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Comment on: Fraunhofer diffraction patterns from apertures illuminated with nonparallel light.

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In a recent paper, Klingsporn¹ uses simple lens theory to relate the Fraunhofer diffraction pattern for nonparallel illumination of an aperture to the well-known results for parallel illumination. We can restate his result in a simpler and yet more general form which is also useful in practical optical design.

For simplicity we consider only slits (rather than circles) for apertures. We also take all angles to be small. First recall the familiar case of a parallel light beam of wavelength λ incident on a slit of width D , undergoing Fraunhofer diffraction, and then being focused to a real image by a positive lens of focal length f . (See Image #1 of Figure 1). Let Δx denote the image width measured from the central maximum to the first zero. Then we have the familiar result

$$\Delta x = f \lambda / D \quad (1)$$

Next reexpress this result. The ratio f/D is commonly called the f -number of the lens. Its inverse, $D/f = \Delta\theta$, is the angular full width of the beam as it converges on the image. We rewrite Eq. (1) in the form

$$\Delta x = \lambda / \Delta\theta \quad (2)$$

so as to express the image width as the wavelength times the f -number.

Now consider a general case where we start with a point source, go through any number of converging and diverging lenses, pass through any number of real or virtual images, scrape on any number of aperture stops located wherever you

please (but not too close to a real image), and are finally brought to focus at a particular real image of interest. (See Image #2 of Figure 1). What is the diffraction width Δx of the image? Consider the beam as it converges to the image; let it have angular full width $\Delta\theta$ as determined by geometrical optics and the aperture stops. Then the general result is that Eq. (2) still holds. We can also define $1/\Delta\theta$ to be the generalized f-number, i.e., the distance from the last lens to the image, divided by the beam diameter at the last lens. Then the generalized interpretation of Eq. (2) is that the image width due to diffraction is the wavelength times the generalized f-number.

To prove this result we may use geometrical optics a la Klingsporn.¹ (See the caption of our Figure 1.) Alternatively, we may rewrite Eq. (2) so that its truth becomes "well known" to the physicist (although less useful to the designer). Multiply both sides of Eq. (2) by $k = 2\pi/\lambda$ and by $\Delta\theta$, and let $k\Delta\theta = \Delta k_x$. Then Eq. (2) becomes

$$\Delta x \Delta k_x = 2\pi. \quad (3)$$

The left side of Eq. (3) is the "volume in phase space" at the image. Its numerical value of 2π is due to our particular choices for Δx (central maximum to first zero) and for Δk_x (corresponding to $\Delta\theta$ being the angular full width). Then the fact that the general interpretation of Eq. (2) is simply obtained from the result (1) for parallel light follows from the well-known result of geometrical optics that volume in phase space is conserved as the beam progresses from one image to the next; and because we can imagine that just before one of these images the beam was parallel and of diameter D before being brought to a focus a distance f from a lens, thus establishing the numerical value of 2π for the conserved phase space.

As an extended application of Eq. (2) consider the interference pattern formed by a system of identical slits having separation d between neighboring

slits. We know that for parallel light incident on the slits the separation Δx between adjacent principal maxima at an image formed by a lens is given by $\Delta x = f \lambda / d$. Now define $\Delta \theta$ to be the angle between adjacent "rays" converging towards the image from adjacent slits. For parallel light $\Delta \theta = d/f$ and we can write $\Delta x = \lambda / \Delta \theta$. For a slit system located anywhere in the system, in converging, diverging, or parallel light, that same formula for Δx gives the interference pattern at the image, provided $\Delta \theta$ (which is generally not d/f) is still taken to be the angle given by geometric optics for adjacent rays converging to the image.

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1. Paul E. Klingsporn, Am.J.Phys. 47, 147 (1979).

Figure 1. Diffraction from apertures illuminated by parallel or nonparallel light. Any one of the three aperture stops shown can be taken to be the limiting aperture. The image width at Image #2 equals that at Image #1 multiplied by the magnification q/p : $\Delta x_2 = (q/p)\Delta x_1 = (q/p)(f/D)\lambda = (q/D')\lambda = \lambda/\Delta\theta_2$.

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