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Supersymmetry breaking in string models and a source of hierarchy (I) [a]

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Abstract

In a certain class of superstring models, supersymmetry is broken in a hidden sector but remains globally conserved in the observable sector of quarks and gluons. We recently identified the symmetry responsible for this property, a nonlocal symmetry closely connected with spacetime duality. This symmetry is broken by chiral and conformal anomalies, which provides the source of supersymmetry breaking in the observable sector and a possibly large hierarchy of scales. We give here details on our analysis, focussing on the identification of the anomalous terms which provide a source of breaking.

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DISCLAIMER

1 Introduction.

The issue of supersymmetry breaking is one of the stumbling blocks for the construction of a realistic string model with definite predictive power. Little progress has been made in this direction in the last years. In a recent paper[1], we suggested a possible way out, connected with the presence of duality symmetry in string models. Very crudely speaking, in the simple model that we studied, the spontaneous breaking of local supersymmetry provides a gravitino mass only a few orders of magnitude smaller than the Planck scale M_{Pl}; but global supersymmetry remains unbroken, being protected by a symmetry connected with duality: thus all matter fields remain massless at this level. Fortunately, at the quantum level, the anomalous behaviour of this symmetry provides for a breaking of global supersymmetry and yields nonzero contributions for the masses of the matter fields; the nonperturbative nature of this breaking generates a large hierarchy between the scales of local and global supersymmetry, thus paving the way for a possible explanation of the hierarchy M_W/M_{Pl} observed in nature.

In this paper and the following[2](respectively numbered I and II), we intend to give a detailed account of our results. This paper will focus on the identification of the symmetry that protects global supersymmetry together with its anomalous behaviour which provides a source of breaking. Paper II deals mainly with the derivation of the effective theory relevant for low energy phenomenology.

The mechanism that we use for breaking local supersymmetry is based on the condensation of gauginos in a hidden sector, such as the one which appears in superstring models compactified on a Calabi-Yau manifold or an orbifold[3] (it can be used in a similar way in 4-dimensional string models).In Section 2, we recall some earlier results which indicate that this does not break supersymmetry globally and we identify the symmetry responsible for it. Preparing the ground for the next Section, Section 3 is a brief summary of what we need to know on the coupling of matter to supergravity, in particular in connection with Kähler invariance which plays a central role in our analysis. Indeed in Section 4, we come to the heart of the matter and describe gaugino condensation in the language of effective Lagrangians. This allows us to include the anomalous contributions which provide the germ for global supersymmetry breaking. Finally, Section 5 studies the connection of our symmetry with the much studied duality symmetry of string theories, introduces Paper II and concludes.

2 The role of $SL(2,\mathcal{R})$ in protecting global supersymmetry.

We first recall some earlier results[4] that seemed to indicate that, at least in a certain class of superstring models, although gaugino condensation in a hidden sector breaks local supersymmetry, it does not break global supersymmetry in the observable sector of quarks and leptons. We will then identify the symmetry responsible for this property.

In this section, we consider a simple compactification[5] of ten-dimensional supergravity that respects the qualitative features of Calabi-Yau compactification[6]. We will return to more realistic models but this particular one provides us with a symmetry structure that is common to a broad class of models. And, as we have already stressed, our proofs will be based mainly on symmetry arguments.

Besides the supergravity multiplet we find two types of "structural" chiral supermultiplets in 4 dimensions:

– a dilaton-type superfield S with scalar component s, whose vacuum expectation value (*vev*) provides the ratio between the Planck scale M_{Pl} and the string scale $M_S = \alpha'^{-1/2}$:

$$\frac{M_{Pl}}{M_S} = < Res >^{1/2} . \tag{2.1}$$

– a modulus-type superfield T with scalar component t, whose *vev* yields the radius R of the compact manifold in string units:

$$M_{comp} \equiv R^{-1} = < Ret >^{-1/2} M_S. \tag{2.2}$$

This superstring model is a grand unified model based on gauge group $\mathbf{G} \otimes \mathbf{H} \subset E_6 \otimes E_8'$, with $\mathbf{G}$ and $\mathbf{H}$ products of simple groups. At the compactification scale, the gauge couplings of all groups are equal: M_{comp} is the grand unification scale

$$M_{GUT} = M_{comp} = \frac{M_{Pl}}{< ResRet >^{1/2}}. \tag{2.3}$$

The common value g of the gauge coupling is actually expressed in terms of the *vev* of Res:

$$\frac{1}{g^2} = < Res > . \tag{2.4}$$

In the gauge nonsinglet sector, there is a sector of fields which are charged under $\mathbf{G} \subset E_6$ and singlet under E_8'. We call it ***observable*** because it is there that one finds the usual gauge fields, quarks and leptons. This observable sector consists of gauge supermultiplets and matter supermultiplets which we denote by Φ^i. The simple model that we consider is not realistic in particular in the sense that there is only one family of quarks and leptons: the index i runs over the components of only one 27 of E_6, possibly not all of them if $\mathbf{G} \neq E_6$ (we take $1 \leq i \leq N$).

There is finally a sector which consists only of the gauge supermultiplets of $\mathbf{H} \subset E_8'$. These fields interact only gravitationally with all the others and thus form a ***hidden*** sector.

As for any four-dimensional model coupled to $N = 1$ supergravity, the couplings in the Lagrangian are fixed by 3 functions[7], the Kähler potential K, the superpotential W and the normalisation of the gauge kinetic terms $f_{\alpha\beta}$. They read respectively:

$$K = -\ln(S+\bar{S}) - 3\ln(T+\bar{T}-|\Phi|^2), \quad |\Phi|^2 = \sum_{i=1}^{N} \Phi^i\overline{\Phi}^{\bar{i}}, \tag{2.5}$$

$$W(\Phi) = c_{ijk}\Phi^i\Phi^j\Phi^k, \tag{2.6}$$

$$f_{\alpha\beta} = S\delta_{\alpha\beta} \tag{2.7}$$

(one can check that (2.4) follows from the last equation).

For example, the scalar potential reads:

$$\begin{aligned} V &= e^{K}[(K^{-1})^{a\bar{b}}(W_a + WK_a)(\bar{W}_{\bar{b}} + \bar{W}K_{\bar{b}}) - 3|W|^2] \\ &= e^{\mathcal{G}}[\mathcal{G}_a(\mathcal{G}^{-1})^{a\bar{b}}\mathcal{G}_{\bar{b}} - 3] + D\text{-terms}, \end{aligned} \tag{2.8}$$

where

$$\mathcal{G} = K + \ln|W|^2. \tag{2.9}$$

and $W_a = \partial W/\partial\Phi^a, \bar{W}_{\bar{a}} = \partial\bar{W}/\partial\overline{\Phi}^{\bar{a}}, \cdots (\Phi^a = S, T, \Phi^i)$. The ground state is reached at $<\Phi^i>=0$ and since then $V=0$ for any value of S and T, these two fields correspond to flat directions of the scalar potential.

At this point, the theory is locally supersymmetric and the gravitino mass $m_{3/2}$ given by

$$m_{3/2}^2 = e^{\mathcal{G}} \tag{2.10}$$

vanishes at the ground state ($<\Phi^i>=0$). One needs to break supersymmetry to lift the degeneracy associated with s and t and to determine the basic scales of the theory (2.1)-(2.3).

As we go down in energy from $M_{comp} = M_{GUT}$, we reach a scale Λ_c where the gauge interaction in the hidden sector becomes strong:

$$\Lambda_c = M_{GUT}e^{-(s+\bar{s})/(4b_0)}, \tag{2.11}$$

where b_0 is the coefficient of the one-loop beta function for the **H** gauge coupling ($\mu dg/d\mu = -b_0 g^3$). At this scale, we expect that the hidden sector gauginos will form condensates[S][c] which will break spontaneously local supersymmetry. This however generates a large cosmological constant and a potential for s monotonically decreasing to zero at $s \to +\infty$ (where supersymmetry is restored).

To overcome this difficulty, it was proposed[3] to introduce a companion supersymmetry breaking mechanism which cancels the cosmological constant (at least at tree level). The usual method is to give a nonzero *vev* to the compact part of the field strength of the antisymmetric tensor B_{MN} present in 10-dimensional supergravity[11]. This field strength is defined in 10 dimensions by ($L, M, N \in \{1\cdots10\}$)

$$H_{LMN} = \partial_L B_{MN} + \partial_N B_{LM} + \partial_M B_{NL} \tag{2.12}$$

[c] We wish to stress here that the situation is that of supersymmetric QCD and not at all the one familiar in QCD. In particular, the one-instanton contribution which we will find below (Eq.(2.13)) is the leading one in supersymmetric QCD, whereas it is drowned in the multi-instanton contribution in QCD. For nice reviews on the subject, see Ref.9,10.

and one sets

$$< H_{lmn} >= \mathbf{c} M_{Pl}{}^3 \epsilon_{lmn}, \tag{2.13}$$

where $l, m, n \in \{1, 2, 3\}$ refer to 3 complex coordinates describing the 6-dimensional compact manifold and ϵ_{lmn} is the completely antisymmetric tensor (for more careful definitions see Ref.4). Note that H_{lmn} does not propagate in 4 dimensions. Also topological arguments similar to the ones that lead to the quantization of the charge of a Dirac monopole indicate that $\mathbf{c}$ obeys a quantization condition[12].

This all boils down to generating s (i.e. gauge coupling) and $\mathbf{c}$-dependent terms in the superpotential:

$$W = \mathbf{c} + \mathbf{h} e^{-\frac{3s}{2b_0}} + W(\Phi). \tag{2.14}$$

The symmetry arguments which lead to (2.14) will be exposed in detail in Section 4 to which we postpone a detailed study of the dynamics of gaugino condensation using an effective Lagrangian approach. For the time being, we will take (2.14) as defining the effective theory below the condensation scale Λ_c.

The vacuum degeneracy associated with s is lifted by supersymmetry breaking. Indeed the potential reads for $\Phi^i = 0$ (still necessary to minimize the potential)

$$V(s, t, \Phi^i = 0) = \frac{1}{(s + \bar{s})(t + \bar{t})^3} |\mathbf{c} + \mathbf{h}(1 + 3\frac{s + \bar{s}}{2b_0}) e^{-\frac{3s}{2b_0}}|^2 \tag{2.15}$$

which determines the *vev* of s:

$$< s >\equiv s_0 = x_0 + i\frac{4n\pi}{3} b_0, \quad n \in \mathcal{Z} \tag{2.16}$$

where

$$\mathbf{c} = -\mathbf{h}(1 + \frac{3x_0}{b_0}) e^{-3x_0/2b_0}. \tag{2.17}$$

At tree level, the t field remains undetermined. Also, at this order, the terms that one would expect from soft supersymmetry breaking in the observable sector (gaugino masses, scalar masses, A-terms) all vanish.

The analysis of the one-loop contributions was undertaken in Refs.13,14,4. The attitude eventually adopted in Ref.4 is that one should not only minimize the potential with respect to the fields s and t but also with respect to the parameters $\mathbf{c}$ and $\mathbf{h}$. The reason behind this is the property of string theory that there is only one fundamental dimensionful parameter in a string model[15], say the string scale M_S. All other parameters are expressed in terms of this scale and *vevs* of scalar fields. Examples of this are provided by Eqs.(2.1)-(2.4). We will see in Section 4 that $\mathbf{h}$ can be similarly interpreted as the expectation value of a scalar (gauge singlet) operator, and $\mathbf{c}$, proportional to $< H_{lmn} >$, is related to $\mathbf{h}$ through the tree-level relation (2.17). The physical interpretation of this property is that, a string theory being a totally constrained system (up to the string coupling α'), all parameters (*vevs*) must adjust themselves so as to minimize the overall vacuum energy in the presence of all quantum corrections to the effective theory.

Under these conditions, one finds that, for a certain range of parameters, the one-loop corrected potential has a stable, nontrivial ($m_{3/2} \neq 0$) minimum; the vacuum energy vanishes and the vacuum is degenerate in one direction in the space of *vevs* (see Fig.1).

This last property is certainly very welcome since the parameter **c** obeys a quantization condition (the topological – global – arguments that lead to it are certainly ignored by the perturbative – local – approach that leads to Fig.1): we are left with a set of discrete vacua along the flat direction.

All scales can be computed in terms of one, say the Planck scale, and **c** (reflecting the left-over degeneracy). In particular one finds[4] at most two orders of magnitude between M_{Pl} and $m_{3/2}\sqrt{b_0 \mathbf{c}}$ which makes one happy not to find any soft supersymmetry breaking term at tree level (terms appearing at this order would be of order $m_{3/2}$). In fact, implementing the condition that the vacuum energy vanishes at one loop generalizes this result to the one-loop level: no gaugino mass, no scalar mass and no A-term are generated by one-loop radiative corrections. This surprising property can be interpreted as the difficulty in sending the information of local supersymmetry breaking in the hidden sector ($m_{3/2} \neq 0$) to the observable sector.

The symmetry responsible for this was identified in Ref.1. In the simple model that we consider, the scalar field kinetic energy has $SU(1,1) \otimes SU(N+1,1)$ symmetry which makes it a no-scale model[16]. A subgroup $SU(1,1) \cong SL(2,\mathcal{R})$ of this symmetry is in fact an exact symmetry of the full Lagrangian. Indeed, under

$$\begin{aligned} S' &= S, \\ T' &= \frac{aT - ib}{icT + d}, \quad ad - bc = 1, \quad a,b,c,d \in \mathcal{R}, \\ \Phi^{i\prime} &= \frac{\Phi^i}{icT + d}, \end{aligned} \tag{2.18}$$

we have (cf Eqs.(2.5),(2.6))

$$K' = K + F + \overline{F}, \tag{2.19}$$
$$W' = We^{-F}, \tag{2.20}$$

with

$$F = 3\ln(icT + d), \tag{2.21}$$

i.e. it can be interpreted as a Kähler transformation and is therefore an invariance of the full supersymmetric Lagrangian (cf. next section).

Eq.(2.18) gives the transformation at the level of superfields. It must be complemented with the transformation law for the θ variables which, having a chiral weight 1, transform under the Kähler transformation (2.19) as (see Section 3):

$$\theta'^{\alpha} = e^{-iImF/2}\theta^{\alpha}. \tag{2.22}$$

This $SL(2,\mathcal{R})$ transformation was identified earlier by Li, Peschanski and Savoy[17] as a symmetry present in a large class of models. We will study in Section 5 its connection with spacetime duality.

We now proceed to give plausibility arguments as to why this symmetry is responsible for the cancellation of soft supersymmetry breaking terms in the observable sector. We first note that, in order to respect the transformation law (2.20), one has to transform the parameters $\mathbf{c}$ and $\mathbf{h}$ according to:

$$\mathbf{c}' = \frac{\mathbf{c}}{(icT+d)^3}, \quad \mathbf{h}' = \frac{\mathbf{h}}{(icT+d)^3}. \tag{2.23}$$

Here we interpret $\mathbf{c}$ and $\mathbf{h}$ as constant superfields; the theory is defined by $\chi^c = F^c = \chi^h = F^h = 0$. Then invariance under (2.23) means that the theory expressed in terms of the primed fields, and defined by $\chi^{c'} = F^{c'} = \chi^{h'} = F^{h'} = 0$, is equivalent to the original theory. Eq.(2.23) corresponds to the correct transformation properties of the fields whose *vevs* are respectively $\mathbf{c}$ and $\mathbf{h}$ (see Section 4). It is also consistent with the attitude advocated above: one should allow $\mathbf{c}$ and $\mathbf{h}$ to vary in order to let the whole system relax to the ground state which minimizes the overall energy.

We then make use of the fact that the full tree level Lagrangian derived from (2.14) is invariant under the transformation (2.18,2.23). In particular, if we place ourselves at the ground state for s, all field-dependent masses can be expressed in terms of the $SL(2,\mathcal{R})$ invariant mass parameter [d]

$$m = \frac{|\mathbf{h}|}{(T+\bar{T}-|\Phi|^2)^{3/2}} M_{Pl}, \tag{2.24}$$

and of $SL(2,\mathcal{R})$ invariant redefinitions of the Φ^i fields: $\hat{\Phi}^i$.

When going to higher orders, the only source of noninvariance arises from the cut-off dependence (giving rise for example at one loop to the trace anomaly; more about this in Section 4). The nature of the cut-off is very special in string models for two specific reasons: i) the cut-off is finite, ii) in its field theoretic version, it is field dependent as any other mass scale. For instance, two typical cut-offs are given by

$$\begin{aligned} \Lambda_{GUT} &= \frac{4M_{Pl}}{[(S+\bar{S})(T+\bar{T}-|\Phi|^2)]^{1/2}}, \\ \Lambda_c &= \Lambda_{GUT} e^{-(S+\bar{S})/(4b_0)}, \end{aligned} \tag{2.25}$$

(note that $<\Lambda_{GUT}>= M_{GUT}$ given in (2.3)) and transform under $SL(2,\mathcal{R})$ according to:

$$\Lambda' = \Lambda\, |icT+d|. \tag{2.26}$$

Since we are mainly concerned with quantities which are zero (through supersymmetry) before gauginos condense, the relevant cut-off for our purpose will be Λ_c. The full quantum potential then reads formally at one loop

$$V_1 = m^4 \tilde{f}(\frac{m^2}{\Lambda_c^2}, \hat{\Phi}^i) \tag{2.27}$$

which we can rewrite[4] using the explicit forms (2.24) and (2.25)

$$V_1 = m^4 f(\zeta, \hat{\Phi}^i), \quad \zeta = \frac{|\mathbf{h}|}{t+\bar{t}-|\Phi|^2}. \tag{2.28}$$

[d] We use here the expression of $\mathbf{c}$ in terms of $\mathbf{h}$ obtained at tree level, Eq.(2.17).

Minimizing with respect to $|\mathbf{h}|$ (see the comments above and Section 4) and t yields, apart from the trivial minimum corresponding to $m = 0$, the following conditions:

$$f(<\zeta>, <\hat{\Phi}^i>) = 0 = \left.\frac{\partial f}{\partial \zeta}(\zeta, <\hat{\Phi}^i>)\right|_{\zeta=<\zeta>} . \tag{2.29}$$

Hence in the case where the potential remains bounded from below, one has the following interesting property: there is a flat direction, i.e. the potential V_1 remains zero in the direction [e]

$$|\mathbf{h}| =<\zeta> (t + \bar{t} - |\Phi|^2). \tag{2.30}$$

This remarkable property implies that global supersymmetry remains unbroken in this direction. Indeed, letting M_{Pl} go to infinity in the final result, we conclude from the fact that $V = 0$ that this flat direction remains globally supersymmetric and that no soft supersymmetry breaking term can thus be generated. Let us stress here that we could not have reached this conclusion if we had put M_{Pl} to infinity before computing radiative corrections, i.e. if we had started with renormalisable terms only: there might be finite terms resulting from the compensations between the $1/M_{Pl}$ factors of the nonrenormalisable terms and the cut-off or the *vevs* that scale with M_{Pl}; these would be lost if we had truncated from the beginning.

The same conclusions could presumably be reached to all orders by following the same line of reasoning. We only sketch the proof here. Suppose that we have proved to the nth order that there exists a flat direction of the type (2.30): $|\mathbf{h}| =<\zeta>^{(n)} (t + \bar{t} - |\Phi|^2)$ (the (n) superscript refers to the fact that $<\zeta>$ has been determined to the nth order). Then no soft supersymmetry breaking masses or couplings are generated to this order in this direction. We conclude that, at the $(n+1)$th level, the potential along this direction can still be formally written as in (2.27): [f]

$$V_{n+1} = m^4 \tilde{f}_{n+1}\left(\frac{m^2}{\Lambda_c^2}, \hat{\Phi}^i\right). \tag{2.31}$$

Following the same argument that led to (2.30), we conclude that there exists a flat direction for the potential: $|\mathbf{h}| =<\zeta>^{(n+1)} (t + \bar{t} - |\Phi|^2)$, where the $(n+1)$th correction to $<\zeta>$ i.e. $(<\zeta>^{(n+1)} - <\zeta>^{(n)})$ is determined precisely by V_{n+1}. No soft supersymmetry breaking term is therefore generated to this $(n+1)$th order and hence to any order.

One should note here that it is the $SL(2,\mathcal{R})$ symmetry that allowed us to express all the masses in terms of the invariant mass m (Eq.(2.24)) and which led us to this conclusion. In this respect, it is interesting to note that, apart from wave function renormalisation terms, the full quantum Lagrangian remains invariant under $SL(2,\mathcal{R})$. The proof was given in Ref.1 but we recall it for the sake of completeness. It actually goes along the same lines

[e] As has been emphasized in Ref.4, one should note however that the *two* conditions (2.29) amount to *one* fine tuning among the parameters describing the potential, more specifically among the parameters that characterize the uncertainties in the regularization of divergences.

[f] At this given $(n+1)$th order, we use the expression of $\mathbf{c}$ in terms of $\mathbf{h}$ obtained at the nth order. This takes care of the $\mathbf{c}$ dependence.

as the preceding one. Define the nonderivative part $\mathcal{L}_{ND}$ of the full quantum Lagrangian $\mathcal{L}_{tot}$:

$$\mathcal{L}_{ND} = \mathcal{L}_{tot} - \mathcal{L}_{KE}, \tag{2.32}$$

which can be written formally

$$\mathcal{L}_{ND} = m^4 F(m, \hat{\Phi}^i, S; \Lambda_c). \tag{2.33}$$

Under $SL(2,\mathcal{R})$,

$$\begin{aligned} \delta\mathcal{L}_{ND} &= \delta\Lambda_c \frac{\partial}{\partial\Lambda_c}\mathcal{L}_{ND} \sim \delta t \frac{\partial}{\partial t}\mathcal{L}_{ND} = \delta t \left[\frac{\partial}{\partial t}\mathcal{L} - \frac{\partial}{\partial t}\mathcal{L}_{KE}\right] \\ &= \delta t \left[\partial_\mu \frac{\partial}{\partial(\partial_\mu t)} - \frac{\partial}{\partial t}\right] \mathcal{L}_{KE}. \end{aligned} \tag{2.34}$$

Hence $\delta\mathcal{L}_1$ cannot contribute to S-matrix elements that do not involve derivative couplings. This is in agreement with the left-over global supersymmetry which allows wave function renormalisation only.

We have established the preceding results more rigorously by deriving Ward identities that will be given elsewhere. The proof makes use of an additional global compact $U(1)_R$ invariance (R-parity) and is similar to the proof of nonrenormalisation theorems for renormalizable theories with global supersymmetry.

3 A refresher on the coupling of matter to supergravity in connection with Kähler invariance.

In order to prepare the ground for the next Section, we describe here the superspace formalism which lies behind the coupling of matter to supergravity. The corresponding Lagrangian was derived by Cremmer *et al.*[18] in terms of the component fields. The structure of the corresponding superspace was identified in Ref.19 and is explained in more details in Ref.20.

As is well known, this coupling is described by the Kähler potential $K(\Phi,\bar{\Phi})$.[g] Correspondingly, Kähler invariance:

$$K(\Phi,\bar{\Phi})' = K(\Phi,\bar{\Phi}) + F(\Phi) + \bar{F}(\bar{\Phi}). \tag{3.1}$$

plays a central role in unravelling the superfield structure. A superfield γ of chiral U(1) weight $\omega(\gamma)$ transforms under a Kähler transformation:

$$\gamma' = \gamma \exp[-\frac{i}{2}\omega(\gamma) Im F] = \gamma \exp[-\omega(\gamma)\frac{F-\bar{F}}{4}]. \tag{3.2}$$

To implement this at the superfield level, one defines a superspace derivative which is covariant with respect to Kähler transformations:

$$\mathcal{D}_{\underline{\alpha}}\gamma = E_{\underline{\alpha}}{}^M \partial_M \gamma + \omega(\gamma) A_{\underline{\alpha}}\gamma = \hat{\mathcal{D}}_{\underline{\alpha}}\gamma + \omega(\gamma) A_{\underline{\alpha}}\gamma, \tag{3.3}$$

[g] For simplicity, in this Section, Φ refers to all chiral supermultiplets in the theory.

where ($\underline{\alpha} = \alpha, \dot\alpha$)

$$A_\alpha = \frac{1}{4}\hat{\mathcal{D}}_\alpha K \quad , \quad A^{\dot\alpha} = -\frac{1}{4}\hat{\mathcal{D}}^{\dot\alpha} K, \tag{3.4}$$

and $\hat{\mathcal{D}}_{\underline{\alpha}}$ is the usual superspace derivative. With the help of this derivative, it is now easy to define component fields with a definite chiral $U(1)$ weight. For instance, in the case of a chiral superfield X of weight ω:

$$\mathcal{D}^{\dot\alpha} X = 0, \tag{3.5}$$

the fermionic field defined as

$$\chi_\alpha = \frac{1}{\sqrt{2}}\mathcal{D}_\alpha X \mid \tag{3.6}$$

has weight $\omega - 1$ and transforms accordingly under a Kähler transformation (cf (3.2)). Other chiral U(1) weights are $\omega(\psi^\alpha_m) = 1, \omega(\lambda^\alpha) = 1$ respectively for a gravitino and a gaugino field.

In this formalism, the superfield Lagrangian takes a simple form. It consists of 3 terms:

(a) a supergravity-chiral matter kinetic term

$$\mathcal{L}_0 = -3\int d^4\theta E = -3\int d^2\Theta\, \mathcal{E}\, \mathcal{R} + h.c., \tag{3.7}$$

where E is the (super)determinant of the supervierbein, $\mathcal{E}$ is the chiral density multiplet of U(1) weight zero and $\mathcal{R}$ a chiral superfield of U(1) weight 2. In (3.7), there is an implicit dependence on the chiral and antichiral superfields $\Phi^i, \overline{\Phi}^{\bar{i}}$ through the dependence of the spinorial derivatives of E on $K(\Phi, \overline{\Phi})$. Indeed,(3.7) yields in particular the kinetic terms for all the components of the (anti)chiral supermultiplets[19,20]:

$$\frac{1}{e}\mathcal{L} = -g_{i\bar{k}} D^\mu \phi^i D_\mu \bar\phi^{\bar{k}} - \frac{i}{2} g_{i\bar{k}}(\bar\chi^{\bar{k}}\bar\sigma^\mu D_\mu \chi^i + \chi^i \sigma^\mu D_\mu \bar\chi^{\bar{k}}) + g_{i\bar{k}} F^i \bar{F}^{\bar{k}} + \cdots \tag{3.8}$$

where $g_{i\bar{k}}$ is the Kähler metric $\partial^2 K/\partial\Phi^i\partial\bar\Phi^{\bar{k}}$ and the component fields are defined through the superspace derivatives introduced earlier:

$$\begin{aligned}
\phi^i &= \Phi^i \mid, \quad \chi^i_\alpha = \tfrac{1}{\sqrt{2}}\mathcal{D}_\alpha \Phi^i \mid, \quad F^i = -\frac{1}{4}\mathcal{D}^2\Phi^i \mid, \\
\bar\phi^{\bar{k}} &= \bar\Phi^{\bar{k}} \mid, \quad \bar\chi^{\bar{k}\dot\alpha} = \tfrac{1}{\sqrt{2}}\mathcal{D}^{\dot\alpha}\bar\Phi^{\bar{k}} \mid, \quad \bar{F}^{\bar{k}} = -\frac{1}{4}\bar{\mathcal{D}}^2\bar\Phi^{\bar{k}} \mid .
\end{aligned} \tag{3.9}$$

(b) a term describing the coupling of Yang-Mills fields to supergravity through a single function $f_{(ab)}(\Phi)$ holomorphic in the chiral superfields:

$$\begin{aligned}
\mathcal{L}_{YM} &= \frac{1}{8g^2}\int d^4\theta \frac{E}{\mathcal{R}} f_{(ab)}(\Phi) W^{\alpha(a)} W^{(b)}_\alpha + h.c. \\
&= \frac{1}{4g^2}\int d^2\Theta \mathcal{E} f_{(ab)}(\Phi) W^{\alpha(a)} W^{(b)}_\alpha + h.c.
\end{aligned} \tag{3.10}$$

(c) a potential term describing the nonderivative interactions of the chiral matter through a single holomorphic function, the superpotential $W(\Phi)$:

$$\begin{aligned}
\mathcal{L}_{pot} &= \frac{1}{2}\int d^4\theta \frac{E}{\mathcal{R}} e^{K/2} W(\Phi) + h.c. \\
&= \int d^2\Theta \mathcal{E} e^{K/2} W(\Phi) + h.c.
\end{aligned} \tag{3.11}$$

Using the transformation laws (3.1) and (3.2) for $\mathcal{R}$ ($\omega(\mathcal{R}) = 2$), one checks that in order for $\mathcal{L}_{pot}$ to be invariant under a Kähler transformation, $W(\Phi)$ has to transform according to[21] ($\int d^4\theta E$ is invariant; likewise $d^2\Theta$ has weight (-2))

$$W'(\Phi) = W(\Phi)e^{-F} \tag{3.12}$$

This is possible only if $W(\Phi)$ is a monomial fonction of the Φ fields as in the case of interest to us (cubic). One should note that the transformation (3.12) respects the holomorphicity of the superpotential: in other words, Kähler transformations on the Φ fields are holomorphic, as it should for fields parametrizing a Kähler manifold. On the other hand, $e^{K/2}W(\Phi)$ has a definite chiral U(1) weight (i.e. satisfies (3.2) with $\omega = 2$) and thus satisfies the chiral superfield constraint:

$$\begin{aligned} 0 = \mathcal{D}^{\dot\alpha}(e^{K/2}W) &= (\hat{\mathcal{D}}^{\dot\alpha} + 2A^{\dot\alpha})e^{K/2}W \\ &= (\frac{1}{2}\mathcal{D}^{\dot\alpha}K + 2A^{\dot\alpha})e^{K/2}W + e^{K/2}\hat{\mathcal{D}}^{\dot\alpha}W \\ &= e^{K/2}\hat{\mathcal{D}}^{\dot\alpha}W, \end{aligned} \tag{3.13}$$

where we have used (3.4). We see that (3.13) expresses only the fact that W is chiral in the usual superspace sense.

Using a specific Kähler transformation ($F = \ln(W)$), one immediately checks that the Lagrangian depends on the single function $\mathcal{G} = K + \ln|W|^2$ (apart from $f_{(ab)}$):

$$\mathcal{L}_{pot} = \int d^2\Theta\mathcal{E}e^{\mathcal{G}/2} + h.c. \tag{3.14}$$

We will not use this formulation here however because Kähler invariance is no longer manifest, since the "gauge" is then fixed by the particular choice made for F. Also the comparison with couplings obtained from string scattering amplitudes is much more straightforward in our original formulation (which is sometimes called for this reason the string basis, to be distinguished from the supergravity basis in (3.14) used by Cremmer *et al.*[18]). One should pay attention to the fact that ,when going from one basis to the other, nonholomorphic factors appear, such as $(\bar{W}/W)^{\omega/4}$ (take (3.2) with $F = \ln W$).

It is important to stress that, apart from this minor modification, the component field Lagrangian obtained from (3.7),(3.10) and (3.11) coincides with the Lagrangian of Cremmer *et al.*[18], without any further redefinition of the component fields[19,20]. The component fields defined in (3.9) are therefore identical to the ones used in Ref.18 and the superfield formalism discussed here <u>is</u> the one which corresponds to the usual component field Lagrangian (although, from a global supersymmetry point of view, it looks rather unfamiliar).

4 Effective Lagrangian describing gaugino condensation.

We have seen in Section 2 that the $SL(2,\mathcal{R})$ invariance of the Lagrangian acts as a custodial symmetry which prevents the information of local supersymmetry breaking in the hidden sector ($m_{3/2} \neq 0$) from being transferred to the observable sector: global supersymmetry remains unbroken. If this was the end of the story, it would be disappointing since no realistic model could be built on such a statement. Luckily enough however, this symmetry turns out to have an anomaly which provides the source of noninvariance that we are looking for. This allows the subsequent breaking of global supersymmetry in the observable sector. Our purpose in this section is to make these statements quantitative and to write the effective theory below the scale of gaugino condensation, including the contribution of the anomaly, i.e. including the terms breaking the $SL(2,\mathcal{R})$ symmetry and leading to supersymmetry breaking in the observable sector. In order to do so, we will use an effective Lagrangian approach which will allow us to implement the effects of the anomalous behaviour of $SL(2,\mathcal{R})$. In fact, we will adapt the method used by Veneziano and Yankielowicz[22] in the case of supersymmetric QCD to the case of matter coupled to $N=1$ supergravity as described in the last Section.

We restrict our attention to the gravitational (including the S and T fields) and hidden gauge sectors. The Lagrangian is given by Eqs.(3.7)(3.10): [h]

$$\begin{aligned} \mathcal{L} &= \mathcal{L}_0 + \mathcal{L}^C_{YM} \\ \mathcal{L}_0 &= -3\int d^2\Theta\, \mathcal{E}\,\mathcal{R} + h.c. \\ \mathcal{L}^C_{YM} &= \frac{1}{4}\int d^2\Theta\, \mathcal{E}\, SW^a_\alpha W^\alpha_a + h.c. \end{aligned} \tag{4.1}$$

The S and T fields appear also implicitely through the dependence of the Kähler potential:

$$K = -\ln(S+\bar{S}) - 3\ln(T+\bar{T}). \tag{4.2}$$

We first consider two classical invariances of the Lagrangian[22]:

(i) global chiral $U(1)_R$

This invariance is simply obtained by considering a purely imaginary Kahler transformation $F = 3i\beta$. From the discussion of the preceding Section, it is obviously an invariance of the Lagrangian $\mathcal{L}$ in Eq.(4.1). One obtains immediately the following transformation laws:

$$\begin{aligned} (i) \qquad\qquad \Theta' &= e^{-\frac{3i}{2}\beta}\Theta, \\ W^{\alpha\prime}_a(x,\Theta') &= e^{-\frac{3i}{2}\beta} W^\alpha_a(x,\Theta), \\ \mathcal{R}'(x,\Theta') &= e^{-3i\beta}\mathcal{R}(x,\Theta), \\ S'(x,\Theta') = S(x,\Theta) \quad &, \quad \mathcal{E}'(x,\Theta') = \mathcal{E}(x,\Theta). \end{aligned} \tag{4.3}$$

(ii) global dilatations

One has the following transformations under global dilatations:

$$\begin{aligned} (ii) \qquad\qquad x' = e^r x \quad &, \quad \Theta' = e^{r/2}\Theta, \\ W^{\alpha\prime}_a(x',\Theta') &= e^{-\frac{3}{2}r} W^\alpha_a(x,\Theta), \\ \mathcal{R}'(x',\Theta') &= e^{-r}\mathcal{R}(x,\Theta), \\ S'(x',\Theta') = S(x,\Theta) \quad &, \quad \mathcal{E}'(x',\Theta') = \mathcal{E}(x,\Theta). \end{aligned} \tag{4.4}$$

[h] Except otherwise stated we set $M_{Pl} = 1$ in this section.

The Yang-Mills action obtained from $\mathcal{L}_{YM}$ is invariant under these transformations but the total action is not because of the presence of $\mathcal{L}_0$; as is well-known, nonrenormalizable terms break scale invariance.

We now turn to the full quantum Yang-Mills Lagrangian which we write

$$\mathcal{L}_{YM} = \mathcal{L}^C_{YM} + \mathcal{L}^Q_{YM}. \tag{4.5}$$

In this expression, $\mathcal{L}^Q_{YM}$ represents the sum over all hidden sector gauge multiplet loops generated by the renormalizable couplings contained in $\mathcal{L}^C_{YM}$ (the S supermultiplet does not propagate in this approximation). At the quantum level, both symmetries are anomalous. Indeed, under (i)

$$\delta\mathcal{L}^Q_{YM} = i\frac{\beta}{2}b_0 \int d^2\Theta\, \mathcal{E} W^\alpha_a W^a_\alpha + h.c. = -\frac{\beta}{2}b_0 F\tilde{F} + \cdots \tag{4.6}$$

and under (ii)

$$\delta S^Q_{YM} = \int d^4x\, \delta\mathcal{L}^Q_{YM} = \frac{r}{2}b_0 \int d^4x \int d^2\Theta . \mathcal{E} W^\alpha_a W^a_\alpha + h.c. = -\frac{r}{2}b_0 \int d^4x F^2 + \cdots \tag{4.7}$$

where b_0 is the coefficient of the one-loop beta function for the hidden sector gauge group: $\mu\partial g/\partial\mu = -b_0 g^3$.

Now because of the coupling of S to $W^a_\alpha W^\alpha_a$ in (4.1), one can define generalized chiral $U(1)$ transformations under which S varies and which are nonanomalous:

$$\begin{array}{lrcl} (i') & \Theta' & = & e^{-\frac{3i}{2}\tau}\Theta, \\ & W^{\alpha\prime}_a(x,\Theta') & = & e^{-\frac{3i}{2}\tau}\, W^\alpha_a(x,\Theta), \\ & S'(x,\Theta') & = & S(x,\Theta) + 2b_0 i\tau \\ & \mathcal{E}'(x,\Theta') & = & \mathcal{E}(x,\Theta). \end{array} \tag{4.8}$$

Similarly for the global dilatations, one defines:

$$\begin{array}{lrcl} (ii') & x' = e^t x & , & \Theta' = e^{t/2}\Theta, \\ & W^{\alpha\prime}_a(x',\Theta') & = & e^{-\frac{3}{2}t} W^\alpha_a(x,\Theta), \\ & S'(x',\Theta') & = & S(x,\Theta) + 2tb_0, \\ & \mathcal{E}'(x',\Theta') & = & \mathcal{E}(x,\Theta). \end{array} \tag{4.9}$$

This last invariance is broken by $\mathcal{L}_0$, because of the transformation law of the S-dependent Kähler potential.

Also, Weyl anomalies such as (4.7) usually arise because infinite quantum corrections must be regulated by introducing a cut-off Λ or by specifying a renormalisation scale μ. In the theories that we consider, this scale is Λ_{GUT} which is itself a dynamical variable. Hence one can restore scale invariance by defining the following transformation:

$$\begin{array}{lrcl} (ii'') & x' = e^u x & , & \Theta' = e^{u/2}\Theta, \\ & W^{\alpha\prime}_a(x',\Theta') & = & e^{-\frac{3}{2}u} W^\alpha_a(x,\Theta), \\ & S'(x',\Theta') & = & S(x,\Theta), \\ & T'(x',\Theta') & = & e^{2u} T(x,\Theta), \\ & \mathcal{E}'(x',\Theta') & = & \mathcal{E}(x,\Theta). \end{array} \tag{4.10}$$

The transformation of the T field is chosen in order that Λ_{GUT} (to be precise, we consider here the scalar component of the superfield defined in Eq.(2.25)) transforms with a Weyl weight unity:

$$\Lambda'_{GUT}(x') = e^{-u}\Lambda_{GUT}(x). \tag{4.11}$$

It follows for the Kähler potential (4.2):

$$K'(x', \Theta') = K(x, \Theta) - 6u. \tag{4.12}$$

Hence this generalized transformation amounts to a Kähler transformation with $F = -3u$. One should note however that, since one also transforms the x variable in (4.10), the superpotential need not be transformed, as can be seen from the general form of the potential term (3.11) and:

$$\int d^4x' \int d^2\Theta' e^{K'/2} = \int d^4x \int d^2\Theta e^{K/2}. \tag{4.13}$$

Now that we have studied the invariances of the underlying Yang-Mills theory, we come to describe the effective theory below the scale of condensation. Indeed, we want to construct an effective Lagrangian in terms of a composite chiral superfield which reproduces the behavior of the original Lagrangian (4.1) under the different symmetries. A natural candidate for this chiral superfield would be the gaugino condensate:

$$U = \frac{1}{4} W_a^{\alpha} W_{\alpha}^{a}. \tag{4.14}$$

However, this field does not have the right Kähler transformation properties to be present as such in a superpotential. Indeed, from our discussion of last section, one sees that U transforms as a chiral superfield of weight $\omega = 2$, that is, using Eq.(3.2):

$$U' = U \exp[(\overline{F} - F)/2]. \tag{4.15}$$

On the other hand, we saw that the chiral superfields which are present in the superpotential should transform holomorphically under a Kähler transformation (they parametrize a Kähler manifold) and U cannot thus appear as such in the superpotential. Obviously (see Section 3) the field H which appears in the superpotential is related to U by

$$U = e^{K/2} f(S) H^3 \tag{4.16}$$

where we have chosen a canonical dimension one for H. Indeed, under a Kähler transformation, H transforms holomorphically:

$$H' = H e^{-F/3} \tag{4.17}$$

and it is a chiral field in the usual sense:

$$\begin{aligned} 0 = \mathcal{D}^{\dot\alpha} U &= (\hat{\mathcal{D}}^{\dot\alpha} + 2A^{\dot\alpha})U = (\frac{1}{2}\hat{\mathcal{D}}^{\dot\alpha} K + 2A^{\dot\alpha})U + e^{K/2}\hat{\mathcal{D}}^{\dot\alpha}(f(S)H^3) \\ &= 3e^{K/2} f(S) H^2 \hat{\mathcal{D}}^{\dot\alpha} H. \end{aligned} \tag{4.18}$$

The function $f(S)$, which represents an effective gauge coupling dependence, will soon be determined.

We have seen earlier that the interactions of a chiral superfield are described by

$$\mathcal{L}_{pot} = \int d^2\Theta\, \mathcal{E} e^{K/2}\, W + h.c. \tag{4.19}$$

and corresponding to the decomposition (4.5) of $\mathcal{L}_{YM}$, we have

$$W = W^C + W^Q. \tag{4.20}$$

Now writing U Eq.(4.16) explicitly in (4.1) (using (4.14)), one sees that:

$$W^C = Sf(S)H^3, \tag{4.21}$$

We thus check that W^C depends precisely on the field H whose Kähler transformation is holomorphic (indeed $W^{C'} = W^C e^{-F}$ as it should, cf Eq.(3.12)).

The classical superpotential W^C reproduces the classical invariances of the underlying Yang-Mills theory if

$$(i) \qquad H'(x,\Theta') = e^{-i\beta} H(x,\Theta)\ ,\ K'(x,\Theta') = K(x,\Theta) \tag{4.22}$$
$$(ii) \qquad H'(x',\Theta') = e^{-r} H(x,\Theta)\ ,\ K'(x',\Theta') = K(x,\Theta) \tag{4.23}$$

As for the full potential W, it reproduces the quantum properties of $\mathcal{L}_{YM}$ in (4.6),(4.7) if

$$(i) \qquad W^Q = e^{3i\beta} W^{Q'} + 2i\beta b_0 S^{-1} W^C, \tag{4.24}$$
$$(ii) \qquad W^Q = e^{3r} W^{Q'} + 2r b_0 S^{-1} W^C. \tag{4.25}$$

The two extra terms (proportional to W^C) are the remnants in the effective theory of the chiral and trace anomalies in the underlying gauge theory. The solution of these two equations is

$$W^Q = 2b_0 f(S) H^3 \ln\left(\frac{H}{\mu(S)}\right), \tag{4.26}$$

i.e.

$$W = 2b_0 f(S) H^3 \ln\left(\frac{H e^{S/(2b_0)}}{\mu(S)}\right), \tag{4.27}$$

where $\mu(S)$ is a S-dependent scale to be determined.

In fact, $f(S)$ and $\mu(S)$ are determined by considering the generalized chiral U(1) transformations (i'). Under these transformations,

$$(i') \qquad W^{C'} = e^{-3i\tau}(W^C + 2ib_0\tau S^{-1} W^C) \tag{4.28}$$

whereas the full quantum Lagrangian is invariant (the symmetry is nonanomalous):

$$(i') \qquad W' = e^{-3i\tau} W. \tag{4.29}$$

How does the H field transform under (i')? In principle, since (i') basically amounts to a Kähler transformation $F = 3i\tau$, one would expect to transform H, which appears in the superpotential, accordingly: $H' = He^{-i\tau}$. However, it is (i), not (i'), which corresponds to a Kähler transformation. In the case of (i'), S transforms ($S' = S + 2b_0 i\tau$) and one has to be careful with the transformation law for H since this field of the effective theory incorporates some gauge coupling dependence (the gauge coupling is S dependent, Eq.(2.4)). In fact, we will see later that H is invariant under (i'):

$$(i') \qquad\qquad H' = H. \tag{4.30}$$

Then (4.28) yields an equation for $f(S')$ in terms of $f(S)$ which is solved by:

$$f(S) = \lambda e^{-3S/2b_0} \tag{4.31}$$

where λ is a constant. Eq.(4.29) yields in turn $\mu(S)$

$$\mu(S) = \mu e^{S/2b_0} \tag{4.32}$$

with μ another constant. The superpotential is then fully determined[23]:

$$\begin{aligned} W^C &= \lambda S e^{-3S/2b_0} H^3 \\ W^Q &= 2b_0 \lambda e^{-3S/2b_0} H^3 \ln\left(\frac{H}{\mu} e^{-S/2b_0}\right) \\ W &= 2b_0 \lambda e^{-3S/2b_0} H^3 \ln\left(\frac{H}{\mu}\right). \end{aligned} \tag{4.33}$$

Now, a transformation law such as (4.30) has no meaning as long as we have not determined the Kähler potential which fixes the normalisation of the H field[i]. In order to do so we will use precisely the $SL(2,\mathcal{R})$ symmetry that was discussed at length in Section 2. Since H appears in the superpotential W^C as a cubic term (note that S does not transform under $SL(2,\mathcal{R})$), its transformation law is identical to the one for a Φ field (Eq.(2.18)):

$$H' = \frac{H}{icT + d}. \tag{4.34}$$

We will only need here the infinitesimal form:

$$\delta H = -icTH \quad , \quad \delta T = -icT^2. \tag{4.35}$$

Correspondingly, the form of the Kähler potential is

$$K = -\ln(S + \bar{S}) - 3\ln(T + \bar{T}) + g\left(\frac{|H|^2}{T + \bar{T}},\ S + \bar{S}\right). \tag{4.36}$$

Indeed, $|H|^2/(T + \bar{T})$ and $S + \bar{S}$ are invariant and $U(1)_{nc}$ amounts to a Kähler transformation, as it should,

$$K \to K + F + \bar{F} \quad , \quad F = 3icT. \tag{4.37}$$

[i] Alternatively we could define $\tilde{H} = He^{-S/2b_0}$ which transforms according to $\tilde{H}' = \tilde{H}e^{-i\tau}$. The superpotential is obtained from (4.33) by replacing H by $\tilde{H}e^{S/2b_0}$. It is only the H or $\tilde{H}$ kinetic term i.e. the H, $\tilde{H}$ dependence of the Kähler potential, that determines whether we are dealing with the same model or not.

It is easy to obtain the variation of the potential term under (4.35). From the explicit form (4.33) of W, we have

$$W' = W(1 - 3icT) - icT2b_0S^{-1}W^C \tag{4.38}$$

or

$$W^Q = e^{3icT}W^{Q'} + 2icTb_0S^{-1}W^C. \tag{4.39}$$

Writing $t = p + iq$, we see from (4.24,4.25) that this corresponds to transformations $(i), (ii)$ with respectively $\beta = cp$, $r = -cq$. The fact that we recover the chiral anomaly term is expected since the Kähler transformation amounts to a chiral U(1). It is more surprising to find a trace anomaly term since we never performed a global dilatation. The origin may be found in the nonanomalous transformation (ii''), Eq.(4.10): the trace anomaly can be cancelled by a shift on the cut-off scale. Indeed, in our case,

$$\Lambda'_{GUT} = e^{-cq}\Lambda_{GUT}, \tag{4.40}$$

as can be seen from the explicit form of (2.25) and (4.35). Hence, following (4.10),(4.11), the corresponding term in (4.39) should be interpreted as the opposite of the trace anomaly arising from the global dilatation $x' = e^{cq}x$ (i.e. $r = cq$).

Now, in the limit of vanishing superpotential (for example letting λ go to zero), the full isometry group must be an invariance of the complete Lagrangian[20,21,24]. This fixes the H dependence of the function g since in our model the isometry group becomes $\frac{SU(N+1,1)}{SU(N+1)\otimes U(1)}$:

$$K = -\ln(S + \bar{S}) - 3\ln(T + \bar{T} - k|H|^2), \tag{4.41}$$

with k a function of $S + \bar{S}$.

In order to determine this function, we need to consider the transformation (ii') which is the only one under which $S + \bar{S}$ varies (Eq.(4.9)). It is well-known[25] that this is an invariance of the theory only up to terms of order $g^2 \sim (S + \bar{S})^{-1}$. Hence we require for the effective theory:

$$\left(e^{K/2}W^C\right)' = e^{-3t}e^{K/2}[W^C + 2tb_0S^{-1}W^C] + O(g^2), \tag{4.42}$$

with H invariant under (ii') as in (4.30). This in turn imposes that δK be itself of order g^2. But

$$\delta K = \left[-\frac{1}{S+\bar{S}} - \frac{3|H|^2}{T+\bar{T}-k(S+\bar{S})|H|^2}\frac{\partial k}{\partial(S+\bar{S})}\right]4tb_0. \tag{4.43}$$

To leading order in g^2, the only reasonable solution is $\partial k/\partial(S + \bar{S}) = 0$, i.e. a mere normalisation constant in front of the $|H|^2$ term in (4.41).

We may now come back to the transformation law (4.30) for the H field. Had we started with a different transformation (say $\tilde{H}' = \tilde{H}e^{-i\tau}$), we would have obtained a different form for the superpotential but also for the Kähler potential (in this case $K = -\ln(S + \bar{S}) - 3\ln(T + \bar{T} - e^{(S+\bar{S})/2b_0}|\tilde{H}|^2)$). The two formulations are obviously equivalent: they correspond to a simple redefinition of the H field (cf. the last footnote).

It is important to check in the end that our results make good physical sense. Indeed, we are now going to see that they have a straightforward physical interpretation. Let us start with our determination of the function $f(S)$ in (4.31). Substituted in Eq.(4.16), this yields:

$$U = \lambda H^3 e^{K/2} e^{-3S/2b_0}. \tag{4.44}$$

Thus writing K explicitly (Eq.(4.41)) and using then (2.3),(2.4) and (2.11),

$$< U > \sim < Res \frac{M_{Pl}^3}{(Res Ret)^{3/2}} e^{-3(s+\bar{s})/4b_0} > = \frac{1}{g^2}\Lambda_c^3. \tag{4.45}$$

On the left-hand side we recognize what the authors of Ref.10 call Λ^{2-loop} (the $1/g^2$ factor is the contribution of the bosonic zero modes) and we can write Eq.(4.45) simply as [j]

$$< \bar{\lambda}\lambda > \sim (\Lambda^{2-loop})^3. \tag{4.46}$$

We are now able to interpret the argument of the logarithm which appears in the quantum piece of the effective superpotential W^Q (Eq.4.33). Using Eq.(4.44),

$$\frac{H}{\mu} e^{-S/2b_0} = \left(\frac{U}{\lambda \mu^3 e^{K/2}}\right)^{1/3}, \tag{4.47}$$

which yields together with Eq.(4.45)

$$\begin{aligned} < \frac{H}{\mu} e^{-s/2b_0} > &= \frac{1}{\mu \lambda^{1/3}} \left(\frac{\Lambda_c^3}{g^2 < Res > M_{Pl}^3 / < Res Ret >^{3/2}}\right)^{1/3} \\ &= \frac{1}{\mu\lambda^{1/3}} \frac{\Lambda_c}{M_{GUT}}. \end{aligned} \tag{4.48}$$

This last expression calls for some important comments:
(i) the scale dependence is completely taken care of by the S and T dependence of the expression (note the importance of the presence of the factor $e^{K/2}$ in (4.44) which takes its origin from Kähler covariance): it thus appears clearly that μ is a mere normalization constant. We will see below to which physical quantity we can relate it.
(ii) we recover in (4.48) the presence of the natural cutoff in the theory i.e. M_{GUT} [4,26]; indeed it is clear that W^C integrates the quantum effects arising from fluctuations of scales μ in the region $\Lambda_c \leq \mu \leq M_{GUT}$; the importance of such a property was discussed at length in Ref.4.
(iii) it is a welcome feature of (4.48) that all explicit S dependence has dropped out.[k] Indeed it is easy to show that S^{-1} is the loop expansion parameter in the 4-dimensional theory; since the tree level Lagrangian expressed in string units is proportional to S, there should be no explicit S dependence at the one-loop level.

[j] A remark on normalizations: our gaugino fields are not properly normalized and we have $< \bar{\lambda}\lambda > = < g^2 (\bar{\lambda}\lambda)_N >$ where the N subscript refers to gauginos with normalized kinetic terms such as the ones used by the authors of Ref.10.

[k] We wish to thank G.Veneziano for making this comment and for discussions that prompted us to look at this point more carefully.

All these arguments make us confident that the results (4.33) and (4.41) accurately describe the theory below the scale of condensation. We can indeed summarize our results in a way which is self-explanatory. Using (4.33),(4.44) and (4.48), we can write:

$$< e^{K/2}W >= \left[\frac{1}{g^2} + b_0 \ln\left(\frac{\Lambda_c^2}{M_{GUT}^2}\frac{1}{\mu^2\lambda^{2/3}}\right)\right] < U > . \tag{4.49}$$

We recognize the renormalisation of the gauge coupling (2.4) from the scale M_{GUT} down to Λ_c.

Restoring the nonsinglet fields Φ^i and the **c** contribution, the functions that determine the tree level potential become:

$$\begin{aligned} K &= -\ln(S+\bar{S}) - \ln(T+\bar{T}-|\Phi|^2-|H|^2) \\ W &= \mathbf{c} + 2b_0\lambda H^3\left(\ln\frac{H}{\mu}\right)e^{-3S/2b_0} + W(\Phi). \end{aligned} \tag{4.50}$$

This potential consists of a sum of positive definite terms:

$$V = V_S + \sum_i \hat{V}_i + \hat{V}_H + D-\text{terms}, \tag{4.51}$$

with (cf. Eq.(2.8))

$$V_S = e^K\left|\mathbf{c} + 2b_0\lambda h^3\left(\ln\frac{h}{\mu}\right)\left(1+\frac{3}{2b_0}(s+\bar{s})\right)e^{-3s/2b_0} + W(\Phi)\right|^2 \tag{4.52}$$

$$\hat{V}_i = \frac{1}{3}\frac{1}{(s+\bar{s})(t+\bar{t}-|\Phi|^2-|h|^2)^2}\left|\frac{\partial W}{\partial\Phi^i}\right|^2 \tag{4.53}$$

$$\begin{aligned} \hat{V}_H &= \frac{1}{3}\frac{1}{(s+\bar{s})(t+\bar{t}-|\Phi|^2-|h|^2)^2}\left|\frac{\partial W}{\partial H}\right|^2 \\ &= \frac{1}{3}\frac{1}{(s+\bar{s})(t+\bar{t}-|\Phi|^2-|h|^2)^2}4b_0^2\lambda^2|h|^4e^{-3\frac{s+\bar{s}}{2b_0}}\left|3\ln\frac{h}{\mu}+1\right|^2. \end{aligned} \tag{4.54}$$

The ground state for h, the scalar component of H, is reached for:

$$< h > = \mu e^{-1/3}, \tag{4.55}$$

and replacing h by its vev in (4.52), which amounts to truncating the theory, one recovers the tree level Lagrangian used in Section 2, i.e. Eq.(2.15) with

$$\mathbf{h} = -\frac{2}{3e}b_0\lambda\mu^3. \tag{4.56}$$

We see that **h** is cubic in the *vev* of the H field. Its transformation law under $SL(2,\mathcal{R})$ intuitively obtained in Section 2 (Eq.(2.23)) is thus consistent with the transformation law for the H field (Eq.(4.34)).

We take this opportunity to justify the transformation law for **c** in Eq.(2.23). In order to do so, we have to place ourselves in the underlying 10-dimensional theory since the field H_{lmn} whose *vev* is **c** (Eq.(2.13)) does not propagate in the 4-dimensional spacetime. It proves easier to work in string units i.e. $M_S = 1$ because the 10-dimensional Lagrangian then takes a very simple form[27]:

$$\frac{1}{e}\mathcal{L}^{(10)} = \phi^{-3}\left(-\frac{3}{4}H_{LMN}H^{LMN} + \dots\right). \tag{4.57}$$

where the dilaton ϕ is expressed in terms of the 4-dimensional fields s and t by:

$$\phi = \frac{Ret}{(Res)^{1/3}}. \tag{4.58}$$

Since $\mathcal{L}^{(10)}$ is necessarily invariant under $SL(2,\mathcal{R})$ (this transformation appears only at the level of compactification), $< H_{lmn}H^{lmn} > \sim |\mathbf{c}|^2$ transforms as $\phi^3 \propto (Ret)^3$ (s is invariant):

$$|\mathbf{c}'| = \left|\frac{\mathbf{c}}{(ict+d)^3}\right|, \tag{4.59}$$

in agreement with the transformation law (2.23). Note that one could have infered in a similar way the transformation law for **h** by looking at the quartic gaugino terms in the 10-dimensional Lagrangian.

Finally, we return to the ground state identification described in Section 2 and argue why we wrote a minimization condition for **h**, i.e. for μ. It turns out that, in contrast to the case of supersymmetric QCD[22], the *vev* $< F^2 >$ of the hidden sector gauge field does not automatically vanish, but itself depends in a nontrivial way on the parameter μ. Although F^2 appears only as a nonpropagating auxiliary field in the effective composite theory, it is an independent degree of freedom of the underlying theory, and its *vev* should relax to a value that minimizes the total vacuum energy. This is what is taken care of when minimizing with respect to μ, or alternatively **h**.

To see this more explicitly, let us compute F^U, the F-component of the U superfield, which reads (cf (3.9) and (3.3,3.4), $\omega(U) = 2$)

$$\begin{aligned} F^U &= -\frac{1}{4}(\hat{\mathcal{D}}^\alpha + A^\alpha)(\hat{\mathcal{D}}_\alpha + 2A_\alpha)U| \\ &= (h^3 K_a F^a + 3h^2 F^H - \frac{3}{2b_0}h^3 F^S)\lambda e^{K/2} e^{-3s/2b_0} + \text{fermion terms}, \end{aligned} \tag{4.60}$$

with the index a running over the $\Phi^a = \Phi^i, S, T, H$ superfields and ($\omega(\Phi^a) = 0$)

$$F^a = -\frac{1}{4}\hat{\mathcal{D}}^2\Phi^a|. \tag{4.61}$$

On the other hand using the definition of U (Eq.(4.14)),we have

$$F^U = -\frac{1}{8}(F^2 - iF\tilde{F}) + \cdots \tag{4.62}$$

Hence $< F^2 >= -8 < F^U >$ is given by (4.60). In this equation, the F^H term vanishes at the h ground state: $< F^H >= 0$ for any value of μ. This is the only contribution in supersymmetric QCD since the first term vanishes in the flat space limit. In our case, $< F^2 >$ depends on μ through the $< F^S >$ term. Already at tree level, if $|\mathbf{c}/\mathbf{h}| \geq 1.2$, the vacuum energy is positive definite; $< V > \propto | < F^S > |^2$ is nonzero and depends on μ. In this case, we expect μ (i.e. $F_{\mu\nu}F^{\mu\nu}$) to relax to a value such that $|\mathbf{c}/\mathbf{h}| \leq 1.2$, so that the vacuum energy is minimized. Once this condition is satisfied, $< F^S >$ vanishes identically and one has to go to the next order. At the one loop level, the relation (4.60) is modified and we expect a corresponding shift in $\mathbf{h}$, i.e. in $< F^2 >$, so as to minimize the one-loop corrected vacuum energy.

5 Connection with spacetime duality. Conclusions.

We have restricted the preceding discussion to the study of a simple compactification model. But the central role played by the $SL(2,\mathcal{R})$ symmetry makes us think that our results can be generalized to a large class of models. The reason is that this symmetry is the continuous version of the spacetime duality which is observed in an increasing number of string models[28]. In the simple case of one dimension compactified on a circle of radius R, duality is associated with the invariance of the string spectrum under the operation: $R \to \sqrt{\alpha'}/R$. In our case, it is described at the level of the effective field theory by the discrete group $SL(2,\mathcal{Z})$, under which the fields transform as in Eq.(2.18), with $a, b, c, d \in \mathcal{Z}$[1,29]. The key point is to understand how the discretization of this symmetry emerges.

We first note that, in our formalism, the continuous $SL(2,R) \otimes U(1)_R$ symmetry is apparently broken by anomalies to a continuous $U(1)_{PQ}$ symmetry: $T \to T + i\gamma$ (in other words, there is no T dependence in the superpotential). However this residual invariance under $U(1)_{PQ}$ is due to the fact that we have neglected the nonrenormalizable couplings of the hidden Yang-Mills supermultiplet in our parametrization of the effects of anomalies. For example, the coupling of the T field to the axial current through the Kähler connection[19]:

$$\Gamma_\mu = \frac{3i}{4}\frac{1}{t+\bar{t}-|\phi|^2}[\partial_\mu(t-\bar{t}) + \varphi^i\partial_\mu\bar{\varphi}^{\bar{i}} - \bar{\varphi}^{\bar{i}}\partial_\mu\varphi^i]$$

will induce its coupling to $F_{\mu\nu}\tilde{F}^{\mu\nu}$ via triangle diagrams of the type in Fig. 2, analogous to those responsible for pion decay to two photons in QCD. Thus we would expect corrections of the form

$$\delta\mathcal{L}_{YM} = \Delta(T)W^\alpha W_\alpha, \tag{5.1}$$

that is, a $U(1)_{PQ}$ noninvariant, T-dependent correction to the (necessarily holomorphic) gauge normalization function. This type of term was actually considered some time ago by Ibáñez and Nilles[30], using arguments based upon the supersymmetrization of anomaly-cancelling terms (in their analysis, $\Delta(T) = \epsilon T$). Coming back to the interpretation of our results, this extra contribution should be added to the ones discussed in Section 4 and for example Eq.(4.49) should read

$$< e^{K/2}W >= \left[\frac{1}{g^2} + b_0 \ln\left(\frac{\Lambda_c^2}{M_{GUT}^2}\frac{1}{\mu^2\lambda^{2/3}}\right) + < \Delta(T) >\right] < U >. \tag{5.2}$$

We recognize what Kaplunovsky[31] calls the threshold correction, i.e. the finite correction terms which arise at one loop due to the contribution of heavy modes. This threshold correction has actually been recently computed in a class of orbifold models[32] where one finds:

$$\Delta(T) = b_0 \ln[\eta(T)^4], \tag{5.3}$$

where $\eta(T)$ is the Dedekind eta function.

This allows us to understand how the discrete symmetry $SL(2, \mathcal{Z})$ emerges in our analysis. Indeed the full expression (5.2) amounts to adding the threshold correction to the effective superpotential in the following way[34]:

$$\begin{aligned} W_{1\text{loop}} &= \mathbf{c} + \lambda e^{-3S/2b_0} H^3 \left[2b_0 \ln\left(\frac{H}{\mu}\right) + \Delta(T)\right] + W(\Phi) \\ &= \mathbf{c} + 2b_0 \lambda e^{-3S/2b_0} H^3 \ln\left[\frac{H}{\mu}\eta(T)^2\right] + W(\Phi). \end{aligned} \tag{5.4}$$

As a modular function, the Dedekind function transforms under (2.18) as:

$$[\eta(T)]' \sim (icT + d)^{1/2}\eta(T), \tag{5.5}$$

only for $a, b, c, d \in \mathcal{Z}$. Thus under $SL(2, \mathcal{Z})$, the full effective superpotential transforms as:

$$(W_{1\text{loop}})' = \frac{1}{(icT + d)^3} W_{1\text{loop}}, \tag{5.6}$$

and the theory remains invariant, as it should (it has been recently proved that duality remains a good symmetry to all orders of perturbation theory[33]). On the other hand, $SL(2, \mathcal{R})$ remains broken by the anomalous terms discussed in Section 4.

The argument can actually be turned around to show that the only possible form for the threshold correction is precisely (5.3)[34]. The basic idea[35,29] is to study how the threshold correction $\Delta(T)$ must transform under $SL(2, \mathcal{R})$ in order to make the full one-loop order correction invariant. This function $\Delta(T)$ being holomorphic, one concludes from its transformation law that it must be expressed in terms of a modular function, which restricts the invariance to the discrete $SL(2, \mathcal{Z})$.

We may now comment on the implications of these results on our analysis of Section 2. When we solve for the *vev* of the H field, we obtain a result similar to Eq.(4.55), except for an extra T dependence through a modular function. This means that in the truncated theory studied in Section 2, the flat direction encountered is replaced by a discrete set of ground states. This "quantization" of the line of minima should be put in parallel with the other quantization condition in the problem, the one found for $\mathbf{c}$[12]. One may hope indeed that this condition is also connected with duality (remember that, order by order, the minimization of the potential yields a relation between **c** and **h**).

Finally, let us come back to the issue of supersymmetry breaking. Our proof that no soft supersymmetry breaking term is generated in the observable sector rests on the continuous $SL(2,\mathcal{R})$ symmetry. The anomalous terms discussed in Section 4 break this custodial symmetry and provide the theory with a seed for soft supersymmetry breaking.[l] It was shown in Ref.1 that this is actually sufficient to generate gaugino masses in the observable sector, in a mass range compatible with the large hierarchy observed in nature (M_W/M_{Pl}). Details of the computation will be presented in Paper (II). Let us just stress here that the analysis goes in two steps. Starting from the effective Lagrangian derived in Section 4, which describes the model (call it $\mathbf{M_1}$) just below the condensation scale:

(i) at low energies, the H superfield is heavy and one has to integrate over the corresponding degrees of freedom.[m] One obtains an effective low energy model ($\mathbf{M_2}$) valid at scales much below Λ_c. No soft supersymmetry breaking terms are generated in this way at tree level but, of course, there are some relics of the noninvariance present in the nonrenormalisable terms.

(ii) nonzero gaugino masses are obtained at the one loop level in this effective model $\mathbf{M_2}$. No scalar mass or A-term are generated to this order (at least if one does not take into account the duality-restoring threshold correction term discussed above). The fact that these gaugino masses are obtained through anomalous terms and two steps of radiative corrections generates in the final answer a large number of mass ratios. More explicitly, we find

$$m_{\tilde{g}} \sim \frac{M_{Pl}}{(16\pi^2)^2}\left(\frac{\Lambda_c}{M_{Pl}}\right)^5\left(\frac{m_{3/2}}{\Lambda_c}\right)\left(\frac{m_H}{\Lambda_c}\right)^2, \tag{5.7}$$

where m_H is the mass of the H supermultiplet. The small hierarchy that one finds in the breaking of soft supersymmetry (typically $m_{3/2} \sim \Lambda_c \sim 10^{-2}M_{Pl}$) is therefore dramatically enhanced and can easily account for the 16 orders of magnitude encountered in nature. The fact that this hierarchy of scales is related to the manifestation of duality in the effective field theory seems to us particularly encouraging.

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Note added: After completion of this paper, we received a preprint by A. Font, L. E. Ibáñez, D. Lüst and F. Quevedo (CERN-TH.5726/90) which treats some of the issues discussed in Section 5.

[l] The fact that the discrete symmetry $SL(2,\mathcal{Z})$ is restored by the threshold coefficient does not play a role here.

[m] Just truncating the theory (i.e. setting $h = \chi^H = 0$) would kill the anomalous terms: this is actually just what we did in Section 2 to obtain the globally supersymmetric theory studied there.

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Figure captions.

Fig.1: Form of the one-loop effective potential corresponding to Eqs.(2.5) and (2.14), as a function of $\mathbf{c}/\pi$ and Ret[4].

Fig.2: Triangle diagrams inducing a coupling of ImT to $F_{\mu\nu}\tilde{F}^{\mu\nu}$ (the fermions involved in the loop are the gauginos, the gravitino and χ^S).

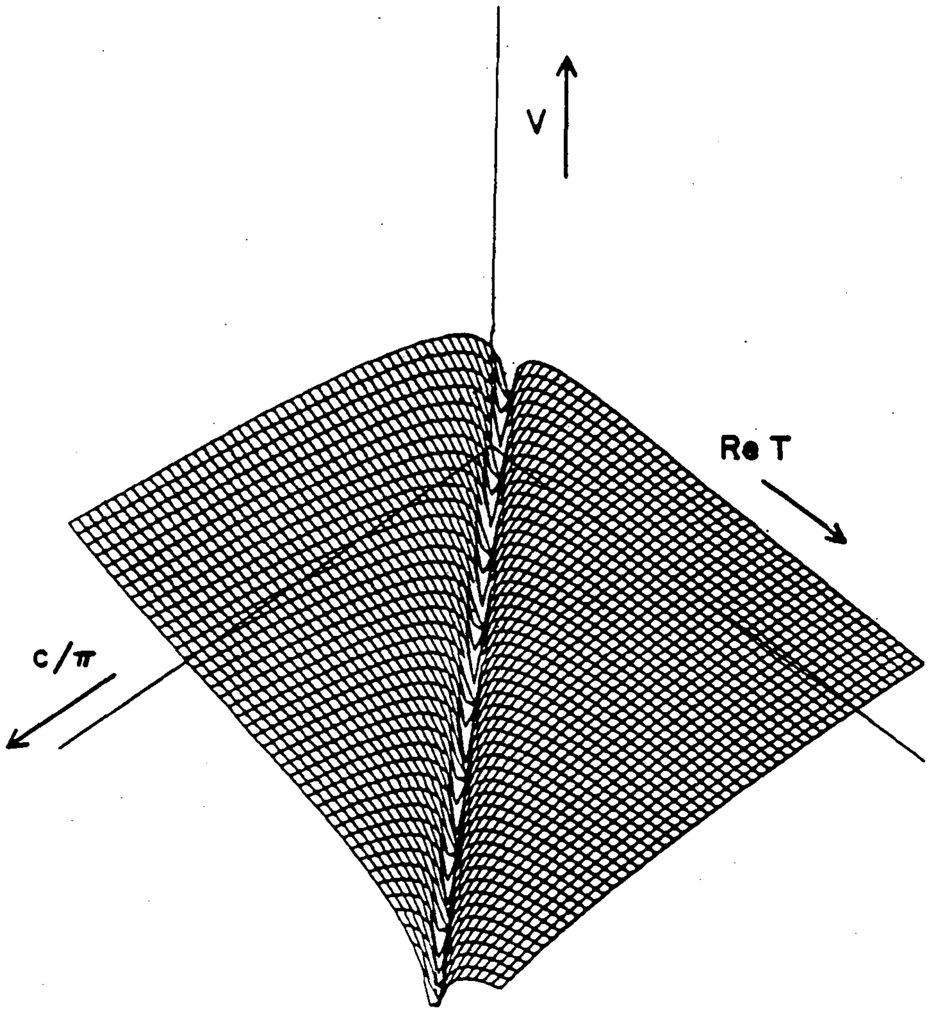

Fig.1

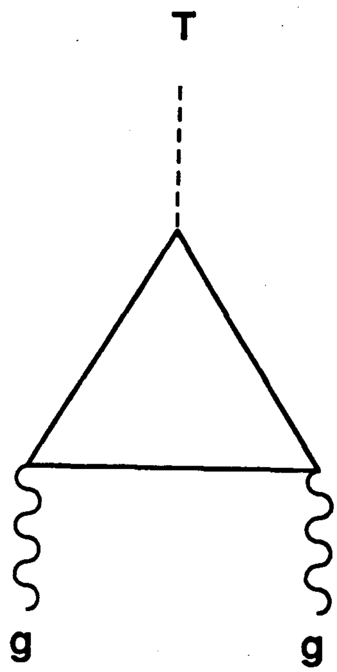

Fig.2