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Analysis and design of interference-limited wireless systems

A dissertation submitted in partial satisfaction of the
requirements for the degree
Doctor of Philosophy

in

Electrical Engineering
(Communication Theory and Systems)

by

Konstantinos Stamatiou

Committee in charge:

Professor James R. Zeidler, Chair
Professor John G. Proakis, Co-Chair
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Professor Lawrence B. Milstein
Professor Geoffrey M. Voelker

2009

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Co-Chair

Chair

University of California, San Diego

2009

It doesn't have to be fun to be fun.

— Barry Blanchard

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Chapter 2, in part, is a reprint of the papers: “A performance analysis of coded FH-OFDMA with pilot-assisted channel estimation”, Kostas Stamatiou and John G. Proakis, as it appeared in the proceedings of IEEE PIMRC, Berlin, 2005; “Performance analysis of a coherent FH-MA system based on Latin squares”, Kostas Stamatiou and John G. Proakis, as it appeared in the IEEE Transactions on Wireless Communications, November 2007.

Chapter 3, in part, is a reprint of the paper “Assessing the impact of physical layer techniques on ad hoc network performance”, Kostas Stamatiou and John G. Proakis, as it appeared in the Elsevier Journal of Physical Communication, March 2008. Chapter 3, in part, has also been submitted for publication to the IEEE Transactions on Wireless Communications, under the title “Channel diversity in random wireless networks”, Kostas Stamatiou, John G. Proakis and James R. Zeidler, May 2009.

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PUBLICATIONS

- K. Stamatiou, J. G. Proakis and J. R. Zeidler, “Channel diversity in random wireless networks,” *submitted to the IEEE Transactions on Wireless Communications*, May 2009
- K. Stamatiou and J. G. Proakis, “Assessing the impact of physical layer techniques on ad hoc network performance,” *Elsevier Journal of Physical Communication*, March 2008
- K. Stamatiou and J. G. Proakis, “Performance analysis of a coherent FH-MA system based on Latin squares,” *IEEE Transactions on Wireless Communications*, November 2007
- K. Stamatiou, F. Rossetto, M. Haenggi, T. Javidi, J. Zeidler and M. Zorzi, “A delay-minimizing routing strategy for wireless multi-hop networks,” in *Proc. IEEE Workshop on Spatial Stochastic Models for Wireless Networks (SPASWIN)*, Seoul, June 2009
- K. Stamatiou, J. G. Proakis and J. R. Zeidler, “Evaluation of MIMO techniques in FH-MA ad hoc networks,” in *Proc. IEEE GLOBECOM*, Washington D.C., November 2007
- K. Stamatiou, J. G. Proakis and J. R. Zeidler, “Information efficiency of ad hoc networks with FH-MIMO transceivers,” in *Proc. IEEE ICC*, Glasgow, June 2007
- K. Stamatiou and J. G. Proakis, “Interference diversity in frequency-hopped systems with soft decoding,” in *Proc. IEEE VTC*, Montreal, September 2006
- K. Stamatiou and J. G. Proakis, “A performance analysis of coded FH-OFDMA with pilot-assisted channel estimation,” in *Proc. IEEE PIMRC*, Berlin, September 2005
- K. Stamatiou and J. G. Proakis, “A performance analysis of coded FH-OFDMA,” in *Proc. IEEE WCNC*, New Orleans, March 2005
- K. Stamatiou, F. Rossetto, M. Haenggi, T. Javidi, J. Zeidler and M. Zorzi, “Delay characterization of random multi-hop networks,” *in preparation*, September 2009
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ABSTRACT OF THE DISSERTATION

Analysis and design of interference-limited wireless systems

by

Konstantinos Stamatiou

Doctor of Philosophy in Electrical Engineering
(Communication Theory and Systems)

University of California, San Diego, 2009

Professor James R. Zeidler, Chair
Professor John G. Proakis, Co-Chair

This thesis is concerned with the performance analysis and design of interference-limited wireless systems. Specifically, we consider wireless systems where transmissions from potential interferers are uncoordinated, i.e., there is no scheduling for the purpose of interference management. Special emphasis is placed on how the spatial distribution of the interferers affects the statistical characteristics of the interference power.

The first topic we address is the performance analysis of the downlink of a cellular frequency-hopping multiple-access system, where coherent detection is employed. Channel estimation is performed with the help of hopping patterns which are broadcasted by the base-station and tracked by different users in the cell. In a time and frequency correlated channel, we identify which patterns result in the best performance in terms of the bit-error-probability at the receiver. We then consider the effect of the interference from the base-station of an adjacent cell and assess the importance of interference side information at the receiver on the bit-error-probability.

The second topic we focus on is the analysis of random single-hop networks. The model we consider is that of a homogeneous Poisson point process of transmitters,

each with a RX at a fixed distance. We evaluate the packet error probability and the network transmission capacity when the channel, consisting of fading and interference, is constant or varying during the packet transmission. The impact of coding, spatial diversity and multiple-stream transmission on the network capacity is identified with closed-form expressions in the small packet error probability regime.

The third and final topic addressed is the evaluation of the mean end-to-end delay in random multi-hop networks. We extend the single-hop network model employed previously to accommodate routes, each consisting of a source, a number of relays and a final destination. Combining tools from the analysis of single-hop networks with queuing theory we derive a closed-form expression for the mean end-to-end delay over the typical route. The number of relays and their placement are determined such that the delay is minimized, for different packet arrival scenarios at the sources.

Chapter 1

Introduction

Interference is an important limiting factor in the performance of wireless systems. In a wireless environment where transmissions take place concurrently, increasing the individual power of each transmission increases the received power from the intended transmitter but also from other, interfering transmitters. The problem is akin to a room full of people who want to communicate with each other; speaking louder and louder does not imply that the people are able to hear each other.

A wireless system generally includes a set of links, each consisting of a transmitter (TX) that wants to communicate with a corresponding (RX). This thesis is concerned with the analysis and design of wireless systems where there is no intelligent scheduling of transmissions from potential interferers. The reasons for considering this scenario are the possible lack of information at the transmitters required for such scheduling or the intention of the system designer to keep the protocol of communication simple. Moreover, the RX only has limited information regarding the interference from concurrent transmissions, i.e., at best, it might be able to estimate its power. Again, the motivation for studying this scenario lies in the unavailability of more information or the ensuing simplicity of the RX and communication protocol design.

In the context described above, we consider physical layer techniques that take advantage of signal diversity at the RX in the time, frequency and spatial domains, or aim at increasing the information rate, or both. Such techniques include primarily error correction coding and multiple-input multiple-output (MIMO) techniques. Special emphasis is placed on how the spatial distribution of the interferers influences the statistical characteristics of the interference seen at the RX. We thus differentiate between

wireless systems with infrastructure, i.e., cellular systems, and no infrastructure at all, i.e., networks with randomly placed nodes, usually referred to as *ad hoc* networks.

The performance of these systems is characterized at the link and system levels. At the link level, we employ typical physical layer metrics such as the bit and packet error probabilities, or the outage and ergodic capacity. At the system level, we consider metrics such as the network throughput, the network transmission capacity and the delay encountered by a packet traveling over multiple links from its source to its final destination. We evaluate these metrics with the goal of providing insight on how they depend on the system parameters and the employed physical layer techniques. The accuracy of our analysis and the effectiveness of the resulting design recommendations are typically tested via simulation.

1.1 System model

1.1.1 Topology

Three basic examples of the wireless systems considered in this thesis are shown in Figs. 1.1, 1.2 and 1.3. The TX and the RX of interest, also referred to jointly as the “typical” TX-RX pair, are denoted as TX_0 and RX_0 , respectively. Other TXs in the network, which cause interference to the transmission of TX_0 are denoted as TX_i , and their respective RXs as RX_i , where $i = 1, 2, \dots$.

In Fig. 1.1, a simple two-cell cellular system is depicted. TX_0 is typically a base-station (BS) which communicates with one of the users in its cell (RX_0). TX_1 is the BS of the neighboring cell, which creates interference at RX_0 by transmitting to one of its own users, RX_1 . Assume that the users of TX_1 are served in a time-division multiple-access (TDMA) fashion. If TX_1 transmits at a constant power to all its users, the interference power seen at RX_0 is constant across time. On the other hand, if TX_1 employs power control (PC) to mitigate path-loss, such that all its users receive the same power irrespective of their distance from it, the interference power at RX_0 varies with time. This is an example of how the spatial distribution of the RXs in the neighboring cell affects the interference power seen at RX_0 .

Fig. 1.2 shows a network where TXs are randomly distributed on the plane and each TX has its own corresponding RX. The interference level at RX_0 depends on the locations of TX_i . If these are drawn randomly according to a two-dimensional process,

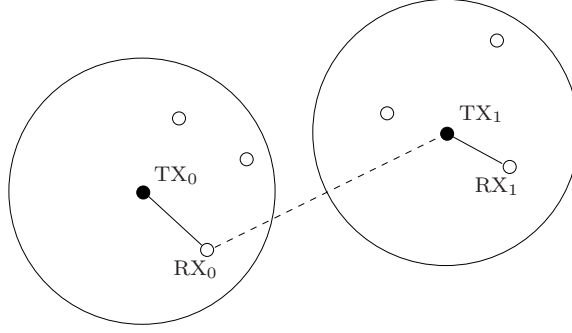


Figure 1.1: An example of a cellular system with two cells. A solid (dashed) line denotes a useful (interfering) signal.

such as a Poisson point process (PPP), the interference power at RX_0 is also a random variable. The network in 1.2 is also referred to as a “single-hop” network, as packets require only one hop to travel from TX_0 to RX_0 . In Fig. 1.3, a “multi-hop” ad hoc network is depicted where a packet travels over multiple links in order to reach its final destination. If the transmission power is fixed, the advantage of multiple hops is that the signal-to-interference-and-noise-ratio (SINR) at each intermediate node is larger compared to a scenario where the source transmits directly to the final destination.

1.1.2 Multiple access

The multiple access (MA) scheme, or, in other words, the set of rules to which the TXs abide in order to access the common medium, is dictated by the infrastructure of the wireless system, as well as the available bandwidth. We consider both narrowband and wideband systems. In the former case, the system bandwidth is roughly equal to the signal bandwidth B , while in the latter case, it is usually a multiple of the signal bandwidth, MB , $M > 0$.

In a narrowband cellular system, MA within the cell can conform to a TDMA scheme, since the BS has the ability to coordinate transmissions. As seen in Fig. 1.1, the source of interference are the transmissions in neighboring cells. In a narrowband ad hoc network where scheduling is not possible, the simplest MA scheme is widely known as ALOHA, i.e., the TXs transmit with a given probability p which is typically a network-wide parameter. A larger p allows the TXs to attempt transmissions to their corresponding RXs more often, however, it also leads to an increased level of interference in the network.

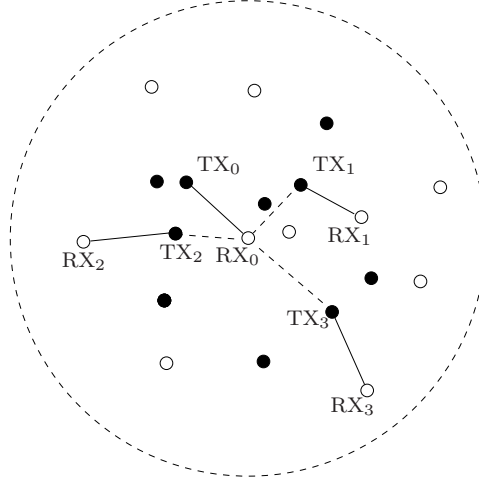


Figure 1.2: An example of a single-hop ad hoc network. A solid (dashed) line denotes a useful (interfering) signal.

In a wideband system, spread-spectrum (SS) MA, otherwise known as code-division MA (CDMA), may be employed. In frequency-hopping (FH) CDMA, or FH-MA, the bandwidth is divided into M orthogonal subbands or “frequencies” of bandwidth B and each TX-RX pair is assigned a hopping pattern. The RX knows this pattern and is synchronized to it throughout communication. We differentiate between “fast” FH, where the frequency changes at the symbol level, and “slow” FH, where, assuming packetized transmission, the frequency changes at the packet level (or every few hundred symbols). In fast FH, the frequency response of the channel and the interference level potentially change within the transmission of a packet. This signal diversity can be exploited through error correction coding in order to enhance the performance. On the other hand, channel estimation and synchronization are more challenging and incur more overhead compared to slow FH, where the RX has more time to adapt to each frequency (provided the channel does not change during the packet interval). In both fast and slow FH, if the patterns are judiciously designed, interference is avoided with increasing probability, as M grows, at the cost of bandwidth. In a cellular system, specifically, and depending on the bandwidth, the BS may assign orthogonal patterns to its users such that intra-cell interference is completely avoided.

The alternative CDMA technique is DS-CDMA. In DS-CDMA, the transmitted signal is multiplied by a pseudorandom code of bandwidth MB . Ideally, the sequences corresponding to different TX-RX pairs are orthogonal and, after correlating the received

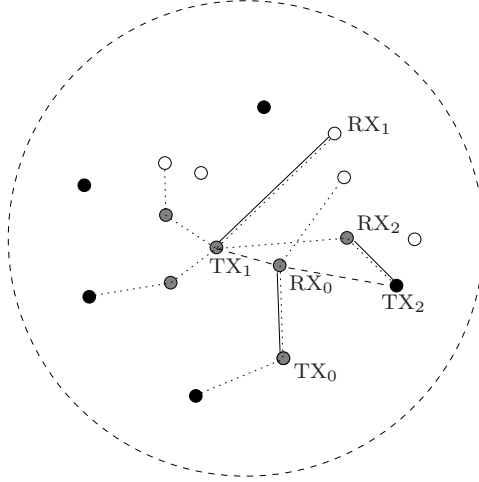


Figure 1.3: An example of a multi-hop ad hoc network. A dotted line denotes a route that consists of a source (black circle), intermediate nodes (grey circles) and a destination (empty circle). A solid (dashed) line denotes a useful (interfering) signal.

signal with the appropriate sequence, MA interference (MAI) is eliminated. However, the lack of synchronization between different TX-RX pairs results in an attenuation of the interference power (and not elimination) by the “processing gain” M . In a cellular system, each BS assigns different codes to the users in its cell; however, due to the lack of synchronization, each BS-RX link is subject to both intra-cell and inter-cell interference. DS-CDMA is also subject to the “near-far” problem, i.e., the fact that the interference power from nearby TXs may overwhelm the RX of interest. In the cellular setting, this problem may be solved with PC, an option which is not available in an uncoordinated ad hoc network. We do not consider DS-CDMA as a MA scheme in this thesis.

1.1.3 Wireless channel

We assume that the wireless channel between a TX and a RX consists of path loss, long-term fading or shadowing, short-term fading or simply “fading”, and noise. Each of these characteristics of the wireless medium is described separately below.

Path loss

If the distance between the TX and the RX is r , we assume that the transmitted signal is attenuated by r^{-b} , where b is the path-loss exponent and, typically, b is between 2 and 6. This model exhibits a singularity as $r \rightarrow 0$. In a cellular system, we can easily

constrain r to strictly positive values. In a random ad hoc network, if TX_i is very close to RX_0 , the above model leads to an artificially small SINR. However, the RX would very likely be heavily interfered even if the singularity was removed, thus it can be verified that there is no apparent effect on the performance analysis that relies on this model. Moreover, in most scenarios considered in this thesis, the density of nodes is such that the event $r \rightarrow 0$ occurs with very small probability, i.e., the network is sufficiently sparse. Finally, this model, despite the singularity, is selected because it is amenable to analysis.

Path-loss is characterized as a long-term random variable, i.e., it is considered constant over the duration of several packet transmissions, as the rate of variation of the node locations is typically much smaller than the rate of data symbol transmission.

Shadowing

Shadowing, or long-term fading, is the result of obstacles blocking the main signal path between the TX and the RX. It is usually modeled as a lognormal random variable S , i.e., $S = 10^{\sigma_s x/10}$, where $x \sim \mathcal{N}(0, 1)$ and, typically, the standard deviation σ_s takes values between 6 and 8. Shadowing is also characterized as a long-term random variable.

Fading

Fading, or short-term fading, models the superposition of the multi-path components at the RX. We assume that the channel frequency response is flat over each subband. As a result, the effect of fading in a given subband is a multiplication of the transmitted signal with a fading coefficient. At each time instant, the fading coefficient H is a random variable (r.v.) which is modeled as circularly symmetric, complex Gaussian with zero mean and variance equal to the average received power, i.e., $H \sim \mathcal{CN}(0, P_r)$, where $P_r = P_t S r^{-b}$ is the average received power and P_t is the transmitted power. The magnitude of the fading coefficient $x = \|H\|$ is distributed as a Rayleigh random variable, i.e.,

$$f(x) = \frac{2x}{P_r} e^{-\frac{x^2}{P_r}}, \quad x > 0. \quad (1.1)$$

The fading coefficients may be correlated or independent across time and frequency depending on the coherence time and coherence bandwidth of the channel, respectively. Moreover, if a TX and/or a RX are equipped with an antenna array, the fading coefficients between all possible antenna pairs are assumed to be independent.

Noise

Since the focus of this thesis is on interference-limited systems, the contribution of background thermal noise at the RX is mostly ignored. When thermal noise is taken into account, it is modeled as additive, white and Gaussian (AWGN).

1.1.4 Signal model

Assume that the TXs and the RXs are equipped with N_t and N antennas, respectively. Let T denote the duration of a symbol transmission. The received signal at RX_0 , over a given subband m , $m = 1, \dots, M$, in the interval $[0, T]$ can generally be expressed as

$$\begin{aligned} \tilde{\mathbf{y}}(t) = & \text{Re} \left\{ \sqrt{\frac{2P_0}{T}} \mathbf{H}_0 \mathbf{x}_0 e^{i2\pi f_m t} \pi(t) \right\} \\ & + \text{Re} \left\{ \sum_{i \in \Pi} \zeta_i \sqrt{\frac{2P_i}{T}} \mathbf{H}_i \mathbf{x}_i e^{i(2\pi f_m t + \phi_i)} \pi(t - t_i) + \tilde{\mathbf{n}}(t) \right\}, \end{aligned} \quad (1.2)$$

where

- $\mathbf{x}_i$ is the symbol vector transmitted by TX_i .
- ζ_i is an indicator random variable which is 1 if TX_i is active in subband m - with a given probability - and 0 otherwise.
- $P_i = P_{t,i} S_i r_i^{-b}$ is the average received power from TX_i , $P_{t,i}$ is the transmitted power from TX_i , and S_i , r_i are the shadowing coefficient and distance between TX_i and RX_0 , respectively.
- $\mathbf{H}_i$ is the fading matrix between TX_i and RX_0 with independent and identically distributed (i.i.d.) entries and $[\mathbf{H}_i]_{k,n} \sim \mathcal{CN}(0, 1)$, $k = 1, \dots, N_t$, $n = 1, \dots, N$.
- Π is the set of interfering TXs.
- f_m is the carrier frequency corresponding to subband m .
- ϕ_i is a phase offset between RX_0 and TX_i and we assume that $\phi_i \sim \mathcal{U}[0, 2\pi)$.
- $\pi(t)$ is a rectangular pulse in $[0, T]$, i.e., $\pi(t) = 1$ for $t \in [0, T]$ and $\pi(t) = 0$, elsewhere; t_i is a time offset between RX_0 and TX_i and we assume that $t_i \sim \mathcal{U}[0, T]$.
- $\tilde{\mathbf{n}}(t)$ is additive white Gaussian noise (AWGN). At time t , $\tilde{\mathbf{n}}(t) \sim \mathcal{CN}(0, \mathbf{I}_N)$.

RX_0 performs matched filtering on $\tilde{\mathbf{y}}(t)$ and samples the output at $t = T$. In complex-baseband representation, the received signal vector can be written as

$$\begin{aligned}\mathbf{y} &= \sqrt{P_0}\mathbf{H}\mathbf{x} + \sum_{i \in \Pi} \zeta_i \sqrt{P_i}\mathbf{H}_i\mathbf{x}_i \left(1 - \frac{t_i}{T}\right) e^{i\phi_i} + \mathbf{n} \\ &= \sqrt{P_0}\mathbf{H}\mathbf{x} + \mathbf{w} + \mathbf{n}.\end{aligned}\tag{1.3}$$

where $\mathbf{n} \sim \mathcal{CN}(0, \mathbf{I}_N)$. Throughout the thesis¹, we make the following assumptions regarding the interference term $\mathbf{w}$.

- $\mathbf{w}$ is Gaussian, with uncorrelated elements, *given* the set of random variables $\{\zeta_i, P_i\}$, i.e., $\mathbf{w} \sim \mathcal{CN}(0, z\mathbf{I}_N)$, where

$$z \triangleq N_t \sum_{i \in \Pi} \zeta_i^2 \left(1 - \frac{t_i}{T}\right)^2 P_i.\tag{1.4}$$

This is a worst-case scenario in terms of analyzing the performance at RX_0 .

- $t_i = 0$, i.e., interfering TXs are synchronized at the symbol level with RX_0 . This is also a worst-case scenario in terms of the interference power seen by RX_0 in $[0, T]$.

The unconditional distribution of $\mathbf{w}$ depends on the distribution of z , which, in turn, is a function of the propagation environment, the MA scheme and the topology of the network.

1.2 Performance metrics

The performance of the typical link is evaluated in terms of the *bit-error probability (BEP)* or the *frame/packet error probability (FEP)*, in case a specific coding and modulation scheme is considered, e.g., a convolutional code in conjunction with Gray mapping and QAM modulation. Otherwise, the performance is characterized in terms of information theoretic metrics such as the *outage probability* and the *ergodic capacity*. The outage probability is defined as the probability that the mutual information of channel (1.3) is smaller than the requested information rate, when the transmitted symbols are selected independently from a Gaussian alphabet, i.e., $\mathbf{x} \sim \mathcal{CN}(0, \mathbf{I}_{N_t})$. The ergodic capacity is evaluated by the expectation of this mutual information, over $\mathbf{H}_0$ and z .

¹With the exception of Chapter 5, where we explicitly take into account the fading of the interfering TXs.

At the system or network level, we define the *network throughput*, as the link throughput multiplied by the number of links in a given area. This metric expresses the trade-off between the spatial density of transmissions and the interference level in the network. In correspondence to the metrics defined in the previous paragraph, the link throughput may be evaluated for a specific coding and modulation scheme or expressed in information theoretic terms.

An alternative system metric is the *transmission capacity*, defined as the maximum network throughput, such that a constraint on the link performance is satisfied. This metric captures the same trade-off as the network throughput but ensures that, in an attempt to pack as many transmissions in space as possible, the performance of the individual links is not compromised. The constraint on the link performance can be dictated by the nature of the application, e.g., transmission of delay-sensitive data.

Both the network throughput and the transmission capacity are single-hop metrics, i.e., they are defined for a given link distance. In an ad hoc network, it is possible that more than one hops are required for a packet to reach its final destination, as the level of interference in the network² renders direct transmission to the final destination impossible. Ultimately, we are interested in the *end-to-end delay*, defined as the total time it takes a packet to travel from the source to the destination. In a network of interfering transmissions, the end-to-end delay as a metric expresses the following trade-off: For a given transmission power, a smaller hopping distance results in a larger received SINR, which implies higher reliability and/or a higher transmission rate over a single hop. On the other hand, each node that is added between the source and the final destination is also the cause of additional delay, since a packet has to be decoded, re-encoded and wait in a queue, before it is transmitted to the next node. A main design issue is therefore the number of hops that minimizes the end-to-end delay for the typical route in the network.

1.3 Thesis overview

This thesis begins with a physical-layer study of the downlink of a FH-MA cellular system (Chapter 2). In Chapter 3, we depart from the cellular architecture and consider the single-hop network scenario depicted in Fig. 1.2, studying the impact of fast FH, coding and spatial diversity on the transmission capacity. Chapter 4 is also a study on

²Or a power constraint in a power-limited regime which is not of interest in this thesis.

single-hop ad hoc networks but from an outage probability perspective and a focus on multiple-stream transmission. Finally, in Chapter 4, we take a “cross-layer” approach and combine analytical results from Chapters 3 and 4 with queueing theory, in order to evaluate the end-to-end delay in the multi-hop scenario depicted in Fig. 1.3.

Our main interest is on interference-limited systems and the impact of the spatial distribution of the interferers on the statistical properties of the interference power. Thus, with the exception of Chapter 2, the contribution of AWGN in (1.2) is considered negligible and disregarded. We now provide a detailed account of the focus of each chapter, describing our motivation and previous related work.

Chapter 2 - Performance analysis of a coherent FH-MA system

This chapter considers a cellular system where the MA scheme consists of a combination of fast FH and Orthogonal Frequency Division Multiple Access (OFDMA). Our study is motivated by the commercial system described in [1], which was also considered for the IEEE 802.20 standard on Mobile Broadband Wireless Access (MBWA).

OFDMA is based on the use of Orthogonal Frequency Division Multiplexing (OFDM) [2] as a MA scheme. Specifically, in the downlink of a generic OFDMA system, each user, according to its bandwidth demand, is assigned a number of subcarriers within an OFDM symbol transmitted by the BS. In the FH-OFDMA system proposed in [1], the subcarrier assignment to users changes every OFDM symbol, according to a set of deterministic hopping patterns. FH-OFDMA is a favorable MA scheme in the sense that it combines the advantages of FH with OFDM. FH can potentially provide frequency diversity, if the channel response varies over the hopping distance, and interference diversity, as described in Section 1.1.1 (in the context of cellular systems, such as GSM, the advantages of frequency and interference diversity, that ensue from combining FH and coding, are analyzed in [3–5]). It also results in fairness, since, with judicious choice of the hopping patterns, the same average inter-cell interference is seen by all users in a cell. Moreover, if the OFDM symbol and cyclic prefix duration are chosen according to the Doppler and delay spreads of the channel, respectively, the subcarriers retain their orthogonality and experience flat-fading upon reception [2]. As a result, the capability for intra-cell orthogonal access is provided and high data rate transmission is possible without the need for equalization. For related work on various flavors of FH-OFDMA, the reader is directed to [6–8].

A large portion of existing work on FH techniques focuses on the performance analysis under MAI and/or partial band interference (PBI)³ when non-coherent modulation is employed [9–12]). Non-coherent modulation comes with the advantage that channel estimation, which may be an involved procedure when the TX frequency changes rapidly, is not required. On the other hand, it incurs a performance penalty, known as the “non-coherent combining loss” [13], which results from the absence of channel state information (CSI) in detection and decoding.

The cellular system described in [1] employs both FH at the symbol level and coherent modulation, on the downlink direction. Therefore, the broad topic addressed in Chapter 2 is the theoretical performance analysis of a coherent FH-MA cellular system, where the required channel estimation is performed with the help of pilot patterns tracked by all the users in the cell. The construction of the patterns is based on orthogonal Latin squares, which follows [1], but was also originally proposed in [14].

The contributions of this chapter are summarized as follows. Assuming convolutional coding and BPSK modulation are employed at the BS, we identify which squares result in the best performance, in terms of the BEP, in a time-frequency correlated channel. The BEP is also expressed as a function of the channel delay spread and the optimum trade-off between frequency diversity and channel estimation quality is demonstrated for a given number of pilot patterns and pilot symbol power. Finally, the BEP is evaluated when the decoder is aware (or not) of the interference power variations from adjacent cells. It is shown that interference side information at the decoder is crucial in harnessing the available interference diversity.

Chapter 3 - Channel diversity in random single-hop networks

This chapter is concerned with the study of interference-limited, single-hop ad hoc networks, an example of which is shown in Fig. 1.2. The model that we employ is that of a homogeneous PPP of TXs, each with a RX at a fixed distance, which is a network-wide parameter. A large body of work [15–23] has employed this model in order to provide analytical performance results for different physical-layer techniques. Its advantage lies in the fact that, due to the properties of the PPP, it allows a statistical characterization of the interference based on the distribution of the node locations. The study of the PPP on the plane or in higher dimensions falls under the general branch of

³This is mostly pertinent to military applications.

applied mathematics called *stochastic geometry* [23].

The goal of this chapter is to explore the benefits of channel diversity in the network setting described above. As in Chapter 2, we consider convolutional coding in conjunction with random FH and maximum-likelihood (ML) decoding at the RX. We analyze the performance of this scheme in terms of the FEP and show that, given a target constraint on the packet error probability, the transmission capacity follows a power law where the exponent is the inverse of the code Hamming distance. Since an averaging over different channel states takes place within a packet, the information rate of the typical TX-RX link is upper-bounded by the ergodic capacity for which we provide tight upper and lower bounds. We also address practical physical layer issues such as channel estimation, power control (PC) and channel correlation, and assess their effect on the performance via simulation. Our work is related to [16, 24], where Reed-Solomon coding combined with FSK modulation and Differential Unitary Space Time Modulation, respectively, is employed to harness the interference diversity in a random network environment. Unlike [16, 24], however, we are able to provide analytical expressions for the FEP that clearly demonstrate the effect of the code diversity order and the size of the antenna array at the RX on the FEP and the transmission capacity.

Chapter 4 - Evaluation of MIMO techniques in random single-hop networks

In this chapter, we consider once more the random single-hop network model employed in Chapter 3. However, we now assume that the channel is constant for the duration of a packet slot. This implies that the packet is transmitted on the same subband and the coherence time of the fading is at least equal to the duration of the packet slot. Under these channel assumptions, an outage model may be employed for packet successes, i.e., a packet is successfully received if the mutual information of the channel realization is greater than the desired information rate.

In contrast to Chapter 3, the fact that the channel is constant across the packet renders the use of MIMO techniques, such as spatial multiplexing, more realistic. Assuming that channel state information is available at the receiver (only), we investigate the problem of determining the optimal number of transmitted streams such that the transmission capacity (defined here as the maximum network throughput per unit area, such that a constraint on the outage probability is satisfied) of the network is maximized.

The trade-off lies in the fact that, introducing more streams can potentially increase the information rate of each link, but also the interference level in the network.

The impact of spatial diversity techniques, such as maximal-ratio-combining (MRC), on the outage probability in a Poisson field of interferers was first explored in [22] where scaling laws for the transmission capacity as a function of the number of transmit/receive antennas and the propagation exponent were derived. Multiple-stream transmission with perfect channel knowledge at the TX was studied in [25] and the optimal number of spatial modes, in terms of maximizing the transmission capacity for a given density, was illustrated. More recently, *multi-user* MIMO techniques such as interference cancelation and space-division multiple-access have been considered in [26–28].

This chapter starts by revisiting the performance of MRC in a Poisson field of interferers. We derive a compact expression for the outage probability and show that N receive antennas provide an approximate gain $N^{-2/b}$ in terms of spatial contention, i.e., the slope of the outage probability as a function of the transmitter density, when the latter is zero. The aforementioned expression is then employed in order to derive exact expressions and approximations of the outage probability - and corresponding transmission capacity - for multiple-stream transmission. We consider the case where each packet is transmitted on the same antenna and the receiver separates the different streams either by zero-forcing (ZF) or zero-forcing with successive interference cancelation (ZF-SIC, also known as VBLAST [29,30]), as well as the case where each packet is divided into segments which are transmitted across all antennas, i.e., DBLAST [30]. In both cases, the number of packets that should be transmitted such that the transmission capacity of the network is maximized are derived, in the small outage probability regime. It is demonstrated that DBLAST offers substantial transmission capacity gains compared to VBLAST.

Chapter 5 - Delay characterization of random multi-hop networks

In the final chapter of the thesis we combine tools from stochastic geometry and queueing theory, in order to study the end-to-end delay performance in a multi-hop random network (Fig. 1.3). One of the main design issues in multi-hop wireless networks is determining the number of hops between the source of information and the final destination [31]. For a given transmission power, a smaller hopping distance results in a larger received SINR, which implies higher reliability and/or a higher transmission

rate over a single hop. However, as argued in [31], this does not necessarily translate to an end-to-end performance benefit, e.g., in terms of delay: each node that is added between the source of information and the final destination is also the cause of additional delay, since a packet has to be decoded, re-encoded and wait in the queue, before it is transmitted to the next node. Moreover, if the intermediate nodes can transmit only one at a time, e.g., to avoid intra-route interference, the throughput might suffer as well. Along the line of thought in [32], a meaningful performance analysis and the design of multi-hop ad hoc networks should be carried out with end-to-end constraints on delay and reliability in mind.

To the best of our knowledge, this is the first work to propose an analytical framework in order to evaluate the end-to-end delay in a random network. Previous work can be divided in two categories. The first body of work focuses on a single-hop network and evaluates heuristic metrics such as the expected progress [15], i.e., the product (packet success probability) $\times$ (hop length), which reflects the trade-off between reliability and hopping distance, or the spatial density of progress [19], i.e., the product (spatial density of successful transmissions) $\times$ (hop length), which combines the notions of network throughput and expected progress. These metrics are defined with the implicit assumption that the destination lies at an infinite distance from the source and are related to the performance at the end-to-end level, without however providing an explicit characterization for it. A second body of work [33–36] is, in a sense, complementary to the first. A well-defined route is considered, where the distance to the final destination and the number of intermediate nodes, or relays, are specified. However, the impact of interference from other transmissions in the network is not taken into account. Assuming a channel model with path-loss, fading and noise, and no delay constraints, [34, 35] determined the end-to-end rate, i.e., the minimum achievable rate over all hops, when a TDMA-access protocol is employed. Alternatively, under a given delay constraint, [36] specified the number of hops and the rate allocation among them, such that the total power consumption is minimized. A similar problem was studied in [33], under an end-to-end success probability requirement.

The multi-hop problem taking into account the interference present in a random network was first considered in [37]. Given an ALOHA medium access control (MAC) scheme, an opportunistic routing strategy was studied, according to which the next node in a route is dynamically selected based on its channel characteristics and its distance to

the final destination. However, the characterization of the end-to-end delay was mainly based on simulations and interference was modeled in the network assuming all relays are backlogged.

In this chapter, we consider a network where each route consists of a source, a number of relays and a destination at a finite distance, and the locations of the sources and relays are determined according to independent Poisson point processes. Given a TDMA/ALOHA medium access control (MAC) protocol, our objective is to determine a relay selection strategy such the mean end-to-end delay in a typical route is minimized. Towards this goal, we analyze an idealized network model where all routes have the same number of hops, the same distance per hop and their own dedicated relays. Using a combination of tools from queueing theory and stochastic geometry, the mean end-to-end delay and throughput are evaluated for a typical route. In the case of backlogged sources, we find that the delay is minimized if the first hop is much longer than the remaining hops, while in the case of non-backlogged sources, if all hops are equidistant. The respective optimal numbers of hops scale sublinearly and linearly with the source-destination distance. Simulating the original network scenario confirms that the analytical results are accurate, provided that an adequate number of relays is present in the network. Sufficient conditions on the medium access probabilities and the traffic load are also derived, such that the queues in the network are stable for any relay placement.

Chapter 2

Performance analysis of a coherent FH-MA system

Motivated by the commercial system outlined in [1], the focus of this chapter is to theoretically evaluate the performance in a *coherent* FH-MA cellular system, where each user is assigned a subband in each time slot and this assignment changes across time according to predetermined hopping patterns. The required channel estimation for coherent detection is performed with the help of pilot symbols, which are transmitted on some of the patterns and are tracked by all the users in the cell. The construction of the patterns is based on orthogonal Latin squares [1, 14]. As explained in these references, patterns constructed from Latin squares are orthogonal within a cell and intersect only once from cell to cell. This results in the desirable practical property of equal average interference levels for all active users. Apart from its practical importance, the Latin squares hopping patterns have theoretical properties which we identify and take advantage of in order to analytically evaluate the BEP over the typical BS-RX link when convolutional codes are employed in order to harness the available frequency and interference diversity.

Regarding frequency diversity, performance results are obtained for different Latin squares under perfect and imperfect channel state information (CSI) conditions. It is shown that, selecting the Latin square appropriately, can have a large impact on the performance experienced by different users. Moreover, under pilot-assisted channel estimation [38–40], it is demonstrated that an increasing frequency selectivity does not necessarily imply better performance; the additional frequency diversity is obtained at

the expense of channel estimation quality.

Interference diversity is created due to the FH and PC of interfering users. We analyze the performance of a receiver that tracks the interference power variations and a receiver that remains oblivious to them. Our results confirm the substantial performance advantage of the former compared to the latter.

The remainder of this chapter is organized as follows. In Section 2.1, we describe our system model including the hopping pattern construction and channel model. In Section 2.2, the pilot-assisted channel estimation scheme is described, followed by the performance analysis in Section 2.3. Numerical results are provided in Section 2.4 and Section 2.5 concludes the paper.

A notational convention employed specifically in this chapter is that an $n \times 1$ vector $\mathbf{x}$ is equivalently represented as $[x_1 \dots x_n]^T$, $(x_1, \dots, x_n)$ and $x_1, \dots, x_n$.

2.1 System model

2.1.1 General

Consider the downlink of a synchronous cellular system. The bandwidth is divided into M flat-fading subbands or, in short, “frequencies”. If T_s is the time-slot duration and $1/T$ the separation between the centers of adjacent bands, then the coordinates $(n_1 T_s, n_2/T)$, with $n_1 \in \mathbb{Z}$ and $n_2 \in 0, \dots, M-1$ specify a time/frequency point. Since T_s and $1/T$ are fixed system parameters, we simply write $\mathbf{n} = (n_1, n_2)$. In the context of a FH-OFDMA system [1, 41], $T_s = T_c + T$ is the duration of the OFDM symbol, T_c is the duration of the cyclic prefix and $1/T$ is the subcarrier separation.

For simplicity, assume there are two circular cells with radius R_0 , a BS at their center and uniformly distributed users as shown in Fig. 1.1 and, in more detail, in Fig. 2.1. The signals transmitted by BS B2 are considered as interference to the signals transmitted by BS B1. Each user is assigned a hopping pattern, which chooses a frequency in each time slot. The patterns within each cell are orthogonal (no intra-cell interference) but interfere across the cells. On top of fading, path-loss attenuates the transmitted signals, which is compensated for with PC by the base-stations. The assumption that the slow channel variation consists solely of path-loss (and not shadowing or their combination) is made for analytical tractability. If r is the distance of a user from its base-station, the power transmitted to it is proportional to r^b , where b is the propagation coefficient

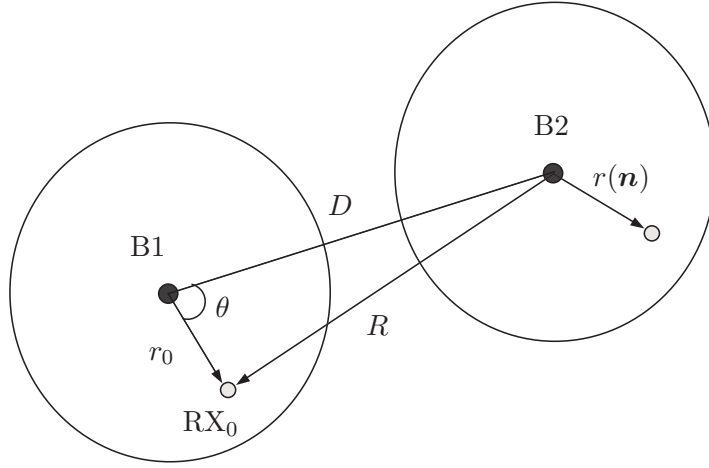


Figure 2.1: Downlink scenario.

(channel-inversion PC).

2.1.2 Transmitter

The binary message corresponding to the user of interest (UI), denoted as RX_0 in Fig. 2.1, is the input to a convolutional encoder. The resulting coded sequence is written column-wise to and read row-wise from a block interleaver with depth β_{in} and span α_{in} . The interleaved bits are then consecutively transmitted on the frequencies, which are specified by the hopping pattern assigned to the user. Since each bit is transmitted at a time/frequency point $\mathbf{n}$, the (interleaved) sequence is denoted as $\{x(\mathbf{n})\}$ ($\{x'(\mathbf{n})\}$) with $x(\mathbf{n}) = \pm 1$ for BPSK modulation. We take α_{in} to be much larger than the span of the dominant error events of the code; it can thus be assumed that the bits of the sequence $\{x(\mathbf{n})\}$ are transmitted every β_{in} slots. We only consider BPSK modulation as the analysis principles for bit-interleaved coded modulation (BICM) remain the same [42].

2.1.3 Hopping patterns

Let M be a prime number. The set of matrices $\mathcal{A} = \{\mathbf{A}^\alpha, \alpha = 1, \dots, M-1\}$, defined by

$$[\mathbf{A}^\alpha]_{i,j} = (\alpha i + j) \bmod M, \quad i, j = 0, \dots, M-1, \quad (2.1)$$

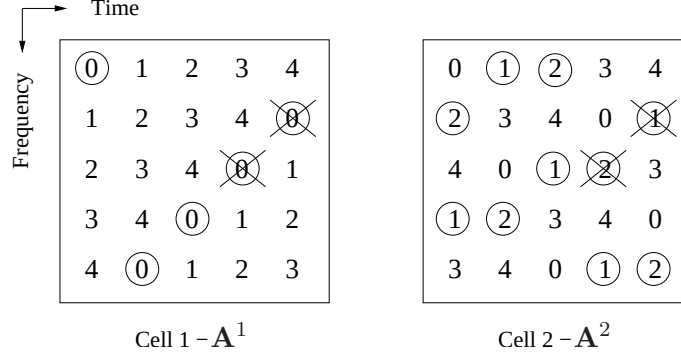


Figure 2.2: The Latin squares $\mathbf{A}^1$ and $\mathbf{A}^2$ for $M = 5$. The interference points of $\mathcal{H}_0^1$ with $\mathcal{H}_1^2$ and $\mathcal{H}_2^2$ are marked.

constitutes a family of $M - 1$ orthogonal Latin squares [14, 42]. As shown in the simple example of Fig. 2.2 for $M = 5$, if the rows of the Latin square correspond to frequencies and its columns to consecutive time slots, M periodic hopping patterns - with period M - can be constructed, corresponding to each of the integers $k = 0, \dots, M - 1$. It is proved in [14] that there is exactly one time/frequency collision for every pair of patterns in different squares. The main consequence of this property is that, assigning a square to each of the cells in a cellular system, different interferers are encountered in successive slots of a pattern. It also holds that, all the patterns in the cell “see” the same total interference power within each period, provided that the set of interfering users does not change during the period of observation. These points are illustrated in Fig. 2.2, where the Latin squares $\mathbf{A}^1$ and $\mathbf{A}^2$ are assigned to cell 1 and cell 2 respectively. We denote pattern k of square $\mathbf{A}^\alpha$ by $\mathcal{H}_k^\alpha$. Assume that a user in cell 1 is assigned the 0th pattern, i.e. $\mathcal{H}_0^1$, and two users in cell 2 are assigned patterns 1 and 2, i.e. $\mathcal{H}_1^2$ and $\mathcal{H}_2^2$. Note that $\mathcal{H}_0^1$ intersects with $\mathcal{H}_1^2$ and $\mathcal{H}_2^2$ at two distinct points during one period.

By the definition in (2.1), setting $k = (\alpha n_2 + n_1) \bmod M$, the frequency occupied by $\mathcal{H}_k^\alpha$ at time n_1 , denoted $f_k^\alpha(n_1)$, is

$$f_k^\alpha(n_1) = ((k - n_1)\alpha') \bmod M,$$

where $\alpha' = (\alpha^{-1}) \bmod M$. Formally, $\mathcal{H}_k^\alpha$ is thus defined as the set of points

$$\mathcal{H}_k^\alpha = \{\mathbf{n} : \mathbf{n} = (n_1, f_k^\alpha(n_1)), n_1 \in \mathbb{Z}\}.$$

If $\mathbf{A}^\alpha$ is infinitely replicated across both dimensions, each pattern $\mathcal{H}_k^\alpha$ becomes a *lattice*

$\mathcal{L}_k^\alpha$, where

$$\begin{aligned}\mathcal{L}_k^\alpha &= \{\mathbf{n} : \mathbf{n} = \mathbf{V}\mathbf{p} + (k, 0), \mathbf{p} \in \mathbb{Z}^2\} = \mathcal{L}_0^\alpha + (k, 0) \\ \mathbf{V} &= \begin{bmatrix} 1 & 1 \\ -\alpha' & M - \alpha' \end{bmatrix}.\end{aligned}\tag{2.2}$$

The lattices $\mathcal{L}_k^\alpha$, $k \neq 0$, are translates of $\mathcal{L}_0^\alpha$. This is a result of the fact that, by construction, $\mathcal{H}_k^\alpha$ is a time-shifted version of $\mathcal{H}_0^\alpha$ by k slots. Using this lattice formulation, $\mathcal{H}_k^\alpha$ is alternatively defined as a “ribbon” of points in $\mathcal{L}_k^\alpha$, i.e.

$$\mathcal{H}_k^\alpha = \{\mathbf{n} : \mathbf{n} = (n'_1, n'_2 \bmod M), \mathbf{n}' \in \mathcal{L}_k^\alpha\}.$$

In Fig. 2.3, lattice $\mathcal{L}_0^2$ and pattern $\mathcal{H}_4^2$ are depicted, when $M = 5$.

The following properties are useful in the performance analysis of Section 2.3.

Property 1. *The difference between the frequencies occupied by $\mathcal{H}_k^\alpha$ in two consecutive time-slots is either $M - \alpha'$ or $-\alpha'$.*

Proof. This follows from the fact that $(f_k^\alpha(n_1 + 1) - f_k^\alpha(n_1)) \bmod M = M - \alpha'$. □

Property 2. *If α_1, α_2 and k_1, k_2 are such that*

$$\alpha_2 = (\alpha_1 \beta_{in}^{-1}) \bmod M, \quad k_2 = (k_1 \beta_{in}^{-1}) \bmod M,\tag{2.3}$$

with $\beta_{in} = 1, \dots, M - 1$, then $f_{k_1}^{\alpha_1}(\beta_{in} n_1) = f_{k_2}^{\alpha_2}(n_1)$.

Proof. We have

$$\begin{aligned}f_{k_1}^{\alpha_1}(\beta_{in} n_1) &= ((k_1 - \beta_{in} n_1) \alpha_1^{-1}) \bmod M \\ &= ((k_2 \beta_{in} - n_1 \beta_{in}) \alpha_2^{-1} \beta_{in}^{-1}) \bmod M \\ &= ((k_2 - n_1) \alpha_2^{-1}) \bmod M = f_{k_2}^{\alpha_2}(n_1).\end{aligned}$$

□

Property 2 implies that using a block interleaver of depth β_{in} combined with the hopping pattern $\mathcal{H}_{k_1}^{\alpha_1}$ means that *effectively* the sequence $\{x(\mathbf{n})\}$ is transmitted on $\mathcal{H}_{k_2}^{\alpha_2}$ without interleaving, but with a slot duration $\beta_{in} T_s$ instead of T_s .

Property 3. Define as a span- S ($1 < S \leq M$) pattern event (PE), $\bar{\mathbf{h}}$, a path that spans S consecutive points in $\mathcal{H}_k^\alpha$

$$\begin{aligned}\bar{\mathbf{h}} &= \mathbf{m}_1, \mathbf{m}_2, \dots, \mathbf{m}_{S-1} \\ \mathbf{m}_i &= (1, -\alpha') \text{ or } (1, M - \alpha'), \quad i = 1, \dots, S-1.\end{aligned}$$

There are only S unique span- S PEs in any pattern $\mathcal{H}_k^\alpha$ of $\mathbf{A}^\alpha$ and their set is denoted as $\mathcal{E}_S^\alpha$. The probability of occurrence of $\bar{\mathbf{h}}$ in $\mathcal{H}_k^\alpha$, denoted as $P(\bar{\mathbf{h}})$, can be computed by counting how many times it occurs within $M + S - 1$ consecutive points of the pattern and then dividing by M .

Proof. Define as a boundary point of $\mathcal{H}_k^\alpha$, a point $(n_1, f_k^\alpha(n_1))$ for which $f_k^\alpha(n_1) = 0$ or $M - 1$. Without loss of generality, suppose that the boundary point is $(n_1, 0)$, for some $n_1 \in \mathbb{Z}$. Consider two PEs, $\bar{\mathbf{h}}_1$ and $\bar{\mathbf{h}}_2$, for which the boundary point is the k_1^{th} and k_2^{th} point, respectively, where $1 \leq k_2 < k_1 \leq S$. It is evident that $\bar{\mathbf{h}}_1$ and $\bar{\mathbf{h}}_2$ cannot be identical (e.g., in Fig. 2.3, $\bar{\mathbf{h}}_1$ and $\bar{\mathbf{h}}_2$, which pass the boundary point of $\mathcal{H}_4^2$ at positions $k_1 = 3$ and $k_2 = 2$, respectively, cannot be the same). Therefore, the PEs that pass the boundary point at positions $1, \dots, S$ are unique and their set is denoted as $\mathcal{E}_S^\alpha$. Due to the mod operation in the definition of $\mathcal{H}_k^\alpha$, a PE $\bar{\mathbf{h}}$ occurring anywhere on $\mathcal{H}_k^\alpha$ must belong to $\mathcal{E}_S^\alpha$. $\square$

Property 3 provides a compact way to describe how much frequency diversity we can expect when using the patterns of a specific square. If $\mathcal{E}_S^\alpha$ contains a PE with small frequency distances between points, that occurs very often, we can intuitively expect a small diversity advantage. In Fig. 2.3, the three span-3 PEs of any pattern in $\mathbf{A}^2$ are shown. They are $\bar{\mathbf{h}}_1 = (1, 2), (1, -3)$ (solid line), $\bar{\mathbf{h}}_2 = (1, -3), (1, 2)$ (dashed line) and $\bar{\mathbf{h}}_3 = (1, 2), (1, 2)$ (dotted line). It is easy to verify that they occur in the pattern with probabilities $P(\bar{\mathbf{h}}_1) = P(\bar{\mathbf{h}}_2) = 2/5$ and $P(\bar{\mathbf{h}}_3) = 1/5$. The fact that, for each square, there is only a finite number of PEs, with corresponding probabilities of occurrence, enables us to exactly take into account the correlation of the channel frequency response in the performance analysis, as will be seen in Section 2.3.

Property 4. If α_1, α_2 and k_1, k_2 are such that $\alpha_2 = M - \alpha_1$ and $k_2 = (k_1 + \alpha_1) \bmod M$ then $f_{k_1}^{\alpha_1}(n_1) = M - 1 - f_{k_2}^{\alpha_2}(n_1)$.

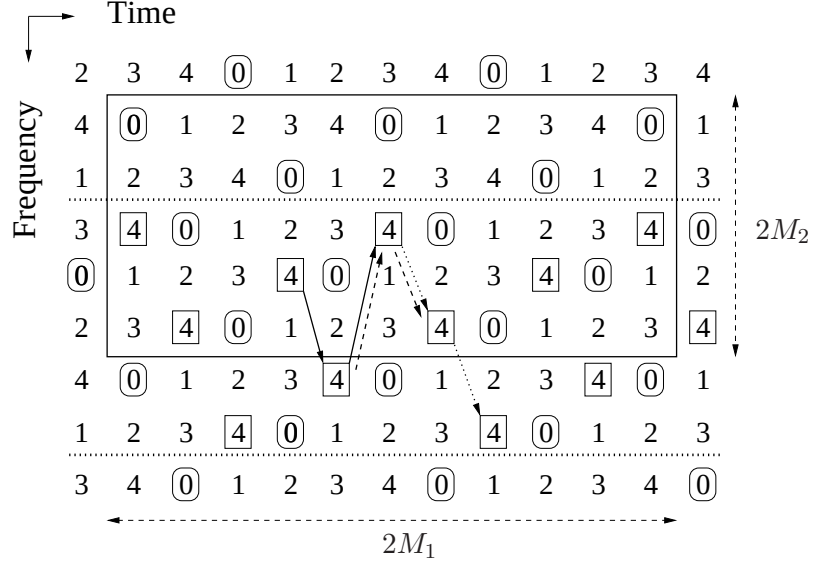


Figure 2.3: The pilot lattice $\mathcal{L}_0^2$ and the data pattern $\mathcal{H}_4^2$ of $\mathbf{A}^2$. The span-3 PEs are $\bar{\mathbf{h}}_1 = (1, 2), (1, -3)$ (solid line), $\bar{\mathbf{h}}_2 = (1, -3), (1, 2)$ (dashed line), $\bar{\mathbf{h}}_3 = (1, 2), (1, 2)$ (dotted line) with probabilities of occurrence $P(\bar{\mathbf{h}}_1) = P(\bar{\mathbf{h}}_2) = 2/5$ and $P(\bar{\mathbf{h}}_3) = 1/5$, and $\mathcal{E}_3^2 = \{\bar{\mathbf{h}}_1, \bar{\mathbf{h}}_2, \bar{\mathbf{h}}_3\}$.

Proof. We have

$$\begin{aligned}
 f_{k_2}^{\alpha_2}(n_1) &= ((k_1 + \alpha_1 - n_1)(M - \alpha_1)^{-1}) \bmod M \\
 &= -(k_1 + \alpha_1 - n_1)\alpha_1^{-1} \bmod M \\
 &= (-1 - (k_1 - n_1)\alpha_1^{-1}) \bmod M \\
 &= M - 1 - ((k_1 - n_1)\alpha_1^{-1}) \bmod M \\
 &= M - 1 - f_{k_1}^{\alpha_1}(n_1).
 \end{aligned}$$

□

Since the patterns of $\mathbf{A}^{M-\alpha_1}$ are symmetric to the patterns of $\mathbf{A}^{\alpha_1}$ with respect to the frequency $(M-1)/2$, we can expect that the two squares result in identical performance.

2.1.4 Channel model

Let UI be assigned the pattern $\mathcal{H}_k^\alpha$. The received sequence over $\mathcal{H}_k^\alpha$ is ¹

$$y(\mathbf{n}) = \sqrt{\gamma}H(\mathbf{n})x(\mathbf{n}) + \sqrt{\xi(\mathbf{n})}H_2(\mathbf{n})x_2(\mathbf{n}) + w(\mathbf{n}), \quad (2.4)$$

where $\mathbf{n} \in \mathcal{H}_k^\alpha$. The received signal-to-noise ratio (SNR) is γ . The channel frequency response at point $\mathbf{n}$ is denoted as $H(\mathbf{n})$ and we assume that $H(\mathbf{n}) \sim \mathcal{CN}(0, 1)$. Its correlation function is defined as $r_H(\mathbf{m}) = \mathbb{E}[H(\mathbf{n})H^*(\mathbf{n} + \mathbf{m})]$, with $r_H(0) = 1$. The choice of $r_H(\mathbf{m})$ is arbitrary; for the numerical results of this paper, we assume that the time variation follows Jakes' model [13], while the multi-path delay profile is exponential with constant $c > 0$. Under these assumptions

$$r_H(\mathbf{m}) = J_0(2\pi m_1 f_D \beta_{in} T_s) \frac{c}{c - j \frac{2\pi m_2}{T}}, \quad (2.5)$$

where f_D is the Doppler spread [13]. Note that the normalized Doppler spread $f_D T_s$ is assumed sufficiently small such that the channel frequency response can be considered constant during a slot. We also define as $\tau_L = \ln(100)/c$, the path delay up to which 99% of the impulse response power is contained; τ_L is thus a measure of the frequency selectivity of the channel.

The term $w(\mathbf{n})$ models the additive noise and we make the customary assumption that $w(\mathbf{n}) \sim \mathcal{CN}(0, 1)$. The inter-cell interference term $I(\mathbf{n}) = \sqrt{\xi(\mathbf{n})}H_2(\mathbf{n})x_2(\mathbf{n})$ consists of the power component $\xi(\mathbf{n})$, the fading coefficient $H_2(\mathbf{n})$ from B2 to UI and the BPSK symbol $x_2(\mathbf{n})$. Conditioned on $\xi(\mathbf{n})$, $I(\mathbf{n}) \sim \mathcal{CN}(0, \xi(\mathbf{n}))$, since $H_2(\mathbf{n})x_2(\mathbf{n}) \sim \mathcal{CN}(0, 1)^2$. The interference power at point $\mathbf{n}$ is given by $\xi(\mathbf{n}) = \gamma R^{-b} r(\mathbf{n})^b \zeta(\mathbf{n})$. Assuming that both base-stations aim at achieving the same received SNR γ at their respective users, $\gamma r(\mathbf{n})^b$ is the power transmitted from B2 to its active user at distance $r(\mathbf{n})$, whose pattern happens to occupy $\mathbf{n}$; R is the distance between B2 and UI, determined by $R^2 = D^2 + r_0^2 - 2Dr_0 \cos \theta$, if r_0 is the distance of UI from B1, D is the distance between the base-stations and θ is the angle shown in Fig. 2.1. In case no user of cell 2 is assigned a pattern that occupies $\mathbf{n}$, the power transmitted by B2 at $\mathbf{n}$ is obviously zero. The event of interference from B2 is modelled by the variable $\zeta(\mathbf{n})$, which is one (zero) with

¹Note that the transmitted sequence is actually $\{x'(\mathbf{n})\}$. However, Property 2 allows us to use this notation, as long as we remember that $\mathcal{H}_k^\alpha$ is an equivalent pattern and the effective slot duration is $\beta_{in} T_s$. Additionally, the phase difference between B2 and UI - see (1.2) - has been assimilated in the fading coefficient $H_2(\mathbf{n})$.

²Here, the Gaussian assumption on the interference in Section 1.1.4 is exact, as $x_2(\mathbf{n})$ is a BPSK symbol.

probability p , where p is the percentage of patterns occupied in cell 2 or the “load” in the system. From Section 2.1.3, the Latin squares construction ensures that all active patterns of cell 2 will interfere with $\mathcal{H}_k^\alpha$ at different points during one period. We can thus make the assumption that, for any two different points $\mathbf{n}_1, \mathbf{n}_2$, the random variable $r(\mathbf{n}_1)$ (and $\zeta(\mathbf{n}_1)$) is independent of $r(\mathbf{n}_2)$ ($\zeta(\mathbf{n}_2)$). Consequently, given R , $\xi(\mathbf{n})$ changes independently across $\mathbf{n}$.

Note that, with respect to a performance measure such as the BEP, R is considered a “long-term” random variable, i.e. it is assumed constant within the interval that the performance is evaluated [5, 43]. This is due to the fact that its variation rate, which depends on how often the position of the user changes, is much smaller compared to the transmission rate. Therefore, $\beta = R^{-b}$ can be considered as the inverse of a *long-term* SIR. On the other hand, $\xi(\mathbf{n})$ is perceived as a “fast” random variable by the decoder, since it changes independently across $\mathbf{n}$. Also note that, in (2.4), we have implicitly ignored the fact that some of the patterns of cell 2 are reserved for pilot transmission. Since all the users share these patterns, the power transmitted on them by B2 is some constant γ'_p and does not follow the model $\gamma r(\mathbf{n})^b$. We ignore this detail on the premise that the number of pilot patterns is small compared to M . Our channel estimation model will be explained in detail in Section 2.2.

Regarding the distributions of distance variables, since the users are uniformly distributed, the probability density function (pdf) of the distance r of a user from its base-station is $f(r) = 2r/R_0^2$, $r \in (0, R_0]$, so the pdf of $r' = r^b$ is

$$f(r') = \frac{2}{bR_0^2} r'^{2/b-1}, \quad r' \in (0, R_0^b]. \quad (2.6)$$

For convenience, we can make the normalization $R_0 = 1$, which reveals that r' is a Beta random variable with parameters $(2/b, 1)$ and mean $E[r'] = 2/(2+b)$ [44].

2.2 Estimation of channel coefficients

Some patterns in each cell are reserved for the transmission of pilot symbols. The indices of the pilot and data patterns are $k_p \in \mathcal{I}_p$, $k \in \mathcal{I}_d$, respectively, with $\mathcal{I}_p \cup \mathcal{I}_d = \{0, \dots, M-1\}$ and $\mathcal{I}_p \cap \mathcal{I}_d = \emptyset$. Note that, due to the Latin squares construction, the pilot patterns are orthogonal to the data patterns. In a sense, using this pilot arrangement matches the multiple access scheme well; some other arrangement, e.g. rectangular, would interfere, possibly irregularly, with the data patterns.

While the data patterns are defined within the finite system bandwidth, we assume that the pilot patterns extend infinitely to form pilot lattices. This assumption implies that around every point on a data pattern, there exists the same number and arrangement of pilot symbols. In this manner, we bypass the difficulty that edge phenomena introduce to the analysis. Having said this, at the pilot locations $\mathbf{n} \in \mathcal{L}_{k_p}^\alpha$, $k_p \in \mathcal{I}_p$, the measurements

$$\tilde{H}(\mathbf{n}) = \sqrt{r_0^{-b}\gamma'_p}H(\mathbf{n}) + I(\mathbf{n}) + w(\mathbf{n})$$

are available, where γ'_p is the *transmitted* pilot SNR. The *received* pilot SINR is therefore

$$\gamma_p = \frac{r_0^{-b}\gamma'_p}{\mathbb{E}[|I(\mathbf{n})|^2] + 1} = \frac{r_0^{-b}\gamma'_p}{\frac{2\beta\gamma p}{2+b} + 1},$$

since

$$\mathbb{E}[|I(\mathbf{n})|^2] = \beta\gamma\mathbb{E}[r^b(\mathbf{n})]\mathbb{E}[\zeta(\mathbf{n})] = \frac{2\beta\gamma p}{2+b}.$$

As illustrated in Fig. 2.3, the estimate of the channel coefficient at point $\mathbf{n}$ on pattern $\mathcal{H}_k^\alpha$,

$$\hat{H}(\mathbf{n}) = \sum_{\mathbf{l} \in \mathcal{W}} c^*(\mathbf{l})\tilde{H}(\mathbf{n} - \mathbf{l}) = \mathbf{c}^H \tilde{\mathbf{H}}(\mathbf{n}), \quad (2.7)$$

is obtained by linear filtering of the measurements $\tilde{H}(\mathbf{n})$ within the rectangular observation window

$$\mathcal{W} = \left\{ \mathbf{l} : \mathbf{n} - \mathbf{l} \in \mathcal{L}_{k_p}^\alpha, |\mathbf{l}_1| \leq M_1, |\mathbf{l}_2| \leq M_2 \right\},$$

where $\tilde{\mathbf{H}} = \tilde{H}(\mathbf{n} - \mathbf{l}_1), \dots, \tilde{H}(\mathbf{n} - \mathbf{l}_{|\mathcal{W}|})$ is the vector of measurements and $\mathbf{c}$ is the vector of corresponding filter coefficients. Note that the set $\mathcal{W}$ depends on k , since the number/positions of the pilots within the window depend on the position of $\mathcal{H}_k^\alpha$ with respect to the pilot lattices.

We define the following correlation functions, which will be used in the performance analysis of Section 2.3

$$\begin{aligned} r_{\hat{H}}(\mathbf{m}) &= E[\hat{H}(\mathbf{n})\hat{H}^*(\mathbf{n} + \mathbf{m})] \\ r_{\hat{H}H}(\mathbf{m}) &= E[\hat{H}(\mathbf{n})H^*(\mathbf{n} + \mathbf{m})]. \end{aligned}$$

After some simple manipulations, we obtain

$$\begin{aligned} r_{\hat{H}}(\mathbf{m}) &= \mathbf{c}^H(\gamma_p \mathbf{R}(\mathbf{m}) + \mathbf{I}_{|\mathcal{W}|})\mathbf{c} \\ r_{\hat{H}H}(\mathbf{m}) &= \gamma_p \mathbf{c}^H \mathbf{r}(\mathbf{m}), \end{aligned}$$

where

$$\begin{aligned} [\mathbf{R}(\mathbf{m})]_{i,j} &= r_H(\mathbf{m} + \mathbf{l}_i - \mathbf{l}_j), \quad i, j = 1, \dots, |\mathcal{W}_k| \\ [\mathbf{r}(\mathbf{m})]_i &= r_H(\mathbf{m} + \mathbf{l}_i), \quad i = 1, \dots, |\mathcal{W}_k|. \end{aligned}$$

For the linear minimum mean-square error (LMMSE) estimator, it is known that

$$\mathbf{c} = (\gamma_p \mathbf{R}(\mathbf{0}) + \mathbf{I}_{|\mathcal{W}|})^{-1} \sqrt{\gamma_p} \mathbf{r}(\mathbf{0}).$$

The channel estimation mean-square error (MSE) on the data pattern $\mathcal{H}_k^\alpha$ is thus

$$\sigma_e^2 = 1 - r_{\hat{H}}(\mathbf{0}) = 1 - \mathbf{r}(\mathbf{0})^H (\mathbf{R}(\mathbf{0}) + \gamma_p^{-1} \mathbf{I}_{|\mathcal{W}|})^{-1} \mathbf{r}(\mathbf{0}). \quad (2.8)$$

In the slightly different context of single-user OFDM, it is shown in [39] that, for an infinite observation window ($M_1, M_2 \rightarrow \infty$), σ_e^2 is the same for every $k \in \mathcal{I}_d$, if the pilot lattice samples the channel frequency response without aliasing. Thus, from a channel estimation perspective, the most desirable Latin squares are the ones with patterns that scan the time/frequency domain as “uniformly” as possible, such that no data pattern is ever too far from the pilot lattice and the same channel estimation quality, in terms of MSE, is obtained for all $k \in \mathcal{I}_d$.

2.3 Performance analysis

The decoder of UI has $\{y(\mathbf{n})\}$ and the estimates $\{\hat{H}(\mathbf{n})\}$ available, for $\mathbf{n} \in \mathcal{H}_k^\alpha$. Its criterion for deciding that the codeword $\hat{\mathbf{x}} = \{\hat{x}(\mathbf{n})\}$ was transmitted is

$$\hat{\mathbf{x}} = \arg \min_{\mathbf{x} \in \mathcal{C}} \left\{ \sum_{\mathbf{n}} \frac{|y(\mathbf{n}) - \sqrt{\gamma} \hat{H}(\mathbf{n}) x(\mathbf{n})|^2}{v(\mathbf{n})} \right\}, \quad (2.9)$$

where $\mathcal{C}$ is the set of all transmitted codewords. The factor $v(\mathbf{n})$ weighs the decoder metric; for the optimal RX in the maximum likelihood sense (denoted R1), it is equal to $v(\mathbf{n}) = \gamma \sigma_e^2 + \xi(\mathbf{n}) + 1$ ³. For a suboptimal RX (denoted R2), which ignores the interference power variations, it is simply $v(\mathbf{n}) = 1$.

Under the criterion in (2.9), the BEP on the hopping pattern $\mathcal{H}_k^\alpha$ can be approximated by

$$P_b \approx \frac{1}{b_c} \sum_{\mathbf{x} \rightarrow \mathbf{x}' \in \mathcal{E}} w_b(\mathbf{x} \rightarrow \mathbf{x}') P(\mathbf{x} \rightarrow \mathbf{x}'), \quad (2.10)$$

³R1 can obtain knowledge of $\xi(\mathbf{n})$ by averaging the magnitude-squared of the interference term in (2.4) over a few periods, assuming that the pattern assignment and positions of the interfering users do not change over the performance evaluation interval.

where b_c is the number of information bits per trellis branch, $\mathcal{E}$ is a set of dominant (small Hamming distance) error events, $w_b(\mathbf{x} \rightarrow \mathbf{x}')$ is the information weight of the error event $\mathbf{x} \rightarrow \mathbf{x}'$ and $P(\mathbf{x} \rightarrow \mathbf{x}')$ is its probability of occurrence or pair-wise error probability (PEP). Assuming that L_b information bits correspond to a frame (or codeword) and that the encoder is trellis-terminated, the FEP can be approximated as [45]

$$\text{FEP} \approx L_b \sum_{\mathbf{x} \rightarrow \mathbf{x}' \in \mathcal{E}} w(\mathbf{x} \rightarrow \mathbf{x}') P(\mathbf{x} \rightarrow \mathbf{x}'), \quad (2.11)$$

where $w(\mathbf{x} \rightarrow \mathbf{x}')$ is the weight of the error event $\mathbf{x} \rightarrow \mathbf{x}'$.

After some manipulations, the PEP can be written as

$$\begin{aligned} P(\mathbf{x} \rightarrow \mathbf{x}') &= P\left(\sum_{l=1}^L 2\text{Re}\left\{\frac{y(\mathbf{n}_{k_l})\hat{H}^*(\mathbf{n}_{k_l})}{v(\mathbf{n}_{k_l})}\right\} \leq 0\right) \\ &= P(\Delta \leq 0), \end{aligned}$$

where S and $L \leq S$ are the span and length of the error event and $\mathbf{k} = k_1, \dots, k_L$ denote the error positions (by definition, $k_1 = 1$ and $k_L = S$). The PEP can be computed by the integral

$$P(\Delta \leq 0) = \frac{1}{2\pi i} \int_{\epsilon - i\infty}^{\epsilon + i\infty} \frac{\Phi_{\Delta}(s)}{s} ds, \quad (2.12)$$

where $\Phi_{\Delta}(s)$ is the moment generating function of Δ and $\epsilon \in \mathbb{R} \cap \text{ROC}\{\Phi_{\Delta}(s)\}$, with $\text{ROC}\{\Phi_{\Delta}(s)\}$ denoting the region of convergence of $\Phi_{\Delta}(s)$. This integral is numerically evaluated in [46] as

$$P(\Delta \leq 0) \approx \frac{1}{M} \sum_{m=1}^{M/2} \text{Re}\{\Phi_{\Delta}(\epsilon + j\epsilon\tau_m)\} + \tau_m \text{Im}\{\Phi_{\Delta}(\epsilon + j\epsilon\tau_m)\}, \quad (2.13)$$

where $\tau_m = \tan((2m-1)\pi/(2M))$ and M is the number of nodes. The evaluation of the BEP thus comes down to determining $\Phi_{\Delta}(s)$.

2.3.1 Analysis on frequency diversity

In the first part of our analysis, we take into account the correlation of the channel frequency response across time and frequency but ignore the effect of inter-cell interference. In this case

$$\Delta = \sum_{l=1}^L 2\text{Re}\left\{y(\mathbf{n}_{k_l})\hat{H}^*(\mathbf{n}_{k_l})\right\} = \mathbf{z}^H \mathbf{Q}_L \mathbf{z},$$

with the definitions

$$\mathbf{Q}_L = \begin{bmatrix} \mathbf{0} & \mathbf{I}_L \\ \mathbf{I}_L & \mathbf{0} \end{bmatrix},$$

$\mathbf{z} = [\mathbf{y}^T \hat{\mathbf{H}}^T]^T$, $\mathbf{y} = y(\mathbf{n}_{k_1}), \dots, y(\mathbf{n}_{k_L})$ and $\hat{\mathbf{H}} = \hat{H}(\mathbf{n}_{k_1}), \dots, \hat{H}(\mathbf{n}_{k_L})$. Given the span- S PE $\bar{\mathbf{h}}$ over which the error event occurred, $\mathbf{z}$ is a quadratic form of the circularly symmetric complex Gaussian random vector $\mathbf{z}$. By Turin's result [47], the moment generating function of Δ is

$$\Phi_{\Delta|\bar{\mathbf{h}}}(s) = \frac{1}{|\mathbf{I} + s\mathbf{R}_z\mathbf{Q}|} = \prod_{l=1}^{2L} \frac{1}{1 + s\lambda_l}, \quad (2.14)$$

where $\{\lambda_l\}$ are the eigenvalues of $\mathbf{R}_z\mathbf{Q}$ and

$$\mathbf{R}_z = E[\mathbf{z}\mathbf{z}^H] = \begin{bmatrix} \gamma\mathbf{R}_H + \mathbf{I}_L & \sqrt{\gamma}\mathbf{R}_{\hat{H}H}^H \\ \sqrt{\gamma}\mathbf{R}_{\hat{H}H} & \mathbf{R}_{\hat{H}\hat{H}} \end{bmatrix}, \quad (2.15)$$

where we have defined $\mathbf{R}_H = E[\mathbf{H}\mathbf{H}^H]$, $\mathbf{R}_{\hat{H}} = E[\hat{\mathbf{H}}\hat{\mathbf{H}}^H]$, $\mathbf{R}_{\hat{H}H} = E[\hat{\mathbf{H}}\mathbf{H}^H]$. If $\bar{\mathbf{h}} = \mathbf{m}_1, \dots, \mathbf{m}_{S-1}$, this implies that

$$\mathbf{n}_{k_j} = \mathbf{n}_{k_i} + \sum_{l=k_i}^{k_j-1} \mathbf{m}_l, \quad j \geq i.$$

The above matrices can now be computed, using our definitions in (2.8), e.g.

$$[\mathbf{R}_{\hat{H}H}]_{i,j} = r_{\hat{H}H} \left(\sum_{l=k_i}^{k_j-1} \mathbf{m}_l \right), \quad i, j = 1, \dots, L, \quad j \geq i \quad (2.16)$$

and obviously $[\mathbf{R}_{\hat{H}H}]_{i,j} = [\mathbf{R}_{\hat{H}H}]_{j,i}^*$ for $j < i$. The matrices $\mathbf{R}_H$, $\mathbf{R}_{\hat{H}}$ are computed in exactly the same manner. Note that all matrices depend on $\bar{\mathbf{h}}$, which determines the distances between the error positions. However, $\mathbf{R}_{\hat{H}}$ and $\mathbf{R}_{\hat{H}H}$ also depend on the pattern index k , which determines the quality of channel estimation. Finally, the unconditional $\Phi_{\Delta}(s)$ can be computed by averaging over all possible PEs, i.e.

$$\Phi_{\Delta}(s) = \sum_{\bar{\mathbf{h}} \in \mathcal{E}_S^\alpha} P(\bar{\mathbf{h}}) \Phi_{\Delta|\bar{\mathbf{h}}}(s).$$

To evaluate the performance under perfect CSI, i.e. $\hat{\mathbf{H}} = \mathbf{H}$, we simply set $\mathbf{R}_{\hat{H}H} = \mathbf{R}_{\hat{H}} = \mathbf{R}_H$ in (2.15). Moreover, in the special case of a large interleaving depth

($\beta_{in} \rightarrow \infty$), it holds that $\mathbf{R}_H = \mathbf{I}_L$ and $\mathbf{R}_{\hat{H}} = \mathbf{R}_{\hat{H}H} = (1 - \sigma_e^2)\mathbf{I}_L$. It is easy to show that $\mathbf{R}_z$ has two distinct eigenvalues λ_1, λ_2 with multiplicity L , for which it holds that

$$\begin{aligned}\lambda_1 + \lambda_2 &= 2\sqrt{\gamma}(1 - \sigma_e^2) \\ \lambda_1 \lambda_2 &= -(1 - \sigma_e^2)(\gamma\sigma_e^2 + 1).\end{aligned}$$

Therefore

$$\begin{aligned}\Phi_{\Delta}(s) &= \left(\frac{1}{1 + s(\lambda_1 + \lambda_2) + \lambda_1 \lambda_2 s^2} \right)^L \\ &\doteq \left(\frac{1}{1 + \frac{4\gamma(1-\sigma_e^2)}{\gamma\sigma_e^2+1}(s - s^2)} \right)^L.\end{aligned}\tag{2.17}$$

The symbol “ $\doteq$ ” denotes “equivalence” rather than strict equality. The second expression of $\Phi_{\Delta}(s)$ is equivalent to the first one, since it is obtained by the linear transformation

$$s \rightarrow \frac{2\sqrt{\gamma}s}{\gamma\sigma_e^2 + 1}.$$

It is apparent from (2.12) that any linear transformation on s leaves the value of the PEP unchanged. From (2.17), it is seen that, under LMMSE channel estimation, the effect of imperfect CSI is reflected on the *effective* SNR

$$\gamma_{ef} = \frac{\gamma(1 - \sigma_e^2)}{\gamma\sigma_e^2 + 1}.$$

As the number of pilot patterns and/or the pilot SINR γ_p are increased, the MSE $\sigma_e^2 \rightarrow 0$, therefore $\gamma_{ef} \rightarrow \gamma$.

2.3.2 Analysis on interference diversity

For this part of the analysis, the effect of interference diversity on the performance is examined, assuming an infinite interleaving depth, such that the fading across the received symbols is independent.

We now write $\Delta = \sum_{l=1}^L \delta_{k_l}$, where $\delta_{k_l} = \mathbf{z}_{k_l}^H \mathbf{Q}_1 \mathbf{z}_{k_l}$, with

$$\mathbf{z}_{k_l} = \frac{1}{\sqrt{v(\mathbf{n}_{k_l})}} [y(\mathbf{n}_{k_l}) \ \hat{H}(\mathbf{n}_{k_l})]^T.$$

Due to the assumption of independence, the time/frequency index $\mathbf{n}_{k_l}$ can be removed and $\Phi_{\Delta}(s) = [\Phi_{\delta}(s)]^L$. From the rule of conditional expectation, $\Phi_{\delta}(s) = \mathbb{E}_{\zeta} [\Phi_{\delta|\zeta}(s)] = \mathbb{E}_{\zeta} [\mathbb{E}_{\xi|\zeta} [\Phi_{\delta|\xi}(s)]]$, or, since ζ is a Bernoulli random variable with probability p

$$\Phi_{\delta}(s) = (1 - p) \Phi_{\delta|\xi=0}(s) + p \Phi_{\delta|\zeta=1}(s),$$

with $\Phi_{\delta|\zeta=1}(s) = \mathbb{E}_{\xi|\zeta=1}[\Phi_{\delta|\xi}(s)]$. Since there is a relatively large number of pilot symbols in the observation window, the channel estimate $\hat{H}(\mathbf{n})$ can be considered approximately Gaussian. Therefore, given ξ , δ is a quadratic form of the circularly symmetric complex Gaussian random vector $\mathbf{z}$. Similarly to the previous paragraph, in the case of LMMSE channel estimation we can show that

$$\Phi_{\delta|\xi}(s) = \frac{1}{1 + \frac{4\gamma(1-\sigma_e^2)}{v} \left(s - \frac{\gamma\sigma_e^2 + \xi + 1}{v} s^2 \right)}. \quad (2.18)$$

In order to simplify the analysis, we now make the assumption of perfect CSI, i.e., $\sigma_e^2 \rightarrow 0$. $\Phi_{\delta|\xi}(s)$ takes the form

$$\text{R1 : } \Phi_{\delta|\xi}(s) = \frac{1}{1 + \frac{4\gamma}{\xi+1}(s-s^2)} \quad (2.19)$$

$$\text{R2 : } \Phi_{\delta|\xi}(s) = \frac{1}{1 + 4\gamma(s - (\xi+1)s^2)}. \quad (2.20)$$

Given $\zeta = 1$, we have $\xi = \beta\gamma r^b$, so the pdf of ξ is

$$f(\xi|\zeta=1) = \frac{1}{\beta\gamma \text{B}(2/b, 1)} \left(\frac{\xi}{\beta\gamma} \right)^{2/b-1}. \quad (2.21)$$

In the case of R1

$$\Phi_{\delta|\zeta=1}(s) = \int_0^{\beta\gamma} f(\xi|\zeta=1) \frac{1}{1 + \frac{4\gamma}{\xi+1}(s-s^2)} d\xi,$$

which, substituting (2.21), can be manipulated into the form

$$\begin{aligned} \Phi_{\delta|\zeta=1}(s) &= \frac{1}{1 + \frac{4}{\beta+\gamma^{-1}}(s-s^2)} \frac{1}{\text{B}(2/b, 1)} \int_0^1 (1-\xi)^{2/b-1} \left(1 - \frac{\beta\xi}{\beta + \gamma^{-1}} \right) \\ &\quad \cdot \left(1 - \frac{\beta\xi}{\beta + \gamma^{-1} + 4(s-s^2)} \right)^{-1} d\xi. \end{aligned}$$

For $\text{Re}\{s\} \in (0, 1)$, invoking identity 3.211, p.287 of [48], we obtain

$$\Phi_{\delta|\zeta=1}(s) = \frac{1}{1 + \frac{4}{\beta+\gamma^{-1}}(s-s^2)} F_1 \left(1, -1, 1, \frac{2}{b} + 1; \frac{\beta}{\beta + \gamma^{-1}}, \frac{\beta}{\beta + \gamma^{-1} + 4(s-s^2)} \right),$$

where $F_1(\lambda, \rho, \sigma, \lambda + \mu; u, v)$ is the first hypergeometric function of two variables. Note that the first term in the above result is (2.19), with $\beta\gamma$ in place of ξ . The second term introduces the effect of the PC adjustments on the interfering users. In the interference-limited regime, i.e., $\gamma \rightarrow \infty$, identity 3.197.3, p.286 of [48] results in

$$\Phi_{\delta|\zeta=1}(s) = \frac{1}{1 + \frac{4}{\beta}(s-s^2)} \frac{2}{2+b} {}_2F_1 \left(1, 1; \frac{2}{b} + 2; \frac{1}{1 + \frac{4}{\beta}(s-s^2)} \right),$$

where ${}_2F_1(\rho, \lambda; \lambda + \mu; u)$ is the Gauss hypergeometric function.

In order to obtain more insight into the performance of R1 under the high SNR regime, we employ the Chernoff bounding technique [13]. The Chernoff bound (CB) is determined by

$$CB = \min_{s \in \mathbb{R}^+} \Phi_{\Delta}(s) \geq P(\mathbf{x} \rightarrow \mathbf{x}').$$

From the previous expression, we have that

$$CB^{R1} = \left[\frac{p}{1 + \frac{1}{\beta}} \frac{2}{2+b} {}_2F_1 \left(1, 1; \frac{2}{b} + 2; \frac{1}{1 + \frac{1}{\beta}} \right) \right]^L.$$

Under a small long-term SIR, i.e. $\beta \rightarrow \infty$, it can be shown that

$${}_2F_1 \left(1, 1; 2/b + 2; \frac{\beta}{\beta + 1} \right) \rightarrow \frac{2+b}{2},$$

thus $CB^{R1} \approx p^L$, which demonstrates that, for large β , the performance of R1 is load-dependent. On the other hand, when $\beta \rightarrow 0$, the hypergeometric term goes to one, so

$$CB^{R1} \approx \left(\beta p \frac{2}{2+b} \right)^L = \left(\beta E[r^b \zeta] \right)^L.$$

This implies that, tracking the interference power variations translates to a “coding gain” $(2p/(2+b))^L$ in the plot BEP vs. β . The coding gain directly depends on the expected value of the interference power variations $E[r^b \zeta] = 2p/(2+b)$ and the length of the error event L .

In the case of R2, the expectation of (2.20) over ξ yields

$$\begin{aligned} \Phi_{\delta|\zeta=1}(s) &= \frac{1}{1 + 4(s - (\beta + \gamma^{-1})s^2)} \frac{1}{B(2/b, 1)} \\ &\cdot \int_0^1 (1 - \xi)^{2/b-1} \left(1 + \frac{4\beta s^2}{1 + 4(s - (\beta + \gamma^{-1})s^2)} \xi \right)^{-1} d\xi. \end{aligned}$$

Employing once more identity 3.197.3, p.286 of [48], we obtain

$$\Phi_{\delta|\zeta=1}(s) = \frac{1}{1 + 4(s - (\beta + \gamma^{-1})s^2)} {}_2F_1 \left(1, 1; \frac{2}{b} + 1; -\frac{4\beta s^2}{1 + 4(s - (\beta + \gamma^{-1})s^2)} \right), \quad (2.22)$$

for all s between the poles of the first term, such that the hypergeometric function is defined. Similarly to R1, the first term is (2.20), with ξ replaced by $\beta\gamma$, and the second term introduces the effect of the interference power variations. Determining the Chernoff bound on the PEP of R2 appears to be more involved. However, having obtained $\Phi_{\Delta}(s)$, the performance of R2 can be evaluated numerically by (2.13).

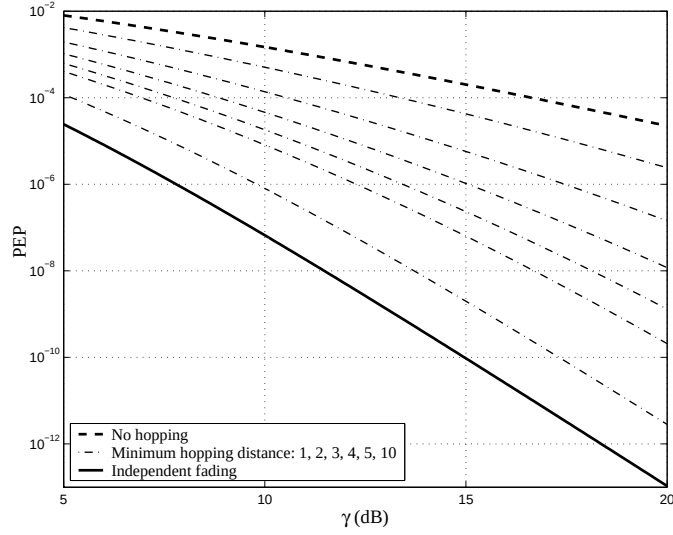


Figure 2.4: PEP vs. SNR for Enc1 and different Latin squares (perfect CSI). The curves from top to bottom correspond to an increasing minimum hopping distance. The PEP in the cases of no hopping and independent fading are also plotted for comparison purposes.

2.4 Numerical Results

The system operating parameters are selected according to [1], i.e. $M = 113$, $T_s = 100 \mu\text{s}$ and $1/T \simeq 11.25 \text{ KHz}$, so the system bandwidth is approximately 1.25 MHz. Unless otherwise stated, UI is situated on the line segment between B1 and B2 and at the border of the cell, i.e. $\theta = 0$ and $r_0 = 1$. Also, $f_D T_s = 0.01$, $\tau_L = 5 \mu\text{s}$, $\gamma = 15 \text{ dB}$, $\gamma_p = 10 \text{ dB}$, $D = 2$ and $b = 4$. Note that a Doppler spread $f_D = 100 \text{ Hz}$ at an operating frequency of 2 GHz corresponds to a speed of approximately 50 km/h. Regarding the channel estimation observation window, we set $M_1 = M_2 = 40$. In the simulation results, edge effects are taken into account and only the pilot symbols within the system bandwidth are used for channel estimation.

The optimum rate 1/2 convolutional codes shown in Table 2.1 and obtained from p.540 of [49] are used. Error events with length up to $d_{\min} + 2$ are taken into account in (2.10) and (2.11).

2.4.1 Frequency diversity

In Fig. 2.4, the PEP corresponding to the shortest error event of Enc1 is plotted vs. SNR, under perfect CSI. The subgroup $\alpha = 1, 56, 38, 28, 45$ of the available $M - 1 =$

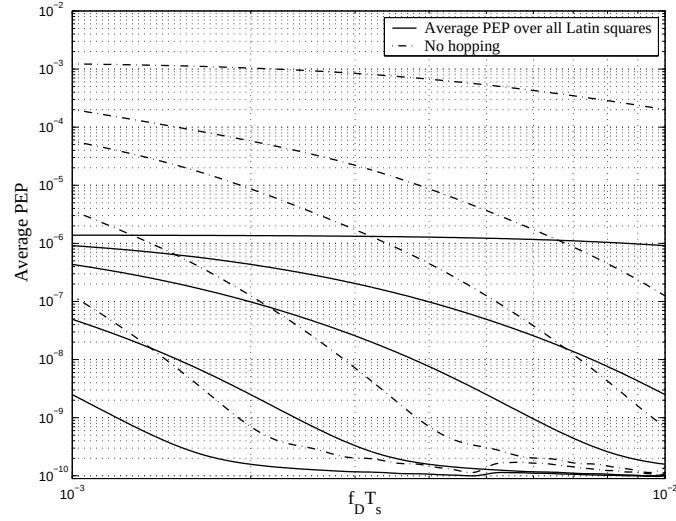


Figure 2.5: Average PEP vs. normalized Doppler spread for Enc1 and different interleaving depths (perfect CSI). Each group of curves from top to bottom corresponds to $\beta_{in} = 1, 10, 20, 50, 100$. The dash-dot curves correspond to the case of no hopping.

112 Latin squares, with corresponding minimum hopping distances 1, 2, 3, 4, 5, 10, is used. As the minimum hopping distance is increased, the performance improves since more frequency diversity is attained. The curve for the case of no hopping, i.e. the case where a user is assigned the same frequency across time and thus benefits only from the channel time variation, serves to illustrate the gain due to frequency hopping. The curve for independent fading provides a performance lower bound.

In Fig. 2.5, the effect of the Doppler spread on the performance of Enc1 is explored for different interleaving depths, under perfect CSI. Recalling Property 2, combining the patterns of a given Latin square with block interleaving effectively yields the patterns of another Latin square. In order to avoid square-specific results, the PEP is averaged over the first $(M - 1)/2$ Latin squares (due to Property 4, we are effectively averaging

Table 2.1: Optimum rate 1/2 convolutional codes in Chapter 2

Encoder	Memory	$g^{(0)}$	$g^{(1)}$	$d_{\min}$
Enc1	3	13	17	6
Enc2	5	53	75	8
Enc3	6	117	155	10

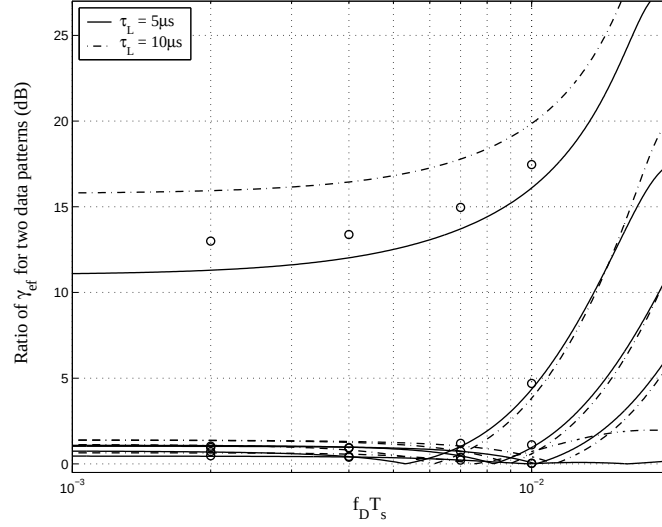


Figure 2.6: Ratio of effective SNRs on the hopping patterns $k = 1$ and $k = (\alpha(M - 1)/2) \bmod M$ vs. normalized Doppler spread for $\alpha = 56, 38, 28, 45, 34$ in that order from top to bottom. The circles correspond to simulation results for $\tau_L = 5 \mu s$.

over all squares). The respective no-hopping curves are also plotted for comparison purposes. For no interleaving, the PEP is practically constant for different Doppler spreads, since diversity is mainly obtained in the frequency domain. As the interleaving depth is increased, an increasing Doppler spread improves the performance since diversity is obtained in the time domain. Note that the performance advantage that hopping offers is larger for small interleaving depths and/or small Doppler spreads, i.e. where time diversity is scarce. As more time diversity becomes available, the gain from frequency hopping becomes marginal. Intuitively, frequency hopping is more beneficial under slow fading or delay-limited (small β_{in}) scenarios.

The effect that the Doppler spread has on the quality of channel estimation is demonstrated in Fig. 2.6. As a measure of fairness, the ratio of the effective SNRs on the patterns $k = 1$ and $k = (\alpha(M - 1)/2) \bmod M$ is plotted vs. $f_D T_s$ for Latin squares $\alpha = 56, 38, 28, 45, 34$, when the $k_p = 0$ pattern ($\mathcal{H}_0^\alpha$) is used for pilot transmission. Note that, for any position on $\mathcal{H}_1^\alpha$, the closest pilot symbol is situated one time-slot away, whereas, for $\mathcal{H}_{(\alpha(M-1)/2) \bmod M}^\alpha$, the position of the closest pilot symbol varies. As the minimum hopping distance is increased, the pilot lattice becomes more uniform in the time-frequency domain and the differences in channel estimation quality between the two data patterns are reduced. Note that insufficient channel sampling, as in the case

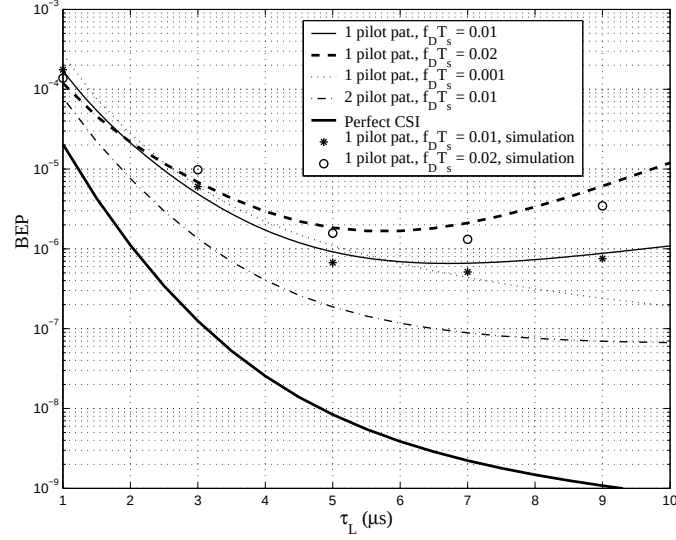


Figure 2.7: BEP vs. channel frequency selectivity for Enc1 and pattern $\mathcal{H}_{97}^{34}$. The set of pilot patterns is $\mathcal{I}_p = \{0, 96\}$.

of $\alpha = 56$, yields large ratio values. An increasing Doppler spread also increases the ratio. A solution is the introduction of more pilot patterns, at the expense of useful data bandwidth. In any case, Fig. 2.6 aims at illustrating the importance of the pilot-lattice structure on the channel estimation quality - and thus the performance - perceived by different users. Any discrepancies between analytical and simulation results are due to the fact that the former are computed based on a pilot lattice assumption.

In Fig. 2.7, the BEP is plotted vs. τ_L , for pattern $k = 97$ of square $\alpha = 34$ ($\mathcal{H}_{97}^{34}$). At a Doppler spread $f_D T_s = 0.01$ and use of one pilot pattern ($\mathcal{H}_0^{34}$), an increasing frequency selectivity leads at first to a performance improvement, then a progressive degradation. The reason is that as τ_L gets larger, more frequency diversity is obtained, however the CSI obtained on $\mathcal{H}_{97}^{34}$ from the pilot pattern $\mathcal{H}_0^{34}$ becomes more inaccurate. When the pilot pattern $\mathcal{H}_{96}^{34}$ is introduced, which is a one-slot shifted version of $\mathcal{H}_{97}^{34}$, the trade-off expectably vanishes. The trade-off also disappears when the Doppler spread is reduced; since the channel varies more slowly, one pilot pattern provides sufficient sampling for the given range of τ_L .

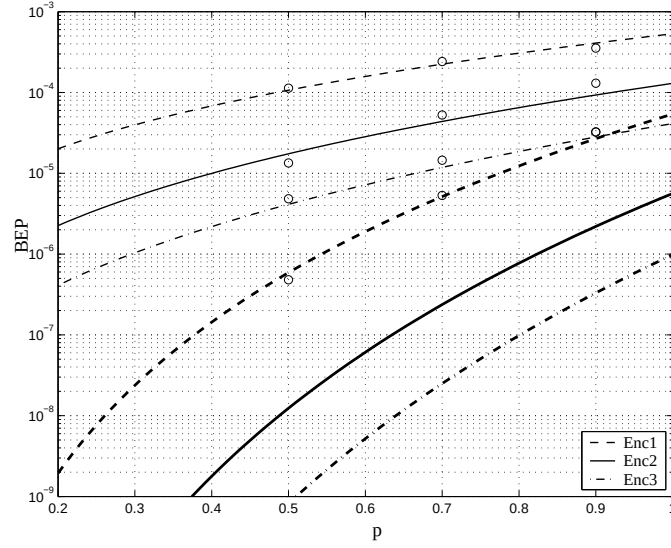


Figure 2.8: BEP vs. load for the three different encoders under high SNR operation ($\gamma \rightarrow \infty$). The thick curves correspond to R1 and the thin ones to R2. The circles correspond to simulation results.

2.4.2 Interference diversity

In Fig. 2.8, the BEP for the encoders of Table 2.1 is plotted vs. the load, under the scenario of independent fading, perfect CSI and high SNR ($\gamma = \gamma_{ef} \rightarrow \infty$). For a given encoder, the gap between the R1 and R2 curves grows as the system load is decreased. However, it is substantial even for high values of p , indicating that tracking the interference power variations due to PC in a heavily loaded system still provides a performance gain. Moreover, we note that the R2 curves for all encoders lie over the R1 curve for Enc1, which illustrates that simply increasing the encoding/decoding complexity, without an appropriate weighting of the decoder metrics according to the random interference power, is not an effective way to improve the performance. In other words, under a receiver total complexity constraint, it is preferable to trade decoding complexity with an interference tracking capability.

Fig. 2.9 shows the dependence of the BEP on the long-term SIR β^{-1} or, indirectly, on the reuse distance D , since $\beta^{-1} = R^b$ and R is an increasing function of D . In agreement with the Chernoff bound analysis in Section 2.3, for the R1 curves, the coding gain is increased as the system load is decreased from $p = 1$ to $p = 0.5$ and then $p = 0.3$ and the improvement is proportional to the load ratios. In the case of R2, the respective

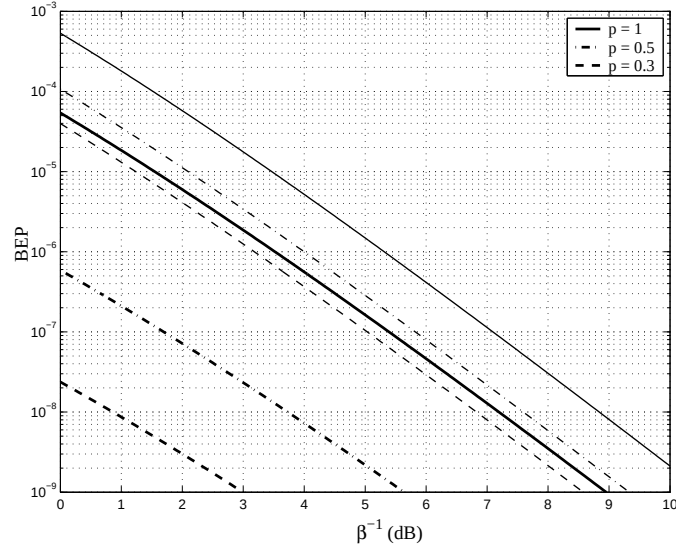


Figure 2.9: BEP vs. long-term SIR for Enc1 under high SNR operation ($\gamma \rightarrow \infty$). The thick curves correspond to R1 and the thin ones to R2.

gains are much smaller.

In Fig. 2.10, the FEP is plotted vs. the propagation exponent b for two different values of the ratio R/R_0 , assuming Enc1 and R1 are employed. The number of information bits is set to $L_b = 400$ (thus the codeword consists of 800 bits since Enc1 is rate $1/2$) and the load is $p = 0.5$. The nature of the dependence on b depends on the position of the user of interest; being “away” from B2, a larger b is beneficial and vice versa.

Fig. 2.11 illustrates the benefits of tracking the interference power variations in terms of the outage probability, defined here as the probability that the BEP takes a value larger than a certain threshold. Similarly to [5], it is computed in a semi-analytical manner, by generating random user positions within a sector $\theta \in [-\pi/3, \pi/3]$, computing the BEP according to (2.10) and counting the number of times it is larger than the threshold, selected here as 10^{-7} . Note that, for a load $p > 0.6$, approximately the same P_{out} is achieved by Enc1 combined with R1, as with the combination of Enc2 with the suboptimal R2.

2.5 Concluding Remarks on Chapter 2

Coherent FH-MA, such as the downlink of the FH-OFDMA system described in [1], is an attractive solution for the physical layer of wide-band wireless applications.

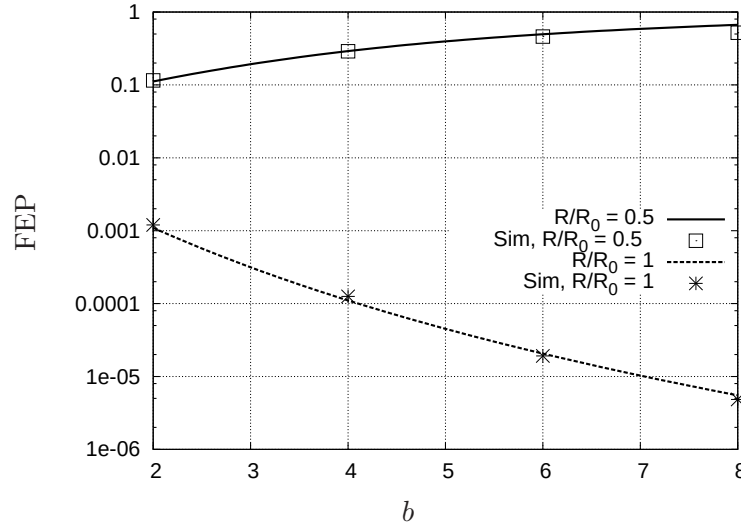


Figure 2.10: FEP vs. b for Enc1, R1 and different values of the ratio R/R_0 ($p = 0.5$, number of information bits in frame $L_b = 400$).

In this chapter, we developed a theoretical framework to evaluate the BEP in a coherent FH-MA cellular system, when the hopping patterns are constructed from Latin squares. The analysis accurately predicts the performance for various system parameters, such as the Doppler and delay spreads, the interleaving depth and the long-term SIR. Taking into account the time/frequency correlation of the channel response, we demonstrated that, under perfect CSI, the performance can vary significantly for different Latin squares, depending on the distance of the hops that occur within the patterns. It was also shown that the performance gains from FH are more apparent under slowly varying channels and/or small interleaving depths.

When pilot-assisted channel estimation is employed, the quality of channel estimation across different users depends on the structure of the pilot lattice. In some squares, using only one pilot pattern can result in large variations of the effective SNR and, consequently, more patterns are needed to sample the channel adequately. Moreover, an increasing frequency selectivity does not necessarily imply better performance; the additional frequency diversity is obtained at the expense of channel estimation quality and a performance trade-off occurs.

Finally, the effect of interference diversity was examined under a simple path-loss and PC model. We explicitly evaluated the performance for a receiver that is aware of the interference power variations and one that isn't. Our results confirmed that interference

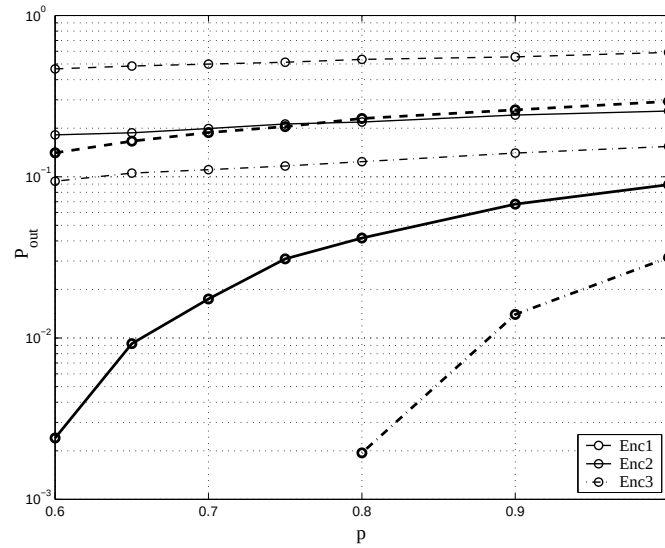


Figure 2.11: Outage probability vs. load for the three different encoders, $\gamma = 15$ dB and a BEP threshold of 10^{-7} . The thick curves correspond to R1 and the thin ones to R2.

side information is of paramount importance to the decoding performance and thus to the system outage probability.

Chapter 2, in part, is a reprint of the papers: “A performance analysis of coded FH-OFDMA with pilot-assisted channel estimation”, Kostas Stamatiou and John G. Proakis, as it appeared in the proceedings of IEEE PIMRC, Berlin, 2005; “Performance analysis of a coherent FH-MA system based on Latin squares”, Kostas Stamatiou and John G. Proakis, as it appeared in the IEEE Transactions on Wireless Communications, November 2007.

Chapter 3

Channel diversity in random single-hop networks

This chapter follows the thread of Chapter 2 and explores the benefits of channel diversity in wideband ad hoc networks. A network of TXs is considered, each with a RX at a fixed distance and locations drawn according to a homogeneous PPP. We demonstrate that large gains in terms of network capacity are possible, by combining FH during packet transmission (fast FH) and error correction coding of modest complexity. Specifically, if L is the Hamming distance of the convolutional code employed at the TX, λ is the density of TXs and M is the number of subbands, we show that, as $\lambda/M \rightarrow 0$, the codeword error probability follows the power law $\eta(\frac{\lambda}{M})^L$, $\eta > 0$. This implies that, for $\epsilon \rightarrow 0$, the transmission capacity is proportional to $\epsilon^{1/L}$ where ϵ is the constraint placed on the codeword error probability. The proportionality constant depends on the geometry of the symbol constellation, the codeword length, as well as the term $N^{2/b}$ where N is the number of RX antennas and $b > 2$, is the propagation exponent. We also derive upper and lower bounds on the ergodic capacity C of the typical TX-RX link; specifically, we show that $C > \frac{1}{\alpha} \log_2 (\mu N^\alpha \frac{M}{\lambda})$, where $\mu > 0$ is an appropriately defined constant.

Practical physical layer issues are discussed such as channel estimation, PC and channel correlation. We demonstrate via simulation that, with an acceptable rate loss due to the transmission of pilot symbols, accurate channel state information (CSI) can be obtained for decoding. With respect to PC, it is shown that channel inversion can actually improve the performance, since the error correction code protects the RX from

the deep fades of its nearby interferers. Finally, the impact of the channel correlation is assessed as the number of subbands and/or the number of dwells is decreased and it is shown that the gains compared to slow FH are still significant.

The rest of this chapter is organized as follows. Section 3.1 introduces our network and physical layer models in detail. In Section 3.2 we derive the statistics of the SIR and determine the performance of the decoder under perfect CSI. The transmission capacity is defined and evaluated in Section 3.3. Section 3.4 discusses practical physical layer considerations and Section 3.5 presents our numerical results. Section 3.6 includes some concluding remarks. A list of symbols commonly used throughout this chapter is listed in Table 3.1.

3.1 System model

3.1.1 General

We consider a network of TXs, each with a RX at a fixed distance R and random orientation. The locations of the TXs are drawn independently according to a homogeneous PPP Π of density λ . The TXs transmit packets to their corresponding RXs concurrently and in a synchronized manner. Typically, the locations of the nodes are constant for at least the duration of a packet.

The bandwidth is divided into M subbands. The channel between a typical TX-RX pair over a subband comprises flat Rayleigh fading and path loss according to the law r^{-b} , where $b > 2$ is the propagation exponent. We assume that the coherence bandwidth of the fading is equal to the width of a subband, while the coherence time can be equal to the duration of a packet slot or the duration of a *dwell*, which will be defined shortly. We also consider an interference-limited scenario, i.e., additive noise at the RX is assumed negligible, such that interference from concurrent transmissions is the only cause for packet errors. Initially, we assume that the power transmitted from all TXs is the same and normalized to one. Issues of PC to compensate for long-term fading, e.g., shadowing, are discussed in Section 3.4.

Assume that a packet corresponds to L_b binary information bits, $b_1, \dots, b_{L_b}$, which are the input to a convolutional encoder of rate $R_c < 1$ ¹. The bits of the output codeword $c_1, \dots, c_{k'}, \dots, c_{L_b/R_c}$ are interleaved and Gray-mapped to symbols

¹We assume that the encoder is trellis-terminated [45], i.e., it is forced to start from and end at the zero-state. This results in a small rate loss which is not taken into account.

Table 3.1: Commonly used symbols in Chapter 3

Symbol	Meaning
R	Distance of typical TX-RX link
λ	Density of TXs
M	Number of subbands
N	Number of RX antennas
b	Path-loss exponent ($b > 2$)
$\alpha = 2/b$	Stability exponent
L_b	Number of information bits per packet
M_c	Number of bits per constellation symbol
R_c	Rate of convolutional code
T_d	Number of data symbols per dwell
T_p	Number of pilot symbols per dwell
D	Number of dwells
L	Hamming distance of convolutional code
P_l	Probability of length- l error event
w_l	Number of length- l error events
P_e	Probability of packet error
C	Ergodic capacity of typical TX-RX link
TC_ϵ	Transmission capacity under constraint $P_e = \epsilon$

$x_1, \dots, x_k, \dots, x_{L_b/(R_c M_c)}$ from a complex PSK or QAM constellation $\mathcal{X}$ of size $|\mathcal{X}| = 2^{M_c}$, zero mean and unit average power. We assume that the one-to-one interleaver mapping $k' \leftrightarrow (k, j_{k'})$, where $j_{k'} = 1, \dots, M_c$ is the position of $c_{k'}$ in the symbol x_k , is known at the RX. Next in the TX chain, the symbol sequence is divided in $D = L_b/(R_c M_c T_d)$ groups² of size T_d and each group is transmitted in a dwell, over a subband which is randomly³ selected with probability $1/M$. If we denote the data symbols of the d^{th} dwell, $d = 1, \dots, D$, as $\mathbf{x}_d^T$ the sequence $\{\mathbf{x}_d^T\}_{d=1}^D$ constitutes a packet. For convenience, we assume all transmissions are synchronized at the dwell level. This is a worst-case scenario in terms of the level of the interference power over a dwell (the issue of asynchronous transmissions is discussed in Section 3.4).

Consider a typical TX and its corresponding RX, both specified by index 0, i.e., TX_0 and RX_0 . If RX_0 is equipped with an antenna array of size $N \geq 1$, the received

²For convenience, we assume that the quotients $L_b/(R_c M_c)$ and $D = L_b/(R_c M_c T_d)$ are integer.

³In reality, the hopping pattern is determined pseudorandomly and is known at the RX. However, the model of random subband selection is convenient for analytical purposes.

data matrix in dwell d is⁴

$$\mathbf{Y}_{d,0} = \mathbf{h}_{d,0} \mathbf{x}_{d,0}^T + R^{b/2} \sum_{i \in \Pi} e_{d,i} r_i^{-b/2} e^{i\phi_i} \mathbf{h}_{d,i} \mathbf{x}_{d,i}^T, \quad (3.1)$$

where $\mathbf{h}_{d,0} \sim \mathcal{CN}(\mathbf{0}, \mathbf{I}_N)$ is the fading vector between TX_0 and RX_0 ; $e_{d,i}$ is the indicator of whether $\text{TX } i$, at distance r_i from RX_0 and denoted as TX_i , occupies the same subband as TX_0 in dwell d , i.e., $e_{d,i} = 1$ with probability $1/M$ and $e_{d,i} = 0$ with probability $1 - 1/M$; $\mathbf{h}_{d,i} \sim \mathcal{CN}(\mathbf{0}, \mathbf{I}_N)$ is the fading vector between TX_i and RX_0 ; $\mathbf{x}_{d,i}^T$ is the group of data symbols transmitted by TX_i in dwell d ; and ϕ_i is a random phase, uniformly distributed in $[0, 2\pi)$, which models the phase offset between the RX_0 and TX_i . Note that the subscript d in the fading vectors indicates that, in general, depending on the coherence time, these may vary independently from dwell to dwell.

Let $\mathbf{W}_{d,0}$ denote the interference term in (3.1). Since the elements of $\mathbf{x}_{d,i}$ are independent and zero-mean, the same holds for the elements of $\mathbf{W}_{d,0}$. Moreover, given $\{r_i\}$, we assume that $[\mathbf{W}_{d,0}]_{n,t} \sim \mathcal{CN}(0, z_{d,0})^5$, $n = 1, \dots, N$, $t = 1, \dots, T_d$, and $z_{d,0} \triangleq R^b \sum_{i \in \Pi} e_{d,i} r_i^{-b}$ is the interference power in dwell d seen by RX_0 . RX_0 can obtain knowledge of $\mathbf{h}_{d,0}$ and $z_{d,0}$ with the help of pilot symbols which are transmitted at the beginning of the dwell. Presently, we assume that they are perfectly known; a straightforward channel estimation algorithm is presented in Section 3.4.

3.1.2 Equivalent channel model and decoding

The reference RX performs MRC, i.e., it evaluates the product $\frac{\mathbf{h}_{d,0}^H}{\|\mathbf{h}_{d,0}\|^2} \mathbf{Y}_{d,0}$. From (3.1), we have

$$\frac{\mathbf{h}_d^H}{\|\mathbf{h}_d\|^2} \mathbf{Y}_d = \mathbf{x}_d^T + \frac{\mathbf{h}_d^H}{\|\mathbf{h}_d\|^2} \mathbf{W}_d, \quad (3.2)$$

where we have omitted the index 0 in order to simplify the notation. As a result, we have the following equivalent channel model for data symbol x_k , $k = (d-1)T_d + 1, \dots, dT_d$, which is transmitted in dwell d

$$y_k = x_k + w_k \quad (3.3)$$

where, $w_k \sim \mathcal{CN}(0, \gamma_k^{-1})$, given the equivalent SIR $\gamma_k = \frac{a_k}{z_k}$. The r.v. a_k is chi-square distributed with $2N$ degrees of freedom, i.e., $a_k \sim \chi_{2N}^2$. Moreover, due to the fact that

⁴We assume that the average received power R^{-b} per antenna is known at RX_0 . We have taken it into account in the interference portion of the received signal because it is convenient in terms of notation in the remainder of the chapter.

⁵The Gaussian assumption for the interference is exact if the elements of $\mathbf{x}_{d,i}$ are selected from a PSK constellation and an approximation if they are selected from a QAM constellation.

the locations of the interferers in each dwell are a realization of a Poisson process on the plane with density λ/M , it is known that z_k is an α -stable random variable with stability exponent $\alpha = 2/b$ [15, 19]. Its moment generating function (mgf) is [15]

$$\Phi_z(s) = \mathbb{E}[e^{-zs}] = e^{-\frac{\lambda\delta}{M}s^\alpha}, \quad s > 0. \quad (3.4)$$

where

$$\delta \triangleq \pi\Gamma(1-\alpha)R^2 \quad (3.5)$$

and $\Gamma(x)$, $x > 0$ is the gamma function. The sequence $\{y_k, \gamma_k\}_{k=1}^{DT_d}$ is the input to the decoder which decides that the codeword $\hat{\mathbf{c}}$ was transmitted according to the simplified ML criterion (eq.(9), [50])

$$\hat{\mathbf{c}} = \arg \min_{\mathbf{c}} \sum_{k'=1}^{L_b/R_c} \gamma_k \min_{x \in \mathcal{X}_{c_{k'}}^{j_{k'}}} \{|y_k - x|^2\} \quad (3.6)$$

where $\mathcal{X}_{c_{k'}}^{j_{k'}}$ denotes the set of constellation symbols that have bit $c_{k'}$ at position $j_{k'}$, where $j_{k'} = 1, \dots, M_c$. The weighting of each distance metric by the respective SIR reflects the confidence of the decoder in that metric.

3.2 Analysis

This section is primarily devoted to the performance analysis of decoder (3.6). In order to round out the analysis, in Section 3.2.4, we also derive upper and lower bounds to the ergodic capacity of channel (3.3). Since the scheme presented in Section 3.1 induces an “averaging” over different channel states within the packet, the ergodic capacity is an upper bound to the information rate of the typical TX-RX link.

3.2.1 Decoder performance

The codeword - or frame - error probability (FEP) of decoder (3.6), P_e , is upper-bounded as [45]

$$P_e \leq L_b \sum_{l=L, L+1, \dots} w_l P_l \quad (3.7)$$

where P_l is the probability of a length- l error event, or pairwise error probability, and w_l is the number of length- l error events. The minimum length of an error event L , i.e., the Hamming distance, as well as the weight distribution $\{w_l\}$ depend on the particular code employed.

We now assume that, due to random interleaving, the sequence of l symbols that corresponds to a sequence of l coded bits encounters *independent* SIR conditions (this assumption is discussed in Section 3.4). From [50, 51], P_l can be upper-bounded as

$$P_l \leq \bar{P}_l = \frac{1}{\pi} \int_0^{\pi/2} \left(\frac{1}{M_c 2^{M_c}} \sum_{(x, x') \in \mathcal{X}} \Phi_\gamma \left(\frac{|x - x'|^2}{4 \sin^2 \theta} \right) \right)^l d\theta \quad (3.8)$$

where $\Phi_\gamma(s)$ is the mgf of the SIR γ , i.e., $\Phi_\gamma(s) = \mathbb{E}[e^{-\gamma s}]$, $s > 0$ (the time index k has been removed as it is of no consequence). (x, x') are all possible $M_c 2^{M_c}$ nearest-neighbor pairs in $\mathcal{X}$ which have complementary bits in position j , $j = 1, \dots, M_c$ [50], e.g., for 4QAM (QPSK) constellations with Gray mapping, all such pairs are at the minimum constellation distance $d_{\min}$. Note that, in the limit of large $E[\gamma]$ ⁶, $\bar{P}_l \simeq P_l$ [50].

From (3.7), a further upper bound to P_e is

$$\bar{P}_e = L_b \sum_{l=L, L+1, \dots} w_l \bar{P}_l. \quad (3.9)$$

The evaluation of $\bar{P}_l$ in (3.8) is possible by numerical integration, provided that $\Phi_\gamma(s)$ is known. The derivation of $\Phi_\gamma(s)$ is the topic of the following subsection.

3.2.2 Statistics of γ

The probability density function (pdf) of γ is given by the following lemma.

Lemma 1. *The pdf of the SIR γ is given by*

$$f_\gamma(\gamma) = \frac{1}{(N-1)!} \frac{e^{-\frac{\lambda \delta}{M} \gamma^\alpha}}{\gamma} \sum_{n=1}^N \frac{|\beta_n^N|}{n!} \left(\frac{\lambda \delta}{M} \gamma^\alpha \right)^n, \quad \gamma > 0 \quad (3.10)$$

where

$$\beta_n^N = \sum_{m=1}^n (-1)^m \binom{n}{m} (\alpha m)_N, \quad n = 1, \dots, N \quad (3.11)$$

and $(\alpha m)_N \triangleq \alpha m \dots (\alpha m - N + 1)$ is the falling sequential product.

Proof. The cumulative distribution function (cdf) of γ , $F_\gamma(\gamma)$, is by definition

$$F_\gamma(\gamma) = \mathbb{P}(a \leq \gamma z) = \int_0^{+\infty} F_a(\gamma z) f_z(z) dz \quad (3.12)$$

⁶In the context of this chapter, this corresponds to the interferer point process in each dwell being sparse, i.e., $\lambda/M \rightarrow 0$.

where

$$F_a(a) = 1 - e^{-a} \sum_{n=0}^{N-1} \frac{a^n}{n!} = 1 - \frac{\Gamma(N, a)}{(N-1)!} \quad (3.13)$$

is the cdf of the chi-square r.v. a and

$$\Gamma(\zeta, x) = \int_x^{+\infty} e^{-t} t^{\zeta-1} dt, \quad x \geq 0 \quad (3.14)$$

is the incomplete gamma function (p.949, [48]). Substituting (3.13) in (3.12) and taking the derivative, the pdf of γ is given by

$$\begin{aligned} f_\gamma(\gamma) &= -\frac{1}{(N-1)!} \int_0^{+\infty} \frac{d\Gamma(N, \gamma z)}{d\gamma} f_z(z) dz = \frac{\gamma^{N-1}}{(N-1)!} \int_0^{+\infty} f_z(z) z^N e^{-\gamma z} dz \\ &= \frac{(-1)^N \gamma^{N-1}}{(N-1)!} \frac{d^N \Phi_z(\gamma)}{d\gamma^N} \end{aligned} \quad (3.15)$$

where we have used the identity (p.951, [48]) $\frac{d\Gamma(\zeta, x)}{dx} = -x^{\zeta-1} e^{-x}$ and the Laplace transform property⁷

$$f_z(z) z^N \xleftrightarrow{\mathcal{L}} (-1)^N \frac{d^N \Phi_z(s)}{ds^N}. \quad (3.16)$$

From the identity for the N^{th} derivative of a composite function (0.430.1, p.24, [48]), after some algebra, we obtain

$$\frac{d^N \Phi_z(s)}{ds^N} = s^{-N} e^{-\frac{\lambda\delta}{M} s^\alpha} \sum_{n=1}^N \frac{\beta_n^N}{n!} \left(\frac{\lambda\delta}{M} s^\alpha \right)^n \quad (3.17)$$

where β_n^N is given by (3.11). From (3.17) and (3.15) we have that

$$f_\gamma(\gamma) = \frac{1}{(N-1)!} \frac{e^{-\frac{\lambda\delta}{M} \gamma^\alpha}}{\gamma} \sum_{n=1}^N \frac{(-1)^N \beta_n^N}{n!} \left(\frac{\lambda\delta}{M} \gamma^\alpha \right)^n. \quad (3.18)$$

In order to derive (3.10) from (3.18), we need to show that $(-1)^N \beta_n^N \geq 0$. Once again, using the identity for the N^{th} derivative of a composite function, β_n^N can be written as the following derivative evaluated at $x = 1$.

$$\beta_n^N = \left. \frac{d^N (1 - x^\alpha)^n}{dx^N} \right|_{x=1}. \quad (3.19)$$

From (3.19), the following iterative relation can be proved for $N \geq 2$

$$\beta_n^N = \sum_{m_1=1}^N \binom{N}{m_1} \beta_1^{m_1} \beta_{n-1}^{N-m_1}. \quad (3.20)$$

⁷This identity is also employed in [22], in order to derive the complementary cumulative distribution function (ccdf) of γ .

By successive application of (3.20), we obtain

$$\frac{(-1)^N \beta_n^N}{N!} = \sum_{m_1=1}^N \sum_{m_2=1}^{N-m_1} \cdots \sum_{m_{n-1}=1}^{N-m_{n-2}-\cdots-m_1} (-1)^{m_1} \beta_1^{m_1} (-1)^{m_2} \beta_1^{m_2} \cdots (-1)^{m_n} \beta_1^{m_n} \quad (3.21)$$

where $m_n = N - m_{n-1} - \cdots - m_1$. However, $(-1)^N \beta_1^N \geq 0$, since, by (3.11), $(-1)^N \beta_1^N = (-1)^{N+1} \alpha(\alpha-1) \cdots (\alpha-N+1)$ and $\alpha = 2/b < 1$. Therefore, $(-1)^N \beta_n^N \geq 0$ for $n = 1, \dots, N$. $\square$

As expected, increasing the spatial diversity order N increases $f_\gamma(\gamma)$, as more positive terms are added to the polynomial in (3.10).

By the definition of the mgf of γ and (3.10), we have

$$\Phi_\gamma(s) = \frac{1}{(N-1)!} \sum_{n=1}^N \frac{|\beta_n^N|}{n!} \left(\frac{\lambda\delta}{M} \right)^n \int_0^{+\infty} \gamma^{\alpha n-1} e^{-\frac{\lambda\delta}{M} \gamma^\alpha - s\gamma} d\gamma. \quad (3.22)$$

This integral can be evaluated numerically for any $s > 0$ using Gauss-Laguerre quadrature.

3.2.3 Approximations

The numerical evaluation of $\bar{P}_l$ using (3.22) provides little insight on how the decoder performance depends on the system parameters. In this section, we examine the decoder performance when $\lambda/M \rightarrow 0$, i.e., the interferer point process in each dwell is sparse. This implies that the network is operated in a regime of small FEP, i.e., typically $\bar{P}_e \leq \epsilon$, with $\epsilon \leq 0.1$.

Let $B(\zeta_1, \zeta_2)$, $\zeta_1, \zeta_2 > 0$ denote the beta function. Our main result is stated in the following proposition.

Proposition 1. *If $\lambda/M \rightarrow 0$, then $\bar{P}_l = \eta(\frac{\lambda}{M})^l + o\left((\frac{\lambda}{M})^l\right)$, where $\bar{P}_l$ is defined in (3.8) and η is a positive constant. Moreover,*

$$\bar{P}_l \simeq \frac{2^{4\alpha l-1}}{\pi} B\left(\alpha l + \frac{1}{2}, \alpha l + \frac{1}{2}\right) \left(\frac{\alpha B(N-\alpha, \alpha) \lambda \pi R^2}{d_{\mathcal{X}}^2 M} \right)^l \quad (3.23)$$

where

$$d_{\mathcal{X}}^2 = \left(\frac{1}{M_c 2^{M_c}} \sum_{x, x' \in \mathcal{X}} \frac{1}{|x - x'|^{2\alpha}} \right)^{-1}. \quad (3.24)$$

Proof. Omitting the term $e^{-\frac{\lambda\delta}{M}\gamma^\alpha}$ in (3.22), an upper bound to $\Phi_\gamma(s)$ for all $s > 0$ is

$$\begin{aligned}\overline{\Phi_\gamma(s)} &= \frac{1}{(N-1)!} \sum_{k=1}^N \frac{|\beta_k^N|}{k!} \left(\frac{\lambda\delta}{M}\right)^k \int_0^{+\infty} \gamma^{\alpha k-1} e^{-\gamma s} d\gamma \\ &= \frac{1}{(N-1)!} \sum_{k=1}^N \frac{|\beta_k^N|}{k!} \left(\frac{\lambda\delta}{M}\right)^k s^{-\alpha k} \Gamma(\alpha k) \\ &= \frac{\alpha B(N-\alpha, \alpha)}{\Gamma(1-\alpha)} \left(\frac{\lambda\delta}{M}\right) s^{-\alpha} + \frac{1}{(N-1)!} \sum_{k=2}^N \frac{|\beta_k^N|}{k!} \left(\frac{\lambda\delta}{M}\right)^k s^{-\alpha k} \Gamma(\alpha k)\end{aligned}\quad (3.25)$$

where we have used the identities $\Gamma(\zeta+1) = \zeta\Gamma(\zeta)$ and [48]

$$B(\zeta_1, \zeta_2) = \frac{\Gamma(\zeta_1)\Gamma(\zeta_2)}{\Gamma(\zeta_1 + \zeta_2)}. \quad (3.26)$$

From (3.25) and (3.8), we can see that, for $\lambda/M \rightarrow 0$, $\bar{P}_l = \eta(\frac{\lambda}{M})^l + o\left((\frac{\lambda}{M})^l\right)$, with η appropriately defined.

Note that the bound in (3.25) is tight as $\lambda/M \rightarrow 0$. Ignoring the higher order terms, for $\lambda/M \rightarrow 0$, we thus have that

$$\Phi_\gamma(s) \simeq \overline{\Phi_\gamma(s)} \simeq \pi\alpha R^2 B(N-\alpha, \alpha) s^{-\alpha} \frac{\lambda}{M}. \quad (3.27)$$

Substituting (3.27) in (3.8)

$$\bar{P}_l \simeq \left(\pi\alpha R^2 B(N-\alpha, \alpha) d_{\mathcal{X}}^{-2} \frac{\lambda}{M} \right)^l \frac{4^\alpha}{\pi} \int_0^{\pi/2} (\sin \theta)^{2\alpha l} d\theta \quad (3.28)$$

where $d_{\mathcal{X}}^2$ is defined in (3.24). Employing the identity (p.412, [48])

$$\int_0^{\pi/2} (\sin \theta)^{2\alpha l} d\theta = 2^{2\alpha l-1} B(\alpha l + 1/2, \alpha l + 1/2)$$

we obtain (3.23). $\square$

Remarks: Proposition 1 states that, for $\lambda/M \rightarrow 0$, the Hamming distance of the convolutional code determines the *diversity order*, i.e., the slope of the curve $\bar{P}_e$ vs. λ/M . Moreover, the spatial diversity order N introduces an array or *coding gain* through the factor $B(N-\alpha, \alpha)^L$. To obtain further insight on this factor, we examine the trend of the beta function for large N . For large ζ , it holds that $\Gamma(\zeta) \sim \sqrt{2\pi}\zeta^{\zeta-1/2}e^{-\zeta}$ ([48], p.945), therefore

$$\frac{\Gamma(N-\alpha)}{\Gamma(N)} \simeq N^{-\alpha} \left(1 - \frac{\alpha}{N}\right)^{N-\alpha-\frac{1}{2}} e^\alpha. \quad (3.29)$$

However, it is easy to verify that $\lim_{N \rightarrow \infty} \left(1 - \frac{\alpha}{N}\right)^{N-\alpha-\frac{1}{2}} = e^{-\alpha}$, so, from (3.26), $B(N - \alpha, \alpha) \simeq \Gamma(\alpha)N^{-\alpha}$. As a result, for large N , the coding gain is proportional to $N^{-L\alpha}$.

A final observation is that, similarly to [50], the parameter $d_{\mathcal{X}}^2$ is the harmonic mean of the minimum squared Euclidean distance between the nearest neighbor pairs defined in Section 3.2.1, raised to the stability exponent α . Assuming unit average energy, for BPSK we have $d_{\mathcal{X},\text{BPSK}}^2 = 2^{-2\alpha}$, while for 4QAM and 16QAM with Gray mapping, we have $d_{\mathcal{X},4\text{QAM}}^2 = 2^{-\alpha}$ and $d_{\mathcal{X},16\text{QAM}}^2 = \frac{3}{4}(\sqrt{\frac{2}{5}})^{-2\alpha} + \frac{1}{4}(2\sqrt{\frac{2}{5}})^{-2\alpha}$, respectively.

3.2.4 Ergodic capacity

The ergodic capacity of (3.3) is

$$C = \int_0^{+\infty} f_{\gamma}(\gamma) \log_2(1 + \gamma) d\gamma. \quad (3.30)$$

A closed form expression appears hard to obtain due to the complicated nature of (3.10); nevertheless, the integral can be evaluated numerically with Gauss-Laguerre quadrature. In [52], an approximation to (3.30) is given for $N = 1$. The following proposition provides upper and lower bounds to C for $N \geq 1$.

Proposition 2. *The ergodic capacity of (3.3) is upper-bounded as*

$$C < \bar{C} = \log_2 \left(1 + N\Gamma \left(\frac{1}{\alpha} + 1 \right) \left(\frac{M}{\lambda\delta} \right)^{1/\alpha} \right) \quad (3.31)$$

and lower bounded as

$$C \geq \underline{C} = \frac{1}{\alpha} \log_2 \left(\frac{M}{e^{\Gamma} \lambda \delta} \right) + \frac{H_{N-1}}{\ln 2}, \quad (3.32)$$

where $\Gamma = 0.577 \dots$ is the Euler-Mascheroni constant and

$$H_n = \begin{cases} \sum_{k=1}^n \frac{1}{k} & n \geq 1 \\ 0 & n = 0 \end{cases} \quad (3.33)$$

is the n^{th} harmonic number. This bound is tight, i.e., $C \simeq \underline{C}$ for $\lambda \rightarrow 0$. A looser lower bound is

$$\underline{\underline{C}} = \frac{1}{\alpha} \log_2 \left(\frac{N^{\alpha} M}{e^{\Gamma} \lambda \delta} \right). \quad (3.34)$$

Proof. The upper bound is derived by noting that $E[\gamma] = E[a]E\left[\frac{1}{z}\right] = NE\left[\frac{1}{z}\right]$, so, for $N = 1$, $E\left[\frac{1}{z}\right] = E[\gamma]$. Setting $N = 1$ in (3.10), we have

$$E\left[\frac{1}{z}\right] = \frac{\lambda\delta\alpha}{M} \int_0^{+\infty} \gamma^{\alpha} e^{-c\gamma^{\alpha}} d\gamma = \left(\frac{M}{\lambda\delta} \right)^{1/\alpha} \Gamma\left(\frac{1}{\alpha} + 1\right).$$

By applying Jensen's inequality⁸ on (3.30), we obtain (3.31).

For the derivation of the lower bound, from (3.15) and (3.30), we obtain

$$\begin{aligned}
\underline{C} &= \frac{(-1)^N}{(N-1)! \ln 2} \int_0^{+\infty} \gamma^{N-1} \Phi_z^{(N)}(\gamma) \ln \gamma d\gamma \\
&= \frac{(-1)^N}{(N-1)! \ln 2} \sum_{k=0}^{N-2} (-1)^k \left[\Phi_z^{(N-k-1)}(\gamma) (\gamma^{N-1} \ln \gamma)^{(k)} \right]_0^{+\infty} \\
&\quad - \frac{1}{(N-1)! \ln 2} \int_0^{+\infty} \Phi'_z(\gamma) (\gamma^{N-1} \ln \gamma)^{(N-1)} d\gamma.
\end{aligned} \tag{3.35}$$

After some algebra, we can show that, for $k = 1, \dots, N-1$, $N > 1$,

$$\begin{aligned}
(\gamma^{N-1} \ln \gamma)^{(k)} &= (N-1) \dots (N-k) \gamma^{N-k-1} \ln \gamma \\
&\quad + \gamma^{N-k-1} \sum_{(l_1, \dots, l_{k-1})} (N-l_1) \dots (N-l_{k-1})
\end{aligned} \tag{3.36}$$

where the summation is taken over all permutations of the vector $(l_1, \dots, l_{k-1})$, $l_j = 1, \dots, k$, $j = 1, \dots, k-1$. When $k = N-1$, then

$$(\gamma^{N-1} \ln \gamma)^{(N-1)} = (N-1)! (\ln \gamma + H_{N-1}) \tag{3.37}$$

where H_n is defined in (3.33). From (3.17) and (3.36), we can show that the first term in (3.35) is zero. Hence

$$\underline{C} = \frac{\lambda \delta \alpha}{M \ln 2} \int_0^{+\infty} \gamma^{\alpha-1} e^{-\frac{\lambda \delta}{M} \gamma^\alpha} \ln \gamma d\gamma + \frac{H_{N-1}}{\ln 2}$$

from which (3.32) follows by use of (4.331.1) on p.602 of [48].

Since the harmonic number is lower bounded as [54]

$$H_N > \ln N + \Gamma + \frac{1}{2(N+1)}$$

we have that

$$H_{N-1} = H_N - \frac{1}{N} > \ln N + \Gamma + \frac{1}{2(N+1)} - \frac{1}{N} > \log N, \quad N \geq 2.$$

The latter inequality holds because, for $N \geq 2$, $\frac{1}{N} - \frac{1}{2(N+1)} < \frac{1}{2} < \Gamma$, $\forall N \geq 2$. As a result, a looser lower bound to the capacity is (3.34). $\square$

Eq. (3.34) shows that $\underline{C}$ is a linear function of $\log_2 \left(\frac{M}{\lambda \delta} \right)$, with slope $1/\alpha = b/2$ and a constant term $\log_2 N - \frac{\Gamma}{\alpha} \log_2 e$.

⁸Jensen's inequality was also employed in [53] in order to derive an upper bound to the ergodic capacity, albeit in a slightly different context.

3.3 Network metrics

Having evaluated the performance at the link level, we now turn our attention to network-wide metrics. Similarly to [18], we define the transmission capacity⁹ TC_ϵ as the maximum spatial density of successful transmissions, multiplied by their rate $R_c M_c$, such that a constraint $P_e = \epsilon$ is satisfied, i.e.,

$$\text{TC}_\epsilon = \lambda_\epsilon (1 - \epsilon) R_c M_c, \quad (3.38)$$

where λ_ϵ is the maximum contention density. A closed-form expression for λ_ϵ may be obtained by noting that, for $\epsilon \rightarrow 0$, we can take into account only Hamming distance error events in (3.9) and, moreover, $P_e \simeq \bar{P}_e$. The constraint that needs to be satisfied is therefore $\bar{P}_e \approx L_b w_L \bar{P}_L = \epsilon$. From Proposition 1, we find that

$$\lambda_\epsilon \approx \left(\frac{\epsilon}{K} \right)^{1/L} \frac{d_{\chi}^2 M}{16^\alpha \alpha B(N - \alpha, \alpha) \pi R^2} \quad (3.39)$$

where

$$K \triangleq \frac{L_b w_L}{2\pi} B(\alpha L + 1/2, \alpha L + 1/2) \quad (3.40)$$

is related to the code parameters and the propagation exponent. The maximum contention density of a coded system is therefore of the order $(\epsilon/L_b)^{1/L}$. This result is a manifestation of the channel diversity harnessed through frequency hopping and coding at the expense of spectrum. Moreover, note that λ_ϵ is proportional to $B(N - \alpha, \alpha)^{-1} \approx N^\alpha$, which is in agreement with the scaling law in [22].

The transmission capacity is upper-bounded by $\text{TC} = \lambda C$, where C is the link ergodic capacity given by (3.30). From Proposition 2, a lower bound to TC is

$$\underline{\text{TC}} = \frac{\lambda}{\alpha} \log_2 \left(\frac{N^\alpha M}{e^\Gamma \lambda \delta} \right). \quad (3.41)$$

Optimizing over λ , it is straightforward to verify that the optimal $\underline{\text{TC}}$ is also proportional to N^α .

3.4 Physical-layer considerations

This section discusses various physical-layer issues with respect to the system model presented in Section 3.1. The influence of these on the decoder performance is assessed via simulation in Section 3.5.

⁹In contrast to [18], where the SIR is constant across a packet, here the probability of packet error P_e is computed by averaging over different channel realizations. We can thus say that, on the average, the probability that a packet is received successfully is $1 - P_e$.

3.4.1 Channel estimation

In this subsection, we discuss the important issue of how the decoder obtains estimates of $\mathbf{h}_d$ and z_d in the d^{th} dwell. Assume that a header of T_p pilot symbols, selected from a complex PSK constellation of zero mean and unit power, and known at the RX, is transmitted at the beginning of the dwell. If this header is denoted as $\mathbf{p}_d^T$, then, similarly to (3.1), the received pilot matrix is

$$\mathbf{Y}_d = \mathbf{h}_d \mathbf{p}_d^T + \mathbf{W}_d \quad (3.42)$$

where $\mathbf{W}_d = R^{b/2} \sum_{i \in \Pi} e_{d,i} r_i^{-b/2} e^{i\phi_i} \mathbf{h}_{d,i} \mathbf{p}_{d,i}^T$ and $[\mathbf{W}_d]_{n,t}$, $n = 1, \dots, N$, $t = 1, \dots, T_p$ are i.i.d. with $[\mathbf{W}_d]_{n,t} \sim \mathcal{CN}(0, z_d)$, given z_d .

An estimate of z_d can be obtained by

$$\hat{z} = \frac{1}{N(T_p - 1)} \left\| \left(\mathbf{I}_T - \frac{1}{T_p} \mathbf{p} \mathbf{p}^H \right) \mathbf{Y}^T \right\|^2, \quad (3.43)$$

where, for convenience, we have removed the subscript d . This estimate is obtained by finding jointly the maximum-likelihood (ML) estimators of $\mathbf{h}$ and z (p.182, [55]), and multiplying the latter by the factor $T_p/(T_p - 1)$ in order to remove the bias. Intuitively, the estimate of the interference power is obtained by projecting the received signal onto the subspace which is orthogonal to the pilot data $\mathbf{p}$. Assuming that the estimate of z is accurate, i.e., $\hat{z} \approx z$, the minimum mean square error (MMSE) estimate of $\mathbf{h}$ is ([55], p.391)

$$\hat{\mathbf{h}} = \frac{1}{z + T_p} \mathbf{Y} \mathbf{p}^* \quad (3.44)$$

and, defining $\boldsymbol{\epsilon} = \mathbf{h} - \hat{\mathbf{h}}$, it holds that $\hat{\mathbf{h}} \sim \mathcal{CN}\left(\mathbf{0}, \frac{T_p}{z + T_p} \mathbf{I}_N\right)$, $\boldsymbol{\epsilon} \sim \mathcal{CN}\left(\mathbf{0}, \frac{z}{z + T_p} \mathbf{I}_N\right)$ and $\hat{\mathbf{h}}, \boldsymbol{\epsilon}$ are independent. We can see that the estimate of $\mathbf{h}$ is accurate if $T_p \gg z$.

Following the estimation of $\mathbf{h}$ and z , the RX performs maximal ratio combining, i.e., it evaluates the product $\frac{\hat{\mathbf{h}}^H}{\|\hat{\mathbf{h}}\|^2} \mathbf{Y}$. From (3.1), we have

$$\frac{\hat{\mathbf{h}}^H}{\|\hat{\mathbf{h}}\|^2} \mathbf{Y} = \mathbf{x}^T + \frac{\hat{\mathbf{h}}^H}{\|\hat{\mathbf{h}}\|^2} (\boldsymbol{\epsilon} \mathbf{x}^T + \mathbf{W}). \quad (3.45)$$

Lumping the channel estimation error term with the interference, we can show that the equivalent channel model follows (3.3), with the SIR defined as

$$\gamma_{k,\text{csi}} = \frac{a_k}{z_k} \left(1 + \frac{1 + z_k}{T_p} \right)^{-1}. \quad (3.46)$$

The sequence $\{y_k, \gamma_{k,\text{csi}}\}_{k=1}^{DT_d}$ is the input to decoder (3.6).

A consequence of channel estimation is the loss of information rate by a factor $\frac{T_d}{T_d+T_p}$, due to the transmission of the pilot symbols in each dwell. This factor must be taken into account when evaluating the transmission capacity.

3.4.2 Correlation of the interference

Eq. (3.8) is based on the assumption that the coded bits encounter independent SIR conditions across the span of an error event in the decoder. The assumption is justified if, (a) there is a sufficient number of dwells¹⁰ such that, due to interleaving, these bits will be transmitted on different dwells, and (b) the SIRs are independent across dwells. Given that the coherence bandwidth of the channel is equal to the width of a subband, the latter assumption is reasonable if the number of frequencies is sufficiently large. As shown in [56], the temporal correlation of the interference in a fixed Poisson network of transmitters and Rayleigh fading is $p/2$, where p is the random access probability. In this thesis, random access is achieved via random frequency hopping, i.e., p can be substituted by $1/M$.

3.4.3 Synchronization

The assumption of synchronization of different TX-RX pairs in the network at the dwell level results in a worst-case scenario in terms of the interference power level. On the other hand, it also results in a best-case scenario in terms of the correspondence of the estimated interference level to that encountered in the remainder of the dwell. In this respect, asynchronous transmissions may require the insertion of more pilot symbols throughout the dwell and, consequently, the loss of rate. The mismatch between the actual and estimated interference power will incur a performance loss, the study of which is beyond the scope of this thesis.

3.4.4 Power control

In [21], it was shown that channel inversion PC increases the outage probability in a random ad hoc network with fading. In order to examine the effect of PC in the context of this chapter, we assume that the typical TX-RX link is subject to lognormal shadowing, i.e., the transmitted signal is multiplied by a r.v. $S = 10^{\sigma_s \xi/10}$, where

¹⁰This is also related to the length of the codeword.

$\xi \sim \mathcal{N}(0, 1)$ and, typically, $\sigma_s = 6 - 8$ dB. Note that S (like the node locations) is assume to be a “long-term” random variable, i.e., it is constant for at least the duration of a packet slot.

In order to take shadowing into account in the analysis of Section 3.2, the definition of the constant δ in (3.5) for the case of channel inversion PC must be modified as $\delta_{\text{pc}} = \pi\Gamma(1 - \alpha)R^2\mathbb{E}[(S^\alpha S'^{-\alpha}]$, where S models the shadowing between TX_i and RX_0 and S' models the shadowing between TX_i and its corresponding RX. In the absence of PC, we have $\delta_{\text{npc}} = \pi\Gamma(1 - \alpha)R^2\mathbb{E}[S^\alpha]S'^{-\alpha}$, where S' now denotes the shadowing between TX_0 and RX_0 . Therefore, in the case of no PC, P_l in (3.8) is conditioned on the realization of S' and the expectation of the former with respect to the latter must be taken in order to obtain the unconditional probability of codeword/packet error.

3.5 Numerical results

In this section, we present numerical results for a network with default parameters $R = 1$ m, $M = 100$, $M_c = 2$ (4QAM), $N = 2$ and $b = 4$. Six rate $R_c = 1/2$ convolutional codes are considered, with memory, minimum Hamming distances and distance spectra listed in Table 3.2 (see [57] for more details). In all plots, the upper bound to the pairwise error probability $\bar{P}_l$ is evaluated by (3.23). We also define the parameter $N_{\text{eff}} = \frac{\lambda\pi R^2}{M}$, i.e., the expected number of TXs in the transmission range per subband, as a measure of the interference level in the network.

In the simulations, we generate a new network realization for every transmitted packet. At the TX, we employ a block interleaver of vertical dimension equal to the number of bits per dwell $M_c T_d$ and horizontal dimension equal to D , such that $M_c T_d$ consecutive codeword bits are guaranteed to be transmitted on different dwells. The coherence time of the fading is taken to be equal to the duration of a packet. In this manner, we can compare the MA scheme proposed with the slow FH scheme advocated in [20].

In Fig. 3.1, we plot the FEP vs. N_{eff}^{-1} for the codes listed in Table 3.2 and $L_b = 800$. The upper bound (3.9) is plotted with a solid line, when error events up to length $L + 4$ are taken into account, and with a dotted line, when only the Hamming distance error events are taken into account. As expected, for each code, the two curves converge as $N_{\text{eff}} \rightarrow 0$. We also simulate the performance of the codes by dividing the packet in $D = 40$ dwells of $T_d = 20$ 4QAM symbols. Fig. 3.1 demonstrates that the dotted

Table 3.2: Optimum rate 1/2 convolutional codes in Chapter 3

Encoder	Memory	L	$w_L, \dots, w_{L+4}$
Enc1	1	3	1,1,1,1,1
Enc2	2	4	2,0,5,0,13
Enc3	3	5	1,0,6,0,16
Enc4	3	6	1,3,5,11,25
Enc5	5	8	2,7,10,18,49
Enc6	7	10	1,6,13,20,64

curve is quite accurate for $P_e < 0.01$ and Enc1-Enc5, however it is slightly optimistic in the case of Enc6. This is attributed to the impact of channel correlation within the packet the effect of which becomes more apparent as the code Hamming distance (hence the span of the error events) becomes larger.

In Fig. 3.2, we plot the simulated FEP vs. N_{eff}^{-1} , for Enc4 and decreasing values of the number of subbands M . As M decreases the diversity order decreases, as the probability that the same fading condition and/or the same interferer are encountered across dwells increases (and the assumption of independent channel conditions thus becomes obsolete). The simulated P_e in the case of slow FH is also shown for comparison; note that even for $M = 4$, the gain of the scheme considered compared to slow FH is still considerable, as the diversity order of the latter is only one.

In Fig. 3.3, we plot the maximum contention density λ_e , evaluated by (3.39), vs. L , for the set of parameters of Fig. 3.1 and different values of the ratio ϵ/w_L . Note that, for $\epsilon/w_L = 0.001$, increasing the diversity order of the code from 3 to 8, results in a tenfold increase of λ_e . The gain comes at an increase of the decoder memory from 1 to 5.

Fig. 3.4 presents the results of Proposition 2 on the link ergodic capacity for two different values of N . As expected, (3.31) and (3.32) become tight as $N_{\text{eff}} \rightarrow \infty$ and $N_{\text{eff}} \rightarrow 0$, respectively.

In Fig. 3.5, we employ the channel estimation scheme of Section 3.4 and examine the effect of imperfect CSI via simulation, when $L_b = 500$, $D = 25$ and $T_d = 20$. For $T_p = 5$ and a rate-loss factor of 20% there is a 1 dB penalty compared to the perfect-CSI case. As T_p is increased the performance loss is reduced, at the expense of information rate, e.g., at $T_p = 10$, the rate-loss factor is 2/3.

Finally, in Fig. 3.6, the simulated FEP is plotted vs. N_{eff}^{-1} for Enc2 and Enc4,

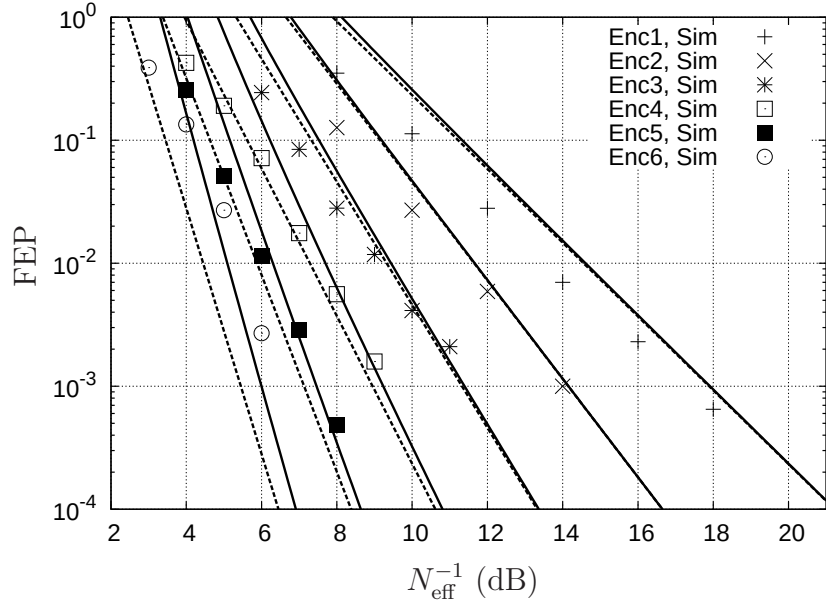


Figure 3.1: FEP vs. N_{eff}^{-1} for the codes in Table 3.2. The solid (dotted) lines depict (3.9) with error events up to length $L + 4$ (L) taken into account (4QAM, $L_b = 800$, $D = 40$, $T_d = 20$, $N = 2$, $b = 4$, perfect CSI).

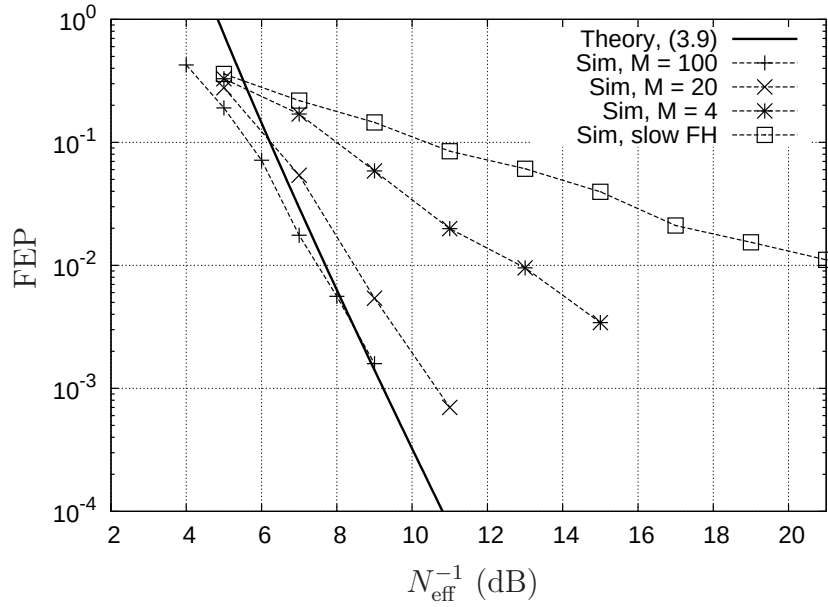


Figure 3.2: FEP vs. N_{eff}^{-1} for Enc4 and $M = 4, 20, 100$. The simulated performance for slow FH is also shown for comparison. (4QAM, $L_b = 800$, $D = 40$, $T_d = 20$, $N = 2$, $b = 4$, perfect CSI).

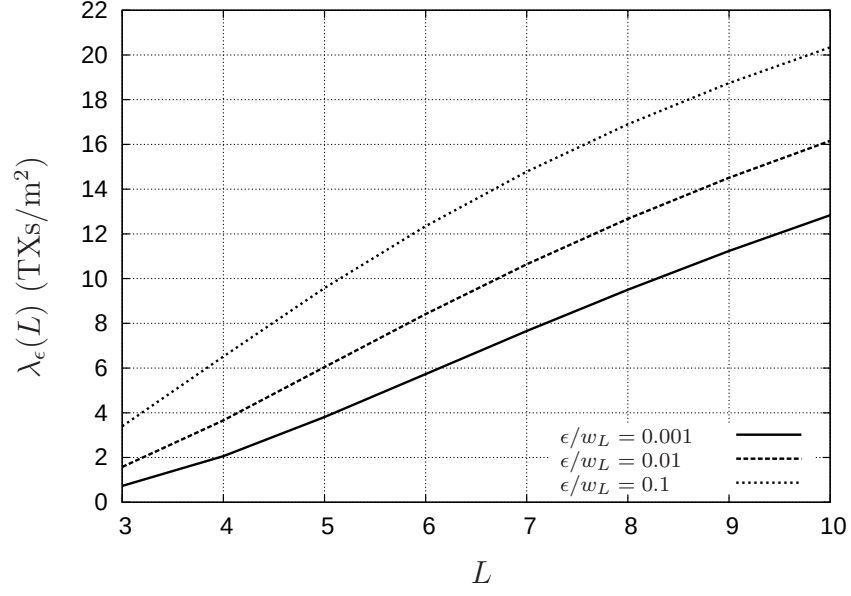


Figure 3.3: Maximum contention density vs. L for different values of ϵ/w_L ($R = 1$ m, 4QAM, $L_b = 800$, $N = 2$, $b = 4$, perfect CSI).

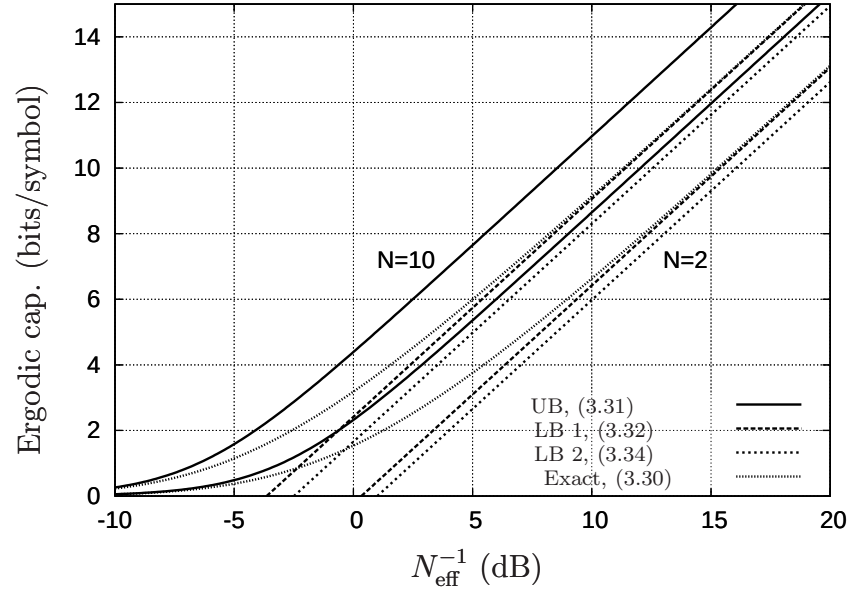


Figure 3.4: Link ergodic capacity vs. N_{eff}^{-1} . The two groups of curves correspond to $N = 2, 10$ ($b = 4$). UB and LB stand for Upper Bound and Lower Bound respectively.

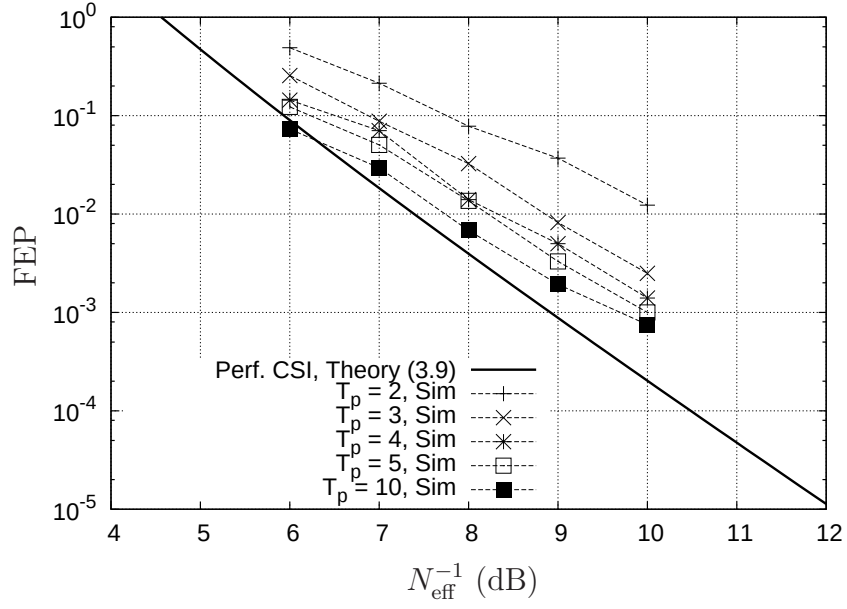


Figure 3.5: FEP vs. N_{eff}^{-1} for Enc1 and different values of T_p (4QAM, $L_b = 500$, $D = 25$, $T_d = 20$, $N = 2$, $b = 4$).

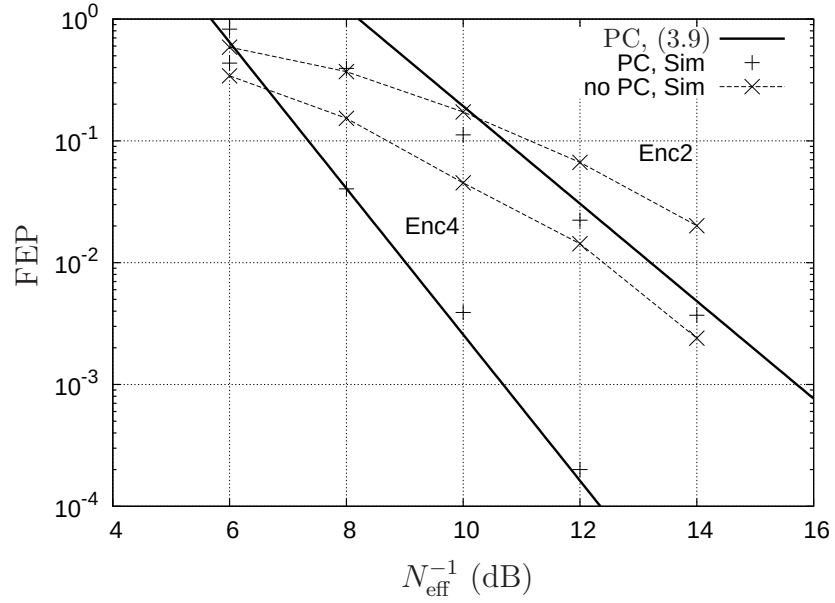


Figure 3.6: FEP vs. N_{eff}^{-1} for Enc2 and Enc4 with/without PC (4QAM, $L_b = 500$, $D = 25$, $T_d = 20$, $N = 2$, $b = 4$, perfect CSI, $\sigma_s = 6$).

for a channel with lognormal shadowing ($\sigma_s = 6$ dB) and shadowing-inversion PC or no PC. For small N_{eff} , PC introduces a substantial gain, e.g., for Enc4 at $N_{\text{eff}} = 10$ dB, this gain is an order of magnitude. The reason for this is that the detrimental effect of shadowing to RX_0 is canceled and, at the same time, the interferer process is sparse enough such that the code protects RX_0 in the unlikely event of a large interfering power from a nearby interferer¹¹. Theoretically, we can see this with the help of Jensen's inequality; given that all shadowing variables are i.i.d., by the definitions of δ_{pc} and δ_{npc} in Section 3.4.4, we have that $\delta_{\text{pc}}^l = \mathbb{E}[S^\alpha]^l \mathbb{E}[S^\alpha]^l < \mathbb{E}[S^{\alpha l}] \mathbb{E}[S^\alpha]^l = \delta_{\text{npc}}^l$. Hence, from Proposition 1, PC should perform better in the small FEP regime, which is verified by Fig. 3.6. On the other hand, as the network becomes very dense, the diversity in the received signal is lost and the decoder is overwhelmed by an increased interference level due to PC. This is more apparent for Enc2 than Enc4, since Enc2 has a smaller Hamming distance. On a final note, in the case of PC, we observe that there is good agreement between (3.9) and the simulation results.

3.6 Concluding remarks on Chapter 3

In this chapter we considered FH during packet transmission and coding, as a physical layer scheme for random wireless networks with uncoordinated transmissions. We demonstrated via analysis and simulation that the transmission capacity scales as $\epsilon^{1/L}$, where ϵ is the constraint placed on the packet error probability and L is the code diversity order. A byproduct of our analysis was the derivation of a compact expression for the pdf of the SIR in a Rayleigh fading and α -stable interference channel, when the RX performs MRC. Upper and lower bounds on the ergodic capacity of this channel were also derived.

Employing a simple channel estimation algorithm based on the transmission of pilot symbols at the beginning of each dwell, we showed that the performance degradation due to imperfect CSI is reasonable, at a rate loss of the order of 20%. The effect of channel inversion PC was also confirmed for a channel with lognormal shadowing and PC was shown to be beneficial if λ/M is sufficiently small. In conclusion, we believe that, given the gains in terms of network capacity at moderate encoding/decoding complexity, even for a small number of subbands, this scheme merits consideration despite the increased overhead compared to a slow FH system.

¹¹This occurs when deep shadowing afflicts the channel between that interferer and its respective RX.

Chapter 3, in part, is a reprint of the paper “Assessing the impact of physical layer techniques on ad hoc network performance”, Kostas Stamatiou and John G. Proakis, as it appeared in the Elsevier Journal of Physical Communication, March 2008. Chapter 3, in part, has also been submitted for publication to the IEEE Transactions on Wireless Communications, under the title “Channel diversity in random wireless networks”, Kostas Stamatiou, John G. Proakis and James R. Zeidler, May 2009.

Chapter 4

Evaluation of MIMO techniques in random single-hop networks

In this chapter, we consider a single-hop random network of MIMO links. We employ the same geometric model as in Chapter 3, i.e., the locations of the transmitters are drawn independently according to a PPP. However, we now assume that the channel is constant during the transmission of a packet and employ an outage-based model for packet successes, i.e., we assume that a packet is successfully received if the channel mutual information over a packet slot is greater than the information rate¹.

The outage performance of single-user MIMO techniques for a channel with fading and noise is well understood [30]. Our objective is to shed light on the performance of some of these techniques in the interference environment of a random network. We first consider single-stream transmission and derive an expression for the outage/success² probability in a Poisson field of interferers. It is shown that N receive antennas provide an approximate gain of $N^{-2/b}$ in terms of spatial contention, i.e., the rate of increase of the outage probability as a function of the transmitter density, when the latter is zero. This result provides an alternative interpretation to the scaling law derived in [22]. We then turn our attention to multiple-stream transmission and derive exact expressions and approximations to the outage probability for different detection techniques at the receiver, namely ZF, ZF-SIC (or VBLAST) and DBLAST. The optimal number of streams

¹Note that, assuming that each packet corresponds to a codeword, this implies a large codeword length.

²The terms “outage” and “success” probability, since complementary, are used interchangeably throughout the chapter.

such that the transmission capacity of the network is maximized is determined for each of these techniques in the small outage probability regime. For DBLAST specifically, it is shown that, for $b \geq 4$, it is optimal to use all transmit antennas, while, for $b < 4$, the number of streams must be judiciously chosen such that the optimal trade-off between network interference and rate increase is achieved. Numerical results showcase that the benefit of DBLAST over VBLAST is significant in terms of the transmission capacity. In both cases, the transmission capacity scales linearly in the number of antennas.

4.1 System Model

As in Chapter 3, assume that the TXs are distributed on the plane according to a homogeneous PPP of density λ and each TX has a corresponding RX at distance R . Time is slotted and transmissions take place concurrently and in a synchronized manner during each slot. The channel between each TX-RX pair over a subband consists of constant flat Rayleigh fading with a coherence time of one slot and path-loss according to the law r^{-b} . The power from each antenna is the same across all TXs and equal to one. Generally, there is a different number of antennas at the TX and the RX; however, for convenience, we assume that N antennas are available at both the TX and the RX.³

Suppose that M antennas are employed for transmission, with $M \leq N$. According to (1.3), the received vector at the typical RX can be written as

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{w}, \quad (4.1)$$

where $\mathbf{H}$ is the $N \times M$ channel matrix between TX and RX, with i.i.d. elements $[\mathbf{H}]_{nm} \sim \mathcal{CN}(0, 1)$; $\mathbf{x} \sim \mathcal{CN}(\mathbf{0}, \mathbf{I}_M)$ is the $M \times 1$ symbol vector transmitted by TX; R_i is the distance between interfering transmitter TX_i and RX; Π is a the PPP of transmitters; and $\mathbf{w}$ is the interference term, modeled as $\mathbf{w} \sim \mathcal{CN}(\mathbf{0}, z\mathbf{I}_N)$, where $z = MR^b \sum_{i \in \Pi} R_i^{-b}$ is the total interference power over a given slot, per RX antenna⁴. As noted in Chapter 3, z is an α -stable random variable with stability exponent $\alpha = 2/b$. Its mgf is therefore given by (3.4) which is repeated here for convenience

$$\Phi_z(s) = \mathbb{E}[e^{-sz}] = e^{-cs^\alpha}, \quad s > 0, \quad (4.2)$$

³This assumption is not unrealistic in an ad hoc network, where a node can be a TX and a RX interchangeably throughout time.

⁴Note that, taking into account the fading from an interferer to a typical RX, the interference is generally correlated across the RX antennas. Assuming the interference is uncorrelated represents a worst-case scenario, which simplifies the subsequent analysis.

where the parameter c is defined as $c \triangleq \lambda\pi R^2\Gamma(1-\alpha)M^\alpha$.

4.2 Single-stream transmission ($M = 1$)

We first consider the transmission of a single stream, i.e., $M = 1$. Defining the desirable information rate as $\mathcal{R} = \log(1 + \theta)$, where θ is an appropriate signal-to-interference-ratio (SIR) threshold, the success probability corresponding to (4.1) is given by [30]

$$\begin{aligned} P_s &= \mathbb{P}\left(\log\left(1 + \frac{a}{z}\right) > \mathcal{R}\right) \\ &= \mathbb{P}\left(\frac{a}{z} > \theta\right), \end{aligned} \quad (4.3)$$

where a is chi-square distributed with $2N$ degrees of freedom, i.e., $a \sim \chi_{2N}^2$. The respective success probability is defined as $P_s = 1 - P_o$.

We now follow a similar procedure to Chapter 3 in deriving the ccdf of the SIR $\gamma = a/z$, $\bar{F}_\gamma(\gamma)$. We have

$$\bar{F}_\gamma(\gamma) = \mathbb{P}(a > \gamma z) = \int_0^{+\infty} \bar{F}_a(\gamma z) f_z(z) dz \quad (4.4)$$

where $\bar{F}_a(a)$ is the ccdf of a and $F_a(a)$ is defined in (3.13). Substituting (3.13) in (4.4), we obtain

$$\bar{F}_\gamma(\gamma) = \Phi_z(\gamma) + \sum_{n=1}^{N-1} \frac{\gamma^n}{n!} \int_0^{+\infty} z^n f_z(z) e^{-\gamma z} dz$$

where $\Phi_z(\gamma)$ is the mgf of γ . From the Laplace transform property in (3.16), it follows that

$$\bar{F}_\gamma(\gamma) = \Phi_z(\gamma) + \sum_{n=1}^{N-1} \frac{\gamma^n}{n!} (-1)^n \frac{d^n \Phi_z(\gamma)}{d\gamma^n}. \quad (4.5)$$

Recalling (3.17) of Chapter 3, it holds that

$$\frac{d^n \Phi_z(\gamma)}{d\gamma^n} = \gamma^{-n} e^{-c\gamma^\alpha} \sum_{k=1}^n \frac{\beta_k^n}{k!} (c\gamma^\alpha)^k \quad (4.6)$$

where β_k^n , $k = 1, \dots, n$ is defined in (3.11). Substituting (4.6) in (4.5) and regrouping terms results in

$$\begin{aligned}
\bar{F}_\gamma(\gamma) &= e^{-c\gamma^\alpha} + e^{-c\gamma^\alpha} \sum_{n=1}^{N-1} \frac{1}{n!} \sum_{k=1}^n \frac{(-1)^n \beta_k^n}{k!} (c\gamma^\alpha)^k \\
&= e^{-c\gamma^\alpha} + e^{-c\gamma^\alpha} \sum_{k=1}^{N-1} \frac{(c\gamma^\alpha)^k}{k!} \sum_{n=k}^{N-1} \frac{(-1)^n \beta_k^n}{n!}.
\end{aligned} \tag{4.7}$$

By the definition of P_s in (4.3), we have that $P_s = \bar{F}_\gamma(\theta)$ or

$$P_s = e^{-c\theta^\alpha} + e^{-c\theta^\alpha} \sum_{k=1}^{N-1} \frac{(c\theta^\alpha)^k}{k!} \sum_{n=k}^{N-1} \frac{|\beta_k^n|}{n!} \tag{4.8}$$

where we have used the property - proved in Lemma 1 of Chapter 3 - that $(-1)^n \beta_k^n \geq 0$. From (4.8), we can see that P_s is a product of the term $e^{-c\theta^\alpha}$ (the success probability for $N = 1$) and a polynomial in $c\theta^\alpha$ of degree $N - 1$ and non-negative coefficients. Clearly, increasing the number of antennas N , increases the success probability as more positive terms are added to the polynomial.

In order to obtain more insight into the effect of $N > 1$ on the success probability, we evaluate the *spatial contention parameter*

$$\eta = - \left. \frac{\partial P_s}{\partial \lambda} \right|_{\lambda=0}, \tag{4.9}$$

defined in [52] for single-antenna networks as the slope of the outage probability as a function of the density λ , at $\lambda = 0$. The larger η is, the sharper the increase of the outage probability as λ increases. Denoting $c' = c\theta^\alpha/\lambda$, from (4.8) we have that

$$\begin{aligned}
\eta &= c' - c' \sum_{n=1}^{N-1} \frac{|\beta_1^n|}{n!} \\
&= c' + c' \sum_{n=1}^{N-1} \frac{(-1)^n (\alpha)_n}{n!} \\
&= c' \sum_{n=0}^{N-1} \frac{(-1)^n (\alpha)_n}{n!} \quad | \quad (\alpha)_0 \triangleq 1 \\
&= c' \frac{(-1)^{N-1}}{(N-1)!} \sum_{n=0}^{N-1} \binom{N-1}{n} (-1)^{N-1-n} (\alpha)_n \\
&= c' \frac{(-1)^{N-1}}{(N-1)!} (\alpha-1)_{N-1}
\end{aligned} \tag{4.10}$$

$$= c' \frac{\Gamma(N - \alpha)}{\Gamma(N)\Gamma(1 - \alpha)} \quad (4.11)$$

where (4.10) stems from the binomial identity for falling sequential products and (4.11) is the result of the successive application of the gamma function property $\Gamma(x+1) = x\Gamma(x)$. As N increases, in (3.29) we showed that $\frac{\Gamma(N-\alpha)}{\Gamma(N)} \simeq N^{-\alpha}$ therefore $\eta \simeq \pi R^2 \theta^\alpha N^{-\alpha}$. It is seen that a spatial diversity order N provides an approximate gain of $N^{-\alpha}$ in terms of spatial contention. This result provides an alternative interpretation to the coding gain factor $N^{-L\alpha}$ in Chapter 3. In simpler terms, the spatial diversity at the RX roughly decreases the SIR threshold θ by a factor N .

The following lemma provides an upper bound on P_s which is tight as $\lambda \rightarrow 0$.

Lemma 1. *For single-stream transmission, P_s is upper-bounded by*

$$\begin{aligned} P_u(\lambda, N) &= \exp\left(-\frac{\Gamma(N - \alpha)}{\Gamma(N)\Gamma(1 - \alpha)} c \theta^\alpha\right) \\ &= \exp(-\eta \lambda), \end{aligned} \quad (4.12)$$

where the equality holds for $N = 1$. Furthermore, $P_u(\lambda, N) \simeq P_s$, for $\lambda \rightarrow 0$.

Proof. Since for $N = 1$, $P_s = e^{-c\theta^\alpha}$, (4.8) holds as an equality for $N = 1$. For $N > 1$, we take the Taylor series expansion of $P_u(\lambda, N)$ over λ and compare individual terms with (4.8). It can be seen that it suffices to prove that

$$A_1 \triangleq \sum_{n=k}^{N-1} \frac{(-1)^n \beta_k^n}{n!} \leq \left(\sum_{n=1}^{N-1} \frac{(-1)^n \beta_1^n}{n!} \right)^k \triangleq A_2, \quad (4.13)$$

with $k = 1, \dots, N - 1$. If $k = 1$, (4.13) holds as an equality. For $k > 1$, it holds that

$$A_2 = \sum_{n_1=1}^{N-1} \cdots \sum_{n_k=1}^{N-1} \delta_1^{n_1} \cdots \delta_1^{n_k} \quad (4.14)$$

where, for convenience, we have defined $\delta_k^n = \frac{(-1)^n \beta_k^n}{n!}$. Moreover, by (3.21), we have that

$$\frac{(-1)^n \beta_k^n}{n!} = \sum_{m_1=1}^n \sum_{m_2=1}^{n-m_1} \cdots \sum_{m_{k-1}=1}^{n-m_{k-2}-\cdots-m_1} \delta_1^{m_1} \delta_1^{m_2} \cdots \delta_1^{m_k} \quad (4.15)$$

where $m_k = n - m_{k-1} - \cdots - m_1$. Substituting (4.15) and (4.14) in (4.13), we can see that (4.13) is a true statement. This is due to the fact that the summation that gives A_1 is over a subset of the terms that are summed to give A_2 .

Finally, by the definition of η , for $\lambda \rightarrow 0$, $P_u(\lambda, N) \simeq 1 - \eta\lambda \simeq P_s$.

□

As a result of Lemma 1, (4.12) can be used as an approximation to P_s in the small outage probability regime.

We now utilize these results in evaluating the transmission capacity of the network, defined as the maximum network throughput per unit area, such that a constraint $P_s \geq 1 - \epsilon$ is satisfied [21], i.e.,

$$\text{TC}_\epsilon = \lambda_\epsilon(1 - \epsilon)\mathcal{R}, \quad (4.16)$$

where the *maximum contention density* λ_ϵ is determined by the constraint $P_s = 1 - \epsilon$. In the small outage probability regime, e.g., typically, $\epsilon \leq 0.1$, we can invoke Lemma 1 to determine an approximation to λ_ϵ , i.e.,

$$\begin{aligned} P_u(\lambda_\epsilon, N) &\approx 1 - \epsilon \\ \exp\left(-\frac{\Gamma(N - \alpha)}{\Gamma(N)}\lambda_\epsilon\pi R^2\theta^\alpha\right) &\approx 1 - \epsilon \\ \lambda_\epsilon &\approx -\frac{\log(1 - \epsilon)}{\pi R^2\theta^\alpha} \frac{\Gamma(N)}{\Gamma(N - \alpha)}. \end{aligned} \quad (4.17)$$

From (4.16) and (4.17), an approximation to the transmission capacity is thus

$$\text{TC}_\epsilon \approx \log(1 + \theta) \frac{(\epsilon - 1)\log(1 - \epsilon)}{\pi R^2\theta^\alpha} \cdot \frac{\Gamma(N)}{\Gamma(N - \alpha)}. \quad (4.18)$$

As seen by (4.18), the transmission capacity of a single-stream system scales as N^α in the number of RX antennas N .

4.3 Multiple stream transmission ($M > 1$)

4.3.1 ZF and ZF-SIC (VBLAST)

Consider the case $M > 1$ and assume that each packet is transmitted over the same antenna during a slot with a rate $\mathcal{R} = \log(1 + \theta)$. If ZF is employed at the RX, the success probability for each stream, P_s^{zf} , is also given by (4.3), with the difference that a is now chi-square distributed with $2(N - M + 1)$ degrees of freedom [30] and $M > 1$ streams are transmitted from each node. As a result, invoking (4.8),

$$P_s^{\text{zf}} = e^{-c\theta^\alpha} + e^{-c\theta^\alpha} \sum_{k=1}^{N-M} \frac{(c\theta^\alpha)^k}{k!} \sum_{n=k}^{N-M} \frac{|\beta_k^n|}{n!}. \quad (4.19)$$

From Lemma 1, we also have that $P_s^{\text{zf}} \leq P_u(\lambda, N - M + 1)$. The transmission capacity is now defined as

$$\text{TC}_\epsilon^{\text{zf}} = \lambda_\epsilon^{\text{zf}} (1 - \epsilon) M \mathcal{R}, \quad (4.20)$$

where the maximum contention density $\lambda_\epsilon^{\text{zf}}$, for small ϵ , is determined by the constraint $P_u(\lambda_\epsilon^{\text{zf}}, N - M + 1) \approx 1 - \epsilon$ or

$$\lambda_\epsilon^{\text{zf}} \approx -\frac{\log(1 - \epsilon)}{\pi R^2 \theta^\alpha} \frac{\Gamma(N - M + 1)}{\Gamma(N - M + 1 - \alpha) M^\alpha}. \quad (4.21)$$

The corresponding transmission capacity is given by

$$\text{TC}_\epsilon^{\text{zf}} \approx \log(1 + \theta) \frac{(\epsilon - 1) \log(1 - \epsilon)}{\pi R^2 \theta^\alpha} \cdot \frac{\Gamma(N - M + 1) M^{1-\alpha}}{\Gamma(N - M + 1 - \alpha)} \quad (4.22)$$

which indicates that the transmission capacity scales as $(N - M + 1)^\alpha M^{1-\alpha}$. This expression can be optimized⁵ over the number of streams M by setting the following derivative to zero

$$\frac{\partial}{\partial M} (N - M + 1)^\alpha M^{1-\alpha} = 0. \quad (4.23)$$

After simple manipulations, we obtain $M_o^{\text{zf}} = (1 - \alpha)(N + 1)$. Since the constraints $M_o^{\text{zf}} \leq N$ and $M_o^{\text{zf}} \in \mathcal{Z}^+$ must also be satisfied, the optimal number of streams is

$$M_o^{\text{zf}} = \min \{ \lceil (1 - \alpha)(N + 1) \rceil, N \} \quad (4.24)$$

where, with a slight abuse of notation $\lceil (1 - \alpha)(N + 1) \rceil$ denotes the closest integer number of streams to $(1 - \alpha)(N + 1)$ that maximizes $\text{TC}_\epsilon^{\text{zf}}$. Note that $(1 - \alpha)(N + 1) \leq N$ holds if and only if $\alpha \geq 1/(N + 1)$ or $b \leq 2(N + 1)$. Equivalently, given N , for a propagation exponent $b > 2(N + 1)$ it is optimal to transmit packets from all antennas. Also, by setting $M = M_o^{\text{zf}}$ in (4.22), we can verify that TC^{zf} scales linearly in N .

Suppose now that the RX employs ZF-SIC, i.e., it cancels out each packet that has already been decoded. The spatial diversity order corresponding to the “worst”

⁵Note that we can also optimize over the SIR threshold θ as in [52].

packet, i.e., the packet that is decoded first is $N - M + 1$. Given an outage constraint ϵ on this worst stream, the maximum contention density is also given by (4.21). Assuming that perfect interference cancelation takes place, i.e., there is no error propagation, the transmission capacity of ZF-SIC or VBLAST is given by

$$\text{TC}_\epsilon^{\text{vb}} \approx \lambda_\epsilon^{\text{zf}} \log(1 + \theta) \sum_{m=1}^M P_u(\lambda_\epsilon^{\text{zf}}, N - m + 1). \quad (4.25)$$

The summation term in (4.25) is the total throughput of all transmitted streams, with corresponding diversity orders $N - M + 1, \dots, N$, ordered from the worst to the best.

4.3.2 DBLAST

In the two previously examined schemes, a packet is transmitted on the same antenna for the duration of a slot, thereby experiencing the same fading conditions across the slot. In DBLAST [30, 58], a packet is separated into segments which are transmitted across the antennas and time, such that each segment experiences different fading conditions. The segments are then detected at the RX by ZF-SIC and, once a packet is decoded, its contribution to the received signal is subtracted. It is known that DBLAST, in conjunction with appropriate coding, approaches⁶ the outage performance of the MIMO Rayleigh channel [30], i.e., for a total transmission rate $M\mathcal{R}$, the outage probability is given by

$$P_o^{\text{db}} = \text{P} \left(\log \det \left(\mathbf{I}_N + \frac{1}{z} \mathbf{H} \mathbf{H}^H \right) < M\mathcal{R} \right). \quad (4.26)$$

In the system model we are investigating, this probability has to be evaluated over the distributions of $\mathbf{H}$ and z . A way to approach this evaluation analytically is to recall that P_o^{db} may be upper-bounded as [58]

$$\begin{aligned} P_o^{\text{db}} &\leq \text{P} \left(\sum_{m=1}^M \log \left(1 + \frac{a_m}{z} \right) < M\mathcal{R} \right) \\ &= \text{P} \left(\prod_{m=1}^M \left(1 + \frac{a_m}{z} \right)^{\frac{1}{M}} < 1 + \theta \right) \end{aligned} \quad (4.27)$$

where $\{a_m\}$ are independent chi-square random variables with $a_m \sim \chi_{2(N-m+1)}^2$. We observe that, for reasonable values of θ (e.g., $\theta = 5 - 20$ dB), an outage roughly occurs

⁶In practice, DBLAST suffers from a rate loss due to initialization.

when the interference power z obtains a large value. In this case, the geometric mean of $\{1 + \frac{a_m}{z}\}_{m=1}^M$ is approximately equal to the arithmetic mean, thus

$$\mathrm{P} \left(\prod_{m=1}^M \left(1 + \frac{a_m}{z} \right)^{\frac{1}{M}} < 1 + \theta \right) \approx \mathrm{P} \left(\frac{1}{z} \sum_{m=1}^M a_m < M\theta \right). \quad (4.28)$$

Moreover, $\sum_{m=1}^M a_m \sim \chi_{2N_{\text{tot}}}^2$, where $N_{\text{tot}} = \sum_{m=1}^M (N - m + 1)$. As a result, an approximation to the success probability for DBLAST, P_s^{db} , can be obtained from (4.8) as follows

$$P_s^{\text{db}} \approx e^{-c(M\theta)^\alpha} + e^{-c(M\theta)^\alpha} \sum_{k=1}^{N_{\text{tot}}-1} \frac{(c(M\theta)^\alpha)^k}{k!} \sum_{n=k}^{N_{\text{tot}}-1} \frac{|\beta_k^n|}{n!}. \quad (4.29)$$

By Lemma 1, for $\lambda \rightarrow 0$, $P_s^{\text{db}} \approx P_u(\lambda, N_{\text{tot}})$, so, under a constraint $P_s^{\text{db}} = 1 - \epsilon$, the optimal contention density for DBLAST is

$$\lambda_\epsilon^{\text{db}} \approx -\frac{\log(1 - \epsilon)}{\pi R^2 \theta^\alpha} \frac{\Gamma(N_{\text{tot}})}{\Gamma(N_{\text{tot}} - \alpha) M^{2\alpha}} \quad (4.30)$$

and the respective transmission capacity is

$$\text{TC}_\epsilon^{\text{db}} \approx \log(1 + \theta) \frac{(\epsilon - 1) \log(1 - \epsilon)}{\pi R^2 \theta^\alpha} \cdot \frac{\Gamma(N_{\text{tot}}) M^{1-2\alpha}}{\Gamma(N_{\text{tot}} - \alpha)}. \quad (4.31)$$

Since $N_{\text{tot}} = \frac{2NM - M^2 + M}{2}$, $\text{TC}_\epsilon^{\text{db}}$ scales as $(2N - M + 1)^\alpha M^{1-\alpha}$. We optimize this expression over M , i.e.,

$$\frac{\partial}{\partial M} (2N - M + 1)^\alpha M^{1-\alpha} = 0 \quad (4.32)$$

and, after some calculations, this yields $M_o^{\text{db}} = (1 - \alpha)(2N + 1)$. Under the constraints $M_o \leq N$ and $M \in \mathcal{Z}^+$, the optimal number of streams is

$$M_o^{\text{db}} = \min \{ \lceil (1 - \alpha)(2N + 1) \rceil, N \} \quad (4.33)$$

where, as in (4.24), with a slight abuse of notation $\lceil (1 - \alpha)(2N + 1) \rceil$ denotes the closest integer number to $(1 - \alpha)(2N + 1)$ that maximizes $\text{TC}_\epsilon^{\text{db}}$. Note that $(1 - \alpha)(2N + 1) \leq N$ is only possible if $\alpha \geq \frac{N+1}{2N+1} > \frac{1}{2}$. This implies that, if $b \geq 4$ (which is a typical value of b for ground propagation) transmission with all antennas maximizes the transmission capacity. Also, by setting $M = M_o^{\text{db}}$ in (4.31), we can verify that $\text{TC}_\epsilon^{\text{db}}$ scales linearly with N .

4.4 Numerical results

In this section, we consider a network with default parameter values $R = 20$ m, $b = 4$, $\theta = 6$ dB. In Fig. 4.1, we plot the theoretical - eq. (4.8) - and simulated success probability as a function of the TX density for a single-stream system with $N = 4$. The agreement of theory and simulation confirm the validity of the analysis in Section 4.2. We also plot the upper bound to the success probability given by (4.12). As shown in Lemma 1, the bound is tight for values of the success probability greater than 0.8 (or, as the TX density becomes progressively smaller).

In Fig. 4.2, the theoretical and simulated success probability of ZF and DBLAST are plotted vs. the TX density for a system with $M = 3$ streams. The agreement between theory and simulation is once again very satisfactory, which, in the case of DBLAST, confirms the validity of the approximations in Section 4.3.2.

Fig. 4.3 shows the dependence of the transmission capacity on the number of transmitted streams for the three MIMO techniques considered in Section 4.3. The number of RX antennas takes two values $N = 4, 8$, the propagation exponent is $b = 3$ and a constraint $\epsilon = 0.1$ is placed on the outage probability. The DBLAST transmission scheme results in higher transmission capacity compared to V-BLAST or simple ZF. Moreover, the gain between VBLAST and simple ZF is marginal, the reason being that, with VBLAST, the maximum contention density is still determined by the subchannel with the smallest diversity order. The optimal number of streams for DBLAST is 3 when $N = 4$, and 6 when $N = 8$. These numbers are in accordance with (4.33). In the case of ZF, the optimal number of streams is 1 when $N = 4$, and 3 when $N = 8$. These numbers are also in accordance with (4.24).

In Fig. 4.4, the propagation exponent takes the value $b = 4$. As predicted in Section 4.3, activating all the TX antennas maximizes the transmission capacity for DBLAST. In the case of VBLAST, the optimal number of streams is dictated by (4.24). Overall, in both Fig. 4.3 and Fig. 4.3, the agreement between theory and simulation is satisfactory.

In Fig. 4.5 and Fig. 4.6, the transmission capacity and the respective - optimal - number of streams are plotted vs. N for DBLAST, ZF and MRC (single-stream transmission) and an outage probability constraint $\epsilon = 0.01$. In the case of DBLAST and ZF, the transmission capacity scales linearly in N , while, in the case of MRC, it scales as $N^\alpha = \sqrt{N}$ (sublinearly). At $N = 4$, DBLAST provides a capacity gain of approximately

1.5 compared to ZF. Moreover, note that for $N \leq 3$, MRC and ZF result in approximately the same transmission capacity. Finally, in the case of ZF, if all TX antennas are employed ($M = N$), the transmission capacity is lower than MRC, which illustrates the importance of optimally selecting the number of streams.

4.5 Concluding Remarks on Chapter 4

In this chapter, we conducted a study of a multiple-antenna single-hop random network, where the locations of the transmitters are determined according to a homogeneous PPP. Assuming channel knowledge at the RX only and that interference from concurrent transmissions is regarded as noise, we evaluated the outage/success probability for single-stream transmission. We then used the results and insights from this analysis in order to evaluate the outage performance as well as the transmission capacity of MIMO techniques such as ZF, VBLAST and DBLAST. We determined the optimum number of streams such that the transmission capacity of the network is maximized in the small outage probability regime (equivalently, a regime where the density of transmitters is small) and quantified the capacity gain of DBLAST over VBLAST.

Overall, the results in this chapter shed light on how MIMO techniques, which are well understood in the single-user context, affect the capacity of a random network. At the heart of the analysis and the resulting design guidelines lies the PPP geometric model, which allows us to take into account how the randomness in the locations of the interfering TXs affects the interference power seen at the typical RX.

Chapter 4, in part, is a reprint of the paper “Evaluation of MIMO techniques in FH-MA ad hoc networks”, Kostas Stamatiou, John G. Proakis and James R. Zeidler, as it appeared in the proceedings of IEEE GLOBECOM, Washington D.C., 2007.

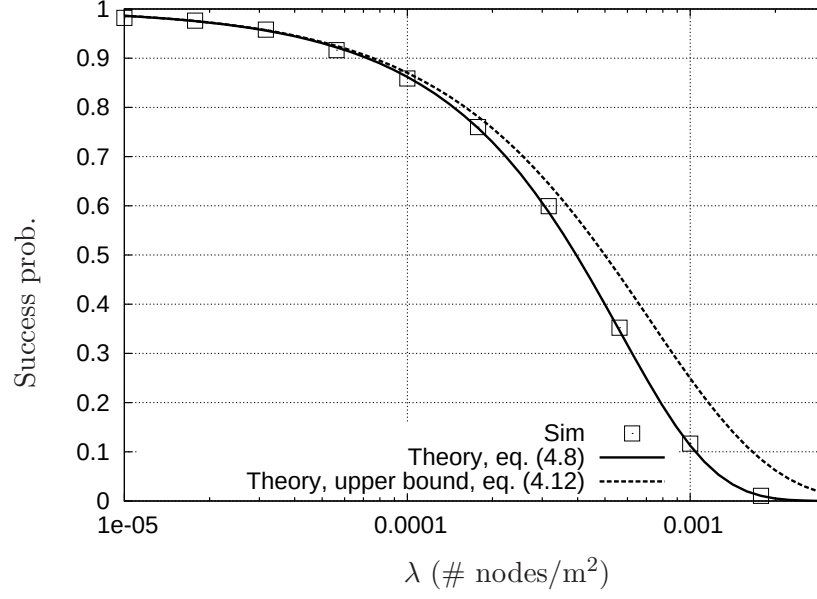


Figure 4.1: Success probability vs. λ for $N = 4$ and $M = 1$ ($R = 20$ m, $b = 4$, $\theta = 6$ dB).

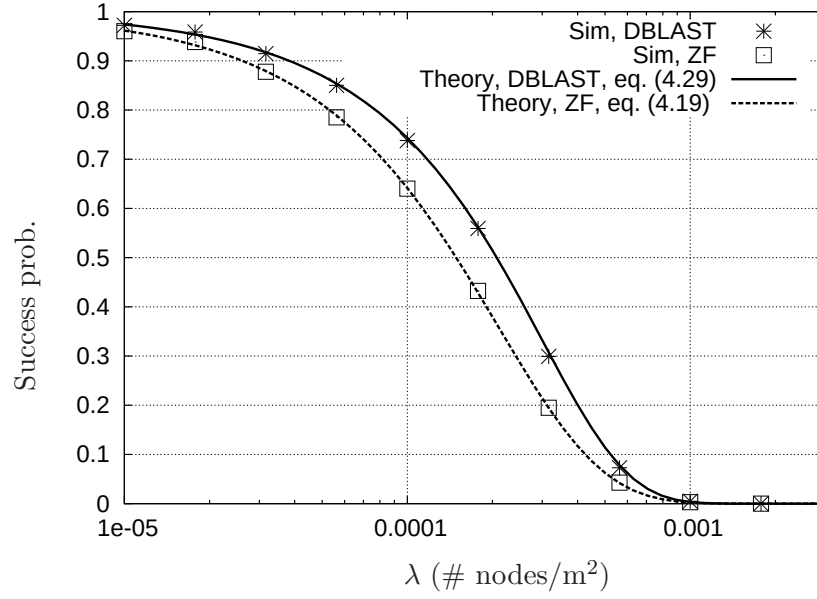


Figure 4.2: Success probability vs. λ for $N = 4$ and $M = 3$ ($R = 20$ m, $b = 4$, $\theta = 6$ dB).

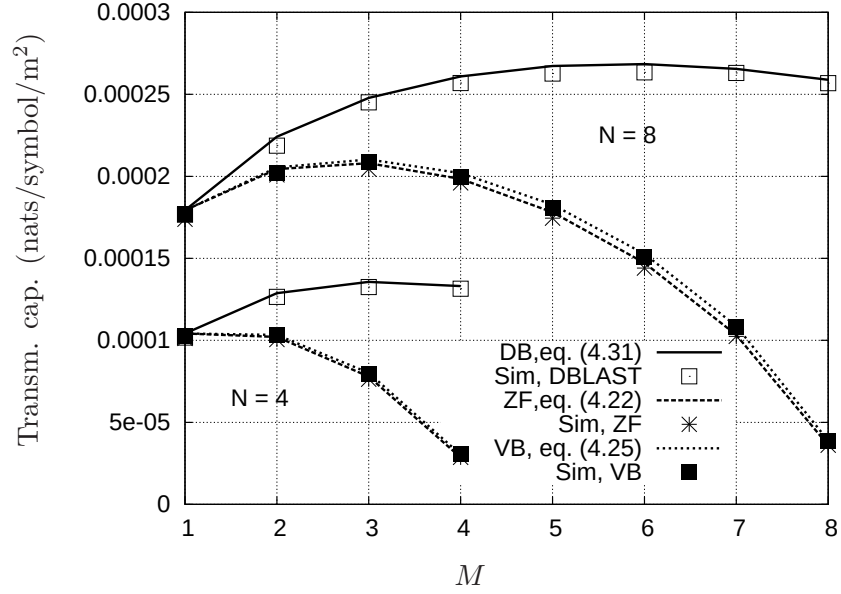


Figure 4.3: Transmission capacity vs. M for $N = 4, 8$ ($R = 20$ m, $b = 3$, $\theta = 6$ dB). To avoid cluttering the figure, DBLAST is denoted as DB and VBLAST as VB.

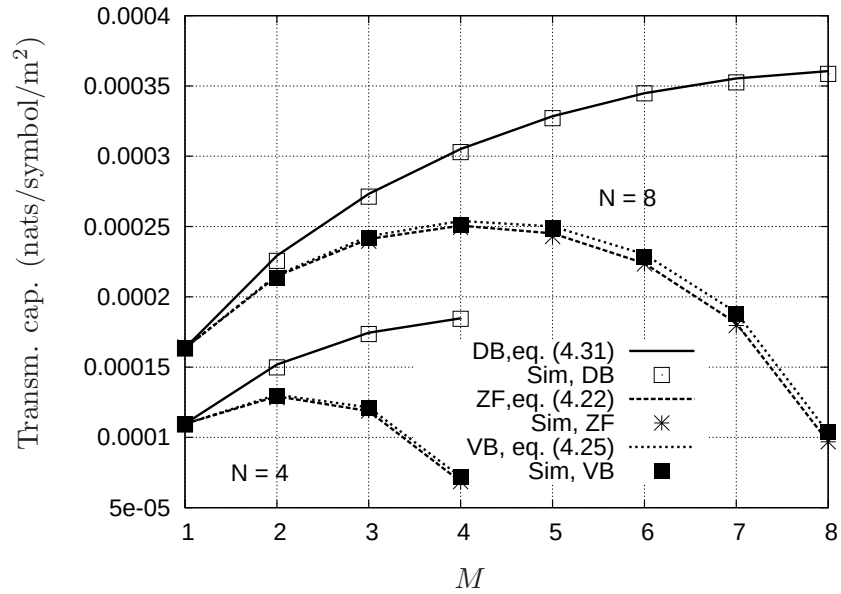


Figure 4.4: Transmission capacity vs. M for $N = 4, 8$ ($R = 20$ m, $b = 4$, $\theta = 6$ dB). To avoid cluttering the figure, DBLAST is denoted as DB and VBLAST as VB.

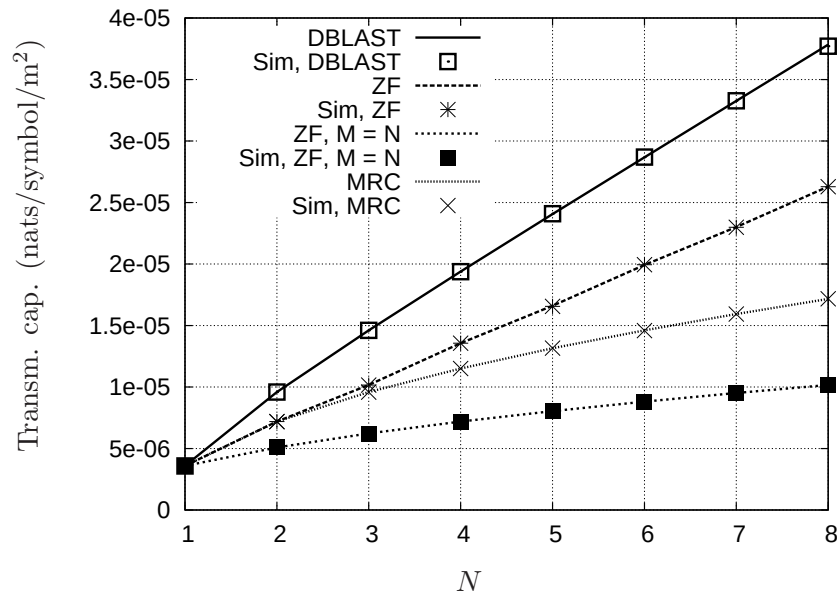


Figure 4.5: Transmission capacity vs. N ($\epsilon = 0.01$, $R = 20$ m, $b = 4$, $\theta = 6$ dB)

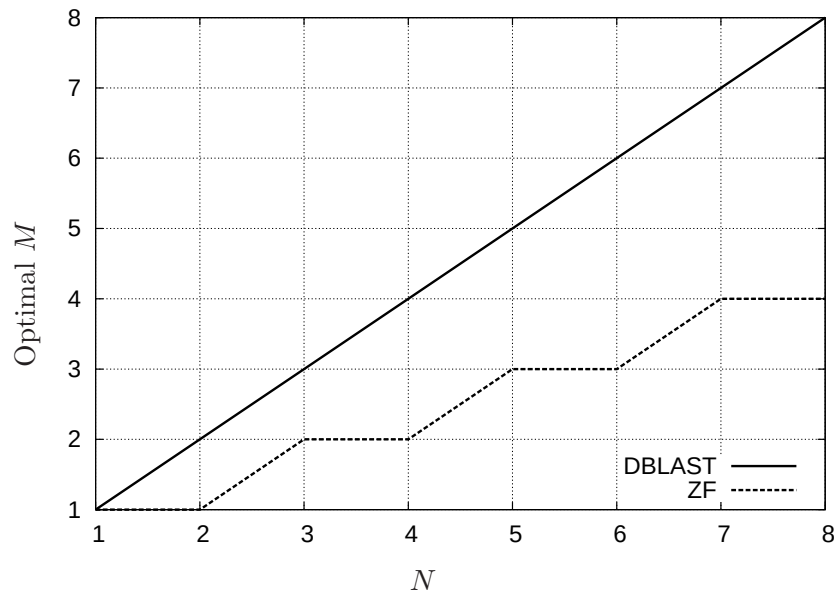


Figure 4.6: Optimal number of streams vs. N ($\epsilon = 0.01$, $R = 20$ m, $b = 4$, $\theta = 6$ dB)

Chapter 5

Delay characterization of random multi-hop networks

In this chapter we depart from the single-hop mentality of Chapters 3 and 4 and combine tools from stochastic geometry and queueing theory in order to study the end-to-end delay performance in multi-hop random networks. We consider a network where each route consists of a source, a number of relays and a destination at a finite distance, and the locations of the sources and relays are determined according to independent Poisson point processes. Nodes are equipped with queues to accommodate the randomness in the delivery of a packet, which is determined by the level of the SIR over each hop. Given a TDMA/ALOHA MAC protocol, we first study an idealized network model where all routes have the same number of hops, the same distance per hop and their own dedicated relays. Assuming that a stationary regime exists for this network, we analytically evaluate the mean end-to-end delay and the throughput for backlogged and non-backlogged sources. A key point in our analysis is taking into account that the interference level over each hop, i.e., the density of transmitting nodes, depends on the packet success probabilities and vice versa. The derivation of closed-form expressions permits the optimization of the mean end-to-end delay with respect to the number and placement of the relays. The benefits of this relay selection - or routing - strategy are then verified in the original network via simulation.

If the sources are backlogged, we find that the delay is minimized if the first hop is much larger than the remaining hops, e.g., in a delay-optimized three-hop route, the first hop covers half the total distance. On the other hand, in the non-backlogged case,

Table 5.1: Commonly used symbols in Chapter 5

Symbol	Meaning
p	source medium access probability (MAP)
p_r	relay MAP
p_a	probability of packet arrival at the source every N slots
λ	density of sources
λ_r	density of relays
R	source-destination distance
N	number of hops
p_n^s	packet success probability over hop n
r_n	distance of hop n

the delay is minimized when all hops are equidistant. We demonstrate that the respective optimal numbers of hops scale sublinearly and linearly with the source-destination distance. We also consider stability issues in a multi-hop random network and derive sufficient conditions on the medium access probability (MAP), and the traffic load in the case of non-backlogged sources, for the rate stability of the network. Simulations show that, when the relays of each route are selected out of a random population, the analytical results are accurate, provided that the density of the relay process is sufficiently large.

The rest of this chapter is organized as follows. Section 5.1 introduces the system model and in Section 5.2, we develop our mathematical framework in order to evaluate the mean delay. In Section 5.3 we consider stability issues and in Section 5.4, we discuss possible extensions to our network model that can accommodate practical scenarios of network operation. Numerical examples and simulation results are presented in Section 5.5 and our conclusions are summarized in Section 5.6. A list of symbols commonly used throughout this chapter is given in Table I.

5.1 System model

5.1.1 Network setting

We consider a network that consists of an infinite number of sources, relays and destinations, referred to indiscriminately as *nodes* (see Fig. 5.1). Each source has its own destination at a distance R and random orientation. The locations of the sources and the relays are drawn independently according to spatially homogeneous Poisson processes of

densities λ and λ_r , respectively. A *route* or *flow* in the network consists of a source, a finite number of relays - whose sole function is to forward the packets that originate at the source - and a final destination.

Each node has an infinite queue, where packets that are received from the previous node in the route can be stored in a first-in, first-out fashion. Time is divided into packet slots. In a route with N hops, the source “receives” a packet from an upper layer of the protocol stack every N slots with probability p_a ; if the sources are assumed to be backlogged, there are always packets to be transmitted in the source queue. Within a route, a TDMA/ALOHA protocol is observed, according to which a node is given the opportunity to transmit every N slots with a certain MAP. In the assigned TDMA slot, a source node is allowed to transmit with probability p , while a relay is allowed to transmit with probability p_r . A packet is received successfully by a node if the SINR in that slot is above a target threshold. If it is not, the transmitting node is informed via an ideal feedback channel and the packet remains at the head of its queue, at least for the next N slots, until it gets another opportunity to transmit ¹.

The network operation starts at some arbitrary time, with an arbitrary number of packets in each queue. For convenience, we assume that different routes are synchronized at the slot level. However, to avoid an artificial periodic behavior, we assume that, for each route, the node which is first allowed to transmit is randomly selected among the source and the relays with probability $1/N$.

Finally, since the population of relays is common to all sources, there is a possibility that, at a given slot, a relay has to be used, either for transmission or reception, by more than one routes. We assume that the conflict is resolved arbitrarily.

5.1.2 Idealized network setting

The analysis of the network model described above is complicated by the fact that the number of relays and their placement differ across routes, as well as the fact that routes might intersect. In order to simplify the model, we let each route in the network have the same number of hops N , the same distance per hop r_n , $n = 1, \dots, N$, and its own set of relays placed on the line between the source and the destination (thus $r_1 + \dots + r_N = R$). Under this idealization, node n of a typical route denotes the receiving node at hop n (node 0 denotes the source and node N the destination). The design

¹This implies that retransmissions are not given priority.

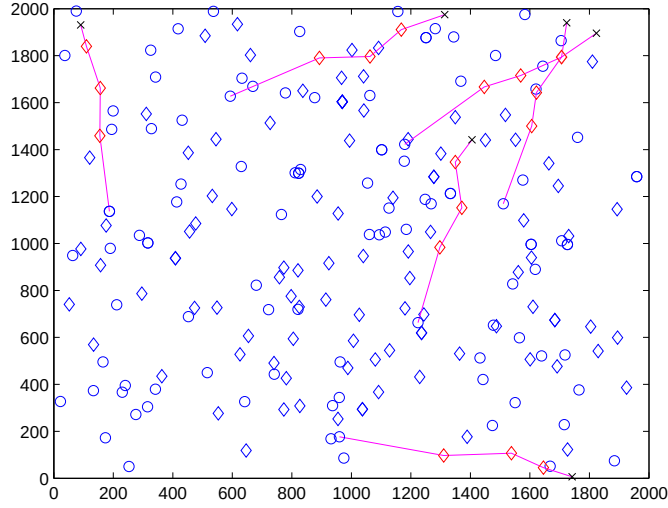


Figure 5.1: A network realization where 10% of the nodes and a few of the routes are shown ($\lambda = 10^{-4}$ sources/m², $\lambda_r = 3\lambda$). Sources are denoted by circles, relays by diamonds and destinations by $\times$'s. The distance to the destination is $R = 800$ m and the number of hops is $N = 4$.

guidelines that stem from the analysis of the idealized model are then used to route packets in a simulation of the original realistic network.

5.1.3 Physical layer

The channel between any two nodes at distance r includes Rayleigh fading and path-loss according to the law r^{-b} , where $b > 2$ is the path-loss exponent. Given the MAC scheme, we assume that the coherence time of the fading coefficient is at least (most) equal to the duration a slot (N slots). We also assume an interference-limited setting, i.e., thermal noise is considered negligible and disregarded, and that all nodes have the same transmit power, which is normalized to one. The SIR at the n^{th} hop of a typical route is therefore

$$\text{SIR}_n = \frac{A_n r_n^{-b}}{\sum_{i \in \Phi_n} B_i d_i^{-b}}, \quad n = 1, \dots, N \quad (5.1)$$

where r_n is the distance of the n^{th} hop; A_n is the fading coefficient between the transmitting and receiving node and exponentially distributed with unit mean; Φ_n is the process of interfering nodes over hop n ; d_i is the distance between the interfering node i and

the receiver, and B_i is the respective fading coefficient, also exponentially distributed with unit mean. We employ an outage formulation for packet successes: a packet is successfully received when the SIR is above a target threshold θ .²

5.1.4 Network metrics

Our metric of interest is the mean end-to-end delay³ D , i.e., the mean total time (in slots) that it takes a packet to travel from the source to the destination in a typical route. Assuming negligible propagation times, D is the sum of the mean *waiting times* and *service times* along the queues of the route. The waiting time at a given node is measured starting from the moment a packet arrives at that node's queue, till it becomes the head-of-line packet. The service time is measured from the moment a packet reaches the head of the queue, till it is successfully received by the next node, and incorporates the access delay associated with the MAC protocol.

In addition to the delay, we are interested in evaluating the overall route throughput RT, defined as the expected number of packets successfully delivered to the destination per slot over a typical route. The respective network throughput per unit area is then defined as $NT = \lambda RT$.

5.2 Analysis

To begin our analysis, we assume that, under certain conditions which are discussed in Section 5.3, the queues approach a stationary behavior in the long run. Moreover, due to the randomness and symmetry present in our network model, we assume that, in this stationary regime, a packet success over the n^{th} hop of a typical route is a Bernoulli event with probability p_n^s , independent of the packet successes in the hops of the same route and all other routes in the network. This assumption is justified when the probabilities of random access p and p_r are sufficiently small, such that the locations of the interferers are completely randomized over each hop, yielding approximately independent interference levels [52].

Under the aforementioned assumptions, a set of fixed-point equations is constructed that provide solutions to the success probabilities $\{p_n^s\}_{n=1}^N$. Note that, in spirit,

²This implies that the receiver regards interference as noise.

³Henceforth, the terms “mean delay” and “delay” will be used interchangeably.

our approach is similar to that introduced in [59], in the context of analyzing the performance of the IEEE 802.11 protocol for wireless local area networks.

In the first part of this section, we use tools from queueing theory in order to express the mean delay over a typical route as a function of p, p_r, p_a and $\{p_n^s\}_{n=1}^N$. In the second part, we evaluate $\{p_n^s\}_{n=1}^N$.

5.2.1 Queueing analysis

We examine the backlogged and non-backlogged source cases separately.

Backlogged sources

The mean waiting and service times at a given relay are functions of the packet arrival and departure probabilities to and from that relay. Each route consists of a set of queues in tandem. However, due to our assumption that packet successes are independent across hops, we can consider any single queue independently of the others [60]. Without loss of generality, assume that we are looking at the queue of the first relay. A packet arrival occurs at the end of slot mN , $m \in \mathbb{Z}$, if the source transmitted in that slot and the transmission was successful. The probability of this event is therefore $p_I = pp_1^s$. Similarly, at the beginning of slot $1 + mN$ - provided that the queue is not empty - a packet departs from the head of the queue with probability $p_O = p_r p_2^s$. This procedure is repeated every N slots.

The queue of the relay is modeled as a Random Walk [61], whose state is the number of packets in the queue at the end of slot mN . The transition probability from state k to state $k + 1$, $k \geq 1$, is $p_I(1 - p_O)$, while, from state $k + 1$ to state k , it is $p_O(1 - p_I)$, for $k \geq 0$. Since in state 0 the queue is empty, the transition probability from state 0 to state 1 is simply p_I . Assuming that the queue is stable, the steady state probability of being in state k , π_k , is found to be

$$\pi_k = \frac{p_I}{p_O(1 - p_I)} \cdot \left(\frac{p_I(1 - p_O)}{p_O(1 - p_I)} \right)^{k-1} \cdot \pi_0, \quad k \geq 1, \quad (5.2)$$

where $\pi_0 = 1 - \frac{p_I}{p_O}$. Obviously, a necessary condition for the stability of the queue is $p_I < p_O$.

The mean waiting time at the first relay, Q_1 , can be computed by Little's theorem, as the average queue size, excluding the head-of-line packet, divided by the arrival

rate [61], in this case p_I/N . Using (5.2), after some calculations we find that

$$Q_1 = N \frac{p_I}{p_O} \frac{1 - p_O}{p_O - p_I}.$$

It is also straightforward to show that the service time for the head-of-line packet is

$$H_1 = N \left(\frac{1}{p_O} - 1 \right) + 1. \quad (5.3)$$

The total time in the queue of the first relay is

$$Q_1 + H_1 = N \frac{p_I}{p_O} \frac{1 - p_O}{p_O - p_I} + \frac{N}{p_O} - N + 1 = N \frac{1 - p_I}{p_O - p_I} - N + 1. \quad (5.4)$$

If the queue at the first relay is stable, i.e., $pp_1^s < p_r p_2^s$, the packet arrival probability to the second relay is pp_1^s . In fact, as long as all intermediate queues are stable, i.e., $pp_1^s < p_r p_n^s$, $n = 2, \dots, N$, the packet arrival probability to relay n is pp_1^s [60]. As a result, we can calculate the total time in the queue of relay n by setting $p_O = p_r p_n^s$ in (5.4).

Adding the service time at the source and the total times at the queues of $N - 1$ relays, the mean end-to-end delay for a packet at the head of the source queue, D_b , is found to be ⁴

$$D_b = \frac{N}{pp_1^s} + N \sum_{n=2}^N \frac{1 - p_r p_n^s}{p_r p_n^s - pp_1^s}, \quad (5.5)$$

The route throughput is determined as follows: a packet is received every N slots by the final destination with probability pp_1^s . Hence, $RT_b = pp_1^s/N$.

Non-backlogged sources

If the sources are not backlogged, then, provided that all queues are stable (including the source queue), the packet arrival probability to all queues is p_a , where p_a has to satisfy $p_a < \min\{pp_1^s, p_r p_2^s, \dots, p_r p_N^s\}$. Following a similar reasoning to the backlogged source case, the mean end-to-end delay for a packet that arrives at the end of the source queue, D_{nb} , is

$$D_{nb} = N \frac{1 - p_a}{pp_1^s - p_a} + N \sum_{n=2}^N \frac{1 - p_r p_n^s}{p_r p_n^s - p_a} \quad (5.6)$$

and the route throughput is $RT_{nb} = p_a/N$.

⁴Since the sources are backlogged, the only meaningful way to define the end-to-end delay is for a packet at the head of the source queue.

5.2.2 Evaluation of packet success probabilities

We now determine the packet success probabilities $\{p_n^s\}_{n=1}^N$, which are required for the evaluation of the mean delay in (5.5) and (5.6).

The definition of the Lambert function is needed in the subsequent analysis and is stated here.

Definition 1. *The Lambert function, $y = W(x)$, is the unique solution to the equation $ye^y = x$, where $x \geq -e^{-1}$ and $y \geq -1$.*

The following lemmas are also useful and are stated without proof.

Lemma 1. *The function $f(x) = xe^{-\mu x}$, $\mu > 0, x \geq 0$, is continuous, with a unique maximum $f_o = (\mu e)^{-1}$ at $x_o = \mu^{-1}$. There are two solutions to the equation $y = f(x)$, $0 \leq y < f_o$, and the smaller of the two is $x_1 = -W(-\mu y)/\mu < \mu^{-1}$.*

Lemma 2. *The function $g(x) = xe^{\mu x}$, $\mu > 0, x \geq 0$, is continuous and strictly increasing. The equation $y = g(x)$, $\mu > 0, x, y \geq 0$, has a unique solution $x_1 = W(\mu y)/\mu$.*

Once again, the backlogged and non-backlogged source cases are presented separately.

Backlogged sources

We begin by noting that the locations of the potential interferers over any hop/slot of the typical route are determined according to a PPP of density λ . This is due to the following facts: (a) the locations of the sources are drawn independently according to a PPP (b) the locations of the relays and the destination of any route are displacements of the source location to a random direction (c) only one node is active per route at a given slot.

We now address the issue of how many of the potential interferers actually transmit packets and are thereby “active”. Under the assumption of independent packet successes, the queue of relay n , $n = 1, \dots, N-1$, of the typical route is not empty at the beginning of its assigned slot with probability $pp_1^s/(p_r p_{n+1}^s)$. Hence the relay transmits a packet with probability $pp_1^s/(p_r p_{n+1}^s) \cdot p_r = pp_1^s/p_{n+1}^s$. As a result the interfering nodes over any hop constitute a “thinned” homogeneous PPP with density

$$\lambda_t = \frac{\lambda p}{N} + \frac{\lambda p}{N} \sum_{k=2}^N \frac{p_1^s}{p_k^s} \quad (5.7)$$

where the first term represents the transmitting sources, while the second term represents the total population of transmitting relays.

According to Corollary 3.2 in [19] (or (4.8) in Chapter 4 for $N = 1$), the probability of successful reception over the n^{th} hop, p_n^s , is given by the expression

$$p_n^s = P(\text{SIR}_n > \theta) = e^{-\lambda_t c r_n^2}, \quad (5.8)$$

where $c = \Gamma(1 + 2/b)\Gamma(1 - 2/b)\pi\theta^{2/b}$ and $\Gamma(x)$, $x > 0$, is the gamma function. Given the Poisson distribution of interfering nodes, the physical meaning of (5.8) is that a packet is successfully received if a disk of area $\lambda_t c r_n^2$ around the intended receiver is free of interfering nodes.

From (5.7) and (5.8), p_n^s therefore satisfies the fixed-point equation

$$p_n^s = e^{-\frac{\lambda}{N} p \left(1 + \sum_{k=2}^N \frac{p_k^s}{p_k^s}\right) c r_n^2}, \quad n = 1, \dots, N. \quad (5.9)$$

Eq. (5.9) captures the coupling effect between concurrent transmissions in a wireless network. The packet success probabilities depend on the interferer density λ_t and vice versa.

The solution of (5.9) over $\{p_n^s\}_{n=1}^N$ for general values of $r_1, \dots, r_N$ appears complicated. Proposition 1 provides the solutions for the packet success probabilities in the setting $r_2 = \dots = r_N$, i.e., when hops $2, \dots, N$ are equidistant.

Proposition 1. *Let $r_2 = \dots = r_N$. Given that the relay queues are stable, the packet success probabilities are given by*

$$\begin{aligned} p_n^s &= e^{-\frac{\lambda}{N} p \left(1 + (N-1) \frac{p_1^s}{p_2^s}\right) c r_n^2}, \quad n = 1, 2 \\ p_2^s &= \dots = p_N^s. \end{aligned} \quad (5.10)$$

Furthermore

$$\frac{p_1^s}{p_2^s} = \begin{cases} \frac{W\left(\frac{\lambda}{N}(N-1)pc(r_1^2 - r_2^2)e^{-\frac{\lambda}{N}pc(r_1^2 - r_2^2)}\right)}{\frac{\lambda}{N}(N-1)pc(r_1^2 - r_2^2)} & r_1 \neq r_2 \\ 1 & r_1 = r_2. \end{cases} \quad (5.11)$$

Proof. Setting $p_2^s = \dots = p_N^s$, (5.10) results directly from (5.9). Dividing the two equations in (5.10) and rearranging terms

$$\frac{p_1^s}{p_2^s} e^{\frac{\lambda}{N}(N-1)pc(r_1^2 - r_2^2) \frac{p_1^s}{p_2^s}} = e^{-\frac{\lambda}{N}pc(r_1^2 - r_2^2)}. \quad (5.12)$$

We have the following cases.

- $r_1 > r_2$: by Lemma 2, (5.12) has a unique solution with respect to p_1^s/p_2^s , given by (5.11).
- $r_1 = r_2$: setting $r_1 = r_2$ in (5.10), it follows that $p_1^s = p_2^s = e^{-\lambda p c r_1^2}$.
- $r_1 < r_2$: by Lemma 1, (5.12) has a solution with respect to p_1^s/p_2^s if and only if

$$p e^{-\frac{\lambda}{N} p c (r_1^2 - r_2^2)} \leq \frac{1}{\frac{\lambda}{N} (N-1) c (r_2^2 - r_1^2) e}.$$

However, this condition holds if the relay queues are stable, which we show as follows. Assuming that all the relay queues in the network are backlogged, a necessary requirement for stability is that

$$p e^{-\frac{\lambda}{N} (p + (N-1)p_r) c r_1^2} < p_r e^{-\frac{\lambda}{N} (p + (N-1)p_r) c r_2^2}.$$

This implies that

$$p e^{-\frac{\lambda}{N} p c (r_1^2 - r_2^2)} < p_r e^{-\frac{\lambda}{N} (N-1) c (r_2^2 - r_1^2) p_r} \leq \frac{1}{\frac{\lambda}{N} (N-1) c (r_2^2 - r_1^2) e}.$$

By Lemma 1, the smaller of the two solutions of (5.12) with respect to p_1^s/p_2^s is given by (5.11). We have ignored the other solution on the basis that it is decreasing in r_2 .

□

Substituting (5.10) and (5.11) in (5.5), we can evaluate D_b . Concluding the analysis of the backlogged sources case, we now show that setting $r_2 = \dots = r_N$ minimizes D_b .

Proposition 2. *The mean delay, given by (5.5), is minimized when $r_2 = \dots = r_N = \frac{R-r_1}{N-1}$.*

Proof. By pairwise division and summation of the expressions in (5.9), we obtain

$$\sum_{n=2}^N \frac{p_1^s}{p_n^s} = \sum_{k=2}^N e^{-\frac{\lambda}{N} p \left(1 + \sum_{n=2}^N \frac{p_1^s}{p_n^s}\right) c (r_1^2 - r_k^2)}. \quad (5.13)$$

Fix r_1 and define $z_n = \frac{r_n^2}{r_1^2} < 1$, $n = 2, \dots, N$. The function $h(z_2, \dots, z_n) \triangleq \sum_{n=2}^N \frac{p_1^s}{p_n^s}$ is strictly increasing in $\{z_n\}$. Indeed, differentiating (5.13) with respect to z_k and rearranging terms, we obtain

$$\frac{\partial h}{\partial z_k} \left(1 + \frac{\lambda}{N} p c r_1^2 \sum_{n=2}^N (1 - z_n) e^{-\frac{\lambda}{N} p (1+h) c r_1^2 (1-z_n)} \right) = \frac{\lambda}{N} p (1+h) c r_1^2 e^{-\frac{\lambda}{N} p (1+h) c r_1^2 (1-z_k)}.$$

Since the right hand side term and the left hand side term in the parenthesis are positive, it follows that $\frac{\partial h}{\partial x_k} > 0$.

From (5.9), it follows that $\{p_n^s\}_{n=1}^N$ are decreasing functions of $r_2, \dots, r_N$, given r_1 . Since D_b in (5.5) is a sum of identical decreasing functions of $p_2^s, \dots, p_N^s$, then, given R , D_b is minimized if $r_2 = \dots = r_N = \frac{R-r_1}{N-1}$. $\square$

Non-backlogged sources

Similarly to the backlogged scenario, the probability that a stable source queue is not empty is $p_a/(pp_1^s)$, so the probability that a source transmits is p_a/p_1^s . The corresponding probabilities for relay n are $p_a/(p_r p_n^s)$ and p_a/p_n^s . The density of interferers at any hop is therefore $\lambda_n = \frac{\lambda}{N} \sum_{k=1}^N \frac{p_a}{p_k^s}$, and, according to (5.8), the success probability over hop n is

$$p_n^s = e^{-\frac{\lambda}{N} \sum_{k=1}^N \frac{p_a}{p_k^s} cr_n^2}, \quad n = 1, \dots, N. \quad (5.14)$$

Following a similar reasoning to Proposition 2, we can show that the delay is minimized if *all* hops are equidistant. For equidistant hops, the solution of (5.14) is given by the following proposition.

Proposition 3. *Let $r_1 = \dots = r_N = R/N$. Given that the queues are stable, the packet success probabilities are given by*

$$\begin{aligned} p_1^s &= -\frac{\lambda p_a cr_1^2}{W(-\lambda p_a cr_1^2)} = e^{W(-\lambda p_a cr_1^2)} \\ p_1^s &= p_2^s = \dots = p_N^s. \end{aligned} \quad (5.15)$$

Proof. Setting $r_1 = \dots = r_N$ in (5.14) results in

$$p_1^s = \dots = p_N^s = e^{-\lambda \frac{p_a}{p_1^s} cr_1^2}. \quad (5.16)$$

By Lemma 2, (5.16) has a solution with respect to $1/p_1^s$, if $\frac{1}{\lambda p_a cr_1^2 e} \geq 1$ or $p_a \leq \frac{1}{\lambda cr_1^2 e}$. This condition is satisfied if the queues are stable. Indeed, assuming the sources are backlogged, a necessary condition for the stability of the source queues is $p_a < pe^{-\lambda p cr_1^2}$. Since $pe^{-\lambda p cr_1^2} \leq \frac{1}{\lambda cr_1^2 e}$, it follows that $p_a \leq \frac{1}{\lambda cr_1^2 e}$. By Lemma 2, (5.15) is the solution to (5.16). The other solution is ignored on the basis that it is increasing in r_1 . $\square$

The mean delay can now be computed by substituting (5.15) in (5.6).

5.3 Discussion on stability

In Sections 5.2.1 and 5.2.2, we evaluated the mean end-to-end delay over a typical route, based on the assumptions that the queues approach a stationary behavior and that packet successes are independent across all hops and routes.

Determining under what conditions the network reaches a stationary regime, i.e., it is *dynamically stable* [19], is a complicated problem. However, we can determine conditions under which the network is *rate stable*, i.e., the queue backlogs remain finite over time. Rate stability is a necessary condition for dynamic stability of the network [62].

Backlogged sources

The following proposition establishes a necessary condition for rate stability when $p = p_r$.

Proposition 4. *Assume $p = p_r$. If the relay queues are rate stable, then $r_1 > r_n$, $n = 2, \dots, N$.*

Proof. Let all the relay queues in the network be backlogged. If the relay queues are rate stable, then, from (5.8), it must be that

$$pe^{-\frac{\lambda}{N}(p+(N-1)p_r)cr_1^2} < p_re^{-\frac{\lambda}{N}(p+(N-1)p_r)cr_n^2}, \quad n = 2, \dots, N,$$

or, equivalently, $r_1 > r_n$, $n = 2, \dots, N$, since $p = p_r$. □

This condition can also be deduced by demanding that the denominators in (5.5) are greater than zero. We now propose a sufficient condition for rate stability.

Proposition 5. *Assume $p = p_r$. The relay queues are rate stable if $r_1 > \sqrt{N}r_n$, $n = 2, \dots, N$.*

Proof. Since the sources are backlogged, the success probability in the first hop is the largest when no potential interfering relay has a packet to transmit. From (5.8), an upper bound to the success probability in the first hop is thus $\bar{p}_1^s = e^{-\frac{\lambda}{N}pcr_1^2}$. In contrast, the success probability in the second hop is the smallest when all nodes that are potential interferers in the second hop have packets to transmit. In this case, the density of interferers is $\frac{N-1}{N}\lambda p + \frac{1}{N}\lambda p = \lambda p$ and a lower bound to the success probability in the second hop is $\underline{p}_2^s = e^{-\lambda pcr_2^2}$. A sufficient condition for the stability of the first relay queue

is therefore $p\bar{p}_1^s < p_r \underline{p}_2^s$ or simply $\bar{p}_1^s < \underline{p}_2^s$. Defining $\underline{p}_n^s = e^{-\lambda p c r_n^2}$ as the lower bound to the success probability of hop n , $n = 2, \dots, N$, a set of sufficient conditions for the stability of *all* relay queues is

$$\bar{p}_1^s < \underline{p}_n^s \Leftrightarrow e^{-\frac{\lambda}{N} p c r_1^2} < e^{-\lambda p c r_n^2} \Leftrightarrow r_1 > \sqrt{N} r_n, n = 2, \dots, N.$$

□

Since packets arrive to the queues of the initially empty relays with positive probability, the packet arrival probability to the backlogged relays progressively decreases due to increasing interference. For this reason, it appears that Proposition 5 places a strict requirement on the distance of the first hop and may be far from necessary. We conjecture that the condition in Proposition 4 is also sufficient for the rate stability of the relay queues, which is corroborated by our simulation results in Section 5.5.

We now let $r_2 = \dots = r_N$ and, in a similar manner to Proposition 5, derive sufficient conditions on p and p_r such that the queues are rate stable for any value of r_1 .

Proposition 6. *Let $r_2 = \dots = r_N$. The relay queues are rate stable if*

$$p < \bar{p} \triangleq \begin{cases} \frac{W\left(\frac{r_2^2 - r_1^2}{(N-1)r_2^2 e}\right)}{\frac{\lambda}{N} c (r_2^2 - r_1^2)} & r_1 \leq \sqrt{N} r_2, \frac{\lambda(N-1)cr_2^2}{N} \geq 1 \quad (5.17a) \\ \frac{W\left(\frac{\lambda}{N} c (r_2^2 - r_1^2) e^{-\frac{\lambda(N-1)cr_2^2}{N}}\right)}{\frac{\lambda}{N} c (r_2^2 - r_1^2)} & r_1 \leq \sqrt{N} r_2, \frac{\lambda(N-1)cr_2^2}{N} < 1 \quad (5.17b) \\ 1 & r_1 > \sqrt{N} r_2 \quad (5.17c) \end{cases}$$

and

$$\underline{p}_r < p_r < \min \left\{ \frac{1}{\frac{\lambda(N-1)cr_2^2}{N}}, 1 \right\}, \quad (5.18)$$

where

$$\underline{p}_r \triangleq \frac{W\left(-\frac{\lambda(N-1)cr_2^2}{N} p e^{-\frac{\lambda}{N} p c (r_1^2 - r_2^2)}\right)}{-\frac{\lambda(N-1)cr_2^2}{N}}. \quad (5.19)$$

The limit $\lim_{r_1 \rightarrow r_2} \bar{p}$ exists and provides the upper bound on p .

Proof. Following the line of thought in the proof of Proposition 5, a sufficient condition for the rate stability of the relay queues is

$$p\bar{p}_1^s < p_r \underline{p}_2^s \Leftrightarrow p e^{-\frac{\lambda}{N} p c r_1^2} < p_r e^{-\frac{\lambda}{N} (p + (N-1)p_r) c r_2^2} \Leftrightarrow p e^{-\frac{\lambda}{N} p c (r_1^2 - r_2^2)} < p_r e^{-\frac{\lambda}{N} (N-1) c r_2^2 p_r}. \quad (5.20)$$

The proof involves making use of Lemmas 1 and 2, in order to determine a set of acceptable values for p and p_r , such that (5.20) is satisfied. The function on the right hand side of (5.20) is $f(p_r)$ of Lemma 1, with $\mu = \frac{\lambda}{N}(N-1)cr_2^2$. By Lemma 1, (5.20) is possible only if

$$pe^{-\frac{\lambda}{N}c(r_1^2-r_2^2)p} < \max_{p_r} f(p_r) = \frac{1}{\frac{\lambda}{N}(N-1)cr_2^2e}. \quad (5.21)$$

We take the following cases.

- $r_1 < r_2$: let $g(p) = pe^{\frac{\lambda}{N}c(r_2^2-r_1^2)p}$. By Lemma 2, (5.21) holds if (5.17a) is satisfied.
- $r_1 = r_2$: by (5.21), it immediately follows that $p < \frac{1}{\frac{\lambda}{N}(N-1)cr_2^2e}$. This is also the limit of (5.17a) for $r_1 \rightarrow r_2$, which can be shown using the Lambert function property $W(x) = xe^{-W(x)}$.
- $r_2 < r_1 \leq \sqrt{N}r_2$: let $f(p) = pe^{-\frac{\lambda}{N}c(r_1^2-r_2^2)p}$. For this range of r_1

$$r_1 \leq \sqrt{N}r_2 \Leftrightarrow \frac{1}{\frac{\lambda}{N}c(r_1^2-r_2^2)e} \geq \frac{1}{\frac{\lambda}{N}(N-1)cr_2^2e} \Leftrightarrow \max_p f(p) \geq \frac{1}{\frac{\lambda}{N}(N-1)cr_2^2e}. \quad (5.22)$$

By Lemma 1, $f(p) = \frac{1}{\frac{\lambda}{N}(N-1)cr_2^2e}$ has two solutions; a sufficient condition for (5.21) to hold is p being smaller than the solution given by Lemma 1. This condition is identical to (5.17a).

- $r_1 > \sqrt{N}r_2$: following the same steps as in (5.22), $f(p) < \frac{1}{\frac{\lambda}{N}(N-1)cr_2^2e}$ for any p . Since p is a probability, it suffices that $p < 1$.

We have determined sufficient constraints on p such that (5.21) is satisfied. By Lemma 1, a sufficient condition for (5.20) to hold is (5.18). We now need to explore if, for $\frac{\lambda(N-1)cr_2^2}{N} < 1$, there are additional constraints on p , such that $\underline{p}_r < 1$ is ensured or

$$W\left(-\frac{\lambda(N-1)cr_2^2}{N}pe^{-\frac{\lambda}{N}pc(r_1^2-r_2^2)}\right) > -\frac{\lambda(N-1)cr_2^2}{N} \Leftrightarrow pe^{-\frac{\lambda}{N}pc(r_1^2-r_2^2)} < e^{-\frac{\lambda(N-1)cr_2^2}{N}}. \quad (5.23)$$

If $r_1 \neq r_2$, then, by Lemmas 1 and 2, (5.23) holds if (5.17b) is satisfied while, if $r_1 = r_2$, it holds if $p < e^{-\frac{\lambda(N-1)cr_2^2}{N}}$, which is also the limit of the bound in (5.17b) when $r_1 \rightarrow r_2$. By comparing the arguments of the Lambert function in (5.17b) and (5.17a), we can show that (5.17b) is a more stringent condition. Moreover, by the property $W(x) = xe^{-W(x)}$, the bound in (5.17b) is an increasing function of r_1 . Since, for $r_1 = \sqrt{N}r_2$, $W\left(\frac{\lambda}{N}c(r_2^2 - Nr_2^2)e^{-\frac{\lambda(N-1)cr_2^2}{N}}\right) = \frac{\lambda}{N}c(r_2^2 - Nr_2^2)$, it follows that, for $r_1 \geq \sqrt{N}r_2$, (5.23) holds for all $p < 1$ and (5.17c) is established. $\square$

Proposition 6 provides an upper bound on p such that rate stability is ensured for $r_1 \leq \sqrt{N}r_2$. Additionally, the upper bound on p_r in (5.18) ensures that the relays do not create excessive interference by transmitting too often, thus causing the packet departure probability from the typical relay queue to become less than the packet arrival probability. The following proposition sheds more light on the relation between $\underline{p}_r$ and p .

Proposition 7. *If $r_1 \leq \sqrt{N}r_2$, then $\underline{p}_r \geq p$, with the equality occurring at $r_1 = \sqrt{N}r_2$. If $r_1 > \sqrt{N}r_2$, then $\underline{p}_r < p$.*

Proof. From (5.19), $\underline{p}_r$ can be written as $\underline{p}_r = \delta p$, where $\delta \triangleq \frac{W(\beta\gamma)}{\beta}$, $\beta \triangleq -\frac{\lambda}{N}(N-1)cr_2^2p$ and $\gamma \triangleq e^{-\frac{\lambda}{N}pc(r_1^2-r_2^2)}$. We want to show that $\delta \geq 1$, when $r_1 \leq \sqrt{N}r_2$, with the equality occurring at $r_1 = \sqrt{N}r_2$, and $\delta < 1$, when $r_1 > \sqrt{N}r_2$. We take the following cases.

- $r_1 \leq \sqrt{N}r_2$ and $r_1 \neq r_2$: The following statements are equivalent due to Definition 1.

$$\delta = \frac{W(\beta\gamma)}{\beta} > 1 \Leftrightarrow W(\beta\gamma) < \beta \Leftrightarrow \beta\gamma < \beta e^\beta \Leftrightarrow \gamma > e^\beta. \quad (5.24)$$

However, the last statement is true, as $e^{-\frac{\lambda}{N}pc(r_1^2-r_2^2)} \geq e^{-\frac{\lambda}{N}(N-1)cr_2^2p} \Leftrightarrow r_1^2 \leq Nr_2^2$, with equality occurring at $r_1 = \sqrt{N}r_2$.

- $r_1 > \sqrt{N}r_2$: If $\beta < -1$, then $\delta = \frac{W(\beta\gamma)}{\beta} < 1$ as $W(\beta\gamma) \in [-1, 0)$. On the other hand, if $\beta \geq -1$, then, following the same steps as in the previous bullet, we can show that $\delta < 1$.
- $r_1 = r_2$: From (5.20), we can see that p and p_r must satisfy the condition $p_r > pe^{\frac{\lambda}{N}(N-1)cr_2^2p_r}$. As the exponential factor is greater than one, it follows that $\underline{p}_r = \delta p$, with $\delta > 1$.

□

In other words, if the distance of the first hop is larger than $\sqrt{N}r_2$, a p_r smaller than p stabilizes the relay queues. On the other hand, if $r_1 \leq \sqrt{N}r_2$, rate stability is guaranteed if the relays transmit with a probability p_r which is larger than p .

Non-backlogged sources

Following a similar reasoning to the backlogged sources case, we establish rate stability conditions for the source and relay queues in the network, when $p = p_r$ (the case $p \neq p_r$ can be easily handled and is therefore omitted).

Proposition 8. *Let p_a be the packet arrival probability to the sources every N slots, and $p = p_r$. The queues are rate stable if and only if*

$$p_a < \min_{n=1,\dots,N} \left\{ p e^{-\lambda p c r_n^2} \right\}. \quad (5.25)$$

Proof. Similarly to Propositions 5 and 6, (5.25) results by demanding that p_a is smaller than the packet departure probability from any node in a typical route, when all queues in the network are backlogged. $\square$

Since $r_1 + \dots + r_N = R$, (5.25) implies that, for rate stability, a strict upper bound to p_a is $\bar{p}_a \triangleq p e^{-\lambda p c \frac{R^2}{N^2}}$. A strict upper bound to the network throughput per unit area is therefore

$$\overline{\text{NT}}_{\text{nb}} = \frac{\lambda \bar{p}_a}{N} = \frac{\lambda p}{N} e^{-\lambda p c \frac{R^2}{N^2}}. \quad (5.26)$$

This function can be optimized over N . Assuming that N can take any positive value, we easily find that $\overline{\text{NT}}_{\text{nb,max}} = \sqrt{\frac{\lambda p}{2c e R^2}}$ and $N_{\text{opt}} = \sqrt{2\lambda p c R}$. It is meaningful to now ask what is the number of hops that minimizes the delay when the system operates at a given percentage of $\overline{\text{NT}}_{\text{nb,max}}$. We address this issue via a numerical example in the next section.

5.4 Extensions to the model of Section 5.1

In this subsection, we discuss some possible extensions to the network model presented in Section 5.1.

5.4.1 Unequal route lengths

In practical networks, it is most likely that different source-destination pairs are at different distances. We can extend the model in Section 5.1 to accommodate this more practical scenario by letting the source-destination distance R be a random variable that takes values $R_1, \dots, R_M$ with probabilities $p^{(1)}, \dots, p^{(M)}$ (which implies that $\sum_{m=1}^M p^{(m)} = 1$). Each route length R_m corresponds to a number of hops N_m , a first-hop distance r_1^m , and a remaining-hop distance $r_2^m = \frac{R - r_1^m}{N_m - 1}$, where, by definition, if $N_m = 1$, then $r_2^m = 0$.

Letting $p = p_r$, similarly to the case of equal-length routes, the total density of

interferers (assuming backlogged sources) is found to be

$$\begin{aligned}\lambda_t &= \lambda p \sum_{m=1}^M p^{(m)} \left(\frac{1}{N_m} + \frac{N_m - 1}{N_m} \frac{p_1^{s,m}}{p_2^{s,m}} \right), \\ \lambda_t &= \lambda p \sum_{m=1}^M p^{(m)} \left(\frac{1}{N_m} + \frac{N_m - 1}{N_m} e^{-\lambda_t c((r_1^m)^2 - (r_2^m)^2)} \right)\end{aligned}\quad (5.27)$$

where $p_1^{s,m} = e^{-\lambda_t c(r_1^m)^2}$ denotes the success probability of the first hop of routes with length R_m . Similarly, $p_2^{s,m} = e^{-\lambda_t c(r_2^m)^2}$ is the success probability of hops $2, 3, \dots, N_m$ and, by definition, $p_2^{s,m} = 1$ if $N_m = 1$. As in the case of equal-length routes, a necessary stability requirement is $r_1^m > r_2^m$. Substituting the expressions for $p_1^{s,m}$ and $p_2^{s,m}$ in (5.27), we can easily show that it has a unique solution over λ_t .

Note that the total density of interferers (5.27) depends on the success probabilities of all route groups, as well as the probabilities $\{p^{(m)}\}$. A delay-optimization problem can therefore be posed as follows: determine the variables $\{r_1^m, N_m\}$, such that the average delay $\bar{D}_b = \sum_{m=1}^M p^{(m)} D_{m,b}$ is minimized, where

$$D_{m,b} = \begin{cases} \frac{N_m}{pp_1^{s,m}} + N_m(N_m - 1) \frac{1 - pp_2^{s,m}}{pp_2^{s,m} - pp_1^{s,m}} & N_m \geq 2 \\ \frac{N_m}{pp_1^{s,m}} & N_m = 1. \end{cases} \quad (5.28)$$

$$(5.29)$$

The above optimization soon becomes computationally intensive as M increases. The optimization is also complicated by the fact that, in general, there is no closed-form solution for λ_t in (5.27). We can however decouple the optimization of different route groups, i.e., let $D_{m,b}$ only depend on r_1^m and N_m as follows.

- Assuming the interfering node from each route is backlogged. In this case, we have $\lambda_t = \lambda$. This assumption results in the most conservative design as λ is the maximum value that λ_t may take.
- Given a route group, say (without loss of generality) 1, and assuming that the interferers of all other route groups $2, \dots, M$ are backlogged. In this case,

$$\lambda_t = \lambda p \left(1 - \frac{N_1 - 1}{N_1} p^{(1)} \right) + \lambda p p^{(1)} \frac{N_1 - 1}{N_1} \frac{p_1^{1,s}}{p_2^{1,s}}. \quad (5.30)$$

In this manner, we are taking into account the fact that the interferers of a given route group do not always have packets to transmit. The resulting system design is still conservative, however less so than the previous design. Note that (5.30) may be solved analytically over λ_t with the use of the Lambert function.

Asynchronous route clocks

The assumption that there is slot synchronization between different routes can be relaxed by assuming an arbitrary time offset that is uniformly distributed in $[0, T_s)$, where T_s is the duration of the slot. The implications of such an asynchronous operation are the following.

- The interference power will vary across a slot. An outage formulation for packet successes should still be employed⁵; if there is coding within the packet across the different interference levels, the interference term in the denominator of (5.1) should be replaced by the average interference power seen over a slot [63].
- Correlations in the interference power will appear across different hops, e.g., if a packet transmission in the first hop is successful, it is likely that will be the case in the second hop. The impact of such correlations on the end-to-end delay appears hard to analyze and should be examined via simulation.

5.5 Numerical results

In this section, we present the analytical results of Section 5.2, along with simulation results, for $\lambda = 10^{-4}$ sources/m², $b = 4$ and $p = p_r = 0.05$ (unless otherwise stated). The desired placement of the relays is on the line between the source and the destination, with $r_2 = \dots = r_N = \frac{R-r_1}{N-1}$.

The simulated network size is 5000×5000 m², which on the average yields 2500 sources (and routes). Since our analysis determines the mean delay and throughput of a typical route, in the simulation, these metrics are evaluated as averages over time, over different routes, as well as over different network realizations. The duration of a simulation for a given network and the number of realizations are chosen in order to provide adequate statistical confidence. In order to minimize the impact of edge effects, metrics are collected only for routes whose sources are situated inside an inner square of size 1100×1100 m².

⁵The closest interfering routes will dominate the interference power, so there is not enough diversity in the interference power to justify the ergodic framework of Chapter 3.

5.5.1 Backlogged sources

Given the desired placement of the relays on the line between the source and the destination, routing in each hop is performed by selecting among the relays that are between the respective desired point and the destination, the one that is closest to that point. This constraint ensures that, if the desired point satisfies the stability constraint, the position of the selected relay will do so as well, as it is even farther away from the source. If the last relay found is closer to the destination than r_2 , then transmission is made directly to the destination. As a result, routes with a smaller number of hops than that theoretically specified may exist in the network. The average end-to-end delay is calculated over those routes as well.

We first explore the impact of the number of hops on the mean delay and the network throughput. In Figs. 5.2 and 5.3, D_b and the respective NT_b are plotted vs. R for different numbers of hops. For each N , the distance of the first hop r_1 is determined such that D_b is minimized. For each R , there is an optimal number of hops that minimizes D_b and the distance between the switching points (values of R for which N and $N + 1$ hops yield the same delay) progressively increases. This points to a sublinear relationship between the optimal N and R (Fig. 5.6). Note that the optimal total service time, shown in a dashed line, provides a rather tight lower bound to the optimal end-to-end delay. This shows that, for optimally placed relays and selected N , the waiting time at each queue is small compared to the service time.

The simulation results, shown with points, correspond to a relay density $\lambda_r = 2(N - 1)\lambda$. We observe that, in the range of R for which a given number of hops is optimum, the match between the theoretical and simulated delay values is satisfactory. As R increases, the discrepancy between theory and simulation for small N becomes worse. This is due to the fact that the average number of crossings between routes increases, so bottlenecks are created at the relays which are used by more than one routes. We confirm that the discrepancy decreases with increasing relay density in Fig. 5.4. A related observation is that the theoretical network throughput values in Fig. 5.3 match the simulation ones quite well even for increasing R ; the reason being that the routes with bottlenecks get very few packets across to the final destination, hence have a minimal contribution to the total end-to-end throughput.

In Fig. 5.5, we examine the sensitivity of the delay to the placement of the first relay, for three different pairs of values (R, N) . The curves correspond to the theoretical

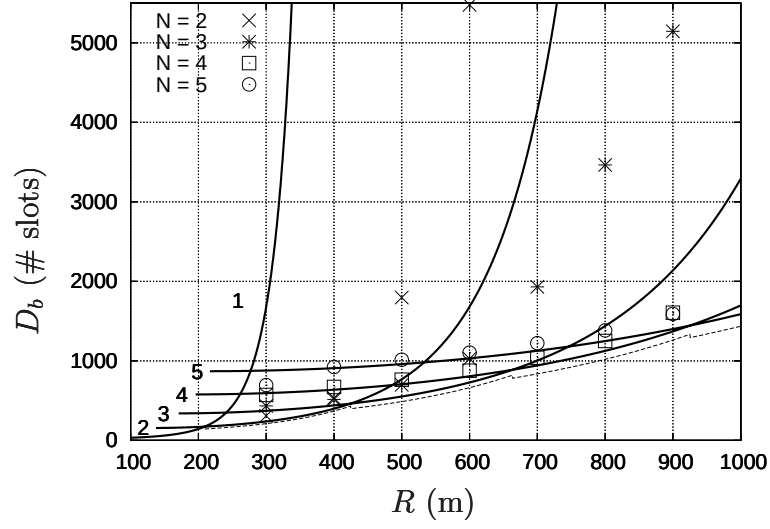


Figure 5.2: D_b vs. R , for $N = 1, \dots, 5$. Simulation points are shown for $\lambda_r = 2(N-1)\lambda$. ($\lambda = 10^{-4}$ sources/m², $b = 4$, $p = p_r = 0.05$)

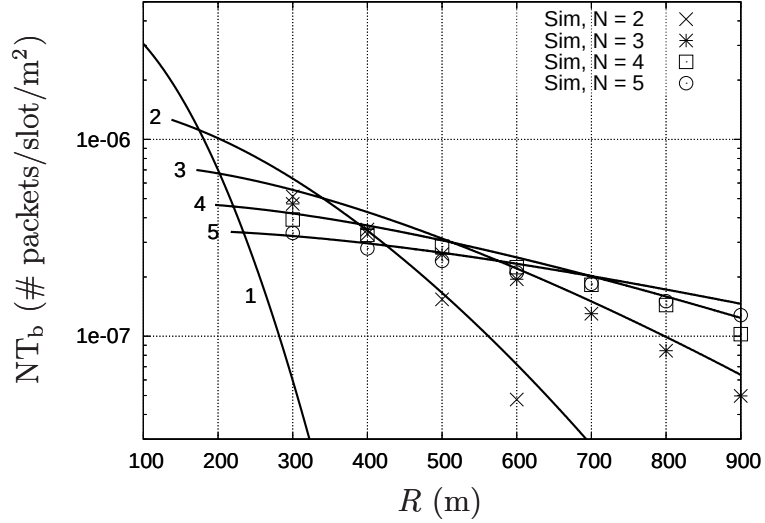


Figure 5.3: NT_b vs. R for $N = 1, \dots, 5$. Simulation points are shown for $\lambda_r = 2(N-1)\lambda$. ($\lambda = 10^{-4}$ sources/m², $b = 4$, $p = p_r = 0.05$)

delay obtained by (5.5). The simulation confirms the validity of the analysis for a relay density $\lambda_r = 2(N-1)\lambda$. The discrepancy at $r_1 = 450$ m for $R = 500$ m is caused by the existence of a large number of two-hop routes in the network, for which the delay is very large, as the - only - relay is placed very close to the destination. The optimal distance

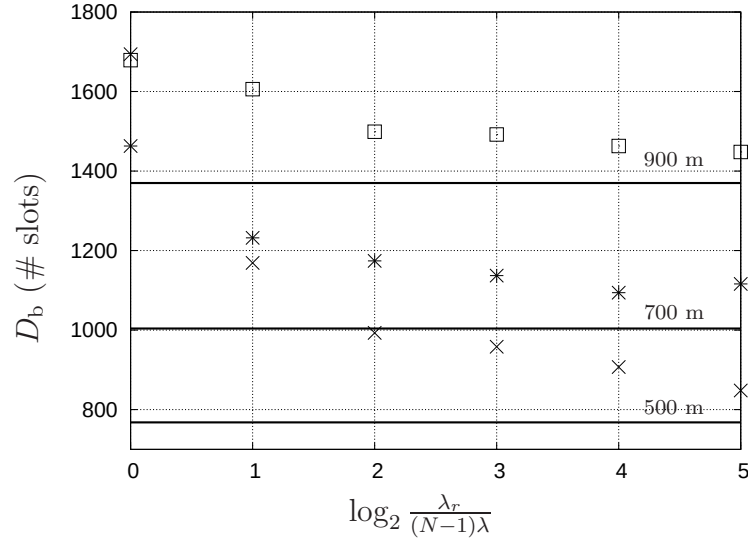


Figure 5.4: D_b vs. $\frac{\lambda_r}{(N-1)\lambda}$, for $R = 500$ m ($N = 2$), $R = 700$ m ($N = 3$) and $R = 900$ m ($N = 4$). Simulation results are shown with points. ($\lambda = 10^{-4}$ sources/m², $b = 4$, $p = p_r = 0.05$)

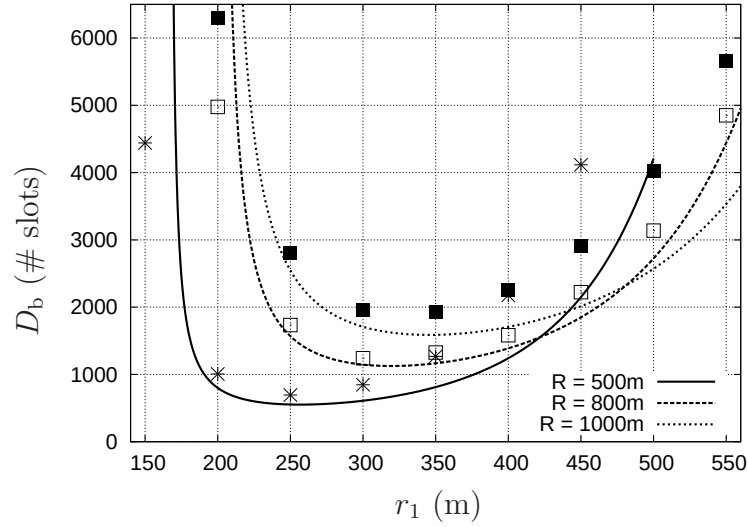


Figure 5.5: D_b vs. r_1 , for $R = 500, 800, 1000$ m ($N = 3, 4, 5$ respectively). Simulation points are shown for $\lambda_r = 2(N-1)\lambda$. ($\lambda = 10^{-4}$ sources/m², $b = 4$, $p = p_r = 0.05$)

of the first hop for $R = 500$ m and $N = 3$ is approximately 250 m, i.e., half the total distance. As $r_1 \rightarrow R$, $p_2^s \rightarrow 1$ and the service time at the source dominates the delay. On the other hand, as $r_1 \rightarrow R/N$, i.e., all hops tend to become equidistant, $p_2^s - p_1^s \rightarrow 0$

and the queueing delay at the relays obtains very large values, e.g., an r_1 approximately 100 m short of the optimal value results in a large delay penalty.

We now determine how D_b scales with R when the number of hops and relay placements are optimally chosen. From Fig. 5.2, we obtain the number of hops that minimizes D_b for each value of R and plot it as a function of R in Fig. 5.6. It is seen that this number scales *sublinearly* with R ; specifically, we find that $N \approx \eta_1 R^{0.74}$, where $\eta_1 > 0$ is determined by the sublinear fit. Likewise, in Fig. 5.7, the optimal distance for the first hop also scales sublinearly with R and we find that $r_1 \approx \eta_2 R^{0.43}$, $\eta_2 > 0$. In Fig. 5.8, we plot the optimal success probabilities vs. R and note that, as R increases, p_1^s progressively decreases, while p_2^s approaches a constant value $\tilde{p}_2^s \approx 0.55$. As a result, solving (5.10) over p_1^s for large values of R , we have the following approximation for p_1^s

$$p_1^s \approx \tilde{p}_2^s \frac{W\left(\frac{\lambda p c r_1^2}{\tilde{p}_2^s} e^{-\frac{\lambda p c}{N} r_1^2}\right)}{\lambda p c r_1^2}.$$

From (5.5), the end-to-end delay can thus be approximated as

$$D_b \approx \frac{\lambda c r_1^2 N}{\tilde{p}_2^s W\left(\frac{\lambda p c r_1^2}{\tilde{p}_2^s} e^{-\frac{\lambda p c}{N} r_1^2}\right)} + N^2 \left(\frac{1}{p_r \tilde{p}_2^s} - 1 \right). \quad (5.31)$$

Taking into account that $p_r \ll 1$, we have

$$D_b \approx \frac{\lambda c \eta_1 \eta_2^2 R^{1.6}}{W\left(\frac{\eta_3 \eta_1}{\tilde{p}_2^s} R^{0.74}\right)} + \frac{\eta_1^2 R^{1.48}}{p_r}, \quad (5.32)$$

where $\eta_3 \triangleq \frac{\lambda p c r_1^2}{N} e^{-\frac{\lambda p c}{N} r_1^2} \leq e^{-1}$. Since $W(x) = \mathcal{O}(\ln x)$ for $x \rightarrow \infty$, it also holds that $W(x) = \mathcal{O}(x)$. Moreover, $W(x) = \Omega(1)$. As a result, the first term in (5.32) is $\Omega(R^{0.86})$ and $\mathcal{O}(R^{1.6})$. The superlinear scaling of D_b with R in (5.32) is a result of the scaling of the optimal r_1 with R , as well as the TDMA nature of the MAC protocol, i.e., the constraint that a node is allowed to transmit only once every N slots. These scaling results indicate that the MAC protocol is tailored to a small number of hops; as R increases, intra-route spatial reuse should be considered, a scenario which is beyond the scope of this thesis. As a side note, since R is present in every expression in the product $\lambda c R^2$, for a given R , we also conclude that the optimal number of hops scales with $(\lambda c)^{0.37}$.

Finally, in Figs. 5.9 and 5.10, we set $R = 800$ m and examine the sufficient rate stability conditions of Proposition 6. The lower and upper bounds on p_r and p ,

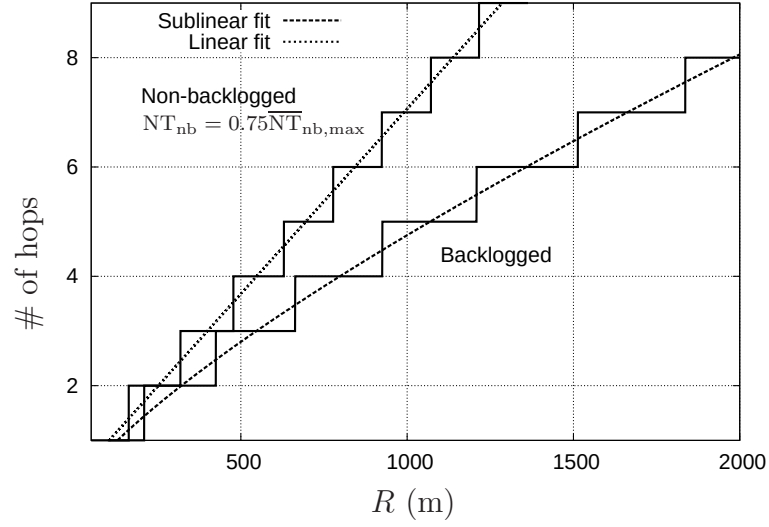


Figure 5.6: Optimal number of hops vs. R for backlogged and non-backlogged sources. ($\lambda = 10^{-4}$ sources/m², $b = 4$, $p = p_r = 0.05$)

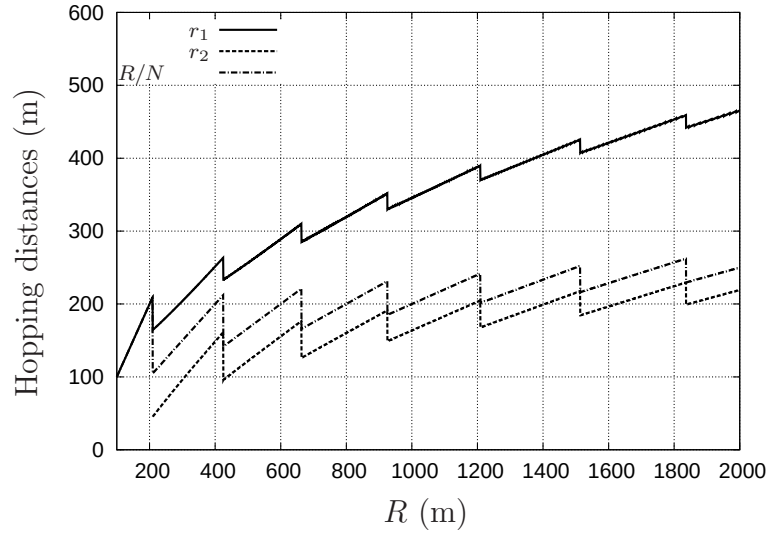


Figure 5.7: Optimal hopping distances vs. R . ($\lambda = 10^{-4}$ sources/m², $b = 4$, $p = p_r = 0.05$)

respectively, are shown in Fig. 5.9. Note that, for $r_1 \leq \sqrt{N}r_2 = 320$ m, $\underline{p}_r \geq p$, as indicated by Proposition 7. The respective delay curves as a function of p , along with simulation results, are shown in Fig. 5.10. For each curve, the value of p_r is set equal to the maximum value of $\underline{p}_r$ over p in Fig. 5.9. As p approaches zero, D_b is dominated by the service time at the source and tends to infinity. For $p > 0$, the sensitivity of the

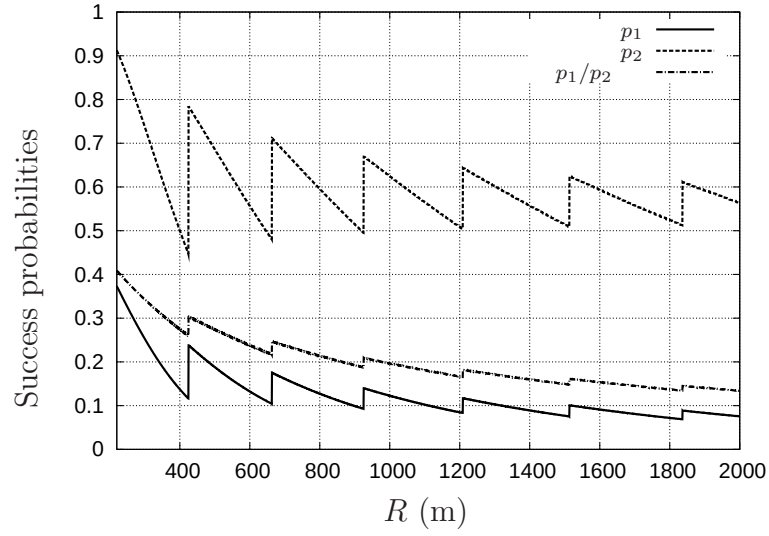


Figure 5.8: Optimal success probabilities vs. R . ($\lambda = 10^{-4}$ sources/m², $b = 4$, $p = p_r = 0.05$)

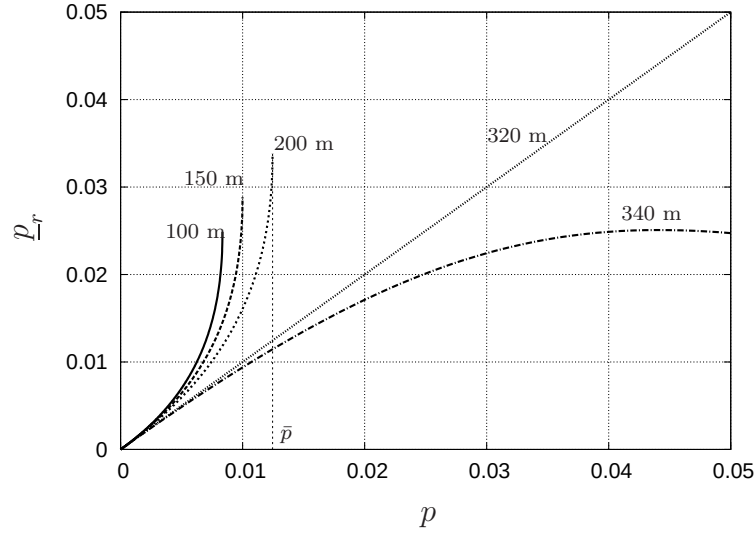


Figure 5.9: p_r vs. p , for $R = 800$ m, $N = 4$ and various placements of the first relay r_1 . ($\lambda = 10^{-4}$ sources/m², $b = 4$)

delay to p depends on the relay placement, i.e., for $r_1 = 100$ m, only a small range of p values around 0.005 yields a delay below 2000 slots. As the relay is moved farther away from the source, this range becomes wider and, for $r_1 \geq 320$ m, the delay varies less with p . Note that, for all values of r_1 , D_b is finite as $p \rightarrow \bar{p}$, which is also verified via the

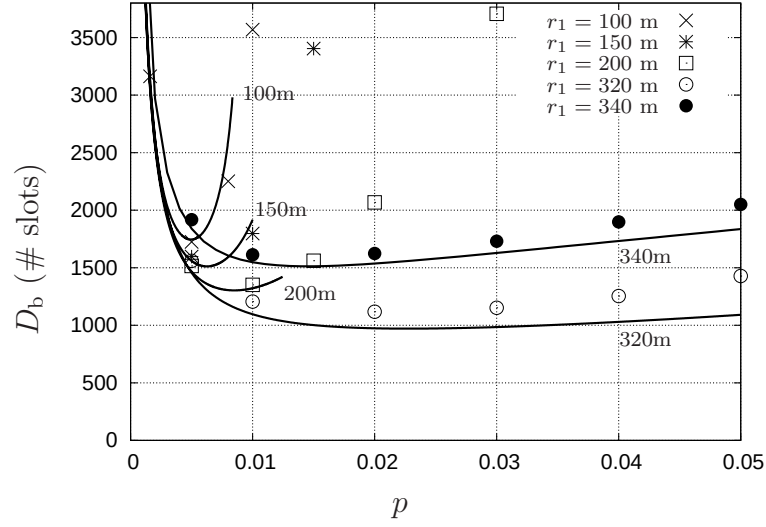


Figure 5.10: D_b vs. p , for $R = 800$ m, $N = 4$ and various placements of the first relay r_1 . Simulation points are shown for $\lambda_r = (N - 1)\lambda$. ($\lambda = 10^{-4}$ sources/m², $b = 4$)

simulation; this provides an indication that the sufficient conditions of Proposition 6 are stringent.

5.5.2 Non-backlogged sources

In Fig. 5.11, we consider a range of R from 50 m to 1200 m and equidistantly placed relays. For each R , the number of hops and traffic load p_a that minimize D_{nb} are found, when NT_{nb} is a certain fraction η of $\overline{NT}_{nb,max}$, i.e., $\frac{\lambda p_a}{N} = \eta \sqrt{\frac{\lambda p}{2ceR^2}}$. The delay is then plotted vs. the optimal number of hops, for $\eta = 0.25, 0.5, 0.75$. The plot confirms that operating the network at a smaller throughput results in a smaller delay and a smaller number of hops (delay-throughput trade-off). In Fig. 5.6, the optimal number of hops is shown for a larger range of R and $\eta = 0.75$; in contrast to the backlogged case, the scaling is linear. This implies that the optimal p_a , thus the packet success probabilities in (5.16), are approximately constant over R . Taking these trends into account in (5.6) we can show that D_{nb} scales with N^2 (or R^2). This behavior is the result of the linear scaling of the optimal N with R , combined with the TDMA character of the MAC protocol.

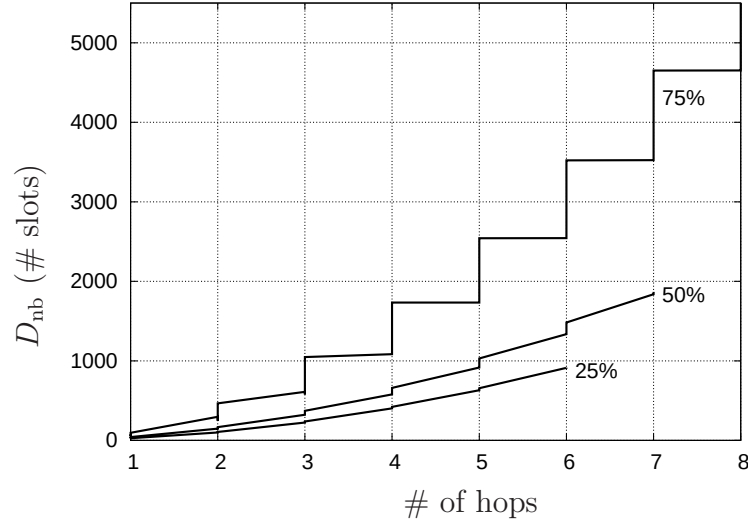


Figure 5.11: D_{nb} vs. optimal number of hops for different fractions of $\overline{NT}_{nb,max}$. (R takes values from 50 m to 1200 m, $\lambda = 10^{-4}$ sources/m², $b = 4$, $p = p_r = 0.05$)

5.6 Concluding remarks on Chapter 5

We have conducted an analytical and simulation study of random, interference-limited, multi-hop networks. Our approach consisted of two steps: first, studying an idealized network model and deriving expressions for the mean end-to-end delay and throughput over a typical route, using tools from basic queueing theory and stochastic geometry; then, employing the obtained design insights to perform routing in a realistic network environment, where the relays are picked from a random population. We determined the number of relays and their placement such that the mean delay is minimized and derived conditions for the rate stability of the network. The simulation results confirmed the validity of the analysis for moderate to high relay densities and illustrated the usefulness of the design guidelines in minimizing the mean end-to-end delay.

Chapter 5, in part, is a reprint of the paper “A delay-minimizing routing strategy for wireless multi-hop networks”, Kostas Stamatiou, Francesco Rossetto, Martin Haenggi, Tara Javidi, James R. Zeidler and Michele Zorzi, as it appeared in the proceedings of the 5th IEEE Workshop on Spatial Stochastic Models for Wireless Networks, Seoul, June 2009.

Chapter 6

Conclusions

In this thesis, we considered three different scenarios of wireless communication, where interference from concurrent transmissions is the main limiting performance factor. A key point in our work is taking into account how the spatial distribution of the interferers determines the statistics of the interference. In this manner, the performance of physical layer techniques, which are traditionally examined in the context of a single link with AWGN, can be assessed in the context of a network where transmissions interfere with each other.

In Chapter 2, we evaluated the BEP on the downlink of a coherent FH-MA cellular system, where coding is employed to exploit the frequency and interference diversity resulting through FH. We developed an analytical framework in order to take into account the effects of the channel time/frequency correlation, imperfect CSI and interference from an adjacent BS. We identified which patterns provide the best frequency diversity gain as well as good channel estimation performance. Given that the interfering BS employs channel inversion PC, we also showed that, for a RX which can measure the interference power, the diversity gain is equal to the code Hamming distance, while the diversity gain is lost for a RX that cannot measure the interference power variations. This difference in performance is most striking for a low system load or probability of interference, but remains significant for a high system load as well.

In Chapter 3, we departed from the cellular architecture and studied a random planar network of TXs, each with a corresponding RX at a fixed distance. As in Chapter 2, we considered a combination of FH at the symbol level with coding, and evaluated the effect of channel diversity on the transmission capacity of the network. It was shown

that, given a target constraint on the packet error probability, the transmission capacity follows a power law where the exponent is the inverse of the code Hamming distance. Our simulation results verified the large gains in network capacity that are possible at the cost of moderate decoding complexity. We concluded that FH during packet transmission, combined with coding, is an attractive physical layer scheme for uncoordinated ad hoc networks, despite the increased overhead and complexity induced by channel estimation and synchronization, compared to FH at the packet level.

In Chapter 4, we considered a block fading and interference model for the same single-hop ad hoc network topology studied in Chapter 3 and focused on the performance analysis of multiple-antenna techniques. We first derived an expression for the outage probability when single antenna is used at each TX and MRC is employed at the RX. Using this result, we derived exact and approximate expressions for the outage probability of multiple-stream transmission techniques such as ZF, VBLAST and DBLAST. The optimal number of streams such that the transmission capacity of the network is maximized was determined for ZF and DBLAST in the small outage probability regime. We also provided numerical results that quantified the capacity gain of DBLAST compared to VBLAST in a random network setting.

Finally, in Chapter 5 we adopted a cross-layer approach and, combining a simple outage-based model for packet successes with queueing theory, we evaluated the mean end-to-end delay in a random multi-hop network. Assuming the nodes observe a TDMA-ALOHA MAC protocol, we determined the optimal number of relays and their placement such that the end-to-end delay is minimized. Specifically, for backlogged sources, it was shown that the optimal distance for the first hop is significantly larger than the remaining hops in the route. The optimality of our relay selection scheme was verified in a practical network setting via simulation.

The capacity of emerging classes of networks such as ad hoc and sensor networks has become a topic of intense investigation. As we demonstrated in Chapters 3 and 4, stochastic geometry provides an analytically tractable way to evaluate the performance of various physical layer techniques, by allowing a characterization of the interference power as a function of the network geometry. Chapter 5 is a first step towards leveraging physical layer information such as the packet error probability, in determining a higher-level metric such as the end-to-end delay in a multi-hop network. Accommodating the cases of intra-route spatial reuse, variable-rate transmission or multiple-stream transmis-

sion - if the nodes have more than one antennas as in Chapter 4 - are all interesting open problems related to the material in Chapter 5. Moreover, the issue of relay utilization by more than one routes, i.e., route crossings, and its impact on the delay and network throughput merit analytical investigation. In this direction, it appears that results from the theory of line segment processes [64] might be useful.

A more general problem statement is the development of a framework for the study of wireless multi-hop networks, which combines elements from stochastic geometry as well as network control theory. The challenge is bridging the existing gap between exact results on single-hop networks and the scaling laws which determine the order-of-growth of the end-to-end rates in a multi-hop network as the number of nodes increases [65].

Appendix A

Nomenclature

A.1 Acronyms/Abbreviations

The acronyms or abbreviations employed throughout the thesis are listed here. The meaning of an acronym or an abbreviation is indicated once, when the respective term is first mentioned in the text.

AWGN	Additive White Gaussian Noise
BEP	Bit Error Probability
BICM	Bit Interleaved Coded Modulation
CSI	Channel State Information
CDMA	Code Division Multiple Access
DBLAST	Diagonal Bell Labs Space-Time architecture
DS	Direct Sequence
FEP	Frame Error Probability
FH	Frequency Hopping
i.i.d.	Independent and identically distributed
MA	Multiple Access
MAC	Medium Access Control
MAI	Multiple Access Interference
ML	Maximum Likelihood
MBWA	Mobile Broadband Wireless Access
MRC	Maximal Ratio Combining
MSE	Mean-square error
MMSE	Minimum MSE
LMMSE	Linear MMSE
OFDM	Orthogonal Frequency Division Multiplexing
OFDMA	Orthogonal Frequency Division Multiple Access
PBI	Partial Band Interference
PEP	Pair-wise Error Probability

PC	Power control
pdf	probability density function
mgf	moment generating function
PPP	Poisson Point Process
RX	Receiver
SIC	Successive Interference Cancellation
SINR	Signal to Interference and Noise Ratio
SIR	Signal to Interference Ratio
SNR	Signal to Noise Ratio
SS	Spread Spectrum
TDMA	Time Division Multiple Access
TX	Transmitter
VBLAST	Vertical Bell Labs Space-Time architecture
ZF	Zero Forcing

A.2 Notation

The following table summarizes the notational conventions employed throughout the thesis.

$\mathbb{Z}$	The set of integer numbers.
$\mathbf{I}_n$	The $n \times n$ identity matrix.
$P(E)$	The probability of event E .
$\mathcal{N}(\boldsymbol{\mu}, \mathbf{C})$	The Gaussian distribution with mean $\boldsymbol{\mu}$ and covariance matrix $\mathbf{C}$.
$\mathcal{CN}(\boldsymbol{\mu}, \mathbf{C})$	The circularly symmetric complex Gaussian distribution. with mean $\boldsymbol{\mu}$ and covariance matrix $\mathbf{C}$.
$\mathcal{U}(x_1, x_2)$	The uniform distribution, $x_1 < x < x_2$.
χ_n^2	The central chi-square distribution with parameter $1/2$ and n degrees of freedom.
$E[\cdot]$	The expectation operator.
$(\cdot)^T$	The transpose operation.
$(\cdot)^H$	The conjugate transpose operation.
$\mathbf{x}$	A vector.
$\mathbf{X}$	A matrix.
$\mathcal{X}$	A set of cardinality $ \mathcal{X} $.
$[\mathbf{X}]_{n,t}$	The element in position (n, t) of matrix $\mathbf{X}$.
$\simeq$	Stands for “asymptotic equality of two functions”.
$\approx$	Stands for “is approximately equal to”.
$\sim$	Stands for “follows the distribution ...”.
$\triangleq$	Stands for “defined as”.
e^x or $\exp()$	The exponential function.
$\log_a(x)$	The logarithmic function, defined for $a, x > 0$
$\text{Re}\{\cdot\}$	The real part of a complex number.

$\text{Im}\{\cdot\}$	The imaginary part of a complex number.
$\ \cdot\ $	Euclidean norm or magnitude of a complex number or a vector.
$\text{mod}M$	The modulo M operation, where $M \in \mathbb{Z}^+$
$B(x_1, x_2)$	The Beta function defined for $x_1, x_2 > 0$
$\Gamma(x)$	The Gamma function, defined for $x > 0$
$\Gamma(n, x)$	The incomplete Gamma function, defined for $n, x > 0$

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