The Interaction of Internal and External Representations in a Problem Solving Task

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Abstract

In these studies I examine the role of distributed cognition in problem solving. The major hypothesis explored is that intelligent behavior results from the interaction of internal cognition, external objects, and other people, where a cognitive task can be distributed among a set of representations, some internal and some external. The Tower of Hanoi problem is used as a concrete example for these studies. In Experiment 1 I examine the effects of the distribution of internal and external representations on problem solving behavior. Experiments 2 and 3 focus on the effects of the structural change of a problem on problem solving behavior and how these effects depend on the nature of the representations. The results of all studies show that distributed cognitive activities are produced by the interaction among the internal and external representations. External representations are not simply peripheral aids. They are an indispensable part of cognition. Two of the factors determining the performance of a distributed cognitive system are the structure of the abstract problem space and the distribution of representations across an internal mind and the external world.

Introduction

The traditional approach to cognition in general and problem solving in particular focuses on an individual's internal mental states. In the traditional view, cognition is exclusively the activity of an internal mind. External objects, if they have anything to do with cognition at all, are at most peripheral aids. There is no doubt that internal factors are important in cognition. They are not, however, the whole story. From the distributed cognition perspective, cognitive activity is distributed across internal human minds, external cognitive artifacts, groups of people, and across space and time (Hutchins, 1990, in preparation; Hutchins & Norman, 1988; Norman, 1988, 1989, 1990).

In this paper, I develop a framework of distributed problem representations to analyze a set of distributed cognitive tasks. My focus is on the nature of external representations and the interactions among internal and external representations. I show that external objects are not simply peripheral aids--they provide a different form of representation. External representations are interwoven with internal representations to produce distributed cognitive activities.

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Distributed Problem Representations

The basic principle to be explored is that the representational system for a problem can be considered as a set, with some members internal and some external. Internal representations are in the mind, as propositions, mental images, or whatever (e.g., multiplication tables, arithmetic rules, etc.). External representations are in the world, as physical symbols (e.g., written symbols, beads of an abacus, etc.) or as external rules, constraints, or relations embedded in physical configurations (e.g., spatial relations of the digits on a piece of paper, physical constraints in an abacus, etc.). Generally, there are one or more internal and external representations involved in any problem.

Figure 1 shows a representational system for a problem with two internal and two external representations. Each internal representation resides in a person's mind and each external representation resides in an external medium. The internal and external representations involved in a given problem together form a distributed representation space mapped to a single abstract problem space that represents the abstract properties of the problem. Each representation in the distributed representation space sets some constraints on the abstract problem space.

The distributed cognition perspective demands the decomposition of the abstract problem space into its internal and external components. In the traditional studies of problem solving, however, many abstract problem spaces having internal and external components were mistakenly treated as solely internal problem spaces. Generally speaking, the abstract problem space of a problem is not equivalent to its internal problem space.

The Tower of Hanoi

The Tower of Hanoi (TOH) problem (Figure 2) was chosen as a concrete example to study distributed cognitive activities in problem solving. It is a well-studied problem (Hayes & Simon, 1977; Kotovsky & Fallside, 1989; Kotovsky, Hayes & Simon, 1985; Simon & Hayes, 1976). Much of the research has focused on isomorphs of the TOH and its problem representations. The basic finding is that different problem representations can have dramatic impact on problem difficulty even if the formal structures are the same. External memory aid is one major factor of problem difficulty. Thus, Kotovsky *et al.* (1985) reported that the *Dish-move* isomorph of the TOH, in which all rules had to be remembered, was harder to solve than the *Peg-move* isomorph, in which one of the rules was embedded in physical configurations. Modifications of these two isomorphs were used in two of the three conditions in Experiment 1 of the present study (*I123* and *I12-E3*).

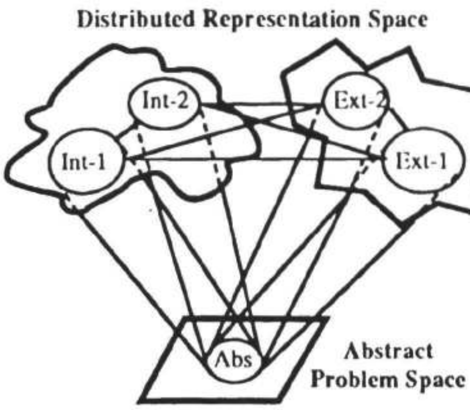

Figure 1. The distributed representation space and the abstract problem space of a problem with two internal and two external representations. The abstract problem space is formed by the conjunction of the internal and external problem spaces.

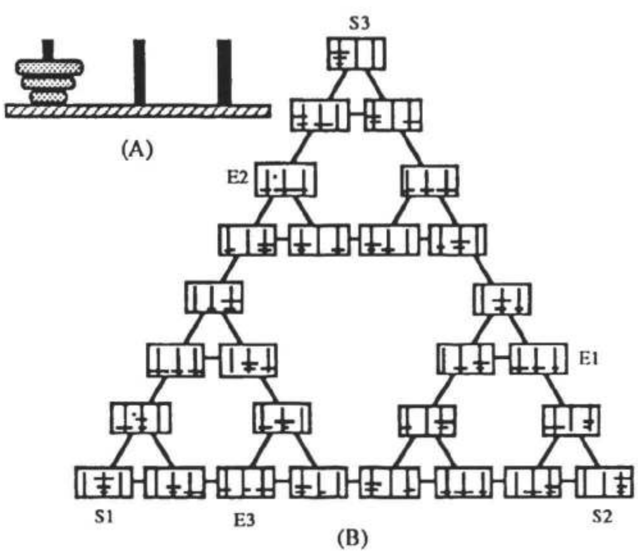

Figure 2 (A) The TOH. The task is to move the disks from one configuration to another, following two rules: only one disk can be transferred at a time (Rule 1) and a disk can only be transferred to a pole on which it will be the largest (Rule 2). (B) The problem space of the TOH. Each rectangle shows one of the 27 possible configurations of the three disks on the three poles. The lines between the rectangles show the transformations from one state to another when the rules are followed. S1, S2, and S3 are three starting states, and E1, E2, and E3 are three ending states. They will be used later.

Internal and External Rules. The TOH problem actually has three rules, not just the two stated earlier. Rule 3 is that only the largest disk on a pole can be transferred to another pole. In the representation shown in Figure 2A, Rule 3 need not be stated explicitly because the physical structure of the disks and poles coupled with Rules 1 and 2 guarantee that it will be followed. But if the disks were not stacked on poles, explicit statement of Rule 3 would be necessary.

In my studies I used four rules:

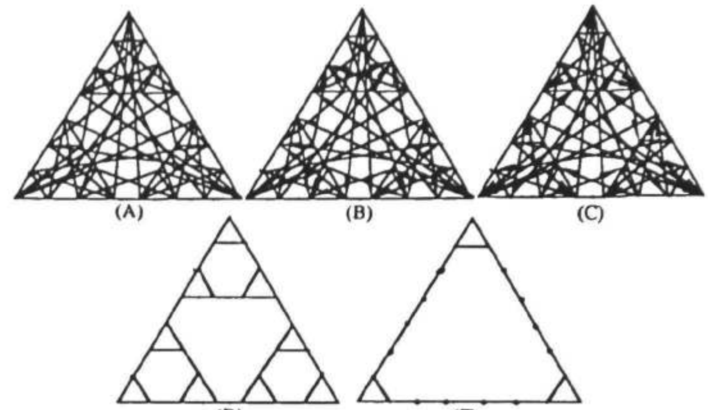

Figure 3 Problem spaces constrained by five sets of rules. (A) Rule 1. (B) Rules 1+2. (C) Rules 1+3. (D) Rules 1+2+3. (E) Rules 1+2+3+4. Lines with arrows are uni-directional. Lines without arrows are bi-directional. The rectangles (problem states) are not shown in this figure for the reason of clarity.

Rule 1: only one disk can be transferred at a time.
Rule 2: a disk can only be transferred to a pole on which it will be the largest.
Rule 3: only the largest disk on a pole can be transferred to another pole.
Rule 4: the smallest disk and the largest disk can not be placed on a single pole unless the medium sized disk is also on that pole.

Any of these four rules can be either internal, memorized, or external, externalized into physical constraints. In the experiments that follow, I varied the numbers of external rules. In one condition, *I123* (Figure 5A), no rule is external. In a second condition, *I12-E3* (Figure 5B), Rule 3 is external. In the *I1-E23* condition (Figure 5C), both Rules 2 and 3 are external. In the *I1-E234* condition (Figure 6D), Rules 2, 3, and 4 are all external.

Internal and External Problem Spaces. A problem space is composed of all possible states and all moves constrained by the rules. Figures 3A-E show the problem spaces constrained by Rules 1, 1+2, 1+3, 1+2+3, and 1+2+3+4, respectively. These five spaces can represent internal problem spaces, external problem spaces, or mixed problem spaces, depending upon how the rules constructing them are distributed. A problem space constructed by external rules is an external problem space, one constructed by internal rules is an internal problem space, one constructed by a mixture of internal and external rules is a mixed problem space. Figure 4 shows the internal, external, and abstract problem spaces of the standard TOH.

Experiment 1

The standard TOH has three rules which can be distributed among internal and external representations. Different distributions may have different effects on problem solving behavior, even if the formal structures are the same. Experiment 1 investigates these effects. My hypothesis is that the more rules are distributed externally, the easier the problem. There are three conditions, isomorphs of the TOH,

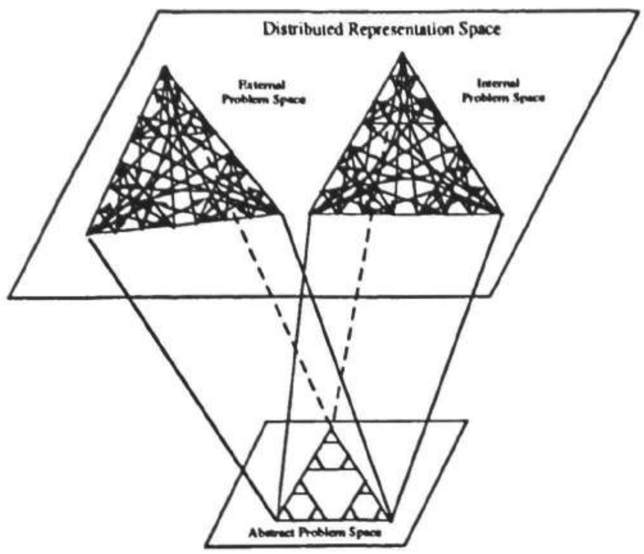

Figure 4. The distributed representation space and the abstract problem space for the standard TOH. The distributed representation space is composed of the internal and the external problem spaces, which are constrained by Rules 1+2 and Rules 1+3. The abstract problem space is the conjunction of the internal and the external problem spaces.

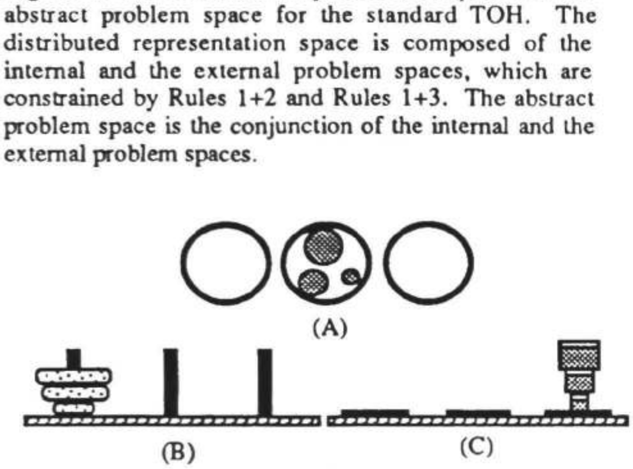

Figure 5. (A) *I123*. No physical constraints. (B) *I12-E3*. The physical constraints (coupled with Rules 1 and 2) guarantee that Rule 3 is followed. (C) *I1-E23*. The cups were filled with coffee. A smaller cup could not be placed on the top of a larger cup (Rule 2), as this would cause the coffee to spill. A cup could not be moved if there was another cup on top of it (Rule 3).

which correspond to three different distributions of the three rules.

In Condition *I123* (*I* = Internal rules and *E* = External rules) condition, Rules 1, 2, and 3 were all internal (Figure 5A). In Condition *I12-E3* (Figure 5B), Rules 1 and 2 were internal, and Rule 3 was external. In Condition *I1-E23* (Figure 5C), Rule 1 was internal, and Rules 2 and 3 were external.

Method

Subjects. The subjects were 18 undergraduate students enrolled in introductory psychology courses at the University of California, San Diego who volunteered for the experiment in order to earn course credit.

Materials. In the *I123* condition, three plastic balls and three porcelain plates were used. In the *I12-E3* condition, three plastic rings and three plastic poles were used. In the *I1-E23* condition, three plastic cups and three paper plates were used. All three cups were filled with coffee.

TABLE 1. THE RESULTS OF EXPERIMENT 1

Measurements	Conditions		
	I123	I12-E3	I1-E23
Times (sec)	131.0	83.0	53.9
Steps	19.7	14.0	11.4
Errors	1.4	0.61	0.22

TABLE 2. THE *p* VALUES OF EXPERIMENT 1

Comparisons	Measurements		
	Times	Steps	Errors
Main Effect	$< .05$	$= .05$	$< .005$
I123 vs. I12-E3	$< .1$	$< .1$	$< .03$
I123 vs. I1-E23	$< .01$	$< .02$	$= .001$
I12-E3 vs. I1-E23	$> .3$	$> .4$	$> .2$

NOTE. Fisher PLSD test was used for the multiple comparisons.

Design. Each subject played all three games, one for each of the three conditions, once in a randomized order. There were six possible permutations for the three games. Each permutation was assigned to a subject randomly. For each subject, the first, the second, and the third games always started at positions S1, S2, and S3 and ended at positions E1, E2, and E3, respectively (see Figure 2B).

Procedure. Each subject read the instructions[3] aloud slowly. Then the subject was asked to repeat all the rules. If a subject could recite all the rules twice without error, he was instructed to start the games. Otherwise he reread the instructions until he reached the criterion. A subject's performance was recorded on a video camera.

Results and Discussion

The average solution times, solution steps, and errors are shown in Table 1. The statistics is shown in Table 2. Problem difficulty measured in solution times, solution steps, and errors for the three problems was consistent. The more rules externalized, the easier the task. The order of difficulty was, from hardest to easiest: *I123* > *I12-E3* ≥ *I1-E23*. The difference between *I12-E3* and *I1-E23* was not statistically significant. All errors made were for internal rules: none were for external rules. Rules, once externalized, seem to be error-proof.

Experiments 2 and 3

Different numbers of rules give rise to different problem spaces. Figure 3 shows that the problem space structure changes with the number of rules. There are at least two rival factors involved. On the one hand, the fewer rules, the more paths there are from an initial state to a final state. Hence, fewer rules might make the problem easier. On the other hand, the more rules, the fewer the choices. The prob-

[3] Due to the maximum length limitation of the paper, the instructions are not shown here. The instructions were three restaurant stories: *Waitress and Oranges*, *Waitress and Donuts*, and *Waitress and Coffee*. For example, in the *Waitress and Coffee* condition, Rule 1 was stated as "only one cup can be moved at a time", and Rules 2 and 3 were not stated because they were external.

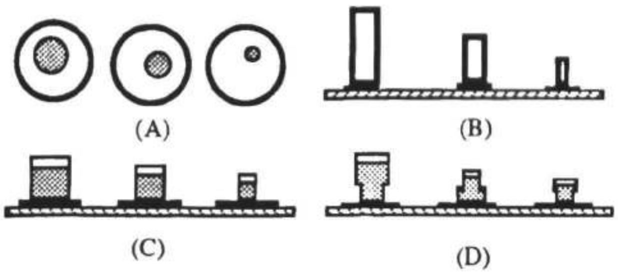

Figure 6. The materials used in Experiment 3. (A) *I1*. (B) *I1-E3*. The sizes of the three straws were such that a smaller straw inside a larger one could not be moved out without the larger straw being moved away first (Rule 3). (C) *I1-E23*. The same as the *I1-E23* in Figure 5C. (D) *I1-E234*. The cups were filled with tea. The sizes of the three cups were such that a smaller cup could not be placed on the top of a larger one (Rule 2), only the topmost cup could be moved (Rule 3), and the largest cup and the smallest cup could not be placed on the top of each other (Rule 4).

lem solver can simply follow where the highly constrained structure forces one to go. So, more rules might make the problem easier. This analysis implies that the problem difficulty might not increase monotonically with the number of rules. In addition, the relationship between the problem difficulty and the number of rules might depend on the nature of the rules (whether internal or external). Experiments 2 and 3 investigate these effects, with Experiment 2 focusing on a change of internal rules and Experiment 3 on a change of external rules.

Both Experiments 2 and 3 have four conditions. In Experiment 2, all rules were internal. Condition *I1* has Rule 1, Condition *I13* Rules 1 and 3, Condition *I123* Rules 1, 2, and 3, and Condition *I1234* Rules 1, 2, 3, and 4. Experiment 3 was exactly the same as Experiment 2, except that Rules 2, 3, and 4 were external rather than internal. Condition *I1* (Figure 6A) had only Rule 1 (internal). Condition *I1-E3* (Figure 6B) had Rule 1 (internal) and Rule 3 (external). Condition *I1-E23* (Figure 6C) had Rule 1 (internal) and Rules 2 and 3 (both external). Condition *I1-E234* (Figure 6D) had Rule 1 (internal) and Rules 2, 3, and 4 (all external).

Method

Subjects. The subjects were 48 (24 for each experiment) undergraduate students enrolled in introductory psychology courses at the University of California, San Diego, who volunteered for the experiment to earn course credit.

Materials. In Experiment 2, exactly the same materials used in the *I123* condition in Experiment 1 were used in all current four conditions. In Experiment 3, materials for Condition *I1* were the same as for Experiment 2. In Condition *I1-E3*, the straws and tiny plates were made from paperboard. Materials for Condition *I1-E23* were the same as for the *I1-E23* condition in Experiment 1. In Condition *I1-E234*, the four cups were made from metal cans and were all filled with tea.

TABLE 3. THE RESULTS OF EXERIMENTS 2 AND 3

	Experiment 2			
	I1	I13	I123	I1234
Times/min. step	2.3	8.8	21.5	18.5
Steps/min. step	1.0	1.9	2.7	1.8
Errors/min. step	0	0.06	0.28	0.26
	Experiment 3			
	I1	I1-E3	I1-E23	I1-E234
Times/min. step	2.5	6.9	9.0	12.5
Steps/min. step	1.0	1.3	1.8	1.9
Errors/min. step	0	0	0	0

TABLE 4. THE *p* VALUES OF EXERIMENTS 2 AND 3

Experiment 2			
Comparisons	Times	Steps	Errors
Main Effect	<.0001	<.0001	=.0001
I1 vs. I13	<.05	<.005	<.06
I1 vs. I123	<.00001	<.00001	<.001
I1 vs. I1234	<.00001	<.01	<.001
I13 vs. I123	<.02	<.01	<.005
I13 vs. I1234	<.005	>.6	<.01
I123 vs. I1234	>.36	<.003	>.75
Experiment 3			
Comparisons	Times	Steps	Errors
Main Effect	<.0001	<.0001	--
I1 vs. I1-E3	<.01	<.1	--
I1 vs. I1-E23	<.0001	<.0001	--
I1 vs. I1-E234	<.00001	<.00001	--
I1-E3 vs. I1-E23	>.18	<.01	--
I1-E3 vs. I1-E234	<.0001	<.0001	--
I1-E23 vs. I1-E234	<.03	>.4	--

NOTE. Fisher PLSD test was used for the multiple comparisons.

Design. The design for Experiments 2 and 3 were the same. Each subject played all four games, once each. There were twenty-four possible permutations for the four games. The twenty-four subjects were assigned to these permutations randomly. Due to a limitation in the number of subjects available, the first, second, third, and fourth games always started at positions S1, S2, S3, and S1 and ended at positions E1, E2, E3, and E1, respectively (see Figure 2B).

Procedure. The procedures for both Experiments 2 and 3 were the same as in Experiment 1.

Results and Discussion

The results are shown in Table 3. The minimum number of steps from the starting state to the final state is 2, 4, 7, and 8 for Conditions *I1* and *I1*, *I13* and *I1-E3*, *I123* and *I1-E23*, and *I1234* and *I1-E234*, respectively. In order to make meaningful comparisons, solution times, solution steps, and errors for each condition were normalized by being divided by the number of minimum steps from the starting state to the final state. The statistics is shown in Table 4.

Experiment 2 shows that when all rules were internal the hardest problem was neither the one with the fewest rules (*I1*), nor the one with the most rules (*I1234*), but the one with an intermediate number of rules (*I123*). When solution times or errors are used as the difficulty measurement, the difficulty order was, from easiest to hardest: *I1* < *I13* < *I1234* ≤ *I123*. The difference between *I1234* and *I123* was not statistically significant. When solution steps are used as the difficulty measurement, the difficulty order remained the same (*I1* < *I13* ≤ *I1234* < *I123*), but in this case the difference between *I13* and *I1234* was not statistically significant.

Experiment 3 shows that when all but one rule were external problem difficulty increased monotonically with the number of rules. When solution times are used as the difficulty measurement, the difficulty order was, from easiest to hardest: *I1* < *I1-E3* ≤ *I1-E23* < *I1-E234*. The difference between *I1-E3* and *I1-E23* is not statistically significant. If solution steps are used as the difficulty measurement, the difficulty order remained the same, but the difference between *I1-E23* and *I1-E234* is not statistically significant. Subjects didn't make any errors in this experiment.

All four rules in Experiment 2 were internal. Rules 2, 3, and 4 in Experiment 3 were external. Comparing the results in these two experiments, we found that the conditions in Experiment 3 with external rules were easier than their counterparts in Experiment 2. This further supports the claim that the more rules externalized, the easier the problem.

General Discussion

A problem can be represented among a set of internal and external representations. Given the same set of rules, the more rules were distributed externally, the easier the problem. Given the same initial and final states, the problem difficulty increased monotonically with the number of rules if most rules were external. When all rules were internal, however, the hardest problem was the one with an intermediate number of rules. In addition to memory aids, external representations play other important roles. They provide a different representation. External representations have the following properties.

External representations provide external memory aids. For example, for all of the games in the present study, the goal problem states didn't need to be remembered, because they were represented by the diagrams placed in front of the subjects.

External representations can provide information which can be directly perceived and used without being interpreted and formulated explicitly. For example, in the *I1-E23* condition, Rules 2 and 3 were not told to the subjects: they were built into the physical constraints and perceived and followed directly. When the subjects were asked to formulate the rules after the games, few could do it.

External representations anchor and structure cognitive behavior. The physical structures in the external world constrain the range of possible cognitive behaviors in the sense that some behaviors are allowed and others prohibited. For example, in the *I1-E23* condition, external Rules 2 and 3 could not be violated. They construct the external problem space and hence structure the cognitive behavior.

External representations change the nature of a task. Norman (1990) proposed that external representations change the nature of a task from the task performer's point of view and enhance the system's ability from the system's (task performer + external representation) point of view. In the *I123* condition, a problem solver had to process three internal rules, while in the *I1-E23* condition the problem solver only had to process one internal rule. The cognitive processes of the problem solver were different in these two conditions. Nevertheless, the performance of the system *I1-E23* was much better than the system *I123*.

Conclusion

The framework of distributed problem representations is useful for analyzing distributed cognitive activities. Under this framework, a problem is represented in a set of internal and external representations, and distributed cognitive activities are produced by the interaction of internal and external representations. External representations are not simply peripheral aids, they are an indispensable part of cognition.

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