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Anomaly cancellation in effective supergravity theories from the heterotic string: Two simple examples [☆]

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Abstract

We use Pauli–Villars regularization to evaluate the conformal and chiral anomalies in the effective field theories from Z_3 and Z_7 compactifications of the heterotic string without Wilson lines. We show that parameters for Pauli–Villars chiral multiplets can be chosen in such a way that the anomaly is universal in the sense that its coefficient depends only on a single holomorphic function of the three diagonal moduli. It is therefore possible to cancel the anomaly by a generalization of the four-dimensional Green–Schwarz mechanism. In particular we are able to reproduce the results of a string calculation of the four-dimensional chiral anomaly for these two models.

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1. Introduction

On-shell Pauli–Villars regularization of the one-loop divergences of supergravity theories was used to determine the anomaly structure of supergravity in [1]. Pauli–Villars regulator fields

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allow for the cancellation of all quadratic and logarithmic divergences [2], as well as most linear divergences [1]. If all linear divergences were canceled, the theory would be anomaly free, with noninvariance of the action arising only from Pauli–Villars masses. However there are linear divergences associated with nonrenormalizable gravitino/gaungino interactions that cannot be canceled by PV fields. The resulting chiral anomaly forms a supermultiplet with the corresponding conformal anomaly, provided the ultraviolet cut-off has the appropriate field dependence, in which case uncanceled total derivative terms, such as Gauss–Bonnet, do not drop out from the effective action. The resulting anomaly term that is quadratic in the field strength associated with the space–time curvature, as well as the term quadratic in the Yang–Mills field strength, was shown in [1] to be canceled by the four-dimensional version of the Green–Schwarz mechanism in Z_3 and Z_7 compactifications, in agreement with earlier results [3]. However, the terms in the anomaly that are quadratic and cubic in the parameters of the anomalous transformation are prescription dependent [4,1]. The choice of PV fields with noninvariant masses used in [1] did not achieve full anomaly cancellation.

Every contribution to the chiral anomaly has a conformal anomaly counterpart, with which it combines to form an “F-term” anomaly. In addition there are “D-term” anomalies associated with logarithmic divergences that have no chiral partner. In a generic supergravity theory, these include terms [1] that are nonlinear in the holomorphic functions $F^i(T^i)$ of the three diagonal Kähler moduli T^i that characterize modular (or T-duality) transformations:

$$T'^i = \frac{a_i - ib_i T^i}{ic_i T^i + d_i}, \quad a_i b_i - c_i d_i = 1, \quad a_i, b_i, c_i, d_i \in \mathbf{Z},$$

$$\Phi'^a = e^{-\sum_i q_i^a F^i(T^i)} \Phi^a, \quad F^i(T^i) = \ln(ic_i T^i + d_i), \tag{1.1}$$

where Φ^a is any chiral supermultiplet other than a diagonal Kähler modulus, and q_i^a are its modular weights. Only terms in the anomaly that are linear in $F = \sum_i F^i$ can be canceled by the Green–Schwarz term.

In addition, in generic supergravity there are anomalous terms that involve the dilaton superfield S in the chiral supermultiplet formulation or L in the linear multiplet formulation [5] for the dilaton. Specifically, one expects [6] a term quadratic in the Kähler field strength

$$X_{\mu\nu} = \left(\mathcal{D}_\mu z^i \mathcal{D}_\nu \bar{z}^{\bar{m}} - \mathcal{D}_\nu z^i \mathcal{D}_\mu \bar{z}^{\bar{m}} \right) K_{i\bar{m}} - i F_{\mu\nu}^a (T_a z^i) K_i, \tag{1.2}$$

where $z^i = Z^i|$ is the scalar component of a generic chiral superfield Z^i , $F_{\mu\nu}^a$ is the gauge field strength, T_a is a gauge group generator, and $K(Z, \bar{Z})$ is the Kähler potential. The term quadratic in $X_{\mu\nu}$ was actually found to vanish in [1], but there remained terms linear in $X_{\mu\nu}$ as well as terms involving the Kähler potential in the nonlinear F^i terms mentioned above. Anomaly cancellation by a Green–Schwarz mechanism, to be outlined in the next section, requires that the operators appearing in the anomaly also appear in the real superfield Ω of the (modified) linearity condition for the superfield L :

$$\left(\bar{\mathcal{D}}^2 - 8R \right) (L + \Omega) = \left(\mathcal{D}^2 - 8\bar{R} \right) (L + \Omega) = 0, \quad \mathcal{D}^2 = \mathcal{D}^\alpha \mathcal{D}_\alpha,$$

$$\bar{\mathcal{D}}^2 = \mathcal{D}_{\dot{\alpha}} \mathcal{D}^{\dot{\alpha}} = (\mathcal{D}^2)^\dagger, \tag{1.3}$$

where \mathcal{D}_α is a spinorial derivative and $R = \bar{R}^\dagger$ is the auxiliary field of the supergravity multiplet whose vev determines the gravitino mass: $\langle R| = \frac{1}{2} m_{\frac{3}{2}}$. The action written in terms of L is related to the action written in terms of S by a superfield duality transformation; the standard derivation of the duality transformation requires that Ω be independent of S . It was shown in

appendix E of the first reference in [1] that the duality transformation still goes through with a slight modification if this is not the case. On the other hand it might perhaps be reasonable to impose

$$\frac{\partial \Omega}{\partial S} = 0, \tag{1.4}$$

which is in fact the case for the chiral anomaly found in the string calculation of [4]. We show that it is possible to eliminate all terms that depend on the full Kähler potential K , as well as all terms nonlinear in F , and to reproduce the result given in [4]. However, as discussed in Appendix B, there may be a residual S -dependent contribution of the part of the “D-term” anomaly that arises from uncanceled logarithmic divergences.

In the following section we outline the four-dimensional Green–Schwarz mechanism. In Section 3 we briefly recall the results of [1] and the differences obtained with the present approach. In Sections 4 and 5 we introduce the relevant set of PV fields, outline the conditions for cancellation of ultraviolet divergences and present our results for Z_3 and Z_7 orbifolds. We summarize our results in Section 6. Some details are relegated to Appendices.

2. The 4-d Green–Schwarz mechanism

The four dimensional version of the Green–Schwarz (GS) mechanism was originally formulated [3] as a means of canceling the anomaly term quadratic in Yang–Mills fields, using the chiral formulation for the dilaton. The classical Lagrangian for the Yang–Mills field strength reads

$$\mathcal{L}_{\text{YM}} = -\sqrt{g} \frac{s}{8} \sum_a \left(F_{\mu\nu}^a - i \tilde{F}_{\mu\nu}^a \right) F_a^{\mu\nu} + \text{h.c.}, \quad s = |S|. \tag{2.1}$$

Under the anomalous modular transformation (1.1) the quantum corrected Lagrangian varies according to

$$\Delta \mathcal{L}_{\text{YM}} = -\frac{\sqrt{g}}{64\pi^2} \sum_{a,i} F^i \left[C_a + \sum_b (2q_i^b - 1) C_a^b \right] \left(F_{\mu\nu}^a - i \tilde{F}_{\mu\nu}^a \right) F_a^{\mu\nu} + \text{h.c.}, \tag{2.2}$$

where C_a is the quadratic Casimir in the adjoint representation of the gauge group factor \mathcal{G}_a and C_a^b is the Casimir for the representation of the chiral supermultiplet Φ^b . In Z_3 and Z_7 orbifolds one has the universality condition:

$$C_a + \sum_b (2q_i^b - 1) C_a^b = 8\pi^2 b \quad \forall \quad i, a, \tag{2.3}$$

with $b = 30/8\pi^2$ in the absence of Wilson lines. The dilaton is classically invariant under the modular transformation (1.1). However if we impose the transformation property:

$$\Delta s = -bF = -b \sum_i F^i(t^i), \quad t^i = T^i, \tag{2.4}$$

the variation of the classical Lagrangian (2.1) cancels (2.2).

Now consider the superspace Lagrangian¹

¹ We use the Kähler superspace formulation of supergravity [5].

$$\begin{aligned} \mathcal{L} &= \int d^4\theta E (S + \bar{S}) \Omega = -\frac{1}{8} \int d^4\theta \frac{E}{R} (\bar{\mathcal{D}}^2 - 8R) (S\Omega) + \text{h.c.} \\ &= -\frac{1}{8} \int d^4\theta \frac{E}{R} S\Phi + \text{h.c.}, \end{aligned} \tag{2.5}$$

where E is the superdeterminant of the supervielbein, Ω is the real superfield appearing in (1.3), Φ is its chiral projection:

$$(\bar{\mathcal{D}}^2 - 8R) \Omega = \Phi, \tag{2.6}$$

and we used superspace integration by parts [5]. When Φ is replaced by the Yang–Mills superfield strength bilinear $W_a^\alpha W_\alpha^a$, (2.5) is just the Yang–Mills Lagrangian that includes the term in (2.1). If, under the modular transformation (1.1) the quantum Lagrangian varies according to

$$\Delta \mathcal{L}_{\text{anom}} = b \int d^4\theta [F(T) + \bar{F}(\bar{T})] \Omega = -\frac{b}{8} \int d^4\theta \frac{E}{R} F(T) \Phi + \text{h.c.}, \tag{2.7}$$

the full Lagrangian is invariant provided

$$\Delta S = -bF(T). \tag{2.8}$$

However the classical Kähler potential for the dilaton is no longer invariant and must be modified:

$$k_{\text{class}}(S, \bar{S}) = -\ln(S + \bar{S}) \rightarrow k(S, \bar{S}) = -\ln(S + \bar{S} + V_{GS}), \tag{2.9}$$

where V_{GS} is a real function of the chiral supermultiplets that transforms under (1.1) as

$$\Delta V_{GS} = b(F + \bar{F}). \tag{2.10}$$

A simple solution consistent with string calculation results [3] is

$$V_{GS} = bg(T, \bar{T}), \tag{2.11}$$

where

$$g(T, \bar{T}) = \sum_i g^i(T^i, \bar{T}^i), \quad g^i = -\ln(T^i + \bar{T}^i) \tag{2.12}$$

is the Kähler potential for the moduli. The modification (2.9) is the 4d GS term in the chiral formulation.

The 4d GS mechanism is in fact more simply formulated in the linear multiplet formalism for the dilaton. The linear superfield L remains invariant, its Kähler potential is unchanged, and one simply adds a term to the Lagrangian. Using (1.3) and (2.6):

$$\begin{aligned} \mathcal{L}_{GS} &= -\int d^4\theta ELV_{GS}, \\ \Delta \mathcal{L}_{GS} &= -b \int d^4\theta ELF + \text{h.c.} = \frac{b}{8} \int d^4\theta \frac{E}{R} F (\bar{\mathcal{D}}^2 - 8R) L + \text{h.c.} \\ &= \frac{b}{8} \int d^4\theta \frac{E}{R} F\Phi + \text{h.c.} = -\Delta \mathcal{L}_{\text{anom}} \end{aligned} \tag{2.13}$$

3. The anomaly in supergravity

As mentioned in the introduction, the quadratic and logarithmic divergences of supergravity can be canceled [2] by a suitable set of Pauli–Villars (PV) supermultiplets. It is straightforward to see [1], by an examination of the quadratic divergences, that not all of these fields can have large PV masses that are invariant under nonlinear transformations on the fields that effect a Kähler transformation, such as the modular transformations (1.1), as we will illustrate with an example below.

It was shown in [1] that modular noninvariant masses can be restricted to a subset of PV chiral supermultiplets Φ^C with diagonal Kähler metric:

$$K(\Phi^C, \bar{\Phi}^C) = f^C(Z, \bar{Z})|\Phi^C|^2. \tag{3.1}$$

In particular, those PV fields that have superpotential couplings to light fields and contribute to the renormalization of the Kähler potential can be chosen to have invariant PV masses. The fields in (3.1) acquire masses through superpotential terms:

$$W(\Phi^C, \Phi'^C) = \mu_C \Phi^C \Phi'^C, \tag{3.2}$$

with μ_C constant (in the absence of threshold corrections, as for the cases considered here). We can define a superfield²

$$\mathcal{M}_C^2 = \exp(K - f^C - f'^C) = \exp(K - 2\bar{f}^C), \quad \bar{f}^C = \frac{1}{2}(f^C + f'^C), \tag{3.3}$$

whose lowest component $m_C^2 = \mathcal{M}_C^2|$ is the Φ^C, Φ'^C squared mass. Then the anomalous part of the one-loop corrected supergravity Lagrangian takes the form [1]

$$\mathcal{L}_{\text{anom}} = \mathcal{L}_0 + \mathcal{L}_1 + \mathcal{L}_r = \int d^4\theta E (L_0 + L_1 + L_r), \tag{3.4}$$

$$L_0 = \frac{1}{8\pi^2} \left[\text{Tr}\eta \ln \mathcal{M}^2 \Omega_0 + K (\Omega_{GB} + \Omega_D) \right], \quad L_r = -\frac{1}{192\pi^2} \text{Tr}\eta \int d \ln \mathcal{M} \Omega_r, \tag{3.5}$$

where $\eta = \pm 1$ is the PV signature,

$$\Omega_0 = -\frac{1}{24} \Omega_{GB} + \Omega_{YM} - \frac{1}{12} G_{\beta\alpha} G^{\alpha\dot{\beta}} + \frac{1}{3} R \bar{R} - \frac{1}{48} \left(\mathcal{D}^2 R + \bar{\mathcal{D}}^2 \bar{R} \right), \tag{3.6}$$

$$\begin{aligned} \Omega_r = & -\frac{\partial}{\partial \ln \mathcal{M}} \left[\frac{1}{4} \left(\mathcal{D}^2 \ln \mathcal{M} \mathcal{D}_{\dot{\beta}} \ln \mathcal{M} \mathcal{D}^{\dot{\beta}} \ln \mathcal{M} + \text{h.c.} \right) - 2G_{\alpha\dot{\beta}} \mathcal{D}^\alpha \ln \mathcal{M} \mathcal{D}^{\dot{\beta}} \ln \mathcal{M} \right. \\ & + \left. \left(\ln \mathcal{M} \left\{ \frac{1}{8} \bar{\mathcal{D}}^2 \mathcal{D}^2 \ln \mathcal{M} + \mathcal{D}^\alpha (R \mathcal{D}_\alpha \ln \mathcal{M}) \right\} + \text{h.c.} \right) \right. \\ & + \frac{1}{2} \mathcal{D}^\alpha \ln \mathcal{M} \mathcal{D}_\alpha \ln \mathcal{M} \mathcal{D}_{\dot{\beta}} \ln \mathcal{M} \mathcal{D}^{\dot{\beta}} \ln \mathcal{M} \\ & \left. - (\ln \mathcal{M})^2 \left(\frac{1}{4} \mathcal{D}^\alpha L_\alpha + \ln \mathcal{M} \mathcal{D}^\alpha X_\alpha \right) \right], \tag{3.7} \end{aligned}$$

² The constants μ_C in (3.2) drop out of the variation $\Delta \mathcal{L}_{\text{anom}}$ of the effective action (3.4), and we ignore them throughout.

with

$$X_\alpha = -\frac{1}{8}(\bar{\mathcal{D}}^2 - 8R)\mathcal{D}_\alpha K, \quad L_\alpha = (\bar{\mathcal{D}}^2 - 8R)\mathcal{D}_\alpha \ln \mathcal{M}, \quad (3.8)$$

$G_{\hat{\beta}\alpha}$ is an auxiliary superfield of the gravity supermultiplet, and Ω_D represents the ‘‘D-term’’ anomaly (see Appendix B) that, together with a contribution to the Gauss–Bonnet term Ω_{GB} :

$$\Omega_{\text{GB}} = -8\Omega_W - \frac{4}{3}\Omega_X - G_{\hat{\beta}\alpha}G^{\alpha\hat{\beta}} + 4R\bar{R}, \quad (3.9)$$

arises from uncanceled total derivatives with logarithmically divergent coefficients as discussed in the introduction. Supersymmetry of these terms requires a field-dependent cut-off:

$$\Lambda = \mu_0 e^{K/4}. \quad (3.10)$$

The constant μ_0 drops out of the effective action (3.4).

The Chern–Simons superfields Ω_W , Ω_X and Ω_{YM} are defined by their chiral projections:

$$\begin{aligned} (\bar{\mathcal{D}}^2 - 8R)\Omega_W &= W^{\alpha\beta\gamma}W_{\alpha\beta\gamma}, & (\bar{\mathcal{D}}^2 - 8R)\Omega_X &= X^\alpha X_\alpha, \\ (\bar{\mathcal{D}}^2 - 8R)\Omega_{\text{YM}} &= W_a^\alpha W_\alpha^a, \end{aligned} \quad (3.11)$$

where $W_{\alpha\beta\gamma}$ is the superfield strength for space–time curvature.

\mathcal{L}_1 is defined by its variation:

$$\Delta L_1 = \frac{1}{8\pi^2} \frac{1}{192} \text{Tr}\eta \Delta \ln \mathcal{M}^2 \Omega'_L = \frac{1}{8\pi^2} \frac{1}{192} \text{Tr}\eta H \Omega'_L + \text{h.c.}, \quad (3.12)$$

where under (1.1) $\ln \mathcal{M}^2$ transforms as

$$\Delta \ln \mathcal{M}^2 = H + \bar{H}, \quad (3.13)$$

with H holomorphic. Defining

$$\begin{aligned} (\bar{\mathcal{D}}^2 - 8R)\Omega_f &= f^\alpha f_\alpha, & (\bar{\mathcal{D}}^2 - 8R)\Omega_{\bar{f}} &= \bar{f}^\alpha \bar{f}_\alpha, & (\bar{\mathcal{D}}^2 - 8R)\Omega_{\bar{f}X} &= \bar{f}^\alpha X_\alpha, \\ f_\alpha &= -\frac{1}{8}(\bar{\mathcal{D}}^2 - 8R)\mathcal{D}_\alpha \ln f, & \bar{f}_\alpha &= -\frac{1}{8}(\bar{\mathcal{D}}^2 - 8R)\mathcal{D}_\alpha \ln \bar{f}, \end{aligned} \quad (3.14)$$

we have

$$\begin{aligned} \Omega'_L &= 192\Omega_f - 128\Omega_{\bar{f}} - 64\Omega_{\bar{f}X}, \\ \Delta L_1 &= \frac{1}{8\pi^2} \text{Tr}\eta H \left(\Omega_f - \frac{2}{3}\Omega_{\bar{f}} - \frac{1}{3}\Omega_{\bar{f}X} \right) + \text{h.c.} \end{aligned} \quad (3.15)$$

The general form of f^C is taken to be

$$\begin{aligned} \ln f^C &= \alpha^C K(Z, \bar{Z}) + \beta^C g(T, \bar{T}) + \delta^C k(S, \bar{S}) + \sum_n q_n^C g^n(T^n, \bar{T}^n), \\ \ln \bar{f}^C &= \bar{\alpha}^C K + \bar{\beta}^C g + \bar{\delta}^C k + \sum_n \bar{q}_n^C g^n, \\ H^C &= (1 - 2\bar{\gamma}^C) F(T) - 2 \sum_n \bar{q}_n^C F^n(T^n), \quad \bar{\gamma}^C = \bar{\alpha}^C + \bar{\beta}^C. \end{aligned} \quad (3.16)$$

The traces in $\Delta\mathcal{L}_{\text{anom}}$ can be evaluated using only PV fields with noninvariant masses or using the full set of PV fields, since those with invariant masses, $H^C = 0$, drop out. The contribution ΔL_0 to the anomaly is linear in the parameters α^C, β^C, q_n^C ; as a consequence the traces are completely determined by the sum rules [2]

$$\begin{aligned}
 N' &= \sum_C \eta^C = -N - 29, & N'_G &= \sum_\gamma \eta_\gamma^V = -12 - N_G, \\
 \sum_C \eta^C \ln f^C &= -10K - \sum_q q_n^p g^n, & &
 \end{aligned}
 \tag{3.17}$$

that are required to assure the cancellation of all quadratic and logarithmic divergences. In (3.17) the index C denotes any chiral PV field, the index γ runs over the Abelian gauge PV superfields that are needed to cancel some gravitational and dilaton-gauge couplings, and the sum over p includes all the light chiral multiplet modular weights with $q_n^S = 0, q_n^{T^i} = 2\delta_n^i$. N and N_G are the total number of chiral and gauge supermultiplets, respectively, in the light sector. All PV fields with noninvariant masses have $\delta = 0$, and most³ with $\delta \neq 0$ have $\alpha = \beta = q_n = 0$. For the purposes of the present analysis we can ignore the latter.

To see that not all the PV chiral multiplets can have invariant masses, there is a quadratically divergent contribution from the light sector given by

$$\mathcal{L}_Q \ni -\sqrt{g} \frac{\Lambda^2}{64\pi^2} (3 + N_G - N) \mathcal{D}^\alpha X_\alpha \Big|, \tag{3.18}$$

where X_α is defined in (3.8). The Pauli–Villars contribution to the operator in (3.18) is

$$\mathcal{L}_Q^{\text{PV}} \ni -\sqrt{g} \frac{\Lambda^2}{64\pi^2} (N'_G - N' - 2\alpha) \mathcal{D}^\alpha X_\alpha \Big|, \tag{3.19}$$

where $\alpha = \sum \eta^C \alpha^C$. The PV chiral multiplets include a subset θ^a with $N'_\theta = N'_G$ which form massive vector supermultiplets with the PV Abelian gauge supermultiplets; these cancel in (3.19). The remainder get superpotential masses as in (3.2). The pair Φ^C, Φ'^C will have an invariant mass if $\ln \bar{f}^C = \bar{\alpha}^C = \frac{1}{2}$, in which case the total contribution of the pair to (3.19) vanishes identically. Therefore chiral fields with noninvariant masses are needed to cancel (3.18).

In contrast to \mathcal{L}_0 , the contributions to the anomaly from \mathcal{L}_1 and \mathcal{L}_r are nonlinear in the parameters α, β, q , and depend on the details of the PV sector. In [1] the PV sector was constructed in such a way that

$$f^C = f'^C = \bar{f}^C \tag{3.20}$$

for the PV fields with noninvariant masses. In this case (3.15) reduces to

$$(\Omega'_L)_{[1]} = 64 \left(\Omega_{\bar{f}} - \Omega_{\bar{f}X} \right) = \Omega_L - 16\Omega_X, \quad (\bar{D}^2 - 8R)\Omega_L = L^\alpha L_\alpha, \tag{3.21}$$

and, for example,

³ There is a set of chiral multiplets in the adjoint representation of the gauge group that has $\ln f = K - k$; these get modular invariant masses though their coupling in the superpotential to a second set with $\ln f = k$. These cancel renormalizable gauge interactions and gauge-gravity interactions, respectively. Together with a third set, that has $f = 1$ and contributes to the anomaly, they cancel the Yang–Mills contribution to the beta-function.

$$\text{Tr}\eta H \Omega_{\bar{f}} = \sum_C \eta_C \left[\left(1 - 2\bar{\gamma}^C\right) F - 2 \sum_n \bar{q}_n^C F^n \right] \times \left(\bar{\alpha}^C X^\alpha + \bar{\beta}^C g^\alpha + \sum_m \bar{q}_m^C g_m^\alpha \right) \left(\bar{\alpha}^C X_\alpha + \bar{\beta}^C g_\alpha + \sum_l \bar{q}_l^C g_\alpha^l \right). \tag{3.22}$$

The Pauli–Villars modular weights q_n^C are related to the modular weights q_n^p of the light fields by the conditions for the cancellation of UV divergences. In the Z_3 and Z_7 orbifolds considered below, the latter satisfy sum rules of the form:

$$\sum_p q_n^p = A_1, \quad \sum_p q_n^p q_m^p = A_2 + B_2 \delta_{mn}. \tag{3.23}$$

The first sum rule in (3.23) assures the universality of the anomaly proportional to $\Omega_0 - \Omega_{\text{YM}}$. However, in the PV sector used in [1] the second equality led to a nonuniversal term:

$$\begin{aligned} \text{Tr}\eta H \Omega_{\bar{f}} \ni & -4 \sum_{p,m,n} q_n^p q_m^p F^n g_m^\alpha \left(\bar{\alpha}^C X_\alpha + \bar{\beta}^C g_\alpha \right) \\ & = -4 \left(A_2 F g^\alpha + B_2 F^n g_n^\alpha \right) \left(\bar{\alpha}^C X_\alpha + \bar{\beta}^C g_\alpha \right). \end{aligned} \tag{3.24}$$

The sum rule cubic in the modular weights is more complicated, but in general leads to additional nonuniversal terms. These can be avoided by imposing $\bar{q}_n^C = 0$ for fields with noninvariant masses, but if (3.20) is imposed we get

$$\text{Tr}\eta H \Omega'_L = F \left(a X^\alpha X_\alpha + b X^\alpha g_\alpha + c g^\alpha g_\alpha \right), \tag{3.25}$$

which does not include the term proportional to

$$F \sum_n g_n^\alpha g_\alpha^n \tag{3.26}$$

found in the string calculation⁴ of [4].

In the following we relax the assumption (3.20), impose $\bar{q}_n^C = 0$, but with $q^C = -q'^C \neq 0$. This still assures a universal anomaly, but allows more freedom in determining its coefficient; in particular, we are able to reproduce the term (3.26).

4. Cancellation of UV divergences

The full set of PV fields needed to regulate light field couplings is described in Section 3 of [1]. Among those, here we are primarily concerned with the set $\dot{Z}^P = \dot{Z}^I, \dot{Z}^A$, with negative signature, $\eta^{\dot{Z}} = -1$, that regulates most of the couplings, including all renormalizable couplings, of the light chiral supermultiplets $Z^P = T^i, \Phi^a$. Covariance of the \dot{Z}^P Kähler metric requires that these fields transform under (1.1) like dZ^P :

$$\dot{Z}^I = e^{-2F^i} \dot{Z}^I, \quad \dot{Z}^A = e^{-F^a} \left(\dot{Z}^A - \sum_j F_j^a \Phi^a \dot{Z}^j \right), \quad F^a = \sum_i F^i (T^i) \tag{4.1}$$

⁴ In fact the four-form $\epsilon^{\mu\nu\rho\sigma} g_{\mu\nu}^n g_{\rho\sigma}^n$ with $g_{\mu\nu}^n = (\partial_\mu t^n \partial_\nu \bar{t}^n) g_{t^n \bar{t}^n}^n - (\mu \leftrightarrow \nu)$, that appears in the chiral part of (3.26), vanishes identically. We find it curious that the authors of [4] neglected to comment on this fact. However the associated conformal anomaly is nontrivial.

Invariance of the full PV Kähler potential for the \dot{Z}^P and covariance of their superpotential under (1.1):

$$K(\dot{Z}') = K(\dot{Z}), \quad W(\dot{Z}') = e^{-F(T)} W(\dot{Z}), \tag{4.2}$$

can be made manifest if we supplement [1] these fields with three additional PV fields \dot{Z}^N , $N = 1, 2, 3$, with Kähler potential

$$K(\dot{Z}^N) = \sum_{i=n} \left| \dot{Z}^N + \dot{a} \chi^n(T^i) \dot{Z}^i \right|^2, \quad \chi^n(T^i) = e^{2F^n} \left(\chi^n(T^i) + F_i^n \right), \tag{4.3}$$

and that transform under (1.1) according to

$$\dot{Z}'^N = \dot{Z}^N - \dot{a} F_i^n(T^i) \dot{Z}^i, \tag{4.4}$$

where \dot{a} is a nonzero constant.⁵

We wish to give these PV fields modular invariant masses. The simplest way to do this is to introduce fields \dot{Y}_P, \dot{Y}_N with the same signature, opposite gauge charges and the inverse Kähler metric. However this would have the effect of canceling the \dot{Z} contributions that are linear in the generalized field strength

$$G_{\mu\nu} = [D_\mu, D_\nu], \tag{4.5}$$

and doubling the quadratic \dot{Z} contributions. Instead we introduce fields \dot{Y}_P, \dot{Y}_N with gauge charges

$$(T_a)\dot{Y} = -(T_a^T)\dot{Z} = -(T_a^T)Z, \tag{4.6}$$

and Kähler potential

$$K(\dot{Y}) = e^{\dot{G}} \left(\sum_A e^{-g^a} |\dot{Y}_A|^2 + \sum_I e^{-2g^i} |\dot{Y}_I - \dot{a} \chi^n(T^i) \dot{Y}_N|^2 + |\dot{Y}_N|^2 \right), \tag{4.7}$$

$$g^a = \sum_n q_n^a g^n, \quad \dot{G} = \dot{\alpha} K + \dot{\beta} g, \quad \dot{\alpha} + \dot{\beta} = 1.$$

(4.7) is modular invariant, and the PV mass superpotential

$$W(\dot{Z}, \dot{Y}) = \dot{\mu} \left(\dot{Z}^A - \dot{a}^{-1} q_n^a \Phi^a \dot{Z}^N \right) \dot{Y}_A + \sum_{i=n} \dot{\mu}_n \left(\dot{Z}^I \dot{Y}_I + \dot{Z}^N \dot{Y}_N \right), \tag{4.8}$$

is covariant, provided under (1.1)

$$\dot{Y}'_A = e^{-F+F^a} \dot{Y}_A, \quad \dot{Y}'_I = e^{-F+2F^i} (\dot{Y}_I + \dot{a} F_i^n \dot{Y}_N), \quad \dot{Y}'_N = e^{-F} \dot{Y}_N. \tag{4.9}$$

It remains to cancel the divergences introduced by the fields \dot{Y} . This was achieved in [1] by an additional set of chiral PV fields, collectively called Ψ , with diagonal metric (3.1), superpotential (3.2), with prefactors (3.16) satisfying (3.20) and $\alpha^\Psi = \delta^\Psi = 0$. In addition $\dot{\alpha} = 0$ in (4.7) was assumed. Here we use a different set of fields, for which we assume only $\delta^C = 0$, as well as allowing $\dot{\alpha} \neq 0$. For this reason we also include in the present analysis the set of fields ϕ^C with prefactors

⁵ Depending on the choice of the functions $\chi^n(T^i)$, one might need to introduce [1] several copies of the sets $\dot{Z}_\lambda^{P,N}$, with constraints on the parameters \dot{a}_λ in such a way that no new divergences are introduced by the fields \dot{Z}^N .

$$\ln f^{\phi^C} = \alpha^C K \tag{4.10}$$

that regulate certain gravity supermultiplet loops. These must be included together with the PV fields introduced below in implementing the sum rules (3.17). We take the following set:

$$\begin{aligned} \Phi^P : \quad & \ln f^{\Phi^P} = \sum_n q_n^P g^n = \alpha^\Phi K + \beta^\Phi g - \ln f^{\Phi'^P}, \quad \alpha^\Phi + \beta^\Phi = 1, \\ \psi^{Pn} : \quad & \ln f^{Pn} = \alpha_\psi^P K + \beta_\psi^P g + q_\psi^P g^n, \quad \alpha_\psi^P + \beta_\psi^P = \gamma_\psi^P, \quad \bar{q}_\psi^P = 0, \\ T^P : \quad & \ln f_T^P = \alpha_T^P K + \beta_T^P g, \quad \alpha_T^P + \beta_T^P = \gamma_T^P. \end{aligned} \tag{4.11}$$

The pairs Φ^P, Φ'^P have modular invariant masses and do not contribute to the anomaly, but they play an important role in canceling certain divergences. In the case of Z_7 orbifolds we take them to be charged under the two $U(1)$'s of that theory. They have no other gauge charges, the ψ^{Pn} are taken to be gauge neutral, and the T^P have a priori arbitrary gauge charges. For those in real representations of the gauge group one can take $T^P = T'^P$. In Appendix A we display a simple solution to the constraints with some T 's in the fundamental and antifundamental representation of the non-Abelian gauge group factors, some with $U(1)$ charges in the Z_7 case, and some gauge singlets.

The quadratic and logarithmic divergences we are concerned with here involve the superfield strengths $-i(T_a)W_\alpha^a, X_\alpha$ and

$$\Gamma_{D\alpha}^C = -\frac{1}{8}(\bar{D}^2 - 8R)D_\alpha Z^i \Gamma_{Di}^C, \tag{4.12}$$

associated with the Yang–Mills, Kähler and reparameterization connections, respectively. Since the theories considered here have no gauge anomalies, cancellation of quadratic divergences requires

$$\text{Tr}\eta\Gamma_\alpha = 0, \tag{4.13}$$

and cancellation of logarithmic divergences requires

$$\text{Tr}\eta\Gamma_\alpha\Gamma_\beta = \text{Tr}\eta\Gamma_\alpha T^a = \text{Tr}\eta(T^a)^2 = 0, \tag{4.14}$$

where $\eta = +1$ for light fields. Cancellation of all contributions linear and quadratic in X_α is assured by the conditions in (3.17) together with (A.5) of Appendix A. The Yang–Mills contribution to the term quadratic in W_α is canceled by chiral fields in the adjoint (see footnote on page 202) that we need not consider here. Finally, cancellation of linear divergences requires cancellation of the imaginary part of

$$\text{Tr}\eta X_\chi = \text{Tr}\eta\phi G \cdot \tilde{G}, \quad \tilde{G}^{\mu\nu} = \frac{1}{2}\epsilon^{\mu\nu\rho\sigma} G_{\rho\sigma}, \tag{4.15}$$

where $G_{\mu\nu}$ is the field strength associated with the fermion connection⁶; for left-handed fermions:

$$G_{\mu\nu} = -\Gamma_{D\mu\nu}^C + iF_{\mu\nu}^a(T_a)_D^C + \frac{1}{2}X_{\mu\nu}\delta_D^C, \tag{4.16}$$

and for a generic PV superfield Φ^C with diagonal metric, its fermion component χ^C transforms under (1.1) as

⁶ Here we neglect the spin connection which is considered in Appendix B.

$$\chi'^C = e^{\phi^C} \chi^C, \quad \phi^C = \left(\frac{1}{2} - \alpha^C - \beta^C \right) F - \sum_i F^i (T^i) q_i^C. \tag{4.17}$$

The full set of conditions is extensive, and we evaluate them in [Appendix A](#). In this section we simply outline how to obtain a universal anomaly using PV regularization. For this purpose we focus on terms contributing to UV divergences that could potentially spoil universality. An important feature in our results is the fact that the expression

$$\epsilon^{\mu\nu\rho\sigma} g_{\mu\nu}^i g_{\rho\sigma}^i = 0, \tag{4.18}$$

vanishes identically, and the expressions

$$\begin{aligned} X^{ij} &= \epsilon^{\mu\nu\rho\sigma} \text{Im} F^i g_{\mu\nu}^i g_{\rho\sigma}^{j \neq i} = 4\epsilon^{\mu\nu\rho\sigma} \text{Im} F^i \partial_\mu g_\nu^i \partial_\rho g_\sigma^j = 4\partial_\rho \left(\epsilon^{\mu\nu\rho\sigma} \text{Im} F^i \partial_\mu g_\nu^i g_\sigma^j \right), \\ X^i &= \frac{1}{2} \epsilon^{\mu\nu\rho\sigma} \text{Im} F^i g_{\mu\nu}^i X_{\rho\sigma} = 4i \partial_\rho \left(\epsilon^{\mu\nu\rho\sigma} \text{Im} F^i \partial_\mu g_\nu^i \Gamma_\sigma \right), \\ X^{ia} &= \epsilon^{\mu\nu\rho\sigma} \text{Im} F^i g_{\mu\nu}^i F_{\rho\sigma}^a = 4\partial_\rho \left(\epsilon^{\mu\nu\rho\sigma} \text{Im} F^i \partial_\mu g_\nu^i A_\sigma^a \right), \end{aligned} \tag{4.19}$$

are total derivatives, where A_μ^a is an Abelian gauge field,

$$g_\mu^i = -\frac{\partial_\mu \bar{t}^i - \partial_\mu \bar{t}^{\bar{i}}}{t^i + \bar{t}^{\bar{i}}}, \quad g_{\mu\nu}^i = \partial_\mu g_\nu^i - \partial_\nu g_\mu^i, \quad \Gamma_\mu = \frac{i}{4} \left(\mathcal{D}_\mu z^i K_i - \mathcal{D}_\mu \bar{z}^{\bar{m}} K_{\bar{m}} \right), \tag{4.20}$$

and $X_{\mu\nu} = 2i (\partial_\mu \Gamma_\nu - \partial_\nu \Gamma_\mu)$ is defined in [\(1.2\)](#).

If, for example, we replaced $g^n(T^n, \bar{T}^{\bar{n}})$ everywhere by the Kähler potential for the n th untwisted sector, a possibility considered in [\[11\]](#), the above would not hold, and we would be unable to obtain a universal anomaly coefficient. Specifically, we would be not be able to cancel the terms cubic in q_n^p that appear in $X_\chi^{\dot{Y}}$, suggesting that the present construction is the only viable possibility. This agrees with the results of [\[4\]](#), where it was found that the untwisted Kähler moduli are the only chiral supermultiplets that appear in the chiral anomaly (see however footnote on page [203](#)).

4.1. Reparameterization curvature terms

The functions $\chi^n(T^i)$ in [\(4.3\)](#) and [\(4.7\)](#) do not contribute to the quantities in [\(4.12\)](#) and [\(4.16\)](#) (see footnote on page [204](#)), and, using [\(3.23\)](#), one obtains

$$\begin{aligned} \text{Tr} \dot{\eta} \Gamma_\alpha^{\dot{Y}} &= -[(N+2)\dot{\beta} - A_1] g_\alpha, \\ \text{Tr} \dot{\eta} \Gamma_\alpha^{\dot{Y}} \Gamma_\beta^{\dot{Y}} &= -2\dot{\alpha} [\dot{\beta}(N+2) - A_1] X_\alpha g_\beta - [\dot{\beta}^2(N+2) - \dot{\beta}A_1 + A_2] g_\beta g_\alpha \\ &\quad - B_2 \sum_n g_\alpha^n g_\beta^n. \end{aligned} \tag{4.21}$$

In addition we have

$$\begin{aligned} X_\chi^{\dot{Y}} &= \frac{1}{2}(N+2)F\dot{G} \cdot \tilde{G} - F \sum_{p,n} q_n^p \dot{G} \cdot \tilde{g}^n + \frac{1}{2}F \sum_{p,m,n} q_m^p q_n^p g^m \cdot \tilde{g}^n \\ &\quad - \sum_{p,n} q_n^p F^n \dot{G} \cdot \tilde{G} + 2 \sum_{p,n,m} q_m^p q_n^p F^m \dot{G} \cdot \tilde{g}^n - \sum_{p,l,m,n} q_l^a q_m^a q_n^a F^l g^m \cdot \tilde{g}^n. \end{aligned} \tag{4.22}$$

In addition to the sum rules listed in (3.23), we have

$$\sum_p q_i^p q_j^p q_k^p = A_3 + B_3 \delta_{ij} \delta_{ik} + C_3 \left[\delta_{ij} \left(\delta_i^1 \delta_k^2 + \delta_i^2 \delta_k^3 + \delta_i^3 \delta_k^1 \right) + \text{cyclic}(ijk) \right] + D_3 \left[\delta_{ij} \left(\delta_i^2 \delta_k^1 + \delta_i^3 \delta_k^2 + \delta_i^1 \delta_k^3 \right) + \text{cyclic}(ijk) \right], \tag{4.23}$$

with $C_3 = D_3$ for Z_3 , $C_3 \neq D_3$ for Z_7 . Then using (4.18) and (4.19), (4.22) reduces to

$$X_\chi^\dot{Y} = \frac{1}{2}(N + 2)F\dot{G} \cdot \tilde{G} - A_1 F\dot{G} \cdot \tilde{g} + \frac{1}{2}A_2 Fg \cdot \tilde{g} - A_1 F\dot{G} \cdot \tilde{G} + 2A_2 F\dot{G} \cdot \tilde{g} - A_3 Fg \cdot \tilde{g} + \text{total derivative}. \tag{4.24}$$

Since $\bar{q}^\Phi = \bar{q}^\psi = 0$, terms cubic in these modular weights do not contribute to X_χ^Φ, X_χ^ψ . Further, since

$$q_m^{Pn} q_l^{Pn} = (q_\psi^P)^2 \delta_m^n \delta_l^n, \tag{4.25}$$

there are no contributions to X_χ^ψ quadratic in ψ modular weights, and since q_ψ^P is independent of n , X_χ^ψ depends only on $F, g_{\mu\nu}$ and $X_{\mu\nu}$. Then imposing

$$\sum_P \eta^P q_n^P = a_1, \quad \sum_P \eta^P q_n^P q_m^P = a_2 + b_2 \delta_{mn}, \tag{4.26}$$

X_χ^Φ can also be made to depend only on $F, g_{\mu\nu}$ and $X_{\mu\nu}$. The terms linear in the Φ and ψ modular weights drop out of $(\text{Tr}\eta\Gamma_\alpha)_{\Phi,\psi}$, and one obtains⁷

$$\begin{aligned} (\text{Tr}\eta\Gamma_\alpha\Gamma_\beta)_\Phi &= \sum_P \eta_\Phi^P G_\alpha^\Phi G_\beta^\Phi - a_1 \left(G_\alpha^\Phi g_\beta + g_\alpha G_\beta^\Phi \right) + 2a_2 g_\alpha g_\beta + 2b_2 \sum_n g_\alpha^n g_\beta^n, \\ G^\Phi &= \alpha^\Phi K + \beta^\Phi g, \\ (\text{Tr}\eta\Gamma_\alpha\Gamma_\beta)_\psi &= \sum_P \eta_\psi^P \left[3G_\alpha^P G_\beta^P + q_\psi^P \left(G_\alpha^P g_\beta + g_\alpha G_\beta^P \right) \right] + B_\psi \sum_n g_\alpha^n g_\beta^n, \\ G^P &= \alpha^P K + \beta^P g, \quad B_\psi = \sum_P \eta_\psi^P (q_\psi^P)^2. \end{aligned} \tag{4.27}$$

To cancel the last term in (4.21) we require

$$2b_2 + B_\psi = B_2. \tag{4.28}$$

The \dot{Y} and Φ fields do not contribute to the anomaly, and the coefficient of the term (3.26) is determined by B_ψ . The remaining terms in (4.21) and (4.22) can be canceled by a combination of the full set of PV fields in (4.10) and (4.11), as shown in Appendix A.

4.2. Yang–Mills field strengths

The gauge charges⁸ and modular weights in Z_3 and Z_7 orbifold compactifications without Wilson lines are given in [4] and Appendix D.5 of [1]. The universality of the anomaly term

⁷ In (4.26) and for Φ in (4.27), the sum is over P only, while for ψ , $\sum_P \equiv \sum_P + \sum_{P'}$, since P and P' are interchangeable in the latter, but not in the former.

⁸ We use the standard charge normalization such that (2.3) is satisfied with $C_a^b = (\text{Tr}T_a^2)_{R(b)}$, where $R(b)$ is the gauge group representation of the chiral supermultiplet Φ^b ; this differs by a factor $\sqrt{2}$ from the normalization used in [4].

quadratic in Yang–Mills fields strengths is guaranteed by the universality condition (2.3), as illustrated in Appendix A. Since gauge transformations commute with modular transformations, a set of chiral multiplets Φ^b that transform according to a nontrivial irreducible representation R of a non-Abelian gauge group factor \mathcal{G}_a have the same modular weights q_n^R such that

$$\sum_{b \in R} q_n^b (T_a)_b^b = q_n^R (\text{Tr} T_a)_R = 0. \tag{4.29}$$

Therefore terms linear in Yang–Mills field strengths occur only for Abelian gauge group factors. There are none in Z_3 , but two in Z_7 , which we refer to as $U(1)_a$, $a = 1, 2$, with charges Q_a . These are anomaly free; their traces vanish when taken over the full spectrum of chiral multiplets. Defining

$$Q_{an} = \sum_b q_n^b Q_a^b, \quad Q_{anm} = \sum_b q_n^b q_m^b Q_a^b, \tag{4.30}$$

we have for Z_7 :

$$\begin{aligned} Q_{1n} &= \frac{1}{2}(8, 2, -10), & Q_{2n} &= \frac{1}{2\sqrt{3}}(12, -18, 6), & n &= (1, 2, 3), \\ Q_{1nm} &= \frac{1}{2}(5, -4, -1), & Q_{2nm} &= \frac{1}{2\sqrt{3}}(-3, -6, 9), \\ & & nm &= (12, 23, 31). \end{aligned} \tag{4.31}$$

These satisfy

$$\sum_n Q_{an} = 0, \quad Q_{anm} = -\frac{1}{2} |\epsilon_{nml}| Q_{al}. \tag{4.32}$$

We wish to cancel the \dot{Y} -loop contribution to logarithmic divergences

$$(\text{Tr} \eta g_\alpha^n T_a)_{\dot{Y}} = - \sum_b q_n^b Q_a^b g_\alpha^n = -Q_{an} g_\alpha^n, \tag{4.33}$$

and, dropping terms proportional to the last expression in (4.19), \dot{Y} contributions to linear divergences:

$$\begin{aligned} X_\chi^{\dot{Y}} &\ni \sum_b \tilde{F}_{\mu\nu}^a Q_a^b \left[g^{n\mu\nu} q_n^b (F - 2q_m^b F^m) + 2q_n^b F^n \left\{ \left(\dot{\alpha} - \frac{1}{2} \right) X^{\mu\nu} + \dot{\beta} g^{\mu\nu} \right\} \right] \\ &= \tilde{F}_{\mu\nu}^a F^1 \left\{ (Q_{a2} g^{2\mu\nu} + Q_{a3} g^{3\mu\nu}) + 2Q_{a1} \left[\left(\dot{\alpha} - \frac{1}{2} \right) X^{\mu\nu} + \dot{\beta} (g^{2\mu\nu} + g^{3\mu\nu}) \right] \right. \\ &\quad \left. - 2(Q_{a12} g^{2\mu\nu} + Q_{a13} g^{3\mu\nu}) \right\} + \text{cyclic}(1, 2, 3) + \text{total derivative}. \end{aligned} \tag{4.34}$$

Using (4.32), (4.34) becomes

$$\begin{aligned} X_\chi^{\dot{Y}} &\ni \tilde{F}_{\mu\nu}^a F^1 \left\{ (Q_{a2} g^{2\mu\nu} + Q_{a3} g^{3\mu\nu}) + 2Q_{a1} \left[\left(\dot{\alpha} - \frac{1}{2} \right) X^{\mu\nu} + \dot{\beta} (g^{2\mu\nu} + g^{3\mu\nu}) \right] \right. \\ &\quad \left. + (Q_{a3} g^{2\mu\nu} + Q_{a2} g^{3\mu\nu}) \right\} + \text{cyclic}(1, 2, 3) + \text{total derivative} \\ &= \tilde{F}_{\mu\nu}^a F^1 \left[(Q_{a2} + Q_{a3} + 2\dot{\beta} Q_{a1}) (g^{2\mu\nu} + g^{3\mu\nu}) + Q_{a1} (2\dot{\alpha} - 1) X^{\mu\nu} \right] \\ &\quad + \text{cyclic}(1, 2, 3) + \text{total derivative}. \end{aligned} \tag{4.35}$$

Now we assign $U(1)_a$ charges Q_a^P and $-Q_a^P$ to Φ^P and Φ'^P , respectively. This gives a contribution to logarithmic divergences

$$2 \sum_P \eta^P q_n^P Q_a^P g_\alpha^n \equiv Q_{an}^\Phi g_\alpha^n. \tag{4.36}$$

Cancellation of (4.33) requires

$$Q_{an}^\Phi = Q_{an}. \tag{4.37}$$

The Φ contribution to linear divergences is

$$\begin{aligned} X_\chi^\Phi &\ni -2 \sum_P \eta^P Q_a^P q_n^P \tilde{F}_{\mu\nu}^a F^n [(\alpha^\Phi - 1)X^{\mu\nu} + \beta^\Phi g^{\mu\nu}] \\ &= -Q_{a1}^\Phi \tilde{F}_{\mu\nu}^a F^1 [(\alpha^\Phi - 1)X^{\mu\nu} + \beta^\Phi (g^{2\mu\nu} + g^{3\mu\nu})] \\ &\quad + \text{cyclic}(1, 2, 3) + \text{total derivative}. \end{aligned} \tag{4.38}$$

To cancel the $X^{\mu\nu}$ term we require

$$\dot{\alpha} = \frac{1}{2}\alpha^\Phi, \quad \dot{\beta} = 1 - \frac{1}{2}\alpha^\Phi = \frac{1}{2}\beta^\Phi + \frac{1}{2}. \tag{4.39}$$

Then

$$\begin{aligned} X_\chi^{\dot{Y}} &\ni \tilde{F}_{\mu\nu}^a F^1 \left\{ [Q_{a2} + Q_{a3} + (\beta^\Phi + 1) Q_{a1}] (g^{2\mu\nu} + g^{3\mu\nu}) + Q_{a1} (\alpha^\Phi - 1) X^{\mu\nu} \right\} \\ &\quad + \text{cyclic}(1, 2, 3) + \text{total derivative} \\ &= \tilde{F}_{\mu\nu}^a F^1 Q_{a1} \left[\beta^\Phi (g^{2\mu\nu} + g^{3\mu\nu}) + (\alpha^\Phi - 1) X^{\mu\nu} \right] \\ &\quad + \text{cyclic}(1, 2, 3) + \text{total derivative} = -X_\chi^\Phi, \end{aligned} \tag{4.40}$$

up to a total derivative.

Note that this is a highly nontrivial result. In addition to the importance of the properties in (4.19), the relations (4.32), that are specific to the Z_7 orbifold we are considering, are crucial to the cancellations in this section. Since the Φ have modular invariant masses, the ψ 's have no gauge charges, and the T 's have n -independent prefactors f^T , no terms linear in the gauge field strengths appear in the anomaly.

Finally we remark that a pair of PV fields Φ^C, Φ'^C with superpotential coupling (3.2) contributes an amount

$$(\phi^C + \phi'^C) C_a^C = \Delta \mathcal{M}^2 C_a^C \tag{4.41}$$

to the coefficient of $F^a \cdot \tilde{F}_a$ in (4.15). This vanishes for pairs with invariant masses, and its form assures that the anomaly arising from PV masses in the regulated theory matches the anomaly due to linear divergences in the unregulated theory. In particular it makes no difference whether or not we assign non-Abelian gauge charges to the Φ^P , and their $U(1)_a$ charges have no effect on the term in the anomaly quadratic in the $U(1)_a$ field strengths.

5. The anomaly in Z_3 and Z_7 orbifolds

In Appendix A we show that is possible to cancel all the ultraviolet divergences from the \dot{Y} fields with a simple choice of the set (4.11) such that the fields with noninvariant masses have the properties

$$\text{Tr}\eta(\ln \mathcal{M})^{n>1} = \Delta \text{Tr}\eta(\ln \mathcal{M})^{n>1} = \text{Tr}\eta(\Delta \ln \mathcal{M})(\bar{f}_\alpha)^{n>0} = 0, \quad (5.1)$$

and the anomaly due to the variation of (3.4) reduces to

$$\begin{aligned} \delta \mathcal{L}_{\text{anom}} &= b \int d^4\theta EF\Omega, \\ \Omega &= \Omega_{\text{YM}} - \frac{1}{24}\Omega_{\text{GB}} - \frac{b_{\text{spin}}}{48b} \left(4G_{\dot{\beta}\alpha} G^{\alpha\dot{\beta}} - 16R\bar{R} + \mathcal{D}^2 R + \bar{\mathcal{D}}^2 \bar{R} \right) \\ &\quad + \frac{1}{30} (\Omega_f + \Omega_D), \end{aligned} \quad (5.2)$$

where (see Appendix B)

$$8\pi^2 b_{\text{spin}} = 8\pi^2 b + 1 = 31. \quad (5.3)$$

The results for the Gauss–Bonnet and Yang–Mills terms are well-established [3] and result from the universality conditions (2.3) and (B.7), as illustrated in the appendices. The only other term in (5.2) that contains a chiral anomaly is Ω_f , which, using the set (4.11) of PV fields, is a priori a product of the chiral superfields X_α , g_α and g_α^n . We show in Appendix A we may choose the PV parameters such that

$$\Omega_f = 30 \sum_n g_n^\alpha g_\alpha^n, \quad (5.4)$$

in agreement with the string calculation of [4].

The anomaly is canceled provided the Lagrangian for the dilaton S, \bar{S} is specified by the coupling (2.5) and the Kähler potential (2.9), or, equivalently, the linear superfield L satisfies (1.3) and the GS term (2.13) is added to the Lagrangian.

6. Conclusions

We have shown that a suitable choice of Pauli–Villars regulator fields allows for a full cancellation of the chiral and conformal anomalies associated, respectively, with the linear and logarithmic divergences in the effective supergravity theories from Z_3 and Z_7 compactification of the weakly coupled heterotic string without Wilson lines. In particular we were able to reproduce the form of the chiral anomaly found in a string theory calculation [4] for these two models.

In a future study we will extend our analysis to an example of Z_3 orbifold compactification with Wilson lines and an anomalous $U(1)$.

Note added in proof. After this work was completed we became aware of earlier studies of anomalies and anomaly cancellation in Z_3 [10] and Z_7 [11] compactifications of the heterotic string.

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Appendix A. Cancellation of ultraviolet divergences and evaluation of the anomaly

In this appendix, we will specify a choice of PV fields that cancel leftover divergences from the invariant mass PV sector of [1] (referred to in what follows as BG) and reproduce the universal chiral anomaly of [4]. Aside from the residual divergences discussed in Appendix B, the PV fields introduced in BG eliminate all the divergences from the light sector of the two string models we are considering, but have some leftover divergences arising from the \dot{Y} fields if one excludes the noninvariant mass PV sector of BG. Since we must alter the noninvariant mass sector of BG to produce a universal anomaly, our strategy here will be to introduce fields with parameters that decouple as much as possible from the BG fields but still cancel the divergences of the \dot{Y} . These new fields replace the BG set that was collectively denoted by Ψ . We expand the sum rules of BG to accommodate the more general Kähler potential of PV fields we consider in (3.16), and find that the sum rules that the PV fields must satisfy to cancel divergences are

$$\sum_C \eta^C = N' = -N - 29 \tag{A.1}$$

$$\sum_\gamma \eta_\gamma^V = N'_G = -12 - N_G \tag{A.2}$$

$$\sum_C \eta^C \alpha^C = -10 \tag{A.3}$$

$$\sum_C \eta^C \left(\beta^C g + \sum_n q_n^C g^n \right) = -A_1 g \tag{A.4}$$

$$\sum_C \eta^C \alpha^C \alpha^C = -4 \tag{A.5}$$

$$\sum_C \eta^C \delta^C = 2 \tag{A.6}$$

$$\sum_C \eta^C \left(\alpha^C \beta^C (X_\alpha g_\beta + g_\alpha X_\beta) + \sum_n \alpha^C q_n^C (g_\alpha^n X_\beta + X_\alpha g_\beta^n) \right) = 0 \tag{A.7}$$

$$\begin{aligned} \sum_C \eta^C \left(\beta^C \beta^C g_\alpha g_\beta + \sum_n \beta^C q_n^C (g_\alpha^n g_\beta + g_\alpha g_\beta^n) + \sum_{n,m} q_n^C q_m^C g_\alpha^n g_\beta^m \right) \\ = -A_2 g_\alpha g_\beta - B_2 \sum_n g_\alpha^n g_\beta^n \end{aligned} \tag{A.8}$$

$$\sum_C \eta^C C_{(G)}^C = C_{(G)}^M \tag{A.9}$$

$$\sum_C \eta^C (T_a)_C^C \alpha^C = 0 \tag{A.10}$$

$$\sum_C \eta^C (T_a)_C^C \left(\beta^C + q_n^C \right) = - \sum_p q_n^p (T_a)_p^p \tag{A.11}$$

$$\sum_C \eta^C \left(\frac{F}{2} - \gamma^C F - \sum_n q_n^C F^n \right) \left(\alpha^C - \frac{1}{2} \right)^2 X \cdot \tilde{X} = -\frac{1}{4} \left(\frac{N}{2} - A_1 \right) F X \cdot \tilde{X} \quad (\text{A.12})$$

$$\begin{aligned} \sum_C \eta^C \left(\frac{F}{2} - \gamma^C F - \sum_n q_n^C F^n \right) \left(\alpha^C - \frac{1}{2} \right) \left(2\beta^C X \cdot \tilde{g} + \sum_n 2q_n^C X \cdot \tilde{g}^n \right) \\ = F \sum_p \left(\frac{A_1}{2} - A_2 \right) X \cdot \tilde{g} \end{aligned} \quad (\text{A.13})$$

$$\begin{aligned} \sum_C \eta^C \left(\frac{F}{2} - \gamma^C F - \sum_n q_n^C F^n \right) \left(\beta^C \beta^C g \cdot \tilde{g} + \sum_n 2\beta^C q_n^C g \cdot \tilde{g}^n + \sum_{n,m} q_n^C q_m^C g^n \cdot \tilde{g}^m \right) \\ = -F \left(\frac{A_2}{2} - A_3 \right) g \cdot \tilde{g} \end{aligned} \quad (\text{A.14})$$

$$\sum_C \eta^C \left(\frac{F}{2} - \gamma^C F - \sum_n q_n^C F^n \right) \left(\alpha^C - \frac{1}{2} \right) (T_a)_C^C = \frac{1}{2} \sum_p \left(\frac{F}{2} - \sum_n q_n^p F^n \right) (T_{(G)})_p^p \quad (\text{A.15})$$

$$\sum_C \eta^C \left(\frac{F}{2} - \gamma^C F - \sum_n q_n^C F^n \right) (T_{(G)})_C^C \beta^C = 0 \quad (\text{A.16})$$

$$\sum_C \eta^C \left(\frac{F}{2} - \gamma^C F - \sum_n q_n^C F^n \right) (T_{(G)})_C^C q_n^C = -\sum_p \left(\frac{F}{2} - \sum_n q_n^p F^n \right) (T_{(G)})_p^p q_n^p \quad (\text{A.17})$$

$$\sum_{C,J} \eta^C \left(\frac{F}{2} - \gamma^C F - \sum_n q_n^C F^n \right) (T_a)_J^C (T_b)_C^J = -\sum_{p,q} \left(\frac{F}{2} - \sum_n q_n^p F^n \right) (T_a)_q^p (T_b)_p^q \quad (\text{A.18})$$

where we have used the sum rules for the light field modular weights (3.23), (4.23) and Eq. (4.18) and Eq. (4.19). F is defined by Eq. (2.4). On the left hand side of each condition we are summing over all PV fields, while the right hand sides correspond to summing over the parameters of the light fields. Since a subset of the BG PV fields already eliminate divergences from the light sectors of the string models, we can recast the above conditions by setting the right hand side of all conditions to zero and summing over only the \dot{Y} , Φ , ϕ , T , and ψ fields. To match the anomaly calculated in SS, we also require

$$0 = \sum_C \eta^C (1 - 2\bar{\gamma}^C) \left(\alpha^C \alpha^C - \frac{2}{3} \bar{\alpha}^C \bar{\alpha}^C - \frac{1}{3} \bar{\alpha}^C \right) \quad (\text{A.19})$$

$$0 = \sum_C \eta^C (1 - 2\bar{\gamma}^C) \left(2\beta^C \alpha^C - \frac{4}{3} \bar{\beta}^C \bar{\alpha}^C - \frac{1}{3} \bar{\beta}^C \right) \quad (\text{A.20})$$

$$0 = \sum_C \eta^C (1 - 2\bar{\gamma}^C) \left(\beta^C \beta^C - \frac{2}{3} \bar{\beta}^C \bar{\beta}^C \right) \quad (\text{A.21})$$

$$0 = \sum_C \eta^C (1 - 2\bar{\gamma}^C)^2 \quad (\text{A.22})$$

$$0 = \sum_C \eta^C q_n^C \alpha^C (1 - 2\bar{\gamma}^C) \tag{A.23}$$

$$0 = \sum_C \eta^C q_n^C \beta^C (1 - 2\bar{\gamma}^C) \tag{A.24}$$

$$0 = \sum_C \eta^C \bar{\alpha}^C (1 - 2\bar{\gamma}^C)^2 \tag{A.25}$$

$$0 = \sum_C \eta^C \bar{\beta}^C (1 - 2\bar{\gamma}^C)^2 \tag{A.26}$$

$$0 = \sum_C \eta^C \bar{\alpha}^C (1 - 2\bar{\gamma}^C)(1 - 2\bar{\alpha}^C) \tag{A.27}$$

$$0 = \sum_C \eta^C \bar{\beta}^C (1 - 2\bar{\alpha}^C)(1 - 2\bar{\gamma}^C) \tag{A.28}$$

$$0 = \sum_C \eta^C \bar{\alpha}^C \bar{\beta}^C (1 - 2\bar{\gamma}^C) \tag{A.29}$$

$$0 = \sum_C \eta^C \bar{\beta}^C \bar{\beta}^C (1 - 2\bar{\gamma}^C) \tag{A.30}$$

$$0 = \sum_C \eta^C \bar{\alpha}^C \bar{\alpha}^C (1 - 2\bar{\gamma}^C)^2 \tag{A.31}$$

$$0 = \sum_C \eta^C \bar{\beta}^C \bar{\beta}^C (1 - 2\bar{\gamma}^C)^2 \tag{A.32}$$

$$0 = \sum_C \eta^C \bar{\alpha}^C \bar{\beta}^C (1 - 2\bar{\gamma}^C)^2 \tag{A.33}$$

$$0 = \sum_C \eta^C \bar{\beta}^C (1 - 2\bar{\gamma}^C) \tag{A.34}$$

$$0 = \sum_C \eta^C \bar{\beta}^C \bar{\beta}^C \bar{\beta}^C (1 - 2\bar{\gamma}^C) \tag{A.35}$$

$$30\delta_{nm} = \sum_C \eta^C q_n^C q_m^C (1 - 2\bar{\gamma}^C) \tag{A.36}$$

$$2 \sum_p q_n^p + 3 - N + N_G = \sum_C \eta^C (1 - 2\alpha^C + 2q_n^C) \tag{A.37}$$

In this second set of conditions, only fields with noninvariant masses contribute to the sums since $\bar{\gamma} = \frac{1}{2}$ for fields with invariant masses. We now describe a particular choice of $\{\Phi, T, \psi\}$ fields that lead to an easily solvable system for their parameters. We must also supplement these fields with the ϕ^C fields, since some of these have noninvariant masses. Starting with divergences related to gauge interactions, we introduce a pair of T fields for each non-Abelian simple factor of the string model gauge group: $(T_{(\mathcal{G})1}^P, T'_{(\mathcal{G})1}^P), (T_{(\mathcal{G})2}^P, T'_{(\mathcal{G})2}^P)$, where \mathcal{G} specifies the simple group factor. We take the $T_{(\mathcal{G})1}$ ($T'_{(\mathcal{G})1}$) to be in the fundamental (antifundamental) representation of \mathcal{G} while the $T_{(\mathcal{G})2}$ are gauge singlets. Then Eq. (A.9) gives

$$C_{\mathcal{G}}^M = 2C_{(\mathcal{G})}^f \sum_P \eta_{T(\mathcal{G})1}^P \tag{A.38}$$

$$C_{(\mathcal{G})}^f N_{(\mathcal{G})} = 2N_{T(\mathcal{G})1} C_{(\mathcal{G})}^f, \quad \sum_P \eta_{T(\mathcal{G})1}^P = \frac{N_{(\mathcal{G})}}{2}, \tag{A.39}$$

for non-Abelian gauge groups and

$$\sum_p Q_a^p Q_a^p = 2 \sum_P (\eta_{\Phi})^P (Q^{\Phi})_a^P (Q^{\Phi})_a^P + 2 \sum_P (\eta_{T_{a1}})^P (Q^T)_a^P (Q^T)_a^P \tag{A.40}$$

for Abelian groups. These are just the conditions needed to cancel the \dot{Y} factor of $C_{\mathcal{G}}^M$. The Φ 's enter in Eq. (A.40) since they are given $U(1)_a$ charges as per the prescription in Section 4.2. Note also that Eq. (A.39) works for the two models considered here since the number $N_{(\mathcal{G})}$ of fundamentals in \mathcal{G} is even for all the gauge groups. We will constrain these T fields so that they do not contribute to any divergences other than those arising from gauge interactions. To do this, we enforce the following

$$(\alpha_{T(\mathcal{G})2}) = (\alpha_{T(\mathcal{G})1}) \tag{A.41}$$

$$(\beta_{T(\mathcal{G})2}) = (\beta_{T(\mathcal{G})1}) \tag{A.42}$$

$$(\alpha'_{T(\mathcal{G})2}) = (\alpha'_{T(\mathcal{G})1}) \tag{A.43}$$

$$(\beta'_{T(\mathcal{G})2}) = (\beta'_{T(\mathcal{G})1}) \tag{A.44}$$

$$(\eta_{T(\mathcal{G})2})^P = -(\eta_{T(\mathcal{G})1})^P \tag{A.45}$$

For the T fields charged under non-Abelian group factors, we impose that $\bar{\gamma}_{T(\mathcal{G})1}^P$ is independent of P so that we can cancel all remaining divergences from non-Abelian interactions by demanding

$$\frac{C_{GS} - C_{(\mathcal{G})}}{2} = \frac{C_{(\mathcal{G})}^f N_{(\mathcal{G})}}{2} (1 - 2\bar{\gamma}_{T(\mathcal{G})1}), \tag{A.46}$$

for every non-Abelian group factor \mathcal{G} , where

$$C_{GS} = 8\pi^2 b = 30 \tag{A.47}$$

is the adjoint Casimir for E_8 , which is the gauge group of the pure Yang–Mills hidden sector of the models considered here. For the Abelian divergences, we will not force $\bar{\gamma}_{T(\mathcal{G})1}^P$ to be independent of P , but we will require Eq. (A.43) and that the primed and unprimed parameters are identical:

$$(\alpha_{T(\mathcal{G})1})^P = (\alpha'_{T(\mathcal{G})1})^P \tag{A.48}$$

$$(\beta_{T(\mathcal{G})1})^P = (\beta'_{T(\mathcal{G})1})^P.$$

With these conditions, the remaining divergences from Abelian interactions are canceled by imposing

$$\frac{C_{GS}}{2} = \sum_M (\eta_{T_{(a)1}})^P (1 - 2\gamma_{T_{(a)1}}^P) (Q^T)_a^P (Q^T)_a^P. \tag{A.49}$$

With the above restrictions, the charged T fields will eliminate only gauge-related divergences and not contribute to any of the other sum rules listed above. Turning to the ψ and ϕ fields, we

choose parameters such that

$$\bar{\gamma}_\psi^P = \bar{\alpha}_\psi^P = \bar{\beta}_\psi^P = 0. \tag{A.50}$$

Then the second set of conditions reduces to

$$0 = \sum_P (\eta_\psi)^P + \sum_C \hat{\eta}^C (1 - 2\bar{\alpha}^C)^2 \tag{A.51}$$

$$0 = \sum_P (\eta_\psi)^P (\alpha_\psi)^P (\alpha_\psi)^P + \sum_C \hat{\eta}^C (1 - 2\bar{\alpha}^C) \left(\hat{\alpha}^C \hat{\alpha}^C - \frac{2}{3} \bar{\alpha}^C \bar{\alpha}^C - \frac{1}{3} \bar{\alpha}^C \right) \tag{A.52}$$

$$0 = \sum_C \hat{\eta}^C \bar{\alpha}^C (1 - 2\bar{\alpha}^C)^2 \tag{A.53}$$

$$0 = \sum_C \hat{\eta}^C \bar{\alpha}^C \bar{\alpha}^C (1 - 2\bar{\alpha}^C)^2 \tag{A.54}$$

$$2 \sum_p q_n^p + 3 - N + N_G = \sum_C \hat{\eta}^C (1 - 2\hat{\alpha}^C) \tag{A.55}$$

$$0 = \sum_P (\eta_\psi)^P \tag{A.56}$$

$$0 = \sum_P (\eta_\psi)^P (\alpha_\psi)^P (\alpha_\psi)^P \tag{A.57}$$

$$0 = \sum_P (\eta_\psi)^P (\alpha_\psi)^P (\beta_\psi)^P \tag{A.58}$$

$$0 = \sum_P (\eta_\psi)^P (\beta_\psi)^P (\beta_\psi)^P \tag{A.59}$$

$$0 = \sum_P (\eta_\psi)^P q_\psi^P \alpha_\psi^P \tag{A.60}$$

$$0 = \sum_P (\eta_\psi)^P q_\psi^P \beta_\psi^P \tag{A.61}$$

$$30 = 2 \sum_P (\eta_\psi)^P q_\psi^P q_\psi^P \tag{A.62}$$

Finally, to cancel all the divergences as required by the sum rules in (A.1)–(A.14), we introduce gauge singlet T fields ($T_3^P, T_3^{\prime P}$) with invariant masses. These fields, along with the Φ and ψ fields, are enough to regulate those divergences of the \dot{Y} fields that do not involve gauge couplings. While we have solved this system to obtain numerical solutions, the results are not particularly enlightening and we will not reproduce them here.

Appendix B. Residual linear and logarithmic divergences

There are two sources of the chiral anomaly involving space–time curvature. The first arises from the spin connection in the fermion covariant derivatives. The three sum rules in (3.17)

assure that the linear divergent terms from the PV fermion spin connection cancel those from the light fields, and the residual anomaly arises from the PV masses, giving a supersymmetric contribution

$$\Delta\mathcal{L}_{\text{spin}} = -b_{\text{sp}} \int EF\Omega_{\text{GB}} + \text{h.c.}, \tag{B.1}$$

which is the variation of the first term in \mathcal{L}_0 in (3.5), with

$$8\pi^2 b_{\text{sp}} = \frac{1}{24} \left(N' - N'_G - 2\alpha + 2 \sum_p q_p^n \right) = \frac{1}{24} \left(2 \sum_p q_n^p + 3 - N + N_G \right) = 31 \quad \forall n \tag{B.2}$$

for Z_3 and Z_7 orbifolds without Wilson lines. The second contribution arises from the affine connection in the gravitino covariant derivative; it has no PV counterpart and is not canceled. However there is a residual conformal anomaly associated with the linear divergence arising from the Gauss–Bonnet term which is a total derivative, and which is uniquely determined [7] by the spins of the particles in the loop. For PV regulated supergravity we have

$$\mathcal{L}_{\text{GB}} = \frac{\sqrt{g} b_{\text{BG}}}{2} \left(r_{\mu\nu\rho\sigma} r^{\mu\nu\rho\sigma} - 4r_{\mu\nu} r^{\mu\nu} + r^2 \right) \ln \Lambda, \tag{B.3}$$

with

$$8\pi^2 b_{\text{GB}} = \frac{1}{48} (N + N' - 3N_G - 3N'_G + 41) = 1. \tag{B.4}$$

The variation of (B.3) forms a supersymmetric operator with the chiral anomaly from the gravitino affine connection provided the cut-off takes the value in (3.10), giving a contribution

$$\Delta\mathcal{L}_{\text{aff}} = b_{\text{GB}} \int EF\Omega_{\text{GB}} + \text{h.c.}, \tag{B.5}$$

which is the variation of the $K\Omega_{\text{GB}}$ term in (3.5), and combines with (B.2) to give

$$\Delta\mathcal{L}_{\text{anom}} \ni -b \int EF\Omega_{\text{GB}} + \text{h.c.}, \tag{B.6}$$

where

$$8\pi^2 b = 8\pi^2 (b_{\text{sp}} - b_{\text{GB}}) = \frac{1}{24} \left(2 \sum_p q_n^p - N + N_G - 21 \right) = 30 \quad \forall n. \tag{B.7}$$

There is also a linear divergence arising from an off-diagonal gravitino–gaugino connection in the fermion covariant derivative. This also combines with an uncanceled logarithmically divergent total derivative to form an anomaly supermultiplet if the cut-off satisfies (3.10). It was shown in Appendix B.3 of [1] that this anomaly can be canceled for a particular choice of masses for certain PV fields that regulate gauge and gravity sector loops.

Finally, there are “D-term” anomalies that arise from uncanceled logarithmically divergent terms with no chiral anomaly counterpart. These were simply dropped in the evaluation of on-shell ultraviolet divergences in [8] and [9]. Since they have no chiral anomaly partner they are more difficult to identify than the above terms. With the cut-off (3.10), the conformal anomaly includes a contribution

$$\Delta\mathcal{L}_{\text{conf}} \ni \frac{\sqrt{g}}{8\pi^2} \text{Re} F K_{i\bar{m}} D^\mu \left[\mathcal{D}_\mu \bar{z}^{\bar{m}} \left(2F^i \Big| R \Big| - D_\nu \mathcal{D}^\nu z^i \right) + D_\nu \mathcal{D}_\mu \bar{z}^{\bar{m}} \mathcal{D}^\nu z^i + \text{h.c.} \right], \tag{B.8}$$

where

$$F^i = -\frac{1}{4} \mathcal{D}^2 Z^i \tag{B.9}$$

is the auxiliary field of the chiral supermultiplet Z^i , that was identified in [1] as arising from a total derivative dropped in the evaluation [8] of UV divergences for gravity coupled to chiral matter. When Yang–Mills couplings are included [9], there are many more terms, and digging out total derivatives is much more difficult. We find the additional light field contribution:

$$\Delta \mathcal{L}_{\text{conf}} \ni -\frac{\sqrt{g}}{8\pi^2} \text{Re} F D^\mu \left[\frac{3}{2} K_{i\bar{m}} \mathcal{D}_\mu \bar{z}^{\bar{m}} D^a (T_a z)^i + \frac{\partial_\mu \bar{s}}{s + \bar{s}} D^a D_a \right] + \text{h.c.}, \tag{B.10}$$

where

$$D^a = -\frac{1}{2} \mathcal{D}_\alpha W_\alpha^a \tag{B.11}$$

is the auxiliary field for the superfield strength W_α^a , and we evaluated the result of [9] using the classical Kähler potential in (2.9) for the dilaton. We also find a contribution [2] from the PV sector

$$\Delta \mathcal{L}_{\text{conf}} \ni \frac{\sqrt{g}}{16\pi^2} \text{Re} F D^\mu \left[K_{i\bar{m}} \mathcal{D}_\mu \bar{z}^{\bar{m}} D^a (T_a z)^i - \frac{\partial_\mu \bar{s}}{s + \bar{s}} (F^i \bar{F}_i - 8R\bar{R}) \right] + \text{h.c.} \tag{B.12}$$

With the classical Kähler potential in (2.9) the equations of motion give

$$F^s | = 2(s + \bar{s}) \bar{R} |, \quad \bar{F}_s | = \frac{2}{s + \bar{s}} R |, \tag{B.13}$$

and the dilaton-dependent contribution can be written

$$\begin{aligned} \Delta \mathcal{L}_{\text{conf}}(s, \bar{s}) = & -\frac{\sqrt{g}}{8\pi^2} \text{Re} F D^\mu \left[(\partial_\mu \bar{s} \square s - \partial_\nu \partial_\mu \bar{s} \partial^\nu s + \text{h.c.}) \right. \\ & \left. + \frac{1}{2} \frac{\partial_\mu \bar{s}}{s + \bar{s}} (F^p \bar{F}_p - 12R\bar{R} + 2D^a D_a) \right]. \end{aligned} \tag{B.14}$$

However we cannot be certain that we have identified all the uncanceled total derivatives. It is also possible that one might be able to modify the PV sector parameter such that the dilaton dependence can be canceled, as was the case for F-term anomaly arising from the off-diagonal gaugino–gravitino connection mentioned above.

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