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Pivots Versus Signals in Elections

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Abstract

In models of voting, both decision theoretic and game theoretic, incentives tend to hinge on the event that a voter is pivotal. In this paper we consider a model in which voters have private information about their preferences over policy and an election is held in each of two periods. In this setting a vote in the first period can have two potential consequences; it can be pivotal in deciding who wins the first period election and it can inform the beliefs that candidates running in the second period use to select equilibrium platforms and policies. We investigate whether the former, pivot, effect or latter, signalling, effect dominates in large electorates.

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1 Introduction

In nearly all formal models of voting the payoff to voting hinges exclusively on pivot events, in which the election is tied and a single vote can determine the outcome. In decision-theoretic models, a voter decides whether and how to vote, based on exogenously set probabilities of ties between candidates. (Downs 1957, Tullock 1967, Riker and Ordeshook 1968, Myerson and Weber 1993). Game theoretic models endogenize the equilibrium probability that a vote is pivotal (Palfrey and Rosenthal 1983; Myerson 1998, 2000; Campbell 1999; Borgers 2004). Other recent research focuses on information that a voter can infer from the fact that he is pivotal, and analyzes electoral equilibria when voters condition on being pivotal (Feddersen and Pesendorfer 1996, 1999; Austen-Smith and Banks 1996).

All of these pivot-based models of elections have two features in common: (i) when a voter is pivotal the action she takes has a large impact on her payoff, but (ii) pivot events are very unlikely. The large impact is due to the fact that in a pivot event, a single vote can determine the outcome of the election. The low probability arises from the fact that in a large election, the odds of two candidates receiving the same number of votes, or differing by exactly one vote, are miniscule.

Although pivot based models dominate the game-theoretic literature on elections, the infrequency of pivot events in all but the smallest elections raises a natural question: is electoral behavior driven by more than just concerns about being pivotal? For example, in the buildup to the 2006 midterm election in the U.S., pundits speculated that growing dissatisfaction with President Bush's handing of the war in Iraq would cost the Republican party its majority in the legislature. While the loss of Senate and House seats may in fact have been the result of a desire simply to change the composition of the legislature, another explanation is that voters cast ballots for Democrats in order to send Bush a message, and encourage him to change policy. For example, CommonDreams.org, a webpage that presents "breaking news and views for the progressive community" posted the following advice for Democrats on January 25, 2006,

> I have three pieces of advice for them. The first is to nationalize this election. Republicans are obscenely vulnerable at the national level, and the faces of Bush, Cheney, DeLay, Abramoff and the rest, along with their incredibly unpopular policies, should be morphed into those of every Republican candidate for representative, senator, governor and dog catcher, from

> Bangor to Burbank and back again. Democrats need to chain the Bush manacle around the ankles of every last one of them, then throw them into Lake Campaign. This election has every possibility of being 1994 in reverse, if Democrats are smart enough (oh god, please don't get me started on that) to do what Republicans did then: forget local issues and run against a severely unpopular national party.

An emerging literature, based on the intuition that voter totals may matter in elections that don't end in a tie, offers an alternative perspective to the dominant pivot-based theories of elections. Both Shotts (2006) and Meirowitz and Tucker (2006) analyze models in which vote totals affect future candidates' beliefs about the distribution of voters. Razin (2003) analyzes a common values model in which vote totals convey information to the winner of an election, and thus affect the policies he enacts. Razin investigates the limiting behavior as the electorate gets large and finds that in the limit voters care about both victory margins and the identity of the winning candidate. Razin characterizes two types of equilibria. In one type of equilibrium, voters' behavior is "conventional" in the sense that a voter whose private signal indicates that candidate L is more likely to be good than candidate R will tend to vote for candidate L. In any limit of these conventional equilibria, the behavior of voters converges to coin flipping.[1] The other equilibria are "unconventional" in the sense that a voter whose private information indicates that L is the better candidate is more likely to vote for candidate R than a voter whose private information indicates that R is the better candidate.

In Razin's model, the structure of the signalling motivation is a primitive of the game and not the result of equilibrium behavior in a subsequent competitive setting. Razin's findings are, thus, suggestive that a desire to influence decisiveness of victory, and not just the identity of the winner, can remain in large elections. But Razin's model is one of mandates, not repeated elections. As such, the question about whether the signal or pivot motivation dominates in repeated large elections remains unanswered.

Based on this second type of reasoning, we analyze a model of elections with both signaling and pivot motivations. Our analysis addresses the question of which effect is dominant in equilibria to games with repeated elections and a large electorate. In our model, each voter has private information about his own policy preferences, and casts a ballot for one of two available alternatives. Following this first election, two candidates compete for a second office, by staking policy positions, and a second election is held. In

[1] This result is Proposition 4, part (i) of Razin.

equilibrium candidates in the second period base their policy positions on beliefs about the distribution of preferences in the electorate. These beliefs are informed by the vote totals in the first election.

1.1 The Model

Consider an electorate with an odd number of voters $n \geq 3$. It will be convenient to use the fact that $n = 2m + 1$ for some integer m. Let the set of voters be N, and let each voter $i \in N$ have an ideal point, $v_i \in [0, 1]$. We assume that the ideal points are iid draws from a strictly increasing, continuously differentiable cumulative distribution function $F(\cdot)$ with continuous density $f(\cdot)$. Each voter's utility over policy, x, in a given period is $u_i(x) = -|x - v_i|$, and the voter's total utility is simply the sum of his policy utility in the two periods.

In the first period election, two fixed alternatives are available. We denote the locations of the alternatives by $L, R \in [0, 1]$. If voters care only about the first period, or are myopic, elimination of weakly dominated strategies yields a unique equilibrium, in which all voters to the left of $x_p = \frac{L+R}{2}$ vote L and all voters to the right of x_p vote R. We call this the *pivot cutpoint.* We, however, are interested in the dependencies across elections, and thus consider a model with 2 periods, building on Shotts (2006).

In the second period, two office motivated candidates select policy platforms and then the electorate votes. The candidates are assumed to know only the distribution $F(\cdot)$ from which the n ideal points are drawn, the size of the electorate, n, and the voters' first period actions. From Calvert (1985), we know that if $F_{median}(\cdot)$ is the distribution of the median voter's ideal point then in the second period subgame in which candidates choose policy platforms and then voters vote, in any Nash equilibrium with weakly undominated voting, the candidates will both locate at $F^{-1}_{median}(\frac{1}{2})$.[2]

In the two-period signalling game that we study, in any Perfect Bayesian equilibrium, the distribution of the median depends on the first-period votes via Bayes' rule (following a profile of votes that occur with positive probability in equilibrium). Given that $F_{median}(\cdot)$ is the distribution given by Bayes's rule following the observed first period voting, in the second period the candidates both locate at the point $F^{-1}_{median}(\frac{1}{2})$. While Shotts (2006) focuses on equilibria with abstention in the first period, we restrict the set of actions available to voters so that they must vote either L or R; this enables us to focus on a

[2] One such equiulibrium has each voter flipping a fair coin when indifferent. Given strategies for the first period, the second period behavior is standard, and well understood (Calvert 1985, Shotts 2006).

particularly simple class of equilibria, involving only a single cutpoint. In particular, we define a class of equilibria in which all voters use the same type specific voting strategy and this strategy is monotone.

In such an equilibrium first-period voting strategies are characterized by a cutpoint x_c with voters to the left ($v_i < x_c$) voting L and voters to the right ($v_i > x_c$) voting R. If voting satisfies this cutpoint, i.e. it is monotone, then the number of votes for R, denoted $\#R$, captures all of the publicly available information about voter ideal points, and is a sufficient statistic for the second-period candidates' problem of inferring the distribution of the median voter's ideal point from first-period behavior. We denote such a posterior distribution as $F_{median}(\cdot \mid \#R; x_c)$.

Before proceeding we provide a few comments about this function. Given that $\#R$ of n voters have ideal points to the right of (greater than) x_c the median is less than x_c if and only if $\#R \leq m = \frac{n-1}{2}$. Similarly the median is greater than x_c if and only if $\#R \geq m+1$. In the former case, the median is the $m+1$ th lowest ideal point of the $n - \#R$ voters with ideal points less than x_c, i.e., the median ideal point is the $m+1$ th order statistic from $n - \#R$ draws from the distribution $H^{-}(x; x_c) = \frac{F(x)}{F(x_c)}$ with support $[0, x_c]$. Similarly in the latter case, the median is the $(m+1-(n-\#R))$'th order statistic from $\#R$ draws from the distribution $H^{+}(x; x_c) = \frac{F(x)-F(x_c)}{1-F(x_c)}$ with support $[x_c, 1]$.

As mentioned, Calvert's result shows that given x_c, $\#R$ and a mapping $F_{median}(\cdot \mid \#R; x_c)$ sequential rationality of the candidates and weakly undominated voting of the voters implies that second period policy is $F^{-1}_{median}(\frac{1}{2} \mid \#R; x_c)$. Accordingly, in characterizing a cutpoint perfect Bayesian equilibrium with weakly undominated second period voting strategies it is sufficient to characterize a first period cutpoint $x_c \in [0,1]$ such that if every voter other than i is using the strategy with cutpoint x_c it is optimal for voter i to do so as well. Checking this condition hinges on the fact that in an equilibrium of this form second period candidates both locate at the point $F^{-1}_{median}(\frac{1}{2} \mid \#R; x_c)$.

The equilibrium cutpoint balances two effects that influence first period voting. The pivot effect captures the incentive to vote for L if $|L - v_i| < |R - v_i|$ and R if the opposite is true. The signalling motivation captures the incentive to vote for L if given i's expectations about the actions of the other voters, increasing $\#R$ is likely to move the second period policy $F^{-1}_{median}(\frac{1}{2} \mid \#R; x_c)$ away from v_i, and to vote for R if increasing $\#R$ is likely to move the second period policy closer to v_i. The pivot effect is the product of the probability that i is pivotal and the payoff difference between the policy L and R. In contrast to the pivot effect, which captures a low probability event with a non trivial payoff in that event, the signalling motivation takes into account the fact that i's vote will *always* have an effect on the second period policy. However, the signalling effect is small in each of the possible realizations of the votes cast by $N\backslash\{i\}$. For some realizations of the votes by $N\backslash\{i\}$ increasing $\#R$ will be attractive to i, while for other realizations of the these votes increasing $\#R$ will be unattractive to i.

The goal of our paper is to compare high impact, low probability pivot events versus low impact, high probability signaling effects and determine which type of effect dominates in large elections. In particular we investigate the limiting behavior of the cutpoint x_c as n tends to infinity. We find that the limiting cutpoint corresponds to the equilibrium cutpoint in a different game in which the first period is irrelevant, (or, equivalently, $L = R$) so that, in the limit, the cutpoint for voter behavior is identical to what it would be if they were motivated purely by signalling concerns. Thus, we find that while equilibrium voting involves a balancing of these two motivations, in a very strong sense, equilibrium voting in large elections is driven by voters' desire to influence the inferences of observers and not by their desire to influence the election at hand.

This result has potentially important implications for pivot-based models of elections, since most of the interesting equilibria in such models rely heavily on the fact that a voter only cares about events in which his vote is pivotal. In our model, in contrast, the effect of such pivot events on equilibrium voter behavior is relatively unimportant compared to the effect of signaling concerns. At the very least, future research needs to take seriously the possibility that pivot events are not the only thing that matter, and that rational voters take into account future effects that their votes will have, even when an election is not exactly tied.

The paper proceeds as follows. In section 2 we present two concrete examples of how signaling and pivot effects work in our model. Section 3 proves equilibrium existence. Section 4 describes the intuition behind our main result, which is proved in Section 5. Section 6 discusses related literature and implications of our result.

2 Two Examples

We start with a decision-theoretic version of the model, in which there is just one voter with utility function $-|x - v_i|$ where x is the policy and v_i is her ideal point. Suppose the exogenously-fixed first period locations are $L = 0$ and $R = 1/2$. The second period candidates believe that the single voter's ideal point is drawn from a uniform distribution on $[0, 1]$. In this case, if the voter's strategy is monotone with cutpoint x_c then the second period policy will be if $\frac{x_c}{2}$ if i votes for L and $\frac{1+x_c}{2}$ if i votes for R. In order for i to be indifferent between voting L and R when her ideal point is $v_i = x_c$, the following equality must hold

$$-|x_c - 0| - \left|x_c - \frac{x_c}{2}\right| = -|.5 - x_c| - \left|x_c - \frac{1 + x_c}{2}\right|.$$

This equality is solved at $x_c = \frac{1}{3}$.

To illustrate how pivot and signaling effects work in the model, we now consider the simplest model where a vote has a probabilistic effect on both first and second period outcomes. While this example cannot resolve the horse race between the signalling and pivot effects as the number of voters gets large, all of the relevant incentives and quantities of interest are present. Consider $n = 3$ voters, $i \in \{1, 2, 3\}$, with policy ideal points that are *i.i.d.* draws from a uniform distribution on $[0, 1]$. Each voter's utility in a given period is $-|x - v_i|$ where x is policy and v_i is his ideal point. The first-period election is between two candidates, with exogenously-fixed policy positions $L = 0$ and $R = 0.5$.

We first consider two benchmark cases: a pure pivot model and a pure signaling model. In a pure pivot model there is a unique voting equilibrium in weakly undominated strategies: a voter votes for the closer candidate, i.e., she votes for L if her ideal point is to the left of $\frac{L+R}{2} = 0.25$ and votes R if her ideal point is to the right of 0.25. So $x_p = 0.25$ is the *pivot cutpoint.*

For a pure signaling model, all that matters is how a vote affects $F^{-1}_{median}(\frac{1}{2} \mid \#R; x_c)$ through $\#R$. If voters only care about the outcome of the second period election then in the three voter example there is a unique equilibrium, specified by the *signaling cutpoint,* $x_s = 1/2$.

To check that this cutpoint is an equilibrium in the game in which only the second period outcome affects voter payoffs we confirm that a voter with $v_i = 1/2$ is indifferent between voting L and R, given the other actors' strategies. Focusing on voter $i = 1$, assume that the other voters are using this cutpoint strategy. The signaling effect of $i's$ first-period vote thus depends on the other voters' first-period actions:

- With probability $F(x_s) \cdot F(x_s) = \frac{1}{2} \cdot \frac{1}{2} = 0.25$ the other two voters vote L. In this case the second-period policy outcome will be 0.25, if i votes L. This is true because the second-period candidates posterior belief given $\#R = 0$ and $x_s = 1/2$ is that all three voters ideal points are uniform draws from $[0, 0.5]$. If i votes R the second-period policy outcome will be $F_{median}^{-1}\left(\frac{1}{2}|1; 1/2\right) = 0.35$, since there is a 50% chance that both of the L voters, and hence the median, will be to the left of 0.35. Thus, the signaling effect of voting R if both other voters vote L is to move the second-period policy outcome from 0.25 to 0.35.

- With probability $F(x_s) \cdot (1 - F(x_s)) + (1 - F(x_s)) \cdot F(x_s) = 2 \cdot \frac{1}{2} \cdot \frac{1}{2} = 0.5$ the other two voters split their votes. In this case if i votes L the second-period policy outcome will be 0.35 and if she votes R the policy outcome will be 0.65.
- With probability $(1 - F(x_s)) \cdot (1 - F(x_s)) = \frac{1}{2} \cdot \frac{1}{2} = 0.25$ the other two voters vote R. In this case if i votes L the second-period policy outcome will be 0.65 and if i votes R the policy outcome will be 0.75.

Thus if a voter with ideal point v_i votes L her expected second-period utility is $-0.25 \cdot |v_i - 0.25| - 0.5 \cdot |v_i - 0.35| - 0.25 \cdot |v_i - 0.65|$. If she votes R her expected utility is $-0.25 \cdot |v_i - 0.35| - 0.5 \cdot |v_i - 0.65| - 0.25 \cdot |v_i - 0.75|$. A voter at $v_i = x_s = 1/2$ is indifferent between voting L and voting R. It is straightforward to confirm that any voter left of $1/2$ strictly prefers to vote L and a voter to the right prefers to vote R.

Three features of the signalling effects are worth noting, since they will show up in our later analysis of large elections. First, which action, L or R, better promotes the voter's policy interests in the second period depends on the other voters' actions as well as the cutpoint x_s. For a voter with $v_i = 1/2$, if the other two voters vote L then voting R is optimal, whereas if the others vote R then voting L is optimal, and if the others split their votes then the voter is indifferent. Second, the different signalling effects are not equally likely to occur, but rather occur with different probabilities. Third, since the other voters' actions are simply draws from a binomial, in a large election, the most likely realized vote totals will be those where L receives a share close to $F(x_s)$ of the votes and R receives a share close to $1 - F(x_s)$ of the votes. All three of these properties of signalling effects hold regardless of the cutpoint x_s, and they also hold for signalling effects given a combined cutpoint x_c.

In a model with both pivot and signaling effects, we solve for the *combined cutpoint* x_c. As shown in Figure 1, in the three-voter example, $x_c \approx 0.35$.[3] For this x_c the *pivot probability* is $2 \cdot 0.35 \cdot (1 -$

[3] It is just coincidence that this value for x_c is the same (to two decimal places) as the second period policy outcome under

$0.35) = 0.455$. For a voter with $v_i = x_c$ the utility difference between the two possible first-period policy outcomes, L and R, is $|v_i - L| - |v_i - R| = |0.35 - 0| - |0.35 - 1/2| = 0.2$. So i receives, in expectation, $0.2 \cdot 0.455 = 0.091$ more first-period utility by voting R than by voting L.

The second-period signaling effect is a bit more complicated to compute:

- With probability $F(x_c) \cdot F(x_c) = 0.35 \cdot 0.35 = 0.1225$ the other two voters vote L. In this case the second-period policy outcome will be 0.175 if i votes L. If i votes R the second-period policy outcome will be 0.244.

- With probability $F(x_c) \cdot (1 - F(x_c)) + (1 - F(x_c)) \cdot F(x_c) = 2 \cdot 0.35 \cdot (1 - 0.35) = 0.455$ the other two voters split their votes. In this case if i votes L the second-period policy outcome will be 0.244 and if i votes R the policy outcome will be 0.537.

- With probability $(1 - F(x_c)) \cdot (1 - F(x_c)) = (1 - 0.35) \cdot (1 - 0.35) = 0.4225$ the other two voters vote R. In this case if i votes L the second-period policy outcome will be 0.537 and if i votes R the policy outcome will be 0.675.

Thus if a voter with ideal point v_i votes L his expected second-period utility is $-0.1225 \cdot |v_i - 0.175| - 0.455 \cdot |v_i - 0.244| - 0.4225 \cdot |v_i - 0.537|$, which equals -0.1486 for $v_i = 0.35$. And if i votes R his expected utility is $-0.1225 \cdot |v_i - 0.244| - 0.455 \cdot |v_i - 0.537| - 0.4225 \cdot |v_i - 0.675|$, which equals -0.2354 for $v_i = 0.35$. The difference is equal to 0.091, so it exactly counteracts the first-period utility gain that the voter at $x_c = 0.35$ receives by voting R rather than L. Thus at x_c the pivot and signaling effects cancel each other out and the voter is indifferent.

This example illustrates the basic tension between pivot and signaling effects in our model. In this three voter example, the equilibrium cutpoint is $x_c = 0.35$, which lies between the pivot cutpoint, $x_p = 0.25$, and the signaling cutpoint, $x_s = 0.5$. The question is how a sequence of cutpoints $\{x_m\}$ will behave in the limit as the population size $n = 2m + 1$ gets large.

The difficulty in answering these questions is that in large elections both the pivot effect and the signaling effect become small; the probability of a pivot event goes to zero and the distance that second-period candidates move in response to a single vote also goes to zero. The question is which converges faster.

one of the action profiles in the pure signalling model. For the general model, they need not be the same.

3 Preliminary Results

In this section we establish two lemmas that are useful in establishing existence of a particular type of equilibria for any n as well as proving the main result about the limiting behavior of this type of equilibrium. We then present the existence result. Our analysis focuses on a particular class of equilibria.

Definition 1 (Symmetric Cutpoint Strategy) *Voters use a symmetric cutpoint strategy if there exists a point $x_c \in [0,1]$ such that for all $i \in N$*

(1) if $x_c = 0$ then vote L if $v_i = 0$, and R otherwise

(2) if $x_c \in (0,1]$, then vote L if $v_i < x_C$, and R if $v_i \geq x_c$.

Given that all other voters use a symmetric cutpoint strategy with cutpoint x_c, optimal behavior for a voter with ideal point v_i depends on the difference in her expected utility between voting R and voting L in the first period. Using $a_i^1 \in \{L, R\}$ to denote voter $i's$ first period action, we can express this difference as

$$u_{dif}(v_i) \equiv u(a_i^1 = R|v_i) - u(a_i^1 = L|v_i) = u_{dif1}(v_i) + u_{dif2}(v_i) \tag{1}$$

where

$$u_{dif1}(v_i) \equiv \binom{2m}{m} F(x_c)^m (1 - F(x_c))^m (|L - v_i| - |R - v_i|) \tag{2}$$

and

$$u_{dif2}(v_i) \equiv \sum_{k=0}^{2m} \binom{2m}{k} (F(x_c))^{2m-k} (1 - F(x_c))^k \begin{pmatrix} \left|F_{median}^{-1}(\frac{1}{2}|k, x_c) - v_i\right| \\ - \left|F_{median}^{-1}(\frac{1}{2}|k+1, x_c) - v_i\right| \end{pmatrix}. \tag{3}$$

Thus $u_{dif1}(v_i)$ captures the first period effect of voting: the pivot probability is $\binom{2m}{m}(F(x_c))^m (1 - F(x_c))^m$ and the utility difference between the two candidates is $|L - v_i| - |R - v_i|$ for a voter with ideal point v_i. Likewise, $u_{dif2}(v_i)$ captures the second period effect: the probability that k other voters vote R is $\binom{2m}{k}(F(x_c))^{2m-k}(1 - F(x_c))^k$ and the utility difference between voting R versus L in this event is $\left|F_{median}^{-1}(\frac{1}{2} \mid k; x_c) - v_i\right| - \left|F_{median}^{-1}(\frac{1}{2} \mid k+1; x_c) - v_i\right|$.

Remark 1: Deriving $F_{median}(y|\#R; x_c)$ for $x_c \in (0,1)$

To understand $u_{dif2}(v_i)$ it is important to see how $F_{median}(y|\#R; x_c)$ depends on $\#R$ and x_c. This function is easy to characterize in terms of order statistics. We note that for fixed $x_c \in (0,1)$, the distribution of the median is constructed as follows. Given that there are $n - \#R$ draws with values strictly less than x_c and $\#R$ draws with values greater than or equal to x_c we know that the median is less than x_c if $\#R$ is strictly less than $m+1$ and it is greater than x_c otherwise. If $\#R < m+1$, the median is the $m+1$ largest of $n-\#R$ draws from the distribution $F(\cdot \mid \{< x_c\})$ which we denote as $H^-(y; x_c) = \max\{0, \min\{1, \frac{F(y)}{F(x_c)}\}\}$. Accordingly if $\#R < m+1$, $F_{median}(y|\#R; x_c) = H^-_{m+1,n-\#R}(y; x_c)$, which is the distribution of the $m+1$ th order statistic from $n-\#R$ draws from the distribution function $H^-(\cdot; x_c)$. Similarly if $\#R \geq m+1$, the median is the $(m+1-(n-\#R))$'th order statistic from $\#R$ draws from the distribution $F(\cdot \mid \{> x_c\})$. This distribution is denoted as $H^+(y; x_c) = \max\{0, \min\{1, \frac{F(y)-F(x_c)}{1-F(x_c)}\}\}$. So if $\#R \geq m+1$ then $F_{median}(y|\#R; x_c) = H^+_{m-(n-\#R),\#R}(y; x_c)$, which is the distribution of the $(m+1-(n-\#R))$'th order statistic from $\#R$ draws from the distribution function $H^+(\cdot; x_c)$.$\blacksquare$

Remark 2: Deriving $F_{median}(y|\#R; x_c)$ for $x_c \in \{0, 1\}$

Now consider extremal cutpoints, $x_c \in \{0, 1\}$. If $x_c = 0$ then according to definition 1, all voters with ideal points in $(0, 1]$ vote R and voters with ideal point $v_i = 0$ vote L. Accordingly, if $\#R < m+1$ then the median voter's ideal point is 0 with probability 1 and $F_{median}(y|\#R; 0)$ is constant at 1 for all $y \in [0, 1]$. In this case, we define $F^{-1}_{median}(\frac{1}{2} \mid \#R; 0) = 0$ and it is clear that equilibrium second period candidate locations are at 0. If $\#R \geq m+1$ then the median is the $(m+1-(n-\#R))$'th order statistic from $\#R$ draws from $F(\cdot)$. This distribution corresponds to $H^+(y; x_c) = \frac{F(y)-F(0)}{1-F(0)}$ with support $[0, 1]$. If $x_c = 1$ then according to definition 1, all voters with ideal point 1 vote R and all voters with ideal points $v_i \in [0, 1)$ vote L. Accordingly, if $\#R \geq m+1$ then the median voter's ideal point is at 1 with probability 1 and $F_{median}(y|\#R; 1)$ is constant at 0 for all $y \in [0, 1)$ and equal to 1 at $y = 1$. In this case we define $F^{-1}_{median}(\frac{1}{2} \mid \#R; 1) = 1$. If $\#R < m+1$ then the median is the $m+1$th order statistic from $n-\#R$ draws from $F(\cdot)$. This distribution corresponds to $H^-(y; x_c) = \frac{F(y)}{F(1)}$ on $[0, x_c]$.$\blacksquare$

One way to see how the distribution function $F_{median}(\cdot|\cdot; \cdot)$ behaves as the arguments $\#R$ and x_c change is to consider the case of the uniform, $F(y) = y$ on $[0, 1]$. Figure 1 plots the function $H^+(y; x_c)$ for two distinct values $x'_c < x''_c$ and figure 2 plots the function $H^-(y; x_c)$ for two distinct values $x'_c < x''_c$.

[Figures 1 and 2 about here – Figure 2 is yet to be made]

Combining remarks 1 and 2 allows us to express the second period policy location as a function of x_c and $\#R$ in a symmetric cutpoint equilibrium

$$\chi(x_c, \#R) = \begin{cases} 0 \text{ if } x_c = 0 \text{ and } \#R < m+1 \\ 1 \text{ if } x_c = 1 \text{ and } \#R \geq m+1 \\ \{x : H^{-}_{m+1,n-\#R}\,(x; x_c) = \frac{1}{2}\} \text{ if } x_c \in (0,1] \text{ and } \#R < m+1 \\ \{x : H^{+}_{m+1-(n-\#R),\#R}\,(x; x_c) = \frac{1}{2}\} \text{ if } x_c \in [0,1) \text{ and } \#R \geq m+1 \end{cases}$$

The first lemma builds on this derivation to establish properties of the distribution of the median and the above mapping when voters use a symmetric cutpoint strategy.

Lemma 1 (Properties of Second Period Policy Outcomes) *If voters use a symmetric cutpoint strategy with cutpoint x_c then*

(1) For each $\#R < m+1$ and $y \in [0,1]$, $F_{median}(y|\#R; x_c)$ is weakly decreasing in x_c for $x_c \in [0, y)$ and strictly decreasing for $x_c \in [y, 1]$ and for each $\#R \geq m+1$ and $y \in (0,1)$, $F_{median}(y|\#R; x_c)$ is weakly decreasing in x_c for for $x_c \in (y, 1]$ and strictly decreasing in $x_c \in [0, y]$.

(2) If $\#R_1 < \#R_2$ (both in $0, 1, 2, ..., n$) then for each $x_c \in (0,1)$, for some set $A_{x_c} \subset [0,1]$ with positive lebesgue measure $F_{median}(y|\#R_1; x_c) > F_{median}(y|\#R_2; x_c)$ if $y \in A_{x_c}$ and $F_{median}(y|\#R_1; x_c)$ $\geq F_{median}(y|\#R_2; x_c)$ for all $y \in [0,1]$. For $x_c \in \{0,1\}$, $F_{median}(y|\#R_1; x_c) \geq F_{median}(y|\#R_2; x_c)$ for all $y \in [0,1]$.

(3) For any $\#R \in \{0, 1, 2, ..., n-1\}$ and $x_c \in [0,1]$, $F^{-1}_{median}(\frac{1}{2} \mid \#R; x_c) \leq F^{-1}_{median}(\frac{1}{2} \mid \#R+1; x_c)$.

(4) $F_{median}(y|\#R; x_c)$ is continuous in x_c on $(0,1)$ for each $\#R \in \{0, ..., n\}$ and $y \in [0,1]$ as well as continuous in y on $[0,1]$ for each $\#R \in \{0, ..., n\}$ and $x_c \in (0,1)$.

(5) The mapping $\chi(x_c, \#R)$ is a function from $[0,1] \times \{1, 2...., n\}$ into $[0,1]$ and it is continuous in x_c.

Proof:

(1) Assume $\#R < m+1$. From our derivation of $F_{median}(y|\#R; x_c)$ in remark 1 this distribution takes on the value 1 if $y \geq x_c$ and $H^{-}_{m+1,n-\#R}\,(x; x_c)$ otherwise. Thus the conclusion that it is weakly decreasing for $x_c \in [y, 1]$ is immediate. Consider $x_c < x_c'$. Since

$$\frac{F(y)}{F(x_c')} < \frac{F(y)}{F(x_c)}$$

$H^{-}(y; x_c') < H^{-}(y; x_c)$ for all $x < x_c$ and thus the former first order stochastically dominates the latter on $[0, x_c]$. This ordering of $H^{-}(y; x_c')$ and $H^{-}(y; x_c)$ implies that the distributions of order statistics,

$H^{-}_{m+1,n-\#R}\,(y;x_c')$ and $H^{-}_{m+1,n-\#R}\,(y;x_c)$ are also ordered by first order stochastic dominance (see for example Theorem 4.4.1 of David and Nagaraja p. 75). An analogous argument holds in the case of $\#R \geq m+1$. For $y < x_c$ the distribution takes on the value 1 and the conclusion that it is weakly decreasing for $x_c \in [0, y)$ is immediate. If $y \geq x_c$ then $F_{median}(y|\#R;x_c) = H^{+}_{m+1-(n-\#R),\#R}\,(y;x_c)$ which is the distribution of an order statistic from $\frac{F(y)-F(x_c)}{1-F(x_c)}$. Since $x_c < x_c'$ the fact that $F(x_c) < F(x_c') < 1$ implies that

$$\frac{F(y)-F(x_c')}{1-F(x_c')} < \frac{F(y)-F(x_c)}{1-F(x_c)}$$

and thus $H^{+}_{m+1-(n-\#R),\#R}\,(y;x_c')$ and $H^{+}_{m+1-(n-\#R),\#R}\,(y;x_c)$ are ordered by first order dominance.

(2) To establish strict monotonicity in $\#R$. Consider two integers, $\#R_1$ and $\#R_2$, with $0 \leq \#R_1 < \#R_2 \leq n$. If $\#R_1 < m+1 \leq \#R_2$ then the support of $F_{median}(\cdot|\#R_1;x_c)$ is $[0, x_c]$ and the support of $F_{median}(\cdot|\#R_2;x_c)$ is $[x_c, 1]$. Since the distribution $F(\cdot)$ is strictly increasing on $[0,1]$, $F_{median}(y|\#R_1;x_c) > 0$ for all $y \in (0, x_c)$ while $F_{median}(y|\#R_1;x_c) = 1$ for all $y \geq x_c$. Similarly, $F_{median}(y|\#R_2;x_c) = 0$ for all $y \leq x_c$ and $F_{median}(y|\#R_2;x_c) < 1$ for $y \in (x_c, 1)$. Thus, $F_{median}(y|\#R_2;x_c) \leq F_{median}(y|\#R_1;x_c)$, with a strict inequality for any $y \notin \{0,1\}$.

Suppose instead that $\#R_1 < \#R_2 < m+1$. The relevant comparison is now between $H_{m+1,n-\#R_1}\,(y;x_c)$ and $H_{m+1,n-\#R_2}\,(y;x_c)$. To see that these two distributions are ordered by first order stochastic dominance, we can partition $n-\#R_1$ draws from $F(\cdot)$ into two sets: first $n-\#R_2$ draws are taken and then another $\#R_2-\#R_1$ are taken. Because $F(\cdot)$ is strictly increasing on $[0,1]$, the probability that one of the $\#R_2-\#R_1$ draws is less than the $m+1$ highest draw of the first $\#R_2-\#R_1$ draws is strictly positive, and thus $H_{m+1,n-\#R_2}\,(y;x_c) < H_{m+1,n-\#R_1}\,(y;x_c)$ for y on $[0,x_c]$. This implies that $F_{median}(y|\#R_2;x_c)$ $\leq F_{median}(y|\#R_1;x_c)$ with a strict inequality if $y \in A_{x_c} = [0, x_c)$ if $\#R_1 < \#R_2 < m+1$. A similar argument holds for $A_{x_c} = (x_c, 1]$ if $m+1 \leq \#R_1 < \#R_2$.

The result for $x_c \in \{0,1\}$ follows from Remark 2.

(3) Follows immediately from (2).

(4) Continuity of $F_{median}(y|\#R;x_c)$ in x_c on $(0,1)$ for each $\#R \in \{0,...,n\}$ and $y \in [0,1]$ as well as continuity in y on $[0,1]$ for each $\#R \in \{0,...,n\}$ and $x_c \in (0,1)$ follows from the assumption that $F(\cdot)$ is strictly increasing and continuously differentiable and the well known fact the distribution of an order statistic from a differentiable distribution function has a density. In particular for $\#R < m+1$ the distri-

bution $F_{median}(y|\#R;x_c)$ has a density $h^-_{m+1,n-\#R}(y;x_c) = k\left[\frac{\partial}{\partial y}\left(\frac{F(y)}{F(x_c)}\right)\right]\frac{F(y)}{F(x_c)}^a(1-\frac{F(y)}{F(x_c)})^b$ for integers k, a, b. For $\#R < m+1$, the distribution $F_{median}(y|\#R;x_c)$ has a density $h^+_{m+1-(n-\#R),\#R}(y;x_c) = k'\left[\frac{\partial}{\partial y}\left(\frac{F(y)-F(x_c)}{1-F(x_c)}\right)\right]\left(\frac{F(y)-F(x_c)}{1-F(x_c)}\right)^{a'}(1-\left(\frac{F(y)-F(x_c)}{1-F(x_c)}\right))^{b'}$ for some k', a', b'. Since we have assumed that $F(\cdot)$ has a continuous density, for fixed $\#R$, as long as $x_c \in (0,1)$ the above densities are well defined and thus the distribution functions are continuous.

(5) To show that $\chi(x_c, \#R)$ is defined on its domain we must show that $\{x : H^-_{m+1,n-\#R}(x;x_c) = \frac{1}{2}\}$ is non-empty if $x_c \in (0,1]$ and $\#R < m+1$ and that $\{x : H^+_{m+1-(n-\#R),\#R}(x;x_c) = \frac{1}{2}\}$ is non-empty if $x_c \in [0,1)$ and $\#R \geq m+1$. In the first case, consider $x_c \in (0,1]$ and $\#R < m+1$. From the proof of part 4 of this lemma we see that $H^-_{m+1,n-\#R}(x;x_c)$ has a continuous density function that is strictly positive as long as $y < x_c$. So the function $H^-_{m+1,n-\#R}(y;x_c)$ is continuous and strictly increasing in y on $[0, x_c]$ with $0 = H^-_{m+1,n-\#R}(0;x_c) < \frac{1}{2} < H^-_{m+1,n-\#R}(x_c;x_c) = 1$. This means that the set $S^-(x_c, \#R) = \{y \in [0,1] : H^-_{m+1,n-\#R}(y;x_c) \in (0,1)\}$ is non-empty for $x_c \in (0,1]$ and $\#R < m+1$. Moreover, by the intermediate value theorem this means that the set $\{x : H^-_{m+1,n-\#R}(x;x_c) = \frac{1}{2}\}$ is non-empty if $x_c \in (0,1]$ and $\#R < m+1$. An analogous argument establishes that $\{x : H^+_{m+1-(n-\#R),\#R}(x;x_c) = \frac{1}{2}\}$ is non-empty if $x_c \in [0,1)$ and $\#R \geq m+1$.

To establish continuity we consider two cases. First assume that $\#R < m+1$. By part 4 of this lemma, for a fixed y, $H^-_{m+1,n-\#R}(y;x_c)$ is continuous in x_c on $(0,1)$ and thus this and the fact that it is strictly increasing (and has a density) in y on a neighborhood of the point $\{x : H^-_{m+1,n-\#R}(x;x_c) = \frac{1}{2}\}$ implies by way of the implicit function theorem that the solution $\chi(x_c, \#R)$ is continuous in x_c if $x_c \in (0,1)$ and $\#R < m+1$. Continuity at $x_c = 0$ follows from the fact $\chi(x_c, \#R) \leq x_c$ if $\#R < m+1$ and thus $\lim_{x_c \to 0} \chi(x_c, \#R) = 0$ and $\chi(0, \#R) = 0$. Continuity at $x_c = 1$ follows from the fact that $H^-_{m+1,n-\#R}(y;1)$ is defined and for each y, $H^-_{m+1,n-\#R}(y;x_c)$ is continuous in x_c at 1. An analogous argument about $H^+_{m+1-(n-\#R),\#R}(x;x_c)$ establishes continuity in the case of $\#R \geq m+1$.■

The next result establishes some properties of the utility difference function in equation 1. Since this result simply uses conclusions from lemma 1 in standard ways the proof is in the appendix.

Lemma 2 (Properties of Utility Difference Function) *If voters use a symmetric cutpoint strategy with cutpoint x_c then*

(1) $u_{dif}(v_i)$ is continuous and weakly increasing in v_i.

(2) (Lpschitz property) $\forall \tilde{v}_i, \hat{v}_i \in [0,1]$, $|u_{dif}(\tilde{v}_i) - u_{dif}(\hat{v}_i)| \leq 3 \cdot |\tilde{v}_i - \hat{v}_i|$.

(3) $u_{dif}(0) \leq 0$ *and* $u_{dif}(1) \geq 0$.

We can now state our first main result. The proof, which applies a standard fixed point argument to the function $u_{dif}(\cdot)$, is in the appendix.

Proposition 1 *There exists an equilibrium in which voters use a symmetric cutpoint strategy in the first period.*

4 Intuition for main result

Having established existence, we now turn to the question of equilibrium behavior in large electorates, i.e., as $m \to \infty$. We suppress the c subscript and let x_m denote the cutpoint in a symmetric cutpoint strategy equilibrium with $n = 2m+1$ voters. Our interest is then in $\lim_{m\to\infty} x_m$ (if it exists). We show that this limit is equal to the point $F^{-1}(\frac{1}{2})$. Since this limit does not depend on the first period candidate locations L and R, it is also the limit of cutpoints for equilibria in the pure signalling game. The proof proceeds by contradiction; we show that if a sequence of cutpoints does not converge to $F^{-1}(1/2)$ then these cutpoints cannot be equilibrium cutpoints because voters at the cutpoint x_m, who must be indifferent in equilibrium, will strictly prefer to vote for one candidate over the other. To be more precise, we show that if any subsequence of equilibrium cutpoints converges to a point other than $F^{-1}(\frac{1}{2})$ then for m sufficiently large a voter with ideal point x_m will strictly prefer to vote for one candidate over the other. This brief section serves as a roadmap for the proof, presenting an informal version of the argument. The next section contains a proof of the main result.

Suppose that in a large electorate voters behave according to a cutpoint $x_m > F^{-1}(1/2)$. We show that a voter at x_m will strictly prefer to vote R. There are three types of effects that the voter must consider:

The first consideration is a pivot effect, which we label *PV*. Since the election is not expected to be a tie, i.e., $x_m \neq F^{-1}(1/2)$, and the population size is large, the probability of this pivot event is exceedingly small in a large electorate.

The second consideration involves bad signalling effects from voting R; whenever more than half of the other voters vote R, the second period policy will be to the right of x_m, so if a voter with $v_i = x_c$

votes R this will move second period policy to the right, i.e., away from his ideal point, as established in part (3) of Lemma 1. However, because $x_m > F^{-1}(1/2)$, more than half of the votes are expected to go to L, and thus in a large electorate bad signalling effects are extremely unlikely to occur. We find an upper bound on the probability-weighted sum of these bad signalling effects, and label it $UBBS$ (upper bound for bad signalling).

The third consideration involves good signalling effects from voting L; whenever more than half of the other voters vote L, the second period policy will be to the left of x_c, so if a voter with $v_i = x_m$ votes R this will move second period policy to the right, i.e., towards his ideal point. Since $x_m > F^{-1}(1/2)$, more than half of the votes are expected to go to L, and thus in a large electorate it is extremely likely that the signalling effect of voting R will be good. We find a lower bound on the probability-weighted sum of these good signalling effects, and label it $LBGS$ (lower bound for good signalling).

We consider the ratio of bad signalling plus pivot effects to good signalling effects, and show that this ratio

$$\frac{PV + UBBS}{LBGS}$$

can be expressed as a limit of the form

$$\frac{P_{tie} + (m+1)\, P_{tie}}{cP} \to 0. \tag{4}$$

In this expression, $P_{tie} = \binom{2m}{m} F(x_m)^m \,(1 - F(x_m))^m$ is the probability of an exact tie among the other $2m$ voters given the cutpoint x_m. In the denominator, P is the probability of a certain type of good signalling effect, and P goes to zero much more slowly than P_{tie}. The c in the denominator is a constant that does not depend on m. At the end of the proof we show that the left hand side of Equation 4 reduces to an expression of the form $(m+2)\, q^{\left(\frac{1}{2}-c_1\right)m}$ with constants $q \in (0,1)$, and $c_1 \in (0, 1/2)$. Thus the limit is 0, which means that for the voter at x_m it will be optimal to deviate and vote R.

5 The convergence result

Our main result is

Proposition 2 $\lim_{m\to\infty} x_m = F^{-1}(1/2)$.

Proof: Assume by way of a contradiction that the cutpoints do not converge to the point $M \equiv F^{-1}(1/2)$. Since $x_m \in [0,1]$, $\forall m$, the Bolzano-Weierstrass Theorem implies that there exists some number $Z \in [0,1]$ with $Z \neq M$ such that a subsequence $\{x_{m'}\} \to Z$. We focus on such a subsequence, ignoring the residual portion of the original sequence, thus the assumption that Proposition 2 is false equates to the claim that $\{x_m\} \to Z$. Either $Z < M$ or $Z > M$. In the remainder of the proof we focus on the latter case; the argument for the former case is virtually identical and is thus omitted. Thus, our goal is to show that there exists a $\bar{m}$ such that if $m > \bar{m}$ then a voter with ideal point x_m has a strict preference to vote for R. Once this claim is established the continuity of the utility functions established in Lemma 2 implies that there exists a $\delta > 0$ such that if $v_i \in (x_m - \delta, x_m + \delta)$ a voter with ideal point v_i prefers to vote for R. Thus for some voters to the left of x_m voting L is not a best response, contradicting the hypothesis that x_m is an equilibrium cutpoint when the population size is $2m+1$. This contradiction means that we cannot have a subsequence of cutpoints converging to $Z \neq M$ and thus the sequence of cutpoints converges to M.

For each m, consider a voter, i, with ideal point x_m. Given that voters to the left of x_m vote L and voters to the right of x_m vote R, the probability of any individual voting R is

$$p_m \equiv 1 - F(x_m).$$

Since $x_m > F^{-1}(1/2)$ we know that $p_m < \frac{1}{2}$.

We start by analyzing the utility function of a voter with ideal point x_m. If exactly m voters other than i vote R then the election is tied, and i's vote is pivotal in determining the first period policy. However, in terms of first period motivations, which depend on the candidate locations L and R, it is not clear whether i prefers to vote L or vote R in the event that he is pivotal. In terms of second period motivations, which depend on candidate locations given $\#R$, it is also unclear whether i prefers to vote L or vote R in the event that he is pivotal.

In contrast, the voter's preferences are clear for events in which he is not pivotal. If $m-1$ or fewer of the other voters vote for R the second policy will be to the left of x_m regardless of i's vote and if at least $m+1$ of the other voters vote for R then the second policy will be to the right of x_m regardless of i's vote (as established in Remark 1). These facts and the monotonicity of the second period policy in $\#R$ (part 3 of Lemma 1) imply that if $m-1$ or fewer of the other voters vote for R then a vote for R

moves the second period policy *closer* to i's ideal point. In contrast, if at least $m+1$ of the other voters vote for R, then a vote for R moves the second period *away* from i's ideal point. Note that in either of these cases, i's vote cannot move policy far enough to leapfrog her ideal point, x_m (see Remark 1).

Following equations 1, 2, and 3, given the conjectured equilibrium for population size $n = 2m+1$, the utility difference between voting R versus voting L for a voter with ideal point v_i in the equilibrium with population size $2m+1$ is

$$u_{dif}^{m}\left(v_i\right) \equiv u_{dif1}^{m}\left(v_i\right) + u_{dif2}^{m}\left(v_i\right).$$

In particular we are interested in the utility difference for a voter with ideal point x_m, i.e.,

$$u_{dif}^{m}\left(x_m\right) \equiv u_{dif1}^{m}\left(x_m\right) + u_{dif2}^{m}\left(x_m\right)$$

which we re-write as

$$\begin{aligned} u_{dif}^{m}\left(x_m\right) \;=\; & \binom{2m}{m}\left(1-p_m\right)^m {p_m}^m \left(\left|L-x_m\right| - \left|R-x_m\right|\right) \qquad (5) \\ & + \sum_{k=0}^{m-1}\binom{2m}{k}\left(1-p_m\right)^{2m-k} {p_m}^k \left(\left|F_{median}^{-1}\left(\frac{1}{2}|k;x_m\right) - x_m\right| - \left|F_{median}^{-1}\left(\frac{1}{2}|k+1;x_m\right) - x_m\right|\right) \\ & + \binom{2m}{m}\left(1-p_m\right)^m {p_m}^m \left(\left|F_{median}^{-1}\left(\frac{1}{2}|m;x_m\right) - x_m\right| - \left|F_{median}^{-1}\left(\frac{1}{2}|m+1;x_m\right) - x_m\right|\right) \\ & + \sum_{k=m+1}^{2m}\binom{2m}{k}\left(1-p_m\right)^{2m-k} {p_m}^k \left(\left|F_{median}^{-1}\left(\frac{1}{2}|k;x_m\right) - x_m\right| - \left|F_{median}^{-1}\left(\frac{1}{2}|k+1;x_m\right) - x_m\right|\right). \end{aligned}$$

The first line of Equation 5 is the pivot effect. The second line represents good signalling effects of voting R, when $m-1$ or fewer other voters vote R. The third line represents the indeterminate signalling effect when the $2m$ other voters split their votes equally between L and R. The fourth line represents bad signalling effects, when $m+1$ or more other voters vote R.

Our ultimate goal is to show that there exists an $\bar{m}$ such that for $m > \bar{m}$, $u_{dif}^{m}\left(x_m\right) > 0$. To simplify the expression in Equation 5, we first simplify each component of the utility function, by finding upper bounds on bad effects, and a lower bound on good effects, of voting R.

Pivot effect. The pivot effect can be either positive or negative, depending on the positions of the two first-period candidates. An upper bound, based on the fact that L, R, and x_m are all in the interval $[0,1]$, will be sufficient:

$$\binom{2m}{m}\left(1-p_m\right)^m {p_m}^m \left(\left|L-x_m\right| - \left|R-x_m\right|\right) > -\binom{2m}{m}\left(1-p_m\right)^m {p_m}^m. \qquad (6)$$

Bad and indeterminate signalling effects. For any $k \in \{1, .., n-1\}$, $F^{-1}_{median}\left(\frac{1}{2}|k; x_m\right) \in (0,1)$ and $F^{-1}_{median}\left(\frac{1}{2}|k+1; x_m\right) \in (0,1)$, so

$$\sum_{k=m+1}^{2m} \binom{2m}{k} (1-p_m)^{2m-k} {p_m}^k \left(\left| F^{-1}_{median}\left(\frac{1}{2}|k; x_m\right) - x_m \right| - \left| F^{-1}_{median}\left(\frac{1}{2}|k+1; x_m\right) - x_m \right| \right)$$
$$+ \binom{2m}{m} (1-p_m)^{m} {p_m}^m \left(\left| F^{-1}_{median}\left(\frac{1}{2}|m; x_m\right) - x_m \right| - \left| F^{-1}_{median}\left(\frac{1}{2}|m+1; x_m\right) - x_m \right| \right)$$
$$> - \sum_{k=m}^{2m} \binom{2m}{k} (1-p_m)^{2m-k} {p_m}^k$$

The binomial expansion is monotonic, i.e., since $p_m < 1/2$, for any $k \in \{m+1, ..., 2m\}$, $\binom{2m}{k} (1-p_m)^{2m-k} {p_m}^k < \binom{2m}{m} (1-p_m)^m {p_m}^m$ so the total of the bad and indeterminate signalling effects must be strictly greater than

$$-(m+1)\binom{2m}{m}(1-p_m)^m {p_m}^m. \tag{7}$$

Good signalling effects. We now develop a lower bound on good signalling effects. Fix any points A and B in the unit interval such that $M < A < B < Z$. For any m, let $A_m \in [M, A]$ represent the largest number less than A such that for some integer $a_m < 2m+1$ it is the case that $A_m = F^{-1}_{median}\left(\frac{1}{2}|a_m; x_m\right)$. Likewise, let $B_m \in [B, Z]$ represent the largest number less than B such that for some integer $b_m < 2m+1$ it is the case that $B_m = F^{-1}_{median}\left(\frac{1}{2}|b_m; x_m\right)$. For the b_m identified in the definition of B_m let $C_m = F^{-1}_{median}\left(\frac{1}{2}|b_m+1; x_m\right)$. By construction, $A_m < A < B_m < B < C_m$

For fixed m the set of profiles for other voters that, given $i's$ vote, can result in a policy between A_m and C_m consists of profiles for which the number of other voters that vote R is in the set $\{a_m, a_m+1, ..., b_m-1, b_m\}$. Although we cannot analytically solve for the policy distance between $F^{-1}_{median}\left(\frac{1}{2}|k; x_m\right)$ and $F^{-1}_{median}\left(\frac{1}{2}|k+1; x_m\right)$ for particular values of k, we do know that

$$\sum_{k=a_m}^{b_m} \left(F^{-1}_{median}\left(\frac{1}{2}|k+1; x_m\right) - F^{-1}_{median}\left(\frac{1}{2}|k; x_m\right) \right) = C_m - A_m > B - A$$

Moreover, for m large enough, $M < A_m$. Since $M < A_m < C_m$ and $p_m < 1/2$ we know from monotonicity of the binomial expansion that the event in which b_m others vote for R is the least likely of the set of profiles of the $2m$ other voters in which i's vote can result in a policy in the interval $[A_m, C_m]$. We thus can re-write the good signalling effects term from Equation 5 as

$$\sum_{k=0}^{m-1} \binom{2m}{k} (1-p_m)^{2m-k} {p_m}^k \left(\left| F^{-1}_{median}\left(\frac{1}{2}|k; x_m\right) - x_m \right| - F^{-1}_{median}\left(\frac{1}{2}|k+1; x_m\right) \right)$$

$$\begin{aligned}
&= \sum_{k=0}^{a_m-1}\binom{2m}{k}(1-p_m)^{2m-k}{p_m}^k\left(\left|F^{-1}_{median}\left(\frac{1}{2}|k;x_m\right)-x_m\right|-\left|F^{-1}_{median}\left(\frac{1}{2}|k+1;x_m\right)-x_m\right|\right)\\
&\quad+\sum_{k=a_m}^{b_m}\binom{2m}{k}(1-p_m)^{2m-k}{p_m}^k\left(\left|F^{-1}_{median}\left(\frac{1}{2}|k;x_m\right)-x_m\right|-\left|F^{-1}_{median}\left(\frac{1}{2}|k+1;x_m\right)-x_m\right|\right)\\
&\quad+\sum_{k=b_m+1}^{m-1}\binom{2m}{k}(1-p_m)^{2m-k}{p_m}^k\left(\left|F^{-1}_{median}\left(\frac{1}{2}|k;x_m\right)-x_m\right|-\left|F^{-1}_{median}\left(\frac{1}{2}|k+1;x_m\right)-x_m\right|\right)\\
&> \sum_{k=a_m}^{b_m}\binom{2m}{k}(1-p_m)^{2m-k}{p_m}^k\left(\left|F^{-1}_{median}\left(\frac{1}{2}|k;x_m\right)-x_m\right|-\left|F^{-1}_{median}\left(\frac{1}{2}|k+1;x_m\right)-x_m\right|\right)\\
&> \binom{2m}{b_m}(1-p_m)^{2m-b_m}p_m^{b_m}\sum_{k=a_m}^{b_m-1}\left(\left|F^{-1}_{median}\left(\frac{1}{2}|k;x_m\right)-x_m\right|-\left|F^{-1}_{median}\left(\frac{1}{2}|k+1;x_m\right)-x_m\right|\right) \quad (8)
\end{aligned}$$

Note that, since $\forall k\in\{a_m,a_m+1,...,b_m\}$, $F^{-1}_{median}\left(\frac{1}{2}|k;x_m\right)<F^{-1}_{median}\left(\frac{1}{2}|k+1;x_m\right)<x_m$,

$$\begin{aligned}
&\sum_{k=a_m}^{b_m}\left(\left|F^{-1}_{median}\left(\frac{1}{2}|k;x_m\right)-x_m\right|-\left|F^{-1}_{median}\left(\frac{1}{2}|k+1;x_m\right)-x_m\right|\right)\\
&\qquad= F^{-1}_{median}\left(\frac{1}{2}|b_m+1;x_m\right)-F^{-1}_{median}\left(\frac{1}{2}|a_m;x_m\right)\\
&\qquad= C_m-A_m\\
&\qquad> B-A
\end{aligned}$$

Thus, we can rewrite Equation 8 to get the following lower bound for good signalling effects:

$$(B-A)\binom{2m}{b_m}(1-p_m)^{2m-b_m}p_m^{b_m}. \tag{9}$$

Having derived bounds on pivot effects, bad signalling effects, and good signalling effects (Equations 6, 7, and 9, respectively) we now substitute these bounds into the utility difference expression in Equation 5 to get

$$u^m_{dif}(x_m)>-\binom{2m}{m}(1-p_m)^m{p_m}^m-(m+1)\binom{2m}{m}(1-p_m)^m{p_m}^m+(B-A)\binom{2m}{b_m}(1-p_m)^{2m-b_m}p_m^{b_m}.$$

To show that there exists an $\bar{m}$, such that for $m>\bar{m}$, $u^m_{dif}(x_m)>0$, it is sufficient to show that

$$\lim_{m\to\infty}\frac{\binom{2m}{m}(1-p_m)^m{p_m}^m+(m+1)\binom{2m}{m}(1-p_m)^m{p_m}^m}{(B-A)\binom{2m}{b_m}(1-p_m)^{2m-b_m}p_m^{b_m}}=0.$$

Combining terms in the numerator, and noting that $B-A$ is strictly greater than zero and unaffected by m, it is sufficient to show that $\lim_{m\to\infty}(m+2)\frac{\binom{2m}{m}(1-p_m)^m{p_m}^m}{\binom{2m}{b_m}(1-p_m)^{2m-b_m}p_m^{b_m}}=0$. For convenience, define

$\Psi_m = (m+2)\,\frac{\binom{2m}{m}(1-p_m)^m {p_m}^m}{\binom{2m}{b_m}(1-p_m)^{2m-b_m} p_m^{b_m}}$. Rearranging yields

$$\begin{aligned}
\Psi_m &= (m+2)\left(\frac{p_m}{1-p_m}\right)^{m-b_m} \frac{\binom{2m}{m}}{\binom{2m}{b_m}} \\
&= (m+2)\left(\frac{p_m}{1-p_m}\right)^{m-b_m} \frac{\frac{2m!}{m!m!}}{\frac{2m!}{b_m!(2m-b_m)!}} \\
&= (m+2)\left(\frac{p_m}{1-p_m}\right)^{m-b_m} \frac{b_m!(2m-b_m)!}{m!m!} \\
&= (m+2)\left(\frac{p_m}{1-p_m}\right)^{m-b_m} \frac{\prod_{j=1}^{m-b_m}(2m-b_m-j+1)}{\prod_{j=1}^{m-b_m}(m-j+1)}.
\end{aligned}$$

Taking the largest of the $m-b_m-1$ terms on the top of the product and the smallest of the $m-b_m-1$ terms on the bottom we see that

$$\Psi_m < (m+2)\left(\frac{p_m}{1-p_m}\right)^{m-b_m} \frac{\prod_{j=1}^{m-b_m}(2m-b_m)}{\prod_{j=1}^{m-b_m}(b_m+1)} = (m+2)\left[\left(\frac{p_m}{1-p_m}\right)\left(\frac{2m-b_m}{b_m+1}\right)\right]^{m-b_m}. \quad (10)$$

To find $\lim_{m\to\infty}\Psi_m$we first simplify the exponent.

Recall that $B_m \in [B, Z]$ is defined as the largest number less than B such that for some integer $b_m < 2m+1$ it is the case that $B_m = F^{-1}_{median}\left(\frac{1}{2}|b_m; x_m\right)$. Since B_m is thus converging to B from below, $F^{-1}_{median}\left(\frac{1}{2}|b_m; x_m\right)$ is converging to $B \in (M, Z)$. Suppose that $\frac{b_m}{2m}$ converges to a number greater than $\frac{1}{2}$. By Remark 1, $b_m > m+1$ implies that $F^{-1}_{median}\left(\frac{1}{2}|b_m; x_m\right) > x_m$ and since we have assumed that $x_m \to Z$, and we have $F^{-1}_{median}\left(\frac{1}{2}|b_m; x_m\right) \to B$ we must have $B > Z$ contradicting the definition of B (that $B < Z$). Also it is clear that $b_m \to 0$ is not possible since $B > 0$ and $B_m \to B$. Thus we have shown that $\lim_{m\to\infty}\frac{b_m}{2m} = c_1$ for some $c_1 \in \left(0, \frac{1}{2}\right)$.

Thus $\lim_{m\to\infty}\frac{m-b_m}{2m} = \frac{1}{2} - c_1$, and if we fix a $\delta = \frac{\frac{1}{2}-c_1}{2}$ there exists a m_1 such that for all $m > m_1$, $\frac{m-b_m}{2m} > \frac{1}{2} - c_1 - \delta$, i.e., $m - b_m > \left(\frac{1}{2} - c_1\right)m$. We substitute in for the exponent in equation 10 to get:

$$\begin{aligned}
\Psi_m &< (m+2)\left[\left(\frac{p_m}{1-p_m}\right)\left(\frac{2m-b_m}{b_m+1}\right)\right]^{\left(\frac{1}{2}-c_1\right)m} \\
&= (m+2)\left[\left(\frac{p_m}{1-p_m}\right)\left(\frac{1-\frac{b_m}{2m}}{\frac{b_m+1}{2m}}\right)\right]^{\left(\frac{1}{2}-c_1\right)m}. \quad (11)
\end{aligned}$$

Expanding out the terms in brackets, $\lim_{m\to\infty} p_m = 1 - F(Z)$ and $\lim_{m\to\infty}\frac{b_m}{2m} = c_1$, so

$$\begin{aligned}
&\lim_{m\to\infty}\left(\frac{p_m}{1-p_m}\right)\left(\frac{2m-b_m}{b_m+1}\right) \\
&= \lim_{m\to\infty}\left(\frac{p_m}{1-p_m}\right)\left(\frac{1-\frac{b_m}{2m}}{\frac{b_m+1}{2m}}\right) \\
&= \frac{1-F(Z)}{F(Z)}\cdot\frac{1-c_1}{c_1}.
\end{aligned}$$

Since $\max\{1-F(z), c_1\} < \frac{1}{2}$, we know that $\frac{1-F(Z)}{F(Z)} \cdot \frac{1-c_1}{c_1} \in (0,1)$ and thus substituting back in to Equation 11 and taking limits

$$\begin{aligned}
\lim_{m\to\infty} \Psi_m &\leq \lim_{m\to\infty} (m+1)\left[\left(\frac{p_m}{1-p_m}\right)\left(\frac{2m-b_m}{b_m+1}\right)\right]^{\left(\frac{1}{2}-c_1\right)m} \\
&= \lim_{m\to\infty} (m+1)\left[\frac{1-F(Z)}{F(Z)} \cdot \frac{1-c_1}{c_1}\right]^{\left(\frac{1}{2}-c_1\right)m} \\
&= 0.
\end{aligned}$$

■

6 Discussion and Related Literature

A few previous papers have examined signaling motivations in elections. Piketty (2000) develops a two period model in which voters with common values communicate policy information to each other as they vote on two referenda. In his model, three exogenously fixed policy alternatives are pitted against each other according to a set agenda: A vs. C in the first referendum and the winner against B in the second. In our model, in contrast, second-period electoral competition is between candidates who can choose any position in the left-right policy spectrum. This difference in the setup of the models has a substantial effect on the role of signaling. In the Piketty model, a voter's action only affects future election outcomes in the event that she is informationally pivotal between alternatives in the second referendum, i.e., the key tradeoff in the Piketty model is between two pivot effects. In our model in contrast, regardless of what the other voters do, a voter's action has some signaling effect on future candidate positions.

Razin (2003) analyzes a model of pivot versus signaling effects in elections with pure common values and incomplete information about a common shock. This contrasts with our private values model, in which candidates have incomplete information about individual voters' preferences. In Razin's model, vote totals affect policies enacted by winning candidates, but the mechanism by which this occurs is different–votes convey to the election winner information about state of the world, whereas in our model votes convey to future candidates information about the distribution of voters. Also, the types of equilibria that arise in the two models are quite different. For example, in Razin's model there is an equilibrium in which signaling effects dominate pivot effects. However, this equilibrium is "unconventional," (Razin's term) in the sense that a vote for a liberal candidate signals that the voter's private information indicates that conservative policies are desirable.

In a few other recently-developed models, voters use votes to signal their preferences, and thus affect policy outcomes. Meirowitz and Tucker (2005) study voting in a parliamentary election and a subsequent presidential election. Voters in the parliamentary election can use their votes to signal dissatisfaction with a presidential candidate, thereby inducing him to expend costly effort to increase his valence. There are a variety of differences between that model and the one we develop here. Most important, our model is fundamentally policy-based rather than valence-based and we are primarily interested in limit results for large electorates whereas Meirowitz and Tucker only analyze a 3-voter case of their model.

In terms of substantive objectives, the most similar research is by Fowler and Smirnov (2007), who analyze a decision theoretic model of voting in situations where vote totals are assumed to affect future policies via an exogenously set reaction function. However, Fowler and Smirnov hard wire the pivot probability at zero so their model cannot be used to analyze the relative importance of pivot and signaling effects.

In terms of technical setup, our model is most similar to Meirowitz (2005), which provides a general analysis of pure signaling motivations in polls, and Shotts (2006), which analyzes abstention in elections with both signaling and pivot motivations in a model with a fixed population size. However, neither of these papers address our the central question: do pivot effects or signaling effects dominate drive the behavior of voters in a large electorate?

One can imagine other signaling motivations in elections. In fact Razin (2003) discusses a model in which the signalling motivation is exogenous or reduced form. The goal of our paper, in contrast, is to consider a game where the signaling motivation was endogenous and see which effect dominates. A natural extension would be to think of settings in which the second period candidates did not converge in equilibrium, e.g., because they face uncertainty about voter preferences and have policy motivations. We conjecture that the proof technique employed in establishing proposition 2 could be extended to address this case, but such an analysis is beyond the scope of this paper.

At a broader level, our result has important implications for theories of elections. Put bluntly, it may be the case that existing electoral models are focusing on the wrong thing. While one-shot pivot-based models have moved beyond simple two candidate competition with uncertainty only about voter preferences to incorporate questions about turnout, exogenous and endogenous valence, three or more candidates and correlated private information on the part of voters, our main result suggests that it might be fruitful to rethink some of these extensions when there is a large electorate and a signalling motivation is present.

7 Appendix

The following Lemma is used in the proof of Proposition 1.

Lemma 2: (1) $u_{dif}(v_i)$ is continuous and weakly increasing in v_i. (2) [Lipschitz property] $\forall \tilde{v}_i, \hat{v}_i \in [0,1]$, $|u_{dif}(\tilde{v}_i) - u_{dif}(\hat{v}_i)| \leq 3 \cdot |\tilde{v}_i - \hat{v}_i|$. (3) $u_{dif}(0) \leq 0$ and $u_{dif}(1) \geq 0$.

Proof: We first prove separately versions of this result for $u_{dif1}(v_i)$ and $u_{dif2}(v_i)$, then combine them to get the desired result for $u_{dif}(v_i) = u_{dif1}(v_i) + u_{dif2}(v_i)$.

For $u_{dif1}(v_i)$, note that $|L - v_i| - |R - v_i|$ is continuous and weakly increasing in v_i since $R > L$ and $\binom{2m}{m}(F(x_c))^m (1 - F(x_c))^m \in [0,1]$. Thus $u_{dif1}(v_i)$ is continuous and weakly increasing in v_i. Also, since the slope of $|L - v_i| - |R - v_i|$ is either one (for $v_i \in (L,R)$) or zero (for $v_i \in (0,L) \cup (R,1)$), $u_{dif1}(v_i)$ satisfies the Lipschitz property with a Lipschitz constant of 1. Finally, since $L < R$, $u_{dif1}(0) \leq 0$ and $u_{dif1}(1) \geq 0$.

For $u_{dif2}(v_i)$, note that by part 3 of Lemma 1, $\forall k \in \{1, ..., 2m\}$, $F^{-1}_{median}\left(\frac{1}{2}|k; x_c\right) \leq F^{-1}_{median}\left(\frac{1}{2}|k+1; x_c\right)$, so $\left|F^{-1}_{median}\left(\frac{1}{2}|k; x_c\right) - v_i\right| - \left|F^{-1}_{median}\left(\frac{1}{2}|k+1; x_c\right) - v_i\right|$ is continuous and weakly increasing in v_i. Thus the probability-weighted sum,

$$\sum_{k=0}^{2m} \binom{2m}{k} (F(x_c))^{2m-k} (1 - F(x_c))^k \left(\left| F^{-1}_{median}\left(\frac{1}{2}|k; x_c\right) - v_i \right| - \left| F^{-1}_{median}\left(\frac{1}{2}|k+1; x_c\right) - v_i \right| \right)$$

is continuous and weakly increasing in v_i. For the Lipschitz property, note that $|u_{dif2}(\tilde{v}_i) - u_{dif2}(\hat{v}_i)|$ equals

$$\left| \begin{array}{c} \sum_{k=0}^{2m} \binom{2m}{k} (F(x_c))^{2m-k} (1 - F(x_c))^k \left(\left|F^{-1}_{median}\left(\frac{1}{2}|k; x_c\right) - \tilde{v}_i\right| - \left|F^{-1}_{median}\left(\frac{1}{2}|k+1; x_c\right) - \tilde{v}_i\right| \right) - \\ \sum_{k=0}^{2m} \binom{2m}{k} (F(x_c))^{2m-k} (1 - F(x_c))^k \left(\left|F^{-1}_{median}\left(\frac{1}{2}|k; x_c\right) - \hat{v}_i\right| - \left|F^{-1}_{median}\left(\frac{1}{2}|k+1; x_c\right) - \hat{v}_i\right| \right) \end{array} \right|$$

$$= \left| \sum_{k=0}^{2m} \binom{2m}{k} (F(x_c))^{2m-k} (1 - F(x_c))^k \left[\begin{array}{c} \left|F^{-1}_{median}\left(\frac{1}{2}|k; x_c\right) - \tilde{v}_i\right| - \\ \left|F^{-1}_{median}\left(\frac{1}{2}|k; x_c\right) - \hat{v}_i\right| - \left|F^{-1}_{median}\left(\frac{1}{2}|k+1; x_c\right) - \tilde{v}_i\right| + \\ \left|F^{-1}_{median}\left(\frac{1}{2}|k+1; x_c\right) - \hat{v}_i\right| \end{array} \right] \right|$$

$$\leq \sum_{k=0}^{2m} \binom{2m}{k} (F(x_c))^{2m-k} (1 - F(x_c))^k \cdot 2 |\tilde{v}_i - \hat{v}_i|$$

$$\leq 2 \cdot |\tilde{v}_i - \hat{v}_i|$$

which simplifies to

$$\left| \sum_{k=0}^{2m} \binom{2m}{k} (F(x_c))^{2m-k} (1 - F(x_c))^k \left(\begin{array}{c} \left|F^{-1}_{median}\left(\frac{1}{2}|k; x_c\right) - \tilde{v}_i\right| - \left|F^{-1}_{median}\left(\frac{1}{2}|k; x_c\right) - \hat{v}_i\right| \\ - \left|F^{-1}_{median}\left(\frac{1}{2}|k+1; x_c\right) - \tilde{v}_i\right| + \left|F^{-1}_{median}\left(\frac{1}{2}|k+1; x_c\right) - \hat{v}_i\right| \end{array} \right) \right|$$

which is less than or equal to

$$\begin{aligned}
& \sum_{k=0}^{2m} \binom{2m}{k} (F(x_c))^{2m-k} (1-F(x_c))^k \cdot 2\left|\tilde{v}_i - \hat{v}_i\right| \\
\leq \quad & 2 \cdot \left|\tilde{v}_i - \hat{v}_i\right|
\end{aligned}$$

For $u_{dif2}(0) \leq 0$, recall that from part 3 of Lemma 1 that $\forall_k$, $F_{median}^{-1}\left(\frac{1}{2}|k; x_c\right) \leq F_{median}^{-1}\left(\frac{1}{2}|k+1; x_c\right)$ so

$$\begin{aligned}
u_{dif2}(0) &= \sum_{k=0}^{2m} \binom{2m}{k} (F(x_c))^{2m-k} (1-F(x_c))^k \left(F_{median}^{-1}\left(\frac{1}{2}|k; x_c\right) - F_{median}^{-1}\left(\frac{1}{2}|k+1; x_c\right)\right) \\
&\leq 0
\end{aligned}$$

By a similar argument $u_{dif2}(1) \geq 0$.

Since $u_{dif1}(v_i)$ and $u_{dif2}(v_i)$ are continuous and weakly increasing in v_i, so is $u_{dif}(v_i) = u_{dif1}(v_i) + u_{dif2}(v_i)$. Since $|u_{dif1}(\tilde{v}_i) - u_{dif1}(\hat{v}_i)| \leq |\tilde{v}_i - \hat{v}_i|$and $|u_{dif2}(\tilde{v}_i) - u_{dif2}(\hat{v}_i)| \leq 2|\tilde{v}_i - \hat{v}_i|$, $|u_{dif}(\tilde{v}_i) - u_{dif}(\hat{v}_i)| \leq 3 \cdot |\tilde{v}_i - \hat{v}_i|$. Finally, since $u_{dif1}(0) \leq 0$ and $u_{dif2}(0) \leq 0$, $u_{dif}(0) \leq 0$, and likewise $u_{dif1}(1) \geq 0$ and $u_{dif2}(1) \geq 0$ imply $u_{dif}(1) \geq 0$.$\blacksquare$

Proposition 1: There exists an equilibrium in which voters use symmetric cutpoint strategies in the first period.

Proof: Consider the correspondence

$$\phi(x_c) = \{v_i \in [0,1] : u_{dif}(v_i) = 0 \text{ when } N \backslash i \text{ use the symmetric cutpoint strategy specified by } x_c\}$$

Note that $\phi(x_c) : [0,1] \to [0,1]$ is nonempty for all $x_c \in [0,1]$, by parts 1 and 3 of Lemma 2 and the Intermediate Value Theorem. Also, since $u_{dif}(x)$ is continuous and weakly increasing, $\phi(x_c)$ is convex-valued. So, to apply Kakutani's fixed point theorem, and conclude that there exists an equilibrium, i.e., an $x_c^* \in \phi(x_c^*)$ all we need to do is to establish that $\phi(x_c)$ is upper hemi-continuous.

Consider a sequence of points $\{x_c^t\} \to \tilde{x}_c$ and a sequence $\{y^t\} \to \tilde{y}$ where $y^t \in \phi(x_c^t), \forall t$. We need to show that $\tilde{y} \in \phi(\tilde{x}_c)$.

For each t following the definition of $u_{dif}(v_i)$ in Equation 1, let $u_{dif}^t(v_i)$ be the utility difference function given cutpoint x_c^t and let $\tilde{u}_{dif}(v_i)$ be the utility difference function given cutpoint $\tilde{x}_c$.

We first note that $\{u_{dif}^t(v_i)\}$ converges pointwise to $\tilde{u}_{dif}(v_i)$. The first part of the utility difference function is

$$u_{dif1}^{t}(v_i) = \binom{2m}{m} \left(F(x_c^t)\right)^m \left(1 - F(x_c^t)\right)^m (|L - v_i| - |R - v_i|)$$

which converges pointwise to

$$\tilde{u}_{dif1}(v_i) = \binom{2m}{m} (F(\tilde{x}_c))^m (1 - F(\tilde{x}_c))^m (|L - v_i| - |R - v_i|)$$

since $\left\{x_c^t\right\} \to \tilde{x}_c$. The second part is

$$\begin{aligned} u_{dif2}^{t}(v_i) \quad &\equiv \quad \sum_{k=0}^{2m} \binom{2m}{k} \left(F(x_c^t)\right)^{2m-k} \left(1 - F(x_c^t)\right)^k \\ &\cdot \left(\left| F_{median}^{-1}\left(\frac{1}{2}|k; x_c^t\right) - v_i \right| - \left| F_{median}^{-1}\left(\frac{1}{2}|k+1; x_c^t\right) - v_i \right| \right) \end{aligned}$$

which by part 5 of Lemma 1 and the fact that $\left\{x_c^t\right\} \to \tilde{x}_c$ converges pointwise to

$$\begin{aligned} \tilde{u}_{dif2}(v_i) \quad &\equiv \quad \sum_{k=0}^{2m} \binom{2m}{k} (F(\tilde{x}_c))^{2m-k} (1 - F(\tilde{x}_c))^k \\ &\cdot \left(\left| F_{median}^{-1}\left(\frac{1}{2}|k; \tilde{x}_c\right) - v_i \right| - \left| F_{median}^{-1}\left(\frac{1}{2}|k+1; \tilde{x}_c\right) - v_i \right| \right). \end{aligned}$$

Now we suppose that $\tilde{y} \notin \phi(\tilde{x}_c)$, and derive a contradiction. If $\tilde{y} \notin \phi(\tilde{x}_c)$ then either $\tilde{u}_{dif}(\tilde{y}) > 0$ or $\tilde{u}_{dif}(\tilde{y}) < 0$. Without loss of generality suppose the former. Then since $u_{dif}^t(v_i)$ converges pointwise to $\tilde{u}_{dif}(v_i)$ there exists T such that for all $t > T$, $u_{dif}^t(\tilde{y}) > \frac{\tilde{u}_{dif}(\tilde{y})}{2}$. By the Lipschitz property in part 2 of Lemma 2, for all $t > T$, $u_{dif}^t(\tilde{y}) - u_{dif}^t(\tilde{y} - \delta) \leq 3 \cdot \delta$ for any $\delta > 0$. Setting $\delta = \frac{\tilde{u}_{dif}(\tilde{y})}{6}$ we have that for $t > T$, $u_{dif}^t(\tilde{y}) - u_{dif}^t(\tilde{y} - \delta) < \frac{\tilde{u}_{dif}(\tilde{y})}{2}$, so $u_{dif}^t(\tilde{y} - \delta) > u_{dif}^t(\tilde{y}) - \frac{\tilde{u}_{dif}(\tilde{y})}{2} > 0$. Thus, since $y^t \in \phi(x_c^t)$, or, equivalently $u_{dif}^t\left(y^t\right) = 0$, and $u_{dif}^t(v_i)$ is weakly increasing in v_i, we conclude that $y^t < \tilde{y} - \delta$ for all $t > T$, which means that $\left\{y^t\right\}$ cannot converge to $\tilde{y}$, a contradiction.■

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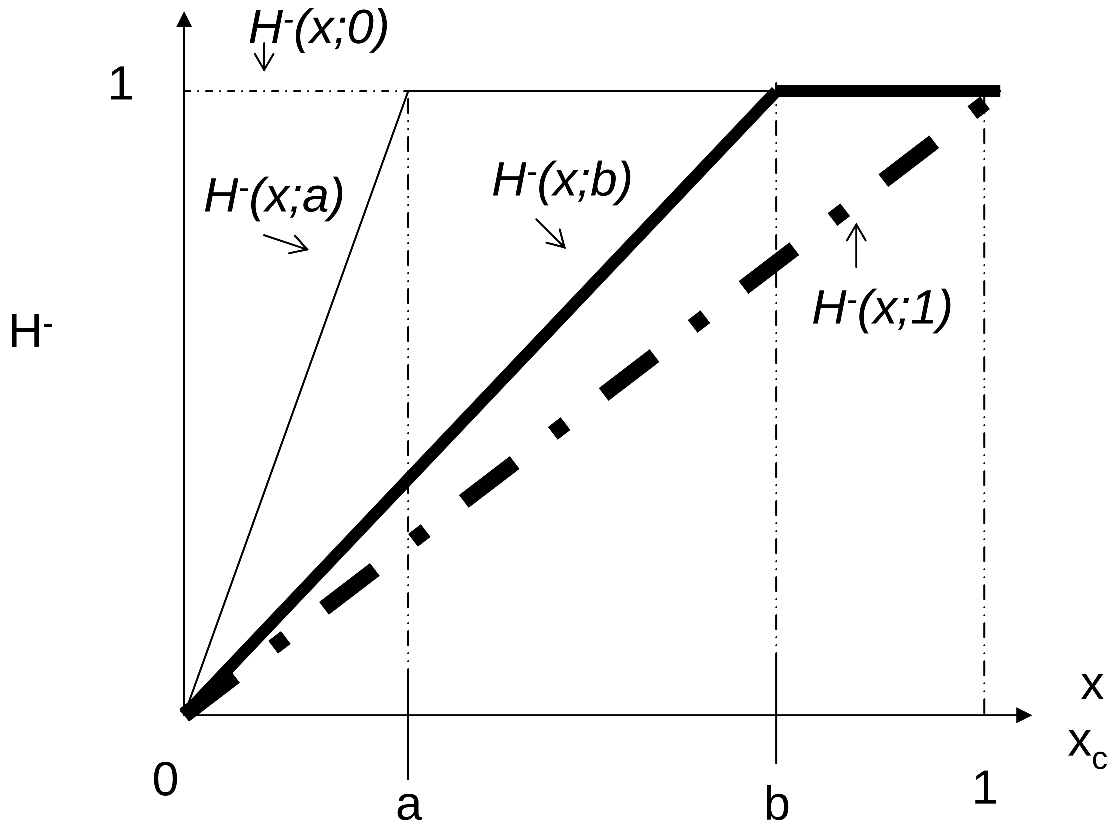
Figure 1



Examples of $H^{-}(x;x_c)$ when $\#R<m+1$ for values of x_c in $\{0,a,b,1\}$