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The Observation of Dispersionless Superluminal Propagation in a Non-Foster Loaded Waveguide and Its Fundamental Limitations

Jiang Long , *Member, IEEE*, and Daniel F. Sievenpiper, *Fellow, IEEE*

Abstract—Superluminal pulse propagation has been observed in past investigations. Conventional methods rely on resonance-based gain/absorption line materials, inevitably inducing significant pulse distortion. Motivated by the negative reactance slope of the non-Foster circuits, we investigated superluminal propagation based on a non-Foster loaded waveguide. Superluminal square-wave pulse transmission with negligible distortion has been observed in time-domain measurements. In addition, the propagation distance limitation for a non-Foster-based superluminal waveguide is derived by analyzing the distortion of the transmitted waveform. We conclude that a distortion-limited superluminal output pulse cannot emerge more than the original pulse duration ahead of the same pulse transmitted in vacuum, in agreement with causality. This investigation provides an alternative solution to the gain/absorption approaches, and is directed toward a more comprehensive understanding of superluminal propagation.

Index Terms—Non-Foster circuit, periodic structure, superluminal propagation.

I. INTRODUCTION

THE implementation of superluminal propagation has both practical and fundamental implications. Its great potential has been studied in terms of communications [1], [2], invisible cloaking [3], [4], and leaky wave antennas [5], [6]. In addition, the investigation on superluminal propagation has also advanced the discoveries on the fundamental physics related to relativity and causality [7]–[11]. Theoretically, ideal superluminal propagation requires an electromagnetic wave to travel in a homogeneous medium with a group velocity defined by a group index $0 < n_g < 1$, as

$$n_g(\omega) = n_r(\omega) + \omega \frac{\partial n_r(\omega)}{\partial \omega} \quad (1)$$

where n_r is the frequency-dependent refractive index and the second term is the derivative of n_r with respect to ω . Normally, most materials have $n_r > 1$ and positive slope with ω . Nevertheless, considerable research efforts have been made toward accomplishing superluminal propagation using engineered materials with narrow gain/absorption frequency bands and anomalous dispersion [11], [12]. Experiments have been extensively reported on the observation of superluminal

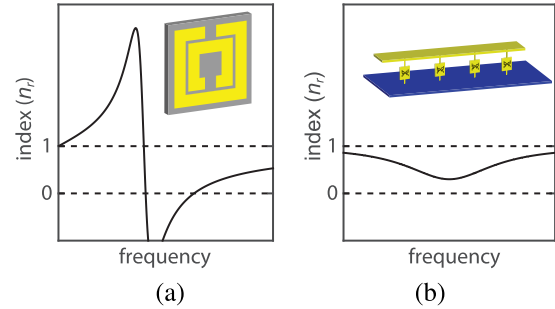


Fig. 1. Illustration of (a) metamaterial and (b) non-Foster circuit-based superluminal index.

or even negative Gaussian pulse transmission over a short distance with the help of dispersive atomic gases [13], photonic bandgap structures [14], Bragg gratings [15], optical cavities [16], metamaterials [17], and so forth. In fact, these experiments were accomplished over a narrow bandwidth with the negative slope of n_r [the second term in (1)] dominating n_g , as illustrated in Fig. 1(a). However, this frequency-dependent n_r inevitably causes dispersion, leading to even smaller distortion-free transmission bandwidth than the superluminal/negative propagation band of the media ($n_g < 1$) itself. Recently, a half-sine wave packet was transmitted through an anomalously dispersive media [18] with a superluminal peak but significant distortion of the beginning and tailing parts, which is primarily attributed to the fact that a half-sine pulse has more frequency components than a Gaussian pulse. That is why significant distortion has been observed for a half-sine wave packet [18] in the experimental demonstration with the anomalously dispersive media, whereas a Gaussian pulse does not exhibit apparent distortion [13]–[17]. Therefore, even more distortion is expected when a square wave pulse propagates in the gain/absorption media compared with a half-sine wave pulse.

To overcome the distortion for square wave pulse transmission, it is desired to reduce the dispersion within the superluminal frequency band. As indicated in Fig. 1(b), a non-Foster loaded material enables $n_g < 1$ by reducing n_r to a less than unity value while keeping it constant [$\partial n_r(\omega)/\partial \omega = 0$] within the superluminal bandwidth, a necessity for dispersionless superluminal propagation. The resultant n_g is close to n_r as the derivative term diminishes, leading to $n_g = n_r < 1$. Compared with the superluminal propagation using an anomalous dispersive gain/absorption band, the non-Foster loading approach is theoretically more promising in terms

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of broad bandwidth, which has vast importance in various aforementioned applications [1]–[6].

Non-Foster loading techniques have been demonstrated in many applications, including electrically small antennas [19]–[22], metasurfaces [23], [24], absorbers [25], invisible cloaks [26], as well as superluminal/fast-wave propagation [27]–[35]. Specifically, propagation has been implemented with different types of non-Foster circuits. Studies in [27] and [28] present an Op-Amp-based non-Foster circuit, achieving a three-unit-cell epsilon-near-zero material. A fast-wave waveguide was accomplished in [29]–[31] at higher frequency with the help of transistor-based non-Foster circuit. However, no more than 10 unit cells are achieved. The current technical difficulty for non-Foster loading approaches is the potential instability [36]–[38]. In particular, due to a growing wave in the propagation direction, periodic non-Foster loaded waveguides in prior studies [28]–[31] have a limited number of unit cells and length. A unilateral non-Foster element is proposed and is demonstrated in squint-free series-fed antenna array [32]–[35]. It utilizes a general amplifier to break the feedback loop of the waveguide, making it unilateral, so that instability is resolved.

In this paper, a smaller non-Foster impedance is utilized to improve the stability. Consequently, a non-Foster loaded waveguide with as many as 40 unit cells is realized. To demonstrate the advantage of the non-Foster loaded superluminal propagation over the gain/absorption material-based approach, time-domain (TD) measurement with a square waveform is performed, demonstrating a superluminal propagation with negligible distortion, which is further corroborated by frequency-domain (FD) measurement.

In addition to the experimental results, different from the limitation on the superluminal propagation distance that is required by stability [37], we derived the theoretical limitation of the non-Foster loading-based superluminal propagation by enforcing a limitation on waveform distortion. It is concluded that the advancement of a distortion-limited superluminal pulse has to be less than the pulse duration, preserving causality. This paper on the non-Foster loading-based superluminal propagation is supplementary to the gain/absorption material-based superluminal propagation, providing an alternative approach.

This paper is organized as follows. Section I is the introduction to the non-Foster-based superluminal propagation. The detailed design of the superluminal waveguide is elaborated in Section II, including the waveguide structure and theoretical simulation. Section III discusses on the non-Foster circuit. Section IV presents the experimental measurement, showing a nondistorted superluminal square pulse transmission. Both TD and FD measurement results are presented, verifying the validity of the measurements. A theoretical discussion on the propagation distance limitation is presented in Section V, and the conclusion is drawn in Section VI.

II. NONDISPERSIVE SUPERLUMINAL WAVEGUIDE

A. Theoretical Analysis

As mentioned earlier, to realize nondispersive superluminal propagation, two conditions should be met: 1) $n < 1$ and

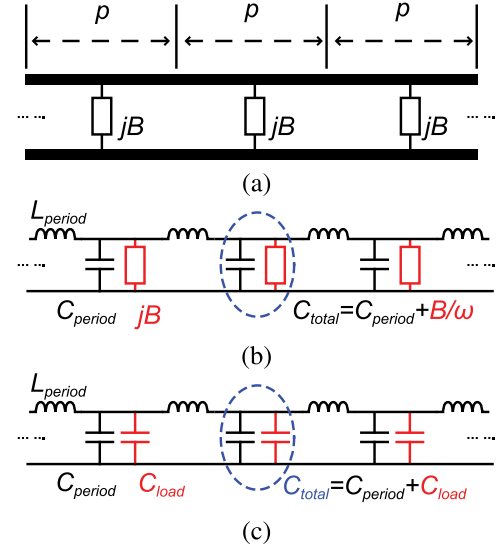


Fig. 2. Illustration of the non-Foster capacitance loaded waveguide. (a) General periodically loaded transmission line. (b) Equivalent distributed lumped element model. (c) Lumped element model with non-Foster capacitor as the load. In (b) and (c), C_{period} and L_{period} are the capacitance and inductance of the transmission line for the length of p , respectively.

2) $(\partial n / \partial \omega) = 0$. Here, we start from the ideal transmission line model with periodical loads, as shown in Fig. 2(a). Provided the periodicity, p , is small, the transmission line can be equivalent to the distributed lumped model as shown in Fig. 2(b), from which the refractive index of an unloaded transmission line can be expressed as

$$n = \frac{c}{v_p} = c \frac{\sqrt{L_{\text{period}} \cdot C_{\text{period}}}}{p} \quad (2)$$

where c is the speed of light, v_p is the phase velocity of the transmission line, and L_{period} and C_{period} are defined as the inductance and capacitance per length of p of the transmission line. In order to make n smaller than 1, reducing C_{period} is one of the approaches. Thus, an impedance jB is periodically loaded to the transmission line with a period of p . As shown in Fig. 2(b), the effective total capacitance for a length of p , C_{total} , can be expressed as

$$C_{\text{total}} = C_{\text{period}} + \frac{B}{\omega} \quad (3)$$

and the achieved refractive index is

$$n = \frac{c}{v_p} = c \frac{\sqrt{L_{\text{period}} \cdot (C_{\text{period}} + B/\omega)}}{p} \quad (4)$$

In order to meet the second requirement, since only B term in (4) is frequency dependent, the most straightforward way is to make it independent of frequency, which leads to

$$\frac{\partial}{\partial \omega} \frac{B}{\omega} = \frac{\partial B}{\partial \omega} \cdot \frac{1}{\omega} - \frac{B}{\omega^2} = 0. \quad (5)$$

Solving the above partial equation (5), it gives

$$B = \omega \cdot K \quad (6)$$

where K is a constant. This clearly implies that in order to remove the frequency dependence of n , B must be linearly proportional to frequency, which is actually a capacitor.

Therefore, the load impedance should be like Fig. 2(c), with loaded impedance being a capacitor, C_{load} . By substituting it to (4), it derives

$$n = \frac{c}{v_p} = c \frac{\sqrt{L_{\text{period}} \cdot (C_{\text{period}} + C_{\text{load}})}}{p}. \quad (7)$$

By considering the first requirement, $n < 1$, it is easy to conclude that C_{load} must be negative, so that the total capacitance C_{total} can get smaller, leading to less than unity refractive index. Therefore, with loaded impedance as a negative capacitor, both requirements of $n < 1$ and $(\partial n / \partial \omega) = 0$ can be satisfied, so that the broadband superluminal can be achieved. Such negative capacitor can be implemented by non-Foster circuits.

B. Design of Superluminal Waveguide

As discussed earlier, a negative capacitor implemented by a non-Foster circuit is used to partially compensate for the intrinsic reactance of the waveguide to boost the phase velocity greater than c , and at the same time to keep it independent of frequency. However, there is a negative real part of non-Foster impedance accompanying the desired non-Foster reactance, causing traveling waves to grow in the propagation direction and further leading to potential instability [37], [38], which limits the achievable number of unit cells and total waveguide length. It should be noted that the stability of the non-Foster circuit application has been extensively discussed in the prior studies [36]–[41], and it is true that the negative conductance is not the only reason for instability. In fact, some negative reactance appearing in the network may also cause instability. However, regarding the non-Foster loaded waveguide application, the negative capacitor is used to partially compensate for the waveguide capacitance, so the waveguide capacitance, C_{period} , as defined in Fig. 2, dominates the reactance appearing in the network. Consequently, if the waveguide itself is low loss, the negative conductance of the non-Foster circuit may cause gain in the wave propagation, leading to potential instability problems.

To stabilize the non-Foster loaded waveguide and allow more non-Foster circuits for longer propagation distance, the growth of propagating waves should be eliminated. Therefore, we intentionally attempted to reduce the negative real part of the non-Foster impedance in order to minimize the associated gain effect. As discussed in Section III, reducing the negative real part inevitably reduces the negative capacitance. To maintain the phase velocity boosting effect, the intrinsic capacitance of the waveguide, C_{period} , should be reduced, because phase velocity is determined by the sum of the intrinsic capacitance, C_{period} , and the loaded non-Foster capacitance, C_{load} , as shown in Fig. 2. Hence, an air-substrate microstrip line structure is employed. Fig. 3 illustrates the experimentally implemented non-Foster loaded waveguide with 40 unit cells. The top conductive trace and its ground plane are printed on two 1-mm-thick substrates (Rogers RO4003C, $\epsilon_r = 3.55$), respectively, and separated by a 3-mm air gap, forming an air-substrate microstrip line. The non-Foster loads are mounted on the ground plane side of the

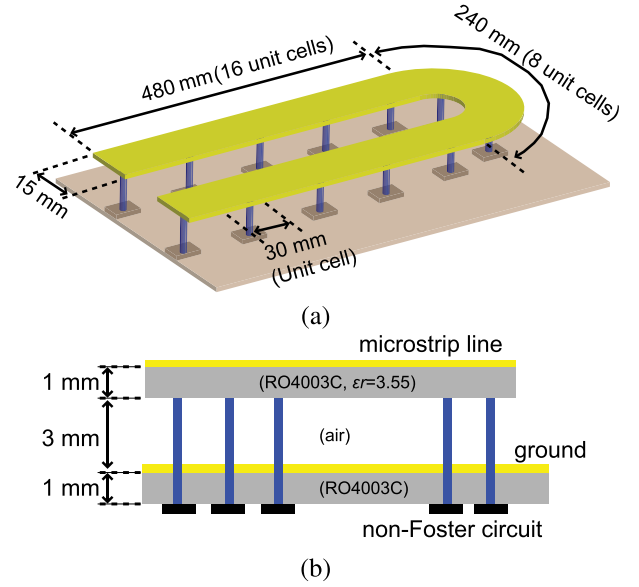


Fig. 3. Experimental design of the non-Foster loaded superluminal waveguide is illustrated in (a) perspective view and (b) side view.

substrate and connected to the microstrip line through vias. The periodicity of the non-Foster load is 30 mm. Due to the fabrication panel size limitation, a U-shaped microstrip line is designed, with 16 unit cells on the two parallel straight sections and 8 unit cells on the curved part. The total length of the waveguide is 1200 mm.

Numerical simulation of the unloaded waveguide with HFSS is performed to extract the S-parameters of the waveguide. The propagation delay θ and the characteristic impedance Z_c are retrieved by [42]

$$Z_c = 50 \times \left(\pm \sqrt{\frac{(1 + S_{11})^2 - S_{21}^2}{(1 - S_{11})^2 - S_{21}^2}} \right) \quad (8)$$

and

$$e^{j\theta} = \frac{S_{21}}{1 - S_{11} \frac{Z_c - 50}{Z_c + 50}} \quad (9)$$

and the refractive index n is derived from θ by

$$n = -\frac{c \cdot \theta}{\omega L} \quad (10)$$

where L is the total length of the waveguide and c represents the speed of light. The retrieved real and imaginary parts of refractive index and the characteristic impedance of a simple microstrip waveguide are presented in Fig. 4, showing a transmission line with $Z_c = 58.7 \Omega$ and an index of 1.07. This further leads to inductance and capacitance of 6.28 nH and 1.82 pF, respectively, for a section of transmission line of 30 mm. Hence, with a -0.5 pF load, the refractive index can be reduced to 0.9. Specifically, an ideal -0.5 pF is implemented in HFSS simulation, which is periodically loaded along the microstrip waveguide, with the periodicity of 30 mm. The retrieved index and characteristic impedance are shown in Fig. 4, implying that a -0.5 -pF capacitor is capable of achieving a refractive index of 0.87 and Z_c about 74 Ω .

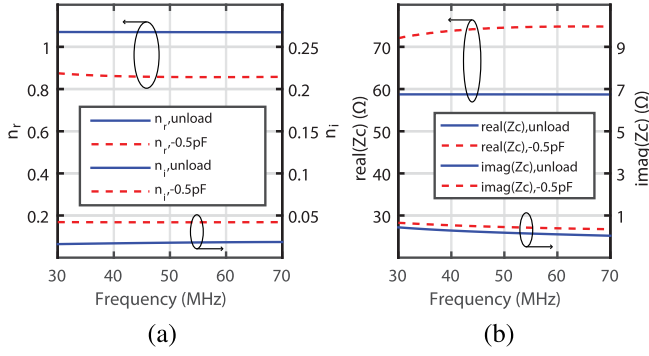


Fig. 4. HFSS simulation results for a simple microstrip line and a microstrip line with -0.5 pF loaded for every 30 mm. Solid lines: simple microstrip waveguide without any loading impedance. Dashed curves: microstrip waveguide with ideal -0.5 -pF loading. (a) Real and imaginary parts of the refractive index. (b) Real and imaginary parts of the characteristic impedance.

Frequency dependence of the refractive index and characteristic impedance is observed in simulation. It is because of the realistic via, which introduces parasitic capacitance at lower frequency but becomes less capacitive as the frequency increases.

III. NON-FOSTER CIRCUIT

A non-Foster circuit has an impedance/admittance that does not follow Foster's theorem for passive components. The -0.5 -pF non-Foster circuit is designed with discrete transistors based on Linvill's negative impedance converter circuit [43], as presented in Fig. 5(a). A similar circuit topology is used as our previous work [30], [31]. Broadcom Ltd. bipolar transistor AT41511 is adopted. A realistic RLC model is included for accounting for the parasitics of the surface mount components. The parasitics of the printed circuit board are also extracted from HFSS simulation. The measurement results show a -0.5 -pF negative capacitor, C_{NIC} , achieved over 30–70-MHz bandwidth. A negative conductance, G_{NIC} , is also generated, which may cause stability issue, but G_{NIC} can be reduced by lowering the gain of the transistor. Specifically, as presented in Fig. 5(c), reducing the biasing voltage at the base of the transistors leads to a weaker negative G_{NIC} . However, C_{NIC} gets smaller as well with the reduced biasing voltage, because dropping the biasing voltage leads to smaller transconductance, consequently, decreasing the loop gain and resulting in smaller achieved negative capacitance. Less negative capacitance is undesired, because a certain value of C_{NIC} is required to compensate for C_{period} in Fig. 2 so as to increase phase velocity. Therefore, a tradeoff between maintaining C_{NIC} and reducing G_{NIC} exists. An extreme case is that the conductance can be positive when V_{bb} is so small that the transistors are OFF, leading to an unconditionally stable circuit, and however, the corresponding C_{NIC} is no longer negative. Our solution to improve the stability is to make C_{NIC} negative, but push it very close to zero, so that the corresponding G_{NIC} can be improved. Therefore, the designed non-Foster circuit has V_{bb} of 0.78 V, corresponding to a constant -0.5 pF. Having such small C_{NIC} , small intrinsic capacitance of the waveguide is needed, leading to our choice of an air substrate in Section II. Note that a negative conductance is still present

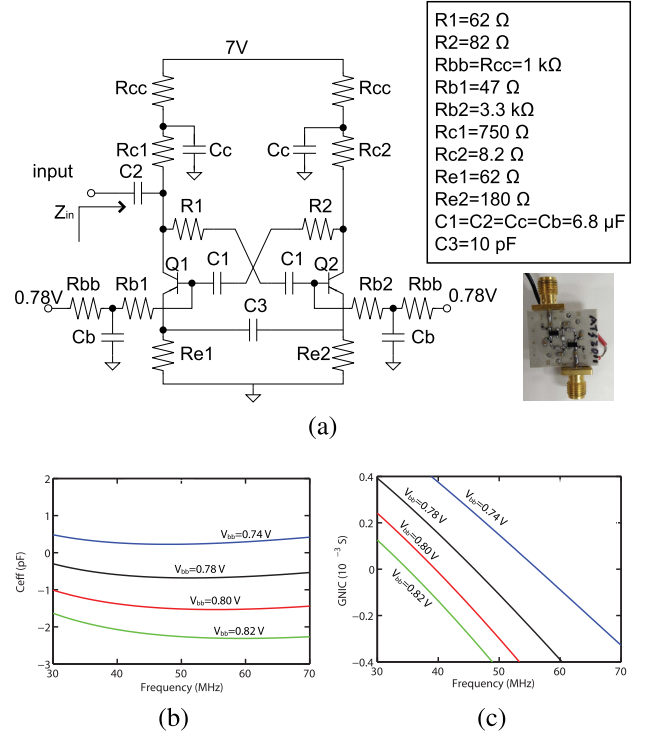


Fig. 5. Non-Foster circuit details. (a) Schematic and fabricated experimental circuit. (b) and (c) Measured input capacitance and conductance with different bias conditions at the base of the transistors.

with -0.5 pF in Fig. 5(c). Nevertheless, the negative conductance is weak enough that the 40 cascaded non-Foster circuits can be stable with a $50\text{-}\Omega$ termination. Stated another way, the unit cell number, 40, in this experimental demonstration is limited by the producible printed circuit board panel size, not the stability. However, this does not mean that waveguides with reduced-negative-resistance non-Foster circuit can be stable at any arbitrary length.

IV. SUPERLUMINAL PROPAGATION MEASUREMENT

This section starts from the FD measurement with the fabricated waveguide, which is demonstrated in agreement with the simulations. The FD measurement results are then used for processing the TD results. The results from both FD and TD are compared, corroborating each other.

Stability is tested in both FD and TD, with the help of spectrum analyzer and oscilloscope. In FD, no spurs are found from dc to 10 GHz. In TD measurement, no oscillation is found in the TD waveform at the output port and dc bias port.

A. Frequency-Domain Measurement

Fig. 6 illustrates the FD measurement setup. A vector network analyzer is used to measure the S-parameters. The refractive index and the characteristic impedance are calculated by (8)–(10). The realistic non-Foster load is included in the simulation to evaluate the superluminal propagation. Specifically, similar to our prior research [44], [45], lumped ports are used at the non-Foster loading position in the HFSS model. The S-parameters of the waveguide with non-Foster loading ports are extracted and cosimulated with the measured

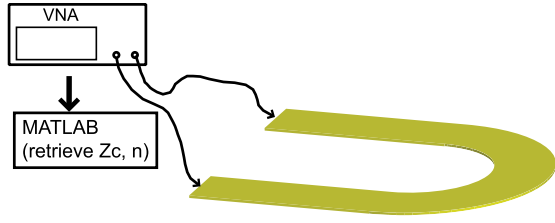


Fig. 6. FD measurement setup.

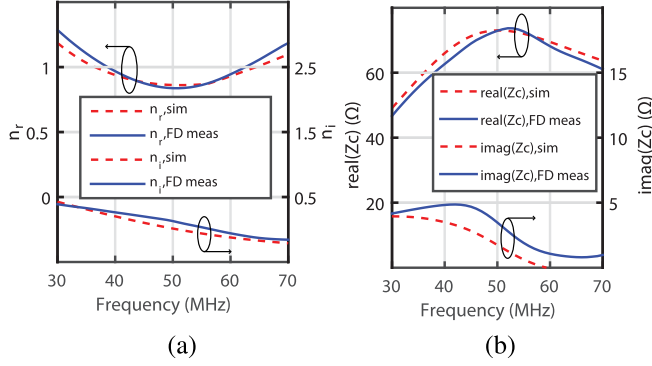


Fig. 7. Numerical simulation and FD measurement results. (a) Real and imaginary parts of the refractive index. (b) Characteristic impedance. Solid lines: measured results. Dashed curves: simulation.

non-Foster impedance in a circuit simulator. The retrieved results are presented in Fig. 7 (dashed curves), which are compared with the FD measurement results (solid lines). From Fig. 7, consistency in FD measurement results and the numerical simulation is observed, indicating the validity of the FD measurement. From Fig. 7, n_r is less than 1 over 39–63 MHz with zero slope around 50 MHz. The waveguide is lossy below 50 MHz, and presents about 1.1-dB gain above 50 MHz, which corresponds to about -0.15 of n_i . This is attributed to the real part of the non-Foster circuit impedance, which is positive below 50 MHz and becomes negative beyond that frequency. This also results in the characteristic impedance peaking around center frequency and falling off at both sides. Interestingly, n_r has its minimum value of 0.84 at 50 MHz with “zero slope” over limited bandwidth. If a signal has a bandwidth within the limited “zero slope” bandwidth, the dispersion will be negligible.

B. Time-Domain Measurement

To clearly observe superluminal propagation of an abruptly changing wave packet, a square wave is modulated on a carrier for testing the implemented waveguide. The TD measurement configuration is illustrated in Fig. 8. A 50-MHz carrier is generated by a signal generator, modulated by a 1- μ s period 50% duty-cycle square pulse, and delivered to a 50- Ω load through the non-Foster loaded waveguide. The input $a(t)$ and output $b(t)$ waveforms, as illustrated in Fig. 8, are sampled with a 4-GSample/s oscilloscope. Subsequently, the square wave is demodulated from the sampled $a(t)$ and $b(t)$, and denoted as $s(t)$ and $r(t)$.

Fig. 9(a) and (b) presents part of the sampled signals $a(t)$ and $b(t)$, and the demodulated waveforms $s(t)$ and $r(t)$, at both input and output. The expanded details of one cycle

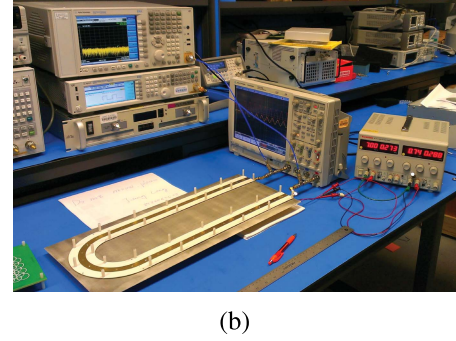
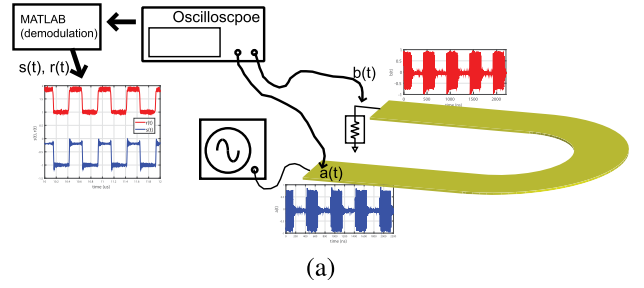


Fig. 8. TD measurement scheme. (a) Experimental configuration. The non-Foster superluminal waveguide is fed by a signal generator and loaded by a 50- Ω load. An oscilloscope is employed for sampling the input and output waveforms, $a(t)$ and $b(t)$, respectively. The sampled data are processed for demodulation to calculate the transmitted and the received pulse shape, i.e., $s(t)$ and $r(t)$, respectively. (b) Picture of the lab setup.

are presented in Fig. 9(c) and (d), clearly illustrating the resemblance of the input and output waveforms. Moreover, the spectrum of $s(t)$ and $r(t)$ are compared in Fig. 10, showing an almost identical spectrum below 20 MHz. This implies the fact that little distortion is introduced by the non-Foster waveguide transmission, as further evidenced in the detailed profiles of the demodulated pulse waveform [i.e., $s(t)$ and $r(t)$] in Fig. 9(d). Next, the delay of the carrier frequency, τ_p , and the delay of the square wave packet, τ_g , are calculated by correlating $a(t)$ with $b(t)$ and $s(t)$ with $r(t)$, respectively. To avoid the inaccuracy of the TD measurement, a set of data are selected from the total 25- μ s-long sampled waveforms by using a 10- μ s window and sliding it with every 0.7- μ s distance [see Fig. 9(b)]. Specifically, the first set of 10 μ s data is taken with a 10- μ s window (from $t = 0$ μ s to $t = 10$ μ s). The data are used for correlation to calculate the time delay, which is reported as the first point in Fig. 9(e). To obtain the second set of data, the 10- μ s window is moved forward by 0.7 μ s (i.e., from $t = 0.7$ μ s to $t = 10.7$ μ s). The data are used for correlation to calculate the second time delay [i.e., the second data point in Fig. 9(e)]. The 10- μ s time window keeps moving forward and the corresponding time delay is recorded, until the window reaches the end of the entire 25- μ s sampled data. Therefore, the last data ($\# = 22$) is calculated based on the sampled data from $t = 0 + 0.7 \times 21 = 14.7$ μ s to $t = 10 + 0.7 \times 21 = 24.7$ μ s. There are 22 points calculated in total. In such a manner, τ_p and τ_g are presented in Fig. 9(e) and (f), from where the averaged τ_p for all sliding windowed samples is 3.146 ns, and τ_g is 3.523 ns.

As seen in Fig. 7, a non-50- Ω transmission line is achieved, so the above-derived time delays, τ_p and τ_g , slightly deviate

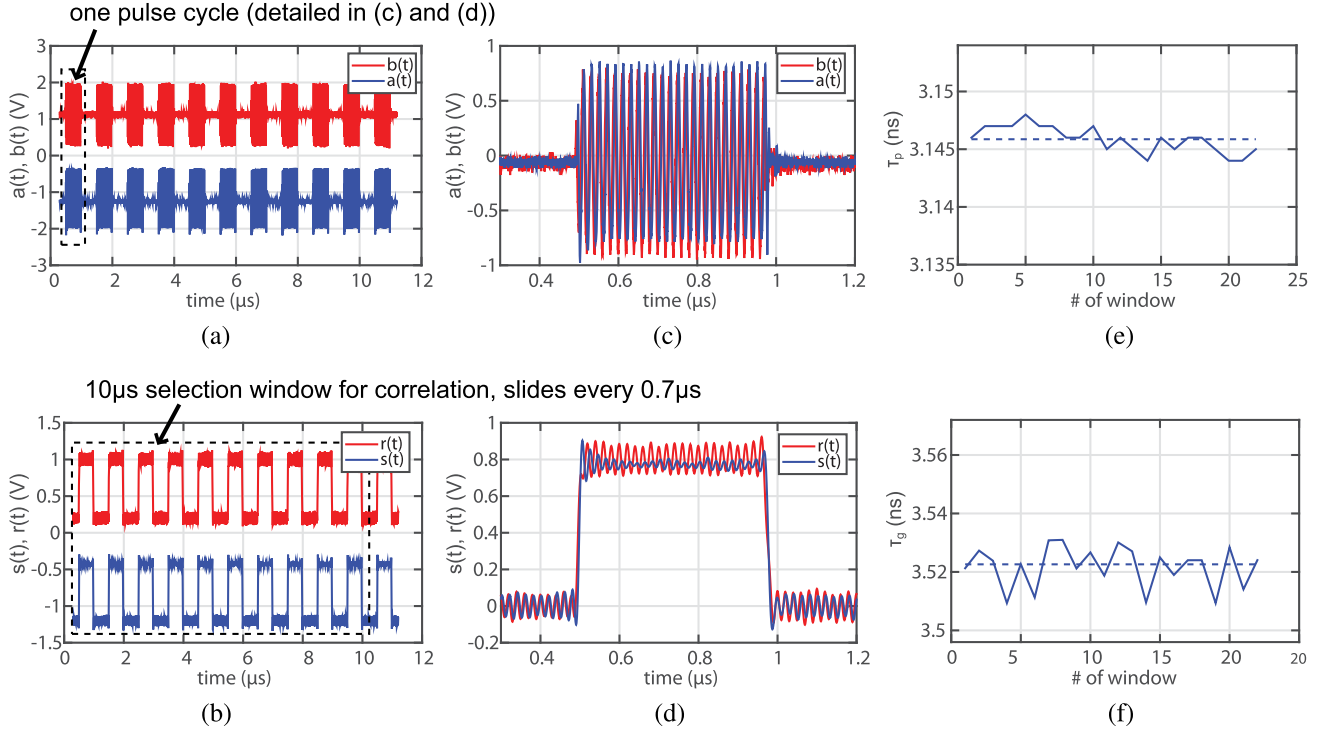


Fig. 9. TD superluminal pulse measurement. The sampled modulated waveforms with a carrier frequency of 50 MHz at (a) input and output and (b) demodulated wave packets are detailed in (c) and (d), respectively. 10- μs data at input and output are windowed from the total 25- μs data and correlated to find (e) time delay τ_p for carrier and (f) τ_g for square pulse, which are further corrected to derive the index n_r and group index n_g . For (a)–(d), the blue curves represent waveforms at input, i.e., $a(t)$ or $s(t)$, and red color is for output, $b(t)$ or $r(t)$.

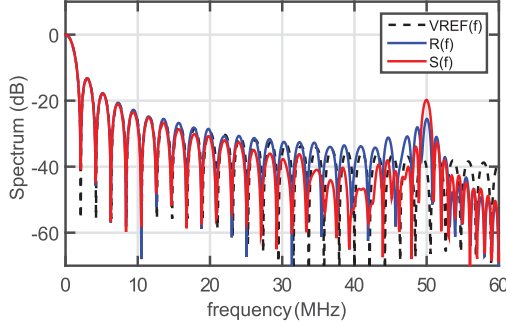


Fig. 10. Normalized spectrum of the input $s(t)$ and output $r(t)$ waveforms in comparison with the ideal square wave pulse.

from the wave propagation delay in a matched waveguide, because the latter is under the assumption that the load impedance equals to the waveguide characteristic impedance, which, however, does not apply to the experimental non-Foster waveguide. A correction factor is thus added as

$$\tau_{\text{wave},p} = \tau_p + \tau_{\text{correction}} \quad (11)$$

and

$$\tau_{\text{wave},g} = \tau_g + \tau_{\text{correction}} \quad (12)$$

where $\tau_{\text{correction}}$ is the function of the propagation constant and characteristic impedance of the non-Foster waveguide. Results obtained from the FD measurement are used to calculate $\tau_{\text{correction}}$. The detailed derivation of $\tau_{\text{correction}}$ is elaborated in the appendix. After correction, the time delays

$\tau_{\text{wave},p}$ and $\tau_{\text{wave},g}$ represent the carrier delay and the waveform packet delay, under the condition that the waveguide is loaded with matched termination. Then, the refractive index is calculated from $\tau_{\text{wave},p}$, and group index is derived from the wave packet delay $\tau_{\text{wave},g}$ as

$$n_r = \frac{\tau_{\text{wave},p}}{L/c} \quad (13)$$

and

$$n_g = \frac{\tau_{\text{wave},g}}{L/c} \quad (14)$$

where L is the total length of the waveguide, 1.2 m, and c is the speed of light. n_r and n_g with different carrier frequencies ranging from 30 to 70 MHz and different square pulsewidth (0.5, 1, and 2 μs) are measured and compared in the FD measurements, as shown in Fig. 11(a) and (b), respectively. Consistency between the TD and FD measurement can be observed, confirming the validity of the measured superluminal propagation results.

Compared with the distorted half-sine waveform reported in [18], distortion has been significantly reduced primarily, because most frequency components of the square wave travel with the same velocity in the non-Foster loaded waveguide, whereas the half-sine wave in [18] experienced different delays for different frequency components in the metamaterial-based medium. This indeed reveals the advantage of the dispersionless non-Foster loaded waveguide to the gain/absorption line-based approaches.

Furthermore, tunability of the superluminal propagation is achieved by varying the biasing voltage. Fig. 12(a) shows the

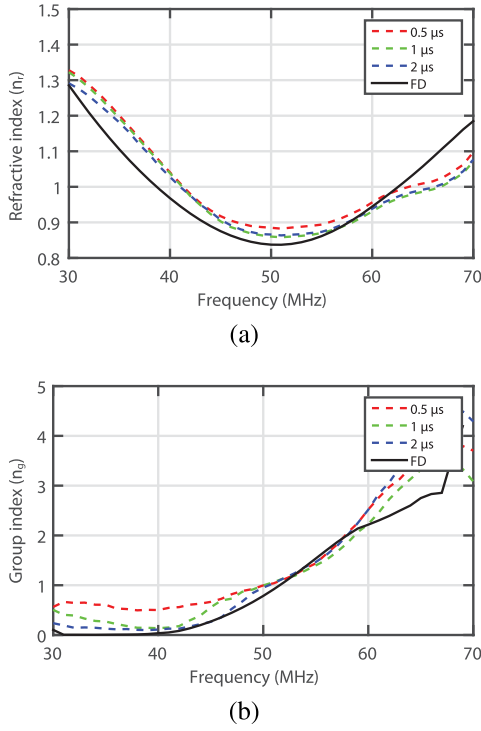


Fig. 11. Measured (a) n_r and (b) n_g with carrier frequency ranging from 30 to 70 MHz, and compared with the FD measurement results (in black solid curves). Different colors are associated with different pulsewidth.

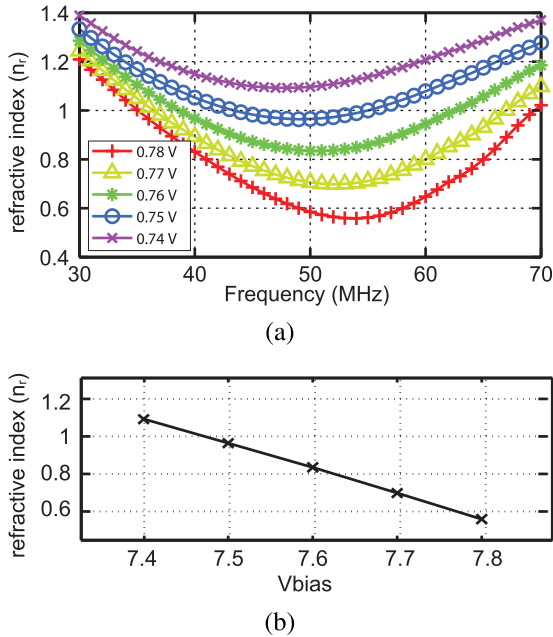


Fig. 12. Tunability of the non-Foster loaded waveguide. By varying the biasing voltage (a) refractive index n_r can be tuned and (b) n_r versus biasing voltage.

FD measurement results on n_r with the non-Foster circuits' biasing voltage ranging from 0.74 to 0.78 V. The larger biasing voltage results in the more negative non-Foster capacitance, and consequently leads to smaller n_r . The variation of the minimum n_r as a function of the biasing voltage is presented in Fig. 12(b), demonstrating a 0.6 (fast wave) to

1.1 (slow wave) tuning range of n_r . Nevertheless, it is seen that smaller n_r leads to narrower bandwidth, manifesting a tradeoff between the achievable bandwidth and superluminal velocity.

In summary, we have developed a 1200-mm non-Foster loaded waveguide with 40 unit cells, which, to the best of our knowledge, is the longest non-Foster loaded waveguide and contains the largest number of the non-Foster unit cells. Relying on such a design, the retrieved n_r indeed manifests a zero slope at around 50 MHz and has less than 10% variation over the bandwidth of 42–58 MHz, corresponding to a 32% fractional bandwidth. The experimental demonstration further confirms that all frequency components travel with almost the same velocity in the non-Foster loaded waveguide, which can never be achieved by gain/absorption line approaches [13]–[17]. Consequently, despite the fact that the spectrum of a square pulse is broader than the Gaussian pulse and even half-sine waveform of previous works, negligible distortion was found by comparing the waveforms at the input and output of the non-Foster loaded waveguide, further confirming the dispersionless property of the non-Foster loaded waveguide. It should be noted that the frequency involved in this experiment is limited by the implemented non-Foster circuit. The non-Foster loading theory itself is not affected by operating frequency.

V. DISCUSSION OF THE THEORETICAL DISTANCE LIMITATION FOR THE NON-FOSTER LOADING-BASED SUPERLUMINAL PROPAGATION

Although the observation of the nondispersive superluminal propagation is demonstrated in this paper, it does follow Einstein's special relativity and causality [30]. Specifically, no signal is able to travel so fast that the tail of a pulse transmitted through a superluminal medium for a certain distance emerges earlier than the head of the same pulse traveling in vacuum with the same distance (i.e., the advanced time of the superluminal pulse is larger than the pulse duration of the same pulse in vacuum). Mitchell and Chiao [7] have demonstrated an advanced pulse response by using lumped circuits and mathematically derived that the response is determined by the past, not the future input. Garrett and McCumber [11] have demonstrated the validity of superluminal propagation in an anomalously dispersive gain/absorption medium with a limited distance. In fact, based on Kramers–Kronig relations, despite that all the frequency components contained in a wave packet propagate in the non-Foster loaded superluminal media with the same speed, they are weighted by different gain/loss factors, consequently introducing distortion. When the medium length is small enough, the output signal after the non-Foster loaded superluminal medium is only slightly different from the original. However, when the medium is long enough, the power of each frequency component of the signal is very different from the original. Thus, the output signal no longer resembles the original, and furthermore, the information contained in the output signal no longer represents that of the original. Consequently, there must be a distance limitation for the non-Foster-based superluminal propagation, preventing the

output signal from becoming completely distorted from the input. This section mathematically demonstrates that in order to preserve the signal profile, the superluminal pulse cannot finish propagation before the emergence of the same pulse transmitted in vacuum, preserving causality, or the maximum advancement of the pulse is less than the pulsewidth.

A. Mathematical Derivation

First, a signal $s(t)$ with narrow bandwidth of bw is modulated on a carrier $\cos(\omega_c t)$ ($bw \ll \omega_c$). Correspondingly, the transmitted waveform is expressed as

$$\begin{aligned} a(t) &= s(t) \cos(\omega_c t) \\ &= \frac{1}{2} s(t) (e^{j\omega_c t} + e^{-j\omega_c t}) \quad (bw \ll \omega_c) \end{aligned} \quad (15)$$

and its Fourier transformation is

$$A(\omega) = \frac{1}{2} [S(\omega - \omega_c) + S(\omega + \omega_c)]. \quad (16)$$

After passing through an L -meter-length non-Foster loaded medium that is described by a transfer function $H(\omega)$, the output signal is

$$B(\omega) = A(\omega)H(\omega) \quad (|\omega \pm \omega_c| < bw) \quad (17)$$

where $H(\omega)$ can be expressed by the complex phase retardation due to the effective material index as

$$H(\omega) = e^{-j\frac{L}{c} \cdot \omega n(\omega)} \quad (18)$$

with $n(\omega)$ being the complex refractive index of the non-Foster loaded media. Therefore, after filtering out the high-frequency components, the demodulated output in FD is

$$\begin{aligned} R(\omega) &= \frac{1}{2} (B(\omega + \omega_c) + B(\omega - \omega_c)) \\ &= \frac{1}{4} S(\omega) (e^{-j\frac{L}{c} \cdot (\omega + \omega_c) n(\omega + \omega_c)} \\ &\quad + e^{-j\frac{L}{c} \cdot (\omega - \omega_c) n(\omega - \omega_c)}) \quad (|\omega| < bw). \end{aligned} \quad (19)$$

Owing to the narrow bandwidth $bw \ll \omega_c$, the term in (19), $(\omega \pm \omega_c)n(\omega \pm \omega_c)$, can be expanded as a Taylor series, and approximated by neglecting all the higher order terms as

$$\begin{aligned} (\omega \pm \omega_c)n(\omega \pm \omega_c) &\approx \pm \omega_c n(\pm \omega_c) \\ &\quad + \omega [n(\pm \omega_c) \pm \omega_c n'(\pm \omega_c)]. \end{aligned} \quad (20)$$

Thus, (19) can be rewritten as

$$\begin{aligned} R(\omega) &\approx \frac{1}{4} S(\omega) [e^{-j\frac{L}{c} (\omega_c n(\omega_c) + \omega [n(\omega_c) + \omega_c n'(\omega_c)])} \\ &\quad + e^{-j\frac{L}{c} (-\omega_c n(-\omega_c) + \omega [n(-\omega_c) - \omega_c n'(-\omega_c)])}]. \end{aligned} \quad (21)$$

As mentioned earlier, the non-Foster loaded waveguide produces a refractive index that has a flat slope at the center frequency of its superluminal band [$\partial n_r(\omega)/\partial \omega|_{\omega_c} = 0$] and converges to 1 at infinite frequency. The waveguide behaves as a normal passive waveguide out of its superluminal frequency band. Taking these features into consideration, the index of the non-Foster loaded waveguide is assumed as

$$n_r(\omega) = 1 - \frac{\frac{n_{r0}}{BW}}{\left(\frac{\omega - \omega_c}{BW}\right)^2 + 1} \quad (\omega > 0) \quad (22)$$

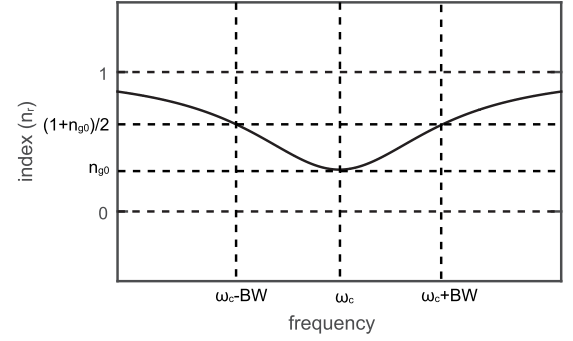


Fig. 13. Illustration of the refractive index of the non-Foster loaded waveguide. It is supposed to have zero slope at center frequency ω_c , and converge to unity at infinite frequency. Here, n_r is mathematically fit by (22).

and it is an even function due to Kramers–Kronig relations. Then, its imaginary part is derived from the Hilbert transformation of n_r to preserve causality [46] as

$$n_i(\omega) = -\frac{\left(\frac{n_{r0}}{BW}\right) \left(\frac{\omega - \omega_c}{BW}\right)}{\left(\frac{\omega - \omega_c}{BW}\right)^2 + 1} \quad (\omega > 0) \quad (23)$$

which is an odd function of ω . In both (22) and (23), ω_c is the center frequency. As indicated in Fig. 13, n_r reaches its minimum value at ω_c . In addition, since the derivative of n_r at ω_c is 0, the corresponding group index, n_{g0} , as defined in (1), is equal to the refractive index at the center frequency ω_c . Specifically, n_{g0} is determined by the parameter n_{r0}/BW , while BW is the bandwidth, where n_r is halfway between 1 and the minimum value. Hence, it is easy to derive

$$n_r(\omega_c) = n_{g0} = 1 - \frac{n_{r0}}{BW} \quad n_i(\omega_c) = 0 \quad (24)$$

and

$$n'_r(\omega)|_{\omega_c} = 0 \quad n'_i(\omega)|_{\omega_c} = \frac{-(1 - n_{g0})}{BW}. \quad (25)$$

Substituting (24) and (25) into (21), and utilizing their symmetry property, $R(\omega)$ can be transformed into

$$R(\omega) = \frac{1}{2} S(\omega) e^{-j\frac{L}{c} \omega \cdot n_{g0}} \cdot \cosh(A) \quad (|\omega| < bw) \quad (26)$$

where

$$A = \frac{L}{c} \omega \cdot \omega_c \frac{1 - n_{g0}}{BW} - j \frac{L}{c} \omega_c n_{g0} \quad (27)$$

and the TD output $r(t)$ is its inverse Fourier transformation. In (26), the exponential term, $e^{-j(L/c)\omega \cdot n_{g0}}$, corresponds to a phase modulation. In particular, this linearly frequency-dependent phase modulation results in a constant time delay in the TD, denoted as $\tau_g = (L/c) \cdot n_{g0}$, further implying that n_{g0} represents the group velocity. In contrast, $\cosh(A)$ is the sum of the two exponential terms, introducing distortion. Particularly, as the propagation distance L increases, the distortion grows exponentially, and eventually, the output signal $r(t)$ gets so distorted that it no longer represents the original input $s(t)$. Hence, there must be a distance limitation to constrain the distortion term within an acceptable range. On the other hand, no distortion is generated if it was a unity constant. Therefore, we define the power difference between

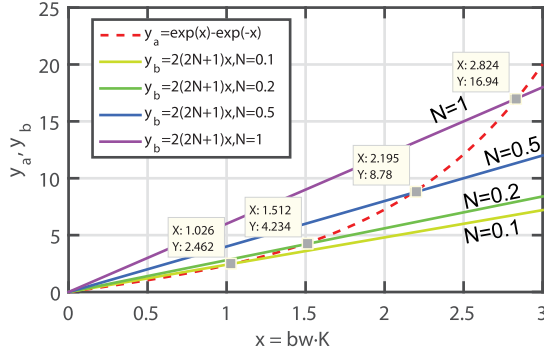


Fig. 14. Solution $bw \cdot K$ of (31) is obtained by graphic method. The red curve is the left-hand side of (31). All other curves represent the right-hand side of (31) with $N = 0.1, 0.2, 0.5$, and 1 . The intersection points represent the solution $bw \cdot K$ with parameter of N .

the frequency-dependent distortion term and the unity constant as

$$P_{\Delta} = \int_{-bw}^{bw} |\cosh(A)|^2 d\omega - \int_{-bw}^{bw} (1)^2 d\omega \leq \frac{1}{2K} (e^{bwK} - e^{-bwK}) - bw = P_{\Delta \max} \quad (28)$$

where

$$K = 2 \frac{L}{c} \cdot \omega_c (1 - n_{g0}) \frac{1}{BW}. \quad (29)$$

As discussed earlier, P_{Δ} should be limited so that the output signal is not distorted too much to be recognized as the original one. However, there is no straightforward approach to quantitatively define the limitation. Here, we restricted the maximum of P_{Δ} , namely, $P_{\Delta \max}$ in (28), as N -folds of the ideal unity integral

$$P_{\Delta \max} = N \times \int_{-bw}^{bw} (1)^2 d\omega \quad (N > 0) \quad (30)$$

which leads to

$$(e^{bwK} - e^{-bwK}) = 2(2N + 1) \cdot bwK. \quad (31)$$

The solution for $bw \cdot K$ is dependent on N , and can be found by a graphic method shown in Fig. 14.

Next, a simulated Gaussian pulse with a -40 dB bandwidth of 20 MHz, that is

$$s_{test}(t) = e^{\frac{(t-\tau)^2 - (2\pi bw)^2}{16 \ln(10^{bw\tau/20})}} \quad (32)$$

where $\tau = 20 \times 10^{-6}$, $bw = 20 \times 10^6$, and $bwr = -40$, which is used to quantify the superluminal transmission. The simulated transfer function for superluminal waveguide is defined as the hyperbolic term in (26), that is,

$$H_{test}(\omega) = \cosh(A) = \frac{e^A + e^{-A}}{2} \quad (33)$$

where A is expressed in (27). The TD responses, $r_{test}(t)$, of the simulated Gaussian pulse through H_{test} with different values of $bw \cdot K$ (as a function of N) are calculated and compared in Fig. 15, where we can observe obvious distortion when $N > 0.5$, or $bw \cdot K > 2.19$.

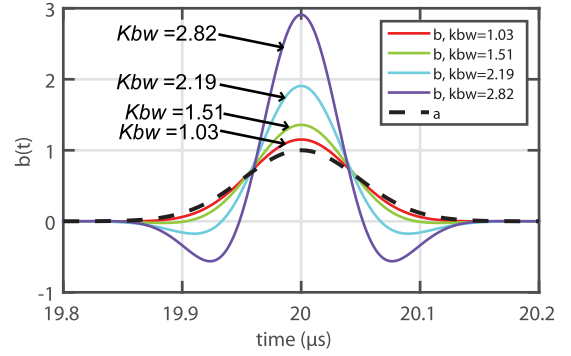


Fig. 15. Response $r_{test}(t)$ of the simulated Gaussian pulse through the transfer function, $H_{test}(\omega)$, defined as (33). The parameter K is based on the solution $bw \cdot K$, where bw is defined in (32). The dotted curve is the input pulse. All the other solid curves are the resulting outputs for different K cases.

Admittedly, $N = 0.5$ is not the exact limit boundary. Nevertheless, it is enough to see that the boundary must be a finite number, since apparent distortion has already been observed when $N = 0.5$. Thus, here we assume $N = 0.5$ as the limitation, and in the following discussion, we demonstrate that this limitation, although relaxed, still leads to the expected conclusion. Therefore, with $N = 0.5$, (31) becomes

$$(e^{bwK} - e^{-bwK}) < 4bwK \quad (34)$$

which leads to

$$Kbw = 2 \frac{L}{c} \cdot \omega_c (1 - n_{g0}) \frac{1}{BW} \cdot bw < 2.19 \quad (35)$$

n_{g0} is the group index of the wave packet as long as the wave packet has a bandwidth relatively small enough compared with the carrier frequency ω_c . Hence, $(L/c) \cdot (1 - n_{g0})$ represents the advanced time of a pulse propagating in the non-Foster loaded superluminal medium compared with that transmitted in vacuum media, denoted as τ_{AD} . In addition, $(1/2bw)$ equals to $(\tau_{pulse}/2\pi)$, and τ_{pulse} is the pulse duration. Consequently,

$$\frac{\tau_{AD}}{\tau_{pulse}} < \frac{2.19}{2\pi} \cdot \frac{BW}{\omega_c}. \quad (36)$$

Since the non-Foster impedance bandwidth is limited, the corresponding superluminal bandwidth BW must be much smaller than the center frequency (also as seen from the bandwidth of the gain/absorption material [11]). Thus,

$$\frac{\tau_{AD}}{\tau_{pulse}} \ll \frac{2.19}{2\pi} = 0.35 \quad (37)$$

which explicitly indicates that the advanced time of the pulse propagating in a non-Foster loaded superluminal waveguide must be far less than 35% of the total transmitted pulse duration. In essence, no pulse can travel so fast that the end of the superluminal pulse occurs before the beginning of the same pulse transmitted through vacuum, preserving causality. It should be noted that (37) is just a loose upper bound, which demonstrates that no one can receive a pulse earlier than the same pulse transmitted in vacuum by a full pulsewidth without distortion. In the following study case, it is found that the maximum advancement of a pulse propagation without significant distortion is far from that limit.

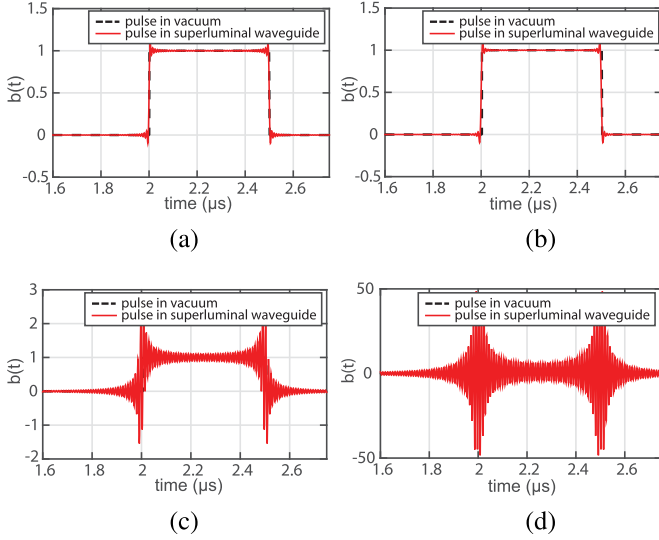


Fig. 16. Testing case with the superluminal media distance of (a) 0.5, (b) 1, (c) 2, and (d) 4 m. The testing propagation media has the minimum index of 0.81 at a center frequency of 50 MHz and a bandwidth of 20 MHz. 1-MHz square wave pulse with 50% duty cycle is used as the testing input signal. It is clear to see that the distortion is increased with the propagation distance.

B. Study Case

To confirm the above hypothesis, we assume that the propagation system has the refractive index with a minimum value of 0.81 at the center frequency of 50 MHz and a 20-MHz bandwidth, which is modeled by a basis function as

$$n_r = 1 - \frac{\frac{n_{r0}}{BW}}{\left(\frac{\omega - \omega_c}{BW}\right)^2 + 1} \quad (38)$$

where $n_{r0}/BW = 0.19$, $\omega_c = 50$ MHz, and $BW = 10$ MHz. Next, we calculated the output of the superluminal system with a square wave pulse input under different propagation distances. The distortion for different propagation distances can be observed in the resulting output waveforms presented in Fig. 16. Specifically, the distortion is insignificant when the propagation distance is less than 1 m, as found in the experiments. As the distance increases, more distortion can be observed. Particularly, when the propagation distance is as long as 4 m, significant distortion has made the output signal hardly be recognized as the input signal. To be noted here, for the testing 1- μ s square pulse with 50% duty cycle, a 4-m distance only corresponds to 0.5% of the pulselength in vacuum, which is well below the limitation quantified in (37). Therefore, the nondistorted superluminal pulse cannot merge before the same pulse in vacuum by a full pulselength, preserving causality.

VI. CONCLUSION

In summary, we have investigated non-Foster circuit-based superluminal propagation. First, a 1200-mm dispersionless superluminal waveguide with 40 non-Foster loads was designed, manufactured, and tested, with a measured refractive index $n_r = 0.84$ over a bandwidth of 42–58 MHz. Next, a square wave packet is transmitted through the waveguide without apparent distortion, demonstrating the dispersionless property. In addition to the experimental demonstration,

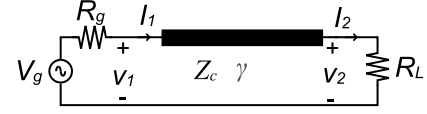


Fig. 17. Illustration of a transmission line with load impedance different from characteristic impedance.

a theoretical analysis on the non-Foster-based superluminal waveguide is carried out for the first time, and a limitation on the superluminal advancement in terms of the pulselength is presented. It is concluded that a distortion-limited superluminal pulse cannot finish before the emergence of the same pulse transmitted in vacuum, reinforcing causality. To put it in another way, the advancement provided by the superluminal guide cannot be more than a pulselength without generating significant distortion. The experimental and theoretical investigation on the non-Foster-based superluminal propagation is supplementary to the gain/absorption material-based superluminal propagation, providing an alternative approach, leading to more comprehensive understanding of the superluminal propagation.

APPENDIX

DERIVATION OF THE CORRECTION FACTOR FOR IMPEDANCE MISMATCH

The phase and group delay directly derived from the correlation of the input and output is close to the wave propagation delay, but still needs a minor correction, owing to the fact that the load impedance is mismatched to the characteristic impedance of the waveguide [47]. Therefore, in order to make it completely accurate, a correction factor is derived to account for the mismatch effect.

The correction factor is the difference between the phase delay measured under the mismatched condition and that corresponding to theoretical wave propagation in a matched waveguide. The former is derived from the transmission line model illustrated in Fig. 17, where the characteristic impedance Z_c is different from the load impedance R_L . The voltages and currents at the input and output are related by

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} \cosh(\gamma L) & Z_c \sinh(\gamma L) \\ Y_c \sinh(\gamma L) & \cosh(\gamma L) \end{bmatrix} \times \begin{bmatrix} V_2 \\ I_2 \end{bmatrix} \quad (39)$$

and the load condition is

$$V_2 = I_2 \times R_L. \quad (40)$$

By solving (39) and (40), it is easy to derive

$$\begin{cases} V_1 = (R_L \cosh(\gamma L) + Z_c \sinh(\gamma L)) \cdot I_2 \\ V_2 = R_L \cdot I_2. \end{cases} \quad (41)$$

Thus, the phase difference between V_2 and V_1 can be obtained, and consequently, the delay between the input and the output under the mismatched condition is

$$\tau_{mismatched} = \frac{\angle V_2 - \angle V_1}{\omega} \quad (42)$$

where ω is the frequency in radians per second. On the other hand, the theoretical delay of an ideal wave in a matched

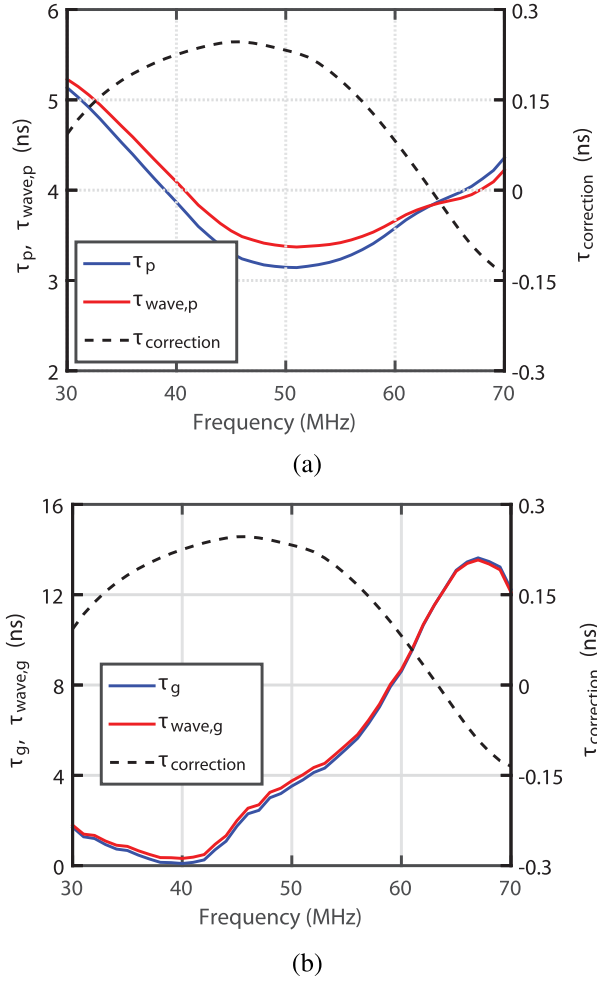


Fig. 18. Comparison of the time delay before and after the calibration with $\tau_{correction}$. τ_p and τ_g represent the time delay derived from the correlation of the sampled TD data. Calibrated data $\tau_{wave,p}$ and $\tau_{wave,g}$ are obtained by correcting τ_p and τ_g with the correction factor $\tau_{correction}$. (a) Time delay between $a(t)$ and $b(t)$. (b) is associated with the time delay between $s(t)$ and $r(t)$. Blue curve is derived directly from TD measurement correlation, dashed curve is the calibration factor derived based on the FD measurement results, and red curve is the time delay after correction.

waveguide is

$$\tau_{theo} = \frac{L}{\omega} \cdot \beta. \quad (43)$$

The correction factor is the difference between (42) and (43)

$$\tau_{correction} = \tau_{theo} - \tau_{mismatched}. \quad (44)$$

In TD measurement, the time delays calculated from the sampled input and output (τ_p and τ_g) are under the mismatched condition. By taking account of the difference between the theoretical ideal delay and the mismatched time delay, as (11) and (12), the calibrated time delays ($\tau_{wave,p}$ and $\tau_{wave,g}$) represent the measured phase and group delay under matched condition. Fig. 18 shows the raw data directly obtained from the correlation of the TD sampled input and output, τ_p and τ_g , the correction factor, $\tau_{correction}$, and the delay after the calibration $\tau_{wave,p}$ and $\tau_{wave,g}$. Minor differences between the raw data and that after calibration are observed, which is because

the difference due to the mismatch effect is not significant, but necessary for accurate calculation. During the calibration, γ and Z_c of the transmission line in (41) is derived from the FD measurement result. The TD result after calibration is compared with the FD result, showing agreement with each other, which validates both the FD and the TD measurement.

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