On the Performance of Optimum Noncoherent Amplify-and-Forward Reception for Cooperative Diversity

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Abstract— **In this paper, we present receiver structures for maximum-likelihood (ML) noncoherent amplify-and-forward (AF) communication links when multiple relay nodes are employed. We consider both on-off keying (OOK) and binary frequency shift keying (BFSK) modulation schemes on Rayleigh fading relay channels with no channel state information. Even for the simple case of having only one relay node, the optimum noncoherent receiver is quite involved, and the ML metric computation requires certain integral evaluations. To lower bound the average bit error rate (BER), we assume that the link between the relay and the destination node is unfaded, a reasonable assumption when there is a strong line-of-sight path between the relay and the destination, and obtain simple closed-form expressions for the average BER with an arbitrary number of relays. Upper bounds on the average BER are also presented by numerically evaluating the Bhattacharyya distance between the likelihood functions. Further, simple suboptimum receiver structures are proposed, for both OOK and BFSK, along with an analytical performance evaluation, and an asymptotic diversity order analysis.**

Keywords: User cooperation, cooperative diversity, relay channels, noncoherent communication.

I. Introduction

Cooperative diversity is attractive for mobile terminals having single-antenna transceivers. A distributed antenna array can be formed by collaboration among M nodes, with a potential to achieve the full diversity order of M. Sendonaris *et al.* in [1] showed that, with transmitter channel state information (CSI), the sum-capacity of an ergodic fading channel can be improved with user cooperation, whereas in [2] Laneman showed that with CSI only at the receiver, the sum-capacity cannot be increased over no-cooperation. The performance of coherent binary PSK (BPSK) signaling with an amplify-and-forward (AF) communication protocol and receiver CSI was studied in [3]. An improved analysis of error probability, using the moment generating function approach, was presented in [4]. Recently, the performance of multi-branch, multi-hop, relay channels was considered in [5] and [6]. The analysis of [3]-[6] showed that with M relay nodes and perfect CSI at the receiver, coherent multi-branch AF reception over independent channels achieves the full diversity order of $M + 1$.

In order to acquire the CSI, the relay channel has to be trained (typically by pilot signaling), which results in a throughput penalty. If the variation of the channel over time is high relative to the signaling duration, then the estimates become outdated. In such a scenario, one is inclined to employ noncoherent detection techniques which do not require knowledge of the instantaneous channel realization. In this context, Chen and Laneman in [7] and [8] studied the performance of noncoherent binary FSK (BFSK) signaling with a decode-and-forward (DF) protocol. They showed that, with M relays, the diversity order achievable with a noncoherent DF protocol is at most $(M/2)+1$ when M is even, and $(M + 1)/2$ when M is odd. That is, with the DF protocol, noncoherent signaling loses approximately half of the available diversity order.

In this contribution, we consider noncoherent communication over Rayleigh fading relay channels with an AF protocol. While neither the relays nor the destination have knowledge of the instantaneous CSI, we assume that the statistical averages of the channel gains are known to them. The amplification gain of the relay is chosen to satisfy an average power constraint. We consider both on-off keying (OOK) and BFSK modulation, and derive the maximum likelihood (ML) noncoherent AF (NCAF) receiver structures at the destination. Unfortunately, even for the case of single relay node, no closed-form expression for the ML NCAF receiver is available, and the ML metric computation requires numerical evaluation of certain integrals. To gain some understanding of the receiver performance, we assume that the relay-to-destination link is unfaded[1]. This is reasonable when there is a strong line-of-sight path from the relay to the destination. With this, we are able to derive simple closed-form expressions for the average bit error rate (BER), with an arbitrary number of relay nodes, that serve as lower bounds on the optimal performance. We derive the upper bounds on the average BER by employing the Bhattacharyya bound [10]. We propose simple suboptimum receivers, for both OOK and BFSK, along with their performance evaluations. We also show that, using asymptotic diversity order analysis [11], with M relay nodes plus a link between the source and the destination, the OOK achieves a diversity order of at least $(M+1)/2$, but never $M+1$, whereas BFSK achieves the full diversity of $M + 1$. However, one of our more surprising results is that for the OOK system, without a relay, the asymptotic diversity analysis predicts a diversity order of less than unity. Since we could not find a good physical interpretation of this result in the context of diversity, it suggests that asymptotic diversity analysis should be used with

[1] A similar assumption is made in [9] for diversity analysis.

caution.

The rest of this paper is organized as follows. We present the system model in Section II. Optimum NCAF receiver structures for OOK and BFSK are formulated in Sections II-A and II-B, respectively. With the assumption that the relay-to-destination link is unfaded, Sections III-A and III-B, respectively, derive the average BER expressions for both OOK and BFSK modulations, whereas Bhattacharyya distance-based upper bounds on the BER are discussed in Section IV. Suboptimum receiver structures are presented in Section V. In Section VI, we study the asymptotic diversity order analysis. Numerical and simulation results are presented in Section VII. Finally, we conclude this work in Section VIII.

II. System Model

Consider a source node S that wishes to communicate with an intended destination node D with the help of M relay nodes, $R_1, \ldots, R_M$. We assume frequency-flat fading on the links between the source and the destination, between the source and the relays, and between the relays and the destination. Let g_1 denote the channel gain on the path from the source to the destination D, and, for $j = 1, \ldots, M$, g_2^j denote the channel gain on the path from the source to the relay node R_j. Also, let g_3^j denote the channel gain on the path from the relay R_j to the destination. We assume that g_1, $\{g_2^j\}$, and $\{g_3^j\}$, $j = 1, \ldots, M$, are zero-mean complex Gaussian random variables (rvs) with variances $E[|g_1|^2] = \Omega_1$, and $E[|g_2^j|^2] = \Omega_2^j$, $E[|g_3^j|^2] = \Omega_3^j$, $j = 1, \ldots, M$, respectively.

The source employs a binary signal constellation, $\mathcal{X}$, and neither the relays nor the destination know the instantaneous channel gains, and hence employ noncoherent demodulation. The relays amplify the signal received from the source to meet their respective average power constraints. The relays and the destination are assumed to know the statistical averages, $\{\Omega_2^j, \Omega_3^j\}_{j=1}^M$ and Ω_1. Our communication protocol is the same as that of [12], where in the first time slot the source broadcasts its signal to the destination and the relays, whereas in the second time slot the relays forward their copies to the destination. That is, if T and W, respectively, denote the message duration and bandwidth required for a single-hop system, then the message duration and bandwidth requirements with M relays are $2T$ and MW, respectively.

Throughout this paper, we employ low-pass equivalent complex baseband signal models so that $\mathcal{X}$, in general, represents a complex-valued constellation. Let $X \in \mathcal{X}$ be the symbol transmitted by the source in the first time slot. Then, the matched-filer (MF) output at the jth relay is

$$r_{RS}^j = g_2^j X + \eta_{RS}^j, \tag{1}$$

where η_{RS}^j is a complex-valued Gaussian noise rv with zero-mean and variance $E[|\eta_{RS}^j|^2] = \sigma_N^2$. The output of the MF at the destination due to the direct link is

$$r_{DS} = g_1 X + \eta_{DS}, \tag{2}$$

where η_{DS} is a complex-valued Gaussian noise rv with zero-mean and variance $E[|\eta_{DS}|^2] = \sigma_N^2$. Relay R_j amplifies the signal r_{RS}^j by a factor A_j, where A_j is chosen to satisfy the constraint $E[|A_j r_{RS}^j|^2] = E_s$. Clearly, $A_j = \sqrt{E_s/(E_s\Omega_2^j + \sigma_N^2)}$. In the second time slot, the relays transmit. The output of the MF at the destination due to the relay R_j is

$$r_{DR}^j = A_j g_3^j r_{RS}^j + \eta_{DR}^j = A_j g_3^j g_2^j X + A_j g_3^j \eta_{RS}^j + \eta_{DR}^j, \tag{3}$$

where η_{DR}^j is a complex-valued Gaussian noise rv with zero-mean and variance $E[|\eta_{DR}^j|^2] = \sigma_N^2$. One of our goals is to derive the optimal receiver structure at the destination, based only on the statistical knowledge of $\Omega_1, \{\Omega_2^j, \Omega_3^j\}_{j=1}^M$. We consider two kinds of binary signal constellations that are amenable to noncoherent detection: 1)OOK modulation, and 2)BFSK modulation.

A. On-Off Keying

With OOK, the signal set is given by $\mathcal{X} = \{0, \sqrt{2E_s}\}$. The signal 0 is transmitted when the bit $b = 0$, whereas $\sqrt{2E_s}$ is transmitted when $b = 1$. The information bits '0' and '1' are assumed equally likely, so the average transmit energy at the output of the source is E_s. When the information bit $b = 1$ is transmitted, the joint pdf of $\{r_{DR}^j\}_{j=1}^M$, and r_{DS}, conditioned on $X = \sqrt{2E_s}$, is given by

$$f_{r_{DR}^1, \ldots, r_{DR}^M, r_{DS}|X=\sqrt{2P}} = E_{g_1}\left[f_{n_{DS}|g_1}\left(r_{DS} - \sqrt{2P}g_1\right)\right] \times \prod_{j=1}^{M} E_{g_2^j, g_3^j}\left[f_{\tilde{n}_{DR}^j|g_2^j, g_3^j}\left(r_{DR}^j - A_j g_2^j g_3^j \sqrt{2P}\right)\right], \tag{4}$$

where $E_U[\cdot]$ denotes the expectation over the rv U, and $\tilde{n}_{DR}^j$, conditioned on g_2^j and g_3^j, is a zero-mean complex Gaussian rv with variance $E[|\tilde{n}_{DR}^j|^2 | g_2^j, g_3^j] = \sigma_N^2(1 + A_j^2|g_3^j|^2)$. Also, $f_{n_{DS}|g_1}(\cdot)$ and $f_{\tilde{n}_{DR}^j|g_2^j, g_3^j}(\cdot)$ are the conditional density functions of n_{DS} and $\tilde{n}_{DR}^j$, respectively. For simplicity, we define $\gamma_1 \triangleq |g_1|^2 E_s/\sigma_N^2$, $\gamma_2^j \triangleq |g_2^j|^2 E_s/\sigma_N^2$, and $\gamma_3^j \triangleq |g_3^j|^2 E_s/\sigma_N^2$, and their respective statistical averages are $\overline{\gamma}_1 \triangleq E[\gamma_1] = \Omega_1 E_s/\sigma_N^2$, $\overline{\gamma}_2^j \triangleq E[\gamma_2^j] = \Omega_2^j E_s/\sigma_N^2$, and $\overline{\gamma}_3^j \triangleq E[\gamma_3^j] = \Omega_3^j E_s/\sigma_N^2$. To proceed further, we need the following lemma:

Lemma 1: If Z is a complex Gaussian rv, having independent real and imaginary parts, with mean $E[Z] = \mathfrak{m}$ and variance $E[|Z - \mathfrak{m}|^2] = \mathcal{N}$, then the expected value of $\exp(-|Z|^2)$ is given by

$$E[\exp(-|Z|^2)] = \frac{1}{1+\mathcal{N}} \exp\left(-\frac{|\mathfrak{m}|^2}{1+\mathcal{N}}\right). \tag{5}$$

Proof: Refer to (Eqn. (2.1-117), [13]). ■

Since $(r_{DS} - \sqrt{2P}g_1)/\sqrt{\sigma_N^2}$ is complex Gaussian with mean $r_{DS}/\sqrt{\sigma_N^2}$ and variance $(2P/\sigma_N^2)E[|g_1|^2] = 2P\Omega_1/\sigma_N^2$, using Lemma 1, the first term in Eqn. (4) simplifies to

$$E_{g_1}\left[f_{n_{DS}|g_1}\left(r_{DS} - \sqrt{2P}g_1\right)\right] = \frac{1}{\pi\sigma_N^2} \times \frac{e^{-\frac{Z_{DS}}{1+2\overline{\gamma}_1}}}{1+2\overline{\gamma}_1}, \tag{6}$$

where $Z_{DS} = |r_{DS}|^2/\sigma_N^2$. Following a similar argument as that for (6), the jth term in the product of the second term in (4) can now be simplified to

$$E_{g_2^j, g_3^j}\left[f_{\tilde{n}_{DR}^j|g_2^j,g_3^j}\left(r_{DR}^j - A_j g_2^j g_3^j \sqrt{2P}\right)\right] = E_{g_3^j}\left[\frac{1}{\pi\sigma_N^2}\frac{e^{-\frac{|r_{DR}^j|^2/\sigma_N^2}{1+A_j^2|g_3^j|^2(1+2\overline{\gamma}_2^j)}}}{(1+A_j^2|g_3^j|^2(1+2\overline{\gamma}_2^j))}\right]. \tag{7}$$

By defining $\lambda(\overline{\gamma}_2^j) = (1+2\overline{\gamma}_2^j)/(1+\overline{\gamma}_2^j)$, and $Z_{DR}^j = |r_{DR}^j|^2/\sigma_N^2$, we can express (7) in the following integral:

$$E_{g_2^j, g_3^j}\left[f_{\tilde{n}_{DR}^j|g_2^j,g_3^j}\left(r_{DR}^j - A_j g_2^j g_3^j \sqrt{2P}\right)\right] = \frac{1}{\pi\sigma_N^2}\int_{x=0}^{\infty}\frac{\exp\left(-x - \frac{Z_{DR}^j}{1+\lambda(\overline{\gamma}_2^j)\overline{\gamma}_3^j x}\right)}{1+\lambda(\overline{\gamma}_2^j)\overline{\gamma}_3^j x}dx. \tag{8}$$

Note that, unfortunately, the integral in Eqn. (8) does not have a closed-form solution.

When $X=0$ is transmitted, we have

$$f_{r_{DR}^1,\ldots,r_{DR}^M,r_{DS}|X=0} = E_{g_1}\left[f_{n_{DS}}(r_{DS})\right] \times \prod_{j=1}^{M} E_{g_2^j,g_3^j}\left[f_{\tilde{n}_{DR}^j|g_2^j,g_3^j}\left(r_{DR}^j\right)\right]. \tag{9}$$

Following the analysis (6) and (7), we can simplify the terms in (9) as

$$E_{g_1}\left[f_{n_{DS}}(r_{DS})\right] = \frac{1}{\pi\sigma_N^2}\exp(-Z_{DS}) \quad \text{and} \tag{10}$$

$$E_{g_2^j,g_3^j}\left[f_{\tilde{n}_{DR}^j}\left(r_{DR}^j\right)\right] = \frac{1}{\pi\sigma_N^2}\int_{x=0}^{\infty}\frac{e^{-x-\frac{z_{DR}^j}{1+\mu(\overline{\gamma}_2^j)\overline{\gamma}_3^j x}}}{1+\mu(\overline{\gamma}_2^j)\overline{\gamma}_3^j x}dx, \tag{11}$$

where $\mu(\overline{\gamma}_2^j) = 1/(1+\overline{\gamma}_2^j)$.

Finally, the log-likelihood ratio of the transmitted bit at the destination is given by

$$\mathrm{LLR}(b) \triangleq \log\left(\frac{f_{r_{DR}^1,\ldots,r_{DR}^M,r_{DS}|X=\sqrt{2P}}}{f_{r_{DR}^1,\ldots,r_{DR}^M,r_{DS}|X=0}}\right) = \mathcal{F}(Z_{DS},\overline{\gamma}_1) + \sum_{j=1}^{M}\mathcal{G}\left(Z_{DR}^j,\overline{\gamma}_2^j,\overline{\gamma}_3^j\right), \tag{12}$$

where

$$\mathcal{F}(Z_{DS},\overline{\gamma}_1) = -\log(1+2\overline{\gamma}_1) + \left(\frac{2\overline{\gamma}_1}{1+2\overline{\gamma}_1}\right)Z_{DS}, \tag{13}$$

and

$$\mathcal{G}\left(Z_{DR}^j,\overline{\gamma}_2^j,\overline{\gamma}_3^j\right) = \log\left(\int_{x=0}^{\infty}\frac{e^{-x-\frac{z_{DR}^j}{1+\lambda(\overline{\gamma}_2^j)\overline{\gamma}_3^j x}}}{1+\lambda(\overline{\gamma}_2^j)\overline{\gamma}_3^j x}dx\right) - \log\left(\int_{x=0}^{\infty}\frac{e^{-x-\frac{z_{DR}^j}{1+\mu(\overline{\gamma}_2^j)\overline{\gamma}_3^j x}}}{1+\mu(\overline{\gamma}_2^j)\overline{\gamma}_3^j x}dx\right). \tag{14}$$

The destination decodes $\widehat{b}=1$ if LLR ≥ 0 and $\widehat{b}=0$ otherwise.

B. Binary FSK

With BFSK signaling the signal constellation is given by $\mathcal{X} = \{\sqrt{P}e^{j2\pi f_1 t}, \sqrt{P}e^{j2\pi f_2 t}\}$, where f_1 and f_2 are two orthogonal frequency tones. When $b=1$ is the transmitted bit, we have $X = \sqrt{P}e^{2\pi f_1 t}$. After matched filtering of the received signals r_{DS} and r_{DR}^j, $j=1,\ldots,M$, by $e^{-2\pi f_1 t}$ and $e^{-j2\pi f_2 t}$ over the signaling duration, we obtain the following low-pass equivalent complex valued outputs as

$$r_{DS,1} \triangleq r_{DS,c,1} + jr_{DS,s,1} = \sqrt{P}g_1 + \eta_{DS,1} \tag{15}$$

$$r_{DS,2} \triangleq r_{DS,c,2} + jr_{DS,s,2} = \eta_{DS,2} \tag{16}$$

$$r_{DR,1}^k \triangleq r_{DR,c,1}^k + jr_{DR,s,1}^k = A_k\sqrt{P}g_2^k g_3^k + A_k g_3^k \eta_{RS,1}^k + \eta_{DR,1}^k \quad k=1,\ldots,M \tag{17}$$

$$r_{DR,2}^k \triangleq r_{DR,c,2}^k + jr_{DR,s,2}^k = A_k g_3^k \eta_{RS,2}^k + \eta_{DR,2}^k \quad k=1,\ldots,M. \tag{18}$$

In (15)-(18), the subscripts 1 and 2 represent the outputs due to correlating with frequencies f_1 and f_2, respectively, whereas the subscripts c and s denote the in-phase and quadrature components, respectively.

Conditioned on f_1 being transmitted, the conditional pdf of $r_{DS,1}, r_{DS,2}, r_{DR,1}^k, r_{DR,2}^k$, $k=1,\ldots,M$, is given by

$$f_{r_{DS,1},r_{DS,2},r_{DR,1}^1,r_{DR,2}^1,\ldots,r_{DR,1}^M,r_{DR,2}^M|f_1} = E_{g_1}[f_{\eta_{DS,1}|g_1}(r_{DS,1}-\sqrt{P}g_1)]f_{\eta_{DS,2}}(r_{DS,2}) \prod_{k=1}^{M} E_{g_2^k,g_3^k}\Big[f_{\tilde{\eta}_{DR,1}^k|g_2^k,g_3^k}(r_{DR,1}^k - A_k\sqrt{P}g_2^k g_3^k)\times f_{\tilde{\eta}_{DR,2}^k|g_2^k,g_3^k}(r_{DR,2}^k)\Big], \tag{19}$$

where $\tilde{\eta}_{DR,1}^k = A_k g_3^k \eta_{RS,1}^k + \eta_{DR,1}^k$ and $\tilde{\eta}_{DR,2}^k = A_k g_3^k \eta_{RS,2}^k + \eta_{DR,2}^k$, conditioned on g_2^k and g_3^k, are two independent complex Gaussian random variables with zero-mean and variances $E[|\tilde{\eta}_{DR,1}^k|^2|g_2^k,g_3^k] = E[|\tilde{\eta}_{DR,2}^k|^2|g_2^k,g_3^k] = \sigma_N^2(1+A_k^2|g_3^k|^2)$.

Using Lemma 1, the terms of (19) can be simplified to

$$E_{g_1}[f_{\eta_{DS,1}|g_1}(r_{DS,1}-\sqrt{P}g_1)] = \frac{1}{\pi\sigma_N^2}\times\frac{e^{-\frac{Z_{DS,1}}{1+\overline{\gamma}_1}}}{1+\overline{\gamma}_1} \tag{20}$$

$$f_{\eta_{DS,2}}(r_{DS,2}) = \frac{e^{-\frac{|r_{DS,2}|^2}{\sigma_N^2}}}{\pi\sigma_N^2} = \frac{e^{-Z_{DS,2}}}{\pi\sigma_N^2} \tag{21}$$

$$E_{g_2^k,g_3^k}\Big[f_{\tilde{\eta}_{DR,1}^k|g_2^k,g_3^k}(r_{DR,1}^k - A_k\sqrt{P}g_2^k g_3^k)\times f_{\tilde{\eta}_{DR,2}^k|g_2^k,g_3^k}(r_{DR,2}^k)\Big] = \frac{1}{\pi^2 N_0^2}\int_{x=0}^{\infty}\frac{dx}{1+\mu(\overline{\gamma}_2^k)\overline{\gamma}_3^k x}\times\frac{e^{-x-\frac{Z_{DR,1}^k}{1+\overline{\gamma}_3^k x}-\frac{Z_{DR,2}^k}{1+\mu(\overline{\gamma}_2^k)\overline{\gamma}_3^k x}}}{1+\overline{\gamma}_3^k x} = \frac{1}{\pi^2 N_0^2}\Psi(Z_{DR,1}^k, Z_{DR,2}^k, \overline{\gamma}_2^k, \overline{\gamma}_3^k). \tag{22}$$

In (22), we have $Z_{DS,1} = |r_{DS,1}|^2/\sigma_N^2$, $Z_{DS,2} = |r_{DS,2}|^2/\sigma_N^2$, $Z_{DR,1}^k = |r_{DR,1}^k|^2/\sigma_N^2$, $Z_{DR,2}^k = |r_{DR,2}^k|^2/\sigma_N^2$, and $\Psi(Z_{DR,1}^k, Z_{DR,2}^k, \overline{\gamma}_2^k, \overline{\gamma}_3^k)$ is simply given by the integral of (22).

When frequency tone f_2 is transmitted, the above analysis remains valid but, since the correlator with $e^{-j2\pi f_1 t}$ would yield only the noise and the correlator with $e^{-j2\pi f_2 t}$ would yield signal-plus-noise, we need to exchange $Z_{DR,1}^k$ and $Z_{DR,2}^k$, and $Z_{DS,1}$ and $Z_{DS,2}$. With this, the LLR of the transmitted bit at the destination is

$$\begin{aligned}\mathrm{LLR}(b) &= \log\left(\frac{f_{r_{DS,1},r_{DS,2},r_{DR,1}^1,r_{DR,2}^1,\ldots,r_{DR,1}^M,r_{DR,2}^M|f_1}}{f_{r_{DS,1},r_{DS,2},r_{DR,1}^1,r_{DR,2}^1,\ldots,r_{DR,1}^M,r_{DR,2}^M|f_2}}\right)\\ &= \log\left(\frac{e^{-\frac{Z_{DS,1}}{1+\overline{\gamma}_1}-Z_{DS,2}}\prod_{k=1}^M \Psi(Z_{DR,1}^k, Z_{DR,2}^k, \overline{\gamma}_2^k, \overline{\gamma}_3^k)}{e^{-\frac{Z_{DS,2}}{1+\overline{\gamma}_1}-Z_{DS,1}}\prod_{k=1}^M \Psi(Z_{DR,2}^k, Z_{DR,1}^k, \overline{\gamma}_2^k, \overline{\gamma}_3^k)}\right)\\ &= \left(\frac{\overline{\gamma}_1}{1+\overline{\gamma}_1}\right)[Z_{DS,1} - Z_{DS,2}]\\ &+ \sum_{j=1}^M \mathcal{H}\left(Z_{DR,1}^j, Z_{DR,2}^j, \overline{\gamma}_2^j, \overline{\gamma}_3^j\right),\end{aligned} \tag{23}$$

where, using the definition of $\Psi(\cdot,\cdot,\cdot,\cdot)$ in (22), we have

$$\begin{aligned}\mathcal{H}\left(Z_{DR,1}^j, Z_{DR,2}^j, \overline{\gamma}_2^j, \overline{\gamma}_3^j\right) &= \log\left(\int_0^\infty \frac{dx e^{-x-\frac{Z_{DR,1}^j}{1+\overline{\gamma}_3^j x}-\frac{Z_{DR,2}^j}{1+\mu(\overline{\gamma}_2^j)\overline{\gamma}_3^j x}}}{(1+\overline{\gamma}_3^j x)(1+\mu(\overline{\gamma}_2^j)\overline{\gamma}_3^j x)}\right)\\ &- \log\left(\int_0^\infty \frac{dx e^{-x-\frac{Z_{DR,2}^j}{1+\overline{\gamma}_3^j x}-\frac{Z_{DR,1}^j}{1+\mu(\overline{\gamma}_2^j)\overline{\gamma}_3^j x}}}{(1+\overline{\gamma}_3^j x)(1+\mu(\overline{\gamma}_2^j)\overline{\gamma}_3^j x)}\right)\end{aligned} \tag{24}$$

The destination decodes $\widehat{b} = 1$ if LLR ≥ 0 and $\widehat{b} = 0$ otherwise.

III. Lower Bounds on the Average BER

Due to the complexity of the detection metrics of (12) and (23), an analysis of the performance of the optimum NCAF is difficult. As a consequence, we now consider a simple case of having a strong line-of-sight path on the relay-to-destination link. An example scenario could be terrestrial communication from the relay to a destination in a rural environment. In this case, we assume that the channel gain from the relay to the destination is unfaded. That is, we have $f_{\gamma_3^j}(x) = \delta(x - \overline{\gamma}_3^j)$, where $\delta(x)$ is the Dirac delta function. With this assumption, we derive simple closed-form expressions for the average probability of bit error. Even if this assumption is not satisfied in practice, the resulting expressions can still serve as lower bounds on the average error performance.

A. On-Off Keying

We first realize that the term $\exp(-x)$ in the integrands of (11) and (22) is due to the fact that we are averaging $E_{\gamma_2^j}[\cdot]$ over the pdf of $\gamma_2^j/\overline{\gamma}_2^j$ which is exponentially distributed with unity mean. Upon replacing $\exp(-x)$ in the integrands of (11) by $\delta(x-1)$, we can simplify the optimum receiver for this special case as

$$\begin{aligned}\mathrm{LLR}(b) &= -\log(1+2\overline{\gamma}_1) + \left(\frac{2\overline{\gamma}_1}{1+2\overline{\gamma}_1}\right) Z_{DS} +\\ &\sum_{j=1}^M \left\{\log\left(\frac{e^{-\frac{Z_{DR}^j}{1+\lambda(\overline{\gamma}_2^j)\overline{\gamma}_3^j}}}{1+\lambda(\overline{\gamma}_2^j)\overline{\gamma}_3^j}\right) - \log\left(\frac{e^{-\frac{Z_{DR}^j}{1+\mu(\overline{\gamma}_2^j)\overline{\gamma}_3^j}}}{1+\mu(\overline{\gamma}_2^j)\overline{\gamma}_3^j}\right)\right\}.\end{aligned} \tag{25}$$

Eqn. (25) can be expressed in a convenient form as

$$\mathrm{LLR}(b) = c_1 Z_{DS} + \sum_{j=1}^M c_2^j Z_{DR}^j - \mathrm{T_h}, \tag{26}$$

where $\mathrm{T_h} = \log(1+2\overline{\gamma}_1) + \sum_{j=1}^M \{\log(1+\lambda(\overline{\gamma}_2^j)\overline{\gamma}_3^j) - \log\left(1+\mu(\overline{\gamma}_2^j)\overline{\gamma}_3^j\right)\}$, $c_1 = 2\overline{\gamma}_1/(1+2\overline{\gamma}_1)$, and, for $j = 1,\ldots,M$, $c_2^j = 2\overline{\gamma}_2^j\overline{\gamma}_3^j/((1+\overline{\gamma}_2^j+\overline{\gamma}_3^j)(1+\lambda(\overline{\gamma}_2^j)\overline{\gamma}_3^j))$. In (Appendix-A, [14]) we derived a simple closed-form expression for the average BER for the detector of (26) which is given by

$$\begin{aligned}\overline{P}_{e,\text{On-Off}} &= \frac{1}{2}\left(1 - F\left(\overline{X}_0, \overline{Y}_0^1, \ldots, \overline{Y}_0^M, \mathrm{T_h}\right)\right) +\\ &\frac{1}{2} F\left(\overline{X}_1, \overline{Y}_1^1, \ldots, \overline{Y}_1^M, \mathrm{T_h}\right),\end{aligned} \tag{27}$$

where

$$\begin{aligned}F(\overline{U}_1, \overline{U}_2, \ldots, \overline{U}_N, \mathrm{T_h}) &= \sum_{j=1}^N \left\{\prod_{i=1, i\neq j}^N \frac{\overline{U}_j}{\overline{U}_j - \overline{U}_i}\right\} \times\\ &\left(1 - \exp\left(-\frac{\mathrm{T_h}}{\overline{U}_j}\right)\right)\end{aligned} \tag{28}$$

$$\overline{X}_0 = \frac{2\overline{\gamma}_1}{1+2\overline{\gamma}_1} \tag{29}$$

$$\overline{Y}_0^j = \frac{2\overline{\gamma}_2^j\overline{\gamma}_3^j}{(1+\overline{\gamma}_2^j)(1+\lambda(\overline{\gamma}_2^j)\overline{\gamma}_3^j)} \qquad j = 1,\ldots,M \tag{30}$$

$$\overline{X}_1 = 2\overline{\gamma}_1 \tag{31}$$

and

$$\overline{Y}_1^j = \frac{2\overline{\gamma}_2^j\overline{\gamma}_3^j}{1+\overline{\gamma}_2^j+\overline{\gamma}_3^j} \qquad j = 1,\ldots,M. \tag{32}$$

In (28), the function $F(\overline{U}_1, \overline{U}_2, \ldots, \overline{U}_N, \mathrm{T_h})$ is defined as the probability that $U_1 + U_2 + \ldots + U_N$ is less than T_h, where $U_1,\ldots,U_N$ are independent exponentially distributed rvs with the mean of U_i being $\overline{U}_i$.

We now study the behavior of (27) at high SNR. Similar to [8], we let $\overline{\gamma}_1 = \mathrm{t}_1\overline{\gamma}$, and, for $j = 1,\ldots,M$, $\overline{\gamma}_2^j = \mathrm{t}_2^j\overline{\gamma}$ and $\overline{\gamma}_3^j = \mathrm{t}_3^j\overline{\gamma}$, so that the average link SNR goes to infinity with $\overline{\gamma}$, while still maintaining a fixed proportionality among them. The variables t_1 and $\{\mathrm{t}_2^j, \mathrm{t}_3^j\}_{j=1}^M$ capture the relay placement and path-loss variability in the relay network. As $\overline{\gamma} \to \infty$, (29)-(32) approach $\overline{X}_0 = 1$, $\overline{Y}_0^j = 1$, $j = 1,\ldots,M$, $\overline{X}_1 = 2\mathrm{t}_1\overline{\gamma}$, and $\overline{Y}_1^j = 2\mathrm{t}_2^j\mathrm{t}_3^j\overline{\gamma}/(\mathrm{t}_2^j + \mathrm{t}_3^j)$, $j = 1,\ldots,M$, respectively. It can be shown that, as $\overline{\gamma} \to \infty$, the threshold $\mathrm{T_h}$ can be approximated as $(M+1)\log(\overline{\gamma})$. In view of $\overline{X}_0 = 1$ and $\overline{Y}_0^j = 1$, $j = 1, \ldots, M$, the first term, $\left(1 - F(\overline{X}_0, \overline{Y}_0^1, \ldots, \overline{Y}_0^M, \mathrm{T_h})\right)$, in (27) is nothing but the probability that the sum of $M+1$ independent and identically distributed (i.i.d.) exponential rvs, each having unity mean, exceeds $\mathrm{T_h}$. This has a well-known closed form, which is given by [13]

$$\left(1 - F(\overline{X}_0, \overline{Y}_0^1, \ldots, \overline{Y}_0^M, \mathrm{T_h})\right) = \int_{\mathrm{T_h}}^{\infty} \frac{e^{-x}x^M}{\Gamma(M+1)}dx$$
$$\approx \frac{e^{-\mathrm{T_h}}\mathrm{T_h}^M}{\Gamma(M+1)} \approx \frac{(M+1)^M}{M!} \times \frac{(\log(\overline{\gamma}))^M}{(\overline{\gamma})^{M+1}} \quad (\overline{\gamma} \to \infty), \tag{33}$$

where in the second step of (33) we employed the asymptotic behavior of the incomplete Gamma function [15]. The third step in (33) is due to $\mathrm{T_h} \approx (M+1)\log(\overline{\gamma})$. To characterize the behavior of $F(\overline{X}_1, \overline{Y}_1^1, \ldots, \overline{Y}_1^M, \mathrm{T_h})$ in (27), we notice that the constituent rvs $\overline{X}_1, \overline{Y}_1^1, \ldots, \overline{Y}_1^M$ have a linear growth in their expected values with $\overline{\gamma}$. Let us define $\mathrm{t_{max}} = \max(\mathrm{t}_1, \mathrm{t}_2^1\mathrm{t}_3^1/(\mathrm{t}_2^1 + \mathrm{t}_3^1), \ldots, \mathrm{t}_2^M\mathrm{t}_3^M/(\mathrm{t}_2^M + \mathrm{t}_3^M))$. The probability $F(\overline{X}_1, \overline{Y}_1^1, \ldots, \overline{Y}_1^M, \mathrm{T_h})$ can be lower bounded as

$$F(\overline{X}_1, \overline{Y}_1^1, \ldots, \overline{Y}_1^M, \mathrm{T_h}) = \mathrm{Prob}(X_1 + Y_1^1 + \cdots + Y_1^M < \mathrm{T_h})$$
$$\geq \mathrm{Prob}\Bigg(\max(\overline{X}_1, \overline{Y}_1^1, \ldots, \overline{Y}_1^M) \times$$
$$(X_1/\overline{X}_1 + Y_1^1/\overline{Y}_1^1 + \cdots + Y_1^M/\overline{Y}_1^M) < \mathrm{T_h}\Bigg)$$
$$= \mathrm{Prob}\left(\frac{X_1}{\overline{X}_1} + \frac{Y_1^1}{\overline{Y}_1^1} + \cdots + \frac{Y_1^M}{\overline{Y}_1^M} < \frac{\mathrm{T_h}}{2\mathrm{t_{max}}\overline{\gamma}}\right). \tag{34}$$

Observe that the rvs $X_1/\overline{X}_1$, $Y_1^1/\overline{Y}_1^1, \ldots, Y_1^M/\overline{Y}_1^M$ are i.i.d. rvs each with unity mean. That is, the sum $\frac{X_1}{\overline{X}_1} + \sum_{j=1}^M \frac{Y_1^j}{\overline{Y}_1^j}$ is Gamma distributed [13]. As a result, (34) simplifies to [13]

$$F(\overline{X}_1, \overline{Y}_1^1, \ldots, \overline{Y}_1^M, \mathrm{T_h}) \geq \int_{x=0}^{\frac{\mathrm{T_h}}{2\mathrm{t_{max}}\overline{\gamma}}} \frac{e^{-x}x^M}{\Gamma(M+1)}dx$$
$$= 1 - e^{-\frac{\mathrm{T_h}}{2\mathrm{t_{max}}\overline{\gamma}}} \sum_{j=0}^{M} \frac{1}{j!}\left(\frac{\mathrm{T_h}}{2\mathrm{t_{max}}\overline{\gamma}}\right)^j$$
$$\geq \frac{(M+1)^{M+1}}{(2\mathrm{t_{max}})^{M+1}(M+1)!}\left(\frac{\log(\overline{\gamma})}{\overline{\gamma}}\right)^{M+1} \quad (\overline{\gamma} \to \infty) \tag{35}$$

Combining (35) with (33), we have

$$\overline{P}_{e,\text{On-Off}} \geq \frac{(M+1)^M}{M!} \times \frac{(\log(\overline{\gamma}))^M}{2(\overline{\gamma})^{M+1}} +$$
$$\frac{(M+1)^{M+1}}{2(2\mathrm{t_{max}})^{M+1}(M+1)!}\left(\frac{\log(\overline{\gamma})}{\overline{\gamma}}\right)^{M+1}$$
$$> \frac{(M+1)^{M+1}}{2(2\mathrm{t_{max}})^{M+1}(M+1)!}\left(\frac{1}{\overline{\gamma}}\right)^{M+1} \quad (\overline{\gamma} \to \infty). \tag{36}$$

B. Binary FSK

By replacing $\exp(-x)$ in the integrals of Eqn. (22) by $\delta(x-1)$, we can simplify the function $\mathcal{H}(\cdot,\cdot,\cdot,\cdot)$ to

$$\mathcal{H}\left(Z_{DR,1}^j, Z_{DR,2}^j, \overline{\gamma}_2^j, \overline{\gamma}_3^j\right) = Z_{DR,1}^j\left(\frac{1}{1+\mu(\overline{\gamma}_2^j)\overline{\gamma}_3^j} - \frac{1}{1+\overline{\gamma}_3^j}\right)$$
$$-Z_{DR,2}^j\left(\frac{1}{1+\mu(\overline{\gamma}_2^j)\overline{\gamma}_3^j} - \frac{1}{1+\overline{\gamma}_3^j}\right)$$
$$= \left(Z_{DR,1}^j - Z_{DR,2}^j\right)\frac{\overline{\gamma}_2^j\overline{\gamma}_3^j}{(1+\overline{\gamma}_3^j)(1+\overline{\gamma}_2^j+\overline{\gamma}_3^j)} \tag{37}$$

With this, we can simplify $\mathrm{LLR}(b)$ of (23) to

$$\mathrm{LLR}(b) = \left(\frac{\overline{\gamma}_1}{1+\overline{\gamma}_1}\right)[Z_{DS,1} - Z_{DS,2}]$$
$$+\sum_{j=1}^{M}\left(\frac{\overline{\gamma}_2^j\overline{\gamma}_3^j}{(1+\overline{\gamma}_3^j)(1+\overline{\gamma}_2^j+\overline{\gamma}_3^j)}\right)\left(Z_{DR,1}^j - Z_{DR,2}^j\right). \tag{38}$$

An analysis of the average error rate of (38) can be performed as follows: Without loss of generality, we assume that the frequency tone f_1 is transmitted. Then $Z_{DS,1}$ is exponentially distributed with mean $1+\overline{\gamma}_1$, whereas $Z_{DS,2}$ is exponentially distributed with mean 1. Similarly, $Z_{DR,1}^j$ is exponentially distributed with mean $1+\overline{\gamma}_3^j$, whereas $Z_{DR,2}^j$ is exponentially distributed with mean $1+\mu(\overline{\gamma}_2^j)\overline{\gamma}_3^j$. Also, note that $Z_{DS,1}, Z_{DS,2}$, $Z_{DR,1}^j$, $Z_{DR,2}^j$, $j = 1, \ldots, M$, are independent rvs. With this, we define the following rvs

$$U_1 = \frac{\overline{\gamma}_1}{1+\overline{\gamma}_1}Z_{DS,1}$$
$$U_{j+1} = \frac{\overline{\gamma}_2^j\overline{\gamma}_3^j Z_{DR,1}^j}{(1+\overline{\gamma}_3^j)(1+\overline{\gamma}_2^j+\overline{\gamma}_3^j)} \quad j = 1, \ldots, M \tag{39}$$
$$\text{and} \quad V_1 = \frac{\overline{\gamma}_1}{1+\overline{\gamma}_1}Z_{DS,2}$$
$$V_{j+1} = \frac{\overline{\gamma}_2^j\overline{\gamma}_3^j Z_{DR,2}^j}{(1+\overline{\gamma}_3^j)(1+\overline{\gamma}_2^j+\overline{\gamma}_3^j)} \quad j = 1, \ldots, M \tag{40}$$

The rvs of Eqns. (39) and (40), respectively, have the following mean values:

$$\overline{U}_1 = \overline{\gamma}_1$$
$$\overline{U}_{j+1} = \frac{\overline{\gamma}_2^j\overline{\gamma}_3^j}{1+\overline{\gamma}_2^j+\overline{\gamma}_3^j} \quad j = 1, \ldots, M, \tag{41}$$
$$\text{and} \quad \overline{V}_1 = \frac{\overline{\gamma}_1}{1+\overline{\gamma}_1}$$
$$\overline{V}_{j+1} = \frac{\overline{\gamma}_2^j\overline{\gamma}_3^j}{(1+\overline{\gamma}_2^j)(1+\overline{\gamma}_3^j)} \quad j = 1, \ldots, M \tag{42}$$

The average BER is then given by

$$\overline{P}_{e,\text{BFSK}} = \text{Prob}\left(\sum_{j=1}^{M+1} U_j < \sum_{j=1}^{M+1} V_j\right) = \sum_{i=1}^{M+1}\sum_{j=1}^{M+1} \kappa_i \zeta_j \left(\frac{\overline{V}_i}{\overline{V}_i + \overline{U}_j}\right), \quad (43)$$

where the details of (43) can be found in (Appendix-B, [14]), and the ζ_j and κ_j of Eqn. (43) are given by

$$\zeta_j = \prod_{i=1, i\neq j}^{M+1} \frac{\overline{U}_j}{\overline{U}_j - \overline{U}_i} \text{ and } \kappa_i = \prod_{j=1, j\neq i}^{M+1} \frac{\overline{V}_i}{\overline{V}_i - \overline{V}_j}. \quad (44)$$

We consider the behavior of (43) at high SNR. The mean values in (41)-(42) simplify to

$$\overline{U}_1 = \mathrm{t}_1 \overline{\gamma} \quad (45)$$

$$\overline{U}_{j+1} = \frac{\mathrm{t}_2^j \mathrm{t}_3^j}{\mathrm{t}_2^j + \mathrm{t}_3^j}\overline{\gamma} \qquad j = 1, \ldots, M, \quad (46)$$

$$\overline{V}_1 = 1 \quad (47)$$

$$\text{and} \quad \overline{V}_{j+1} = 1 \qquad j = 1, \ldots, M. \quad (48)$$

Define $V = \sum_{j=1}^{M+1} V_j$. At high SNR, from (47) and (48), V is a sum of $M+1$ i.i.d. exponential random variables of unity mean, and hence V is Gamma distributed [13]. Using (45)-(48), the expression in (43) can be simplified as

$$\overline{P}_{e,\text{BFSK}} = \text{Prob}\left(\sum_{j=1}^{M+1} U_j < V\right)$$
$$\geq E\left[\text{Prob}\left(\max\left(\overline{U}_1, \ldots, \overline{U}_{M+1}\right) \times \left\{\sum_{j=1}^{M+1} \frac{U_j}{\overline{U}_j}\right\} < v\right) \middle| V = v\right] \quad (49)$$

Since $\max\left(\overline{U}_1, \ldots, \overline{U}_{M+1}\right) = \mathrm{t}_{\max}\overline{\gamma}$, and $\sum_{j=1}^{M+1} \frac{U_j}{\overline{U}_j}$ is again Gamma distributed, (49) can be simplified as

$$\overline{P}_{e,\text{BFSK}} \geq \int_{v=0}^{\infty} dv \frac{e^{-v} v^M}{\Gamma(M)} \times \int_{u=0}^{\frac{v}{\mathrm{t}_{\max}\overline{\gamma}}} du \frac{e^{-u} u^M}{\Gamma(M+1)}$$
$$= \int_{v=0}^{\infty} dv \frac{e^{-v} v^M}{\Gamma(M)} \times e^{-\frac{v}{\mathrm{t}_{\max}\overline{\gamma}}} \sum_{j=M+1}^{\infty} \frac{1}{j!}\left(\frac{v}{\mathrm{t}_{\max}\overline{\gamma}}\right)^j$$
$$= \frac{1}{\Gamma(M+1)} \sum_{j=M+1}^{\infty} \frac{1}{j!} \frac{1}{(\mathrm{t}_{\max}\overline{\gamma})^j} \times \frac{\Gamma(M+j+1)}{(1+\frac{1}{\mathrm{t}_{\max}\overline{\gamma}})^{M+j+1}}$$
$$\geq \frac{(2M+1)!}{M!(M+1)!}\left(\frac{1}{\mathrm{t}_{\max}\overline{\gamma}}\right)^{M+1} \qquad (\overline{\gamma} \to \infty). \quad (50)$$

IV. Upper Bounds on the Average BER

In this section, we present upper bounds on the average BER for both OOK and BFSK signals on noncoherent relay channels. To accomplish this task, we use the likelihood functions

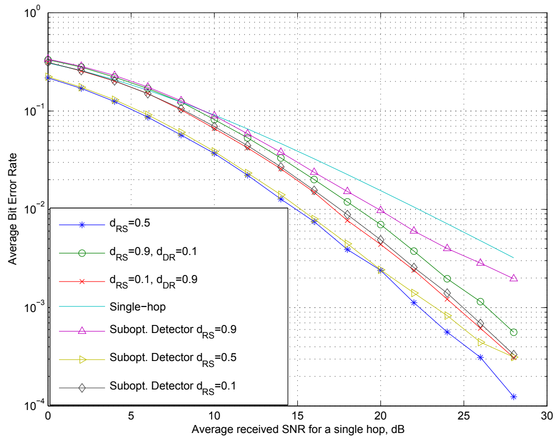

Fig. 1. Average probability of error for OOK modulation with noncoherent demodulation. Three cases of relay placements are considered: a)Relay close to the source, b)relay at the midpoint between the source and the destination, and c)relay close to the destination. Also shown is the analytical error probability performance of a system with no relay and the performance with the suboptimum detector of Eqn. (62)

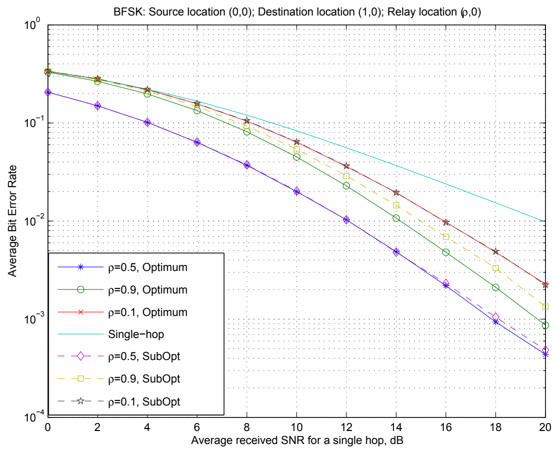

Fig. 2. Average probability of error for FSK modulation with noncoherent demodulation. Three cases of relay placements are considered: a)Relay close to the source, b)relay at the midpoint between the source and the destination, and c)relay close to the destination. Also shown is the analytical error probability performance of a system with no relay and the performance achieved by the suboptimum detector of Eqn. (63).

of the transmitted bits along with the Bhattacharyya bound. The Bhattacharyya upper bound on the probability of error in discriminating two hypotheses, H_0 and H_1, is given by [10]

$$\overline{P}_b \leq \int_{z=-\infty}^{\infty} \sqrt{f_{\mathrm{H}_1}(z) f_{\mathrm{H}_0}(z)}\, dz, \quad (51)$$

where $f_{\mathrm{H}_j}(z)$ is the likelihood function for the hypothesis H_j, $j \in \{0, 1\}$.

For OOK, when the signal is present, the conditional density function, f_{H_1}, takes the form of $f_{Z_{DS}, Z_{DR}^1, \ldots, Z_{DR}^M | X=\sqrt{2E_s}}$, whereas f_{H_0}, when the signal is absent, takes the form of $f_{Z_{DS}, Z_{DR}^1, \ldots, Z_{DR}^M | X=0}$. Since $f_{Z_{DS}, Z_{DR}^1, \ldots, Z_{DR}^M | X=\sqrt{2E_s}} = f_{Z_{DS} | X=\sqrt{2E_s}} \times \prod_{j=1}^{M} f_{Z_{DR}^j | X=\sqrt{2E_s}}$ and $f_{Z_{DS}, Z_{DR}^1, \ldots, Z_{DR}^M | X=0} = f_{Z_{DS} | X=0}$

$\times \prod_{j=1}^{M} f_{Z_{DR}^j|X=0}$, the integral in (51) can be expressed as

$$\overline{P}_{b,On-Off} \le \int_{Z_{DS}=0}^{\infty} \sqrt{\frac{1}{1+2\overline{\gamma}_1} \exp\left(-\frac{Z_{DS}}{1+2\overline{\gamma}_1} - Z_{DS}\right)} dZ_{DS} \times$$
$$\prod_{j=1}^{M} \int_{Z_{DR}^j=0}^{\infty} \sqrt{\mathsf{A}(Z_{DR}^j, 1, \lambda(\overline{\gamma}_2^j)\overline{\gamma}_3^j)\mathsf{A}(Z_{DR}^j, 1, \mu(\overline{\gamma}_2^j)\overline{\gamma}_3^j)} dZ_{DR}^j, \quad (52)$$

where

$$\mathsf{A}(Z, a, b) = \int_{x=0}^{\infty} \frac{\exp(-ax)}{1+bx} \exp\left(-\frac{Z}{1+bx}\right) dx = \frac{1}{b}\mathsf{A}(Z, a/b, 1). \quad (53)$$

Except for the first term, which can be evaluated as $\sqrt{1+2\overline{\gamma}_1}/(1+\overline{\gamma}_1)$, (52) does not appear to have a closed-from. As a result, one must resort to numerical integration. At high SNR, the functions $\mathsf{A}(Z_{DR}^j, 1, \lambda(\overline{\gamma}_2^j)\overline{\gamma}_3^j)$ and $\mathsf{A}(Z_{DR}^j, 1, \mu(\overline{\gamma}_2^j)\overline{\gamma}_3^j)$ in the jth integral of the product of (52) can be approximated as

$$\mathsf{A}(Z_{DR}^j, 1, \lambda(\overline{\gamma}_2^j)\overline{\gamma}_3^j) \approx \frac{1}{2\mathsf{t}_3^j\overline{\gamma}} \mathsf{A}\left(Z_{DR}^j, 0, 1\right) \quad (54)$$

and $$\mathsf{A}(Z_{DR}^j, 1, \mu(\overline{\gamma}_2^j)\overline{\gamma}_3^j) \approx \mathsf{A}\left(Z_{DR}^j, 1, \frac{\mathsf{t}_3^j}{\mathsf{t}_2^j}\right). \quad (55)$$

The first term of (52), $\sqrt{1+2\overline{\gamma}_1}/(1+\overline{\gamma}_1)$, at high SNR, can be approximated as $\sqrt{2/\overline{\gamma}_1} = \sqrt{2/\mathsf{t}_1}/\sqrt{\overline{\gamma}}$. Using this, along with (54) and (55), the high SNR version of (52) is

$$\overline{P}_{b,\mathrm{On}-Off}\,(\text{High SNR}) \le \frac{1}{\sqrt{\overline{\gamma}}}\sqrt{\frac{2}{\mathsf{t}_1}} \prod_{j=1}^{M} \left\{ \frac{1}{\sqrt{2\mathsf{t}_3^j\overline{\gamma}}} \int_{Z_{DR}^j=0}^{\infty} \sqrt{\mathsf{A}\left(Z_{DR}^j, 0, 1\right) \mathsf{A}\left(Z_{DR}^j, 1, \frac{\mathsf{t}_3^j}{\mathsf{t}_2^j}\right)} dZ_{DR}^j \right\}$$
$$= \frac{1}{(\overline{\gamma})^{\frac{M+1}{2}}} \left[\sqrt{\frac{2}{\mathsf{t}_1}} \prod_{j=1}^{M} \frac{1}{\sqrt{2\mathsf{t}_3^j}} \times \int_{Z_{DR}^j=0}^{\infty} \sqrt{\mathsf{A}(Z_{DR}^j, 0, 1)\mathsf{A}(Z_{DR}^j, 1, \mathsf{t}_3^j/\mathsf{t}_2^j)} dZ_{DR}^j \right]. \quad (56)$$

For the case of BFSK, the Bhattacharyya bound, analogous to (52), can be written as

$$\overline{P}_{b,BFSK} \le \int_{Z_{DS,1}=0}^{\infty} \int_{Z_{DS,2}=0}^{\infty} \frac{e^{-[Z_{DS,1}+Z_{DS,2}]\frac{(2+\overline{\gamma}_1)}{2(1+\overline{\gamma}_1)}}}{(1+\overline{\gamma}_1)} dZ_{DS,1} \times$$
$$dZ_{DS,2} \times \prod_{j=1}^{M} \left\{ \int_{Z_{DR,1}^j=0}^{\infty} \int_{Z_{DR,2}^j=0}^{\infty} \sqrt{\Psi(Z_{DR,1}^j, Z_{DR,2}^j, \overline{\gamma}_2^j, \overline{\gamma}_3^j)} \times \sqrt{\Psi(Z_{DR,2}^j, Z_{DR,1}^j, \overline{\gamma}_2^j, \overline{\gamma}_3^j)} dZ_{DR,1}^j\, dZ_{DR,2}^j \right\}, \quad (57)$$

where the function $\Psi(\cdot,\cdot,\cdot,\cdot)$ is given in (22). Only the first term in (57) has a closed-form solution, and is given by $4(1+\overline{\gamma}_1)/(2+\overline{\gamma}_1)^2$. However, similar to (52), (57) must be evaluated numerically. At high SNR, the first term $4(1+\overline{\gamma}_1)/(2+\overline{\gamma}_1)^2$ in (57) behaves as $\frac{4}{\mathsf{t}_1\overline{\gamma}}$, and the functions $\Psi(Z_{DR,1}^j, Z_{DR,2}^j, \overline{\gamma}_2^j, \overline{\gamma}_3^j)$ and $\Psi(Z_{DR,2}^j, Z_{DR,1}^j, \overline{\gamma}_2^j, \overline{\gamma}_3^j)$, with the help of (22), can be simplified to

$$\Psi(Z_{DR,1}^j, Z_{DR,2}^j, \overline{\gamma}_2^j, \overline{\gamma}_3^j) \approx \frac{1}{\mathsf{t}_3^j\overline{\gamma}} \int_{u=0}^{\infty} \frac{e^{-Z_{DR,1}^j - \frac{Z_{DR,2}^j}{1+u}}}{1+u} du$$
$$= \frac{1}{\mathsf{t}_3^j\overline{\gamma}} \Theta(Z_{DR,1}^j, Z_{DR,2}^j) \quad (58)$$

and $$\Psi(Z_{DR,2}^j, Z_{DR,1}^j, \overline{\gamma}_2^j, \overline{\gamma}_3^j) \approx \frac{1}{\mathsf{t}_3^j\overline{\gamma}} \int_{u=0}^{\infty} \frac{e^{-Z_{DR,2}^j - \frac{Z_{DR,1}^j}{1+u}}}{1+u} dt$$
$$= \frac{1}{\mathsf{t}_3^j\overline{\gamma}} \Theta(Z_{DR,2}^j, Z_{DR,1}^j), \quad (59)$$

where

$$\Theta(Z_1, Z_2) = \int_{t=0}^{\infty} \frac{e^{-Z_1 - \frac{Z_2}{1+t}}}{1+t} dt. \quad (60)$$

Using (58) and (59), (57) becomes

$$\overline{P}_{b,\mathrm{BFSK}}\,(\text{High SNR}) \le \frac{4}{\mathsf{t}_1\overline{\gamma}} \prod_{j=1}^{M} \left\{ \int_{Z_{DR,1}^j=0}^{\infty} \int_{Z_{DR,2}^j=0}^{\infty} \frac{1}{\mathsf{t}_3^j\overline{\gamma}} \times \sqrt{\Theta(Z_{DR,1}^j, Z_{DR,2}^j)\Theta(Z_{DR,2}^j, Z_{DR,1}^j)} dZ_{DR,1}^j\, dZ_{DR,2}^j \right\}$$
$$= \frac{1}{(\overline{\gamma})^{M+1}} \left[\frac{4}{\mathsf{t}_1} \prod_{j=1}^{M} \frac{1}{\mathsf{t}_3^j} \int_{Z_{DR,1}^j=0}^{\infty} \int_{Z_{DR,2}^j=0}^{\infty} \left\{ \sqrt{\Theta(Z_{DR,1}^j, Z_{DR,2}^j)} \times \sqrt{\Theta(Z_{DR,2}^j, Z_{DR,1}^j)} dZ_{DR,1}^j\, dZ_{DR,2}^j \right\} \right]. \quad (61)$$

V. Suboptimum Receivers

In this section, we present easy to implement suboptimum receivers for both OOK and BFSK. For OOK, we propose the following detector:

$$\mathrm{LLR}(b)\,(\mathrm{OOK}) = c_1 Z_{DS} + \sum_{j=1}^{M} c_2^j Z_{DR}^j - \mathrm{T_h}, \quad (62)$$

where $\mathrm{T_h}$, c_1, and, for $j = 1, \ldots, M$, c_2^j, are defined in Section III-A. It is to be noted that (62) is optimum only when the relay-to-destination link is unfaded. Performance analysis of (62) is carried out in (Appendix-C, [14]).

For BFSK, we propose the following suboptimum receiver:

$$\mathrm{LLR}(b) = \left(\frac{\overline{\gamma}_1}{1+\overline{\gamma}_1}\right)[Z_{DS,1} - Z_{DS,2}] + \sum_{j=1}^{M} \left(\frac{\overline{\gamma}_2^j\overline{\gamma}_3^j}{(1+\overline{\gamma}_3^j)(1+\overline{\gamma}_2^j+\overline{\gamma}_3^j)}\right)\left(Z_{DR,1}^j - Z_{DR,2}^j\right). \quad (63)$$

Again, (63) is optimal only when the relay-to-destination link is unfaded. In (Appendix-D, [14]) we present an analysis on the performance of (63).

VI. Asymptotic Diversity Order Analysis

In this section, we consider the use of the expression from [11] for the asymptotic diversity orders of the systems discussed in this paper. As will be seen, it is unclear as to how much credence one should put on the results of this type of analysis. However, we present them because similar results for other systems are prevalent in the literature (see, e.g., [8], [9]).

From [11], the asymptotic diversity order of a system with M relay nodes is

$$\mathsf{d}_M \triangleq - \lim_{\overline{\gamma}\to\infty} \frac{\log \overline{P}_b^{(M)}(\overline{\gamma})}{\log(\overline{\gamma})}, \tag{64}$$

where $\overline{\gamma}$ is the average SNR, and $\overline{P}_b^{(M)}(\overline{\gamma})$ is the average BER for an M-relay system with an SNR of $\overline{\gamma}$. This expression can be used for the noncoherent relay channels. For BFSK, using the lower bound of (49) and the upper bound of (61), on the BER, in (64), we conclude that

$$\begin{aligned} & \mathsf{d}_{M,BFSK} \;\leq\; M+1, \\ \text{and}\quad & \mathsf{d}_{M,BFSK} \;\geq\; M+1, \end{aligned} \tag{65}$$

respectively. That is, BFSK achieves a full diversity order of $M+1$. Furthermore, in [14] we show that the suboptimum BFSK receiver of (63) also achieves a full diversity of $M+1$ as $\overline{\gamma}\to\infty$. Using the lower bound on the BER for OOK, (36), in (64) yields the following upper bound on the diversity order:

$$\mathsf{d}_{M,OOK} < M+1, \tag{66}$$

Also, the upper bound on the BER of (56) yields the following lower bound on $\mathsf{d}_{M,OOK}$:

$$\mathsf{d}_{M,OOK} \geq (M+1)/2. \tag{67}$$

Taken together, (66) and (67) show that OOK achieves a diversity order of at least $(M+1)/2$, but cannot achieve the full diversity of $M+1$. In particular, for a noncoherent OOK system without the relay, upon setting $M=0$ in (66), the asymptotic diversity order is less than unity. This can be explained by the fact that the noncoherent ML receiver does not adapt the decision threshold, $\mathrm{T_h}$, to the instantaneous channel gains, and hence results in a loss of performance[2]. This would seemingly be attributed to a loss in diversity if the asymptotic diversity result was blindly applied. However, since a diversity order of less than unity does not appear to have a sensible physical interpretation, care should be exercised when attempting to interpret performance behavior based solely upon asymptotic diversity analysis.

[2] At high SNR, the average BER of the noncoherent OOK receiver with direct transmission is given by $\log(\overline{\gamma})/\overline{\gamma}$ [16]. Our lower bound on the average BER with M relays (cf. (36)), is consistent with the results of [16].

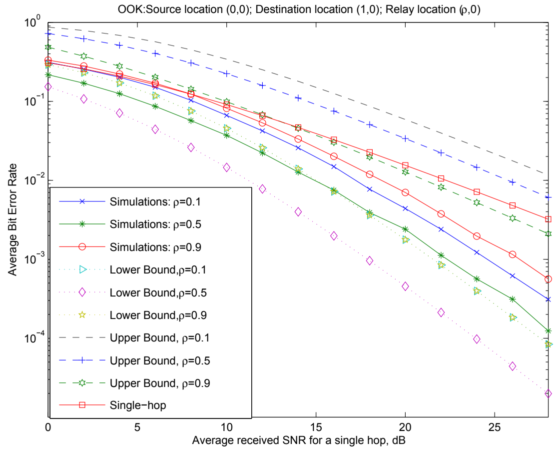

Fig. 3. Comparison of the average BER for OOK modulation with noncoherent demodulation. The curves with legend "Upper Bound" correspond to Bhattacharyya distance between the likelihood functions, whereas the curves with legend "Lower Bound" correspond to the assumption that the link between the relay and the destination is unfaded. The curves with legend "Simulations" are essentially the same as that of the simulation results of Fig. 1.

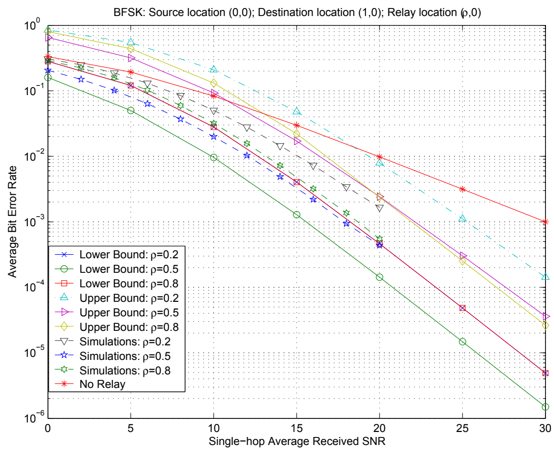

Fig. 4. Comparison of the average BER for BFSK modulation with noncoherent demodulation. The curves with legend "Upper Bound" correspond to Bhattacharyya distance between the likelihood functions, whereas the curves with legend "Lower Bound" correspond to the assumption that the link between the relay and the destination is unfaded.

VII. Results and Discussion

We now present some numerical and simulation results to illustrate the performance of receivers for OOK and BFSK signal sets, as derived in the previous sections. Results are shown for $M=1$ relay. The distance between the source and the destination is set to unity, and the relay is placed on the line joining the source and the destination. The path loss exponent η is set to four, so that we have $\overline{\gamma}_1 = d_{DS}^{-\eta}\overline{\gamma}/2 = \overline{\gamma}/2$, $\overline{\gamma}_2^1 = d_{RS}^{-\eta}\overline{\gamma}/2$, and $\overline{\gamma}_3^1 = d_{DR}^{-\eta}\overline{\gamma}/2$, where $\overline{\gamma}$ is the SNR without a relay. The factor $1/2$ is due to the power split between the source and the relay. Fig. 1 shows the average BER of the optimum OOK receiver of Eqn. (12) as a function of the single-hop SNR, $\overline{\gamma}$. Three scenarios of the relay placement are considered: 1)relay is closer to the source than to the destination, with $d_{RS}=0.1$, 2)relay is closer to the destination than to the source, with $d_{RS}=0.9$, and

3)relay at the midpoint between the source and the destination with $d_{RS} = 0.5$. Also shown is the average BER for single-hop transmission, which is obtained analytically by evaluating Eqn. (27) with $M = 0$, and the BER performance obtained by using the suboptimum detector of Eqn. (62). From Fig. 1, we observe that placement of the relay at the midpoint uniformly minimizes the average BER, whereas relay placement close to the destination results in worse BER performance. In fact, at lower values of $\overline{\gamma}$, single-hop transmission performs slightly better than the case with $d_{RS} = 0.9$, which can be attributed to an increase in the noise amplification at the relay. Compared with single-hop performance, as $\overline{\gamma}$ increases, we notice from Fig. 1 an improved performance with a single relay node. We also conclude from Fig. 1 that the suboptimum detector performs reasonably well, compared with the ML NCAF receiver, when the relay is close to the source. This can be explained by the fact that noise amplification at the relay is less severe when the relay is close to the source.

The average BER for the optimum BFSK receiver of Eqn. (23) is plotted in Fig. 2 as a function of $\overline{\gamma}$. The placement of the relay is the same as that of Fig. 1. The suboptimum receiver as given in (63) is also considered. The following observations can be made from Fig. 2. First, with optimum reception, BER performance with $d_{RS} = \rho = 0.5$ is uniformly better than with $\rho = 0.1$ and 0.9, which is due to the fact that, with $\rho = 0.5$, the noise amplification at the relay is roughly balanced by the strong signal from the relay to the destination. The suboptimum receiver of (62) has identical performance as that of the optimum one (over the plotted range of SNR values) at $\rho = 0.1$ and performs very close to the optimum one at $\rho = 0.5$, whereas its performance is inferior to the optimum one at $\rho = 0.9$. This is due to the fact that, for relay placement closer to the destination that to the source, the suboptimum receiver suffers from more noise amplification than the optimum one. Also, notice from Fig. 2 that, over the range of the plotted average SNR, $\overline{\gamma}$, performance with relay is uniformly better than the single-hop transmission.

Finally, we present upper and lower bounds on the average BER performance of the NCAF receivers. Fig. 3 shows the average BER performance with OOK modulation. In Fig. 3, the upper bound is obtained by evaluating (52), whereas the lower bound is given by (27). Fig. 4 plots the average BER for BFSK using the upper bound of (57), and the lower bound of (43). The average BER is parameterized by $\rho \in \{0.1, 0.5, 0.9\}$ in Fig. 3, and by $\rho \in \{0.2, 0.5, 0.8\}$ in Fig. 4. The lower bounds in Figs. 3 and 4 show that the placement of relay at the midpoint between the source and the destination is optimal, whereas relay placement close to the source yields the same performance as that of placement close to the destination. This can be explained with the observation that Eqns. (27) and (43) are symmetric with respect to $\overline{\gamma}_2^j$ and $\overline{\gamma}_3^j$. That is, by exchanging $\overline{\gamma}_2^j$ and $\overline{\gamma}_3^j$, the resulting average BER does not change. For OOK, the upper bounds in Fig. 3 indicate that single-hop transmission has better performance over the relay-based one when $\rho \in \{0.1, 0.5\}$, whereas with BFSK Fig. 4 shows that, at high SNR, the relay-based system performs better than the single-hop system.

VIII. Conclusion

We have presented ML receiver structures for noncoherent amplify-and-forward communication when multiple relay nodes are employed. We considered both OOK and BFSK modulation schemes on Rayleigh fading channels with no receiver CSI. It was observed that, even for the simplest case of having only one relay node, the optimum noncoherent receiver is quite involved, and the ML metric computation requires evaluation of certain integrals. Next, we presented lower and upper bounds on the average BER, and also proposed simple suboptimum receivers along with their performance evaluation. Our asymptotic diversity analysis showed that, with M relay nodes, and a link between the source and the destination, OOK achieves a diversity order of at least $(M + 1)/2$, whereas BFSK achieves the full diversity of $M + 1$.

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