline on our analysis we assume that expectations are rational; that is,

$$\mathbb{E}_t^*\left[\frac{P_{K,t+1}}{P_{t+1}}\right] = \mathbb{E}_t\left[\frac{P_{K,t+1}}{P_{t+1}}\right] = \Theta_t, \tag{1.18}$$

where the expectation $\mathbb{E}$ is taken with respect to the true probabilities in a rational expectations equilibrium.

Because there is no aggregate investment in our model, capital represents an input in fixed supply. We interpret P_K to be the the average price of assets traded in the stock market and changes in P_K represent self-fulfilling shifts in perceptions of financial wealth.

1.6 The Equations of the Model

The equilibrium of our model is described by the following seven equations. To obtain these equations, we used two facts. First, factor incomes are proportional to GDP,

$$\frac{R_t K_t}{P_t} = \alpha Y_t \quad \text{and} \quad \frac{W_t L_t}{P_t} = (1-\alpha) Y_t \tag{1.19}$$

and second, in a symmetric equilibrium, total employment equals individual employment and each firm employs one unit of capital,

$$L_t = \bar{L}_t \quad \text{and} \quad K_t = 1. \tag{1.20}$$

Now we turn to a description of each of the seven equations that comprise our model.

$$\frac{1-\alpha}{1+\beta+\delta}\left(\beta - \frac{\delta}{i_t}\right) Y_t = \frac{P_{K,t}}{P_t}. \tag{1.21}$$

Equation (1.21) describes equilibrium in the asset markets. It equates the demand for interest bearing assets by the young (the young's real savings function S_t^y) to the real value of the single unit of capital ($P_{K,t}/P_t$) available in the economy. Since government bonds are in zero supply, the young's savings must be equal to the purchases of capital sold by the old generation. Equation (1.21) is our analog of the IS curve.

$$\frac{M_{t+1}^*}{P_t} = \frac{(1-\alpha)\delta}{1+\beta+\delta}\left(\frac{1+i_t}{i_t}\right) Y_t. \tag{1.22}$$

Equation (1.22) is the money market clearing condition and it is our equivalent of the LM curve. Here M_{t+1}^* is the stock of money exogenously determined by the central bank and available for the young generation to hold as part of their optimal portfolio.

$$\mathbb{E}_t\left[\frac{\beta}{C_{t+1}^o}\left(\frac{1+i_t}{P_{t+1}/P_t} - \frac{\left(P_{K,t+1} + \alpha P_{t+1} Y_{t+1}\right)/P_{t+1}}{P_{K,t}/P_t}\right)\right] = 0. \tag{1.23}$$

Equation (1.23) is the no-arbitrage condition (NAC) between the money interest rate and the return to capital. This equation represents the assumption that physical capital and government bonds pay the same rate of return and it has no analog in the simplest version of the IS-LM model.

$$P_t C_t^o = \alpha P_t Y_t + P_{K,t} + M_t^*. \tag{1.24}$$

Equation (1.24) is the expenditure function of the old. It says that the old's expenditure on consumption must be equal to the income plus principal from selling capital plus the value of the money held by the old.

$$Y_t = \left[\left(1 - \frac{L_t}{\Gamma} \right) L_t \right]^{1-\alpha}. \tag{1.25}$$

Equation (1.25) is the social production function. This equation serves only to determine employment and it plays the role of the 45 degree line in the Keynesian Cross model.

Next, real GDP is the sum of the consumption of the two generations

$$Y_t = C_t^y + C_t^o. \tag{1.26}$$

Finally, we add a seventh equation, the belief function (1.27).

$$\mathbb{E}_t \left[\frac{P_{K,t+1}}{P_{t+1}} \right] = \Theta_t. \tag{1.27}$$

The belief function distinguishes our model from the New Keynesian approach and it replaces the New Keynesian Phillips curve. In the absence of this new element, the other six equations would not uniquely determine the seven endogenous variables $\{Y_t, P_t, i_t, P_{K,t}, L_t, C_t^y, C_t^o\}$. The belief function is an equation that determines how much households are willing to pay for claims on the economy's capital stock. It represents the aggregate state of confidence or 'animal spirits' and, in combination with the other six equations of the model, the belief function selects an equilibrium.

In our comparative statics exercises in Section 1.8, we compare two alternative specifications for the belief function. In one specification we assume that,

$$\Theta_t = \Theta \quad \text{for all } t. \tag{1.28}$$

We call this assumption *fixed beliefs* and it amounts to the assumption that, in the collective view of asset market participants, the stock market has some fixed real value measured in terms of the CPI.

In a second specification we assume that

$$\Theta_t = \frac{P_{K,t}}{P_t}. \tag{1.29}$$

We call this assumption *adaptive beliefs* and it amounts to the assumption that, in the collective view of market participants, the real value of the stock market is a random walk. This second assumption, which is a better description of the actual behavior of stock market prices, has a non-standard implication that we draw attention to in Section 1.10.3. It implies that unanticipated shocks to the money supply can have permanent effects on the steady state unemployment rate.[16]

Equations (1.21), (1.22), (1.23), (1.24), (1.26), and (1.27) determine aggregate demand. Given beliefs $\{\Theta_t\}$ and monetary policy M_t^*, these equations select an equilibrium sequence for $\{Y_t, P_t, i_t, P_{K,t}, C_t^y, C_t^o\}$ and equation (1.25) determines how much labor firms need to hire to satisfy aggregate demand. Since employment is determined recursively, in the subsequent parts of the paper we dispense with equation (1.25) in our discussion of equilibrium.

1.7 The IS-LM-NAC Representation of the Steady-State

In this section, we show that the steady-state equilibrium of our model admits a representation that is similar to the IS-LM representation of the *General Theory* developed by Hicks and Hansen. The IS-LM model is a static construct in which the price level is predetermined. To provide a fully dynamic model, Samuelson

[16] Farmer (2012c, 2015) finds evidence that the real value of the stock market and the unemployment rate can be parsimoniously modeled as co-integrated random walks. Our work in this paper provides one possible theoretical model that can explain this finding.

closed the IS-LM model by adding a price adjustment equation that later New-Keynesian economists replaced with the New-Keynesian Phillips curve.

We take a different approach. We select an equilibrium by closing the labor market with a belief function. Our model consists of the IS curve, the LM curve and the NAC curve. The NAC curve is a new element that equates the return to capital to the nominal interest rate. And unlike the interpretation of animal spirits that was popularized by George Akerlof and Robert Shiller (2009), pessimistic animal spirits are fully rational. The people in our model are rational and have rational expectations but they are, sometimes, unable to coordinate on a socially efficient outcome.

The following equations characterize the steady-state equilibrium:

$$\text{IS:} \quad \frac{1-\alpha}{1+\beta+\delta}\left(\beta - \frac{\delta}{i}\right) Y = \Theta, \tag{1.30}$$

$$\text{LM:} \quad \frac{M}{P} = \frac{(1-\alpha)\delta}{1+\beta+\delta}\left(\frac{1+i}{i}\right) Y, \tag{1.31}$$

$$\text{NAC:} \quad i = \frac{\alpha Y}{\Theta}. \tag{1.32}$$

Equations (1.30) – (1.32) determine the three unknowns: Y, i and P, for given values of M and Θ. We treat $\Theta = \mathbb{E}[P_K/P]$ as a new exogenous variable that reflects investor confidence about the real value of their financial assets and by making Θ exogenous we provide a new interpretation of Keynes' idea that equilibrium is selected by 'animal spirits'.

In (Y, i) space, the IS and NAC curves determine Y and i and the price level adjusts to ensure that the LM curve intersects the IS and NAC curves at the steady state. We illustrate the determination of a steady state equilibrium in Figure 1.2.

The IS curve, equation (1.30), is downward sloping and its position is determined by animal spirits, Θ. In a steady state equilibrium, beliefs about future

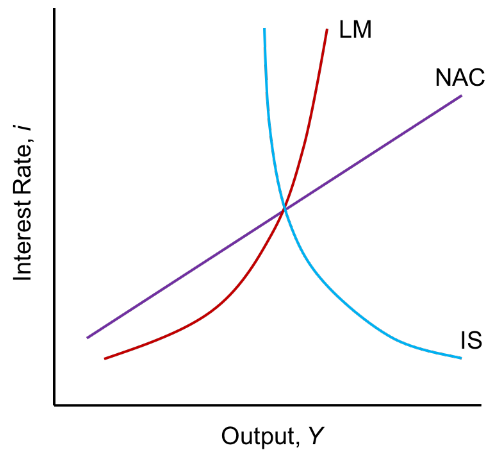

Figure 1.2: The IS-LM-NAC Representation of the Steady State

wealth are self-fulfilling. When people feel wealthy, they *are* wealthy. Beliefs about wealth determine consumption, and firms hire as much labor as necessary to satisfy demand. The value of capital in a rational expectations equilibrium adjusts to match the beliefs.

1.8 Two Comparative Static Exercises

In this section we ask how shifts in exogenous driving variables affect the equilibrium values of Y, i and P. We conduct two comparative static exercises. In the first exercise we increase Θ from a low value to a higher value at some date, $t = 1$, and we assume that it remains constant thereafter. In the second exercise, we hold Θ fixed forever and we increase the stock of money.[17]

Consider first, the experiment of an increase in the belief about the value of financial wealth. A greater value of Θ influences output through two channels. Firstly, since consumers believe, correctly, that they are wealthier, real consumption of goods and services increases. The IS curve shifts to the right. Moreover, higher asset prices reduce the interest rate and the NAC curve becomes flatter.

[17]In section 1.10 we consider an alternative model of expectation formation in which the belief about the future value of capital is equal to its current realized value.

These effects are illustrated in Figure 1.3.

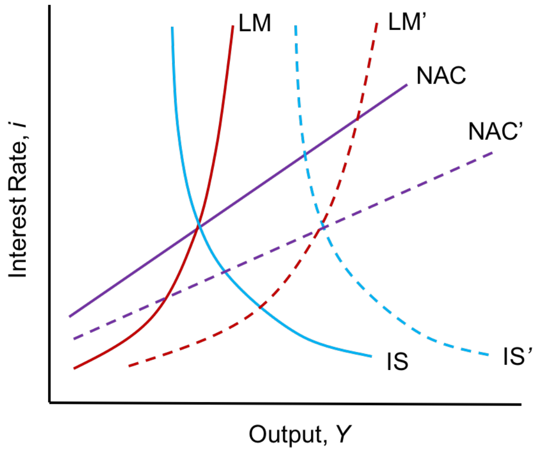

Figure 1.3: An Increase in Confidence

As people become more confident, the IS curves shifts to the right beginning at the solid IS curve and ending at the dashed IS curve. At the same time, the NAC curve shifts down and to the right, from the solid NAC to the dashed NAC curve. Because output increases, the demand for real money balances increases, and the price level must be lower in the new steady state equilibrium. This is reflected on Figure 1.3 by a rightward shift in the LM curve. Because the class of Cobb-Douglas utility functions implies a unitary elasticity of intertemporal substitution, the intertemporal substitution effect and the income effect cancel each other out and, at the new equilibrium, the interest rate remains unchanged.

Consider next, the effect of an increase in the stock of money, which we illustrate on Figure 1.4.

Equations (1.30) and (1.32) determine the equilibrium values of output and the interest rate independently of the stock of money. The demand for real balances depends only on Y and i and, once these variables have been determined, the price level, P, adjusts to equate the real value of the money supply to the real value of money demand. It follows that changes in the supply of money will cause proportional changes in the price level and the nominal value of wealth, leaving output and the interest rate unchanged.

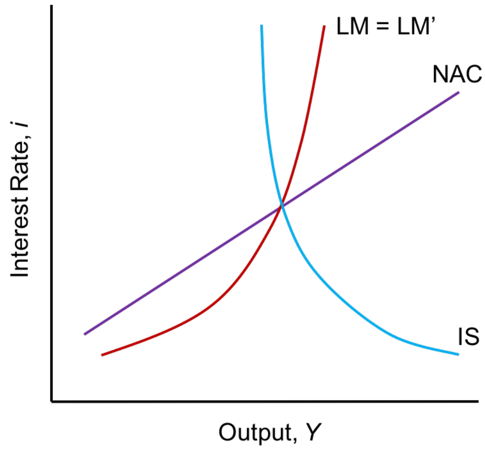

Figure 1.4: An Increase in the Money Supply

Figure 1.4 illustrates the effects of a change in M on a graph. The LM curve after the increase in the money supply is identical with the LM curve before the change, illustrating the concept that money, in our model, is neutral. However, as we will show in Section 1.10, this result depends on the form of the belief function. If beliefs about the future value of financial wealth depend on the current realized value of wealth, an increase in the money supply may have a permanent real effect on output through its effect on business and consumer confidence.

1.9 Dynamic Equilibria

In this section we shift from a comparison of steady states to a description of complete dynamic equilibria. To study the equilibria of the complete model, we use the algorithm, GENSYS, developed by Christopher Sims (2001). First, we choose a constant sequence $\{M, \Theta\}$ to describe policy and we log-linearize the dynamic equations around a steady state. Let

$$x_t \equiv \left[y_t, \tilde{i}_t, p_t, p_{K,t}, \mathbb{E}_t\left[y_{t+1}\right], \mathbb{E}_t\left[p_{t+1}\right], \mathbb{E}_t\left[p_{K,t+1}\right]\right]' \tag{1.33}$$

be log deviations of the endogenous variables from their steady state values. Let

$$\varepsilon_t \equiv [m_t, \theta_t]' \tag{1.34}$$

log deviations of the exogenous variables from their initial values and define three new variables,

$$\eta_t^1 \equiv p_t - \mathbb{E}_{t-1}[p_t], \tag{1.35}$$

$$\eta_t^2 \equiv p_{K,t} - \mathbb{E}_{t-1}[p_{K,t}], \tag{1.36}$$

$$\eta_t^3 \equiv y_t - \mathbb{E}_{t-1}[y_t]. \tag{1.37}$$

These new variables represent endogenous forecast errors. Next, we log-linearize equations (1.21) – (1.23) and equation (1.27) and we append them to equations (1.35) – (1.37). That leads to the following linear system of seven equations in seven unknowns,

$$\Gamma_0 x_t = \Gamma_1 x_{t-1} + \Psi \varepsilon_t + \Pi \eta_t, \tag{1.38}$$

The matrix Ψ is derived from the linearized equations and it explains how shocks to M and shocks to Θ influence each of the equations of the model.

Once we have provided a model of beliefs, the steady state of our system is determinate. For every specification of the belief function, equation (1.27), there is a unique steady state. In this sense, our animal spirits model is similar to any dynamic stochastic general equilibrium model. For a given specification of fundamentals, there is a unique predicted outcome.

But the fact that the model, augmented by a belief function, has a unique steady state, is not enough to uniquely determine a dynamic equilibrium. To establish uniqueness of a dynamic equilibrium, we must show that for every representation of fundamentals, where fundamentals now include beliefs, there is a unique dynamic path converging to the steady state. The uniqueness or nonuniqueness of dynamic equilibria is determined by the properties of the matrices Γ_0 and Γ_1, in equation (1.38).

To establish the properties of a dynamic equilibrium, we must provide a calibrated version of the model since determinacy of equilibrium is, in general, a numerical question. To study determinacy, we used the calibration from Table 1.1.

Table 1.1: Calibration

Parameter	Definition	Value
α	Share of capital in output	.33
β	Subjective discount rate	.50
δ	Coefficient on real money balances in utility	.05

For this calibration, we found that our model has one degree of indeterminacy. In words, that implies that for any set of initial conditions there is a one dimensional continuum of dynamic paths, all of which converge to a given steady state. In practice, it means that the rational expectations assumption is not sufficient to uniquely determine all three of the forecast errors, η_t, as functions of the fundamental shocks, ε_t. When the model displays dynamic indeterminacy, there are many ways that people may use to forecast the future, all of which are consistent with a rational expectations equilibrium (Farmer, 1991, 1999).

Following Farmer (2000), we resolve this indeterminacy by selecting a particular equilibrium for which

$$\eta_t^1 \equiv p_t - \mathbb{E}_{t-1}[p_t] = 0. \tag{1.39}$$

This equation is a special case of equation (1.35). In words, this assumption means that money prices are set one period in advance. It is important to note that price stickiness does not violate the property of rational expectations. The equilibrium with sticky prices is one of many possible equilibria of the economy where agents form self-fulfilling beliefs about wealth and it is an equilibrium that explains an important property of the data: Unanticipated monetary shocks have real short run effects and they feed only slowly into prices.

In our model, the equilibrium is selected by the way that people form beliefs. How should we view the choice of an equilibrium with predetermined prices? Farmer has argued elsewhere (Farmer, 1999) that when there are multiple equilibria, we should allow the data to determine how people form beliefs in the real world. It may be, for example, that a small but unmodeled cost of changing prices leads market participants to an equilibrium where prices are predetermined. Here, we choose to display the properties of the predetermined price equilibrium and we refer the reader to the paper by Farmer and Nicolò (2018) which provides evidence that a predetermined price equilibrium of this kind is a good fit to US data.

1.10 Three Dynamic Experiments

In this section, using the parameter values from Table 1.1, we analyze three dynamic experiments. In the first experiment, we begin from a steady state, and we ask how a permanent unanticipated increase in confidence affects the endogenous variables of the model. In the second and third experiments, we ask how a permanent unanticipated increase in the stock of money affects the economy.

In our second experiment, the belief of households about the future real value of the stock market is invariant to its current value. In our third experiment, households expect the future real value of the stock market to be equal to its current value. We refer to the alternative assumptions in experiments two and three as fixed and adaptive beliefs.

In the case of fixed beliefs, the experiment of increasing the money supply, has the same long-run effects that it would have in a classical model in which output is supply determined: Money is neutral. In contrast, if households form their beliefs adaptively, a permanent increase in the money supply has a permanent effect on output. Money is non-neutral because it increases the real value of the stock market in the short run and that increase is translated, through a confidence effect, into a permanent increase in beliefs about the value of the stock market.

Is it reasonable to think that a change in a nominal variable may have *permanent* real effects? We think so. Farmer (2012a) and Farmer and Nicolò (2018) have estimated a model of the US economy in which beliefs about future income growth are equal to current income growth and they have shown that a belief function of this kind outperforms standard New-Keynesian models closed by a Phillips curve. In their model, the central bank sets the money interest rate and changes in the interest rate have a permanent effect on the unemployment rate by shifting the economy from one equilibrium to another. In our model a similar shift from one steady state equilibrium to another is achieved by an increase in the money supply which raises share prices and has a permanent effect on animal spirits.

1.10.1 Experiment 1: A Shock to Confidence

Figure 1.5 displays the dynamic paths of eight variables in response to a one time increase in beliefs about the future value of capital. We call this a shock to confidence.

Panel (a) depicts the value of beliefs about the future real value of the stock market, $\mathbb{E}_t\left[P_{K,t+1}/P_{t+1}\right]$. This is the variable we refer to as Θ. In our first experiment, Θ increases by one percent and it remains one percent higher for ever after. Panel (b) shows the value of the money supply, which we hold fixed for this experiment.

Panel (c) shows that, in period 2, output increases and remains permanently higher by one percent. This occurs because rational forward-looking consumers increase their spending on goods and services and firms respond by hiring addi-

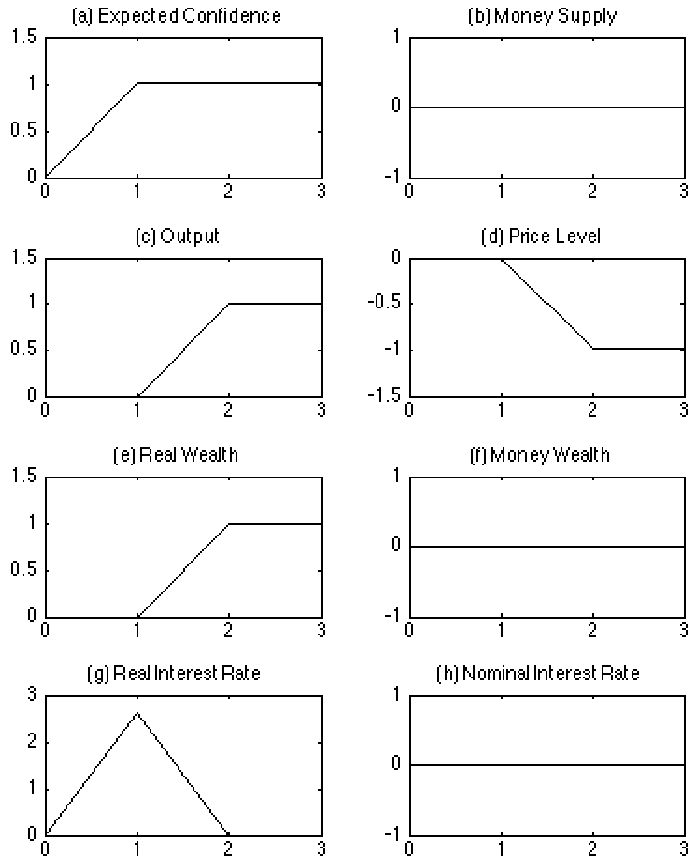

Figure 1.5: A Permanent Shock to Confidence

tional workers to produce these goods. Panel (d) shows that the price level falls and stays permanently lower. Greater output increases the demand for real money balances and the price level must fall to equate the demand and supply of money.

Panel (e) shows that, in period 2, the realized value of real stock-market wealth increases by one percent. That follows from the rational expectations assumption; people expected the value of share prices to increase and, in a rational expectations equilibrium, that belief is supported by the way that people form their beliefs in period 2 and in all subsequent periods. From panel (g), we see that the real interest rate jumps up in period 1 and reverts to its steady-state value thereafter. Because the price level and the money interest rate do not adjust in the first period, the real interest rate adjustment is achieved by a self-fulfilling adjustment to the expected future price level.

We want to draw attention to several features of these impulse responses. First, although adjustment to a confidence shock is delayed, the delay lasts for only one period. That follows from the stylized nature of a model in which there are no endogenous propagation mechanisms. Second, prices do not respond at all in the first period. In the New-Keynesian model, prices are sticky because of adjustment costs or restrictions on choice. Although we are not averse to the possibility that restrictions of this kind may be important in the real world, they are not an essential element of our theory. In our model, prices are fixed because people rationally anticipate that output, not prices, will respond to unanticipated shocks.

If models in this class are to be taken seriously as descriptions of data, they must be tied down by an assumption about how beliefs are formed. To give the model empirical content, one must assume that the belief function remains time invariant at least over the medium term. If that assumption holds, the parameters of the belief function can be estimated in the same way that econometricians estimate preference parameters. See Farmer (2012a) and Farmer and Nicolò (2018) for examples of empirical exercises that estimate a version of this model on U.S. data. These papers tie down the equilibrium of the theoretical model by treating the covariance of prices with contemporaneous variables as a parameter of the belief function. In the empirical work of Farmer (2012a) and Farmer and Nicolò (2018), it is the sticky price equilibrium that best explains data.

1.10.2 Experiment 2: A Shock to the Money Supply with Fixed Beliefs

In subsections 1.10.2 and 1.10.3 we show that the way economic agents form beliefs about the future matters for the long-term effect of monetary shocks.

In subsection 1.10.2, we consider the case of fixed beliefs, which we model with

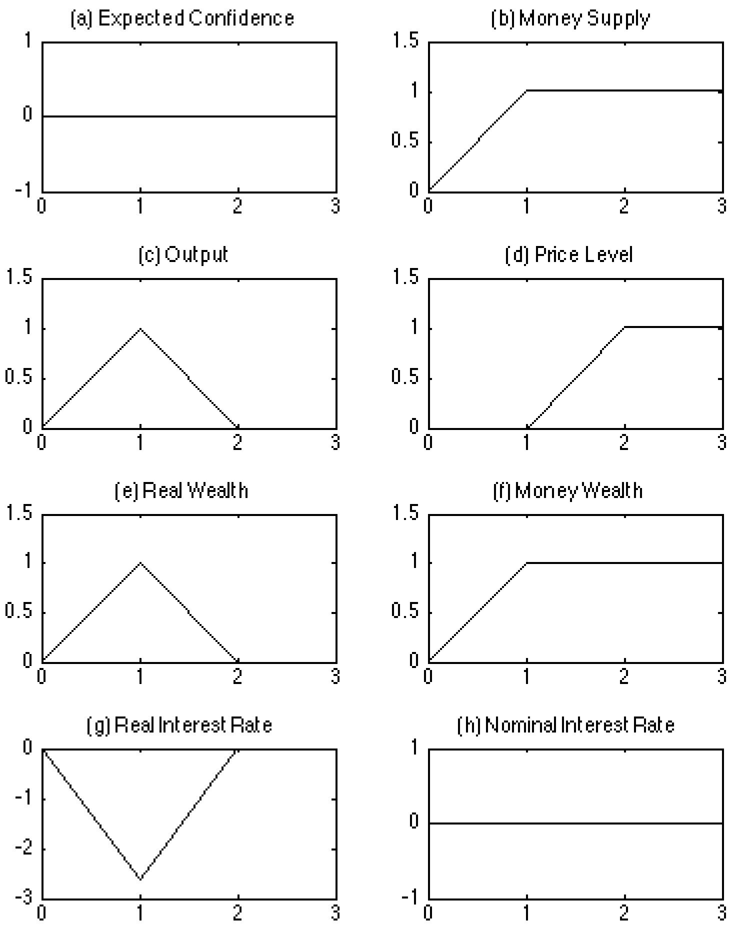

Figure 1.6: A Permanent Shock to the Supply of Money under Fixed Beliefs

equation (1.40),

$$\mathbb{E}_t \left[\frac{P_{K,t+1}}{P_{t+1}} \right] = \Theta. \tag{1.40}$$

Figure 1.6 displays the dynamic paths for the variables of this economy in response to a shock to the money supply when beliefs are modeled in this way. This shock is reflected in Panel (b) which depicts the time path for M. We assume that at date 1, M increases by one percent and that it remains one percent higher forever after. Panel (a) reflects our assumption that beliefs are fixed.

Panel (c) shows that output increases temporarily in the first period by one percent. This happens because prices are predetermined and are unable to adjust until period 2. Instead, the increase in the money supply causes an increase in aggregate demand that is met by a corresponding temporary increase in output and employment. Firms hire more workers to satisfy the increased aggregate demand.

Panel (d) shows that prices respond in period 2 and remain 1 percent higher. This increase neutralizes the increase in the money supply and is consistent with the return to steady state of output reflected in panel (c). Panels (e) and (f) show that the real value of shares in the stock market increases and then returns to its original value. In contrast, the money value of shares in the stock market goes up by one percent and remains permanently higher. From panel (g) we see that the real interest rate falls in period 1 and panel (h) shows that the money interest rate remains constant during the entire exercise.

1.10.3 Experiment 3: A Shock to the Money Supply with Adaptive Beliefs

To model adaptive beliefs, we replace equation (1.40), with equation (1.41),

$$\mathbb{E}_t\left[\frac{P_{K,t+1}}{P_{t+1}}\right] = \frac{P_{K,t}}{P_t}. \tag{1.41}$$

When beliefs are adaptive, households expect the real value of the stock market to be a random walk. This is a special case of a more general model in which beliefs are formed by the following adaptive expectations equation,[18]

$$\mathbb{E}_t\left[\frac{P_{K,t+1}}{P_{t+1}}\right] = \lambda\left(\frac{P_{K,t}}{P_t}\right) + (1-\lambda)\mathbb{E}_{t-1}\left[\frac{P_{K,t}}{P_t}\right], \quad \lambda \in [0,1].$$

Figure 1.7 displays the dynamic paths for the variables of this economy in response to a shock to the money supply when beliefs about the real value of

[18] We have restricted ourselves to the special case of $\lambda = 1$ because Farmer (2012a) estimated a model that allows λ to lie in the interval $[0,1]$ and found that empirically, the data favors a model where $\lambda = 1$.

shares in the stock market are determined by a random walk. We assume that M increases by one percent and that it remains one percent higher for ever after. The shock to the money supply is reflected in Panel (a).

The increase in the stock of money causes an increase in the money price of financial assets; this is shown in Panel (f). Because the price of goods is predetermined, the increase in the nominal share price is also an increase in its real price as shown in Panel (e). Panel (b) shows that beliefs about the future real value of shares respond to this monetary shock and they remain permanently one percent higher in all subsequent periods.

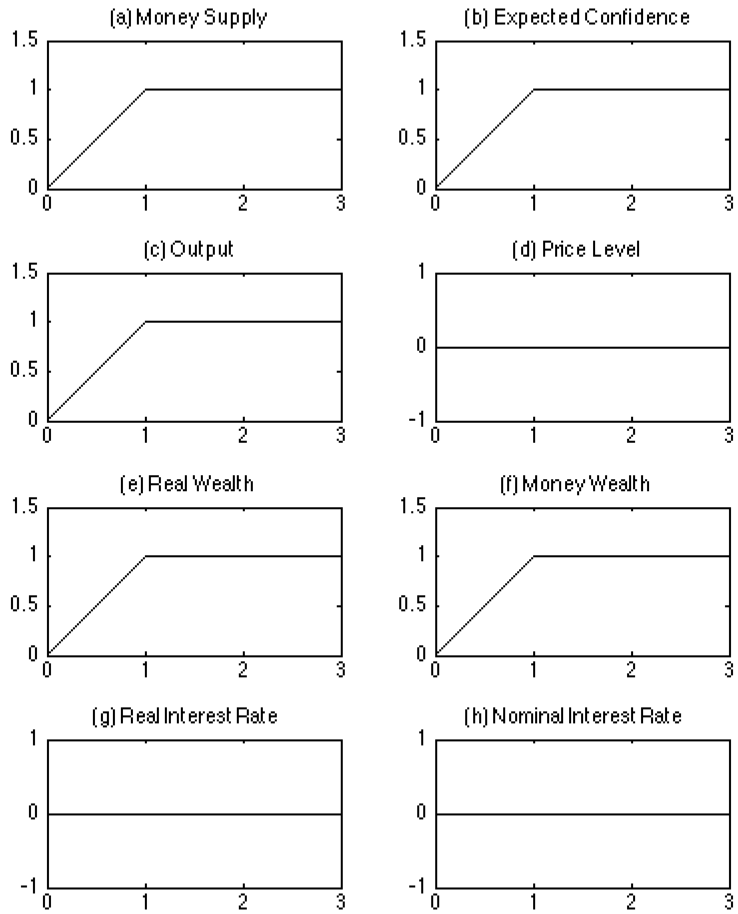

Figure 1.7: A Permanent Shock to the Supply of Money under Adaptive Beliefs

Panel (c) shows that the increase in the real value of the stock market triggers an increase in output that is sustained because of the effect of the increase in the money supply on beliefs, as reflected in Panel (b). Panels (d), (g) and (h) show that the price level, and the real and nominal interest rates do not respond at all to a one off permanent increase in the money supply which is reflected entirely in changes to output and in the real value of financial assets.

1.11 Conclusion

We have proposed a fresh way of thinking about the monetary transmission mechanism. By integrating Keynesian economics with general equilibrium theory in a new way, we have provided an alternative to the IS-LM framework that we call the IS-LM-NAC model. Our new model provides an alternative narrative to New-Keynesian economics to explain how macroeconomic policy influences prices and employment.

Our approach differs from New Keynesian economics in two fundamental ways. First, our model displays dynamic indeterminacy. We focus on a dynamic path with predetermined prices to show that changes in the money supply may affect real economic activity even if all nominal prices are perfectly flexible. Second, our model displays steady state indeterminacy that arises as a consequence of search frictions in the labor market. We replace the classical search assumption that firms and workers bargain over the wage, with the Keynesian search assumption that beliefs about the future value of the stock market select a steady-state equilibrium. In our view, beliefs should be treated as a new fundamental of the model. The belief function advances our understanding of why the unemployment rate is so persistent in real world data.

Finally, we have presented a simple graphical apparatus that can be used by policy makers to understand how policy affects the economy. A new element, the NAC curve, connects the interest rate to current and expected future values of the stock market and it explains how 'animal spirits' influence economic activity.

The IS-LM-NAC framework provides a rich new approach to policy analysis that explains the short-run and long-run effects of policy, without the assumption that prices are prevented from moving by artificial barriers to price adjustment.

1.12 Extension: Generalizing our Approach

A referee has pointed out that our graphical method relies on the assumption that only current income enters decision rules and asks if our model generalizes to models of permanent income or models with long-lived agents. The answer is yes.[19]

1.12.1 The Demand Side of the Economy

Here, we adapt the Blanchard perpetual youth model (Blanchard, 1985) to deliver a version of the IS-LM-NAC model in steady state. To keep the presentation simple, we assume that there is no aggregate uncertainty. The case of aggregate uncertainty can be handled by modeling the pricing kernel using the work of Farmer et al. (2011) and adapting the methods described in Farmer (2018).

Time is discrete and indexed by $t = 0, 1, 2, \ldots$. There is a total mass of population equal to 1. Every agent faces a constant probability λ of surviving into the next period. At the beginning of each period, a fraction $1 - \lambda$ of agents die, and an equal mass of individuals is born. Generations are indexed by the date of birth s. As in Blanchard (1985), we assume the existence of annuities and

[19] A second issue that was raised by a referee is whether our model would survive the introduction of produced capital. We suspect that the answer is yes. But the model would need to be more elaborate in other dimensions. In a one-sector model with reproducible capital, the relative price of capital is pinned down by technology and economic fluctuations cannot be driven simply by the relative price of capital as they are in the model we present in this paper. To give full justice to a model with reproducible capital it would seem to us, that either one should build a two-sector model, or one would need to drop the assumption of a static labor market by allowing for labor as a state variable. Either of those variations would permit the value of the stock-market to diverge from the value of capital and allow the introduction of self-fulfilling beliefs about the relative price of an asset to drive business cycle fluctuations.

a competitive set of life insurance companies that pay every agent an additional return to financial wealth in return for a claim on the agent's wealth upon death. Free entry guarantees zero profit of the insurance companies.

Conditional on surviving, agents discount the future at rate $0 < \beta < 1$. Each generation's preferences are defined over sequences of consumption $\{C_t^s\}_{t=s}^{\infty}$ and real money balances $\{M_{t+1}^s/P_t\}_{t=s}^{\infty}$,

$$\sum_{\tau=0}^{\infty} (\beta\lambda)^{\tau} \left((1-\delta) \log C_{t+\tau}^s + \delta \log \left(\frac{M_{t+1+\tau}^s}{P_{t+\tau}} \right) \right),$$

where consumption and money holdings are indexed by the generation s for $s \leq t$.

Households hold three assets: money balances, M_{t+1}^s, that provide liquidity services and enter the utility function, government bonds B_{t+1}^s that pay the rate of return i_t, and shares S_{t+1}^s of a representative firm that has a value of F_t and that pays dividends D_t. The flow budget constraint takes the form:

$$P_t C_t^s + M_{t+1}^s + B_{t+1}^s + S_{t+1}^s F_t = W_t L_t^s + \frac{1}{\lambda} \left[S_t^s (D_t + F_t) + M_t^s + (1 + i_{t-1}) B_t^s \right].$$

The term $1/\lambda$ in this expression represents the annuity premium paid by the life insurance company in return for a claim on the agent's wealth in the event of death. We also assume that people are born with zero wealth; that is, $S_t^t = 0$, $M_t^t = 0$, and $B_t^t = 0$.

Under the no arbitrage condition and assuming no uncertainty, the dollar return to holding a government bond equals the dollar return to holding stock:

$$1 + i_t = \frac{D_{t+1} + F_{t+1}}{F_t}.$$

Because preferences are given by a Cobb-Douglas utility function, spending on consumption and money holding are each equal to a constant fraction of each agent's wealth:

$$P_t C_t^s = (1-\delta)(1-\beta\lambda)\left[\frac{1}{\lambda}(S_t^s(D_t+F_t)+M_t^s+(1+i_{t-1})B_t^s)+H_t^s\right],$$

$$\frac{i_t}{1+i_t}M_{t+1}^s = \delta(1-\beta\lambda)\left[\frac{1}{\lambda}(S_t^s(D_t+F_t)+M_t^s+(1+i_{t-1})B_t^s)+H_t^s)\right],$$

where H_t^s is the nominal value of human wealth defined recursively as

$$H_t^s = W_t L_t^s + \frac{\lambda}{1+i_t}H_{t+1}^s.$$

Define the following aggregate variables:

$$C_t = \sum_{s=-\infty}^{t}(1-\lambda)\lambda^{t-s}C_t^s, \quad H_t = \sum_{s=-\infty}^{t}(1-\lambda)\lambda^{t-s}H_t^s,$$

$$B_t = \sum_{s=-\infty}^{t}(1-\lambda)\lambda^{t-s}B_t^s, \quad S_t = \sum_{s=-\infty}^{t}(1-\lambda)\lambda^{t-s}S_t^s,$$

$$M_t = \sum_{s=-\infty}^{t}(1-\lambda)\lambda^{t-s}M_t^s.$$

where recall that, in period t, the newborns have zero assets.

The demand side of the economy is completely described by the following three equations:

$$P_t C_t = (1-\delta)(1-\beta\lambda)\left[S_t(D_t+F_t)+M_t+(1+i_{t-1})B_t+H_t\right],$$

$$\frac{i_t}{1+i_t}M_{t+1} = \delta(1-\beta\lambda)[S_t(D_t+F_t)+M_t+(1+i_{t-1})B_t+H_t],$$

$$H_t = W_t L_t + \frac{\lambda}{1+i_t}H_{t+1},$$

plus the no-arbitrage condition

$$1 + i_t = \frac{D_{t+1} + F_{t+1}}{F_t}.$$

1.12.2 The Supply Side of the Economy

The production sector is comprised of a continuum of identical perfectly competitive firms. Because all firms will make the same decisions, we consider the problem of a representative producer.

The firm owns a unit of non-reproducible capital and hires labor to produce goods. The production function is given by

$$Y_t = X_t^{1-\alpha},$$

where X_t is labor used in production.

In period t, the firm hires L_t workers that must be assigned to production, X_t, and recruiting, V_t,

$$L_t = X_t + V_t.$$

Each worker assigned to recruiting hires q_t workers,

$$L_t = q_t V_t.$$

The firm maximizes its value which equals the discounted present value of its cash flow, all of which is paid out as dividends,

$$D_t = P_t Y_t - W_t L_t.$$

The value maximization problem is constrained by the following two equations,

$$L_t = X_t + V_t,$$

and

$$L_t = q_t V_t.$$

Because we assume that workers are fired and rehired every period, the firm's problem reduces to a sequence of static maximization problems with the following first order condition which holds in every period,

$$(1-\alpha) P_t Y_t = W_t L_t.$$

Dividends, in the optimal solution, are given by

$$D_t = \alpha P_t Y_t.$$

At the aggregate level, there exists a matching technology,

$$q_t \bar{V}_t = (\Gamma \bar{V}_t)^{\frac{1}{2}},$$

and, in equilibrium, $\bar{L}_t = L_t$, $\bar{V}_t = V_t$, and $L_t = (\Gamma V_t)^{1/2}$.

1.12.3 Equilibrium

An equilibrium is a sequence of consumption, production, employment, asset holdings and prices such that the above equations hold, the goods market clears, $C_t = Y_t$, the demand for money equals the fixed supply, $M_{t+1} = M^*_{t+1}$, debt is in zero net supply, $B_{t+1} = 0$, and the stock of capital is fixed, $S_{t+1} = 1$. We allow unemployment to be different from the social optimum and we close the model by assuming that agents form beliefs about the real value of the stock market,

$$\frac{F_{t+1}}{P_{t+1}} = \Theta_t.$$

1.12.4 Steady state

To derive the model equations in the steady state we first find expressions for Q, V, X and Y as functions of steady-state employment, L. These functions are derived as follows. Using the labor recruiting equation,

$$L = qV,$$

the labor allocation identity,

$$X + V = L,$$

plus the aggregate matching function and the production function we attain the following steady-state relationships

$$q = \frac{\Gamma}{L}, \quad V = \frac{L^2}{\Gamma}, \quad X = L - \frac{L^2}{\Gamma}, \quad Y = \left(L - \frac{L^2}{\Gamma}\right)^{1-\alpha}. \tag{1.42}$$

Because the production function is non-monotonic in L we assume that in any search equilibrium, the economy is on the rising part of the production function; that is, $L < \frac{\Gamma}{2}$. Given this assumption, for any

$$Y < \left(\frac{\Gamma}{2}\left(1 - \frac{\Gamma}{2}\right)\right)^{1-\alpha},$$

we can find a unique value of L and, from equations (1.42) we can find expressions for steady-state q, V and X . Further, from the first-order conditions of individual firms we have,

$$\frac{WL}{P} = (1 - \alpha)Y,$$

and

$$\frac{D}{P} = \alpha Y.$$

By substituting these expressions into the demand equations for consumption and real balances and using goods market clearing, $C = Y$, the assumption that debt is in zero net supply, $B = 0$, and the fact that there is a single non-reproducible unit of capital, $S = 1$, we arrive at the following four equations in the five variables Y, F/P, M/P, H/P, and i,

$$Y = (1-\delta)(1-\beta\lambda)\left[\alpha Y + \frac{F}{P} + \frac{M}{P} + \frac{H}{P}\right], \tag{1.43}$$

$$\frac{i}{1+i}\frac{M}{P} = \delta(1-\beta\lambda)\left[\alpha Y + \frac{F}{P} + \frac{M}{P} + \frac{H}{P}\right], \tag{1.44}$$

$$\frac{H}{P} = (1-\alpha)Y + \frac{\lambda}{1+i}\frac{H}{P}, \tag{1.45}$$

$$1 + i = \frac{\alpha Y + F/P}{F/P}. \tag{1.46}$$

To close the system we assume that beliefs about the stock market (animal spirits) are given by the expression $F/P = \Theta$. These four equations determine the four real variables: Y, i, M/P, and H/P. The steady state price level, P, is determined by the nominal money supply M.

Summarizing this discussion, we can characterize steady-state equilibrium in a long-lived agent model with three equations. Two of these equations, the LM curve and the no-arbitrage equation, are identical with the two-period-lived model. These are given by the expressions,

$$\frac{M}{P} = \frac{\delta}{1-\delta}\frac{1+i}{i}Y. \tag{LM Curve}$$

and,

$$i = \frac{\alpha Y}{\Theta}. \tag{NAC Curve}$$

Note that the LM curve has a positive slope in space (Y, i) space.

To obtain the IS curve, substitute for M/P and H/P in equation (1.43) and divide both sides by Y to give:

$$1 = (1-\delta)(1-\beta\lambda)\left[\alpha + \frac{\Theta}{Y} + \frac{\delta}{1-\delta}\frac{1+i}{i} + \frac{1-\alpha}{1-\frac{\lambda}{1+i}}\right]. \qquad \text{(IS Curve)}$$

By totally differentiating this equation and evaluating the partial derivatives at the steady state one readily verifies that the IS curve defines a downward sloping relationship in (Y, i) space. Further, one verifies that increases in confidence, which we attribute to Θ, shifts the IS curve to the right.

CHAPTER 2

The Forward Premium Anomaly: Overlapping Generations, Multiple Equilibria, and Shocks to Beliefs

2.1 Introduction

Uncovered Interest Parity (UIP) is a central concept in international finance. Based on the principle of no arbitrage opportunities, UIP states that exchange rates of high interest rate countries should depreciate over time. Under UIP, investors engaging in the carry trade, i.e. those who borrow in a low interest rate currency and invest in a high interest rate currency, should not be able to earn systematic excess returns. Competitive international financial markets should eliminate profitable opportunities, and any non-zero interest differential should be offset by the subsequent dynamics of the exchange rate. In the data, the opposite is true: high interest rate currencies tend to appreciate, opening up profitable opportunities for the carry trade. Systematic observed violation of UIP is known as the forward premium anomaly. Standard open economy models, in which movements in the exchange rate are driven by fundamentals are unable to generate a large and volatile risk premium. In contrast, I show that a model driven by non-fundamental shocks (sunspots) *can* produce substantial movements in the risk premium and that these movements are able to account for the profitability of the carry trade.

Another empirical regularity concerns the connection between the current interest differential and future excess returns on the carry trade. UIP predicts that today's interest differential should not be able to predict the profitability of the carry trade in the future. However, the data demonstrate a strong correlation between current interest differentials and future excess returns: at a horizon of one year and up, high interest rate currencies earn negative excess returns. Furthermore, co-existence of these two empirical findings, described above, constitutes a third puzzle. A theory of the joint determination of exchange rates and interest rates must explain why high interest rates predict positive excess returns on the carry trade at short horizons but negative excess returns at longer horizons. Models that attempt to rationalize the correlation between high interest rates and negative excess returns typically assume that the UIP condition holds.

This paper explores a novel theoretical mechanism to explain the joint determination of exchange rates, interest rates and risk premia. A standard explanation of the first empirical regularity suggests that the carry trade involves foreign exchange risk and that, as a consequence, investors demand a risk premium (e.g. Backus et al. (1995). However, the second empirical observation implies that high interest rates must be associated with a relatively small future risk. A complete model must explain the reversal in the sign of the covariance between the interest rate and excess returns over longer horizons.

This paper presents a simple stylized model to rationalize the observed link between the interest rate differential, the exchange rate and the risk premium. In my model, exchange rates are much more volatile than the underlying fundamentals. I propose a new source of fluctuations in exchange rates: volatile shocks to beliefs about exchange rates. I assume that agents have rational expectations and, to justify this assumption, I rely on the multiplicity of dynamic equilibria caused by incomplete participation of investors in the financial market. Following the literature on sunspots and self-fulfilling prophecies[1], I introduce incomplete participation with a model of overlapping generations. The use of incomplete participation to explain financial market anomalies is not new in the literature (e.g. Alvarez and Jermann, 2001; Storesletten et al., 2007). Here, I apply it in a novel way. Agents who are currently alive form expectations about the decisions of future newborns. These expectations are summarized in expectations about the future exchange rate. Due to the multiplicity of equilibria, a range of beliefs can be supported in equilibrium.[2] Each dynamic path is associated with different self-fulfilling beliefs about the future exchange rate. I impose a structure for how self-fulfilling beliefs are formed, and I allow these beliefs to be subject to shocks. Shocks to beliefs shift the economy from one equilibrium to another, causing fluctuations in the exchange rate.

To keep the structure simple, I assume that there is no intrinsic uncertainty in the model: endowments, preferences and policy are time and state invariant and known by everyone. The only source of uncertainty is shocks to beliefs. These shocks affect the current state of the economy but also determine expectations about the future. Shocks to beliefs are sunspot shocks (as in Cass and Shell, Cass and Shell), they are not related to the fundamentals. Instead, they appear as investor sentiments that might be explained as the result of narratives. The beliefs of agents will turn out to be correct, as long as future agents continue to form beliefs in the same way. Shocks to beliefs affect the economy directly by redistributing wealth across the two countries and changing domestic and foreign interest rates. This mechanism leads to foreign exchange risk that causes high interest currencies to pay a risk premium in equilibrium. Following Farmer (1999, 2013) and Farmer and Platonov (2019), I introduce a belief function that describes the formation of beliefs about the real exchange rate, and I assume that these

[1] This literature was pioneered by Cass and Shell (1983), Azariadis (1981), Farmer and Woodford (1997).

[2] See also Geanakoplos and Polemarchakis (1991) and Demichelis and Polemarchakis (2007) on multiplicity of equilibria in economies with overlapping generations.

beliefs are subject to time-varying volatility. The assumption of time-varying stochastic volatility of the belief function is new to this paper, and it is this assumption that allows me to solve the forward premium anomaly.

Violation of uncovered interest parity is explained as follows. When agents believe that the exchange rate will appreciate, they prefer to borrow in a foreign currency and invest in the domestic currency. When all else is equal, the expected appreciation will deliver profit from the carry trade. However, agents, recognizing that there is foreign exchange risk, will demand excess returns to compensate them for this risk. This risk premium causes the domestic interest rate to increase relative to the foreign interest rate. If agents believe that periods with high uncertainty precede periods of low uncertainty, they will expect negative excess returns in the future. This reversal in volatility explains the sign reversal of expected returns over longer horizons.

To test my theory empirically, I investigate the ability of lagged expectations to help explain the UIP anomalies. In my model, the period $t-1$ expectation about the period t exchange rate is a state variable. What agents believed the equilibrium would be, matters for the subsequent dynamics of the economy. Typically, past expectations appear as a state variable in multiple equilibria models and they do not enter as state variables models with a unique equilibrium driven by fundamentals. A multiple equilibria model is, for this reason, a natural candidate to explain the UIP anomalies.

When testing UIP empirically, I show that estimates of past expectations, generated by a vector error correction model, explain much of the deviation from UIP. In the case of Japan, the forward premium anomaly and the reversal puzzle disappear completely once past expectations are controlled for. For the other G7 countries, the covariance between interest differentials and excess returns becomes substantially closer to zero when past expectations are taken into account. These findings suggest that past expectations matter for the determination of the current exchange rate and the interest differential.

There are two key assumptions in the model. First, incomplete participation generates a multiplicity of dynamic equilibria. This allows me to impose a rule that selects the equilibrium, and I postulate it in a form of beliefs about the real exchange rate. Following Farmer (1999), I call it a belief function. Introduction of the belief function does not violate the rational expectations assumption; it simply puts a Markovian structure on the manifold of equilibria. Second, I assume that shocks to beliefs have time-varying volatility (in the spirit of Bansal and Yaron, 2004) and that this volatility depends on the history of the realized shocks, as in the Cox-Ingersoll-Ross process (Cox et al., 1985). This makes interest rates correlate with the risk premium.

The overlapping generations structure rationalizes another puzzle, the Backus-Smith puzzle (Backus and Smith, 1993). In the data, relative aggregate consumption exhibits a weakly negative correlation with the real exchange rate. The vast majority of international economics models produce a positive correlation because the real exchange rate is determined by the relative supply of goods. In my model, the exchange rate is determined by the relative *demand* for goods. When domestic consumption is high, due to home bias in consumption, the price of the domestic good is high relatively to the price of the foreign good, and the real exchange rate is low.

Engel (2016) and Valchev (2017) document the puzzle regarding the reversal in the sign of the covariance between the interest differential and the risk premium, and Bacchetta and van Wincoop (2018), Engel (2016), Itskhoki and Mukhin (2017), and Valchev (2017) offer explanations of this puzzle. This paper contributes to this literature. My model differs crucially in what drives uncertainty. Bacchetta and van Wincoop (2018) consider delayed portfolio adjustment, Engel (2016) studies shocks to assets of various liquidity, Valchev (2017) looks at interaction of monetary and fiscal policy, and Itskhoki and Mukhin (2017) assume the risk premium is subject to financial shocks that hit the Euler equation directly. In contrast, my model is driven by shocks to self-fulfilling beliefs. Papers that explain why high interest rates are associated with low risk usually assume that UIP holds, and papers that account for the violation of UIP cannot explain the reversal. My paper can explain the forward premium anomaly *and* the reversal in a single framework.

More generally, my paper bridges three literatures: Survey-based evidence for violation of UIP, theoretical explanations based on the time-varying risk premium under rational expectations and non-rational expectations. Backus et al. (1995), Bansal and Shaliastovich (2013), Colacito and Croce (2013), Farhi and Gabaix (2015), Gabaix and Maggiori (2015), and Verdelhan (2010) build a model where volatile fundamentals drive the risk premium. Alvarez et al. (2009) study a model where the risk premium arises due to incomplete participation of investors in the financial markets. However, Burnside et al. (2010) shows that fundamentals cannot explain the observed excess returns on the carry trade. Froot and Frankel (1989) and Bussiere et al. (2018) use surveys about the expected exchange rate to conclude that UIP is violated due to expectational errors and not uncertainty about fundamentals. Bacchetta and van Wincoop (2007), Burnside (2011), Ilut (2012), and Gourinchas and Tornell (2004) abandon rational expectations. This paper connects these literatures by using the machinery developed for unique equilibria models but letting the exchange rate be driven by beliefs, and the risk premium be associated with volatile exchange rates, not the fundamentals of the economy.

The mechanism of my model relies on the existence of a multiplicity of dynamic equilibria (dynamic indeterminacy). The indeterminacy is of the same kind as in Farmer and Woodford (1997) where the overlapping generations demography makes expectations about the future self-fulfilling. I also rely on the indeterminacy of exchange rates in an overlapping generations economy of the kind introduced by Kareken and Wallace (1981). I extend their analysis by introducing a belief function. My work generalizes the models of dynamic indeterminacy surveyed in Farmer (2016b) by allowing for time-varying volatility in the belief process.

The paper is organized as follows. Section 2.2 summarizes the evidence on interest rate differentials and excess returns. Section 2.3 develops an intentionally stylized two-country model with international trade and capital flows. Section 2.4 establishes the existence of dynamic indeterminacy and introduces the belief function. Section 2.5 studies the stochastic properties of the simulated model, the response of the economies to the shocks and demonstrates the main result. Section 2.6 discusses the chosen modeling approach and its implications.

2.2 Evidence on Uncovered Interest Parity

This section reproduces the well-documented evidence on violation of uncovered interest parity (UIP) for nominal and real interest rates, and nominal and real exchanges rates. Violation of UIP has been widely documented, e.g. Bilson (1981), Fama (1984), Backus et al. (1995), Engel (1996, 2016) and others. Empirical tests of UIP have received a lot of attention in the literature for two reasons. First, most open economy models predict UIP and rely on its implications. Second, UIP robustly fails for many high-income low-inflation country pairs. Understanding why UIP fails is detrimental for understanding the dynamics of the exchange rates.

2.2.1 Nominal Interest Rates and Nominal Exchange Rates

Let s_t be the nominal exchange rate of a foreign currency vis-a-vis the U.S. dollar expressed as the amount of dollars per unit of the foreign currency. As is common in the literature, the United States is always the home country. Let i_t and i_t^* be the monthly riskless money interest rate in the home and foreign countries respectively. These interest rates are determined in period t and pay off in period $t+1$.

An investor faces a choice of investing in a domestic safe asset or a foreign safe asset. The total return on the foreign safe asset, however, is random as it includes depreciation of the domestic currency. A standard textbook indifference condition for a risk-neutral investor states that, if investors are risk neutral, the following condition must hold:

$$1 + i_t = (1 + i_t^*) \frac{\mathbb{E}_t \left[s_{t+1} \right]}{s_t},$$

or approximately in logs,

$$\mathbb{E}_t \left[\Delta \log s_{t+1} \right] = i_t - i_t^*.$$

This condition states that, on average, the two investment opportunities are expected to deliver the same rate of return. If, for instance, the domestic interest rate exceeds the foreign interest rate by 1%, rational investors should expect depreciation of 1%, so that there are no expected gains from carry trade. The expression

$$\rho_{t+1} = \Delta \log s_{t+1} + i_t^* - i_t,$$

represents excess returns on carry trade, which should on average be equal to zero and should not exhibit a non-zero correlation with the interest rate differential $i_t^* - i_t$. If there were systematic correlation, investors would take on this profitable opportunity, and the exchange rate would adjust in such way that positive excess returns are eliminated.

Definition 1. *Uncovered Interest Parity (UIP) holds if in the regression (called the Fama regression),*

$$\rho_{t+1} = a + b(i_t^* - i_t) + u_{t+1} \tag{2.1}$$

Table 2.1: The Fama Regression

Country	Slope $\hat{b}$	90% Interval	Intercept $\hat{a}$	90% Interval
Canada	2.271	(1.186, 3.355)	−.045	(−.250, .160)
France	1.216	(−.171, 2.603)	−.028	(−.346, .290)
Germany	2.091	(.599, 3.583)	.192	(−.136, .520)
Japan	3.713	(2.390, 5.036)	.924	(.504, 1.343)
United Kingdom	3.198	(1.170, 5.225)	−.410	(−.768, −.051)
G6	2.467	(.769, 4.164)	.054	(−.184, .292)

Note: Estimates of the regression $\rho_{t+1} = a + b(i_t^* - i_t) + u_{t+1}$. Under UIP, $b = 0$. The confidence intervals are based on the Newey-West robust standard errors. Monthly data, 1979:6–2009:10. Reproduced from Engel (2016).

the slope coefficient b is equal to zero.[3][4]

Table 2.1 displays estimates of the Fama equation (2.1) for a variety of high-income low-inflation countries and reproduces the findings previously documented in the literature. For all currencies the point estimates of b are positive and, except for France, statistically different from zero. For France, the confidence interval lies primarily above zero, although it includes zero.[5] Positive slopes mean that high interest rate currencies tend to systematically earn excess returns and that the exchange rate does not adjust to cancel the expected profit from the carry trade. In fact, when $b > 1$, there are not only expected excess returns due to the interest rate differential, but also the exchange rate moves in the opposite direction than predicted by theory. For instance, for Canada the slope estimate of 2.271 means that when the interest rate in Canada denoted i_t^* exceeds the interest in the United States denoted i_t, by 1%, an investor borrowing in the U.S. dollars and investing

[3] Some versions of this definition impose $a = 0$ and $b = 0$.

[4] An equivalent way to define UIP is to require that, in the regression $\Delta \log s_{t+1} = \tilde{a} + \tilde{b}(i_t - i_t^*) + u_{t+1}$, the slope coefficient $\tilde{b} = 1$. Note that these parameters are related to those in (2.1) by $a = \tilde{a}$ and $b = 1 - \tilde{b}$.

[5] Before adoption of the euro, Germany and France were part of the European Exchange Rate Mechanism and kept their exchange rates fixed to each other. Provided that Germany represents a stronger economy than the French one, monetary policy in France can be considered as adjusting the exchange rate of the French frank to the Deutschemark. This explains why the confidence interval for the slope estimate for France includes zero.

in Canadian dollars is expected to earn 2.271% profit, where 1% comes from the interest rate differential and 1.271% comes from the favorable depreciation of the U.S. dollar. For all the pairs of currencies, except the yen and the pound sterling, the intercept is statistically indistinguishable from zero.

Lemma 1. *Under UIP, if the slope parameter b in the Fama equation* (2.1) *is greater than 1/2, excess returns ρ_{t+1} must be more volatile than the exchange rate $\Delta \log s_{t+1}$:*

$$var(\rho_{t+1}) > var(\Delta \log s_{t+1}).$$

Proof. Under the null that UIP holds, the probability limit of the estimator $\hat{b}$ in the Fama regression (2.1) is given by

$$\text{plim } \hat{b} = \frac{\text{cov}(\rho_{t+1}, i_t^* - i_t)}{\text{var}(i_t^* - i_t)} = \frac{\text{cov}(\Delta \log s_{t+1}, \rho_{t+1} - \Delta \log s_{t+1})}{\text{var}(\rho_{t+1} - \Delta \log s_{t+1})}.$$

The requirement that plim $\hat{b} > 1/2$ is equivalent to $\text{var}(\rho_{t+1}) > \text{var}(\Delta \log s_{t+1})$. □

Lemma 1 states that an explanation of UIP must explain why excess returns ρ_{t+1} (and, under rational expectations, expected excess returns $\mathbb{E}_t [\rho_{t+1}]$) are more volatile than the exchange rate $\Delta \log s_{t+1}$. This constitutes a challenge for models where uncertainty is related to the fundamentals.

2.2.2 UIP and the Real Interest Rates

This section documents the violation of UIP and the forward premium anomaly for real interest rates. Because the ex-ante (expected) real interest rates are unobserved, they must be estimated before testing UIP. The methodology of this section is based on Engel (2016).

Define the real exchange rate

$$\log q_t = \log s_t + \log p_t^* - \log p_t$$

where p_t and p_t^* are the consumer price indices. Let the ex-ante real interest rate be

$$r_t = i_t - \mathbb{E}_t \left[\Delta \log p_{t+1} \right], \quad r_t^* = i_t^* - \mathbb{E}_t \left[\Delta \log p_{t+1}^* \right].$$[6]

The ex-ante counterpart of the Fama regression with real interest rates would be the equation,

$$\mathbb{E}_t \left[\Delta \rho_{t+1} \right] = \alpha + \beta (r_t^* - r_t) + v_t \tag{2.2}$$

where v_t is the error term.

To estimate the expected real depreciation rate and the ex-ante real interest rate, I construct a vector error correction model (VECM) in three variables: the nominal exchange rate, the nominal interest differential, and the price differential. Let

$$x_t = \begin{bmatrix} \log s_t \\ i_t - i_t^* \\ \log p_t^* - \log p_t \end{bmatrix}.$$

The empirical model takes the form

$$\Delta x_t = A x_{t-1} + A_0 + \sum_{l=1}^{L} A_l \Delta x_{t-l} + e_t. \tag{2.3}$$

where A, A_0 and A_l, $l = 1, \ldots, L$ are the conforming matrices of the coefficients to be estimated, and e_t is the error term. Matrix A is constrained in such a way that the real exchange rate is stationary. Having estimated the vector error correction model, I obtain the estimates of the expected excess returns $\hat{\mathbb{E}}_t \left[\rho_{t+1} \right]$ and of the

[6] This is an approximation of the real interest rate as this definition omits the inflation risk premium. Ang et al. (2008) study the inflation risk and the real interest rates in the United States in detail.

Table 2.2: The Fama Regression with Real Interest Differential

Country	Model A $\hat{\beta}$	Model A 90% Interval	Model B $\hat{\beta}'$	Model B 90% Interval	$\hat{\gamma}$	90% Interval
Canada	.735	(.140, 1.295)	.380	(−.142, .902)	.186	(−.014, .387)
France	1.328	(.825, 1.831)	.790	(.277, 1.303)	.542	(.365, .719)
Germany	1.592	(.569, 2.615)	.580	(.084, 1.076)	.525	(.384, .667)
Japan	2.233	(1.719, 2.887)	−.198	(−.714, .319)	.888	(.786, .990)
United Kingdom	1.843	(1.347, 2.335)	.571	(.279, .864)	.774	(.680, .867)
G6	1.886	(1.201, 2.562)	.918	(.322, 1.314)	.647	(.497, .797)

Note: Model A: $\hat{\mathbb{E}}_t[\rho_{t+1}] = \alpha + \beta(\hat{r}_t^* - \hat{r}_t) + v_t$ and β is reported. Model B: $\hat{\mathbb{E}}_t[\rho_{t+1}] = \alpha' + \beta'(\hat{r}_t^* - \hat{r}_t) + \gamma\hat{\mathbb{E}}_{t-1}[\rho_t] + w_t$ and β' and γ are reported. The point estimates and the confidence intervals are bootstrapped. $L = 3$ in the VECM. Monthly data, 1979:6–2009:10.

real interest differential $(\hat{r}_t^* - \hat{r}_t)$ by using

$$\hat{\mathbb{E}}_t[\Delta x_{t+1}] = \hat{A}x_t + \hat{A}_0 + \sum_{l=1}^{L} \hat{A}_l \Delta x_{t-l+1} \tag{2.4}$$

where the "hats" denote the OLS estimates of the parameters of the empirical model and of the implied expectations. Estimates of j-step ahead expectations $\hat{\mathbb{E}}_t[\Delta x_{t+j}]$ are constructed in the same way.

The estimates of the Fama slope β in real terms from (2.2) are presented in column (A) of Table 2.2. The slopes are statistically above 1/2, except for Canada. For Canada, the point estimate is positive and greater than 1/2; the confidence interval excludes zero. This confirms the violation of UIP in real terms for all the countries in the sample and existence of the forward premium anomaly, when the real exchange rates and real interest rates are considered, and suggests that this regularity is not a purely monetary phenomenon.

2.2.3 UIP and Expectations

This section presents the main empirical contribution of the paper. It shows that the Forward Premium Anomaly arises due to the process for market expectations.

The positive estimates of the Fama slope $\hat{\beta}$ could be caused by an omitted confounding factor that is correlated with the error term v_t in the Fama regression (2.2). Under the null hypothesis that UIP holds, i.e. the true parameter $\beta = 0$, the estimator $\hat{\beta}$ measures the covariance of the interest rate differential with the expected excess returns:

$$\text{plim } \hat{\beta} = \frac{\text{cov}(\hat{r}_t^* - \hat{r}_t, v_t)}{\text{var}(\hat{r}_t^* - \hat{r}_t)}.$$

If the covariance of the error term with the interest rate differential is large and positive, the resulting estimates will be positive.

I find that past expectations about the excess returns are negatively correlated with the real interest differential, driving the Fama estimates negative, and that they explain the Forward Premium Anomaly and from 70% to 100% of the violation of UIP as measured by an increase in the slope estimate of the Fama regression β in (2.2). I estimate the Fama equation again where I add the expectations made at time $t-1$ about the excess returns earned in period t:

$$\hat{\mathbb{E}}_t [\rho_{t+1}] = \alpha' + \beta'(\hat{r}_t - \hat{r}_t^*) + \gamma \hat{\mathbb{E}}_{t-1} [\rho_t] + w_t. \tag{2.5}$$

The estimated coefficients β' and γ are presented in Table 2.2, column (B). It shows that for all countries in the sample the inclusion of past expectations makes the coefficient on the interest smaller and closer to zero. Controlling for past expectations, positive interest differentials predict depreciation, consistent with UIP. For Canada and Japan, the slope becomes statistically indistinguishable from unity, so that the past expectations fully explain the violation of UIP. For Germany, the lower bound of the confidence interval does not reach zero but is very close to it. For the other countries, the slope is still statistically above zero, but, nevertheless, the estimates turn smaller than one and closer to zero.

The violation of UIP implies ex-ante (and ex-post) profitability of the carry trade. This section shows that profitability is explained by past expectations. For all the countries the estimates of γ are statistically positive. One possible explanation for this result is that investors extrapolate past expectations and that they earn excess returns simply because they expected profits in the past. Patterns of this kind have been found for other assets (e.g. Greenwood and Shleifer, 2014).

2.2.4 The Reversal

UIP states that on average (and in expectation) excess returns on the carry trade must be zero. Consequently, the current interest differential must not be able to predict future excess returns at any horizon.

Engel (2016) and Valchev (2017) investigate how the current interest rate differential predicts excess returns in the future. They document the reversal of the Fama slope over time. Consider the regression

$$\hat{\mathbb{E}}_t\left[\rho_{t+j}\right] = \tilde{\alpha}_j + \tilde{\beta}_j(\hat{r}_t^* - \hat{r}_t) + v_{jt}. \tag{2.6}$$

Under UIP, $\tilde{\beta}_j = 0$ for any horizon j and $\tilde{\beta}_j > 0$ means profitability of the carry trade in the way that was documented before.

Engel and Valchev find that the estimate of the slope $\tilde{\beta}_j$ is positive and diminishes to zero on a horizon of around one year ($j = 12$). It then turns negative, reaching its minimum within a range of $(-.8,\ -.2)$, depending on the country, and slowly converging to zero. Fig. 2.1 reproduces this finding. It plots the estimates of $\tilde{\beta}_j$ against the time horizon j. The current interest differential predicts positive excess returns (in line with the violation of UIP) on a horizon up to one year and negative excess returns thereafter. Engel interprets this finding as follows: a higher domestic interest rate predicts bigger risk in the short run and much smaller risk thereafter.

I show that much of such behavior of the expected excess returns can be attributed to the process for market expectations. Following the approach in the previous section, I estimate the same regression, as (2.6), with an addition of past expectations about excess returns:

$$\hat{\mathbb{E}}_t\left[\rho_{t+j}\right] = \tilde{\alpha}_j' + \tilde{\beta}_j'(\hat{r}_t^* - \hat{r}_t) + \tilde{\gamma}_j \hat{\mathbb{E}}_{t-1}\left[\rho_{t-1+j}\right] + w_{jt}. \tag{2.7}$$

and I display the results on two figures. Fig. 2.2 plots the estimates of $\tilde{\beta}_j'$ and their confidence intervals against the time horizon j, along with the confidence bounds for $\tilde{\beta}_j$ from regression (2.6) indicated as the gray area. Fig. 2.3 plots the estimates of $\tilde{\gamma}_j$ and their confidence bounds.

Fig. 2.2 clearly shows that the inclusion of past expectations explains a big portion of the dynamics of the excess returns. When past expectations are taken into consideration, the Fama slopes become much closer to zero. The difference in $\tilde{\beta}_j$ and $\tilde{\beta}_j'$ is statistically significant. For Japan, past expectations make the Fama slope insignificant at all horizons, thus fully accounting for the violation of UIP. For other countries, the estimates of $\tilde{\beta}_j'$ still display the reversal but of a much smaller magnitude.

Similarly to the previous section, I find that investors tend to extrapolate past expectations into the future. This can be seen on Fig. 2.3. It shows the coefficients on past expectations. For all of the countries in the sample, the coefficient γ_t' is statistically positive and close to one. For Japan, the coefficient is indistinguishable from one. This implies that investors engaging in the carry trade with the yen and the U.S. dollar, tend to impose their past expectations onto the future. This is also true for Canada and France. For the rest of the countries, the importance of past expectations declines as the horizon widens, but it remains very high.

The evidence from the first part of this paper suggests that past expectations play a crucial role in the determination of exchange rates and interest rates. This provides a natural scope for multiple equilibria models where past expectations appear as a state variable.

2.3 The Model

In this section I construct a model that features a real endowment two-country two-good two-period overlapping generations model. Time t is discreet and never ends. Let $j = 1, 2$ index the generations. Foreign variables will be marked by asterisk. The two countries are identical except that they differ in their values for the subjective discount factor: $\beta \neq \beta^*$. I will assume $\beta > \beta^*$ and will refer to the first country as home and the other country as foreign.

2.3.1 Uncertainty

There is no intrinsic uncertainty; preferences, endowments, and policy are known by every agent and are constant over time. The only source of uncertainty is the sunspot variable ε_t that is distributed normally with zero mean, standard deviation of 1, and zero autocorrelation of any order.

The exogenous state, z_t, depends on the realization of ε_t and represents shocks to beliefs about the real exchange rate q_t:

$$\log q_{t+1} = \mathbb{E}_t \left[\log q_{t+1}\right] + z_{t+1}. \tag{2.8}$$

The state z_{t+1} evolves according to the Cox et al. (1985) law:

$$z_{t+1} = \sigma \varepsilon_{t+1} \sqrt{z_t + \lambda_1 z_{t-1} + \ldots + \lambda_p z_{t-p} + \varphi} \tag{2.9}$$

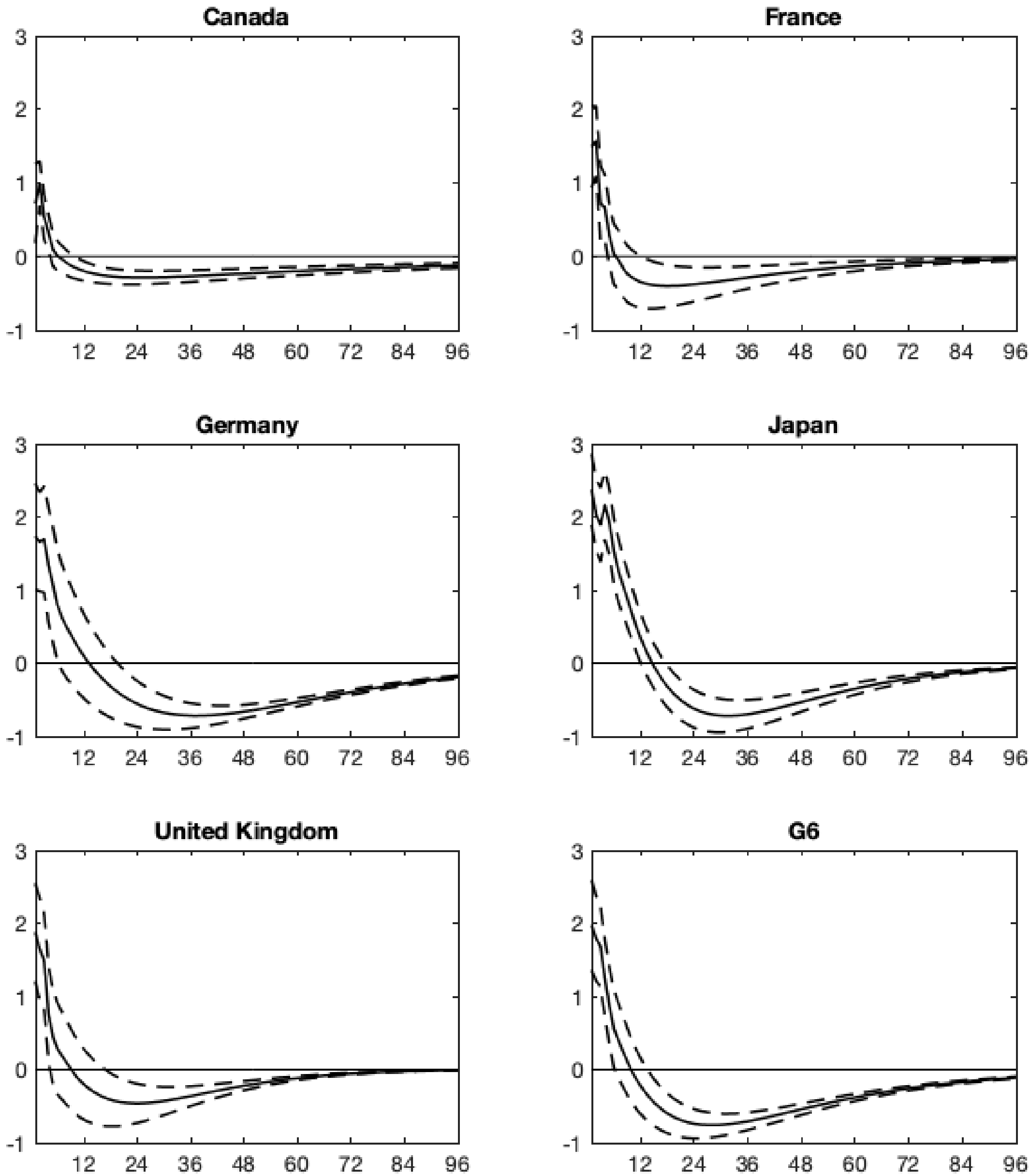

Figure 2.1: The Fama Slope over Various Horizons

Note: The slope estimate $\hat{\beta}_j$ from the regression $\hat{\mathbb{E}}_t [\rho_{t+j}] = \alpha_j + \beta_j(\hat{r}^*_t - \hat{r}_t) + v_{jt}$. The 95% confidence intervals are based on the Newey-West standard errors. The horizontal axis measures the horizon j in months. The vertical axis measures the slope estimate $\hat{\beta}_j$. Monthly data, 1979:6–2009:10.

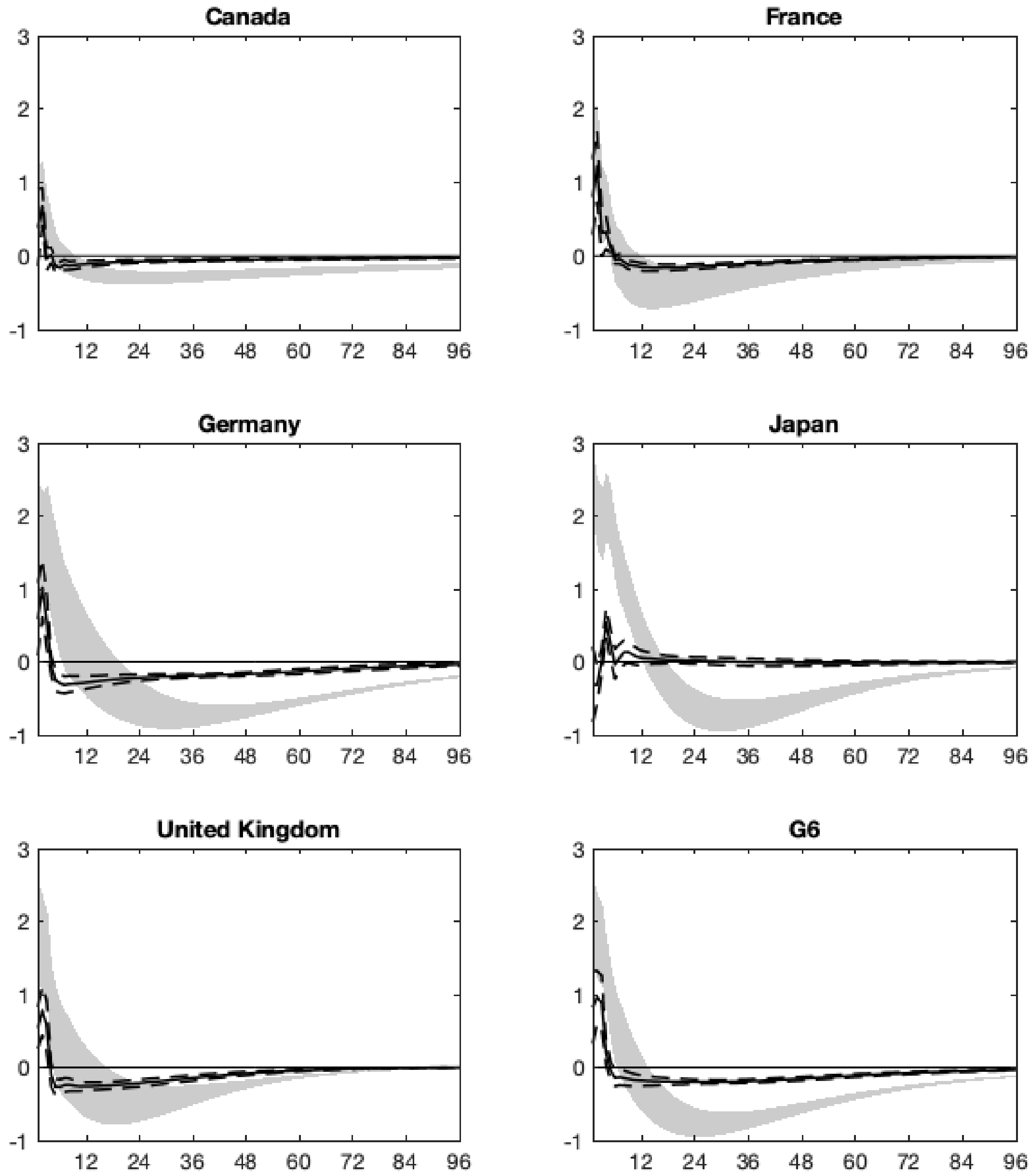

Figure 2.2: The Fama Slope over Various Horizons Controlling for Past Expectations

Note: The slope estimate $\hat{\beta}'_j$ from the regression $\hat{\mathbb{E}}_t\left[\rho_{t+j}\right] = \alpha'_j + \beta'_j(\hat{r}^*_t - \hat{r}_t) + \gamma_j \hat{\mathbb{E}}_{t-1}\left[\rho_{t-1+j}\right] + w_{jt}$. The 95% confidence intervals are based on the Newey-West standard errors. The horizontal axis measures the horizon j in months. The vertical axis measures the slope estimate $\hat{\beta}'_j$. The gray area represents the confidence bounds from Fig. 2.1. Monthly data, 1979:6–2009:10.

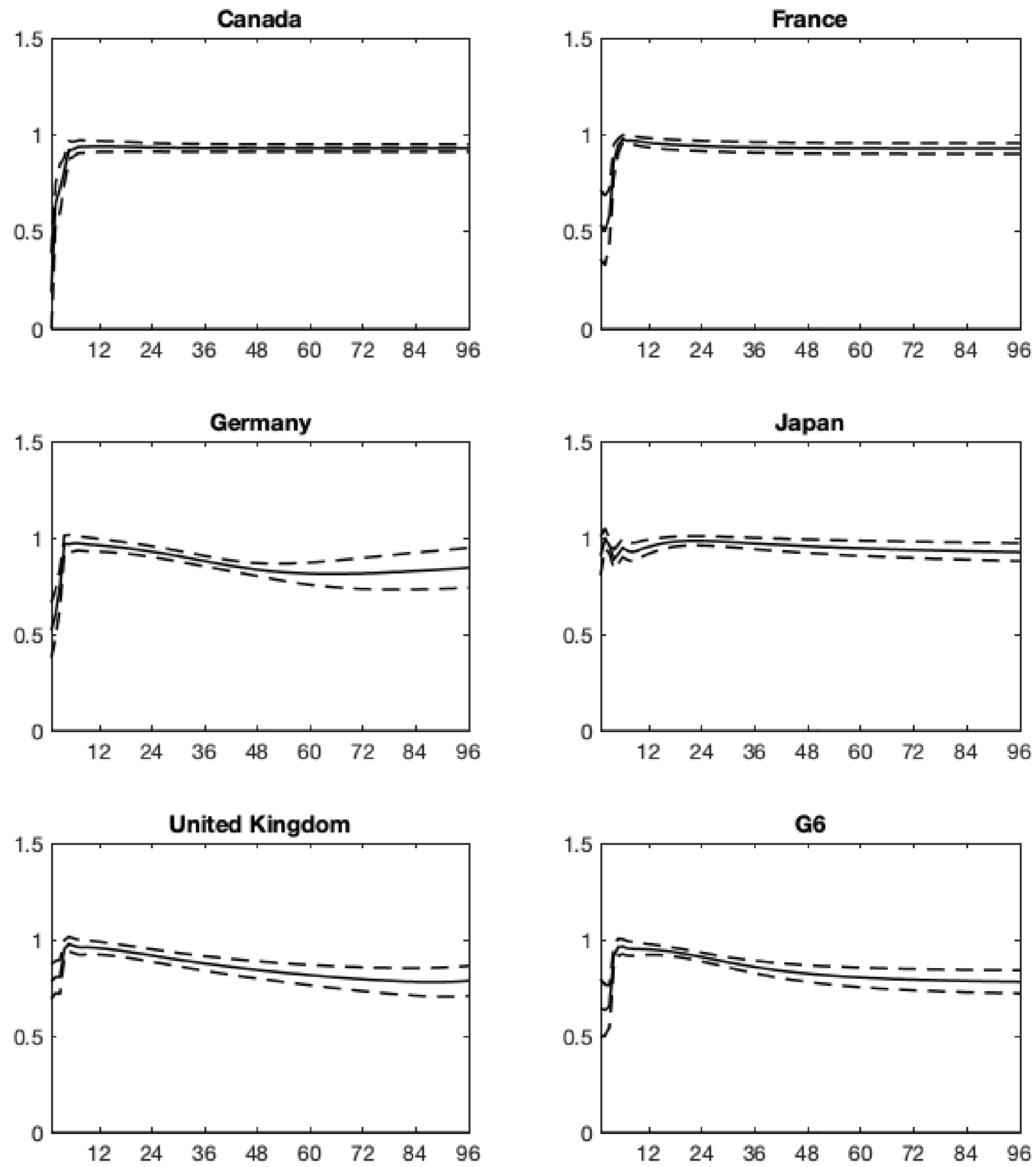

Figure 2.3: Past Expectations in the Fama Regression over Various Horizons

Note: The slope estimate $\hat{\gamma}_j$ from the regression $\hat{\mathbb{E}}_t\left[\rho_{t+j}\right] = \alpha_j' + \beta_j'(\hat{r}_t^* - \hat{r}_t) + \gamma_j \hat{\mathbb{E}}_{t-1}\left[\rho_{t-1+j}\right] + u_{jt}$. The 95% confidence intervals are based on the Newey-West standard errors. The horizontal axis measures the horizon j in months. The vertical axis measures the slope estimate $\hat{\gamma}_j'$. Monthly data, 1979:6–2009:10.

where $\sigma > 0$ and $\varphi > 0$.[7] This law of motion implies the following conditional distributions of z_{t+1}:

$$z_{t+1}|_t \sim \mathcal{N}\left(0, \sigma^2(z_t + \lambda_1 z_{t-1} + \ldots + \lambda_p z_{t-p} + \varphi)\right),$$

i.e. the conditional mean of z_{t+1} is equal to zero and its conditional variance is time-varying: it depends on the history. The zero conditional mean of z_{t+1} ensures that the expectations are rational.[8]

The unconditional distribution of z_{t+1} is characterized by the moments

$$\mathbb{E}[z_{t+1}] = 0, \quad \text{var}(z_{t+1}) = \sigma^2\varphi.$$

For the baseline model, I will consider the specification with $p = 0$. Under this convenient assumption, the exogenous state is summarized by a single variable z_t. In general, however, the state is described by z_t and its lags.

Let history $h_t = (z_0, z_1, \ldots, z_t)$ be a collection of realized states up to time t. Every variable x in the model depends on the history: $x_t = x(h_t)$ and all future realizations are indexed by the future state $x_{t+1}(h_t, z)$ which will be written simply as $x_{t+1}(z)$. The rational expectations are defined as the mathematical expectation over all possible future realizations of $x_{t+1}(h_t, z)$ given the observed history h_t:

$$\mathbb{E}_t[x_{t+1}] = \int_z x_{t+1}(h_t, z) dF(h_t, z)$$

where F is the distribution of z_{t+1} described above. All other moments of x_{t+1} are defined in a similar way.

[7] Under this specification, there is a positive probability that the expression inside the square root becomes negative. As Backus et al. (1998) mention, this probability shrinks to zero as the time interval becomes infinitesimally small. The presence of a constant φ will affect only means of the variables. In simulations, whenever the expression inside the square root becomes negative, I replace it with a very small number.

[8] To see this, take the time t conditional expectation from both sides of (2.8) to obtain an identity.

2.3.2 Agents

There are two tradable goods: a and b. In each country, the young generation is endowed with the local good $\bar{y}_1$ and $\bar{y}_1^*$ respectively, and the old generation is endowed with $\bar{y}_2$ and $\bar{y}_2^*$ units of the local good. I will assume $\bar{y}_1 = \bar{y}_1^*$ and $\bar{y}_2 = \bar{y}_2^*$. The goods are perishable, so that the total endowments of $(\bar{y}_1 + \bar{y}_2)$ and $(\bar{y}_1^* + \bar{y}_2^*)$ must be consumed each period. The agents trade goods internationally and consume a basket of domestic and imported goods.

Each generation j purchases a consumption basket which is a CES aggregator of the domestic good a_{jt} and the foreign good b_{jt}:

$$c_{jt} = \left(\omega^{\frac{1}{\eta}} \left(a_{jt}\right)^{\frac{\eta-1}{\eta}} + (1-\omega)^{\frac{1}{\eta}} \left(b_{jt}\right)^{\frac{\eta-1}{\eta}}\right)^{\frac{\eta}{\eta-1}}. \tag{2.10}$$

The goods are assumed to be imperfect substitutes, with the elasticity of substitution $\eta \geq 1$ (the case $\eta = 1$ would correspond to a Cobb-Douglas aggregator). The home good is the numeraire, and all the prices are quoted in terms of the home good. The price of the foreign good (the relative price) is p_t and the prices of the consumption goods are respectively p_t^c and p_t^{c*}. The prices of the consumption goods must satisfy

$$p_t^c = \left(\omega + (1-\omega)\left(p_t\right)^{1-\eta}\right)^{\frac{1}{1-\eta}}. \tag{2.11}$$

$$p_t^{c*} = \left(\omega\left(p_t\right)^{1-\eta} + (1-\omega)\right)^{\frac{1}{1-\eta}}. \tag{2.12}$$

Agents of each generation in each country solve the static problem of minimizing expenditure on the consumption basket. The implied demand functions are

$$a_{jt} = \omega\left(\frac{1}{p_t^c}\right)^{-\eta} c_{jt}, \qquad b_{jt} = (1-\omega)\left(\frac{p_t}{p_t^c}\right)^{-\eta} c_{jt}. \tag{2.13}$$

$$b_{jt}^* = \omega\left(\frac{p_t}{p_t^{c*}}\right)^{-\eta} c_{jt}^*, \qquad a_{jt}^* = (1-\omega)\left(\frac{1}{p_t^{c*}}\right)^{-\eta} c_{jt}^*. \tag{2.14}$$

The utility function of the young generation is defined over the consumption basket in the first period c_{1t} and a set of consumption baskets in the second period realized under all possible possible future states.

$$\frac{c_{1t}^{1-\theta}}{1-\theta} + \beta \mathbb{E}_t \left[\frac{c_{2t+1}^{1-\theta}}{1-\theta} \right]. \tag{2.15}$$

with a constant parameter of risk aversion $\theta > 0$ and with $\theta = 1$ being log utility.

Asset markets are sequentially complete. There is a whole set of state-contingent Arrow securities $\xi_{t+1}(z)$ that deliver a unit of the consumption basket upon the realization of a particular state z. The probability-adjusted price of such a security (the stochastic discount factor) is denoted m_{t+1}. The period budget constraints are given by

$$p_t^c c_{1t} = \bar{y}_1 + \mathbb{E}_t \left[m_{t+1} \xi_{t+1} \right], \tag{2.16}$$

$$p_t^c c_{2t} = \bar{y}_2 + \xi_t. \tag{2.17}$$

For the foreign country, the equations are similar:

$$p_t^{c*} c_{1t}^* = p_t \bar{y}_1^* + \mathbb{E}_t \left[m_{t+1} \xi_{t+1}^* \right], \tag{2.18}$$

$$p_t^{c*} c_{2t}^* = p_t \bar{y}_2^* + \xi_t^*. \tag{2.19}$$

Since the young generation has access to a complete set of contingent securities, there exist two present-value budget constraints:

$$p_t^c c_{1t} + \mathbb{E}_t \left[m_{t+1} p_{t+1}^c c_{2t+1} \right] = \bar{y}_1 + \mathbb{E}_t \left[m_{t+1} p_{t+1} \right] \bar{y}_2. \tag{2.20}$$

$$p_t^{c*} c_{1t}^* + \mathbb{E}_t \left[m_{t+1} p_{t+1}^c c_{2t+1}^* \right] = p_t \bar{y}_1^* + \mathbb{E}_t \left[m_{t+1} p_{t+1} \right] \bar{y}_2^*. \tag{2.21}$$

The young agents decide how much to consume and how much to save by maximizing (2.15) subject to (2.20). The first order conditions are given by the following Euler equations that hold state by state:

$$m_{t+1}(z) = \beta \left(\frac{c_{2t+1}(z)}{c_{1t}} \right)^{-\theta} \frac{p_t^c}{p_{t+1}^c(z)}, \tag{2.22}$$

$$m_{t+1}(z) = \beta^* \left(\frac{c_{2t+1}^*(z)}{c_{1t}^*} \right)^{-\theta} \frac{p_t^{c*}}{p_{t+1}^{c*}(z)}. \tag{2.23}$$

Equations (2.22) and (2.23) state that consumers equalize the intertemporal marginal rate of substitution calculated in terms of the home good to the stochastic discount factor. Note that there is no expectations operator as, due to the market completeness, agents are able achieve this equality in any possible state of the world.

2.3.3 Government

The government issues state-contingent bonds $\xi_{t+1}^g(z)$ that can be positive (government is in debt) or negative (government holds wealth). The bonds are rolled over from a period to period. For simplicity, the government neither collects taxes nor purchases consumption goods. This implies the following law of motion for public debt:

$$\mathbb{E}_t \left[m_{t+1} \xi_{t+1}^g \right] = \xi_t^g. \tag{2.24}$$

The price of the bonds is determined only by their resale value. The young agents will purchase government bonds only if they expect that the future young generation will buy these bonds from them.

The role of the government in this economy is passive. The only sustainable policy will be issuing as many bonds as is demanded by the private sector. I impose a terminal condition that public debt remains bounded in equilibrium.

2.3.4 Real Exchange Rate, Real Interest Rates, and Risk Sharing

The real exchange rate q_t is defined as the relative price of home and foreign consumption goods:

$$q_t \equiv \frac{p_t^{c*}}{p_t^c}, \tag{2.25}$$

so that an increase in q_t means real depreciation for the home country.

The Euler conditions (2.22) and (2.23) lead to the following risk sharing condition:

$$\beta \left(\frac{c_{2t+1}}{c_{1t}} \right)^{-\theta} \frac{p_t^c}{p_{t+1}^c} = \beta^* \left(\frac{c_{2t+1}^*}{c_{1t}^*} \right)^{-\theta} \frac{p_t^{c*}}{p_{t+1}^{c*}}. \tag{2.26}$$

Define m_{t+1}^c and m_{t+1}^{c*} the stochastic discount factors quoted in terms of home and foreign consumption goods respectively. The risk sharing condition can be rewritten as

$$m_{t+1}^c \frac{q_{t+1}}{q_t} = m_{t+1}^{c*}. \tag{2.27}$$

Finally, define the real interest rates as follows:

$$\exp(-r_t) = \mathbb{E}_t \left[m_{t+1}^c \right], \tag{2.28}$$

$$\exp(-r_t^*) = \mathbb{E}_t \left[m_{t+1}^{c*} \right]. \tag{2.29}$$

Under the log-normal distribution of the innovations in the model (and as a second-order approximation in general), the risk sharing condition and the definition of the real interest rates are combined to obtain an equation that relates the expected depreciation rate, the interest rate differential, and the risk premium:

$$\mathbb{E}_t \left[\Delta \log q_{t+1} \right] = r_t - r_t^* + \frac{1}{2} \left(\text{var}_t \left(\log m_{t+1}^c \right) - \text{var}_t \left(\log m_{t+1}^{c*} \right) \right). \tag{2.30}$$

Note that this equation relies only on the existence of the stochastic discount factors and on sequential completeness of the financial markets and does not depend on the structure of the model.

Equation (2.30) states that the expected depreciation rate, $\mathbb{E}_t[\Delta \log q_{t+1}]$, consists of two parts: the interest rate differential, $(r_t - r_t^*)$, and the risk premium equal to one half of the difference in conditional variances of the log stochastic discount factors. Rewrite the risk sharing condition to arrive at an analogue of the Fama regression:

$$\mathbb{E}_t[\rho_{t+1}] \equiv \mathbb{E}_t[\Delta \log q_{t+1}] + r_t^* - r_t = \frac{1}{2}\left(\text{var}_t\left(\log m_{t+1}^c\right) - \text{var}_t\left(\log m_{t+1}^{c*}\right)\right) \quad (2.31)$$

This equation states that the expected excess returns are proportional to the differential of the conditional variances of the stochastic discount factors. To explain the forward premium anomaly, the expression on the right-hand side of the equation above must be positively correlated with the real interest differential $r_t^* - r_t$. In equilibrium, both $r_t^* - r_t$ and $\mathbb{E}_t[\rho_{t+1}]$ will be driven by the single state z_t and, as I show, have the correlation of the right sign.

2.3.5 Beliefs

The model described above admits a multiplicity of dynamic equilibria. To put a structure on the manifold of dynamic paths, I introduce a belief function about the real exchange rate:

$$\log q_{t+1} = \mathbb{E}_t[\log q_{t+1}] + z_{t+1} \quad (2.32)$$

where z_{t+1} is the factor that drives the beliefs. In the absence of intrinsic uncertainty, the variable z_{t+1} represents self-fulfilling shocks to the relative price q_{t+1}. Were there no multiplicity of equilibria, the only stationary equilibrium would impose zero loading on the sunspot factor z_{t+1}.

Once an assumption is made about the formation of beliefs, the dynamic equilibrium becomes unique. This way, the belief function selects a particular dynamic equilibrium.

2.3.6 Equilibrium

Definition 2. *A sequential rational expectations equilibrium in this economy is a collection of history-dependent consumption allocations* $\{c_{1t}, c_{2t}\}$ *and* $\{c^*_{1t}, c^*_{2t}\}$*, relative prices* $\{p_t, p^c_t, p^{c*}_t, q_t\}$*, the stochastic discount factor* $\{m_t\}$ *and quantities of contingent assets* $\{\xi_{t+1}(z), \xi^*_{t+1}(z), \xi^g_{t+1}(z)\}_z$ *such that, given the law of motion of the exogenous state,*

$$z_{t+1} = \sigma\varepsilon_{t+1}\sqrt{z_t + \varphi}, \quad \varepsilon_{t+1} \sim iid\ \mathcal{N}(0, 1), \tag{2.33}$$

the following equations hold (some of them repeated here for convenience):

1. *The young generations' Euler equations hold:*

$$m_t = \beta \left(\frac{c_{2t}}{c_{1t-1}}\right)^{-\theta} \frac{p^c_{t-1}}{p^c_t}, \tag{2.34}$$

$$m_t = \beta^* \left(\frac{c^*_{2t}}{c^*_{1t-1}}\right)^{-\theta} \frac{p^{c*}_{t-1}}{p^{c*}_t}. \tag{2.35}$$

2. *The young generations' present value budget constraints hold:*

$$p^c_t c_{1t} + \mathbb{E}_t\left[m_{t+1} p^c_{t+1} c_{2t+1}\right] = \bar{y}_1 + \mathbb{E}_t\left[m_{t+1}\right]\bar{y}_2, \tag{2.36}$$

$$p^{c*}_t c^*_{1t} + \mathbb{E}_t\left[m_{t+1} p^{c*}_{t+1} c^*_{2t+1}\right] = p_t \bar{y}^*_1 + \mathbb{E}_t\left[m_{t+1} p_{t+1}\right]\bar{y}^*_2. \tag{2.37}$$

3. *The old consume their wealth:*

$$p^c_t c_{2t} = \bar{y}_2 + \xi_t, \tag{2.38}$$

$$p_t^{c*} c_{2t}^* = p_t \bar{y}_2^* + \xi_t^*. \tag{2.39}$$

4. *Relative prices satisfy*

$$p_t^c = \left(\omega + (1-\omega)\left(p_t\right)^{1-\eta}\right)^{\frac{1}{1-\eta}}, \tag{2.11}$$

$$p_t^{c*} = \left(\omega \left(p_t\right)^{1-\eta} + (1-\omega)\right)^{\frac{1}{1-\eta}}, \tag{2.12}$$

$$q_t = \frac{p_t^c}{p_t^{c*}}. \tag{2.55}$$

5. *Goods markets clear:*

$$\sum_{j=1}^{2} \omega c_{jt} \left(\frac{1}{p_t^c}\right)^{-\eta} + \sum_{j=1}^{2} (1-\omega) c_{jt}^* \left(\frac{1}{p_t^{c*}}\right)^{-\eta} = \bar{y}_1 + \bar{y}_2, \tag{2.40}$$

$$\sum_{j=1}^{2} \omega c_{jt}^* \left(\frac{p_t}{p_t^{c*}}\right)^{-\eta} + \sum_{j=1}^{2} (1-\omega) c_{jt} \left(\frac{p_t}{p_t^c}\right)^{-\eta} = \bar{y}_1^* + \bar{y}_2^*. \tag{2.41}$$

6. *The market for all Arrow securities clears: for any future z,*

$$\xi_{t+1}(z) + \xi_{t+1}^*(z) + \xi_{t+1}^g(z) = 0. \tag{2.42}$$

7. *The dynamics of the exchange rate q_t satisfies the belief function:*

$$\log q_{t+1} = \mathbb{E}_t \left[\log q_{t+1}\right] + z_{t+1}. \tag{2.43}$$

The budget constraint of the government (the law of motion for ξ_{t+1}^g as in (2.24)) will be satisfied by Walras' law. The bond holdings will be determined recursively, and I omit them in further discussion.

The model admits a recursive representation. Since keeping track of the histories makes simulations complicated, below I define the recursive rational expectations equilibrium. The allocations that satisfy the conditions of the recursive equilibrium will also satisfy the conditions of the sequential equilibrium. Here I will use the conventional notation by dropping the time t subscripts and marking the next-period variables with a prime.

The state of the economy consists the exogenous state z and endogenous state $(\mathbb{E}q_{-1}, \Omega)$ where $\mathbb{E}q_{-1}$ is past expectations about the current relative price p and Ω is the relative wealth of the young evaluated in terms of marginal utility in common units,

$$\Omega_t = \frac{c_{1t}^{-\theta}}{c_{1t}^{*-\theta}} \frac{p_t^{c*}}{p_t^c}. \tag{2.44}$$

By the risk sharing condition (2.26), the current distribution of wealth Ω_t will impose a condition on the allocation of consumption in the next period: for any future state,

$$\Omega_t = \frac{\beta}{\beta^*} \frac{c_{2t+1}^{-\theta}}{c_{2t+1}^{*-\theta}} \frac{p_{t+1}^{c*}}{p_{t+1}^c}. \tag{2.45}$$

Definition 3. *A recursive stochastic equilibrium is a set of decision rules* $c_1(z, \mathbb{E}q_{-1}, \Omega)$, $c_2(z, \mathbb{E}q_{-1}, \Omega)$, $c_1^*(z, \mathbb{E}q_{-1}, \Omega)$ *and* $c_2^*(z, \mathbb{E}q_{-1}, \Omega)$, *the pricing functions* $p(z, \mathbb{E}q_{-1}, \Omega)$, $q(z, \mathbb{E}q_{-1}, \Omega)$, $p^c(z, \mathbb{E}q_{-1}, \Omega)$ *and* $p^{c*}(z, \mathbb{E}q_{-1}, \Omega)$, *and the laws of motion* $\Omega' = \mathcal{G}(z, \mathbb{E}q_{-1}, \Omega)$ *and* $\mathbb{E}q = \mathcal{H}(z, \mathbb{E}q_{-1}, \Omega)$ *such that, given the law of motion of the exogenous state*

$$z' = \sigma\varepsilon'\sqrt{z + \varphi}, \quad \varepsilon' \sim \mathcal{N}(0, 1), \tag{2.46}$$

1. *The decision rules for consumption solve the young's utility maximization problem:*

$$\begin{aligned} & p^c(z, \mathbb{E}q_{-1}, \Omega)c_1(z, \mathbb{E}q_{-1}, \Omega) \\ & + \beta\mathbb{E}_{z'|z}\left[\left(\frac{c_2(z', \mathbb{E}q, \Omega')}{c_1(z, \mathbb{E}q_{-1}, \Omega)}\right)^{-\theta} \frac{p^c(z, \mathbb{E}q_{-1}, \Omega)}{p^c(z', \mathbb{E}q, \Omega')} p^c(z', \mathbb{E}q, \Omega')c_2(z', \mathbb{E}q, \Omega')\right] \\ & = \bar{y}_1 + \beta\mathbb{E}_{z'|z}\left[\left(\frac{c_2(z', \mathbb{E}q, \Omega')}{c_1(z, \mathbb{E}q_{-1}, \Omega)}\right)^{-\theta} \frac{p^c(z, \mathbb{E}q_{-1}, \Omega)}{p^c(z', \mathbb{E}q, \Omega')}\right] \bar{y}_2 \end{aligned} \tag{2.47}$$

$$
\begin{aligned}
&p^{c*}(z,\mathbb{E}q_{-1},\Omega)c_1^*(z,\mathbb{E}q_{-1},\Omega) \\
&+\beta^*\mathbb{E}_{z'|z}\left[\left(\frac{c_2^*(z',\mathbb{E}q,\Omega')}{c_1^*(z,\mathbb{E}q_{-1},\Omega)}\right)^{-\theta}\frac{p^{c*}(z,\mathbb{E}q_{-1},\Omega)}{p^{c*}(z',\mathbb{E}q,\Omega')}p^{c*}(z',\mathbb{E}q,\Omega')c_2^*(z',\mathbb{E}q,\Omega')\right] \\
&\qquad = p(z,\mathbb{E}q_{-1},\Omega)\bar{y}_1^* \\
&+\beta^*\mathbb{E}_{z'|z}\left[\left(\frac{c_2^*(z',\mathbb{E}q,\Omega')}{c_1^*(z,\mathbb{E}q_{-1},\Omega)}\right)^{-\theta}\frac{p^{c*}(z,\mathbb{E}q_{-1},\Omega)}{p^{c*}(z',\mathbb{E}q,\Omega')}p(z',\mathbb{E}q,\Omega')\right]\bar{y}_2^*. \quad (2.48)
\end{aligned}
$$

2. *The marginal utilities of the old satisfy*

$$
\Omega = \frac{\beta}{\beta^*}\frac{c_2(z,\mathbb{E}q_{-1},\Omega)^{-\theta}}{c_2^*(z,\mathbb{E}q_{-1},\Omega)^{-\theta}}\frac{p^{c*}(z,\mathbb{E}q_{-1},\Omega)}{p^{c}(z,\mathbb{E}q_{-1},\Omega)}. \quad (2.49)
$$

3. *The pricing functions satisfy*

$$
q(z,\mathbb{E}q_{-1},\Omega) = \mathbb{E}q_{-1} + z, \quad (2.50)
$$

$$
p^c(z,\mathbb{E}q_{-1},\Omega) = \left(\omega + (1-\omega)p(z,\mathbb{E}q_{-1},\Omega)^{1-\eta}\right)^{\frac{1}{1-\eta}}, \quad (2.51)
$$

$$
p^{c*}(z,\mathbb{E}q_{-1},\Omega) = \left(\omega p(z,\mathbb{E}q_{-1},\Omega)^{1-\eta} + (1-\omega)\right)^{\frac{1}{1-\eta}}. \quad (2.52)
$$

4. *The law of motion* $\Omega' = \mathcal{G}(z,\mathbb{E}q_{-1},\Omega)$ *is given by*

$$
\Omega' = \frac{c_1(z,\mathbb{E}q_{-1},\Omega)^{-\theta}}{c_1^*(z,\mathbb{E}q_{-1},\Omega)^{-\theta}}\frac{p^{c*}(z,\mathbb{E}q_{-1},\Omega)}{p^{c}(z,\mathbb{E}q_{-1},\Omega)}. \quad (2.53)
$$

5. *The law of motion* $\mathbb{E}q = \mathcal{H}(z,\mathbb{E}q_{-1},\Omega)$ *is given by*

$$
\mathbb{E}q = \int_{z'} q(z',\mathbb{E}q,\Omega')dF(z'|z). \quad (2.54)
$$

The following propositions characterize the static relationships in the model and reduce the dimensionality of the model.

Lemma 2. *The real exchange rate* q_t *depends solely on the relative price* p_t*:*

$$q_t = \left(\frac{\omega p_t^{1-\eta} + (1-\omega)}{\omega + (1-\omega) p_t^{1-\eta}} \right)^{\frac{1}{1-\eta}}. \tag{2.55}$$

Under home bias in consumption (i.e. $\omega > .50$*), the real exchange rate is increasing in the relative price* p_t*.*

Lemma 2 states that there is a one-to-one mapping between the relative price p_t and the real exchange rate q_t. It also establishes a positive link between the relative price of endowments and the relative price of consumption. When the domestic endowment is relatively expensive (low p_t), the domestic currency will also be relatively expensive. This conclusion comes from the assumption of the home bias. When $\omega > .50$, consumers put a bigger weight on the local good in consumption. When the price of the local good increases, the price of the consumption basket increases due to the appreciation of the bigger component in it.

Next, define the total consumption in each country

$$c_t = c_{1t} + c_{2t}, \tag{2.56}$$

$$c_t^* = c_{1t}^* + c_{2t}^*. \tag{2.57}$$

Lemma 3. *Under the goods market clearing conditions, the total quantities of consumption depend only on the relative price* p_t*. The corresponding consumption functions* $c_t = c(p_t)$ *and* $c_t^* = c^*(p_t)$ *are given by*

$$c(p_t) = \frac{\omega - (1-\omega) p_t^{\eta}}{2\omega - 1} p^c(p_t)^{-\eta}, \tag{2.58}$$

$$c^*(p_t) = \frac{\omega p_t^{\eta} - (1-\omega)}{2\omega - 1} p^{c*}(p_t)^{-\eta}. \tag{2.59}$$

In addition, under home bias in consumption (i.e. $\omega > .50$*), the home consumption* $c(p_t)$ *is a decreasing function of the relative price and the foreign consumption* $c^*(p_t)$ *is an increasing function in the relative price* p_t*.*

Lemma 3, together with Lemma 2, states that the international allocation of consumption is uniquely determined by the relative price of consumption q_t which, in turn, is determined by the past beliefs $\mathbb{E}_{t-1}[\log q_t]$, and the shock to beliefs z_t. Fig. 2.4 illustrates how the international allocation of consumption is determined. It plots the production possibilities frontier, the combination of all consumption goods (c_t, c_t^*) that can be produced given the total endowments in this two-country economy. Its slope is negative due to resource constraints, and, under home bias, the production possibility frontier is concave. At equilibrium, the marginal rate of transformation (the slope of the production possibilities frontier must be equal to the real exchange rate (the relative price of the two consumption). Point E is the endowment allocation that can be optimally chosen under a certain exchange rate. Suppose the realized exchange rate is relatively low. On Fig. 2.4, it is denoted q_{low}. The domestic good is relatively more expensive than the foreign good. At equilibrium, since the domestic good is owned by the domestic consumers, their wealth increases, and the total consumption of the domestic good increases, and the total consumption of the foreign good decreases. This allocation is indicated as point A on Fig. 2.4. Similarly, the allocation B corresponds to the outcome when the foreign total consumption is relatively bigger than the domestic total consumption, and this allocation will be supported only if the exchange rate is relatively big denoted q_{high}, and the price of the foreign good is expensive. The real interest rates, r_t and r_t^*, and expectations about the depreciation of the exchange rate, $\mathbb{E}_t[\Delta \log q_{t+1}]$, will determine the intergenerational allocation of consumption (c_{1t}, c_{2t}) and (c_{1t}^*, c_{2t}^*).

Lemma 4. *The dynamical system* (2.47) – (2.53) *can be reduced to a two-dimensional dynamical system in two variables: relative wealth* Ω_t *and expectations about the*

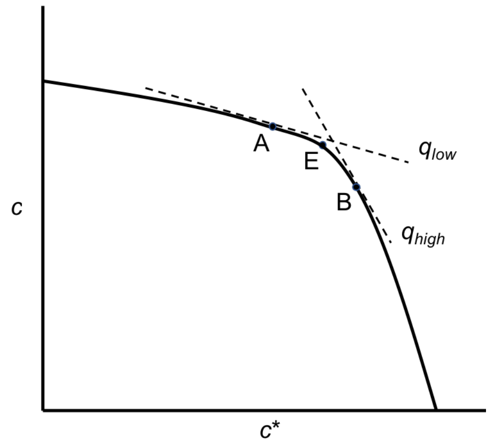

Figure 2.4: The Production Possibilities Frontier and the Determination of International Allocation of Consumption

future relative price $\mathbb{E}_t [\log q_{t+1}]$, *driven by the exogenous process* z_t.

Proof. The proof is trivial: eliminate all the static relations from the definition of the recursive equilibrium. □

As the following section shows, there are multiple dynamic equilibria; that is, there are multiple decision rules and pricing functions, all of which are consistent with the equilibrium conditions.

2.4 Multiplicity of Equilibria and Beliefs

This section establishes the fact that the dynamic system from section 3.6 exhibits a multiplicity of dynamic equilibria near one of the two steady states. The belief function (3.24) serves as an equilibrium selection mechanism.

2.4.1 Calibration

Because there is no closed-form solution, the model needs to be solved numerically. I adopt the calibration presented in Table 2.3. This is an intentionally highly stylized model, and the chosen values of the parameters are only illustrative.

Table 2.3: Calibration

Parameter	Interpretation	Value
β	Home subjective discount factor	.80
β^*	Foreign subjective discount factor	.75
θ	Coefficient of relative risk aversion	3
η	Elasticity of substitution across goods	1.50
ω	Weight of domestic good in consumption	.95
$(\bar{y}_1, \bar{y}_2)$	Endowment profile in home country	(.80,.20)
$(\bar{y}_1^*, \bar{y}_2^*)$	Endowment profile in foreign country	(.80,.20)
σ	Scale factor in the Cox-Ingersoll-Ross process	.50
φ	Shift factor in the Cox-Ingersoll-Ross process	1

The subjective discount factors for home and foreign consumers are set equal to .80 and .75 respectively. This implies that home consumers are more patient than foreign consumers, and home consumers will accumulate wealth by selling goods to foreign consumers. Because the young generations discount future utility at different rates, they will hold different portfolios of assets. This will cause significant international redistribution of wealth in response to the sunspot shocks.

The coefficient of relative risk aversion θ is set at a reasonable value of 3. This degree of risk aversion will be sufficient for accumulation of precautionary savings and will lead to a bigger redistribution of wealth across agents. The elasticity of substitution and the home bias are chosen so as to make sure that aggregate consumption responds little to fluctuations in the real exchange rate. Section 5.4 discusses the link between aggregate consumption and the real exchange rate in greater detail.

The endowment profiles $(\bar{y}_1, \bar{y}_2) = (\bar{y}_1^*, \bar{y}_2^*) = (.80, .20)$ show that the young generation has a significantly bigger endowment than the old. As discussed below, this assumption is necessary to make sure that the indeterminate steady state exists.

The parametrization of the Cox-Ingersoll-Ross process is chosen so to as generate the slope of the Fama regression of $-.25$.

2.4.2 Deterministic Steady States

The model developed in this paper is an extension of the canonical pure exchange overlapping-generations model studied in the textbook by Ljungqvist and Sargent (2004, Chapter 9). There are typically two deterministic steady states. To see this, recall that for any generation j,

$$p_t^c c_{jt} = a_{jt} + p_t b_{jt},$$

and the same holds for the foreign economy. Write the present-value budget constraints evaluated at the steady state and rearrange terms:

$$(a_1 + pb_1 - \bar{y}_1) + m\,(a_2 + pb_2 - \bar{y}_2) = 0 \tag{2.60}$$

$$(a_1^* + pb_1^* - p\bar{y}_1^*) + m\,(a_2^* + pb_2^* - p\bar{y}_2) = 0. \tag{2.61}$$

Add them up and use the goods market clearing conditions to obtain

$$(m-1)\left[(a_1 + a_1^* - \bar{y}_1) + p\,(b_1 + b_1^* - \bar{y}_1^*)\right] = 0. \tag{2.62}$$

The term in the second parentheses is the total young's excess demand for goods. The equation above shows that there are two steady states. The first steady state is where $m = 1$ and there is intergenerational trade as the excess demand of the young is not necessarily equal to zero. The second steady state is intergenerational autarky, where the excess demand of the young is equal to zero: although there is intra-generational trade, there is no trade across generations.

2.4.3 Multiplicity of Dynamic Equilibria

Theorem 1. *Under the calibration from Table 2.3, the steady state with $m = 1$ is dynamically determinate and the steady state where $m \neq 1$ is dynamically*

Table 2.4: Decisions Rules and Dynamic Indeterminacy

Argument of the decision rule	Determinate steady state	Indeterminate steady state
$\log\tilde{\Omega}_{t-1}$	non-zero	non-zero
$\mathbb{E}_{t-1}\left[\log\tilde{q}_t\right]$	zero	non-zero
z_t	zero	non-zero

indeterminate. In addition, around the indeterminate steady state there is a one dimensional manifold of dynamic equilibria indexed by z_t*.*

Proof. Consider a reduced dynamical system given by the equations from the definition of the recursive equilibrium, without the belief function.

According to Lemma 4, the entire model can be reduced to a two-dimensional system. The relative wealth Ω_t is an endogenous state variable and the expected exchange rate $\mathbb{E}_t\left[\log q_{t+1}\right]$ is a jump variable. The state z_t evolves exogenously and may or may not affect the dynamics, depending on the dynamical properties of the equilibrium.

According to the Stable Manifold Theorem (see, e.g. Perko, 2013), the stability properties of a non-linear model are equivalent to those of the linearized model. Therefore, postulate the linear solution of the form

$$\log\tilde{\Omega}_t = g_{10} + g_{11}\log\tilde{\Omega}_{t-1} + g_{12}\mathbb{E}_{t-1}\left[\log\tilde{q}_t\right] + g_{13}z_t \tag{2.63}$$

$$\mathbb{E}_t\left[\log\tilde{q}_{t+1}\right] = g_{20} + g_{21}\log\tilde{\Omega}_{t-1} + g_{22}\mathbb{E}_{t-1}\left[\log\tilde{q}_t\right] + g_{23}z_t \tag{2.64}$$

where the variables with tilde represent deviations from a steady state and the parameters g_{ij} are to be determined in equilibrium. Note that if a steady state is determinate, past expectations must not matter and loading on the sunspot shock z_t must be zero. If a steady state is indeterminate, past expectations will matter and loading on the sunspot shock will be distinct from zero. These conditions are summarized in Table 2.4.

I solve for the undetermined coefficients by log-linearizing the equations of the model and matching the coefficients. To take into account the fact that the conditional volatility of the future unknown variables may be of the first order in magnitude, I use the following approach:

$$\begin{aligned}\mathbb{E}_t\left[\Omega_{t+1}\right] &= \Omega\mathbb{E}_t\left[\exp(\log\tilde{\Omega}_{t+1})\right]\\ &= \Omega\exp\left(\mathbb{E}_t\left[\log\tilde{\Omega}_{t+1}\right]+\frac{g_{13}^2}{2}\mathrm{var}_t(z_{t+1})\right)\end{aligned}$$

where the second line holds due to conditional normality of z_{t+1} and where $\mathrm{var}_t(z_{t+1})$ is given by the assumptions of the model.

Solving for the decision rules at the two steady states produces the result of the theorem. □

2.5 The Model's Implications

2.5.1 Impulse Response Functions for a Shock to Beliefs

Before demonstrating the main result of this paper on the forward premium anomaly, I illustrate the mechanism of how the shocks to beliefs affect the equilibrium and propagate internationally. This subsection studies the impulse response functions to the sunspot shock ε_t.

Fig. 2.5 displays the impulse response functions to a shock to beliefs about the real exchange rate. The economy is initially at the steady state and in period 1, the real exchange rate appreciates by 1%. For all subsequent times, $\varepsilon_t = 0$. All variables are measured in percentage deviations from the deterministic steady state.

The shock to the exchange rate is self-fulfilling: if agents believe that the domestic good is becoming more expensive, the wealth of the young domestic

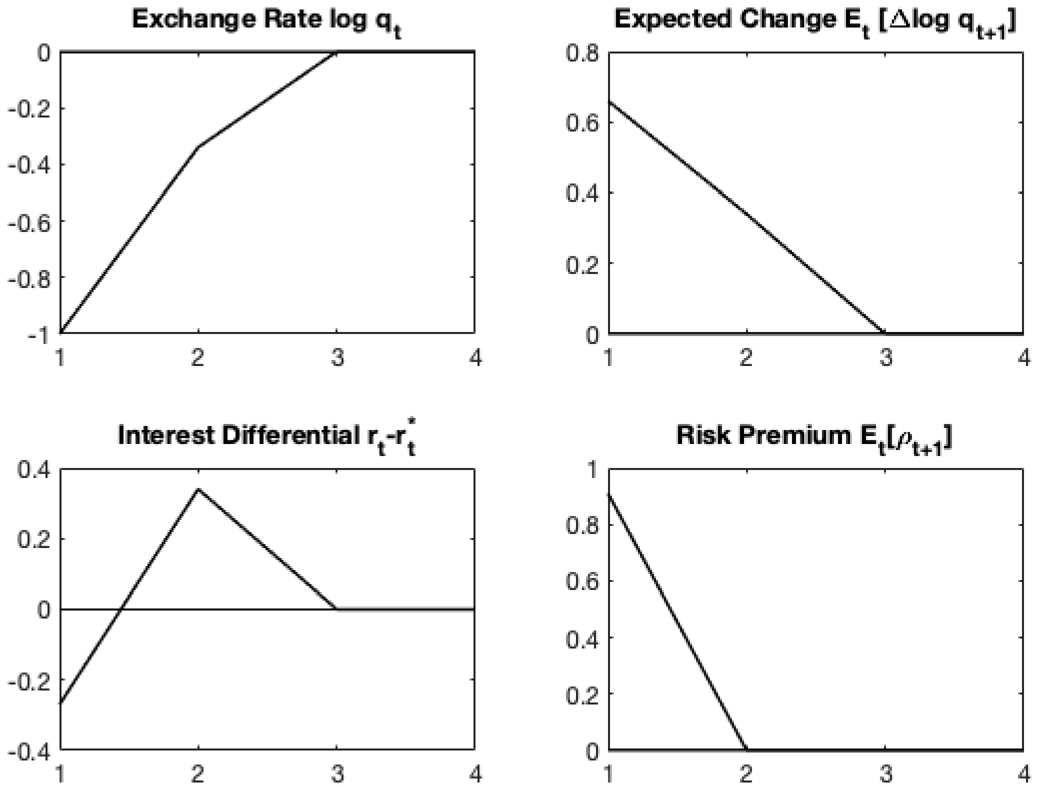

Figure 2.5: Shock to the Exchange Rate: Impulse Response Functions

agents increases relative to the wealth of the foreign young agents. Due to home bias in consumption, the domestic young agents increase demand for the domestic good, and it, indeed, appreciates relatively to the foreign good. Subsequently, the exchange rate depreciates and returns to the steady state.

Since the domestic agents become wealthier, they increase savings in the foreign economy. However, they understand the future exchange rate may change, and they demand excess returns. This explains why the interest rate differential $r_t^* - r_t$ becomes positive. Summing up the two effects, expected depreciation and the positive interest rate differential take place, which explains the positive Fama slope.

Starting from period 2, there is no more uncertainty, and the expected excess returns drop to zero. Since the exchange rate continues to depreciate, there is a

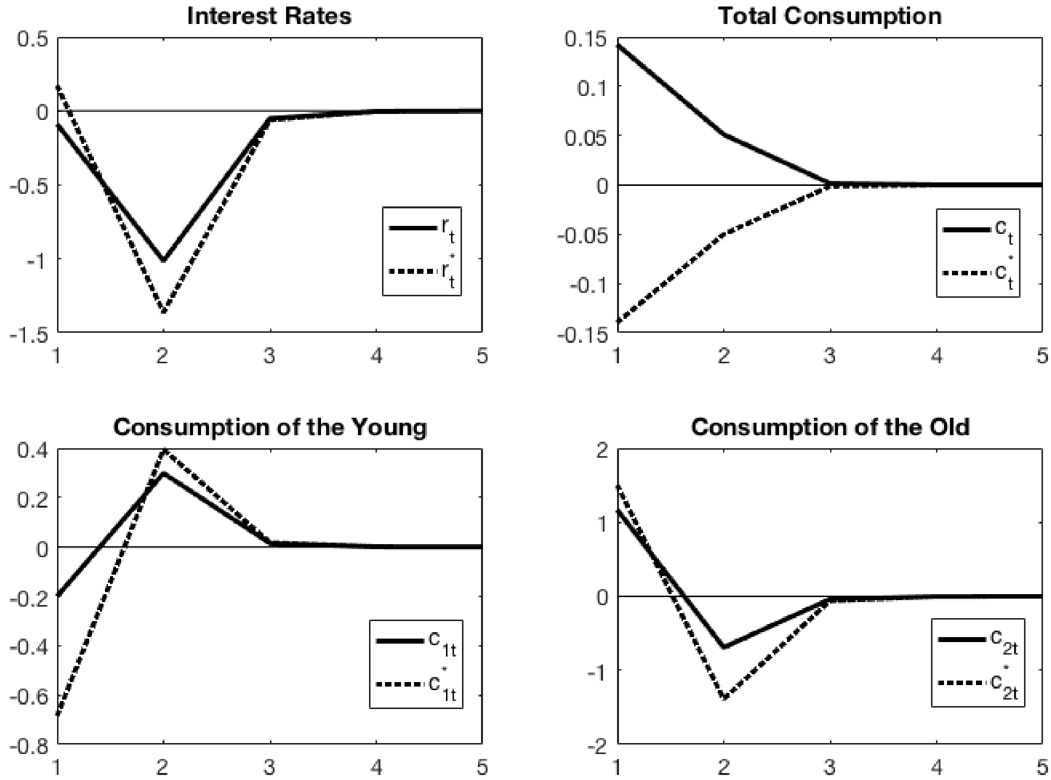

Figure 2.6: Shock to the Exchange Rate: Impulse Response Functions (cont.)

negative interest rate differential that makes the returns on domestic and foreign investments equal to each other.

Fig. 2.6 studies the adjustment of the economy in greater detail. Due to the specification of uncertainty, a shock to the exchange rate leads to two effects: immediate redistribution of wealth and increased volatility of consumption in the future. Since the domestic young agents become wealthier, their consumption exceeds consumption of the foreign young. At the same time, the risk averse young agents respond to the increased risk by cutting consumption and making precautionary savings. As a result, the level of consumption of the young in both countries drops. As the young increase demand for bonds held by the old generation, the resale price of the bonds rises, and consumption of the old increases in both countries. Due to the redistribution of wealth, total consumption shifts towards the domestic economy (see Lemma 3).

Table 2.5: Simulated Moments and the Data

Moment	Data						Model
	Canada	France	Germany	Japan	U.K.	G6	
$\text{std}(\Delta \log q_t)$	1.97	3.19	3.27	3.44	3.12	2.57	1.14
$\text{std}(r_t^* - r_t)/\text{std}(\Delta \log q_t)$	.06	.07	.05	.06	.06	.06	.33
$\text{std}(\mathbb{E}_t \rho_{t+1})/\text{std}(\Delta \log q_t)$	3.99	3.55	5.22	4.41	4.31	4.32	1.23
$\text{AR}(q_t)$	.98	.98	.98	.98	.97	.98	.34
$\text{AR}(\Delta \log q_t)$	$-.06$	.04	.04	.07	.09	.06	$-.23$
$\text{AR}(rr_t^* - rr_t)$	.54	.60	.75	.58	.60	.78	$-.43$
$\text{AR}(\mathbb{E}_t \rho_{t+1})$	.23	.62	.57	.86	.83	.73	.07
Fama slope β	$-.26$	$-.33$	$-.59$	-1.23	$-.84$	$-.89$	$-.25$

Note: std = standard deviation, AR = first-order autocorrelation. The Fama slope β is as in Table 2.2, column (A).

Starting from period 2, it becomes apparent that there is no more uncertainty about the real exchange rate, and the need in precautionary savings disappears. Consumption of the young rises, and consumption of the old falls. According to the Euler equation, higher current consumption and lower future expected consumption is supported by smaller interest rates.

2.5.2 The Stochastic Properties of the Model

The stochastic properties of the model qualitatively match those found in the data. Table 2.5 compares the second moments generated by the model and in the data. First, the exchange rate is a volatile process: the standard deviation of the exchange rate is bigger than that of the interest rate differential. Second, excess returns are even more volatile than the exchange rate. Third, the model generates a persistent exchange rate, as in the data, and the mechanism relies only on an endogenous propagation mechanism. Finally, the model reproduces a negative Fama slope.

2.5.3 Reversal

Consider the general case again as in (2.9). Given the recursive structure of the model, the interest rate differential depends on Ω_{t-1}, $\mathbb{E}_{t-1}[\log q_t]$, and z_t and its lags up to order p. From the equation for the risk premium (2.31) it follows that the risk premium depends only on the conditional variance of the future state, $\text{var}_t(z_{t+1})$ which is given by (2.9). The future risk premia, $\mathbb{E}_t[\rho_{t+j}]$, $j > 1$, will be exactly equal to zero for $j > p$ and will depend on the coefficients $\lambda_1, \lambda_2, \ldots, \lambda_p$ for $j \leq p$. Therefore, the Fama slope in the regressions of $\mathbb{E}_t[\rho_{t+j}]$ on $(r_t^* - r_t)$ will depend on the parameters λ_1, λ_2, ... In a more realistic model, these parameters could be estimated. Fig. 2.7 shows the reversal in $\tilde{\beta}_j$ as in (2.6) generated by the model when $\lambda_1 = -.15$ and all subsequent coefficients are set equal to zero.

The reversal in the Fama slope arises due to the reversal in the conditional volatility of the shocks to beliefs. From Fig. 2.6 it follows that, in period 1, a shock to the exchange rate occurs, and wealth is shifted towards one of the countries. At the same time, as the conditional volatility of the exchange rate increases, investors demand a risk premium. The Fama slope at a horizon of 1 is positive. In period 2 the interest differential reverses as wealth tends to equalize and capital flows change the direction. In a stochastic equilibrium, to generate the reversal in the Fama slope, the reversal in the risk premium must be predictable. Under the assumption $\lambda_1 < 0$, in period 1 investors expect the interest differential to reverse in period 2 and, as a results, they expect the risk premium to change the sign. If $\lambda_1 = 0$, there would be no correlation between period 1 and period 2 risk premia, and the expected period 2 risk premium would be equal to zero.

2.5.4 The Backus-Smith Puzzle

Backus and Smith (1993) document a weakly negative correlation between relative consumption (ratio of domestic aggregate consumption to foreign aggregate

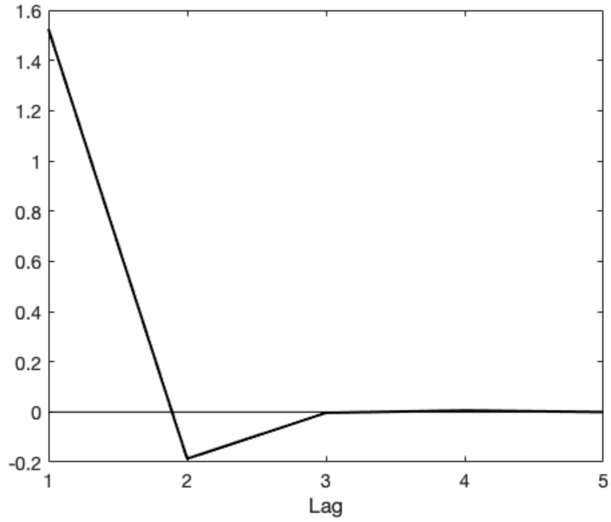

Figure 2.7: The Reversal of the Fama Slope in the Model

consumption) and the real exchange rate. This is a puzzle because in any model with a representative agent and complete markets, the risk sharing condition implies a strictly positive correlation: the domestic goods must be relatively cheap to support relatively high domestic consumption (Chari et al., 2002).

In my model, the exchange rate is demand-determined. When domestic consumption is high, due to home bias in consumption, demand for domestic goods will also be relatively high. Domestic goods will be expensive relative to foreign goods, and the exchange rate will be relatively low. Note that a low exchange rate supports redistribution of wealth towards the domestic consumers because the domestic consumers hold the endowment of the domestic goods.

The explanation of the Backus-Smith puzzle in my model holds under the assumption of complete financial markets. Corsetti et al. (2008) and Itskhoki and Mukhin (2017) are examples of the models that rely on market incompleteness.

To see the mechanism of this model clearly, log-linearize the two consumption functions (2.58) and (2.59) and the real exchange rate (2.55) around a symmetric

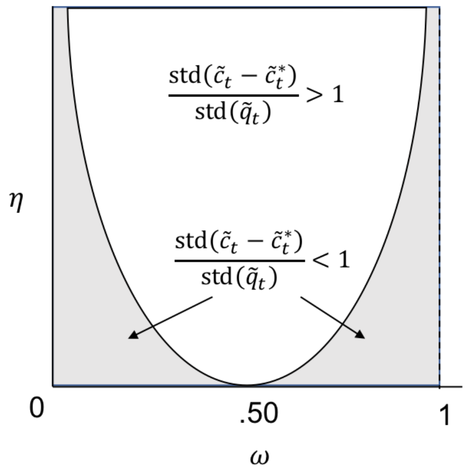

Figure 2.8: Parameter Space where $\text{std}(\tilde{c}_t - \tilde{c}_t^*) < \text{std}(\tilde{q}_t)$

point $c = c^* = 1$ and $q = 1$ to obtain

$$\log c_t = -\frac{\eta\omega(1-\omega)}{(2\omega-1)^2} \log q_t, \tag{2.65}$$

$$\log c_t^* = \frac{\eta\omega(1-\omega)}{(2\omega-1)^2} \log q_t. \tag{2.66}$$

It follows immediately that

$$\log c_t - \log c_t^* = -\frac{2\eta(1-\omega)}{(2\omega-1)^2} \log q_t. \tag{2.67}$$

Provided $\eta > 0$ and $\omega < 1$, relative consumption $(\log c_t - \log c_t^*)$ is negatively correlated with $\log q_t$. In a model driven only by one shock, this correlation is equal to -1 exactly. Addition of other shocks may reduce this correlation.

Equation (2.67) also gives insight on the volatilities of relative consumption and of the real exchange rate. Relative consumption will be less volatile than the real exchange rate if the coefficient on $\log q_t$ in (2.67) is smaller than 1. This is true if the home bias ω is sufficiently large (provided $\omega > .50$) and if the elasticity of substitution η is sufficiently low. Figure 2.8 shows the areas in the parameter space (ω, η) where relative consumption is less volatile than the real exchange rate.

2.6 Conclusion and Discussion

The empirical evidence on exchange rates and interest rate differentials produces a long-standing puzzle: the forward premium anomaly (violation of uncovered interest parity). Standard international macro theory predicts that the domestic currency must tend to depreciate when the domestic interest rate is greater than the foreign interest rate. The opposite is found for advanced economies in the data. In addition, the data exhibits that positive interest differentials predict future negative excess returns on the carry trade. Uncovered interest parity states that there shouldn't be any strong correlation between interest differentials and excess returns.

This paper builds a model of the risk premium paid the on carry trade. In the model, the exchange rate is determined by self-fulfilling beliefs. Shocks to beliefs generate uncertainty and therefore risk premia. Changes in the real exchange rate move wealth across countries and affect real interest rates, consumption allocation, and savings decisions. When investors believe that the domestic currency will appreciate, given everything else equal, they prefer to borrow at a foreign interest rate and purchase assets denominated in the domestic currency because the expected appreciation adds to the profit from carry trade. However, investors realize that the future exchange rate is uncertain, and they demand a risk premium. This makes the domestic interest rate greater than the foreign interest rate. If agents believe that periods with high uncertainty precede periods of low uncertainty, they will expect negative excess returns in the future. This reversal in volatility explains the sign reversal of expected returns over longer horizons found in the data.

Two assumptions are crucial for the results of the model: incomplete participation and the time-varying volatility of shocks to beliefs. Incomplete participation in the form of overlapping generations produces an indeterminate dynamic equilibrium. The indeterminacy is resolved by introduction of a belief function. The time-varying uncertainty makes agents demand a risk premium that is correlated with the fundamentals of the economy.

Several points deserve discussion and clarification.

Why does the explanation of the data on exchange rates and interest rates need to rely on multiple equilibria? First, although the belief function could have been treated as a behavioral object, this could have introduced "irrational" behavior into the model. The assumption of multiple equilibria brings market psychology to the model without violating the rational expectations assumption. The formation of beliefs, subject to shocks, bridges the rational expectations macroeconomics with the literature on behavioral patterns found in investors and other market participants.

Second, Beyer and Farmer (2007a) show observational equivalence of determinate and indeterminate linear models. Consider two models: a determinate model driven by shocks to fundamentals, and an indeterminate model driven by sunspots. Without identifying assumptions, there is nothing in the data that could tell the two models apart. The belief function is itself a fundamental object. Once one takes this position, the equilibrium becomes determinate and the shocks to beliefs are themselves fundamental. From this viewpoint, sunspot generated dynamics are equivalent to the dynamics generated by a combination of the fundamental shocks in a more complex model.

Third, because there are multiple equilibria, there may be other equilibria that exhibit a different link between interest rates and exchange rates. Under a different belief function, the predictions of the model would be different. There are many belief functions that are not equivalent to each other. Following Farmer (1999), here I take the belief function as a primitive and give it the status that standard models attribute to preferences, endowments and technology. Beliefs can be studied and estimated in the same way as any other primitives of the model, and the correct belief function becomes an empirical question.

Fourth, there are two steady state equilibria in the model. The dynamics is analyzed around the indeterminate steady state. The indeterminate steady state is dynamically inefficient. The other, determinate, steady state Pareto dominates the indeterminate steady state. A reasonable question would be: Why is the inferior equilibrium considered in the model? None of the results crucially depend on the dynamic inefficiency of the steady state. Such a modeling framework was chosen to demonstrate the mechanism of how beliefs can explain the observed co-movements of foreign excess returns and interest rates. The results can be extended to more complicated environments where the steady state is dynamically efficient and indeterminate (e.g. a three-period overlapping generations model as in Kehoe and Levine, 1983).

Fifth, the model assumes the existence of government "bubbly" bonds (similarly to Martin and Ventura, 2012) and no other saving technologies. Indeed, in the present model introduction of any other saving technology, such as social security or physical capital accumulation, is likely to eliminate the indeterminate equilibrium. Storage would preserve the indeterminacy of the steady state provided the return to storage is smaller than the interest rate at the efficient steady state. Storage would simply put a lower bound on the equilibrium interest rates. Kehoe and Levine (1983) is an example of a model in which no other savings technologies are likely to be used in equilibrium. Explicit addition of a nominal asset would preserve the multiplicity of equilibria.

Finally, newborns play a crucial role. Once more periods of life are added, the share of newborns becomes smaller. When preferences are described by expected utility, it will be harder to obtain a large and volatile risk premium in a multiple period model, retaining the assumption of deterministic (not necessarily constant) total endowment. However, preliminary results show that the introduction of Epstein-Zin preferences to an overlapping generations model ensures that even a small fraction of newborns in the total population is sufficient to cause significant redistribution of wealth across the existing agents.

CHAPTER 3

Confidence Crashes and Stagnation in the Eurozone

3.1 Introduction

Real economic activity in the eurozone has been stagnating since 2010 when the eurozone was hit by a debt crisis. The stagnation in the eurozone differs from typical business cycle fluctuations. It demonstrates unemployment persisting at high rates and no recovery in real GDP. Economic models that rely on price stickiness and on the existence of the unique long-run unemployment rate fail to predict crashes like the 2010 eurozone crisis and absence of fast recovery in real economic activity. I propose an original way of thinking about the Eurozone crisis and the recession it caused in a general equilibrium model with rational expectations.

I combine a model of a two-country monetary union with general equilibrium, labor market search and matching frictions, and asset pricing in a new way. By allowing for multiple equilibria on the labor market, I argue that beliefs about the prices of assets play a crucial role in the determination of the unemployment rate, real economic activity and international trade. With multiple equilibria, beliefs about asset prices become self-fulfilling. Sudden downward revisions of the beliefs, so called 'animal spirits', can trigger persistent economic recessions. In such economies, there may be no self-correction until beliefs recover. To put discipline on how equilibrium is selected, we propose the belief function, a forecasting rule, as a new fundamental.

This paper contributes to the bigger subject of financial crises and economic recessions. First, the model is a formalization of the crisis narrative that views the eurozone crisis as a self-fulfilling crisis triggered by a loss of confidence in the value of assets held by the peripheral countries of the eurozone. The peripheral countries (Greece, Ireland, Italy, Portugal, and Spain) relied on international borrowing and accumulated large debts. In 2010, investors revised their beliefs about solvency of these countries. The stock market crashes spread to the core countries, such as Austria, Belgium, France, Germany, and the Netherlands. In our model, the financial crisis is caused by self-fulfilling beliefs about the value of assets. Bacchetta and van Wincoop (2016), Benhabib et al. (2016), Perri and Quadrini (2014), Martin and Rey (2006) use a similar approach.

Second, I contribute to the large literature on causes and mechanisms of the Great Recession, sometimes referred to as secular stagnation (Caballero et al., 2008; Eggertsson et al., 2016; Schmitt-Grohé and Uribe, 2017 among many others). The standard explanation for the slow recovery of the world economy after the 2008 International Financial Crisis is the liquidity trap. Our explanation does not rely on the zero lower bound. Instead, we view the multiple equilibria on the labor market as a vehicle to capture persistence of high unemployment and the absence of automatic recovery. Third, we connect the asset prices and real economy in à la Lucas tree model, where investors' 'animal spirits' cause stock market crashes and economic recessions (Farmer 2012b, 2013). The model in this paper displays contagion, when asset price crashes transmit internationally (Benhabib et al., 2016; Van Wincoop, 2013; Kodres and Pritsker, 2002; Moser, 2003).

The approach of this paper builds heavily on Farmer (2010b, 2012b, 2013 and Farmer and Platonov (2019). It differs from the standard New Keynesian paradigm in two ways. First, our model exhibits steady state indeterminacy. Due to search and matching frictions on the labor market, the labor market equilibrium is indeterminate. I assume away Nash bargaining over wage and let the unemployment rate be determined by the aggregate demand. In our view, the secular stagnation is not a deviation from the unique steady state; this is a new steady state. Second, each steady state exhibits dynamic indeterminacy. There are many dynamic paths that lead to the same steady state. To select a particular dynamic equilibrium, we assume that prices are set one period in advance. Price stickiness is an equilibrium feature of the model.

The paper is organized as follows. Section 3.2 presents the model. Section 3.3 discusses the steady state equilibria. Section 3.4 discusses the dynamic equilibria and shows how a shock to confidence propagates over time and internationally. Section 3.5 concludes.

3.2 The Model

3.2.1 Structure

The analysis of the Eurozone crisis is built on the interaction between two ex-ante identical countries (regions), Home and Foreign. The conuntries are in a monetary union, they share the same currency and have access to the common loanable funds market. Each country can issue or buy bonds on this market. The countries are ex-ante identical. The foreign variables are marked with an asterisk.

Time t is discrete and never ends. Each country is populated by a unit continuum of overlapping generations as in the Blanchard (1985) 'perpetual youth' model. Every consumer has an infinite horizon but faces an age-invariant probability of death. At the beginning of each period, a new cohort of consumers is born to keep the population constant. As in Blanchard (1985), there exists a perfect annuitites market. Through this market, wealth of those who did not survive into the next period is redistributed to the existing consumers. The financial markets are sequentially complete. Consumers derive utility from a final nontradable consumption good, assembled from the local and imported intermediate goods, and from real money holdings.

In each country, there is a unit of non-reproducible physical capital. Physical capital plays two roles. First, combined with labor, capital produces a country-specific tradable good. Second, being a physical asset, capital has a market value interpreted as the value of the stock market. Labor is immobile across countries.

The labor market is characterized by search and matching frictions, as in Farmer (2012b). Firms need to withdraw labor from production and devote resources to recruiting activities. Similar to the tragedy of commons, search has negative externality on other firms. Generally, there is a positive equilibrium rate of unemployment, with a unique socially optimal level of unemployment.[1]

Search and matching frictions on the labor market lead to multiplicity of equilibria. Instead of allowing for bargaining over wage with constant weights as a selecting mechanism, I add a new fundamental: the belief function, as in Farmer (2013). The belief function is a rule used by economic agents to coordinate on the value of capital. The belief function does not contradict the rational expectations hypothesis because, in a model with multiple equilibria, beliefs are self-fulfilling: any level of confidence can be supported in a rational-expectations equilibrium. The belief function picks a particular equilibrium out of many. When consumers believe they are rich, they demand more goods (both local and imported). Firms hire more labor to satisfy the demand. National wealth rises, and beliefs fulfill themselves. Exogenous changes in the beliefs about the value of capital ('animal spirits') generate volatility and lead to crashes of the stock market and economic recessions.

[1]Stabilizing macro policy should be aimed at maintaining the unemployment rate at the socially optimal rate. Fiscal interventions and characterization of the optimal policy are beyond the scope of this paper.

3.2.2 Demand Side

Consumers face a constant probability of surviving into next period λ, and with probability $1-\lambda$ they die. Let $s = 0, 1, 2, \ldots$ index the date of birth of an agent. Consumers discount the future at rate $0 < \beta < 1$ and maximize

$$\mathbb{E}_t \sum_{\tau=0}^{\infty} (\beta\lambda)^{\tau} \left[(1-\delta) \log c_{t+\tau}^s + \delta \log \frac{M_{t+\tau+1}^s}{P_t} \right], \tag{3.1}$$

where C_t^s is the consumption basket of a consumer born at time s, a combination of domestic and foreign goods:

$$c_t^s = \left(\omega^{\frac{1}{\nu}} (c_{H,t}^s)^{\frac{\nu-1}{\nu}} + (1-\omega)^{\frac{1}{\nu}} (c_{F,t}^s)^{\frac{\nu-1}{\nu}} \right)^{\frac{\nu}{\nu-1}}, \tag{3.2}$$

and M_{t+1}^s / P_t are real money balances held by the consumer.

The money price of the domestic good is $P_{H,t}$ and the price of the foreign good is $P_{F,t}$. Expenditure minimization implies the following demand functions and the domestic consumer price index P_t:

$$c_{H,t}^s = \omega \left(\frac{P_{H,t}}{P_t} \right)^{-\nu} c_t^s, \tag{3.3}$$

$$c_{F,t}^s = (1-\omega) \left(\frac{P_{F,t}}{P_t} \right)^{-\nu} c_t^s, \tag{3.4}$$

$$P_t = \left(\omega P_{H,t}^{1-\nu} + (1-\omega) P_{F,t}^{1-\nu} \right)^{\frac{1}{1-\nu}}. \tag{3.5}$$

Each consumer has one unit of labor inelastically supplied on the domestic labor market. Due to existence of search and mathcing frictions, a consumer may be employed or unemployed. Under the assumption that there is perfect insurance within each generation, the intertemporal budget constraint can be written in real terms as

$$\lambda\left(S_t k^s_{t+1} + \mathbb{E}_t\left[Q_{t+1}P_{H,t+1}a^s_{t+1}\right] + M^s_{t+1}\right) + P_t c^s_t$$
$$= P_{H,t}a_t + (S_t + D_t)\,k^s_t + \frac{W_t}{P_t}L^s_t + \frac{M^s_t}{P_t}. \quad (3.6)$$

Here S_t is the money market price of capital, k^s_t is ownership in the representative firm that pays nominal dividends D_t, a^s_{t+1} represents purchases of Arrow securitites denomenated in the units of the domestic good, and the price of the Arrow securitites is Q_{t+1}; $W_t L^s_t$ is nominal labor income.

Completeness of financial markets requires that the following no-arbitrage conditions hold:

$$\mathbb{E}_t\left[Q_{t+1}\frac{(S_{t+1}+D_{t+1})/P_{H,t+1}}{S_t/P_{H,t}}\right] = 1. \quad (3.7)$$

According to this condition, up to the first order approximation, the rate of return on the stock market must be equal to the expected real interest rate (defined as $1/Q_{t+1}$).

Because the preferences are logarithmic, the spending on consumption and money holding constitute a constant fraction of consumer's wealth:

$$P_t c^s_t = (1-\delta)(1-\beta\lambda)\left[(S_t+D_t)k^s_t + P_t a^s_t + P_t h^s_t + M^s_t\right] \quad (3.8)$$

$$\frac{\lambda i_t}{1+i_t}M^s_{t+1} = \delta(1-\beta\lambda)\left[(S_t+D_t)k^s_t + P_t a^s_t + P_t h^s_t + M^s_t\right], \quad (3.9)$$

where i_t is the nominal interest rate that represents the opportunity cost of holding money, and h^s_t is real human wealth that follows the law of motion

$$h^s_t = \frac{W_t}{P_t}L^s_t + \lambda\mathbb{E}_t\left[Q_{t+1}h^s_{t+1}\right]. \quad (3.10)$$

Let aggregate variables denote the aggregated lower-case variables. The aggregate demand of the economy is completely described by the following equations:

$$c_t = (1-\delta)(1-\beta\lambda)\left[\frac{S_t}{P_t} + \frac{D_t}{P_t} + A_t + H_t + \frac{M_t}{P_t}\right], \tag{3.11}$$

$$\frac{\lambda i_t}{1+i_t}\frac{M_{t+1}}{P_t} = \delta(1-\beta\lambda)\left[\frac{S_t}{P_t} + \frac{D_t}{P_t} + A_t + H_t + \frac{M_t}{P_t}\right], \tag{3.12}$$

$$H_t = \frac{W_t}{P_t}L_t + \lambda\mathbb{E}_t\left[Q_{t+1}H_{t+1}\right], \tag{3.13}$$

where the total amound of capital, K_t is set equal to 1.

Finally, because consumers can trade the Arrow securitites internationally, the risk sharing condition holds for any two subsequent periods t and $t+1$ and for any realized states:

$$\begin{aligned}&\left(\frac{\lambda C_t}{\lambda C_{t+1} - (1-\lambda)(1-\beta\lambda)H_{t+1}}\right)^{-\theta}\frac{P_{t+1}}{P_t}\\ &\qquad = \left(\frac{\lambda C_t^*}{\lambda C_{t+1}^* - (1-\lambda)(1-\beta\lambda)H_{t+1}^*}\right)^{-\theta}\frac{P_{t+1}^*}{P_t^*}.\end{aligned} \tag{3.14}$$

The risk sharing condition states that the stochastic discount factors must be equalized across the two countries.[2]

3.2.3 Supply Side

The supply side is comprised of a representative firm. The firm is owned by the domestic households. It hires labor to produce goods, payes a fraction of revenue in wages and the rest in dividends.

There are search and matching friction, as in Farmer (2012b). The firm uses

[2]For the derivation of the stochastic discount factor in a perpetual youth model, see Farmer et al. (2011).

capital K and hires labor L_t, and it faces the production technology

$$Y_t = \left(\alpha^{\frac{1}{\nu}} K_t^{\frac{\nu-1}{\nu}} + (1-\alpha)^{\frac{1}{\nu}} L_{pt}^{\frac{\nu-1}{\nu}}\right) \tag{3.15}$$

where L_{pt} is the amount of labor engaged in production.[3]

The firm does not use all hired labor in production because search is costly; the firm must devote labor to hire workers. If there are L_{rt} recruiters, the following equations hold:

$$L_t = L_{pt} + L_{rt}, \tag{3.16}$$

$$L_t = q_t L_{rt}. \tag{3.17}$$

The endogenous variable q_t is a measure of labor market tightness. It shows how many workers one recruiter can hire. This variable is taken as given by firms.

The optimization problem of the firm is static: it maximizes dividends,

$$D_t = P_{H,t} Y_t - W_t L_t, \tag{3.18}$$

by choosing how much labor to hire L_t and how to allocate this labor. The first order condition delivers,

$$W_t L_t = P_{H,t}\left(Y_t - \alpha^{\frac{1}{\nu}} Y_t^{\frac{1}{\nu}}\right), \tag{3.19}$$

$$D_t = \alpha^{\frac{1}{\nu}} P_{H,t} Y_t^{\frac{1}{\nu}}. \tag{3.20}$$

At the macro level, there exists a matching function,

$$L_t = m(L_{rt}), \tag{3.21}$$

[3] I depart from the Cobb-Douglas production function to avoid the problem of the 'peculiar financial equilibria,' in which stock markets turn out to be perfectly correlated across countries. For discussion, see Cass and Pavlova (2004).

where $m'(\cdot) > 0$ and $m''(\cdot) < 0$. This function determines the equiliblrium labor market tightness,

$$q_t = \frac{m(L_{rt})}{L_{rt}}. \tag{3.22}$$

The described structure of the economy makes labor L_t and the labor allocation L_{pt} and L_{rt} residual variables. The aggregate demand Y_t uniquely determines labor L_t, and the factor payments can be expressed solely in terms of the aggregate demand, as shown above. Therefore, labor will be omitted in the subsequent analysis of the model.

3.2.4 Monetary Policy

There exists a central bank that issues the single currency held in both countries. The money supply is denoted M_{t+1}^{cb}. For the sake of simplicity, the sequence of the money supply is considered exogenously constant.

Due to market completeness, the nominal interest rate—the rate of return on nominal claims that are in zero net supply—must satisfy

$$\frac{1}{1+i_t} = \mathbb{E}_t \left[Q_{t+1} \frac{P_t}{P_{t+1}} \right]. \tag{3.23}$$

3.2.5 Beliefs

Economic agents form self-fulfilling expectations about the real value of the stock market, S_t/P_t and S_t^*/P_t respectively. As in Farmer and Nicolò (2018), these expectations imply that the stock markets are martingales: the expectations are adaptive:[4][5]

$$\mathbb{E}_t \left[\frac{S_{t+1}}{P_{t+1}} \right] = \frac{S_t}{P_t} \exp(\varepsilon_t), \tag{3.24}$$

[4]For an extensive discussion of belief functions, see Farmer (2016a).

[5]The martingale process is a special case of adaptive expectations, $\mathbb{E}_t [S_{t+1}/P_{t+1}] = (S_t/P_t)^\rho \left(\mathbb{E}_{t-1} [S_t/P_t]\right)^{1-\rho} \exp(\varepsilon_t)$ for $0 \leq \rho \leq 1$. In this case, the expectations depend on the current realization of the stock market and the past expectations.

$$\mathbb{E}_t\left[\frac{S^*_{t+1}}{P^*_{t+1}}\right] = \frac{S^*_t}{P^*_t}\exp(\varepsilon^*_t), \tag{3.25}$$

where ε_t and ε^*_t represent independent and serially uncorrelated shocks to expectations, or confidence shocks.

Due to the presence of multiplicity of equiliria, the belief functions (3.24) and (3.25) are rational, in spite of their adaptive nature.

3.2.6 Definition of the Equilibrium

A competitive rational-expectations equilibrium is a sequence of production quantities $\{Y_t, Y^*_t\}$, consumption $\{C_t, C^*_t\}$, prices $\{P_{H,t}, P_{F,t}, P_t, P^*_t, S_t, S^*_t\}$, human wealth $\{H_t, H^*_t\}$, realized contingent payments, prices of the Arrow securitites and nominal interest rates $\{A_{t+1}, Q_{t+1}, i_t\}$ for all possible future states, and of money holdings $\{M_{t+1}, M^*_{t+1}\}$ such that, given the exogenous confidence shocks $\{\varepsilon_t, \varepsilon^*_t\}$ and monetary policy $\{M^{cb}_{t+1}\}$, the following conditions hold:

1. Consumers' behavior is described by the demand functions (3.11), (3.12), (3.13) and their equivalents for the second country
2. The goods markets clear:

$$\omega C_t\left(\frac{P_{H,t}}{P_t}\right)^{-\eta} + (1-\omega)C^*_t\left(\frac{P_{H,t}}{P^*_t}\right)^{-\eta} = Y_t, \tag{3.26}$$

$$\omega C^*_t\left(\frac{P_{F,t}}{P^*_t}\right)^{-\eta} + (1-\omega)C_t\left(\frac{P_{F,t}}{P_t}\right)^{-\eta} = Y^*_t \tag{3.27}$$

3. The nominal interest rate satisfies (3.23)
4. The money market clears:

$$M_{t+1} + M^*_{t+1} = M^{cb}_{t+1} \tag{3.28}$$

5. The no-arbitrage condition (3.7), its equivalent for the second country, and the risk sharing condition (3.14) hold

6. The Arrow securities are in zero net supply:

$$P_{H,t}A_t + P_{F,t}A_t^* = 0 \tag{3.29}$$

7. The stock market value satisfies the the belief functions (3.24) and (3.25)

8. Necessary transversality conditions hold

These equilibrium conditions are standard with one exception. The innovation of the paper is the introduction of the belief functions that govern the value of capital, and allowing for the shocks to beliefs about the future value of capital.

3.3 Steady State Equilibria

By giving up Nash bargaining over wage with constant bargaining powers, we leave the labor market equilibrium indeterminate. However, once we introduce the belief function and once we allow beliefs about asset prices to determine the aggregate demand, the equilibrium becomes unique. Our model is isomorphic to a model with Nash bargaining where the bargaining weights are dependent on the aggregate demand.

The steady state in our model has two distinctive features. First, beliefs about the value of assets are self-fulfilling. If economic agents believe prices of assets must be high, the economy will be in equilibrium with high output and consumption, low unemployment, and high values of capital. Second, any unemployment rate can be sustained at the steady state. We call this property the steady-state indeterminacy.[6] For given beliefs, there is a unique unemployment rate. Since

[6] Farmer (2016a) discusses the evolution of the models with determinate and indeterminate steady states.

Table 3.1: Parameter Values

Parameter	Interpretation	Value
α	Coefficient on capital in the production function	0.33
β	Time discount factor	0.99
λ	Probability of surviving	0.99
δ	Coefficient on money in utility	0.01
ω	Home bias in consumption	0.95
η	Elasticity of substitution between capital and labor	0.50
ν	Elasticity of substitution between domestic and foreign goods	5.00

a range of beliefs can be sustained at the steady state, there is a continuum of steady states indexed by the beliefs about the value of capital. An immediate implication of the steady state indeterminacy is that there is no tendency for the economy to self-correct in response to shocks.

Because of the complexity of the model, it is impossible to obtain the closed-form solution for the steady state. We use the numerical solutions and demonstrate the comparative statics graphically. We adopt the parameter values provided in Table 3.1 and plot the steady states on Fig. 3.1.

On Figure 3.1, each panel represents a three-dimensional diagram where the x-axis and y-axis measure the exogenous beliefs about the real value of capital in the steady state, S/P and S^*/P^* respectively, and the z-axis measures an endogenous variable. The contour lines are depicted below each surface.

Panels (a) and (b) display output in each country. The total amount of goods is increasing in beliefs about the stock market in both countries, but the effect is asymmetric. When domestic consumers believe they are wealthy, they increase consumption of both, domestic and imported, goods. Due to home bias in consumption, the effect of the domestic stock market on the aggregate demand is stronger than of the foreign stock market.

Panels (c) and (d), (e) and (f) show the positive effect of the two stock markets on consumption and real money balances for each country. As the value of capital

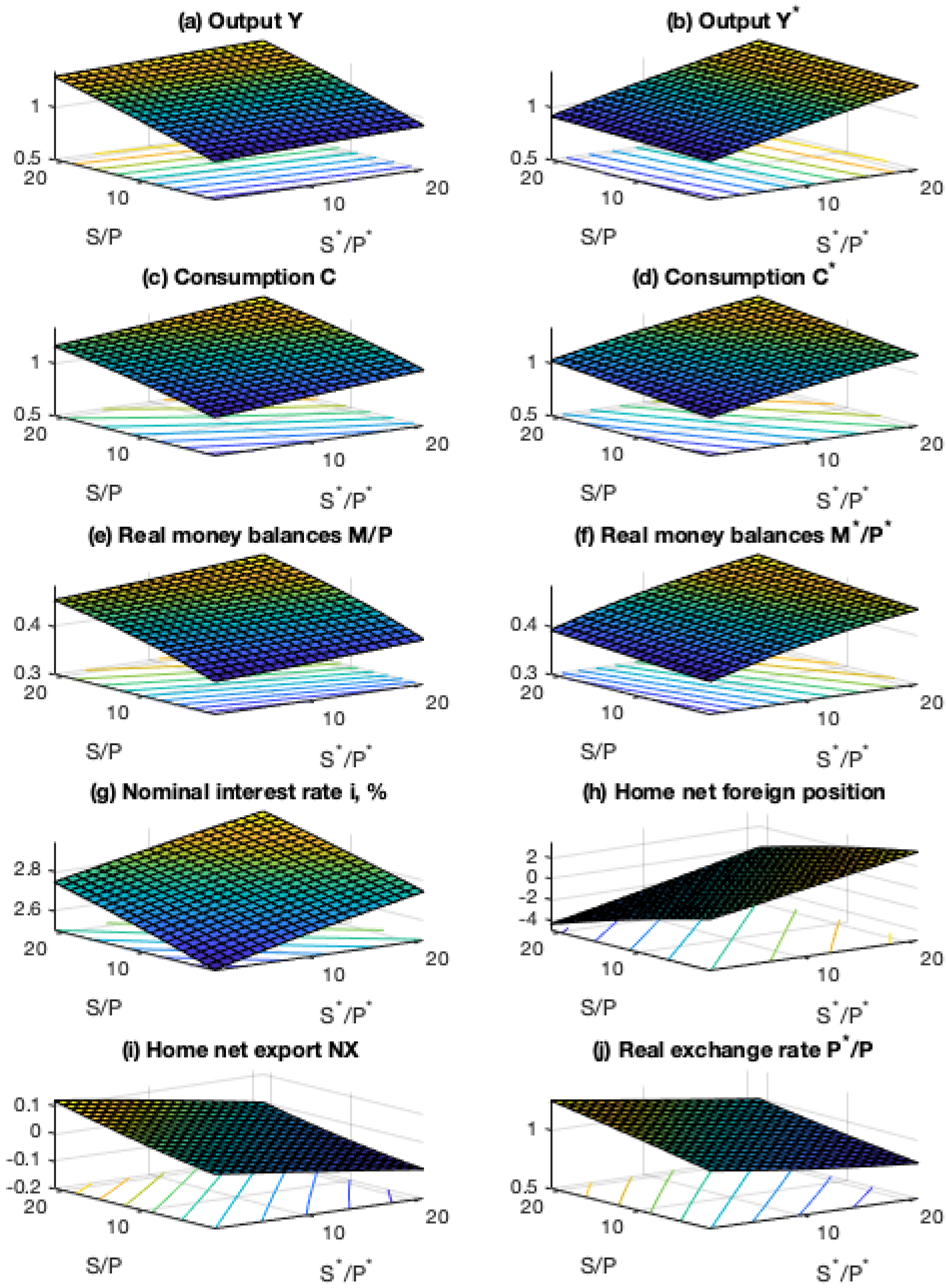

Figure 3.1: Steady-State Comparative Statics: the Role of Beliefs in the Determination of Equilibrium

becomes larger in either country, the aggregate demand rises, and consumption increases in both countries. Because consumers need money to purchase goods, real money balances increase too. Again, home bias in consumption makes the cross-country effects of the stock markets assymetric: domestic variables are more sensitive to the domestic stock market rather than the foreign stock market.

Panel (g) shows the single nominal interest rate. Because there is zero inflation at the steady state, the nominal interest rate is equal to the real interest rate. The interest rate is positively and symmetrically related to the stock market in each country. The positive relation comes from the assumption that the money supply is helod constant at the steady state. As the aggregate demand increases, the demand for real balances increases, and the interest rate rises to clear the money market. The symmetry is based on the assumption that there is a single loanable funds market, and the interest rates must be equalized across countries.

Panel (h) shows the net foreign asset position of the Home country. For the Foreign country, the net foreign asset position is equal to the negative of the Home foreign asset position. As the domestic stock market rises, the Home country takes a negative foreign asset position on the international financial market by issuing bonds and selling them to the Foreign country. The domestic consumers need currency to purchase consumption, and they trade bonds for currency. Because there is a single currency market, the response of the foreign asset position is symmetric to changes in domestic or foreign beliefs about the stock market.

Panels (i) and (j) display Home's net export and terms of trade, respectively. Domestic net export is positively related to the domestic stock market and negatively to the foreign stock market. This comes from the fact that foreign debt must be supported by future trade balance surpluces. As a result, there must be depreciation of the terms of trade to promote positive trade balance. Since, at the steady state, international trade is determined by the net foreign asset position, both net exports and the terms of trade respond symmetrically to the domestic and foreign stock markets.

3.4 Dynamic Equilibria and Confidence Crashes

This section considers the dynamic equilibrium of our model. It describes how the economy moves between the steady states in response to the shocks to expectations. Because the object of interest in this section is deterministic convergence to the steady state, the model is log-linearized around the socially optimal steady state. The calibration from Table 3.1 is preserved.

The dynamic equilibrium is characterized by dynamic indeterminacy. To select the unique dynamic path, the prices are assumed to be predetmined and set one period in advance, based on rational expectations, as in Farmer (2000).

The belief functions (3.24) and (3.25) demonstrate that the expectations about the future value of the stock market are weighted averages of the past expectations and the current value. The adaptive nature of expectations allows to incorporate the feedback response of expectations to the economy: not only expectations determine the dynamics of the economy, but also the current equilibrium affects the expectations.

To illustrate the dynamic properties of the model, this sectionstudies the impact of a confidence shock ε_t. In period 0 the economy is at the steady state. In period 1, beliefs at Home are hit by a negative 1% shock. The subsequent dynamics of the economy is deterministic. Figure 3.2 shows the dynamic path of the economy.

Panel (a) displays the dynamics of the expectations about the future stock market, and panel (b) displays the realized value of the stock market. Although the shock was as large as -1%, the decline in expectations is almost 2%. The additional effect comes from the endogenous response of the current value of the stock market, as depicted on panel (b). Pessimistic expectations about the future

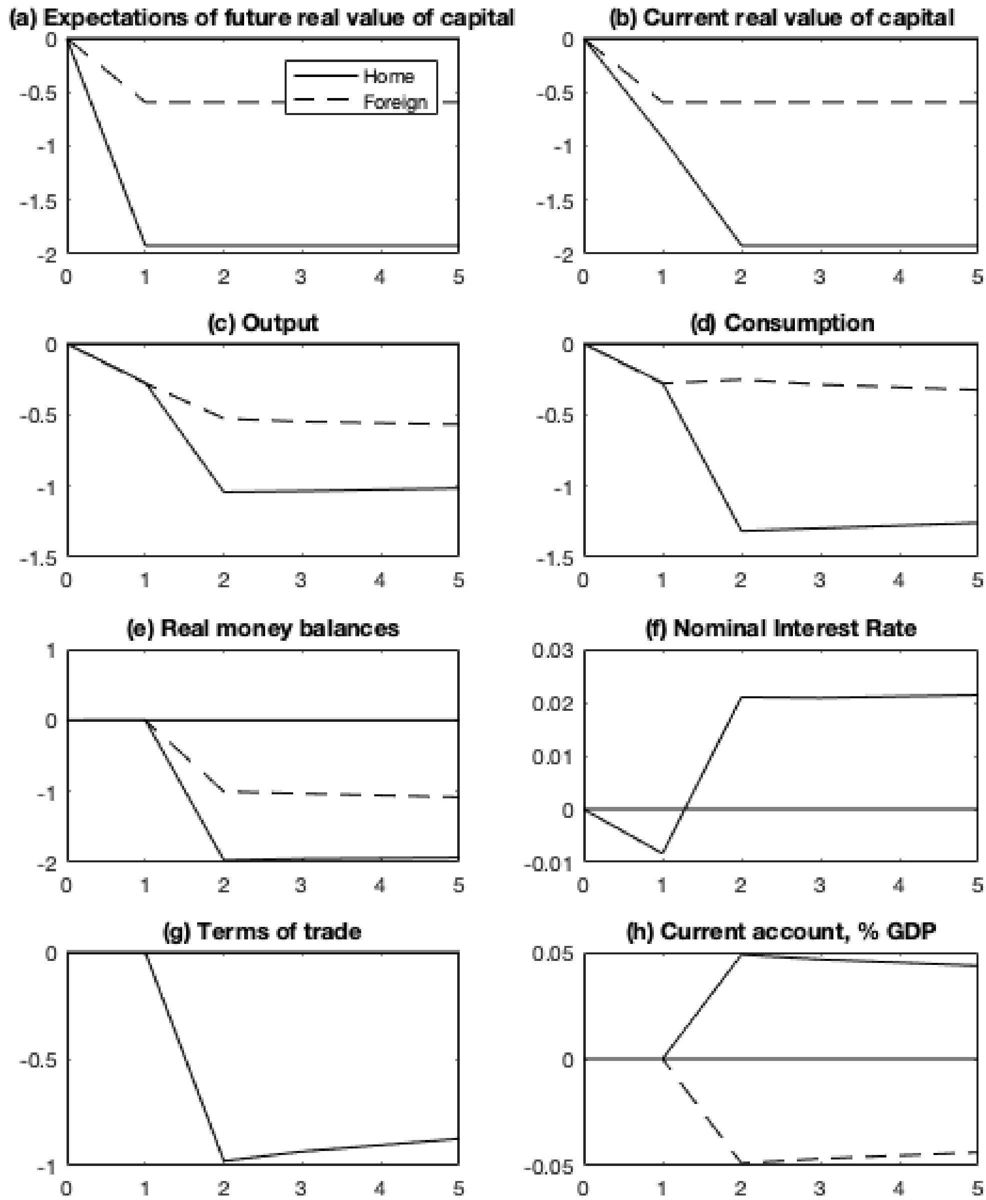

Figure 3.2: Impulse Response Functions: Response of the Economy to a 1% Crash in Home's Confidence.

translate into an immediate crash of the stock market today due to the contagion effect. Panels (a) and (b) shows that both stock markets crash in period 1, even though there was a shock only to the Home economy. The stock market never recovers, and the economy turns out to be in a new steady state.

Panels (c) and (d) display the dynamics of output and consumption. Since the value of the stock market determines consumers' wealth, the stock market crash leads to a decrease in the aggregate demand and equilibrium output in both countries. The effect on output lasts two periods. In period 1, prices are predetermined and cannot adjust, causing contagion. In period 2, prices adjust, and the recession deepens.

Panel (e) shows the real money balances. The real money balances depend on the aggregate demand, and as the aggregate demand decreases, consumers reduce the demand for money.

Panel (f) shows the dynamics of the nominal interest rate. In period 1, the interest rate declines. Since there is no uncertainty after period 1, forward-looking consumers correctly predict a decline in the domestic stock market, between periods 1 and 2. Due to the no-arbitrage condition, the nominal interest rate decreases. In period 2, the interest rate rises above the initial steady-state level and remains permanently higher that in period 0. The increase is caused by the implied dynamics of nominal prices: since the domestic goods are in smaller supply, they become relatively more expensive.

Panel (g) shows the dynamics of the terms of trade for the Home country. The terms of trade remain unchanged in period 1 due to the assumption that prices are predetermined. In period 2, the terms of trade appreciate to clear the goods markets, where the domestic goods are in smaller supply and appreciate relatively to the supply of foreign goods.

Panel (h) shows the dynamics of current accounts. Because prices are fixed in period 1, the current accounts respond only in period 2. In period 2, the Home country runs a positive current account; domestic consumers trade excess currency for bonds, and the Home country experiences capital outflow. As the terms of trade gradually return to the steady state value, the current account declines to zero.

The dynamic experiment presented above illustrates the logic of prolonged stagnation in the Eurozone based on a confidence crash. In 2010, investors became pessimistic about the future of the Peripheral countries, and the stock markets in Periphery crashed. Core's asset prices responded to the shock to confidence in Periphery. The channel for contagion is international trade. When agents in Periphery become pessimistic about the value of their assets, production and consumption in Periphery fall. For consumers in Core, Periphery's goods become relatively more expensive, so they reduce import. The wealth effect makes them reduce consumption and hence production in Core. The decline of output in Core is smaller than in Periphery for two reasons. First, there is substitution effect: both economies switch to the Core goods against the goods produced in Periphery. Second, the two countries share the same currency and prefer to hold money proportional to consumption. As consumption falls in Periphery, Core holds more currency than Periphery, which prevents big drops in Core's output.

3.5 Conclusion

This paper proposes a two-country model of a monetary union to rationalize the long-lasting stagnation in the Eurozone. By combining search and matching frictions on the labor market, abandoning the Nash bargaining over wage, and allowing for self-fulfilling beliefs about asset prices, I build a model that is consistent with the stylized facts about the Eurozone crisis and stagnation. First, the trigger of the crisis is a negative shock to confidence, to the beliefs about the value of assets in Periphery. Second, the model demonstrates financial contagion. Pessimism about the value of assets in Periphery spreads over to the core countries, causing an economic collapse all over the monetary union. Third, in the model, confidence crashes have permanent effects: high unemployment and low output can persist indefinitely long, without tendency to self-recover unless confidence recovers.

The results rely on two distinctive features of the adopted approach. First, the model displays steady state indeterminacy caused by multiple equilibria on the labor market. This breaks the assumption that the unemployment rate returns to its unique long-run optimal rate. Second, the model displays dynamic indeterminacy. The analysis is focused on a dynamic equilibrium with predetermined prices. Price stickiness creates nominal persistence. Self-fulfilling beliefs propel the shocks over time.

This paper develops as a rich environment for policy analysis. The model suggests that managing the single nominal interest rate for the central bank is not sufficient for recovery. While the interest rate affects *relative* production, consumption and the unemployment rate, the *level* of production and of other variables depends on the level of expectations about the stock market. A successful policy should be able to address the level of the stock market, not only the rate of return on capital. The future work will add fiscal policy to study the optimal stabilizing, monetary and fiscal policies, and interaction between them, and the role of labor mobility in a monetary union.

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