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INELASTIC EVENTS INDUCED BY 32 MEV PROTONS ON HELIUM

Jacob Benveniste

(Thesis)

March 31, 1952

Berkeley, California

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ABSTRACT

A study of inelastic events induced by the bombardment of helium nuclei with 52 Mev protons has been made.

No low energy group of protons has been found which would indicate the existence of an excited state of the helium nucleus which would be stable to decay by particle emission (excitation energy less than ≈ 20 Mev). Considerations on the shape of the continuum of protons from the reactions

yield a maximum cross section of .4 mb/ster., for the detection of a single excited helium level as high as 23.3 Mev above the ground state. Further, the observed extent of the continuum requires the existence of at least two levels of about 1 Mev half width, separated by about 1 Mev.

The angular variation of the differential cross section for the production of deuterons from the reaction

has been measured. When the principle of detailed balancing is invoked, good agreement with the prediction of the theory of S. T. Butler is obtained for r_0 (range of nuclear forces plus radius of He^3 nucleus) $= 4.2 \times 10^{-13}$ cm.

INTRODUCTION

Evidence for the nature of nuclear forces comes from several sources. From binding energy measurements, especially of light nuclei, and from nucleon-nucleon scattering experiments, one may arrive at some conceptions of the character of the forces acting between elementary particles. Application of these ideas to the solution of more complex systems should then serve as a check on the accuracy of these concepts.

Thus, in 1936, it was shown theoretically¹ that, on the basis of what was then known about nuclear forces, the He^4 nucleus could be expected to have at least one excited level which was stable to particle disintegration. However, the analysis was based on sum rules valid only for neutron-proton potentials of the ordinary type. It was pointed out that if new sum rules are set up considering Majorana forces, the upper limit on the excitation energies which can be deduced from them are too high to be useful.

Bethe and Bacher² deduced the probable existence of two stable excited levels, a $1P$ state at ≈ 10 Mev and a $3P$ state at ≈ 10 Mev, and concluded that this was compatible with the experimental data available to 1936.

More recently, King and Goldstein,³ pointed out that the observed $1/v$ behavior of the He^3 slow neutron absorption cross section indicated the absence of a bound excited state of the compound He^4 nucleus not too far below 20.5 Mev; furthermore, in view of the large (≈ 5000 barns) $\text{He}^3(n,p)\text{H}^3$ cross section⁴ in the $1/v$ region, the existence of a broad unstable level in He^4 above 20.5 Mev is in the realm of possibility.

One of the more provocative recent experiments in this field is that of Argo,⁵ et al, at Los Alamos. The yield of γ -rays from the

reaction $p(p,\gamma)He^4$ was observed as a function of the incident proton energy. A cross section which increased more rapidly than could be explained by barrier penetration was interpreted as evidence for the existence of an excited level in the He^4 nucleus. Only the rising portion of the yield curve was observed in this work, due to the limitation that 2.5 Mev was the highest proton energy available. However, this curve was fitted with a Breit-Wigner single level resonance formula, and the constants evaluated. The presumed level was assigned an excitation energy 21.6 Mev and a half-width ≈ 1 Mev.

Shortly after the first announcement⁶ of these results, Professor W. K. H. Panofsky suggested that, with the 32 Mev proton linear accelerator, we were ideally equipped to resolve the question of the existence of an excited He^4 level by the method of inelastic scattering of protons from helium.

The inelastic scattering process may be described in the following way. As illustrated in Fig. 1, a proton incident on a target nucleus (A,Z) in its ground state E_0 , may be considered to form a compound nucleus $(A+1, Z+1)$ in an excited state. One of the ways in which the excited compound nucleus might decay is by re-emitting a proton. If the proton (P) comes off with energy equal to that of the incident proton, the target nucleus is left in its ground state; however, if the target nucleus possesses an excited level E_n , the compound nucleus may decay to this level by emitting a proton (p') of less energy than that of the incident proton. This proton is said to be inelastically scattered. Thus, to search for excited levels of the target nucleus, one requires detectors with which to analyze the energy spectrum of scattered particles.

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Bombardment Geometry

A diagram of the bombardment geometry is shown in Figure 2. The 32 Mev proton beam from the linear accelerator was first passed through the premagnet collimator C_1 , usually set at about $1/3$ in. x $1/8$ in., although it was sometimes made smaller when counting rates became excessive (near elastic peak). After being steered into the 20° port by the analyzer magnet M, the beam was further collimated to $1/8$ in. diameter at C_2 and J_3^* , before it entered the scattering chamber S. Then, having passed through the scattering chamber, the beam was collected in a Faraday cup and integrated.

Singly ionized hydrogen molecules (H_2^+), since they have half the e/m of protons, could come down the linear accelerator with half the velocity of protons and keep in step with the accelerating electric field by spending an extra cycle in each drift tube. Thus they would emerge from the machine with about 16 Mev, and, having the same momentum as 32 Mev protons, would not be separated from the main beam by the analyzer magnet. This situation, however, is easily remedied by inserting a "stripping foil" (1/4 mil nylon) in the beam between C_1 and M. Since the cross section for stripping the hydrogen molecule is high, a small amount of matter very efficiently breaks up the 16 Mev, H_2^+ into two 8 Mev protons which are then removed from the main beam at the analyzer magnet.

Alignment of the Scattering Chamber

In the beginning the adjustable collimator C_2 was reduced in size to about $1/16$ in. x $1/16$ in., and the beam was maximized by steering

*This collimator was designed by Roger Wallace.

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through this hole directly into the Faraday cup. Then a long pipe (about 5 ft.) was fastened to the system at C_2 , and a "picture" of the beam was burned into a glass plate fastened to an ext foil at the other end of the long pipe from C_2 . The function of the length of pipe was not only to provide us with a lever arm for finding the beam line, but also, since it was part of the line vacuum, to eliminate air scattering, which would tend to broaden the beam and thus make its center more difficult to find. A set of cross-hairs on a rigid stand was moved into position so that the cross-hairs were directly over the center of the black spot left on the glass plate by the beam. Now, the small hole at C_2 and the cross-hairs represented two points on the beam line, and after removing the long pipe and glass plate, it was a simple matter to sight a telescope along the beam line. Then the collimator C_3 was moved in and locked into position along the line of sight of the telescope.

The platform on which the scattering chamber was mounted was scribed with radial lines every $7\frac{1}{2}^\circ$. In lining up the scattering apparatus, the table was first oriented so that the $0-180^\circ$ line was accurately along the beam line. Then by means of closely fitting polystyrene inserts in the entrance and exit tubes, the scattering chamber was aligned and secured to the table. It is felt that this alignment procedure gave assurance that the beam passed within 15 mils of the center of the scattering chamber.

To give assurance that the collimator before the scattered particle detector was "looking at" that portion of the target gas through which the beam passed, a dummy collimator-detector was made up which duplicated the actual apparatus in its important dimensions. A $1/8$ in. diameter steel rod centered in the scattering chamber by means of inserts in the entrance and exit tubes simulated the beam. Looking through the collimator one could shim up the dummy until the steel rod was seen to be in the center of the

collimator field of view. Since the beam was 1/8 in. high and the collimator slits, 1/4 in. high, the dummy could be quite adequately aligned by eye. Finally, the dummy was replaced by the actual detector.

Vacuum and Gas-Handling System

A schematic diagram of the vacuum and gas system is given in Figure 3.

Before a bombardment, the chamber was evacuated, flushed thoroughly with the target gas, then pumped down to < 10 microns. After isolating the chamber from the pumps, helium was slowly introduced through the charcoal-liquid air trap--the rate of flow being adjusted at the regulator. The combination of charcoal and liquid air is capable of filtering out all gases but He, Ne, and H^2 . Since the producer specifies the bottled He to be 99.95 percent pure, with less than .03 percent H_2 , target impurity was considered inconsequential. Actually, since the impurity is hydrogen, a spectrum of scattered particles as measured in this work, would show the contributions from He and H^2 as separate.

The chamber was filled with the target gas to a pressure of about 1 atmosphere. This pressure was maintained at a constant differential with respect to atmospheric pressure by means of an oil filled manometer through which the target gas bubbled continuously during the course of a run. The pressure differential was of the order of 10 in. of oil ("Litton" diffusion pump oil-- 0.9 gm/cm^3), i.e. about 2 percent of the total pressure. The atmospheric pressure was read before and after a run to 0.1 mm on a precision mercury barometer. Difference in altitude between the barometer and scattering chamber introduced a correction of 4.85 mm Hg.

The gas temperature was measured with an accurate thermometer in thermal contact (maintained by a water cup) with the scattering chamber. The fact that the chamber and the target gas are in thermal equilibrium with each other (within 1° C) was established by Panofsky and Fillmore.⁷

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Beam Integrator

The beam was collected in a Faraday cup placed in a vacuum chamber. The charge was placed on a low leakage condenser connected to the grid of a 5803. Figure 4 is a simplified schematic diagram* of the integrating electrometer. The practically 100% feedback feature of the circuit assures (1) stability of operation, (2) minimum loss of the collected charge since the grid of the 5803 is always at ground potential, and (3) minimizes the effect of the cable capacitance between the Faraday cup and 5803 grid. It turns out that the effective capacitance collecting charge is C plus cable capacitance gain of amplifier. Since the cable capacitance is 500 μf and the amplifier gain, 500, the correction to C is negligible. The feedback voltage, or a fraction thereof, is applied to a Leeds and Northrup recording voltmeter which is self-calibrating against a standard cell. The voltage reading is believed to be accurate within 0.2 percent.

The condenser used was compared on a General Radio Co. impedance bridge with a G.R. standard condenser whose capacitance was known within 0.1 percent. Several measurements independently made by two observers establishes the value of C as 0.1040 ± 0.3 percent microfarads.

Electrons which might be coming down with the proton beam were kept from entering the Faraday cup by two permanent horseshoe magnets placed just beyond the scattering chamber. A second set of magnets provided a flux density of about 200 gauss just outside the Faraday cup to prevent secondary electrons from emerging.

Design and Characteristics of the Detectors

Scattered particles are detected in a telescope of three gas proportional counters. Between the second and third counters is an aluminum foil 5.82 mg/cm^2 thick. By mixing pulses from the first two counters in coincidence and from the last counter in anti-coincidence, the telescope counts all

*This was kindly prepared by Mr. George Milian of Mr. H. D. Fransworth's Electronics Group.

NOTE: "C" IS USED FOR INTEGRATING ELECTROMETER
 "R" IS USED FOR CURRENT ELECTROMETER

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particles which stop in the foil. Hence, referring to the thickness of this foil as ΔR , if we place an absorber R in front of the telescope, it detects those particles having a range between R and $R + \Delta R$. Thus by changing R one obtains a differential range spectrum of the scattered particles.

The proportional counters are of the multiple-wire type--two mil wolfram wires spaced $1/4$ inch apart over an area 2 inches in diameter,* suspended with polystyrene spacers between thin aluminum ground planes $1/2$ in. apart. A trio of counters was designed as a unit; i.e. three counters were formed by alternating aluminum foils and multiple wire grids contained in a single chamber. The entire assembly was attached to a flange through which Kalar seals brought high voltage and signal leads. This flange was also the top of the counter gas chamber; thus the counter could be assembled and checked in the open before the box was closed and the chamber sealed.

All counter components were first scrubbed with acetone, then washed in a detergent and thoroughly rinsed with water. After assembly the chamber was outgassed for several days, flushed thoroughly with the counter mixture, then filled to $1/2$ atmosphere with a mixture of 96 percent argon and 4 percent CO_2 .

The counters were operated at about 950 volts and the linear amplifier gains were adjusted so that the largest pulses were not quite overloading. Under these conditions the coincidence counting rate (i.e., front two counters in coincidence and the last counter in anti-coincidence) as a function of discriminator voltage on the front two counter yields the dependence shown in Figure 5. The plateau is seen to be quite adequate. Later it will be shown that a slight decrease in linear amplifier gain provides a means for distinguishing protons from deuterons.

*The author is indebted to Earl Hostetter for constructing these grids to his specifications. See Appendix I for basis for this choice of dimensions.

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Jitter is the term applied to the change in relative time between the pulses formed by a charged particle in two adjacent counters. If this were excessive, it would (a) lose counts, or (b) require the use of such long coincidence gates that only relatively low counting rates could be accommodated. One way to measure jitter is to observe the coincidence counting rate as a function of the delay introduced between the gates formed by pulses from the two counters. The results of this measurement showed that the jitter was certainly less than $1/5$ microsecond, and hence introduced no problems.

The subsequent choice of 2μ sec. gates for coincidence was made on the basis of the requirement of positive operation of the mixer-scaler.

Electronics

A block diagram of the electronics associated with the counters is shown in Figure 6.

The general requirements of the electronics were well fulfilled with standard components.*

Stability of the equipment was insured in the following ways: (1) the high voltage supply was monitored continuously by measuring a small fraction of the high voltage with a potentiometer and standard cell. In the course of a run, the high voltage remained constant to within 0.1 percent. (2) the 110 volt a.c. for the linear amplifier filaments was supplied through a SOLA constant voltage transformer. (3) strong blowers on the variable gate units maintained the long-time stability of the pulse-height discriminators.

Pulses from proportional counters 1 and 2 generated 2μ sec. gates which were fed into the coincidence channels of the mixer scaler, while pulses from proportional counter 3 generated 3μ sec. gates, straddling the 2μ sec. gates of channels 1 and 2 in time, which were fed into the anti-coincidence channel of the mixer scaler. Counting rates were low enough

*These were designed and built by the Electronics Group under the direction of H. D. Farnsworth.

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to make pile-up or dead-time corrections completely unnecessary. It was possible to monitor the accidental coincidences by splitting the output of linear amplifier 2, introducing an arbitrary delay into another variable gate unit, and feeding this delayed gate into a separate mixer-scaler with the undelayed gates of the other channels. The mixer-scalers were effectively gated to count only during the beam by feeding a "beam gate" into the third coincidence channel.

Calculation of Solid Angle Subtended by Two Slits

It is assumed:

1. Line source of scattered particles.
2. Collimator for scattered particles is formed by two slits, of width A , a distance l apart.
3. Height of front slit is greater than or equal to that of back slit.

From Figure 7, the solid angle available to those particles scattered from that portion of the line source between s and $s + ds$ is:

$$d\Omega = H \frac{ds}{S} \int \frac{\cos \alpha}{r^2} da \quad (1)$$

where H is the height of the back slit, and S is the total scattering length seen by the collimator. From the geometry, we see:

$$r = (l + t + s \cos \phi) \frac{1}{\cos \alpha} = (R_0 + s \cos \phi) \frac{1}{\cos \alpha} \quad (2)$$

Hence,

$$d\Omega = \frac{H}{S} ds \int \frac{\cos^3 \alpha}{(R_0 + s \cos \phi)^2} da \quad (3)$$

To eliminate the unwieldy integration over s , we seek a relation between b and s :

$$\tan \alpha = \frac{a-b}{l} = \frac{a - \frac{A}{2} + s \sin \phi}{R_0 + s \cos \phi} \quad (4)$$

$$b = a - \frac{l(a - \frac{A}{2} + s \sin \phi)}{R_0 + s \cos \phi}$$

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$$dt = \frac{l \left[R_0 \sin \phi - l \left(1 - \frac{A}{2} \right) \cos \phi \right]}{(R_0 + s \cos \phi)^2} ds \quad (5)$$

Further,

$$\cos^2 \alpha = \frac{l^2}{(a - b)^2 + l^2} \quad (6)$$

So one obtains:

$$\begin{aligned} d\Omega &= + \frac{H}{S} \frac{l^2 da}{\left[R_0 \sin \phi - \left(a - \frac{A}{2} \right) \cos \phi \right]} \int_0^A \frac{db}{\left[(a - b)^2 + l^2 \right]^{3/2}} \\ &= - \frac{H}{S} \frac{l^2 da}{\left[\frac{a - A}{2} \right]} \int_0^{a-A} \frac{dm}{(m^2 + l^2)^{3/2}} \end{aligned} \quad (7)$$

if one lets $m = a - b$.

The integral yields:

$$\frac{1}{l^2} \left[\frac{a - A}{\left[(a - A)^2 + l^2 \right]^{1/2}} - \frac{a}{\left[a^2 + l^2 \right]^{1/2}} \right]$$

Expanding the bracket and keeping terms of order no higher than $\frac{A^2}{l^2}$, this becomes:

$$\frac{1}{l^2} \left[- \frac{A}{l} \left(1 - \frac{A^2}{2l^2} + \frac{3}{2} \frac{Aa}{l^2} - \frac{3}{2} \frac{a^2}{l^2} + \dots \right) \right]$$

Writing the remaining bracket in Eq. (7) as:

$$R_0 \sin \phi \left[1 - \frac{a - \frac{A}{2}}{R_0} \cot \phi \right]$$

and expanding, one obtains

$$\begin{aligned} S d\Omega &= \frac{H A da}{l R_0 \sin \phi} \left[1 - \frac{a - \frac{A}{2}}{R_0} \cot \phi + \left(\frac{a - \frac{A}{2}}{R_0} \right)^2 \cot^2 \phi \right] \\ &\quad \times \left[1 - \frac{A^2}{2l^2} + \frac{3}{2} \frac{Aa}{l^2} - \frac{3}{2} \frac{a^2}{l^2} + \dots \right] \end{aligned} \quad (8)$$

Hence,

$$S\Omega = \frac{H A^2}{l R_0 \sin \phi} \left[1 - \frac{A^2}{4l^2} + \frac{A^2}{12 R_0^2} \cot^2 \phi + \dots \right] \quad (9)$$

This turns out to be a special case of the Critchfield formula.⁸

Table I

For this case:

$$H = 1/4 \text{ in.}$$

$$A = 1/4 \text{ in.}$$

$$l = 4 \text{ in.}$$

$$R_0 = 10 \text{ in.}$$

$$\Omega s = \frac{.99215}{\sin \phi} \times 10^{-3} \left[1 - .00098 + .00005 \cot^2 \phi - \dots \right]$$

ϕ	Ωs
10	6.716
15	3.834
22.5	2.590
30	1.983
37.5	1.629
45	1.402
52.5	1.250
60	1.144
67.5	1.072
75	1.026
82.5	1.000
90	.9911

Correction Due to Finite Beam Width - The solid angle calculation was based on the assumption of an infinitely thin line source. The solid angle contribution of a line source turns out to be:

$$\omega \approx \frac{1}{r \sin \phi}$$

If the beam has a width $2\Delta R$, then the average solid angle accessible to particles scattered from all parts of the finite beam is:

$$\bar{\omega} = \frac{1}{2 \frac{\Delta R}{\sin \phi}} \int_{R - \frac{\Delta R}{\sin \phi}}^{R + \frac{\Delta R}{\sin \phi}} \omega dr \quad (10)$$

where ϕ is the scattering angle or the observation.

$$\begin{aligned} \bar{\omega} &= \frac{1}{2 \Delta R} \left[\ln \left(R + \frac{\Delta R}{\sin \phi} \right) - \ln \left(R - \frac{\Delta R}{\sin \phi} \right) \right] \\ &= \frac{1}{2 \Delta R} \left[\ln R + \frac{\Delta R}{R \sin \phi} - \frac{1}{2} \left(\frac{\Delta R}{R \sin \phi} \right)^2 + \frac{1}{3} \left(\frac{\Delta R}{R \sin \phi} \right)^3 + \dots \right. \\ &\quad \left. - \ln R + \frac{\Delta R}{R \sin \phi} + \frac{1}{2} \left(\frac{\Delta R}{R \sin \phi} \right)^2 + \frac{1}{3} \left(\frac{\Delta R}{R \sin \phi} \right)^3 + \dots \right] \\ &= \frac{1}{R \sin \phi} \left[1 + \frac{1}{3} \left(\frac{\Delta R}{R \sin \phi} \right)^2 + \dots \right] \quad (11) \end{aligned}$$

For our case ($\Delta R = 1/16$ in., $R = 10$ in., and in the worst case $\phi = 15^\circ$) the correction is:

$$\frac{1}{3} \left(\frac{1}{16 \times 10 \times 1/4} \right)^2 = \frac{1}{3} \left(\frac{1}{40} \right)^2 = \frac{1}{4800}$$

i.e. about .02 percent and hence negligible in this work.

Correction Due to Rapid Variation of Cross Section with Angle - If the scattering cross section changes rapidly over the angular interval selected by the collimator slits, a correction to the observed counting rate must be made.

The fraction of particles, scattered at an angle ϕ from a cross section, accepted by the slit system is $f(\phi - \phi_0)$, where ϕ_0 is the angle selected by the

centerline of the slit system and the beam direction. In this case*

$$\begin{aligned} f(\phi - \phi_0) &= l (\phi - \phi_0) + A & \phi < \phi_0 \\ &= -l (\phi - \phi_0) + A & \phi > \phi_0 \end{aligned}$$

where A is the width of the slits, and l , the distance between slits.

If $N(\phi)d(\phi)$ is the number of particles scattered into the angle $\phi(d\phi)$, the number of particles accepted by the slit system at the angle ϕ_0 is:

$$\mathcal{N}(\phi_0) = \frac{l}{A^2} \int N(\phi) f(\phi - \phi_0) d\phi \quad (12)$$

where $\frac{l}{A^2}$ is the normalization factor.

Expanding $N(\phi)$ about ϕ_0 :

$$N(\phi) = N(\phi_0) + N'(\phi_0) (\phi - \phi_0) + \frac{N''(\phi_0)}{2} (\phi - \phi_0)^2 + \dots \quad (13)$$

where the primes indicate differentiation with respect to ϕ . Now,

$$\begin{aligned} \mathcal{N}(\phi_0) &= N(\phi_0) \frac{l}{A^2} \left[\int_{-\frac{A}{l}}^0 l \phi + A d\phi + \int_0^{\frac{A}{l}} -l \phi + A d\phi \right] \\ &\quad + \frac{N''(\phi_0) l}{2 A^2} \left[\int_{-\frac{A}{l}}^0 (l \phi + A) \phi^2 d\phi + \int_0^{\frac{A}{l}} (-l \phi + A) \phi^2 d\phi \right] + \dots \end{aligned} \quad (14)$$

where a change of variable (from $\phi - \phi_0$ to ϕ) has been made. Since $N(\phi - \phi_0)$ is an even function, all coefficients of odd derivatives of $N(\phi)$ are zero.

Finally, to second order terms,

$$\mathcal{N}(\phi_0) = N(\phi_0) \left(1 + \frac{N''(\phi_0)}{N(\phi_0)} \frac{1}{12} \frac{A^2}{l^2} \right) \quad (15)$$

If the cross section goes as a power law, e.g., $N(\phi) \simeq \phi^{-n}$

$$\mathcal{N}(\phi_0) = N(\phi_0) \left(1 + \frac{n(n+1)}{\phi_0^2} \frac{1}{12} \frac{A^2}{l^2} \right) \quad (16)$$

Here $N(\phi_0)$ represents the "true" counting rate at a scattering angle ϕ_0 , while $\mathcal{N}(\phi_0)$ is the observed counting rate at a finite angular interval

*See Appendix II.

centered at ϕ_0 . So we write:

$$N(\phi_0) = \mathcal{N}(\phi_0) \left(1 - \frac{n(n+1)}{\phi_0^2} \frac{1}{12} \frac{\Delta^2}{\ell^2} \right) \quad (17)$$

If $n = 3$, the worst case at $\phi_0 = 15^\circ$ yields a correction term:

$$\frac{3(4)}{(.262)^2} \frac{1}{12} \left(\frac{1}{2 \times 4} \right)^2 = .057$$

Errors Affecting the Cross Section Measurement - The differential cross section

of an observed process is proportional to:

$$\frac{d\sigma}{d\Omega} \approx \frac{T N}{P C V (\Omega S)} \quad (18)$$

where T is the temperature in degrees absolute and P , the pressure of the gas, C is the capacitance of the condenser which is charged by the collected beam to a voltage V , ΩS is the solid angle of scattering, determined by the geometry, and N is the number of particles detected.

In the course of a run one measures the number of particles which hit a aluminum foil of thickness ΔR , i.e., each point of the spectrum represents $\frac{dN}{dR} \Delta R$. Hence, to determine the number of particles scattered into the detector as a result of a given reaction, one measures the area under the spectrum and divides this by ΔR :

$$N = \frac{1}{\Delta R} \int \frac{dN}{dR} \Delta R \quad (19)$$

The area measurement is a function of counting statistics and will be taken up later. For now, it is noted that Eq. (18) becomes:

$$\frac{d\sigma}{d\Omega_a} \approx \frac{T}{P C V (\Omega S) \Delta R} \quad (20)$$

Combining the several contributing errors quadratically, the estimate of error on the differential cross section is given by:

$$\delta \frac{d\sigma}{d\Omega_a} = \frac{d\sigma}{d\Omega_a} \left[\left(\frac{\delta T}{T} \right)^2 + \left(\frac{\delta P}{P} \right)^2 + \left(\frac{\delta C}{C} \right)^2 + \left(\frac{\delta V}{V} \right)^2 + \left(\frac{\delta \Omega S}{\Omega S} \right)^2 + \left(\frac{\delta \Delta R}{\Delta R} \right)^2 \right]^{1/2} \quad (21)$$

When we say that Ω is certainly known within 0.3 percent, we shall refer to $\frac{\delta \Omega}{\Omega} = .003$ as the "outside" error. Since all the contributing errors are of the outside type, the total estimated error is also an "outside" error.

In a previous section it was shown:

$$\Omega s \approx \frac{A^2}{R_o \tan \phi}$$

Hence,

$$\left(\frac{\delta \Omega s}{\Omega s}\right)^2 = \left(\frac{\delta H}{H}\right)^2 + \left(\frac{2 \delta A}{A}\right)^2 + \left(\frac{\delta l}{l}\right)^2 + \left(\frac{\delta R_o}{R_o}\right)^2 + \left(\frac{\delta \phi}{\tan \phi}\right)^2 \quad (22)$$

For this experiment:

$$\frac{\delta H}{H} = 0.2 \text{ percent}$$

$$\frac{\delta A}{A} = 0.2 \text{ percent}$$

$$\frac{\delta l}{l} = 0.1 \text{ percent}$$

$$\frac{\delta R_o}{R_o} = 0.5 \text{ percent}$$

ϕ	15	22.5	30	37.5	45	52.5	60	67.5	75	82.5	90
$\frac{\delta \phi}{\tan \phi} = 4.3$	2.1	1.5	1.1	.87	.67	.50	.36	.23	.11	0	

This yields:

ϕ	15	22.5	30	37.5	45	52.5	60	67.5	75	82.5	90
$\frac{\delta s}{s}$	11.5	7.84	5.68	4.36	3.21	2.50	1.70	1.05	.50	.17	.10

Further:

$$\frac{\delta T}{T} = 0.4 \text{ percent}$$

$$\frac{\delta P}{P} = .15 \text{ percent}$$

$$\frac{\delta C}{C} = 0.7 \text{ percent}$$

$$\frac{\delta V}{V} = 0.2 \text{ percent}$$

$$\frac{\delta \Delta R}{\Delta R} = 2.0 \text{ percent}$$

Hence, for the total estimated error, one obtains:

ϕ	15	22.5	30	37.5	45	52.5	60	67.5	75	82.5	90
$\delta \frac{d\sigma}{d\Omega_a}$	3.94	3.0	2.6	2.4	2.3	2.25	2.2	2.2	2.2	2.15	2.15

This represents the estimated error in the measurement of the absolute cross section as a result of contributions from all sources of error except statistics.

The total estimated error in the relative cross section is given by:

$$\delta \frac{d\sigma}{d\Omega_r} = \frac{d\sigma}{d\Omega_r} \left[\left(\frac{\delta T}{T} \right)^2 + \left(\frac{\delta V}{V} \right)^2 + \left(\frac{\delta \phi}{\tan \phi} \right)^2 \right]^{1/2} \quad (2)$$

This yields:

ϕ	15	22.5	30	37.5	45	52.5	60	67.5	75	82.5	90
$\delta \frac{d\sigma}{d\Omega_r}$	3.3	2.1	1.5	1.1	0.9	.7	.54	.41	.30	.22	.2

for the estimated error of the relative cross section measurement as a result of contributions from all sources of error except statistics.

RESULTS AND CONCLUSIONS--SEARCH FOR EXCITED He⁴ LEVEL

A typical spectrum is shown in Figure 8. The peak at 1,100 mg/cm² represents elastically scattered protons; that at 30 mg/cm², elastically scattered alphas. The interesting group of particles is that at 100 mg/cm². If they are deuterons, they are products of the reaction:

If they prove to be protons, they indicate the existence of an excited level of the helium nucleus.

Identification of Low-Energy Peak

A. From the kinematics of reaction (24) the energy of the deuterons is:

$$E_d = \frac{2}{25} E \cos 2\phi + \frac{3}{5} \left(\frac{4}{5} E - 18.3 \right) + \frac{4}{25} E \sqrt{6 \left(1 - \frac{22.9}{E} \right)} \cos \phi \sqrt{1 - \frac{\sin^2 \phi}{6 \left(1 - \frac{22.9}{E} \right)}} \quad (25)$$

where E is the energy in the laboratory system of the incident protons at the center of the scattering chamber and ϕ is the angle which the scattered deuteron makes with the beam direction.

E may be quite accurately determined in the following way: The range of the elastically scattered protons includes, in addition to the aluminum absorber, a distance equal to the radius of the scattering chamber in the target gas, the entrance foil to the counter telescope, and the counter filling gas. The range in the gases may be reduced to mg/cm² of aluminum according to the relation:

$$R(\text{mg/cm}^2 \text{ Al}) = \frac{\left(\frac{dE}{dx} \right)_E \frac{\text{Mev}}{\text{cm. gas}}}{\left(\frac{dE}{dx} \right)_E \frac{\text{Mev}}{\text{mg/cm}^2 \text{ Al}}} \Delta R (\text{cm gas}) \quad (26)$$

The non-aluminum portion of the range is equivalent to about 14 mg/cm² aluminum, and is the least precisely known contribution to the total range. However, the energy of the elastic protons is not sensitive to small uncertainties in the total range and is considered to be practically as

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precisely known as the range energy data from which it is derived.⁹ Then from the relation expressing the energy of protons elastically scattered from helium:

$$E_P = \frac{E}{25} \left[16 + \cos 2 \phi + 8 \cos \phi \sqrt{1 - \frac{1}{16} \sin^2 \phi} \right] \quad (27)$$

one can find E. From range measurements of the elastic peak at seven different angles,

$$E = 31.5 \pm 0.1 \text{ Mev}$$

On the other hand, the precision with which the energy of the low-range particles depends strongly on how well known the non-aluminum portion of the range is known. It turns out that in addition to the calculations based on distances, gas pressures, and rates of energy loss, one has an excellent, independent, experimental check of this contribution: The energy of the elastically scattered alpha-particle is:

$$E_{He^4} = \frac{16}{25} E \cos^2 \phi \quad (28)$$

and is as well known as E. The experimentally observed range of the alpha particle must be that corresponding to E_{He^4} from the range-energy curves. Thus, the difference between this range and the aluminum absorbers plus counter foils is the non-aluminum contribution to the range. Actually, a small correction to this measurement must be made to account for the fact that the ratio of the $\frac{dE}{dx}$'s is slightly different for different particles.

In Table II, the column headed E_d lists the expected deuteron energies from the reaction (24) as given by Eq. (25). The column headed $E_d(\text{obs})$ contains the energies of the particles having the observed ranges, assuming the particles to be deuterons.

TABLE II

ϕ_{lab}	E_d	E_d (obs)
15.	12.41 $\pm$.15	12.40 $\pm$.05
22.5	11.47 $\pm$.15	11.25 $\pm$.05
30.	10.54 $\pm$.15	10.35 $\pm$.05
37.5	9.29 $\pm$.1	9.32 $\pm$.07
45.	7.94 $\pm$.1	7.35 $\pm$.07
52.5	6.53 $\pm$.1	6.27 $\pm$.1
60.	5.25 $\pm$.1	5.40 $\pm$.1

The agreement is seen to be quite good. However, it still remains to be seen whether the scattering data are explainable on the basis of inelastically scattered protons.

If there is an excited level of the helium nucleus,

the inelastic proton energy will have the angular distribution:

$$E_{p'} = \frac{E}{25} \cos 2\phi + \frac{16}{25} E \left(1 - \frac{5}{4} \frac{\epsilon}{E}\right) + \frac{8}{25} E \sqrt{1 - \frac{5}{4} \frac{\epsilon}{E}} \cos \phi \sqrt{1 - \frac{\sin^2 \phi}{16 \left(1 - \frac{5}{4} \frac{\epsilon}{E}\right)}} \quad (30)$$

where ϵ is the energy above the ground state of the presumed excited state. To determine a consistent value for ϵ , one measures the range of the low energy group of particles at some angle, say 45° . From the range-energy relations one can determine the energy of a proton having the range of the particles observed; then inserting this E_p , and the previously determined E into Eq. (7), one finds that $\epsilon = 21.2$ Mev. The column headed E_p , in Table III lists the results of Eq. (30), using this value of ϵ . $E_{p, (obs)}$ is the energy of a proton having the range experimentally measured.

TABLE III

ϕ	E_p	$E_p(\text{obs})$
15	$8.15 \pm .15$	$9.20 \pm .05$
23.5	$7.72 \pm .15$	$8.30 \pm .05$
30	$7.16 \pm .15$	$7.70 \pm .05$
37.5	$6.51 \pm .10$	$6.93 \pm .05$
45	$5.77 \pm .10$	$5.80 \pm .07$
52.5	$5.02 \pm .10$	$4.85 \pm .1$
60	$4.27 \pm .10$	$4.03 \pm .10$

The discrepancy is seen to be quite significant.

B. Another identification of the low-energy peak can be made by comparing the $\frac{dE}{dx}$ of these particles with that of the elastic protons. It will be recalled from the description of the construction of the proportional counter telescope that all counted particles entering the counters have the same residual range. The $\frac{dE}{dx}$ of deuterons and protons having the same residual range differs by about 10 percent; hence, one expects a difference of about 30 percent in the pulse height observed in the front counters. Inserting enough absorber before the counter telescope to see just the elastic protons and plotting counting rate vs. discriminator voltage on the front two counters yields the curve of Fig. 9 (a).

Figure 9 (b) is a similar curve obtained with the absorber required to see just the particles of the low energy peak. The $\frac{dE}{dx}$ of the particles of the low energy peak is seen to be about 25 percent larger than that of protons, hence these particles are certainly deuterons.

DISCRIMINATOR SETTING ON COUNTERS 1 AND 2 (APPROX. VOLTS)

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Consideration of the Continuum

We now turn our attention to the continuum on the low-range side of the deuteron peak. These particles have been identified as protons by means of a $\frac{dE}{dx}$ measurement, and from a comparison of the range and energy of the high energy cut-off. (This latter measurement was performed by making a pulse height analysis with a NaI (Tl) crystal.)

The most reasonable sources for a continuum of protons of the observed range are the reactions:

where p' and p'' represent inelastically scattered protons. It is clear that observation of p' would yield information on the states of the $(n+\text{He}^3)$ and $(p''+\text{H}^3)$ systems which could also be referred to as the excited states of the He^4 nucleus (He^{4*}). For example, if the wave function describing the $n+\text{He}^3$ system became unusually large for a given relative momentum, the probability that the proton (p') would come off with a related momentum (given by the conservation laws) would be correspondingly large. Reactions (31a) and (31b) may be rewritten:

with somewhat less probability for the mode:

The spectrum of p' should be a peak with an energy spread commensurate with the breadth of the excited He^4 level, while that of p'' would appear rather spread out due to the motion of its parent He^{4*} , and its area would be somewhat less than half that of the p' peak. Thus we can see qualitatively that if there were an excited He^4 level, the spectrum of

protons would be expected to appear as shown in Fig. 10(a).

Fig. 10(a)

if the excitation energy were ≈ 22 Mev, or as shown in Fig. 10(b),

Fig. 10(b)

if the excitation energy were ≈ 25 Mev.

Note: Only 25.4 Mev are available in the c.m. system, so one can investigate the level structure of He^4 only to this energy.

If there were an excited level, therefore, one would expect to be able to detect a bump in the continuum. Actually, no bump (exhibiting the proper range-angle dependence) was discernible; therefore, all one can do is calculate the maximum cross section for the formation of an excited state by stimulating the maximum area of a peak which might have been masked by the counting statistics. A calculation of this sort for the spectrum obtained at 22.5° (see Fig. 1) yields:

$$\left(\frac{d\sigma}{d\Omega}\right)_{1\lambda} \leq .34 \text{ millibarns/steradian}$$

Unfortunately, the intrusion of elastically scattered alpha particles masks some of the low-range portion of the small angle spectra where it would be desirable to hunt for a low-energy group of protons. For example, at 22.5° it would not be possible to detect a small group of protons having a range less than 50 mg/cm^2 (5 Mev). Reference to the kinematical equation shows that a group of protons of this energy would result if there were a level in He^4 at 23.3 Mev.

Finally, making the transformation to the c.m. system, we may conclude that the differential cross section for the formation of an excited level in He^4 as high as 23.3 Mev above the ground state by bombarding alpha particles with protons is certainly less than 0.1 mb/ster. at 45° (c.m.).

If the excitation energy were as high as 24-25 Mev, the maximum cross section which might have escaped detection via the p^+ protons is 0.4 mb/ster.

If there exists a single excited level of the He^4 nucleus in the energy region and with the cross section indicated by Argo, et al.,⁷ a low-energy proton group would have been seen. The absence of such a low-energy proton group confirms and extends the findings of some other workers in this field.

The most natural extension to the work of the Los Alamos group was performed by Falk and Phillips,¹⁰ who extended the excitation curve for the reaction $\text{H}^3(p,\gamma)\text{He}^4$ to incident proton energies of 3.4 Mev. No resonance is observed; rather, the γ -ray yield continues to increase in the way predicted by the calculations of Flowers and Mandl.¹¹ However, the existence of a weak (up to 30 percent), broad resonance superposed on the rising yield curve is not ruled out.

Allred,¹² in studying the reaction $\text{He}^3(d,p)\text{He}^4$, sought a group of low energy protons as evidence for the existence of an excited He^4 level. He reports an upper limit to the cross section for observing these protons

at a c.m. angle of 102.7° of $0.2 \times 10^{-27} \text{ cm}^2$, implying the absence of an excited He^4 level below 20.9 Mev.

Finally, the continued high level of the inelastically scattered protons, as far as we have been able to see, requires the existence of at least two levels, each having a half-width of about a million volts and a separation of about a million volts. However, whether there are just two levels or several closely spaced levels contributing to this region cannot be decided from this work.

RESULTS AND CONCLUSIONS--PICK-UP DEUTERONS

It is interesting to compare the angular dependence of the differential cross section for the production of deuterons from the reaction $\text{He}^4(p,d)\text{He}^3$ with that predicted by the Butler theory.¹³ Butler considers an incident deuteron to be stripped, with the neutron, say, deposited in the target nucleus. He finds that the angular distribution of the remaining proton depends in a distinctive way on the angular momenta and parities of the states of the initial and final nuclei. To date, the theory has been applied successfully to the assignment of angular momentum and parity to one nucleus when that of the other was known.^{14,15,16,17,18,19}

Here, we are concerned with the inverse reaction, so we invoke the principle of detailed balancing which tells us that in the c.m. system, the differential cross section for the inverse reaction is directly proportional to that for the direct reaction; further, the proportionality factor includes only statistical weights, so that the angular dependences of the two differential cross sections are the same. Hence, applying the Butler theory to the reaction inverse to ours, $\text{He}^3(d,p)\text{He}^4$, will yield to us directly the angular dependence of the differential cross section for observation of our deuterons. Since the spins and parities of the initial and final nuclei are known, comparison of the theoretical and experimental data might be considered a check of the theory; or if the theory is considered to be a reasonable approximation to the truth, and if one believes the principle of detailed balancing, the theory may serve to fix the "size" of the He^3 nucleus.

If the captured neutron carries with it zero angular momentum, Butler

gives:

$$\sigma(\theta) \approx \left\{ \frac{1}{k^2 + a^2} - \frac{1}{k^2 + (a+b)^2} \right\}^2 \left| \left(\chi_s + \frac{2}{2r} \right)_{r_0} \left(\frac{r}{2} \right)^{1/2} J_{1/2}(2r) \right|^2 \quad (53)$$

$$\approx \left\{ \frac{1}{k^2 + a^2} - \frac{1}{k^2 + (a+b)^2} \right\}^2 \left| \frac{\chi_s}{2} \sin Zr_0 + \cos Zr_0 \right|^2$$

where

$$k^2 = \frac{k_d^2}{4} + k_{ps}^2 - k_d k_{ps} \cos \theta$$

$$\Delta^2 = \frac{k_d^2}{4} + k_{ps}^2 - 2 k_d k_{ps} \cos \theta$$

$$k_d^2 = \frac{2m_d}{\hbar^2} E_d$$

$$k_{ps}^2 = \frac{2m_p}{\hbar^2} E_p$$

$$i\chi_s = k_{ns}$$

$$k_{ns}^2 = \frac{2m_n}{\hbar^2} E_n$$

$$E_n = E_d - \epsilon - E_p$$

ϵ is deuteron binding energy.

$$\left. \begin{aligned} a &= .35 \times 10^{-13} \text{ cm}^{-1} \\ b &= 1.4 \times 10^{-13} \text{ cm}^{-1} \end{aligned} \right\} \text{ are parameters of internal wave function of the deuteron}$$

r_0 is the interaction radius between the neutron and the target nucleus (He^3).

Figure 12 is a comparison of the theoretical curve for $r_0 = 4.2 \times 10^{-13} \text{ cm}$,

with a curve drawn through the experimental points. In spite of the somewhat

meager experimental data, the fit, as determined by the coincidence of maxima

and minima, is considered rather good. The increasing divergence of the

two curves at large angles is not unexpected in view of the meaning of r_0 .

By definition, the proton does not penetrate the region of radius r_0 about

the target nucleus; however, this assumption is expected to be violated in collisions involving high momentum transfers, i.e., large scattering angles.

Butler and Symonds²⁰ compare the theoretical angular distributions for the reactions $T(d,n)He^4$ and $He^3(d,n)He^4$ for 10.5 Mev deuterons with the experimental curves ofrolley, et al.,²¹ and Alfred¹² and find $r_0 = 4.3 \times 10^{-13}$ cm. The difference is not considered significant.

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APPENDIX I - SCATTERING OF CHARGED PARTICLES IN THICK ABSORBERS.

The proportional counter telescope detects charged particles with a residual range of about 15 mg/cm^2 . To slow the particle from its original energy to that at which it may be counted, it is passed through some aluminum absorbers in which it may be scattered. To get some idea of the sensitive area required of the counters in order that not too many particles would be lost by scattering, an investigation of the scattering of charged particles considering energy loss was made.

We begin with Rossi and Greisen's²² derivation of the diffusion equation for the distribution function of particles in an absorber and proceed with Eyges's²³ treatment considering energy loss.

Fig. 13

Consider the projection of motion of particles on the (t, y) plane and let $F(t, y, \theta) dy d\theta$ be the number of particles at thickness t having lateral displacement (y, dy) and traveling at an angle $(\theta, d\theta)$ with the t -axis.

For symmetry reasons, F also describes the distribution in the (t, z) plane. Since deflections in the two orthogonal directions are independent of each other, the number of particles having lateral displacement with components (y, dy) and (z, dz) and angular deflection with components $(\theta_y, d\theta_y)$ and $(\theta_z, d\theta_z)$ is:

$$F(t, y, \theta_y) F(t, z, \theta_z) dy dz d\theta_y d\theta_z \quad (34)$$

Let $\rho_{\Delta t}(\theta)d\theta$ be the probability that a particle traversing thickness Δt will be deflected through the angle $(\theta, d\theta)$; deflection θ is not necessarily caused by single scattering.

By definition $\rho(\theta)$ satisfies the following conditions.

$$\rho_{\Delta t}(\theta) = \rho_{\Delta t}(-\theta) \quad (35)$$

$$\int_{-\infty}^{\infty} \rho_{\Delta t}(\theta)d\theta = 1 \quad (36)$$

$$\int_{-\infty}^{\infty} \theta \rho_{\Delta t}(\theta)d\theta = 0 \quad (37)$$

$$\int_{-\infty}^{\infty} \theta^2 \rho_{\Delta t}(\theta)d\theta = \langle \theta^2 \rangle_{\text{ave. } \Delta t} = \frac{1}{2} \frac{E_s^2}{p^2 \beta^2} \Delta t \quad (38)$$

where $E_s = 21$ Mev, p is the momentum and β , the velocity of the incident particle. The integrals may be extended to infinity in both directions since $\rho(\theta)$ has a sharp maximum at $\theta = 0$ and decreases rapidly on both sides of zero.

In passing from t to $t + \Delta t$, F changes because of changes in both the space and angular distributions. The space distribution changes because particles traveling at an angle θ undergo a lateral displacement $\theta \Delta t$ in the layer Δt . Scattering in this layer is a second order effect and is disregarded. It follows that particles having lateral displacement y at $t + \Delta t$ are those having lateral displacement $y - \theta \Delta t$ at t . Hence, neglecting a change in the angular distribution:

$$F(t + \Delta t, y, \theta) = F(t, y - \theta \Delta t, \theta) = F(t, y, \theta) - \frac{\partial F}{\partial y} \theta \Delta t \quad (39)$$

To calculate the effect of a change in the angular distribution, consider two angular intervals $(\theta, d\theta)$ and $(\theta', d\theta')$. There are $F(t, y, \theta') dy d\theta'$ particles in $(\theta', d\theta')$ at thickness t with lateral displacement (y, dy) , and

a fraction $\rho_{\Delta t}(\theta - \theta')$ of these is scattered into the angular interval $(\theta, d\theta)$ while traversing a layer Δt . Hence, neglecting the change in the space distribution,

$$F(t + \Delta t, y, \theta) = \int_{-\infty}^{\infty} F(t, y, \theta') \rho_{\Delta t}(\theta - \theta') d\theta' \quad (40)$$

Since $\rho_{\Delta t}(\theta - \theta')$ is different from zero only for small values of the argument, F may be developed in a Taylor's series of $(\theta - \theta')$:

$$F(t, y, \theta') = F(t, y, \theta) + \frac{\partial F}{\partial \theta} (\theta - \theta') + \frac{1}{2} \frac{\partial^2 F}{\partial \theta^2} (\theta - \theta')^2 + \dots \quad (41)$$

Dropping terms of order higher than the second, and taking account of the properties of $\rho_{\Delta t}(\theta)$, one obtains:

$$\begin{aligned} F(t + \Delta t, y, \theta) &= F(t, y, \theta) \int_{-\infty}^{\infty} \rho_{\Delta t}(\theta - \theta') d\theta' + \frac{\partial F}{\partial \theta} \int_{-\infty}^{\infty} (\theta - \theta') \rho_{\Delta t}(\theta - \theta') d\theta' \\ &\quad + \frac{1}{2} \frac{\partial^2 F}{\partial \theta^2} \int_{-\infty}^{\infty} (\theta - \theta')^2 \rho_{\Delta t}(\theta - \theta') d\theta' + \dots \\ &= F(t, y, \theta) + \frac{1}{4} \frac{E_s^2}{p^2 \ell^2} \frac{\partial^2 F}{\partial \theta^2} \Delta t \end{aligned} \quad (42)$$

Thus the total change in F in a layer Δt is.

$$\Delta F = - \frac{\partial F}{\partial y} \theta \Delta t + \frac{1}{4} \frac{E_s^2}{p^2 \ell^2} \frac{\partial^2 F}{\partial \theta^2} \Delta t \quad (43)$$

or, one may write the P.E. for the distribution function:

$$\frac{\partial F}{\partial t} = - \theta \frac{\partial F}{\partial y} + \frac{1}{w^2} \frac{\partial^2 F}{\partial \theta^2} \quad (44)$$

where

$$w = \frac{2 p \ell}{E_s} \quad (45)$$

Note: For large values of p , w represents 2 λ of the incident particle in units of E_s ; for small p , w is $4 \times E$ of the incident particle in units of E_s .

This is the diffusion equation for the distribution function F . Now, after Bygges,²⁵ y and γ are assumed $\gamma(t)$ only, i.e., neglect the fact that a particle at t has traveled somewhat further than $\gamma(t)$ due to deviations caused by scattering.

Applying the Fourier transforms:

$$F(t, y, \theta) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} G(t, x, \gamma) e^{i(xy + \gamma\theta)} dx d\gamma \quad (43)$$

$$G(t, x, \gamma) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} F(t, y, \theta) e^{-i(xy + \gamma\theta)} dy d\theta \quad (47)$$

to the differential Equation (44).

$$\frac{\partial F}{\partial t} = \frac{1}{2\pi} \iint \frac{\partial G}{\partial t} e^{i(xy + \gamma\theta)} dx d\gamma \quad (48)$$

$$\frac{\partial F}{\partial y} = \frac{1}{2\pi} \iint i x G e^{i(xy + \gamma\theta)} dx d\gamma \quad (49)$$

$$\frac{\partial^2 F}{\partial \theta^2} = \frac{1}{2\pi} \iint -\gamma^2 G e^{i(xy + \gamma\theta)} dx d\gamma \quad (50)$$

One gets:

$$\frac{\partial G}{\partial t} = -i x \theta G - \frac{\gamma^2}{w^2} G \quad (51)$$

or, since

$$\frac{\partial G}{\partial \gamma} = -i \theta G$$

$$\frac{\partial G}{\partial t} = x \frac{\partial G}{\partial \gamma} - \frac{\gamma^2}{w^2(t)} G \quad (52)$$

Introducing the new variables

$$\xi = t + \frac{\gamma}{x} \quad (53)$$

$$t' = t \quad (54)$$

$$\frac{\partial G}{\partial t} = \frac{\partial G}{\partial \xi} \frac{\partial \xi}{\partial t} + \frac{\partial G}{\partial t'} \frac{\partial t'}{\partial t} = \frac{\partial G}{\partial \xi} + \frac{\partial G}{\partial t'} \quad (55)$$

$$\frac{\partial G}{\partial \gamma} = \frac{\partial G}{\partial \xi} \frac{\partial \xi}{\partial \gamma} = \frac{\partial G}{\partial \xi} \frac{1}{x} \quad (56)$$

The equation for G becomes:

$$\frac{\partial G}{\partial t'} = - \frac{r^2(\xi - t')^2}{w^2(t')} G \quad (57)$$

The solution of this equation is:

$$G = H(\xi) e^{-x^2 \int_k^{t'} \frac{(\xi - \gamma)^2}{r^2(\gamma)} d\gamma} \quad (58)$$

where $H(\xi)$ is the "constant" of integration, and for convenience the lower limit in the integral has been taken to be some fixed number k . The boundary condition is that at $t = 0$ there is one particle incident normal to $y = 0$; i.e.,

$$F(0, y, \theta) = \delta(y) \delta(\theta) \quad (59)$$

Hence,

$$G(0, x, \gamma) = \frac{1}{2\pi} \quad (60)$$

In terms of the original variables,

$$G(t, x, \gamma) = H(t + \frac{\gamma}{x}) e^{-x^2 \int_k^t \frac{(t + \frac{\gamma}{x} - \gamma)^2}{r^2(\gamma)} d\gamma} \quad (61)$$

so, from Eq. (60):

$$H(\frac{\gamma}{x}) = \frac{1}{2\pi} e^{x^2 \int_1^0 \frac{(\frac{\gamma}{x} - \gamma)^2}{r^2(\gamma)} d\gamma} \quad (62)$$

yields the functional form of H . Hence G is completely determined as:

$$G(t, x, \gamma) = \frac{1}{2\pi} e^{-x^2 \int_0^t \frac{(\frac{\gamma}{x} + t - \gamma)^2}{r^2(\gamma)} d\gamma} \quad (63)$$

Defining three new functions of t as:

$$A_0(t) = \int_0^t \frac{d\gamma}{r^2(\gamma)} \quad (64)$$

$$A_1(t) = \int_0^t \frac{(t - \gamma)}{r^2(\gamma)} d\gamma \quad (65)$$

$$A_2(t) = \int_0^t \frac{(t - \gamma)^2}{r^2(\gamma)} d\gamma \quad (66)$$

one can write:

$$G(t, x, y) = \frac{1}{2\pi} e^{-(A_0 y^2 + 2A_1 y x + A_2 x^2)} \quad (67)$$

Putting this into Eq. (46), and integrating over x and y ,

$$F(t, y, \theta) = \frac{1}{4\pi B^{1/2}(t)} e^{-\frac{A_2 \theta^2 - 2A_1 y \theta + A_0 y^2}{4B}} \quad (68)$$

where

$$B(t) = A_0 A_2 - A_1^2 \quad (69)$$

Integrating over y ,

$$\int_{-\infty}^{\infty} F(t, y, \theta) dy = \frac{1}{2(\pi A_0)^{1/2}} e^{-\frac{\theta^2}{4A_0}} \quad (70)$$

Integrating over θ ,

$$\int_{-\infty}^{\infty} F(t, y, \theta) d\theta = \frac{1}{2(\pi A_2)^{1/2}} e^{-\frac{y^2}{A_2}} \quad (71)$$

If w^2 is constant, it is easily shown that these solutions reduce to those of Fermi as given in Rossi and Greisen.²²

From Eq. (71) we see that the mean-square value of the displacement is,

$$\overline{y^2} = 2 A_2 \quad (72)$$

here A_2 is defined in Eq. (66). A sample calculation of this displacement for 30 Mev protons follows:

The columns of Table IV contain the computations of the integrals of $F_2(t)$. Although t and η in Eq. (66) are to be expressed in radiation lengths of the absorber material, these terms appear as g/cm^2 in the table because they are most conveniently obtained in this form from the range-energy curves. The proper conversion factor is introduced after the graphical integration.

From Fig. 14,

$$A_2 = 1.234 \times 10^{-3} \text{ cm}^2$$

$$\sqrt{y^2} = \sqrt{2 A_2} = .05 \text{ cm.}$$

The magnitude of the root-mean-square projected displacement gives a measure of the required lateral dimensions of the absorbers. Those used were 5/8" wide, so no losses are expected from this source.

TABLE IV

$E = 30$ Mev protons.

Residual Range of Protons which enter Counters $\approx 15 \text{ mg/cm}^2$

$$t = 1165 - 15 = 1150 \text{ mg/cm}^2$$

γ	$(t - \gamma)$	$(t - \gamma)^2$	$\frac{(t - \gamma)^2}{w^2}$
	1150	1.32×10^6	40.5×10^3
50	1100	1.21×10^6	39.7×10^3
100	1050	1.10×10^6	37.3×10^3
150	1000	1.00×10^6	36.4×10^3
200	950	$.90 \times 10^6$	34.4×10^3
250	900	$.81 \times 10^6$	32.9×10^3
300	850	$.722 \times 10^6$	31.5×10^3
350	800	$.64 \times 10^6$	30.2×10^3
400	750	$.563 \times 10^6$	28.3×10^3
450	700	$.49 \times 10^6$	26.9×10^3
500	650	$.423 \times 10^6$	25.2×10^3
550	600	$.36 \times 10^6$	23.5×10^3
600	550	$.303 \times 10^6$	22.0×10^3
650	500	$.25 \times 10^6$	20.1×10^3
700	450	$.202 \times 10^6$	18.2×10^3
750	400	$.16 \times 10^6$	16.3×10^3
800	350	$.122 \times 10^6$	14.4×10^3
850	300	$.09 \times 10^6$	12.6×10^3
900	250	$.0625 \times 10^6$	10.9×10^3
950	200	$.04 \times 10^6$	8.8×10^3
1000	150	$.0225 \times 10^6$	6.86×10^3
1050	100	$.01 \times 10^6$	4.33×10^3
1100	50	2.5×10^3	2.27×10^3
1150	0	0	0

From Eq. (70) we see that the mean-square projected scattering angle is given by:

$$\overline{\theta^2} = 2 A_0 \quad (73)$$

where A_0 is defined in Eq. (64). From Table V and Fig. 15,

$$A_0 = .778 \times 10^{-2}$$

$$\overline{\theta^2} = 2 A_0 = 1.5 \times 10^{-2} \text{ radians}$$

$$\begin{aligned} \sqrt{\overline{\theta^2}} &= 1.25 \times 10^{-1} \text{ radians} \\ &= 7.15^\circ \end{aligned}$$

TABLE V

E = 30 Mev protons

(n_0/cm^2)	E(Mev)	$W = \frac{dE}{d\Omega}$	W^2	$\frac{1}{W^2}$
0	30	5.71	32.6	3.07×10^{-2}
50	29	5.52	30.5	3.28×10^{-2}
100	28.5	5.43	29.5	3.39×10^{-2}
150	27.5	5.24	27.5	3.64×10^{-2}
200	26.9	5.12	26.2	3.82×10^{-2}
250	26.0	4.96	24.6	4.07×10^{-2}
300	25.1	4.79	22.9	4.36×10^{-2}
350	24.2	4.61	21.2	4.72×10^{-2}
400	23.4	4.46	19.9	5.02×10^{-2}
450	22.4	4.27	18.2	5.50×10^{-2}
500	21.5	4.10	16.8	5.95×10^{-2}
550	20.5	3.91	15.3	6.54×10^{-2}
600	19.5	3.71	13.8	7.24×10^{-2}
650	18.5	3.52	12.4	8.07×10^{-2}
700	17.5	3.33	11.1	9.01×10^{-2}
750	16.4	3.13	9.8	10.20×10^{-2}
800	15.3	2.91	8.5	11.77×10^{-2}
850	14.0	2.67	7.1	14.10×10^{-2}
900	12.6	2.40	5.75	17.40×10^{-2}
950	11.2	2.13	4.55	22.0×10^{-2}
1000	9.9	1.88	3.54	28.2×10^{-2}
1050	8.1	1.54	2.37	42.2×10^{-2}
1100	5.8	1.10	1.21	82.6×10^{-2}
1120	4.7	.895	.80	$125. \times 10^{-2}$
1140	3.3	.620	.395	$253. \times 10^{-2}$
1150	2.5	.476	.227	$440. \times 10^{-2}$
1165	0

We may derive a simple, approximate expression for this mean square scattering angle with energy loss, in the following way:

An approximate form of the projected mean square scattering angle^{22,24} considering zero energy loss is:

$$\overline{\theta^2}_{dx} = \frac{2\pi r_0^2 Z^2 N}{\left(\frac{E}{mc^2}\right)^2} \ln 181 Z^{-1/3} dx \quad (74)$$

$$\overline{\theta^2}_{dE} = 2\pi r_0^2 Z^2 N (mc^2)^2 \ln 181 Z^{-1/3} \frac{dE}{E^2 \frac{dE}{dx}} \quad (75)$$

For energies above 2 Mev, the analytic expression:

$$-\frac{dE}{dx} = \frac{\pi e^4 Z^2}{mc^2} B \quad (76)$$

$$B = Z \ln \frac{2mv^2}{I} \quad (77)$$

is rather good. Non-relativistically:

$$B = Z \ln \frac{4mE}{I} \quad (78)$$

and

$$\overline{\theta^2}_{dE} = -R \frac{dE}{E \ln \frac{4mE}{I}} \quad (79)$$

where:

$$R = \frac{n}{N} \frac{Z}{Z} \ln 181 Z^{-1/3} \quad (80)$$

Hence, for finite energy loss $E = E_f - E_o$,

$$\begin{aligned} \overline{\theta^2}_E &= -R \int_{E_o}^{E_f} \frac{dE}{E \ln \frac{4mE}{I}} \\ &= R \ln \left(\frac{\ln \frac{4mE_o}{I}}{\ln \frac{4mE_f}{I}} \right) \end{aligned} \quad (81)$$

For protons ($z = 1$) being absorbed in aluminum ($Z = 13$, $I = 149.5$ ev)

$$R = \frac{1}{1840} \frac{13}{1} \ln \frac{101}{2.35} = 3.07 \times 10^{-2}$$

Protons just entering the counter telescope have a residual range of about 15 mg/cm^2 (2.5 Mev), hence if they were originally of 30 Mev:

$$\begin{aligned} \overline{\theta^2}_{30} &= 3.07 \times 10^{-2} \ln \left(\frac{\ln \frac{4}{1840} \frac{30 \times 10^6}{149.5}}{\ln \frac{4}{1840} \frac{2.5 \times 10^6}{149.5}} \right) \\ &= 1.61 \times 10^{-2} (\text{rad})^2 \end{aligned}$$

$$\begin{aligned} \sqrt{\overline{\theta^2}_{30}} &= 1.27 \times 10^{-1} \text{ rad} \\ &= 7.3^\circ \end{aligned}$$

in excellent agreement with the value obtained by the method of the distribution function and the empirical range-energy curves.

This is the worst case of scattering encountered. The counters were designed to have an aperture of plus or minus two root-mean-square scattering angles ($\pm 15^\circ$) so that we could expect to count 95 percent of these particles. Virtually no correction is necessary in counting lower energy particles.

APPENDIX II - CALCULATION OF LINE PROFILE GIVEN BY PROPORTIONAL COLLIDER TELESCOPE

A mono-energetic stream of particles entering the detector will not yield an infinitely thin line spectrum, but will be broadened out because of several effects. We shall now consider some of the contributions to the line breadth and calculate the resultant line shape.

I. Contribution of Finite Range "Bite" Accepted by the Counter Telescope.

Since we do not know where in the slimmer foil the particle stops, all depths in the foil are equally probable.

Fig. 16

II. Contribution of Range Straggling.

Two effects contribute to the straggling in range of a homogeneous group of particles; the distribution of energy loss on collision and the variation in the distance between successive collisions. To a sufficient approximation the distribution in range is Gaussian:

$$I(R)dR = -\frac{\pi}{4s^2} (R-R_0)^2 \quad (62)$$

where R_0 is the mean range, and s is the difference between mean and extrapolated range.

$$s^2 = \frac{\pi}{2} \overline{(R-R_0)^2} \quad (63)$$

Figure 17 is from the work of Livingston and Bethe.²⁵ Although the ordinate represents s/R for protons, this ratio for other particles may be determined

in the following way: Livingston and Bethe show that

$$s = M^{1/2} Z^{-2} g(v) \quad (84)$$

where M is the mass, Z , the charge of the particle in question, and $g(v)$ is some function of the velocity. Hence one may write:

$$\frac{s}{R} \frac{Z^2}{M} M^{1/2} = g(v) \quad (85)$$

Furthermore, from the known dependence:

$$\frac{dE}{dx} = Z^2 f(v) \quad (86)$$

may be deduced:

$$R = \frac{\int \frac{dE}{dE}}{\frac{dE}{dx}} = \frac{M}{Z^2} \int \frac{v dv}{f(v)} \approx \frac{M}{Z^2} h(v) \quad (87)$$

Hence

$$\frac{s}{R} M^{1/2} = F(v) \quad (88)$$

The straggling of particles having the same velocity is related by:

$$\frac{s_1}{R_1} M_1^{1/2} = \frac{s_2}{R_2} M_2^{1/2} \quad (89)$$

or, said in another way, the straggling of particles of mass M , charge Z , and range R may be expressed in terms of the straggling of protons of range $\frac{Z^2 R}{M}$;

$$\left(\frac{s}{R} \right)_{M,Z,R} = M^{-1/2} \left(\frac{s}{R} \right)_{1,1} \frac{Z^2 R}{M} \quad (90)$$

In addition to these instrumental effects, contributions to the line breadth will obtain if the incident protons are not mono-energetic.

III. Contribution due to Energy Spread of the Incident Beam.

We calculate this approximately on the basis of the dispersion of the analyzing magnet and the finite size of the collimator. Assuming the beam enters the magnet in an infinitely thin line, we take liberties with the

geometry (Fig. 18) so we can write:

Fig. 18

$$\frac{r_1 - r_2}{\sin \Delta \theta} = \frac{r_2}{\sin \theta_1} \quad (21)$$

$$\begin{aligned} \Delta \theta &= \frac{r_1 - r_2}{r_2} \sin \theta_1 \\ &= \frac{\Delta r}{r} \sin \theta = \frac{\Delta E}{E} \sin \theta \end{aligned}$$

$$\Delta E = E \frac{\Delta \theta}{\sin \theta} \quad (22)$$

Fig. 19

If one considers the finite width of the beam entering the magnet one obtains a correction of the nature of a very slight slanting of the sides of the above distribution. In this case it turns out to be hard

without considering.

Fig. 20

IV. Contribution of Finite Width of Acceptance Slits:

For light target nuclei, the energy of the scattered particle will depend on the angle of scatter.

If the first slit is uniformly illuminated, (i.e., the number of particles striking the first slit between x and $x+dx$ and making an angle θ between θ and $\theta+d\theta$ with the normal to the slit depends only on the scattering length contribution),

$$N(\theta, x) dx = \frac{K}{\sin(\phi - \theta)} d\theta dx \quad (93)$$

This is a good assumption if the differential cross section for the scattering process varies little over the acceptance angle of the slits. Then the number of particles which pass through the two slits and make an angle $\theta(d\theta)$ is:

Fig. 21

$$N(\theta)d\theta = d\theta \int_{-\frac{A}{2}}^{-l \tan \theta + \frac{A}{2}} \frac{K}{\sin(\phi - \theta)} dx = \frac{Kd\theta}{\sin(\phi - \theta)} \left[A - l \tan \theta \right] \quad \theta > 0 \quad (94)$$

Since, in this case,

$$\theta_{\max} = \tan^{-1} \frac{1}{\frac{A}{l}} \quad (95)$$

one is safe in approximating, for not too small ϕ ,

$$\begin{aligned} N(\theta)d\theta &\approx \frac{Kd\theta}{\sin \phi - \theta \cos \phi} [A - l\theta] \\ &\approx \frac{Kd\theta}{\sin \phi (1 - \theta \cot \phi)} (A - l\theta) \\ &\approx \frac{K}{\sin \phi} (1 + \theta \cot \phi)(A - l\theta)d\theta \end{aligned} \quad (96)$$

Thus, neglecting terms of order θ^2 ,

$$N(\theta)d\theta \approx \frac{K}{\sin \phi} [A - l(1 - \frac{A}{l} \cot \phi) \theta] d\theta \quad \theta > 0 \quad (97a)$$

and

$$N(\theta)d\theta \approx \frac{K}{\sin \phi} [A + l(1 - \frac{A}{l} \cot \phi) \theta] d\theta \quad \theta < 0 \quad (97b)$$

Fig. 22

Since E is a function of θ

$$E(\phi + \theta) - E(\phi) = \left(\frac{\partial E}{\partial \theta} \right)_{\phi} \theta + \left(\frac{\partial^2 E}{\partial \theta^2} \right)_{\phi} \frac{\theta^2}{2} + \dots \quad (98)$$

$N(E)$ will be linear in E when the second and higher order terms are inconsequential.

Fig. 23

V. Contribution of Gas Scattering.

Multiple scattering in the target may cause the detector to see particles it would ordinarily have not, and to miss particles it would have seen. To get an idea of the amount of line broadening due to this effect, one might calculate the root mean square scattering angle from the Williams formula; then

$$\Delta E = \left(\frac{\partial E}{\partial \alpha} \right)_{\phi} \sqrt{\alpha^2} \quad (99)$$

For α in the scattering chamber $\sqrt{\alpha^2} = \frac{32 \times 10^{-3}}{E}$ radians, and since $\left(\frac{\partial E}{\partial \alpha} \right)_{\phi}$ is of the order 1 Me /rad, it is apparent that this contribution will be negligible.

VI. Contribution of Finite Target Width.

Since the scatter may occur anywhere in the target, spread in the energies of both the incident and scattered particles will be produced by absorption in the target. The magnitude of the inhomogeneity introduced into the energy of the scattered particles will be a function of the energy, angle of observation, mass and charge of the particle, and although it is easily evaluated in the specific case, an expression describing the general case would be rather cumbersome. In range, the inhomogeneity is of the order of once or twice the target thickness.

It will be noted that some contributions to the line profile were more easily expressed in terms of energy spread and others, in terms of

range spread. Actually, since this apparatus is a differential range measuring device, it would make more sense to express the line profile in terms of range alone, so that it might be compared with the raw data. It is expected that if the region of interest is small enough so that the range energy curve may be considered linear, the form of the distributions will be the same in the two representations. Otherwise, appropriate corrections must be made.

It turns out that, for the case of scattering of Helium gas target, the major contributions to the line width are I, II, and IV. In certain circumstances the other effects (especially III and VI) may contribute appreciably; however, it is felt that when small, the additional broadening due to these may be adequately approximated, or when large, a separate analysis may be made. So in the interest of avoiding complications which affects the final result only little, we shall neglect those in calculating the theoretical line shape.

The fold of two functions $f(x)$ and $g(x)$ is defined as:

$$F(x') = \int_{-\infty}^{\infty} f(x-x') g(x) dx \quad (10)$$

The extension to the definition of the fold of three functions is obvious, as is the property that the result of the folding operation is independent of the order in which the folding is done.

First contributions I and IV will be considered. The saw-toothed contribution will be represented by:

$$\begin{aligned} g(x) &= ax + b & x &\leq 0 & a, b, c &\geq 0 \\ &= -cx + b & x &\geq 0 \end{aligned} \quad (10)$$

The square-wave contribution may be decomposed into two step functions, one positive and one negative, as shown in Fig. 24.

Fig. 31

The first step-function is given by:

$$\begin{aligned} f(x') &= 1 & x' \leq 0 \\ &= 0 & x' > 0 \end{aligned} \quad (102)$$

and its fold with the saw-tooth yields:

$$F(x') = \int_{-\frac{1}{2}}^{x'} (ax + b) dx + \int_0^{x'} (-ax + b) dx \quad (103)$$

$$\begin{aligned} & \quad \quad \quad x' \leq 0 & \quad \quad \quad \frac{b}{c} \geq x' \geq 0 \end{aligned}$$

$$= \frac{ax'^2}{2} + bx' + \frac{b^2}{2c} + \left[-\frac{ax'^2}{2} + bx' \right]$$

$$= \frac{ax'^2}{2} + bx' + \frac{b^2}{2c} \quad x' \leq 0 \quad (104a)$$

$$= -\frac{ax'^2}{2} + bx' + \frac{b^2}{2c} \quad \frac{b}{c} \geq x' \geq 0 \quad (104b)$$

The second step-function is given by:

$$\begin{aligned} f(x') &= -1 & x' \leq -W + \frac{b}{c} \\ &= 0 & x' > -W + \frac{b}{c} \end{aligned} \quad (105)$$

and its fold with the saw-tooth yields:

$$F(x') = - \left[\int_{-\frac{1}{2}}^{-W+x'} (ax + b) dx + \int_0^{-W+x'} (-ax + b) dx \right] \quad (106)$$

$$\begin{aligned} & \quad \quad \quad x' - W \leq 0 & \quad \quad \quad \frac{b}{c} > -W + x' \geq 0 \end{aligned}$$

$$= - \left[\frac{a(x'-W)^2}{2} + b(x'-W) + \frac{1}{2c} \right] \quad x'-W \leq 0 \quad (107a)$$

$$= - \left[\frac{1}{2c} - \frac{a(x'-W)^2}{2} + b(x'-W) \right] \quad \frac{1}{c} > x'-W \geq 0 \quad (107b)$$

Here, we have assumed that $W < \frac{1}{c}$, in agreement with the experimental conditions.

$F(x')$ takes on the following forms in the specific regions:

$$I \quad F(x') = \frac{ax'^2}{2} + cx' + \frac{1}{2c} \quad -\frac{1}{c} < x' < -\frac{1}{c} + W \quad (108a)$$

$$II \quad F(x') = W + cx'W - \frac{ax'^2}{2} \quad -\frac{1}{c} + W < x' < 0 \quad (108b)$$

$$III \quad F(x') = 1 - \frac{ax'^2}{2} - \frac{a(x'-W)^2}{2} \quad 0 < x' < W \quad (108c)$$

$$IV \quad F(x') = W - cx'W + \frac{ax'^2}{2} \quad W < x' < \frac{1}{c} \quad (108d)$$

$$V \quad F(x') = \frac{1}{2c} + \frac{a(x'-W)^2}{2} - b(x'-W) \quad \frac{1}{c} < x' < \frac{1}{c} + W \quad (108e)$$

$F(x') = 0$ elsewhere.

Field of Gaussian with $F(x')$ in region I:

$$G_1(r) = \int_{-\frac{b}{a}}^{-\frac{b}{a}+W} e^{-a(r-x')^2} F(x') dx' \quad (109a)$$

$$= \int_{-\frac{b}{a}}^{-\frac{b}{a}+W} e^{-a(r-x')^2} \left(\frac{a}{2} x'^2 + cx' + \frac{1}{2c} \right) dx'$$

$$= -\frac{1}{2a} (ar + b + W) e^{-a(r+\frac{b}{a}-W)^2} + \frac{1}{2a} (r+\frac{b}{a}) e^{-a(r+\frac{b}{a})^2}$$

$$- \left(\frac{a}{2a} + \frac{1}{2c} \right) \int_{-\frac{b}{a}}^{r+\frac{b}{a}-W} e^{-a x'^2} dx'$$

From region II:

$$\begin{aligned}
 G_2(r) &= \int_{-\frac{b}{a}+W}^0 e^{-a(r-x')^2} (bW + ax'W - \frac{a}{2} W^2) dx' \\
 &= -\frac{aW}{2a} \left[e^{-ar^2} - e^{-a(r+\frac{b}{a}-W)^2} \right] - (bW + arW - \frac{a}{2} r^2) \int_{r+\frac{b}{a}-W}^r e^{-ax^2} dx
 \end{aligned} \quad (109b)$$

From region III:

$$\begin{aligned}
 G_3(r) &= \int_0^W e^{-a(r-x')^2} (bW - \frac{c}{2} x'^2 - \frac{a(x'-W)^2}{2}) dx' \\
 &= \frac{1}{4a} (2aW - (a+c)r) e^{-ar^2} + \frac{1}{4a} (r(a+c) - W(a-c)) e^{-a(r-W)^2} \\
 &\quad + \left(\frac{a+c}{4a} ar - bW + \frac{a}{2} W^2 + \frac{a+c}{2} r^2 \right) \int_r^{r-W} e^{-ax^2} dx
 \end{aligned} \quad (109c)$$

From region IV:

$$\begin{aligned}
 G_4(r) &= \int_W^{\frac{b}{c}} e^{-a(r-x')^2} (b - cx'W + \frac{cW^2}{2}) dx' \\
 &= \frac{cW}{2a} \left[e^{-a(r-\frac{b}{c})^2} - e^{-a(r-W)^2} \right] - (bW - cWr + \frac{cW^2}{2}) \int_{r-W}^{r-\frac{b}{c}} e^{-ax^2} dx
 \end{aligned} \quad (109d)$$

From region V:

$$\begin{aligned}
 G_5(r) &= \int_{\frac{b}{c}}^{\frac{b}{c}+W} e^{-a(r-x')^2} \left(\frac{b^2}{2c} + \frac{c}{2} (x'-W)^2 - b(x'-W) \right) dx' \\
 &= \frac{1}{4a} (b+cW-cr) e^{-a(r-\frac{b}{c}-W)^2} + \frac{1}{4a} (cr-b-2cW) e^{-a(r-\frac{b}{c})^2} \\
 &\quad - \left(\frac{c}{2} r^2 + \frac{b^2}{2c} + bW + \frac{c}{2} W^2 - (cW+b)r + \frac{c}{4a} \right) \int_{r-\frac{b}{c}}^{r-\frac{b}{c}-W} e^{-ax^2} dx
 \end{aligned} \quad (109e)$$

Finally, the fold of the three contributions considered here is given by:

$$G(r) = \sum_i G_i(r) \quad (110)$$

To compare the predicted line shape $G(r)$ with that actually observed, we shall evaluate the constants for the deuterons scattered at 30° in the lab system:

- (1) Since it is arbitrary, we shall choose $b = 1$.
- (2) From the kinematics, Eq. (25), the geometry of the collimating slits, and the range-energy relation:

$$\frac{1}{a} = \frac{1}{c} = 9.75 \text{ mg/cm}^2$$

- (3) From the straggling curve,

$$\sigma = 0.377$$

- (4) Up to now, the finite range "bite" of the counters has been considered to be the thickness of the foil between counters No. 2 and No. 3. More careful consideration, however, shows that this cannot be the whole story. If one compares, for example, the energy loss in the second counter of particles which stop in the near and far extremities of the foil, he finds a ratio of about 2.5 to 1. (In the course of our runs, the discriminators for the front two counters were set so that they accepted pulses over a range of slightly more than 3 to 1.) Hence, it is clear that particles stopping somewhere in the second counter will yield pulses large enough to be counted. If we assume that the rate of energy loss of particles in the second counter is uniform, we may conclude that particles penetrating at least $1/3$ the thickness of the second counter will be counted; i.e., to the foil thickness must be added the back $2/3$ of the second counter,

$\approx 0.76 \text{ mg/cm}^2$. One might also want to consider the fact that after passing through the absorber, the particles have scattered and will not enter the thin foil normally. However, as has been shown for the worst case, $\sqrt{d^2} \approx 7^\circ$, so the error introduced by neglecting this effect will be in all cases less than one percent. Hence, we obtain:

$$W = 5.82 + 0.76 = 6.58 \text{ mg/cm}^2 \pm 2\%$$

By way of comparison, contribution III, the major one neglected, will be evaluated:

$$\Delta E = 2E \frac{\Delta \theta}{\sin \theta}$$

where $E = 31.5 \text{ Mev}$ and $\theta = 20^\circ$. Since the beam collimator has $1/8''$ diameter and is located at least 7 feet beyond the analyzing magnet,

$$\Delta \theta = \frac{1/8}{7 \times 12}$$

so,

$$\Delta E = .25 \text{ Mev.}$$

From the kinematics, the deuteron energy distribution corresponding to this incident energy distribution is:

$$\Delta E_d = .15 \text{ Mev.}$$

Hence, the range spread is:

$$\Delta R = 2.3 \text{ mg/cm}^2$$

For neutrons at 30° ,

$$b = 1 \quad (\text{arbitrarily chosen})$$

$$\frac{1}{a} = \frac{1}{c} = 9.75$$

$$W = 6.58$$

$$a = .377$$

$$G_1(r) = -.664 \left(\frac{r}{9.75} + 1.675 \right) e^{-.377(r+3.17)^2} + .664 \left(\frac{r}{9.75} + 1 \right) e^{-.377(r+9.75)^2} \\ - \left(\frac{r^2}{19.5} + r + 5.03 \right) 1.442 \left[\frac{2}{\sqrt{\pi}} \int_0^{.614(r+3.17)} e^{-n^2} dn - \frac{2}{\sqrt{\pi}} \int_0^{.614(r+9.75)} e^{-n^2} dn \right]$$

$$G_2(r) = -.895 \left(e^{-.377r^2} - e^{-.377(r+3.17)^2} \right) \\ -(4.58+.675r) 1.442 \left[\frac{2}{\sqrt{\pi}} \int_0^{.614r} e^{-n^2} dn - \frac{2}{\sqrt{\pi}} \int_0^{.614(r+3.17)} e^{-n^2} dn \right]$$

$$G_3(r) = .138 (1.58 - r) e^{-.377r^2} + .138r e^{-.377(r-6.58)^2} \\ + (.138r^2 - .675r - 1.22) 1.442 \left[\frac{2}{\sqrt{\pi}} \int_0^{.614(r-6.58)} e^{-n^2} dn - \frac{2}{\sqrt{\pi}} \int_0^{.614r} e^{-n^2} dn \right]$$

$$G_4(r) = .895 \left(e^{-.377(r-9.75)^2} - e^{-.377(r-6.58)^2} \right) \\ -(5.80 - .675r) 1.442 \left[\frac{2}{\sqrt{\pi}} \int_0^{.614(r-9.75)} e^{-n^2} dn - \frac{2}{\sqrt{\pi}} \int_0^{.614(r-6.58)} e^{-n^2} dn \right]$$

$$G_5(r) = -.664 \left(\frac{r}{9.75} - 1.675 \right) e^{-.377(r-16.33)^2} + .664 \left(\frac{r}{9.75} - 2.35 \right) e^{-.377(r-9.75)^2} \\ - \left(\frac{r^2}{19.5} - 1.675r + 18.75 \right) 1.442 \left[\frac{2}{\sqrt{\pi}} \int_0^{.614(r-16.33)} e^{-n^2} dn - \frac{2}{\sqrt{\pi}} \int_0^{.614(r-9.75)} e^{-n^2} dn \right]$$

TABLE VI

Line Profile Computations

r	$G_1(r)$	$G_2(r)$	$G_3(r)$	$G_4(r)$	$G_5(r)$	$G(r)$
-11	.055					.05
-10	.174					.17
-9	.427					.43
-8	.874					.87
-7	1.55	.004				1.55
-6	2.46	.051				2.51
-5	3.35	.417				3.77
-4	3.42	1.81	.004			5.23
-3	2.27	4.82	.060			7.15
-2	.862	7.31	.553			8.72
-1	.177	7.82	2.615			10.61
0	.017	5.37	6.985			12.38
1		2.19	11.67			13.80
2		.477	14.39			14.57
3		.063	15.18	.012		15.25
4			14.77	.148		14.82
5			13.48	.980		14.46
6			9.82	3.42		13.24
7			4.90	6.67	.050	11.62
8			1.45	7.96	.353	9.76
9			.309	6.31	1.31	7.93
10			.02	3.31	2.74	6.07
11				.97	3.29	4.26
12				.183	2.81	2.99
13				.016	1.78	1.80
14					.705	.70
15					.32	.32
16					.11	.11

Figure 25 shows the theoretical shape of the deuteron peak, and superimposed on this are the experimentally observed points with their probable errors. The overall fit is seen to be quite good; the slight departure on the low range side of the peak is due in small part to the neglected contributions to the theoretical width, but mainly to the intrusion of the low energy continuum of protons.

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REFERENCES

1. E. Feenberg, Phys. Rev. 49, 328 (1936).
2. H. A. Bethe and R. F. Bacher, Rev. Mod. Phys. 3, 147 (1932).
3. L. D. P. King and L. Goldstein, Phys. Rev. 75, 1306 (1949).
4. J. H. Joyn and R. F. Bacher, Phys. Rev. 75, 1358 (1949).
5. H. V. Argo, H. T. Gittings, A. Hemmendinger, G. A. Jarvis, and R. F. Taschek, Phys. Rev. 75, 691 (1950).
6. R. F. Taschek, invited paper, Seattle Meeting of A.P.S., June 27-29, 1949.
7. W. K. H. Panofsky and F. L. Fillmore, Phys. Rev. 79, 57 (1950).
8. J. C. Allred, L. A. 981, Appendix II.
9. J. H. Smith, Phys. Rev. 71, 32 (1947).
10. C. E. Falk and F. C. Phillips, Phys. Rev. 83, 433 (1951).
11. B. H. Flowers and F. L. Phillips, Proc. Roy. Soc. (London) A206, 131 (1951).
12. J. C. Allred, Phys. Rev. 84, 695 (1951).
13. S. T. Butler, Proc. Roy. Soc. (London), A208, 559-79 (1951).
14. L. Schecter, Phys. Rev. 85, 695 (1951).
15. H. E. Burrows, C. F. Powell, and J. Rotblatt, Proc. Roy. Soc. (London), A209, 431 (1951).
16. H. E. Burrows, C. F. Powell, and J. Rotblatt, Proc. Roy. Soc. (London), A209, 478 (1951).
17. H. E. Burrows, W. M. Gibson, and J. Rotblatt, Proc. Roy. Soc. (London), A209, 439 (1951).
18. E. J. Marge, H. E. Burrows, W. M. Gibson, and J. Rotblatt, Proc. Roy. Soc. (London), A210, 534 (1952).
19. W. M. Gibson, and E. E. Thomas. Proc. Roy. Soc. (London), A210, 543 (1952).
20. S. T. Butler and J. L. Symonds, Phys. Rev. 83, 858 (1951).
21. J. E. Drolley, Jr., J. L. Fowler, and E. J. Stovall, Jr., Phys. Rev. 82, 502 (1951).
22. F. Rossi and K. Greisen, Rev. Mod. Phys. 13, 267 (1941).
23. L. Eyles, Phys. Rev. 74, 1534 (1948).
24. E. J. Williams, Proc. Roy. Soc. (London), A169, 531 (1939).
25. A. S. Livingston and H. A. Bethe, Rev. Mod. Phys. 9, 283 (1937).