# **Uncertainty-Quantified Damage Identification for High-Rate Dynamic Systems**

Zihan Wu and Michael D. Todd

Department of Structural Engineering, University of California San Diego, 9500 Gilman Drive, La Jolla CA 92093-0085 USA.

### ABSTRACT

This paper proposes an uncertainty-quantified damage identification framework that models a high-rate dynamic system using a Gaussian process regression embedded within a nonlinear autoregressive (GP-NARX) model. The output of the GP-NARX model consists of a set of normal distributions at prediction time points with first-order statistics, facilitating the development of a dynamic threshold for outlier detection. A novel weight-updating approach is introduced which weighs the outliers with real-time frequency changes to evaluate and quantify the probability of multiple damage features as time evolves. The framework is further studied using experimental measurements from shock-loaded electronic packages. The results show that the proposed approach can effectively classify different structure characteristics, including damage-sensitive features.

Keywords: High-rate dynamics, Uncertainty quantification, Gaussian process, Input space, Structural health monitoring

## INTRODUCTION

High-rate dynamic systems are defined as systems being subject to high-speed and high-amplitude events (e.g., blast and shockload), which normally occur over a millisecond time scale. These events are often accompanied by structural changes (such as damage) along with nonstationary characteristics that confound observable data [1]. The failure of rapid damage prognosis and mitigation strategies may lead to loss of system functionality or other severe consequences [2]. In most cases, model-based state prediction for such systems is impractical due to uncertainty in the structures' underlying physical behavior and lack of training data sets. Therefore, structural health monitoring algorithms must be capable of identifying the damage under unmodeled nonlinear effects while considering the presence of significant uncertainty and noise. As an alternative, the Gaussian process regression (GPR) is a data-driven approach that may be combined with Bayesian probabilistic inference to build a model directly from observations. In contrast to a deterministic technique, the probabilistic framework includes the stochasticity of the system which can accommodate a measure of uncertainty and other non-deterministic effects. In this work, a GPR based framework is implemented to detect and classify different damage features using different experimental measurements obtained from two shock test results.

### **EXPERIMENTAL SETUP**

A series of drop tower shock tests were developed to simulate the high-rate dynamic system as shown in Fig.1. The electronic packages consist of circuit boards with high-g accelerometers which are able to accurately measure accelerations of 120,000  $g_n$  [3], where  $1 g_n = 9.81 \text{ m/s}^2$ . According to the presence of significant structure failure during the shock load, the corresponding measurements are further defined as "undamaged" and "damaged", shown in Fig. 1 (b) and (c), respectively. Although both measurements are highly nonlinear with large uncertainties (i.e., unmodeled system noise, high-rate interaction between circuit boards, and unknown changing interior boundary conditions), a significant structural failure was introduced during the "damaged" test where the RC circuit on the bottom board functionally failed (i.e., it was not able to be charged).

## NARX MODEL WITH AUTOREGRESSIVE FEEDBACK

A nonlinear autoregressive exogenous input (NARX) model is first applied to study the boundary condition change in the "undamaged" dataset, which uses the measurements from accel 1 and accel 4 as the input and the output of the model, respectively. The prediction  $\hat{y}$  at time *t* is modeled by the input and the output at the previous steps

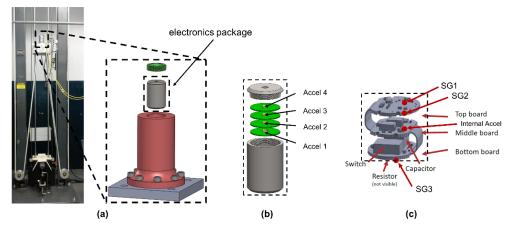
$$\hat{y}(t) = f(y(t-1), y(t-2), \dots, y(t-n_y), u(t-1), u(t-2), \dots, u(t-n_y))$$
<sup>(1)</sup>

The input space of the model is determined by embedding measurement data with embedding parameters (dimension and lag), which are selected to preserves the essential dynamics of the system without excessive computational burden. Because the system dynamics keep changing during the shock load and between consecutive tests, such fixed-parameter NARX model inevitably produces an increasing error, which can be further classified as consistent shifting error and inconsistent shifting error. A GPR-based autoregressive (AR) model is designed to identify the error pattern *e* produced by the NARX model as

$$f(e) \sim GP(m(e), k(e, e')) \tag{2}$$

$$\begin{bmatrix} f(e) \\ f(\hat{e}) \end{bmatrix} \sim N\left(\begin{bmatrix} m(e) \\ m(\hat{e}) \end{bmatrix}, \begin{bmatrix} k(e,e') & k(\hat{e},e) \\ k(\hat{e},e) & k(\hat{e},\hat{e}') \end{bmatrix}\right)$$
(3)

where e is considered as an autoregressive term and GP stands for the Gaussian process. The AR model minimizes the prediction error caused by the consistent condition changes while remains sensitive to the inconsistent shifting error, which itself is not explainable by the consistent condition changes.



**Fig.1** Drop tower shock tests of highly-instrumented electronic packages: (a) drop tower and its mounting fixture, (b) electronic package for "undamaged" dataset, and (c) electronic package for "damaged" dataset.

## **OUTLIER WEIGHT UPDATING**

The GPR is embedded into a NARX model directly for "damaged" dataset which accommodates more flexibility in capturing the damage-sensitive features among confounding unmodeled events. The output of the GP-NARX model consists of a set of normal distributions at prediction time points: the mean value  $\mu$  presents the prediction and the variance  $\sigma$  as the confidence level reflects the similarity between the training input space and testing input space. The outlier for damage detection is defined as the observations that locate outside the predefined confidence level ( $2\sigma$  in this case). A weight updating approach is introduced as a physics-informed data-driven model that uses the internal acceleration as an additional indicator to evaluate the detected outliers. The specific frequency events in short time windows are "learned" by a continuous wavelet transform (CWT), which is obtained from the rate of change of the input u(t) and compared with the rate of change of the internal acceleration in parallel. The framework identifies the outliers caused by internal aberrant signal (known as "false positives") and classifies the damage features with different sensitivity to frequency components, shown in Fig. 2.

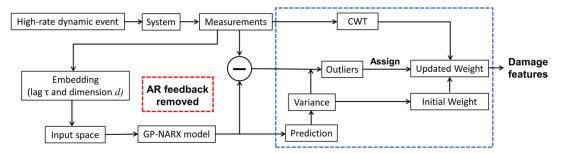


Fig.2 Damage identification framework.

# RESULTS

As shown in Fig. 3(a), the predictions with large variance indicate the presence of system shifting where both consistent and inconsistent shifting are captured. Despite the large variance given by the prediction, the GPR-based model has a much better performance on the consistent shifting part. The inconsistent shifting is then reflected by the model as a prediction with large variance and large error. The model has worse performance in predicting the inconsistency of the system as it magnifies the error compared to the model without AR feedback. Because the inconsistency shifting of the system is more likely a damage feature to identify, we believe the model can prevent the system from some "false-negative" predictions. Updating the weight of the outliers with CWT of internal acceleration, the outstanding score (at 10.4 *ms*) indicates that significant system damage is observed. It is reasonable to guess that the RC circuit was disbonded from the bottom board at that time, which finally led to the functional failure of its chargeability.

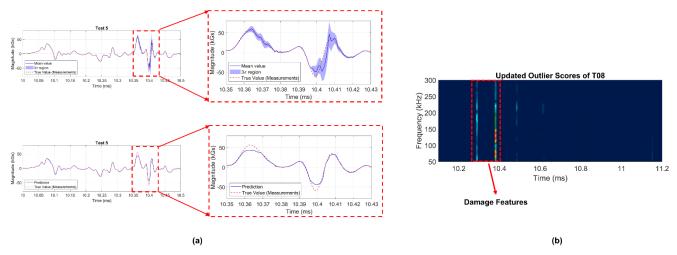


Fig.3 (a) Prediction results from "undamaged" dataset, and (b) damage visualization in the time-frequency domain.

# CONCLUSION

This work proposed an uncertainty-quantified damage identification framework with two strategies of implementing GPR into high-rate system modeling. The precision of the prediction is increased compared to fixed-parameter models. The framework can accommodate a measure of uncertainty and other non-deterministic effects which be combined with Bayesian probabilistic inference to build a model directly from observations.

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