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Author

Moschkovich, Judit N

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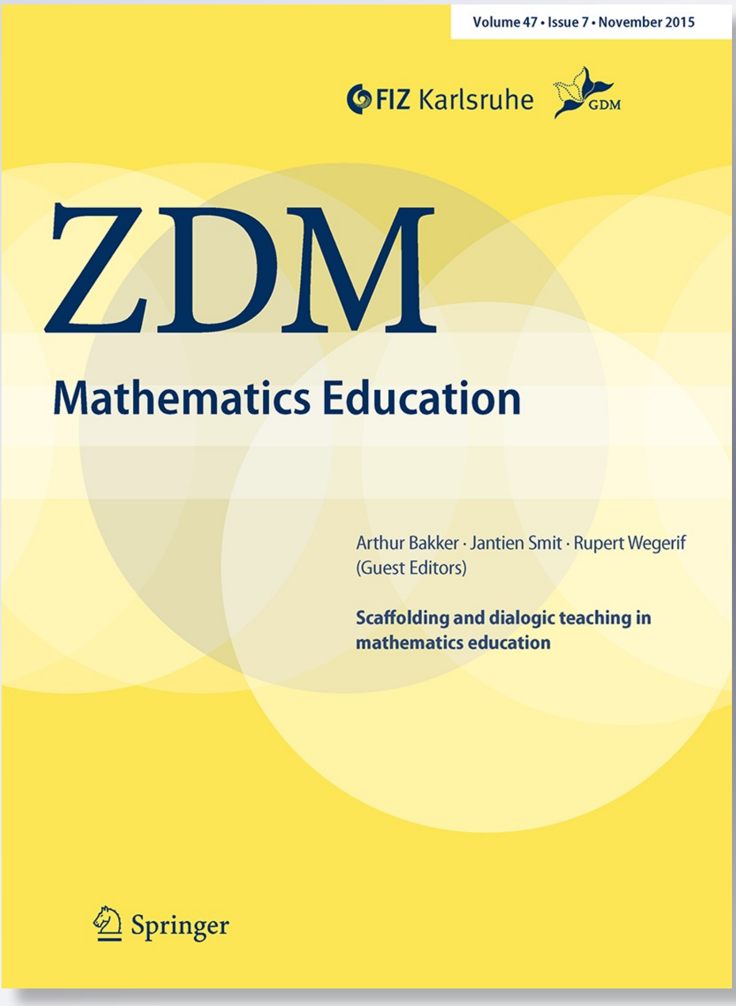

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ORIGINAL ARTICLE

Scaffolding student participation in mathematical practices

Judit N. Moschkovich[1]

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Abstract The concept of scaffolding can be used to describe various types of adult guidance, in multiple settings, across different time scales. This article clarifies what we mean by scaffolding, considering several questions specifically for scaffolding in mathematics: What theoretical assumptions are framing scaffolding? What is being scaffolded? At what level is scaffolding implemented? What is the setting for scaffolding? And lastly, how can scaffolding manage the tension between providing appropriate calibrated support while also providing opportunities beyond learners' current understandings? The paper describes how attention to mathematical practices can maintain a sociocultural theoretical framing for scaffolding and move scaffolding beyond procedural fluency. The paper first specifies the sociocultural theoretical assumptions framing the concept of scaffolding, with particular attention to mathematical practices. The paper provides three examples of scaffolding mathematical practices in two settings, individual and whole-class. Lastly, the paper considers how two teacher moves during scaffolding, proleptic questioning and revoicing, can serve to provide appropriate calibrated support while also creating opportunities beyond current proficiency.

Keywords Sociocultural · Mathematical practices · Scaffolding

✉ Judit N. Moschkovich
jmoschko@ucsc.edu

[1] University of California, Santa Cruz, Santa Cruz, CA, USA

1 Introduction

The concept of scaffolding can be used to describe different types of adult guidance, with different purposes, in multiple settings, and across varied time scales. To clarify scaffolding in mathematics education research and practice, discussions of this construct need to be specific in describing the "what, why, and how" of scaffolding (Pea, 2004). We can begin to clarify what we mean by scaffolding in mathematics education by specifying the theoretical assumptions used to frame scaffolding. We can also be specific regarding the different levels (micro, meso, or macro) or settings (individual, small group, whole class) for scaffolding. And perhaps most importantly, we can describe scaffolding processes specifically for mathematics. This article considers two questions as they apply specifically to scaffolding in mathematics learning and teaching: (1) what is being scaffolded? In particular, how can scaffolding support students in developing more than procedural fluency? And (2) how is scaffolding accomplished? In particular, how can scaffolding provide appropriate calibrated support while also providing opportunities beyond learners' current understandings?

The article will first specify the sociocultural theoretical assumptions that frame the concept of scaffolding. These assumptions are crucial if the concept of scaffolding is to be implemented as a sociocultural construct, rather than with behaviorist assumptions about learning and teaching. The paper also specifies the time scales and settings for the examples of scaffolding, illustrating the scaffolding of mathematical practices at two different time scales, micro and meso, and in two different settings, individual tutoring and whole-class. The article addresses the first question, *what* is scaffolded with examples of scaffolding student participation in mathematical practices. For mathematics learning and teaching, there are many possibilities for what to scaffold: procedural skills, conceptual understanding, metacognitive strategies, and mathematical practices. The examples provided here focus on scaffolding mathematical practices to illustrate how attention to mathematical practices can maintain a sociocultural theoretical framing for scaffolding and use scaffolding beyond rote procedural skills in mathematics.

To address the second question regarding *how* scaffolding is accomplished, examples illustrate two ways that scaffolding can provide both appropriate calibrated support while also providing opportunities beyond current student competencies, proleptic questioning and revoicing. Two examples of scaffolding interactions illustrate how these two teacher (or adult) moves can support student participation in mathematical practices and provide exposure to academic language.

Scaffolding is a concept used to describe how learning is mediated by interactions with more expert others (Bruner & Sherwood, 1975; Wood, Bruner & Ross, 1976). Stone (1998a) identified four key features of scaffolding:

1. The adult takes responsibility for encouraging a child to become involved in a "meaningful and culturally desirable activity beyond the child's current understanding or control" (p. 349).
2. The adult engages in diagnosing the learner's current level of understanding or proficiency and calibrates the appropriate support to be provided.
3. The adult provides a range of types of support.
4. The support is temporary and fades over time.

A description of the characteristics of scaffolding comes from Wood et al. (1976):

> "…six tutor actions that constitute the process of scaffolding: (1) recruiting interest in the task; (2) reducing the degrees of freedom (simplifying the task); (3) maintaining direction toward the goals of the task; (4) marking critical features; (5) controlling frustration; and (6) modeling the preferred procedures by demonstrating, so that the learner can 'imitate it back' (Smit et al., p. 98)."

Possible distinctions among different types of scaffolding include differences between designed and interactional scaffolding (Smit & van Eerde, 2011); cognitive, metacognitive, and affective scaffolding (Leiss & Weigand, 2005); settings such as whole class (Smit, van Eerde, & Bakker, 2013), a group of students, or a single student; and whether the intervention takes place at the beginning, in the middle, or at the end of a task (Leiss & Weigand, 2005). A crucial distinction specific to mathematics learning is whether the pedagogical purpose or goal of scaffolding is to support students in acquiring procedural fluency (Kilpatrick, Swafford, & Findell, 2001), developing conceptual understanding, or participation in classroom discussions focused on mathematical practices (for an example see Moschkovich, 2007). Procedural fluency has been defined as skill in carrying out procedures flexibly, accurately, efficiently, and appropriately. Conceptual understanding has been defined as the comprehension of mathematical concepts, operations, and relations.[1] Mathematical practices include problem solving, sense making, reasoning, modeling, abstracting, generalizing, and looking for patterns, structure, or regularity (Moschkovich, 2004, 2007, and 2013).[2]

Another important distinction for types of scaffolding (or pedagogical scaffolding as van Lier calls it) is that support can be provided at different levels or times scales (van Lier, 2004):

(a) Macro: the design of long-term sequences of work or projects, with recurring tasks-with-variations over a protracted time period;
(b) Meso: the design of individual tasks as consisting of a series of steps or activities that occur sequentially or in collaborative construction;
(c) Micro: contingent interactional processes of appropriation, stimulation, give-and-take in conversation, collaborative dialogue (Swain, 2000), and so on.

The metaphor of scaffolding can thus be used to describe expert guidance in many different situations, including those that involve more than one adult and one child. Applying the concept loosely to activity at different levels (micro, meso, and macro), in different settings (individual or collective), or for different pedagogical purposes (to support procedural fluency, conceptual understanding, or participation in classroom discussions), makes it difficult to review research or make recommendations for practice. For example, scaffolding at the meso or macro levels may not include all the central characteristics of micro scaffolding and scaffolding in a whole class setting is likely to function differently than with a single student. In mathematics, scaffolding can focus on supporting students' opportunities to practice procedural skills, develop conceptual understanding, or participate in mathematical practices. Scaffolding of procedural skills is likely to be accomplished through different kinds of interactions than scaffolding of student

[1] For a discussion of differences between procedural fluency and conceptual understanding, see Kilpatrick, Swafford, & Findell (2001).

[2] These practices are described in the NCTM Standards and in the Common Core State Standards (2010). For a discussion of differences between procedural fluency and mathematical practices, see Moschkovich (2013).

participation in a mathematical discussion. In the first case, there would be a tight focus on the goal of accomplishing steps in an arithmetic procedure. In the second case, the interactions might be more open ended and allow for building on student ideas.

Although the study of scaffolding can be grounded in the four features and six characteristics of scaffolding provided above, these have limitations. First, the four features and six characteristics seem to refer, in particular, to micro scaffolding that involves one adult working face to face with a learner or learners and particularly, that the adult is in dialogue with the learner. It is important to examine scaffolding interactions in whole class settings and consider how micro scaffolding works in such settings. Most importantly for mathematics learning and teaching, these general descriptions do not address the details of scaffolding for learning mathematics. It is crucial to examine scaffolding interactions that focus on different mathematical goals, such as supporting student participation in mathematical practices or in a classroom discussion, in contrast to accomplishing a set of procedural steps.

The article first describes the broader theoretical assumptions framing scaffolding from a sociocultural perspective by focusing on mathematical practices (Sect. 2). Section 3 provides an example of prolepsis (Stone, 1998b) during micro scaffolding of mathematical practices in an individual setting. Section 4 describes how teacher revoicing (O'Connor & Michaels, 1993) in a whole class setting managed the tension between providing calibrated support while also providing opportunities beyond students' current proficiency in mathematical practices and academic language. The fifth section of the paper describes how to design scaffolding focused on mathematical practices at the meso level in a whole class setting. The article concludes with a summary and instructional implications.

2 A sociocultural theoretical framework for scaffolding mathematical practices

Several theoretical constructs are important for maintaining a Vygotskian perspective on scaffolding. This section introduces mathematical practices, appropriation, prolepsis, and revoicing, in preparation for using these constructs in later sections.

2.1 Mathematical practices

Work in mathematics education in the last 25 years has assumed that mathematics instruction in schools needs to parallel, at least in some ways, the practices of mathematicians (for example Cobb, Wood, & Yackel 1993; Lampert, 1986 and 1990; Schoenfeld, 1992). This work emphasizes classroom activities that emulate academic mathematical practices and include aspects of mathematicians' practices, such as making conjectures or generalizations and subjecting these to review and refutation by a (classroom) community. Students should have opportunities to participate in mathematical practices such as abstracting, generalizing, and constructing arguments (NCTM, 1989). Students are expected to make conjectures, agree or disagree with the conjectures made by their peers or the teacher, and engage in public discussion and evaluation of claims and arguments made by others (NCTM, 1989).

In my own research, I have used a Vygotskian theoretical framing (Vygotsky, 1978; Wertsch, 1979, 1985) to describe how students participate in mathematical practices during tutoring (Moschkovich, 2004) or classroom discussions (Moschkovich, 1999). This perspective assumes that social interaction that leads to learning involves joint activity (not just any type of interaction). I use the terms *practice* and *practices* in the sense used by Scribner (1984) for a practice account of literacy to "… highlight the culturally organized nature of significant literacy activities and their conceptual kinship to other culturally organized activities involving different technologies and symbol systems …" (p. 13). This definition requires that *practices* be culturally organized in nature and involve symbol systems. From this perspective, mathematical practices are social and cultural, because they arise from communities and mark membership in communities. They are also cognitive, because they involve thinking, and they are also semiotic, because they involve semiotic systems (signs, tools, and meanings).

Many researchers have used the concept of mathematical practices. For example, Cobb, Stephan, McClain, & Gravemeijer (2001) define mathematical practices as the "taken-as-shared ways of reasoning, arguing, and symbolizing established while discussing particular mathematical ideas" (Cobb et al., p. 126). In contrast to social norms and socio-mathematical norms, mathematical practices are specific to particular mathematical ideas. Schoenfeld (2012, 2014) has also referred to mathematical practices, describing these as one of the "two main (and deeply intertwined) aspects to doing mathematics: mathematical content and mathematical practices" (2014, p. 500). Mathematical practices are similar to mathematical habits of mind (Schoenfeld, 2012) such as "a predilection to explore, to model, to look for structure, to make connections, to abstract, to generalize, to prove" (2012, p. 592).

A Vygotskian perspective has implications for using the concept of mathematical practices, including (a) goals are an implicit yet fundamental aspect of mathematical practices; (b) discourse is central to participation in mathematical practices; and (c) meanings for words are situated and constructed while participating in practices. In my work I have used the Vygotskian perspective summarized above to frame analyses of mathematical practices. In this article, I use an example from a study that examined how interaction with a tutor (Moschkovich, 2004) supported learner appropriation of mathematical practices, in part through tutor scaffolding. The first example (Sect. 3) uses an excerpt from that previous study to illustrate a focus on mathematical practices and proleptic questioning. In the second example (Sect. 4), I use an excerpt from a sociocultural analysis of students' participation in a mathematical discussion (Moschkovich, 1999 and 2007) to illustrate teacher revoicing during scaffolding that supported mathematical practices.

2.2 Appropriation, prolepsis, and revoicing

The notion of "focus of attention" comes from Rogoff's (1990) definition of appropriation. Central features of appropriation (Rogoff, 1990) include achieving a joint focus of attention, developing shared meanings, and transforming what is appropriated. Rogoff suggests that "intersubjectivity may be especially important for learning to participate in practices that are implicit or 'inaccessible' cognitive processes that are difficult to observe or explain" (p. 143). Rogoff distinguishes between what she calls "skills" and "shifts in perspective." Echoing the distinction between procedural mathematical skills and mathematical practices, Rogoff defines skills as "the integration and organization of information and component acts into plans for action under relevant circumstances" (p. 142). In contrast, shifts of perspective (and participation in mathematical practices), involve "giving up an understanding of a phenomenon to take another view contrasting with the original perspective" (p. 142).

Two constructs have been used to describe adult guidance during scaffolding, prolepsis and revoicing. Stone (1998b) described scaffolding as involving a *proleptic* interactional dynamic:

> "In a scaffolding situation, the child is led to participate in an activity whose full meaning has yet to be fulfilled. That is, the child is acting in anticipation of full understanding and must develop an understanding from the actions in which he or she is led to engage" (Stone, 1998b, p. 353).

Stone (1998b) and van Lier (2004) have suggested that prolepsis, or "attributing intent before its true onset, and capitalizing on incipient skills and understandings as they show signs of emerging" (van Lier, 2004), is an essential aspect of micro scaffolding. The first example (excerpt in Sect. 3) illustrates how micro scaffolding in an individual setting (Moschkovich, 2004) involved prolepsis, in particular a tutor move I label "proleptic questioning." The example describes how "proleptic questioning" attributed shared meaning before its onset and supported a student's participation in mathematical practices.

Revoicing (O'Connor & Michaels, 1993) is a teacher move describing how an adult, typically a teacher, rephrases a student's contribution during a discussion, expanding or recasting the original utterance (Forman, McCormick, & Donato, 1997). Revoicing has been used to describe teacher talk moves in several studies (for example, Enyedy et al., 2008; Herbel-Eisenmann et al., 2009). The second example (excerpt in Sect. 4) illustrates how scaffolding during a whole class discussion involved revoicing that supported mathematical practices and academic language.

Proleptic questioning and revoicing are certainly not the only possible types of scaffolding, they are examples of a vaster array of adult moves during scaffolding. These two particular types of scaffolding were selected for theoretical reasons. First, since prolepsis has been identified as an essential aspect of micro scaffolding, proleptic questioning provides an example of a proleptic adult move. Second, revoicing, which can also be proleptic, provides an example of a teacher move that can serve to manage the tension between providing appropriate calibrated support while also providing opportunities beyond the learners' current proficiency, but not through direct instruction. Most importantly, both of these examples were selected because they illustrate teacher (or tutor) moves that focus on mathematical practices, thus serving to maintain a sociocultural framing for scaffolding.

3 Example 1: micro scaffolding mathematical practices through prolepsis

The first example comes from a case study of one student exploring functions through interaction with a tutor (Moschkovich, 2004) while using graphing software (Schoenfeld, 1990). That case study used a Vygotskian perspective and the concept of appropriation (Newman, Griffin, & Cole, 1989; Rogoff, 1990) to describe the impact that interaction with a tutor had on a learner, focusing on mathematical practices. The analysis of two tutoring sessions illustrated how the tutor introduced three tasks (estimating y-intercepts, evaluating slopes, and exploring parameters).[3]

The previous analysis (Moschkovich, 2004) described how a learner appropriated two mathematical practices crucial for working with functions (Breidenbach, Dubinsky, Nichols, & Hawks, 1992; Even, 1990; Moschkovich, Schoenfeld, & Arcavi, 1993; Schwarz and Yerushalmy, 1992;

[3] The study also described how appropriation functioned through the focus of attention, meaning for utterances, and goals for these three tasks, showing how the learner actively transformed some goals.

Sfard, 1992): "treating lines as objects" and "connecting a line to its equation" (in the form $y = mx + b$). The analysis showed that the student appropriated these two mathematical practices; she came to see, talk about, and act as if a line is an object that can be manipulated and as if lines are connected to their equations. I argued that *what* the learner appropriated were not procedural skills, but these two mathematical practices, central for success in using and exploring functions.

The previous analysis addressed questions specific to the scaffolding and subsequent appropriation of mathematical practices: What *particular* aspects of mathematical practices did the learner appropriate? The case study described how the learner, by solving problems jointly with a tutor, appropriated the focus of attention, meanings for utterances, and goals for carrying out new tasks. The focus of attention, meanings, and goals were not evident in the interactions as explicit tutor knowledge or directions. Rather, ways of seeing, talking, and acting were implicitly embedded in the scaffolding the tutor provided. The analysis showed how micro scaffolding supported the development of shared meanings for utterances and focus of attention on particular aspects of graphs. Scaffolding in these interactions followed a cycle of the tutor setting new tasks, the learner engaging in new tasks, participation in joint problem solving activity, and then tutor support fading as the learner progressed.[4] That analysis showed that the student appropriated the goals for four tasks initially introduced through joint problem solving with the tutor (generating equations, estimating the y-intercept, evaluating a slope, and exploring parameters). Although in the beginning of the tutoring sessions the student did not set these goals independently, she later initiated and carried out these tasks successfully on her own. The example below illustrates how the tutor used proleptic questioning for one task, evaluating slopes.

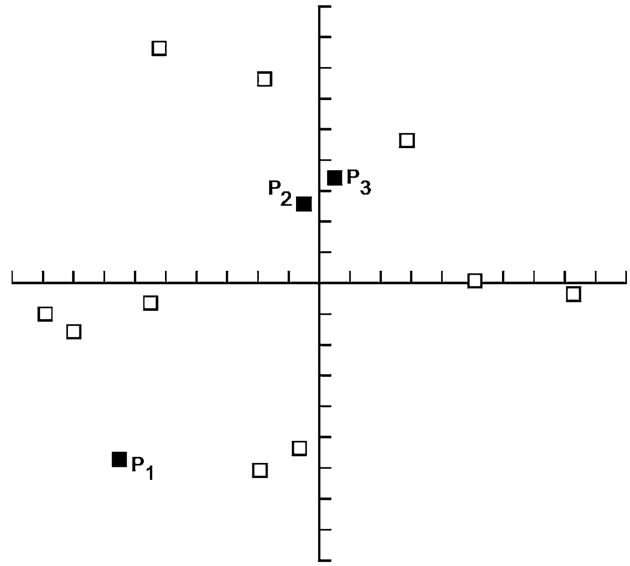

Fig. 1 Generating an equation

3.1 Scaffolding mathematical practices through proleptic questioning

This example illustrates how prolepsis functioned during scaffolding. A proleptic question "Now does that look like the right number?" led the student to participate in an activity whose full meaning was not initially shared by the student. The tutor introduced a new task, evaluating a slope, asking the student a question about the slope number she had calculated. The student did not initially understand the question. In later tutoring sessions, she moved to accepting the goal as valid when set by the tutor. Ultimately, by the end of the tutoring sessions, she initiated the task, set the goal, and correctly carried out the task independently.

During the student's first attempt to hit a set of blobs, the tutor introduced a new task, evaluating a slope. The student was trying to hit three blobs in Fig. 1 (P_1, P_2, and P_3). Two of these blobs straddle the y-axis. P_3 in the first quadrant had coordinates (0.5, 3.5) and P_2 in the fourth quadrant had coordinates (−0.5, 2.5).

When attempting to hit the set of two blobs in Fig. 1 (P_1, P_2, and P_3), the student first calculated the slope for the line passing through two of these three blobs. After she produced the number +1 for the slope, the tutor asked her to evaluate the result of her calculation.

1. Student: (writing down the result of her first slope computation) OK, I think it's negative 1. Negative 1 over negative 1, and so that means that the slope is one!
2. Tutor: Now does that look like the right number?

[4] Throughout the sessions, the tutor fostered executive control activities, such as revising and evaluating, crucial for competent problem solving in this domain (Brown et al. 1983; Schoenfeld, 1985). The tutor also provided corrections, proleptic instruction (Stone, 1993), and guiding questions to scaffold student goal setting. Initially, the tutor was the problem poser, goal setter, critic, and evaluator; he asked for and suggested plans and overtly engaged in goal setting, checking, and evaluation. As tutoring proceeded, the student assumed some of these herself, setting new problems, developing new goals, transforming incomplete or inappropriate goals to reflect more content knowledge, checking results, and evaluating solutions. Thus, the student appropriated many of the executive control activities first experienced in interaction.

3. Student:	What do you mean the right number?
4. Tutor:	Well, do you have any sense of what the slope should come out to be?
5. Student:	Yes, it should be negative.
6. Tutor:	Does that look about right?
7. Student:	No. OK, it can't be right. (She proceeds to check her slope calculation.)

With the question in line 2, "Now does that look like the right number?" the tutor introduced a new task, evaluating a slope, asking the student to check the result of her slope calculation by comparing it to an expectation. This expectation is based on knowledge about the orientation of lines–lines with positive slopes rise to the right and lines with negative slopes rise to the left.

The question, "Now does that look like the right number?" is an example of prolepsis because it assumes that the meaning was shared. The question presupposes not only an understanding of the meaning of the phrase "the right number," but also an understanding of the task's overarching goal, to compare the result of a computation with an expectation. We can conclude that the student did not, in fact, understand the first question since she explicitly asked the tutor what he meant by "the right number." The tutor clarified the meaning of his first question with a second question (line 4), "Do you have any sense of what the slope should come out to be?" This second rephrased question makes two things explicit, the meaning of the phrase "the right number" and the goal of comparing the computed slope number with an expectation. The second question, which is perhaps less proleptic or less dependent on the student's assumed understandings, provided the student with an opportunity to construct a shared meaning for the phrase "the right number" and for the overarching goal for evaluating a slope.

Following the tutor's second question, the student accepted the goal set by the tutor and carried out the task. She first stated her (incorrect) expectation that the slope of a line passing through the selected blobs should be negative, and then compared her expectation to her computed slope. At this point, the student's knowledge of the relationship between the sign of m and the orientation of a line was reversed. She described lines rising to the right as having a negative slope and lines rising to the left as having a positive slope. Nevertheless, she evaluated her calculation (+1) against her (incorrect) expectation for the line she wanted to produce through the selected blobs: "It should be negative." She then proceeded to rework her calculation. This evaluation, the comparison of a computed slope value with the inclination of the expected line through the selected blobs, was thus a goal first set through tutor scaffolding that involved prolepsis, in particular proleptic questioning. In this example, proleptic questioning supported the student in developing the meaning for an utterance and a new goal.

In subsequent interactions during the first tutoring session, the student continued to produce equations to hit selected blobs. The tutor continued to provide scaffolding by prompting the student to compare her expectation for the slope with the orientation of a line produced on the screen. This comparison led to the student to self-correct her matching of the slope sign and the orientation of a line. In a few more interactions, the tutor continued to provide scaffolding by setting the goal to evaluate the slope. As tutor support faded, he continued to set the goal but provided less support for accomplishing the task. During these interactions the student did not set the goal or initiate the task of evaluating a slope, but she readily accepted this as a valid goal and provided an appropriate response. The student responses in subsequent interactions show that she and the tutor came to use a shared meaning for later questions in other game contexts with different blobs. For example, when in one instance the tutor asked "Does that (your calculation) seem right?," the student proceeded to check her calculation with an expected orientation for the line. In another instance the tutor asked "Does it make sense that this one (the slope number) would be negative?" and the student responded appropriately to that question.

By the second tutoring session, tutor support for setting or carrying out this task had disappeared. The student independently initiated the task of evaluating her slope values with the orientation of lines on the screen. In one instance, the student independently evaluated the sign of a calculated slope saying, "But still the line is going this way (gesture in a negative slope direction), and I wanted the line to go the other way (gesture in a positive slope direction), right? So then…that means the slope isn't negative, it's positive." In another interaction during the second tutoring session, she independently compared the orientation of a target line with the sign of the slope resulting from her computation. In a later interaction the student independently evaluated the slope of a line produced on the screen: she identified the slope as the problem, correctly evaluated the slope of the line through the blobs as being negative rather than positive (as was the slope in her equation), and she changed the sign of m in the equation.

The interactions above show how prolepsis functioned during micro scaffolding. First, the tutor introduced a new goal, evaluating the slope, through proleptic questioning. Through this initial prolepsis the student and the tutor developed a shared meaning for an utterance and a new goal. Through repeated tutor scaffolding and subsequent fading, the student began to initiate this task and set the goals to accomplish it independently. After successive tutor scaffolding and fading, the student initiated and successfully and independently completed the task of evaluating a slope.

3.2 Mathematical practices

The question "Now does that look like the right number?" when intended to mean "Do you have any sense of what the slope should come out to be?" presupposes several aspects of an expert view of this domain. First, the question presupposes an understanding of the task's overarching goal, to compare the result of a computation with an expectation for how a line will look on a graph. It also presupposes treating both equations and lines as objects, rather than treating an equation as a process that involves an input that generates an output: when *m* changes in an equation (an object), the orientation of the line (an object) changes accordingly. Lastly, it presupposes a perspective of lines as objects that have different orientations depending on their slopes.[5] In order to decide whether a computation for the slope of a line is the right number, one has to treat the line as an object that can both be imagined and manipulated. The slope must also be seen as existing in two connected representations, the graph and the equation. To decide whether a computation for the slope was the right number, one needs to imagine the orientation of the line produced by that slope.

This task thus required that the student participate in two mathematical practices, treating the equation as an object and treating the expected line as an object. The tutor scaffolded the student's participation in these mathematical practices through proleptic questioning, assuming the student understood the meaning of an utterance and the goal for a task, and engaging the student in joint problem solving that involved these mathematical practices. Through interaction with the tutor, the student came to focus her attention on the orientation of an imagined line and to interpret the meaning of the question "Does that look like the right number?" as "What should the slope be?" These two mathematical practices were emergent during these interactions and are also representative of expert mathematical practices.

4 Example 2: scaffolding mathematical practices through revoicing

The second example comes from a lesson in a fourth grade bilingual classroom (33 students, urban school in California). In general, this teacher introduced topics first in Spanish and then later in English, using materials in both languages. Desks were arranged in tables of four and students could work together. Students had been working for several weeks on a unit on two-dimensional geometric figures. Instruction had focused on the properties of quadrilaterals and had included vocabulary such as the names of different quadrilaterals in both languages. Students had been talking about shapes and had been asked to point, touch, and identify different shapes. The teacher described this lesson as an English as a second language (ESL) mathematics lesson, where students would be using English to discuss different shapes.

Below is an excerpt from the transcript for this lesson that involves descriptions of a rectangle. Conversational turns are numbered and brackets indicate non-verbal information.

1. Teacher: Let's see how much we remembered from Monday. Hold up your rectangles… high as you can. [students hold up rectangles] Good, now. Who can describe a rectangle (for me)? Eric, can you describe it? [a rectangle] Can you tell me about it?

2.Eric: A rectangle has… two… short sides, and two… long sides.

3. Teacher: Two short sides and two long sides. Can somebody tell me something else about this rectangle? If somebody didn't know what it looked like, what, what… how would you say it?

4. Julian: Parallel(a). [holding up a rectangle].

5. Teacher: It's parallel. Very interesting word. Parallel, wow! Pretty interesting word, isn't it? Parallel. Can you describe what that is?

6. Julian: Never get together. They never get together [runs his finger over the top length of the rectangle].

7. Teacher: OK, what never gets together?

8. Julian: The parallela… they… when they, they get, they go, they go higher [runs two fingers parallel to each other first along the top and base of the rectangle and then continues along those lines] they never get together.

9. Antonio: Yeah!

10. Teacher: Very interesting. The rectangle then has sides that will never meet [runs fingers along top and base of an invisible rectangle] those sides will be parallel [motions fingers vertically in parallel lines]. Good work. Excellent work.

Julian's pronunciation in turns 4 and 8 can be interpreted as a mixture of English and Spanish, the word "parallel"

[5] For a discussion of object and process perspectives of functions, see Moschkovich, Schoenfeld, & Arcavi (1993). The game environment and the tutoring were designed, in part, to introduce students to the object and process perspectives for functions; these were novel for this student.

pronounced in English, and the added "a" pronounced in Spanish.[6] In Spanish, the word parallel would agree with the noun (line or lines), in both gender (masculine or feminine) and number (plural or singular). For example, "parallel lines" translates to "líneas paralelas" and "parallel sides" translates to "lados paralelos." The grammatical structure in turn 8 can thus also be interpreted as a mixture of Spanish and English. The apparently singular "parallela" (turn 8) was followed by the plural "when they go higher."

The excerpt illustrates several teacher moves to support student participation in a discussion: asking for clarification, accepting and building on what students say, probing what students mean, and revoicing student statements. In turn 5, the teacher accepted Julian's response, revoicing it as "It's parallel," and probed what Julian meant by "parallela." In turn 10, the teacher revoiced Julian's contribution in turn 8: "the parallela, they" became "sides," and "they never get together" became "will never meet, will be parallel."

Julian	Teacher
The parallela, they	Sides
Never get together	Will never meet
	Will be parallel

A teacher's revoicing can support student participation in a discussion. First, it can facilitate student participation in general, by accepting a student's response, using it to make an inference, and allowing the student to evaluate the accuracy of the teacher's interpretation of the student contribution (O'Connor and Michaels, 1993). This teacher move allows for further student contributions in a way that the standard classroom initiation–response–evaluation (IRE) pattern (Mehan, 1979; Sinclair & Coulthard, 1975) does not.

Second, revoicing can support student participation in mathematical practices. Revoicing can build on students' own use of mathematical practices or a student contribution can be revoiced to reflect new mathematical practices. There were several mathematical practices evident in Julian's original utterance in line 8. Julian was abstracting, describing an *abstract* property of parallel lines, and generalizing, making a *generalization* that parallel lines will *never* meet. In this case, the teacher's revoicing made Julian's claim more precise, introducing a new mathematical practice, attending to the precision of a claim. In line 10, the teacher's claim is more precise than Julian's claim because the second claim refers to the sides of a quadrilateral, rather than any two parallel lines.

Revoicing also provides opportunities for students to hear more formal mathematical language. The teacher revoiced Julian's everyday phrase "get together" as "meet" and "will be parallel," both closer to academic language. This revoicing seemed to impact Julian, who used the term "side(s)" twice when talking with another student in a later interaction, providing some evidence that revoicing supported this student's participation in both mathematical practices and more formal academic language.

5 Example 3: designing scaffolding for a whole class setting

Focusing on mathematical practices is important for both analyzing and designing scaffolding. Although a teacher may not be able to have individual scaffolding conversations with each student, it is possible to attend to mathematical practices when designing whole class scaffolding. This section shifts from using scaffolding as a descriptive construct to scaffolding as the "design of individual tasks as consisting of a series of steps or activities that occur sequentially or in collaborative construction" (van Lier, 2004). The next example shows how a focus on mathematical practices can be implemented when designing whole class meso level scaffolding.

Below is the outline for the task "Reading and Understanding a Math Problem.[7]" The purpose of this task is to support students in making sense of a mathematics word problem while learning to read, understand, and extract relevant information from a word problem. In addition to these questions, the task requires a mathematics word problem.

5.1 Reading and understanding a math problem

1. Read the problem out loud to a peer. Try to answer this question. What is the problem about?
2. Read the problem again. Talk to your partner about these questions: What is the question in the problem? What are you looking for?
3. Read the problem a third time. Talk to your partner about these questions.

   (a) What information do you need to solve the problem? (What do you want to know?)
   (b) What information do you have? (What do you know?)
   (c) What information are you missing? (What don't you know?)

[6] Julian uttered "parallela" (turn 4) with hesitation and his voice trailed off. It is impossible to tell whether he said "parallela" or "parallelas."

[7] Adapted from handout by Harold Asturias, available online at Understanding Language http://ell.stanford.edu.

(d) Draw a diagram of the problem and label all the information you know.

4. (If useful for this problem) Draw a diagram, act the problem out, or use objects to represent the problem situation.

The organization for this task places students in different situations. Students can first work individually, reading or attempting to read the problem. Students can then work in pairs, talking together and answering the questions orally and in writing. Pairs of students can then present responses or diagrams to the class, and students can add details to their own diagram as they view and interpret other diagrams. If the task were used several times in a unit for a long-term sequence of work, it would then provide *macro* level scaffolding.

The task would not necessarily involve *micro* scaffolding through dialogue with an adult, unless the teacher engages in instructional conversations with individuals or pairs. In general, scaffolding at the meso level in a collective setting may not involve contingent interactional processes of appropriation, give-and-take in conversation, or collaborative dialogue. Challenges in implementing this task include maintaining a sociocultural framing, including characteristics of micro scaffolding in an individual setting, and supporting students' participation in mathematical practices, not on setting a series of algorithmic steps. In the next two sections, I examine how this task could be used either to focus on procedural fluency or to scaffold student participation in mathematical practices.

5.2 Designing scaffolding focused on procedural fluency

Let us imagine how this task might work in a classroom when used to make sense of the word problem below[8]:

> Jane, Maria, and Ben each have a collection of marbles. Jane has 15 more marbles than Ben, and Maria has two times as many marbles as Ben. All together they have 95 marbles. Find how many marbles Maria has.

If the teacher or the students interpret the goal for solving word problems as translating text to arithmetic computation, then the task could deteriorate into underlining key words and translating those to arithmetic operations. When implementing this task in a whole class setting one important characteristic of micro scaffolding to maintain is to diagnose and adjust instruction to students' current levels. Diagnosis, however, could easily focus only on procedural fluency. For example, a teacher could prepare to address students' levels of procedural skills by predicting and monitoring how students might approach the problem.

This problem can be solved arithmetically (without algebraic symbols) or algebraically; the scaffolding will depend on the solution strategy used. If the learner decides to guess an answer and check that answer, they might need support in accurately applying arithmetic operations. For example, a student could guess that Maria has 20 marbles, then Ben would have 10 (20 divided by 2), and Jane would have 25 ($10 + 15$). Scaffolding would then be calibrated to support this approach. Students would be asked to read the word problem again to check whether this guess works in the original word problem, when adding $20 + 10 + 25$ to equal 55. Since this is less than 95, the guess for the number of marbles that Maria has should be larger.

Another approach would be to organize numerical guesses in a table. If the student decides to write an equation, then scaffolding would focus on supporting procedural skills in algebra, required for a solution that arrives at one of several possible linear equations, depending on which quantity is chosen to be the variable. For x defined as the number of marbles Maria has, the equation would be $95 = x + x/2 + (x + 15)$. Solving that equation for x might also require scaffolding in manipulating the algebraic symbols.

5.3 Designing scaffolding focused on mathematical practices

Solving this word problem entails not only procedural fluency (arithmetic or algebraic), but also competencies in *mathematical practices*, and these practices should not be ignored when implementing this task. This task can also be implemented to support student participation in an important mathematical practice, making sense of a problem.

Reading the word problem involves not only alphabetic literacy (reading text) but also competence in mathematical discourse—reading and comprehending a mathematical text. Word problems are a particular genre of mathematical discourse. The purpose and structure of the text are specific to this genre and different than for texts in other content areas; reading this word problem involves different literacy skills than reading other school texts. The purpose of the text is not to tell a story, make an argument, or persuade the reader. Instead, the text provides a situation to be modeled with mathematics. The structure of the text is that some information is given that describes a real world situation and sets the stage, and then there is a question posed for the reader. Although while reading the word problem one has to extract

[8] From "Creating Equations 1" Mathematics Assessment Resource Service, University of Nottingham. Available at http://map.mathshell.org/materials/tasks.php?taskid=292#task292.

multiple pieces of information, the focus is only on the mathematical aspects of the situation—what the unknown quantity is, how the quantities are related, etc. The reader must disregard all other aspects of the text or the situation (the setting, who the protagonists are, why they collect marbles, etc.) that are not relevant to the mathematical solution.

Furthermore, "reading" the word problem cannot be separated from the *mathematical practice* of "making sense" of the word problem. As one reads, one also needs to make mathematical sense of what one is reading: focus only on the mathematically relevant information, make sense of each quantity, how quantities are related, and model the situation mathematically (using numbers, arithmetic operations, or algebraic symbols). One also needs to make metacognitive strategic decisions (Schoenfeld, 1992), for example decide what strategy to use (making a guess and checking that guess, writing an equation, etc.) or what arithmetic computation will model the relationship among the quantities.

What other mathematical practices could this task support? If a student uses a "guess and check" strategy and organizes guesses in a table, they might use the mathematical practices "Look for and make use of structure" or "Look for and express regularity in repeated reasoning" to then generate an equation. Opportunities for students to participate in mathematical practices in this task (or any task) depend on the activity structure for the task. Whether a task includes other mathematical practices depends not on the task as it is written, but on the activity structure for the task. For example, if the activity structure required that students write down an explanation for their solution, discuss their individual solution in pairs or groups, arrive at a common group solution, and then present their group solution and explanation, then this task would provide opportunities for other mathematical practices, such as constructing viable arguments and critiquing the reasoning of others.

This second focus for implementing scaffolding with this task provides a sharp contrast to the first focus on procedural skills. The second focus includes not only cognitive and metacognitive aspects of mathematical activity—mathematical reasoning, thinking, concepts, and metacognition—but also sociocultural aspects—participation in mathematical practices. Most importantly, these components work in unison. Maintaining a sociocultural perspective when implementing scaffolding with this task requires maintaining a view of scaffolding that includes student participation in mathematical practices.

6 Summary

The examples of scaffolding presented here have addressed the "what, why, and how" of scaffolding (Pea, 2004) by illustrating how two teacher moves (the how), proleptic questioning and revoicing, can support student participation in mathematical practices (the what). Mathematical practices were used to analyze scaffolding in two settings, individual and whole class. The examples also showed how two teacher moves, proleptic questioning and revoicing, during scaffolding can manage the tension between providing appropriate calibrated support while also providing opportunities beyond learners' current proficiency.

A focus on mathematical practices was also used to designing instruction that includes scaffolding. The third example shows that a focus on mathematical practices is possible not only during individual micro scaffolding, but also when designing meso level scaffolding for a whole class. Although a teacher may not be able to have individual scaffolding conversations with each student, attending to mathematical practices when designing whole class scaffolding is possible. Opportunities for students to participate in mathematical practices during whole class scaffolding depend on the activity structure for the task. Whether a task includes mathematical practices depends not on the task as it is written, but on the activity structure provided for the task and the classroom norms for what constitutes a mathematical solution, explanation, justification, or argument. When designing meso scaffolding for whole class settings, the activity structure for a task sets expectations for what students will attend to. If the teacher provides a structure focusing on mathematical practices rather than on procedural skills, then whole class scaffolding can provide opportunities for students to engage in a variety of mathematical practices: providing explanations, justifications, and arguments, as well as using multiple modes (oral and written) and symbol systems (tables or diagrams, etc.).

When designing scaffolding it is important to consider how to manage the tension between calibrated support and opportunities for new competencies. The first example illustrates how proleptic questioning can provide opportunities for a learner to participate in mathematical practices beyond their current repertoire. The second example illustrates how revoicing can support students' own participation in mathematical practices while also inserting new mathematical practices into a discussion. The second example also illustrates how revoicing can build on students everyday language while providing exposure to formal academic language.

The second and third examples highlight how scaffolding can address the linguistic complexity of oral contributions and written text. It is important to prepare students to deal with typical written texts in mathematics, such as word problems, and also other students' oral or written explanations. The goal of instruction should not necessarily be to "reduce the language demands" of a word problem or a student explanation, but instead to provide scaffolding for students to learn how to manage complex written text and oral explanations. Tasks and discussions that allow students to engage with academic language (with support) can provide opportunities for students to participate in mathematical practices and use academic language.

The sociocultural perspective on scaffolding mathematical practices is not only analytical (for researchers), but also practical (for the teacher). Teachers need not explicitly use the label "sociocultural" or describe the assumptions underlying a sociocultural theoretical perspective. However, for teachers to implement scaffolding with a sociocultural framing focused on mathematical practices, teacher education would need to provide experiences that distinguish different framings for scaffolding. Teachers would need to see, hear, and discuss different ways to implement scaffolding, for example contrasting behaviorist and sociocultural enactments. In order to focus on mathematical practices, teacher education should provide video or transcript examples of teacher moves, such as revoicing and proleptic questioning, that focus on mathematical practices.

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