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Heiner Meldner

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PREDICTIONS OF NEW MAGIC REGIONS AND MASSES FOR SUPER HEAVY NUCLEI FROM CALCULATIONS WITH REALISTIC SHELL MODEL SINGLE PARTICLE HAMILTONIANS

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ABSTRACT

The single particle part of a realistic shell model Hamiltonian is considered to contain a nonlocal potential similar to the classic Van Vleck nuclear potential. Using an ansatz for the Hamiltonian suggested by rather general treatments of many-fermion systems it was possible to get a consistent fit to 1) nuclear charge density distributions including isotope effects, 2) 1s proton levels as measured in (e,e'p) scattering, 3) total binding energies or nuclear mass defects, while 4) reproducing the shell model level sequences, i.e. spin assignments. Extrapolations to nuclei off the stability valley and to super heavy nuclei appear to be reasonable. In the latter case some evidence was found for a double magic nucleus at proton number 114 and neutron number 184 with a total binding energy around 2200 MeV. There is no indication of a double magic nucleus with Z=126, N=184 close to the line of beta stability. However, some support was found for a relatively long-lived compound nucleus with these nucleon numbers by investigating metastable proton states.

Nuclear single particle Hamiltonians with <u>local</u> potentials (e.g. of the Woods-Saxon or Nilsson type) satisfactorily reproduce the experimental spin sequences and equilibrium deformations but cannot account for the deepest bound proton levels observed in recent (e,e'p) scattering measurements 1). These levels are around 80 MeV in the region of Calcium. Also it is well known that calculations with such local potentials lead to total nuclear binding energies that are at least one order of magnitude smaller than the observed mass defects 2). A measure of the nonlocality required to reproduce these data is the so-called effective mass of a nucleon subject to the short range forces of a residual nucleus. It was found to be around 0.5 in many investigations 3,2). An effective mass of this magnitude corresponds to a nonlocality range of 0.8±0.2 fm or 1.5±0.2 fm respectively for Yukawa or Gaussian form factors in nonlocal potentials of the classic Van Vleck type 4).

As is discussed in detail elsewhere 5), the simplest reasonable equation for nuclear single particle wave functions φ_ν has the form

$$(1)\quad \left(\frac{\hbar^2}{2m}\frac{\partial^2}{\partial\vec{r}^2} - \varepsilon_\nu\right)\varphi_\nu(\vec{r}) = \int_{(\infty)} d^3\vec{r}'\, u(\vec{r},\vec{r}')\left[1 + \sigma b_{m_t}^2 \vec{\nabla}_\rho \cdot (\vec{p}\times\vec{s})\right]\left[\rho_{-m_t}\left(\frac{r+r'}{2}\right) + \chi\rho_{m_t}\left(\frac{r+r'}{2}\right)\right]\varphi_\nu(\vec{r}')$$

$$+ \left(\frac{1}{2} - m_t\right)\frac{Z-1}{Z}e^2\int_{(\infty)} d^3\vec{r}'\,\frac{\rho_{-\frac{1}{2}}(r')}{|\vec{r}-\vec{r}'|}\,\varphi_\nu(\vec{r})$$

$$\rho_{m_t}(r) \equiv \sum_k \left|\varphi_{k,m_t}(\vec{r})\right|^2$$

The sum over k includes all occupied levels; k contains all quantum numbers to specify a bound nucleon except the third component m_t of the i-spin. $\vec{\nabla}_\rho \cdot (\vec{p} \times \vec{s})$ is the usual spin-orbit operator 6), and the parameters σ and χ measure respectively the strengths of the spin-orbit and i-spin portions of the kernel. The factor b corresponds to a radius constant; see (5) below. A properly chosen function u will lead to self-consistent densities ρ which fit those measured in pion- and electron-nucleus scattering. For simplicity, only spherically symmetric ρ's are considered in (1).

In the results presented here, the output values ρ agreed with the corresponding input data for central density and slope (or rms radius) to within a few percent. Iterative calculations designed to obtain complete self-consistency are in progress. Here the u was taken to be a sum of two Yukawa functions; hence one has for a spherical nucleus the equations

$$(2)\quad u(\vec{r},\vec{r}\,') = \sum_{i=1}^{2} V_i \frac{e^{-\frac{|\vec{r}-\vec{r}\,'|}{a_i}}}{|\vec{r}-\vec{r}\,'|}\, a_i \;, \qquad h_\ell(r,r') = 2\pi r r' \int_{-1}^{1} dz\, P_\ell(z)\, u(\vec{r},\vec{r}\,') \;, \quad z = \frac{\vec{r}\vec{r}\,'}{rr'}$$

$$(3)\quad \left[\frac{\hbar^2}{2m}\left(\frac{d^2}{dr^2} - \frac{\ell(\ell+1)}{r^2}\right) - \varepsilon_{n\ell j m_t}\right] u_{n\ell j m_t}(r) = \int_0^\infty dr'\, h_\ell(r,r') \Big\{ \Big[\rho_{-m_t}\Big(\frac{r+r'}{2}\Big) + \chi\, \rho_{m_t}\Big(\frac{r+r'}{2}\Big)\Big]$$

$$+ \sigma \frac{1}{2}\Big(j(j+1) - \ell(\ell+1) - \frac{3}{4}\Big) b_{m_t}^2 \Big[\rho'_{-m_t}\Big(\frac{r+r'}{2}\Big) + \chi\, \rho'_{m_t}\Big(\frac{r+r'}{2}\Big)\Big]\Big\} u_{n\ell j m_t}(r')$$

$$+ \Big(\frac{1}{2} - m_t\Big) \frac{Z-1}{Z} e^2 \Big[\frac{1}{r}\int_0^r dr' r'^2 \rho_{-\frac{1}{2}}(r') + \int_r^\infty dr' r' \rho_{-\frac{1}{2}}(r')\Big] u_{n\ell j m_t}(r)$$

u_{nljm_t} are the usual radial wave functions, and l is the orbital, j the total angular momentum and n the radial quantum number for the nucleon in question. ρ' is the derivative of the density with respect to the argument. The integro-differential equation (3) was solved numerically by a finite-difference method; the eigenvalues and -vectors were determined by use of the "Wielandt inverse iteration" 7).

The input density distributions were taken to be normalized Fermi functions. For such distributions simple approximate expressions (which have accuracies of better than 10^{-4} in all cases discussed here) are available for the half and equivalent radii in terms of the central density $\rho(0)$ and the surface thickness a 8):

$$(4) \qquad \rho_{m_t}(r) = \rho_{m_t}(0)\left[1 + \exp\left((R_{m_t} - r)/a\right)\right]^{-1} , \qquad B_{m_t} = 4\pi\int_0^\infty dr\, r^2 \rho_{m_t}(r)$$

$$(5) \qquad R = b B^{\frac{1}{3}} - \frac{\pi^2 a^2}{3b} B^{-\frac{1}{3}} , \qquad b \equiv \left(\frac{4\pi}{3}\rho(0)\right)^{-\frac{1}{3}}$$

$$(6) \qquad R_{eq} \equiv \left(\frac{20\pi}{3}\int_0^\infty dr\, r^4 \rho(r)\right)^{\frac{1}{2}} = b B^{\frac{1}{3}} + \frac{5\pi^2 a^2}{6b} B^{-\frac{1}{3}} - \frac{7\pi^4 a^4}{8b^3} B^{-1}$$

In (5) and (6) the index m_t is omitted. $B_{\frac{1}{2}}$ is simply the neutron number N and $B_{-\frac{1}{2}}$ the proton number Z.

A crude fit was obtained with the input data of table 1. For some nuclei on the line of beta stability one gets the neutron levels shown in figure 1 and the proton levels of figure 2. The experimental points in the latter figure refer to recent (e,e'p) scattering measurements[1)]. The extrapolations shown in these figures suggest closed shells at N=184 and Z=114. The same result was obtained in simpler calculations with local potentials two years ago[9)]. The hypothetical super-heavy nucleus with these nucleon numbers was investigated in more detail by varying within their reasonable limits the parameters of importance for level sequences. The computer program in addition allows an approximate calculation of metastable levels or resonances, and these are shown as dashed lines in figures 1-4.

In this fit the calculated total binding energies agree with nuclear mass defects to within 5 percent. The charge density distributions reproduce recent electron scattering data[10)] on Ca and Pb with a χ^2 of the same order of magnitude as is obtained in a fit with local potentials[11)]. The Johnson-Teller effect[12)], i.e. larger rms radii for neutron than for proton densities, is exhibited for the heavier nuclei in spite of the decreasing central density of protons.

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FIGURE CAPTIONS

Fig. 1. Neutron single particle levels for spherical nuclei close to the line of beta stability.

Fig. 2. Proton single particle levels for spherical nuclei close to the line of beta stability. The two experimental points give the 1s levels found in (e,e'p) scattering on S^{32} and Ca^{40}.

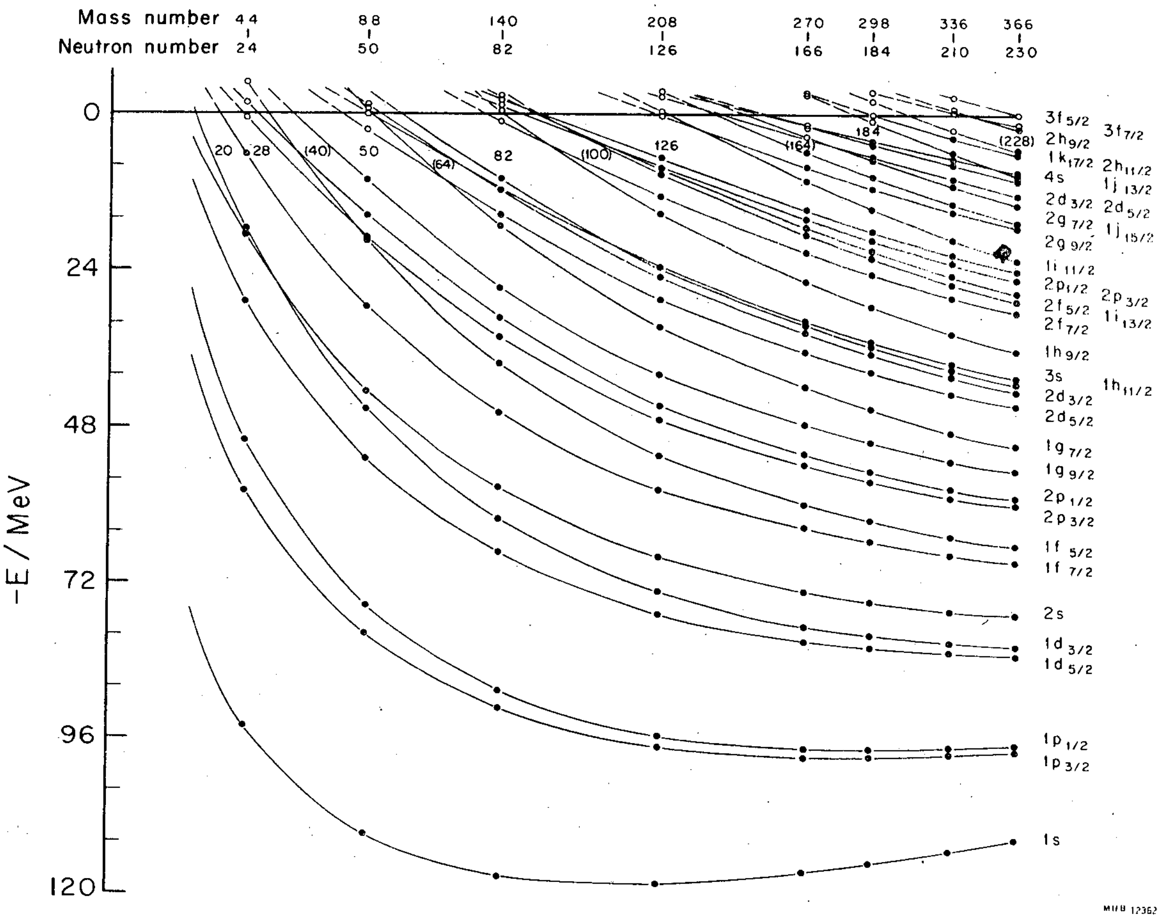
Fig. 3. Neutron level shifts due to variations of the spin orbit interaction strength σ and the surface thickness a for the hypothetical nucleus with $N = 184$, $A = 298$.

Fig. 4. Proton level shifts due to variation of the spin orbit interaction strength σ and the surface thickness a for the hypothetical nucleus with $Z = 114$, $A = 298$.

Table 1

V_1/MeV	V_2/MeV	a_1/fm	a_2/fm	a/fm	σ	χ
30.0	-11.3	0.3	0.9	0.6	0.3	0.6

$4\pi\rho_{\frac{1}{2}}(0)/\mathrm{fm}^{-3} = 1.15$, $\quad 4\pi\rho_{-\frac{1}{2}}(0)/\mathrm{fm}^{-3} = 1.15 - 0.0035 * Z$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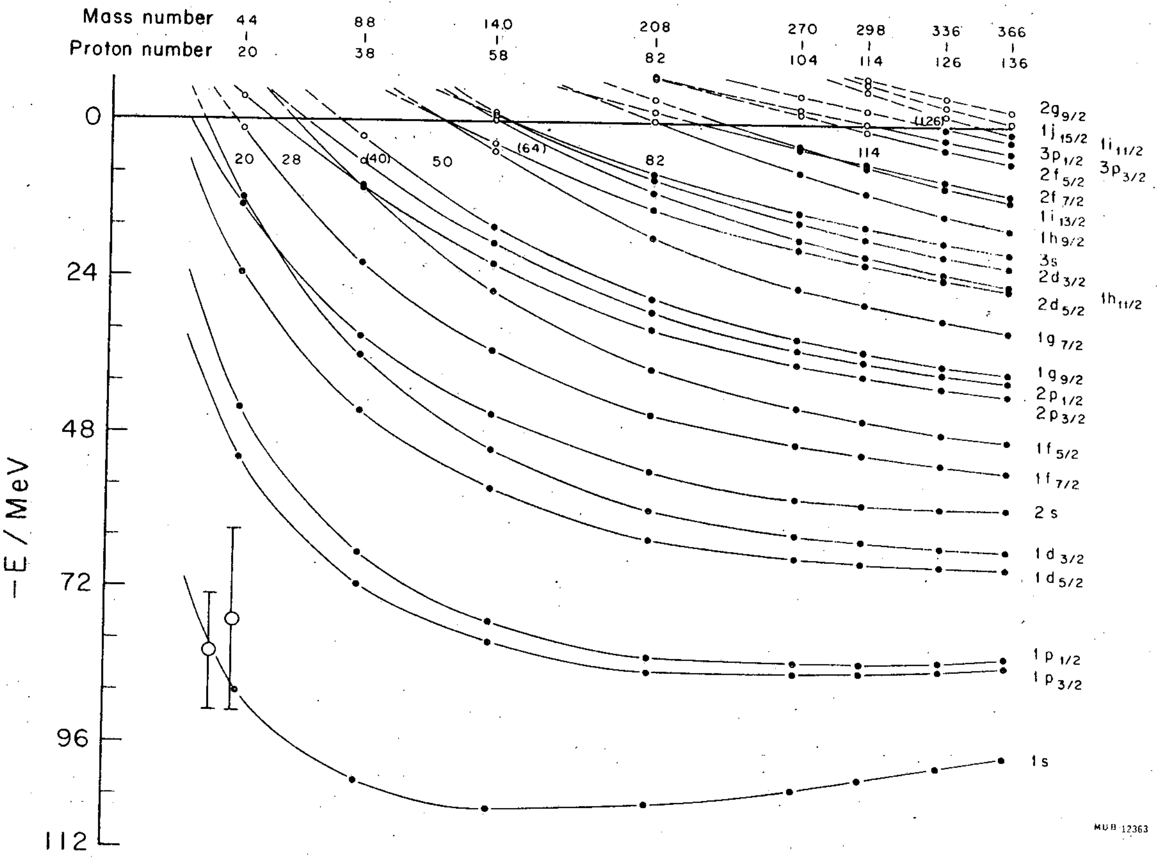
Neutron number 24 50 82 126 166 184 210 230
-E / MeV
0
24
48
72
96
120
20 28 (40) 50 (64) 82 (100) 126 (164) 184 (228)
3f5/2 3f7/2
2h9/2
1k17/2 2h11/2
4s 1j13/2
2d3/2 2d5/2
2g7/2 1j15/2
2g9/2
1i11/2
2p1/2 2p3/2
2f5/2 1i13/2
2f7/2
1h9/2
3s 1h11/2
2d3/2
2d5/2
1g7/2
1g9/2
2p1/2
2p3/2
1f5/2
1f7/2
2s
1d3/2
1d5/2
1p1/2
1p3/2
1s
MUB 12362


Mass number
Proton number
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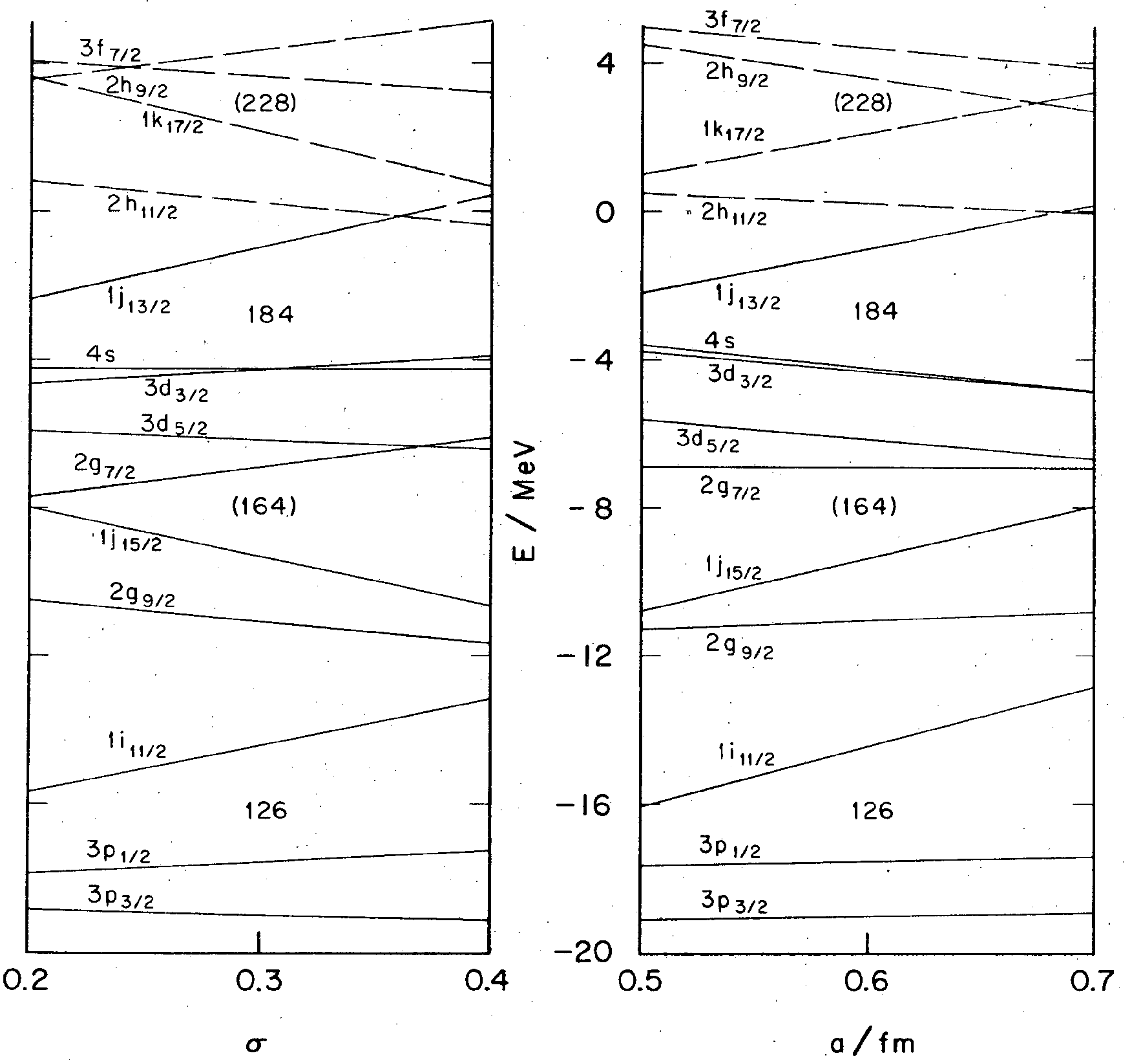
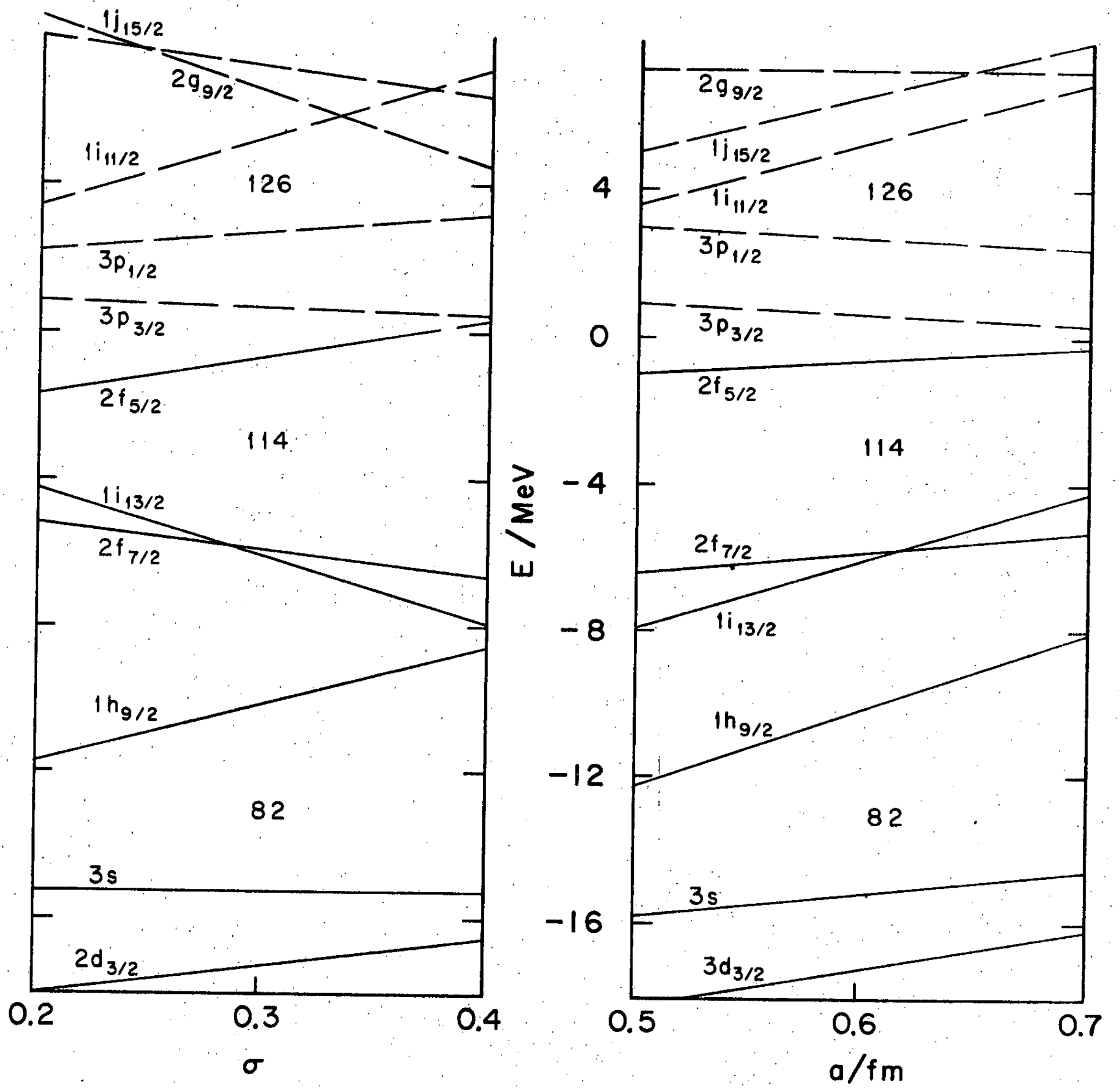
20
88
38
140
58
208
82
270
104
298
114
336
126
366
136
−E / MeV
0
24
48
72
96
112
20
28
(40)
50
(64)
82
114
(126)
2g 9/2
1j 15/2
1i 11/2
3p 1/2
3p 3/2
2f 5/2
2f 7/2
1i 13/2
1h 9/2
3s
2d 3/2
2d 5/2
1h 11/2
1g 7/2
1g 9/2
2p 1/2
2p 3/2
1f 5/2
1f 7/2
2 s
1d 3/2
1d 5/2
1p 1/2
1p 3/2
1s
MUB 12363



3f7/2
2h9/2
1k17/2
(228)
2h11/2
1j13/2
184
4s
3d3/2
3d5/2
2g7/2
(164)
1j15/2
2g9/2
1i11/2
126
3p1/2
3p3/2
4
0
-4
-8
-12
-16
-20
E / MeV
0.2
0.3
0.4
σ
0.5
0.6
0.7
a / fm

MUB-12360


1j15/2
2g9/2
1i11/2
126
3p1/2
3p3/2
2f5/2
114
1i13/2
2f7/2
1h9/2
82
3s
2d3/2
0.2
0.3
0.4
σ
2g9/2
1j15/2
1i11/2
126
3p1/2
3p3/2
2f5/2
114
2f7/2
1i13/2
1h9/2
82
3s
3d3/2
0.5
0.6
0.7
a/fm
4
0
-4
-8
-12
-16
E /MeV

MUB-12361