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Coordination and Incentive Contracts in Project Management under Asymmetric Information

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Abstract

We study the problem of the manager of a project consisting of two sub-projects or tasks which are outsourced to different subcontractors. The project manager earns more revenue from the project if it is completed faster, but he cannot observe how hard subcontractors work, only the stochastic duration of their tasks. We derive the optimal linear incentive contracts to offer to the subcontractors when the tasks are conducted in series or in parallel. We compare them to the fixed-price contracts often encountered in practice, and discuss when incentive contracts lead to bigger performance improvement. We characterize how the incentive contracts vary with the subcontractors' risk aversion and cost of effort, the marginal effect of subcontractor effort, and the variability of task durations. We find that this dependence is sometimes counter-intuitive in nature. For instance, for parallel tasks, if the first agent's task is on the critical path and his variability increases, the project manager should induce the first agent to work *less* hard and the second agent to work harder.

Keywords: project management, incentive contracting, asymmetric information, moral hazard.

1 Introduction

Managing projects is increasingly becoming a critical activity for many companies. Moreover, in many industries, the trend towards outsourcing means that projects often consist of sub-projects, each performed by different subcontractors. In some sectors, such as construction, motion pictures, and aerospace, this has long been the case; in others, this trend has recently become more pronounced, as in pharmaceuticals, information systems, and toys as rapid change has become essential on survival requirement for most organizations (Klastorin, 2004). Chemical firms hire engineering firms to design their plants; these engineering firms, in turn, hire subcontractors for the actual construction work. Aircraft manufacturers increasingly outsource design of major subsystems (such as wings, engines and control systems) to suppliers. Boeing recently outsourced key R&D and manufacturing activities for the wings of its new 7E7 aircraft to a consortium of Japanese suppliers (The Economist, Oct. 9, 2004). Flavin et al. (2001) estimate that by 2004, nearly 42% of spending on drug development by pharmaceutical firms will be on outsourced activities, including early-stage discovery and clinical trials.

In many cases, a company can earn higher revenues if a project is completed faster, or pays a penalty if it is delayed. Airbus faces a six-month delay in delivering the first A380s to Singapore Airlines and the Air France-KLM Group, leading to penalties of over \$1 million for each delayed plane (Los Angeles Times, June 15, 2005). In the case of pharmaceuticals, each day of delay in getting a new drug to market can cost \$1 million or more in lost revenues (Squires 2002).

In all these examples, the project owner prefers earlier completion, but often cannot observe how hard the subcontractors try to shorten the duration of their sub-projects. In practice, project managers sometimes offer subcontractors a duration-based incentive contract, hence passing part of the penalty for delayed completion on to the subcontractors. Surprisingly often, though, contracts just specify a fixed price, regardless of duration.

As an example of the large potential benefits of incentive contracts that tie payments to duration, consider the Northridge earthquake (January 17, 1994), which damaged two main

elevated freeways in Los Angeles, the interstate 5 and 10, creating severe gridlock. The repair contract was awarded to Clint C. Meyers, who promised to have the 10 freeway open for use in a record time of six months, while other contractors had projected the repair would take at least 18 months to 2 years. As the City of Los Angeles claimed losses in excess of \$1 million per day for every day that the freeway was out of commission, Meyers proposed a contract with an incentive plan. For every day the freeway remained closed beyond the 6 month target, he would pay the city \$200,000. But for every day the freeway opened ahead of schedule, the city would pay Meyers an additional \$200,000. On several occasions, the contractor incurred additional expenses to keep the project on schedule. For instance, he paid for inspectors to be on-site constantly to monitor and approve each step rather than waiting until the end to do a final inspection. He passed part of the time-related bonus and penalty on to the subcontractors. When a shipment of material was delayed, Meyers paid for express shipment rather than incur several weeks' delay. As a results, the 5 freeway was reopened 33 days ahead of schedule, earning the contractor a bonus of \$4.95 million, while the 10 freeway was reopened 66 days ahead of schedule, earning the contractor a bonus of \$14.8 million (Boarnet, 1998). Just how extraordinary this performance is becomes clear by comparing it to the aftermath of the Loma Prieta earthquake near San Francisco on October 17, 1989: the viaducts took several years to rebuild and some have still not been reconstructed (Chang and Nojima, 2001).

From this example, and many others, it is clear that subcontractors have significant discretion in deciding how much effort and resources to spend to accelerate completion of their work or to mitigate the consequences of unexpected problems. In this paper, we let the project manager offer linear duration-based incentive contracts to two subcontractors, who work in series or in parallel. We compare the case where the project manager cannot observe the subcontractors' effort with the first-best case, and the optimal linear contract with the optimal fixed-price contract. We also show how the optimal effort levels and contracts vary with key problem parameters: the subcontractors' cost of effort, the reduction in completion time per unit effort, the variability in completion time, and the subcontractors' risk aversion. Consider for instance, a simplified case, the development of a new airplane, where the engines and wings are developed

by suppliers. We address the following research questions:

- If the duration of wing development is more uncertain than in previous projects, due to lack of experience with the new wing material being used, should the project manager induce the wing manufacturer to exert more or less effort to reduce the duration?
- If the wing manufacturer is less diversified and therefore more risk averse than the engine manufacturer, should the project manager induce the wing manufacturer to exert more or less effort?
- If it is more costly to accelerate wing development than engine development, should the project manager induce the wing manufacturer to exert more or less effort?

Our results can be summarized as follows. First, we confirm that incentive contracts are always at least (weakly) superior to fixed-price contracts, resulting in shorter expected project duration and higher expected profits for the principal. If the agents have high costs of effort, if sub-project durations are highly variable, if agents are highly risk averse or if the marginal effect of effort is low, then the difference between the incentive and fixed-price contracts is small. Second, we confirm that in the serial case the optimal contracts for the two agents are independent of one another, while in the parallel case, the optimal contracts depend on the characteristics of both agents and both sub-projects. Our third and main result shows that, in some cases, the behavior of the optimal effort levels is counter-intuitive. For example, if the variability of the duration of the first sub-project increases and the first sub-project is likely to be on the critical path, the optimal contract will induce a switch of effort *away* from the first agent to the second agent.

The contribution of this paper is to formally introduce decentralized decision making, asymmetric information, and incentive contracts into a project management setting. We model this situation as a principal-multi-agent problem. In our parallel sub-projects case, the interaction between the agents' tasks and the principal's revenue is more complex than what is usually

considered in the existing literature on principal-multi-agent problems. To ground this study in practice, we conducted informal interviews with project managers from various industries. We relate their anecdotal experiences to our analytical results in the conclusions.

This paper is organized as follows. In Section 2 we review relevant literature in project management and in economics. Section 3 introduces the basic model and notation. Sections 4 and 5 analyze the cases with serial and parallel tasks, respectively. In Section 6 we report some numerical experiments. Section 7 concludes the paper and suggests some areas for future research.

2 Literature

Our paper draws on two bodies of literature: the operations management (OM) literature on project management, and the economics literature on moral hazard problems.

Project management has been studied extensively by both academics and practitioners, often with a focus on scheduling interdependent activities. This work started with the development of two activity network techniques: the Critical Path Method (CPM) and the Project Evaluation and Review Technique (PERT), both in the late 1950s. Both have since spawned substantial bodies of work. Elmaghraby (1995), Brucker et al. (1999), Tavares (2002) and Williams (2003) review this literature, most of which considers problems in deterministic environments. Recently, Herroelen and Leus (2005) review project scheduling models under uncertainty.

The subfield of project scheduling closest to our work is the time-cost trade-off problem, where activity durations can be shortened but at a cost. The planner can minimize project completion time subject to a budget constraint. Kelley (1961) and Fulkerson (1961) independently laid the foundations for this area. Since then, the literature has considered variations of the problem where the time-cost relationships are continuous (see Moder et al. 1983 for references) or discrete (see De et al. 1995 for references). We contribute to this literature by placing it in a decentralized setting, adding information asymmetry and contracting issues.

Our work is also related to the project contracting literature. Elmaghraby (1990) is the first to analyze project bidding in a probabilistic framework. Paul and Gutierrez (2005) study three commonly occurring forms contracts for projects consisting of a single task and assigned to a single contractor: fixed price, cost plus percentage and menu contracts. They use stochastic ordering to study which type of contract results in lower expected price with risk neutral or risk averse contractors and with collusion between contractors. Gutierrez and Paul (2000) identify cases when splitting subprojects among multiple subcontractors, or pooling, assigning them to a single subcontractor, are preferred.

The economics literature on principal multi-agent models is vast, see e.g., Holmstrom (1982), Demski and Sappington (1984), Mookherjee (1984), McAfee and McMillan (1991), Itoh (1991) and Che and Yoo (2001). Our work builds on this literature and extends it by including the case with parallel tasks, in which the output of the productive technology depends on the individual agents' outputs in a non-additive manner.

In recent years, many scholars have applied agency theory to problems in operations management. Cachon (2003) and Tsay (1999) review the contracting literature in supply chain management, Baiman et al. (2005) use agency theory in an assembly system. Their weakest link property is similar to our case with parallel sub-projects, but they focus on adverse selection rather than moral hazard. Gibbons (2005) reviews incentive contracting models in economics and relates them to supply chain problems. Iyer et al. (2005) use an agency model for setting specifications in a collaborative product development context.

Applications of agency theory to project management are very few. Sommer and Loch (2003) use agency theory to characterize an incomplete contracts in projects with trial and error learning under ambiguity. Later in a separate paper, they derive adjustable contracts under a similar setting (Sommer and Loch, 2004) and ask how the project manager should adjust the contract as ambiguity is resolved in the project. In both papers, they consider project as a single entity. Our focus, by contrast, is on coordinating two sub-projects.

3 Model and notation

A project manager or principal (*she*) is in charge of a project that consists of two tasks (or sub-projects). We will consider the case where these tasks must be performed in sequence (serial case) or simultaneously (parallel case). In the serial case, we do not let the sub-projects overlap. The project is completed when both tasks are completed. The principal hires two subcontractors (agents) to work on the two tasks.

Both agents can exert effort to accelerate their respective tasks; task completion times depend stochastically on the agent's effort level. The principal cannot observe these effort levels, only the resulting task completion times. The principal can offer the agents fixed-price contracts or incentive contracts; in the latter case, the payment to the agent consists of a fixed part and a variable part that depends on the task duration. The agents have outside options; if an agent's expected profit does not meet the corresponding reservation profit level, the agent will not participate, the entire project does not take place, and all parties receive their reservation profit levels. This leads to the standard individual rationality constraints below. The agents will choose effort levels that are optimal from their perspective, given the incentive contract offered. The principal incorporates this behavior into her contract design, from which the usual incentive-compatibility constraints follow.

The duration of task i , d_i , depends on agent i 's effort level e_i through $d_i(e_i) = g_i(e_i) + \epsilon_i$, where ϵ_i is a random component. The constant component $g_i(e_i)$ is decreasing in effort, so $g_i'(e_i) < 0$. Agent i 's cost of effort $C_i(e_i)$ is convex increasing, so $C_i'(e_i) > 0$ and $C_i''(e_i) > 0$, and his profit is $\Pi_i = W_i(d_i) - C_i(e_i)$. The agents are risk-averse with exponential utility functions $U_i(x_i) = -e^{-k_i x_i}$, where k_i is agent i 's coefficient of risk aversion. Let $\underline{\pi}_i$ be the profit associated with agent i 's outside opportunity, and $\underline{u}_i = U(\underline{\pi}_i)$ the corresponding reservation utility level.

The principal is risk-neutral and maximizes her expected profit. Her revenue $R(d)$ is decreasing in the project duration, so $R'(d) < 0$. The total project duration is $d = d_1 + d_2$ if the tasks are serial and $d = \max(d_1, d_2)$ if the tasks are parallel. The principal offers the agents duration-

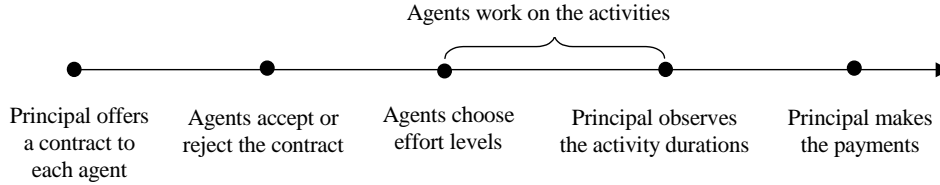


Figure 1: Contract time line

based contracts $W_i(d_i)$. The principal's profit is then given by $P = R(d) - W_1(d_1) - W_2(d_2)$. Below, we will use linear functions for the relationships between revenue, duration and efforts, and we restrict ourselves to linear incentive contracts, so $W_i(d_i) = w_{0i} - w_{1i}d_i$. In the absence of risk aversion, linear contracts are optimal. From Mirrlees (1974) we know that linear contracts are not optimal for the single agent version of this problem with risk aversion, but also that the optimal contracts are very difficult to derive. For the parallel case, we were indeed unable to do so. We have not encountered nonlinear contracts in practice, except threshold contracts, which we defer for future work. The freeway case from the introduction is an example of the type of linear contract that we consider here. Under fixed-price contracts, $W_i(d_i) = w_{0i}$.

The sequence of events is given in Figure 1. At the beginning of the project, the principal offers a take-it-or-leave-it contract to both agents simultaneously. Both agents accept or reject their contract. If they both accept, they then choose their effort levels simultaneously. In the serial case, the second agent could also choose his effort level after the first agent has completed his task. This makes no difference, as the principal does not renegotiate the contract after observing the first task duration. Task durations are realized and observed by the principal, who receives her payoff from the project and pays the agents based on their realized task durations. The parties have symmetric information at the time of contracting, there is no renegotiation, and after the contract is written, only the agents take action.

To maximize the principal's expected profit while satisfying the individual rationality and

incentive-compatibility constraints, we solve:

$$\begin{aligned}
\max P &= E[R(d(e_1, e_2))] - E[W_1(d_1(e_1))] - E[W_2(d_2(e_2))] \\
\text{s.t. } E[U(W_i(d_i(e_i)) - C_i(e_i))] &\geq \underline{u}_i & i = 1, 2 \quad (\text{IR}) \\
E[U(W_i(d_i(e_i)) - C_i(e_i))] &\geq E[U(W_i(d_i(e'_i)) - C_i(e'_i))] \quad \text{for all } e'_i & i = 1, 2 \quad (\text{IC})
\end{aligned}$$

We follow the usual modeling assumptions of the time-cost trade-off literature and assume that the principal's revenue depends linearly on duration, i.e., $R(d) = r_0 - r_1 d$. We also assume that $g_i(e_i) = d_{0i} - d_{1i} e_i$, so agent i 's expected task duration is linear in effort. Realized duration is $d_i(e_i) = d_{0i} - d_{1i} e_i + \epsilon_i$, where ϵ_i is normally distributed with mean 0 and variance σ_i^2 , small enough that we can ignore negative durations. Agent i 's cost of effort is quadratic, i.e., $C_i(e_i) = \frac{1}{2} c_i e_i^2$; using the linear cost of effort functions common in the time-cost trade-off literature leads to trivial extreme-point solutions.

Let μ_{w_i} and $\sigma_{w_i}^2$ be the resulting mean and variance of the agents' payments $W_i(d_i)$. Then the agents' expected utility is given by:

$$E[U_i(\Pi_i)] = E[U(W_i(d_i) - \frac{1}{2} c_i e_i^2)] = U(\mu_{w_i} - \frac{1}{2} c_i e_i^2 - \frac{1}{2} k_i (\sigma_{w_i})^2). \quad (1)$$

This follows because whenever $U(\pi) = -e^{-k\pi}$ with π normally distributed with parameters μ_π and σ_π^2 , then $E[U(\pi)] = U(\mu_\pi - \frac{1}{2} k \sigma_\pi^2)$. Since $U(\pi)$ is monotone increasing, we can replace the inequality $E[U(\pi)] \geq \underline{u}_i$ with $\mu_\pi - \frac{1}{2} k \sigma_\pi^2 \geq \underline{\pi}_i$. In (1), the term on the right-hand side within the utility function is agent i 's certainty equivalent which has the following interpretation:

$$\begin{aligned}
\mu_{w_i} - \frac{1}{2} c_i e_i^2 - \frac{1}{2} k_i (\sigma_{w_i})^2 &= E[W_i(d_i)] - \frac{1}{2} c_i e_i^2 - \frac{1}{2} k_i \text{Var}(W_i(d_i)) \\
&= \text{expected payment} - \text{cost of effort} - \text{risk premium}
\end{aligned}$$

Given effort choice e_i , a certain payment of $\mu_{w_i} - \frac{1}{2} k_i (\sigma_{w_i})^2$ would yield the same utility to agent i as accepting the uncertain payment $W_i(d_i)$, which in expectation is equal to μ_{w_i} .

4 Serial tasks

We first start with the serial case where the two tasks are performed sequentially. This case is straightforward and is included only as a contrast to the parallel case, which is considerably more complex and discussed later. We derive the first-best solution and the optimal linear and fixed-price contracts.

4.1 First-best contract

In the first-best case, the principal can observe effort levels and can contract directly on them.

The principal's optimization problem is then formulated as:

$$\begin{aligned} \max_{W_i, e_i} P_{FB} &= E[R(d_1(e_1) + d_2(e_2))] - E[W_1(d_1(e_1))] - E[W_2(d_2(e_2))] \\ \text{s.t.} \quad E[W_i(d_i(e_i))] - \frac{1}{2}c_i e_i^2 - \frac{1}{2}k_i(w_{1i}\sigma_i)^2 &\geq \underline{\pi}_i \quad i = 1, 2 \quad (IR) \end{aligned}$$

Since the principal's revenues are linear in project duration, and total project duration is the sum of the individual task durations, expected revenues can be expressed as $E[R(d_1(e_1) + d_2(e_2))] = E[R(d_1(e_1))] + E[R(d_2(e_2))] - r_0$. The problem can be decomposed into two single agent problems:

$$\begin{aligned} \max_{W_i, e_i} P_{FB,i} &= E[R(d_i(e_i))] - E[W_i(d_i(e_i))] \\ \text{s.t.} \quad E[W_i(d_i(e_i))] - \frac{1}{2}c_i e_i^2 - \frac{1}{2}k_i(w_{1i}\sigma_i)^2 &\geq \underline{\pi}_i \end{aligned}$$

and $P_{FB} = P_{FB,1} + P_{FB,2} - r_0$. To solve this problem we first show that the constraint is binding and substitute the expected payment into the objective function.

Proposition 1 *The first-best effort levels in the serial case are given by $e_i^{FB} = \frac{r_1 d_{1i}}{c_i}$. The corresponding contracts satisfy $w_{0i}^{FB} = \underline{\pi}_i + \frac{1}{2}c_i(e_i^{FB})^2 = \underline{\pi}_i + \frac{1}{2}\frac{(r_1 d_{1i})^2}{c_i}$ and $w_{1i}^{FB} = 0$ when agent*

i chooses the optimal effort level, and $w_{0i}^{FB} = -\infty$ otherwise.

All proofs are provided in the Appendix. When effort levels are contractible, it is optimal for the project manager to offer a fixed-price contract in exchange for the optimal effort level, hence insulating the agent from all risk.

4.2 Optimal linear contract under asymmetric information

Here, the principal can no longer observe the agents' effort levels, only the task durations. The principal now solves the following problem:

$$\begin{aligned} \max_{W_i, e_i} P_{SB} &= E[R(d_1(e_1) + d_2(e_2))] - E[W_1(d_1(e_1))] - E[W_2(d_2(e_2))] \\ \text{s.t.} \quad E[W_i(d_i(e_i))] - \frac{1}{2}c_i e_i^2 - \frac{1}{2}k_i(w_{1i}\sigma_i)^2 &\geq \underline{\pi}_i \quad i = 1, 2 \quad (IR) \\ E[W_i(d_i(e_i))] - \frac{1}{2}c_i e_i^2 - \frac{1}{2}k_i(w_{1i}\sigma_i)^2 &\geq \\ E[W_i(d_i(e'_i))] - \frac{1}{2}c_i (e'_i)^2 - \frac{1}{2}k_i(w_{1i}\sigma_i)^2 &\text{ for all } e'_i \quad i = 1, 2 \quad (IC) \end{aligned}$$

As above, this problem can be decomposed into two single-agent problems. To solve each sub-problem we determine the effort level that satisfies the IC constraint and substitute it into the IR constraint. The rest of the solution procedure is identical to that for the first-best contract.

Proposition 2 *The principal's optimal linear contract for the serial case is given by*

$$\begin{aligned} w_{1i}^* &= \frac{r_1}{1 + c_i k_i \left(\frac{\sigma_i}{d_{1i}}\right)^2}, \\ w_{0i}^* &= \underline{\pi}_i + w_{1i}^* d_{0i} - \frac{(w_{1i}^* d_{1i})^2}{2c_i} + \frac{1}{2}k_i (w_{1i}^* \sigma_i)^2. \end{aligned}$$

The corresponding effort levels in the serial case are given by $e_i^ = \frac{w_{1i}^* d_{1i}}{c_i}$.*

Substituting w_{1i}^* into e_i^* confirms that the effort levels are lower than the first-best effort levels. We compare the results under full and asymmetric information in more detail below,

after deriving the optimal fixed-price contract.

4.3 Fixed-price contract

Here we restrict w_{1i} to be zero, so $W_i(d_i) = w_{0i}$ and hence $\mu_{w_i} = w_{0i}$ and $\sigma_{w_i} = 0$. The agent's expected utility is now $U_i(\mu_{w_i} - \frac{1}{2}c_i e_i^2 - \frac{1}{2}k_i(\sigma_{w_i})^2) = U_i(w_{0i} - \frac{1}{2}c_i e_i^2)$. It is easy to show that $e_i^{FP} = 0$, so the agents do not exert any effort to accelerate their tasks and the principal only pays the agents enough to satisfy their reservation utility level, $W_i(d_i) = w_{0i}^{FP} = \underline{\pi}_i$. Such fixed-price contracts are obviously simple to implement which presumably explains their widespread use in practice.

4.4 Serial tasks: comparing first-best, optimal linear, and fixed-price cases

	first-best (full info.)	optimal linear (asym. info.)	fixed-price (asym. info.)
e_i	$\frac{r_1 d_{1i}}{c_i}$	$\frac{r_1 d_i}{c_i(1+c_i k_i(\frac{\sigma_i}{d_{1i}})^2)}$	0
d_i	$d_{0i} - \frac{(d_{1i})^2 r_1}{c_i}$	$d_{0i} - \frac{(d_{1i})^2 r_1}{c_i(1+c_i k_i(\frac{\sigma_i}{d_{1i}})^2)}$	d_{0i}
w_{0i}	$\underline{\pi}_i + \frac{1}{2}c_i(\frac{r_1 d_{1i}}{c_i})^2$	$\underline{\pi}_i + w_{1i}^* d_{0i} - \frac{(w_{1i}^* d_{1i})^2}{2c_i} + \frac{1}{2}k_i(w_{1i}^* \sigma_i)^2$	$\underline{\pi}_i$
w_{1i}	0	$\frac{r_1}{1+c_i k_i(\frac{\sigma_i}{d_{1i}})^2}$	0
W_i	$\underline{\pi}_i + \frac{(r_1 d_{1i})^2}{2c_i}$	$\underline{\pi}_i + \frac{(r_1 d_{1i})^2}{2c_i(1+c_i k_i(\frac{\sigma_i}{d_{1i}})^2)}$	$\underline{\pi}_i$
R	$r'_0 + \sum_{i=1,2} \frac{(r_1 d_{1i})^2}{c_i}$	$r'_0 + \sum_{i=1,2} \frac{(r_1 d_{1i})^2}{c_i(1+c_i k_i(\frac{\sigma_i}{d_{1i}})^2)}$	r'_0
P	$r''_0 + \sum_{i=1,2} \frac{(r_1 d_{1i})^2}{2c_i}$	$r''_0 + \sum_{i=1,2} \frac{(r_1 d_{1i})^2}{2c_i(1+c_i k_i(\frac{\sigma_i}{d_{1i}})^2)}$	r''_0

where $r'_0 = r_0 - r_1(d_{01} + d_{02})$ and $r''_0 = r'_0 - (\underline{\pi}_1 + \underline{\pi}_2)$

Table 1: Summary of results for serial tasks.

Table 1 summarizes the results for the serial case. First, as expected, the first-best contract results in higher effort levels and hence shorter expected project duration and higher expected profit for the principal than the optimal linear contract under asymmetric information. This difference is the smallest for projects with low task duration variability, when the marginal effect

of effort is low, when the cost of effort is low, and when agents are less risk averse. In the extreme cases where task durations are deterministic ($\sigma_i = 0$) or the agents are risk neutral ($k_i = 0$), then the first two columns are exactly the same. Second, expected project duration under the optimal linear contract is always shorter than under a fixed-price contract and the principal's profits are correspondingly higher. But we also observe that in projects with higher task duration variability, more risk-averse agents, or higher cost of effort, the difference between the fixed-price and optimal linear contracts is smaller. Third, we observe that one agent's payment does not depend on the characteristics of the other agent or the other task, all as expected.

5 Parallel tasks

In the case with parallel tasks, project completion time depends on the longest task, so the objective function now depends on the expected value of the maximum of the two (normally distributed) task completion times. We will use the following result (Clark 1961):

Lemma 3 *If X_1, X_2 are normally distributed with mean μ_1 and μ_2 and variance σ_1^2 and σ_2^2 respectively, and $X = \max(X_1, X_2)$, then $E[X] = \mu_1\Phi(\alpha) + \mu_2\Phi(-\alpha) + \beta\phi(\alpha)$ where $\beta = \sigma_1^2 + \sigma_2^2$, $\alpha = \frac{\mu_1 - \mu_2}{\beta}$, $\phi(x) = \frac{1}{\sqrt{2\pi}}e^{-\frac{x^2}{2}}$, and $\Phi(x) = \int_{-\infty}^x \phi(t)dt$.*

5.1 First-best contract

The principal solves the following problem:

$$\begin{aligned} \max_{W_i, e_i} \quad & P_{FB} = E[R(\max(d_1(e_1), d_2(e_2)))] - E[W_1(d_1(e_1))] - E[W_2(d_2(e_2))] \\ \text{s.t.} \quad & E[W_i(d_i(e_i))] - \frac{1}{2}c_i e_i^2 - \frac{1}{2}k_i \text{Var}(W_i(d_i(e_i))) \geq \underline{\pi}_i \quad i = 1, 2 \text{ (IR)} \end{aligned}$$

We cannot decompose this problem into two single agent problems but the solution procedure is similar to that for the single agent case.

Proposition 4 *The first-best effort levels in the parallel case satisfy:*

$$\begin{aligned} r_1 d_{11} \Phi(\theta_{FB}) - c_1 e_1^{FB} &= 0, \\ r_1 d_{12} \Phi(-\theta_{FB}) - c_2 e_2^{FB} &= 0. \end{aligned} \tag{2}$$

where e_i^{FB} are the first-best effort levels, $\theta_{FB} = \frac{d_{01} - d_{11} e_1^{FB} - d_{02} + d_{12} e_2^{FB}}{\sigma_1^2 + \sigma_2^2}$ and $\Phi(\cdot)$ is the cumulative probability function of the standard normal distribution. The corresponding contracts satisfy $w_{0i}^{FB} = \underline{\pi}_i + \frac{1}{2} c_i (e_i^{FB})^2$ and $w_{11}^{FB} = w_{12}^{FB} = 0$ when agent i chooses the optimum effort level, and $w_{0i}^{FB} = -\infty$ otherwise.

The first-order conditions depend on the cumulative probability function for the standard normal distribution, $\Phi(\cdot)$. Due to the complex structure of $\Phi(\cdot)$ we cannot obtain closed-form solutions for the optimal contract parameters, but we can obtain analytical results on how the optimal effort levels change with σ_i , k_i , c_i , and d_{1i} .

Proposition 5 *The optimal effort levels under the first-best contract (full information) for agents 1 and 2 obey the following comparative statics:*

e_1^{FB}	e_2^{FB}	<i>in</i>
\downarrow if $\theta_{FB} > 0$ \uparrow if $\theta_{FB} < 0$	\uparrow if $\theta_{FB} > 0$ \downarrow if $\theta_{FB} < 0$	<i>variability of duration of task 1, σ_1</i>
<i>unchanged</i>	<i>unchanged</i>	<i>risk aversion of agent 1, k_1</i>
\downarrow	\downarrow	<i>cost of effort of agent 1, c_1</i>
<i>undetermined</i>	\uparrow	<i>marginal effect of agent 1's effort, d_{11}</i>

where $\theta_{FB} = \frac{d_{01} - d_{11} e_1^{FB} - d_{02} + d_{12} e_2^{FB}}{\sigma_1^2 + \sigma_2^2}$

The above results show that in the parallel case the effort levels and the contract parameters for both agents depend, as expected, on characteristics of both agents and both tasks, unlike in the serial case. The first row is intriguing. The condition $\theta_{FB} > 0$ is that the expected duration of task 1 must be greater than that of task 2, i.e., in expectation, task 1 is the critical path.

We see that an increase in the variability of task 1 will lead to a shift of effort *away* from agent 1 to agent 2 if task 1 is already expected to be critical, and hence an increase in the expected duration of task 1. The reason for this is that task 2 is cheaper to accelerate (otherwise task 1 would not be critical); higher variability for task 1 increases the probability that task 2 will in fact become critical, making it worthwhile to spend more to further accelerate task 2. The rest of the results are as expected and consistent with the results in the serial case.

5.2 Optimal linear contract under asymmetric information

The principal's problem is now given by:

$$\begin{aligned}
\max_{W_i, e_i} P_{SB} &= E[R(\max(d_1(e_1), d_2(e_2)))] - E[W_1(d_1(e_1))] - E[W_2(d_2(e_2))] \\
\text{s.t. } E[W_i(d_i(e_i))] - \frac{1}{2}c_i e_i^2 - \frac{1}{2}k_i \text{Var}(W_i(d_i(e_i))) &\geq \underline{\pi}_i & i = 1, 2 \quad (\text{IR}) \\
E[W_i(d_i(e_i))] - \frac{1}{2}c_i e_i^2 - \frac{1}{2}k_i \text{Var}(W_i(d_i(e_i))) &\geq \\
E[W_i(d_i(e'_i))] - \frac{1}{2}c_i (e'_i)^2 - \frac{1}{2}k_i \text{Var}(W_i(d_i(e'_i))) &\text{ for all } e'_i & i = 1, 2 \quad (\text{IC})
\end{aligned}$$

To solve this problem, we first find the Nash equilibrium defined by the IC constraints, substitute the equilibrium effort level solutions in the IR constraints, and then follow the same steps as for the first-best solution.

Proposition 6 *The principal's optimal linear contract and corresponding effort levels for the parallel case satisfy*

$$\begin{aligned}
w_{0i}^* &= \underline{\pi}_i + w_{1i}^* d_{0i} - \frac{1}{2} \frac{(w_{1i}^* d_{1i})^2}{c_i} + \frac{1}{2} k_i (w_{1i}^* \sigma_i)^2 \\
w_{1i}^* &= \frac{c_i e_i^*}{d_{1i}} \\
r_1 d_{11} \Phi(\theta_{OL}) - e_1^* c_1 - k_1 e_1^* \left(\frac{c_1 \sigma_1}{d_{11}}\right)^2 &= 0 \\
r_1 d_{12} \Phi(-\theta_{OL}) - e_2^* c_2 - k_2 e_2^* \left(\frac{c_2 \sigma_2}{d_{12}}\right)^2 &= 0
\end{aligned} \tag{3}$$

where $\theta_{OL} = \frac{d_{01}-d_{11}e_1^*-d_{02}+d_{12}e_2^*}{\sigma_1^2+\sigma_2^2}$.

The first-order conditions again depend on the cumulative probability function for the standard normal distribution, $\Phi(\cdot)$. Since we again cannot obtain closed-form solutions for the optimal contract parameters, we derive analytical results on how the optimal effort levels change with σ_i , k_i , c_i , and d_{1i} .

Proposition 7 *The optimal effort levels under the optimal linear contract (asymmetric information) for agents 1 and 2 obey the following comparative statics:*

e_1^*	e_2^*	<i>in</i>
\downarrow if $\theta_{OL} > \Lambda$ \uparrow if $\theta_{OL} < \Lambda$	\uparrow if $\theta_{OL} > \Theta$ \downarrow if $\theta_{OL} < \Theta$	<i>variability of duration of task 1, σ_1</i>
\downarrow	\downarrow	<i>risk aversion of agent 1, k_1</i>
\downarrow	\downarrow	<i>cost of effort of agent 1, c_1</i>
<i>undetermined</i>	\uparrow	<i>marginal effect of agent 1's effort, d_{11}</i>

where $\theta_{OL} = \frac{d_{01}-d_{11}e_1^*-d_{02}+d_{12}e_2^*}{\sigma_1^2+\sigma_2^2}$, $\Theta = \frac{d_{11}k_1c_1e_1^*}{(d_{11})^2+k_1c_1\sigma_1^2}$,

$$\Lambda = \frac{-2k_1 \frac{c_1}{d_{11}} e_1^*(\sigma_1^2+\sigma_2^2)[r_1 \frac{(d_{12})^3}{c_2(\sigma_1^2+\sigma_2^2)} \phi(\theta_{OL}) + (d_{12}+k_2\sigma_2^2 \frac{c_2}{d_{12}})]}{(d_{12}+k_2\sigma_2^2 \frac{c_2}{d_{12}}) \frac{r_1(d_{11})^2}{c_1} \phi(\theta_{OL})}$$

The first row here shows a similar result as in the first best contract, although the degree to which task 1's expected duration must exceed that of task 2 is greater as $\Theta > 0$. The rest of the results are again as expected and similar to those for the first best contract.

If both agents do not suffer from uncertainty, either by being risk-neutral or by having deterministic task duration, i.e., if $k_1\sigma_1 = 0$ and $k_2\sigma_2 = 0$, then the effort levels under asymmetric information are the same as the first-best effort levels.

5.3 Fixed-price contract

Here, we restrict w_{1i} to be zero, so $W_i(d_i) = w_{0i}$ and hence $\sigma_{w_i} = 0$. The agents' expected utility is $U_i(w_i - \frac{1}{2}c_i e_i^2)$, so their optimal effort level is zero, as before. Since the agents do not exert any effort to accelerate their tasks, the principal only pays them enough to satisfy their reservation utility level, so $W_i(d_i) = w_{0i}^{FP} = \underline{\pi}_i$.

5.4 Special case: identical agents and identical tasks

We can obtain closed-form solutions for the special case where the agents and their tasks are identical, i.e., $d_{01} = d_{02} = d_0$, $d_{11} = d_{12} = d_1$, $\sigma_1^2 = \sigma_2^2 = \sigma^2$, $c_1 = c_2 = c$, and $\theta = \frac{(d_1)^2(w_{12}^* - w_{11}^*)}{2c\sigma^2}$. The propositions below give the optimal effort levels and contracts under full information and under asymmetric information with linear and fixed-price contracts.

Corollary 8 *With symmetric agents and tasks, the first-best effort levels are given by $e_1^{FB} = e_2^{FB} = \frac{r_1 d_1}{2c}$. The optimal effort levels under asymmetric information are given by $e_1^* = e_2^* = \frac{r_1 d_1}{2c(1+ck(\frac{\sigma}{d_1})^2)}$. The variable components of the optimal linear contract satisfy $w_{11}^* = w_{12}^* = \frac{r_1}{2(1+ck(\frac{\sigma}{d_1})^2)}$.*

	first-best (full info.)	optimal linear (asym. info.)	fixed-price (asym. info.)
e_i	$\frac{r_1 d_1}{2c}$	$\frac{r_1 d_1}{2c(1+ck(\frac{\sigma}{d_1})^2)}$	0
d_i	$d_0 - \frac{(d_1)^2 r_1}{c}$	$d_0 - \frac{(d_1)^2 r_1}{2c(1+ck(\frac{\sigma}{d_1})^2)}$	d_0
w_{0i}	$\underline{\pi} + \frac{(r_1 d_1)^2}{2c}$	$\underline{\pi} + w_{1i}^* d_0 - \frac{(w_{1i}^* d)^2}{2c} + \frac{1}{2}k(w_{1i}^* \sigma)^2$	$\underline{\pi}$
w_{1i}	0	$\frac{(r_1/2)}{(1+ck(\frac{\sigma}{d_1})^2)}$	N/A
W_i	$\underline{\pi} + \frac{(r_1 d_1)^2}{2c}$	$\underline{\pi} + \frac{((r_1/2)d_1)^2}{c(1+ck(\frac{\sigma}{d_1})^2)}$	$\underline{\pi}$
R	$r'_0 + \frac{(r_1 d_1)^2}{c}$	$r'_0 + 2\frac{((r_1/2)d_1)^2}{c(1+ck(\frac{\sigma}{d_1})^2)}$	$r'_0 - r_1 \sigma^2$
P	$r''_0 + \frac{(r_1 d_1)^2}{2c}$	$r''_0 + \frac{(r_1 d_1)^2}{2c(1+ck(\frac{\sigma}{d_1})^2)}$	$r''_0 - r_1 \sigma^2$

where $r'_0 = r_0 - r_1 d_0$ and $r''_0 = r'_0 - 2\underline{\pi}$.

Table 2: Summary of results for parallel tasks with symmetric agents and tasks

Table 2 summarizes the results for the parallel case with symmetric agents and tasks. The expressions for this special case are highly similar in structure to those for the symmetric version of the serial case. The term r_1 , the marginal revenue the principal gains when the project duration is reduced by one time unit, in the payment and profit expressions in the serial case is replaced by $(r_1/2)$. In the serial case the principal achieves a one-day earlier project completion whenever either one of the tasks is finished a day early, so he only needs to share this marginal revenue r_1 with one agent. By contrast, in the parallel case, in order to finish the project a day earlier both tasks have to be finished a day earlier, so the principal has to share the marginal revenue r_1 with two agents.

Note also that under the fixed-price contract the principal's revenue R^* and profit P^* are now decreasing in task duration variability, unlike the serial case where they were constant. This is because expected project completion time does not depend on the variability of the individual task durations in the serial case, but increases in that variability in the parallel case.

6 Numerical experiments

To illustrate the performance of various contract types and the difference between them we performed a set of numerical experiments based on a 3^4 factorial design, where four parameters can each take any of three levels. We use the 3^k instead of 2^k factorial design to uncover the non-linear relationships. We vary the same four key parameters as in the sensitivity analysis in Propositions 5 and 7: d_{11} , the marginal effect of effort on the duration of task 1; σ_1 , the standard deviation of the duration of task 1; k_1 , the coefficient of risk aversion of agent 1; and c_1 , the cost of effort for agent 1. The corresponding parameters for agent 2 and task 2 are kept constant at their medium level. For each combination we calculate the optimal values for principal's profit P , the payments to the agents W_1 and W_2 , and task durations d_1 and d_2 under the first-best, optimal linear and fixed-price contracts in the serial and parallel cases.

In the serial case, we use the expressions in Table 1 to determine the optimal values for P ,

W_1, W_2, d_1 and d_2 . Since we do not have closed form solutions in the parallel case we use the non-linear mathematical programming solver engine in XPRESS to find the optimal contracts directly from the original problem formulation. Using the optimality conditions derived in Section 5 gives the same results.

The full results for each combination of parameter values in the serial and parallel cases are tabulated in Tables 1 and 2 in the online Appendix. We focus on a subset of these combinations to graphically illustrate the interactions between the factors and the principal's profits and payments. We set d_{11} and k_1 at their medium levels and let σ_1 and c_1 each take three values. Figure 2 shows the changes in profit and payments for the serial case. We observe that, as expected, the principal's profits decrease as the cost of effort for the first agent increases, and this decrease is steeper when the first agent's variability is low. This can be explained as follows: when task duration variability is high, a risk-averse agent will not exert much effort, so the impact of an increase in cost of effort on chosen effort level will be low. We also confirm that the principal's first-best profit does not depend on task duration variability in the serial case. The first agent's payment follows similar trends as the principal's profit; the second agent is not affected by changes in the first agent's costs or variability.

The interactions in the parallel case are more intriguing and sometimes counter-intuitive, as shown in Figure 3. We observe that the principal's profit decreases in c_1 , as in the serial case. However, the principal's first-best profit is now also decreasing in σ_1 , because the variability of the first task affects the expected project duration. If c_1 is low, i.e., when agent 2's task is the critical path, then the principal responds to an increase in σ_1 by increasing his payment to agent 1, hence inducing higher effort and shorter expected duration of task 1. Conversely, when c_1 is high, so that agent 1's task is the critical path, an increase in σ_1 leads to a reduction in payment to agent 1, hence lower effort and longer duration. In short, and surprisingly, the principal seems more concerned about reducing expected duration in response to an increase in uncertainty if the task is *not* on the critical path. This is consistent with the analytical results in Propositions 5 and 7.

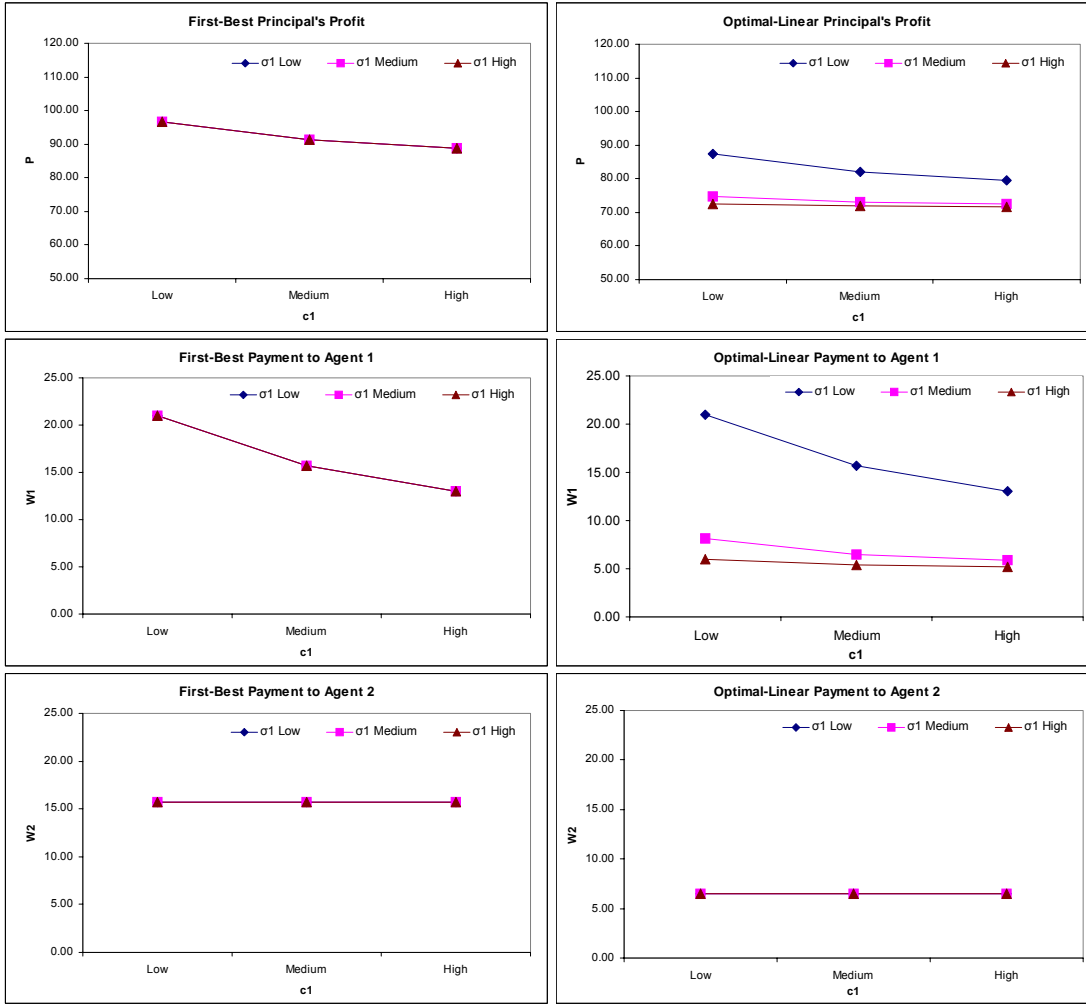


Figure 2: Serial case: σ_1 and c_1 interaction graphs for principal's profit and wages for both agents.

We also explore when the optimal linear contract yields much higher profits than the fixed-price contract, and when fixed-price contracts might be considered reasonable. We use a subset of the full results to graphically illustrate the principal's profits under the three contract types, letting each of σ_1 , k_1 , d_{11} and c_1 individually take either high or low values, while keeping the others at their medium value. As seen in Figure 4 the principal's expected profit under the optimal linear contract is indeed always higher than under the fixed-price contract. However, the difference is smaller for higher values of σ_1 , k_1 and c_1 and lower values of d_{11} . These results were also proven analytically in the serial case. Figure 4 shows that, in our numerical experiments, the same trends occur in the parallel case, where we do not have analytical results. Interestingly,

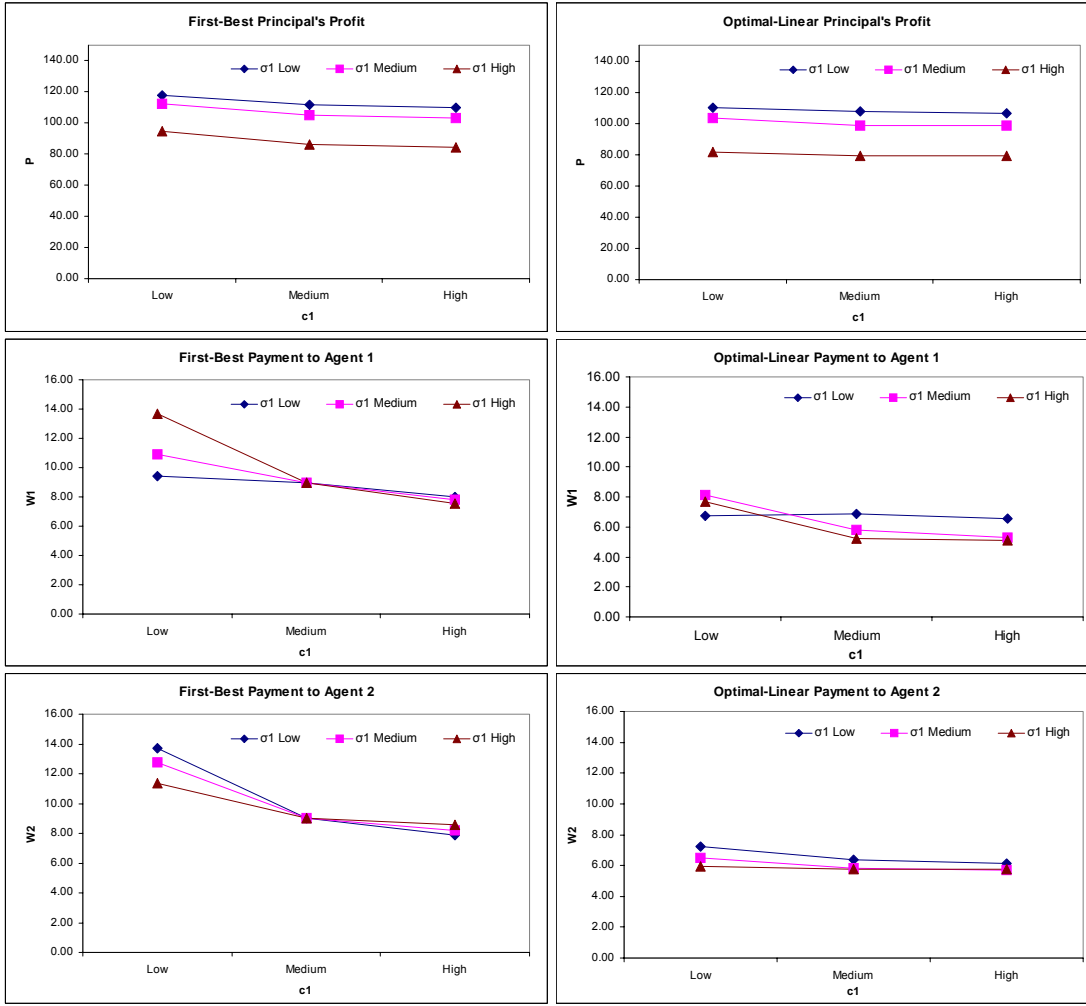


Figure 3: Parallel case: σ_1 and c_1 interaction graphs for principal's profit and wages for both agents.

the cost to the principal of information asymmetry is considerably smaller in the parallel case than in the serial case. This is because in the serial case, the principal can afford to offer an inefficient contract to one of the agents while focusing her resources on the critical agent; in the parallel case, completion time of both agents always matters. While we do not explore this issue further here, this indicates that information asymmetry provides another reason to favor parallel rather than serial structures whenever possible.

Under the same scenarios, Figure 5 shows that the expected project duration is always shorter under the incentive contract than under the fixed price contract. However, the difference is smaller for lower values of σ_1 and higher values of k_1 and c_1 and lower values of d_{11} .

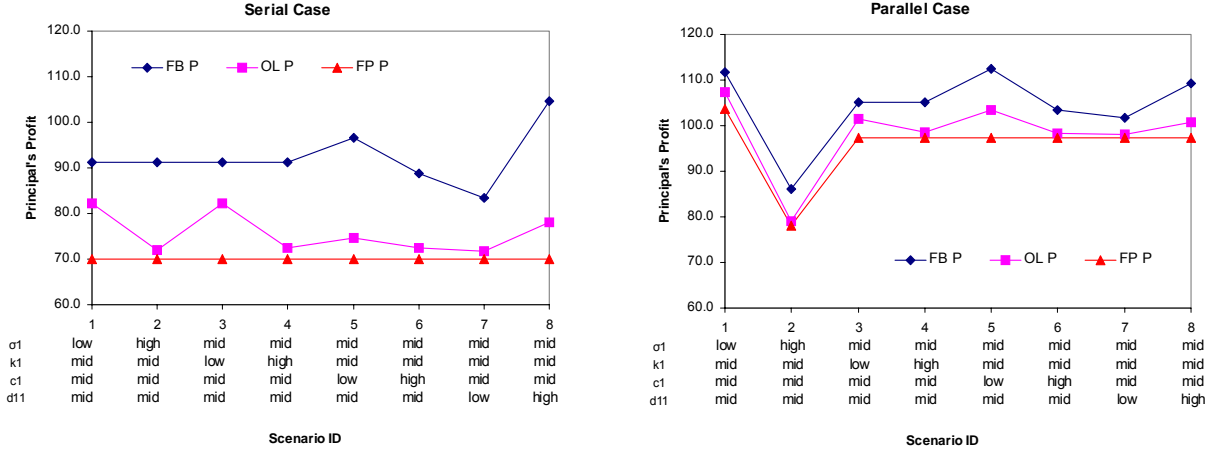


Figure 4: Principal's expected profit (P) under first-best (FB), optimal-linear (OL), and fixed-price (FP) contracts

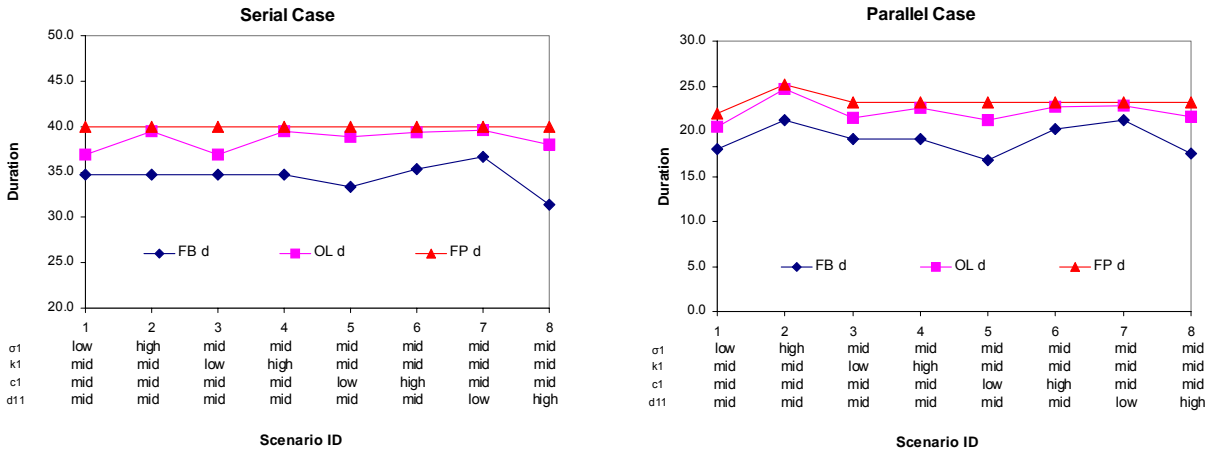


Figure 5: Expected project duration (d) under first-best (FB), optimal-linear (OL), and fixed-price (FP) contracts

Table 3 presents average percent differences in principal's expected profit and expected project duration for all 81 scenarios used in the numerical analysis. Clearly, the case with symmetric information (first-best) results in superior performance compared to the other two cases. Similarly, the optimal linear contract is more profitable than the fixed price contract. The results again suggest that the cost of information asymmetry is greater in the serial case than the parallel case.

Average increase in the principal's expected profit (P)

	FB vs OL	OL vs FP	FB vs FP
	$\frac{P_{FB}-P_{OL}}{P_{OL}}$	$\frac{P_{OL}-P_{FP}}{P_{FP}}$	$\frac{P_{FB}-P_{FP}}{P_{FP}}$
Serial Case	18.7%	13.9%	34.5%
Parallel Case	6.3%	4.0%	10.7%

Average increase in expected project duration (d)

	FB vs OL	OL vs FP	FB vs FP
	$\frac{d_{FB}-d_{OL}}{d_{OL}}$	$\frac{d_{OL}-d_{FP}}{d_{FP}}$	$\frac{d_{FB}-d_{FP}}{d_{FP}}$
Serial Case	-9.5%	-6.1%	-15.1%
Parallel Case	-13.5%	-4.6%	-17.4%

Table 3: Average differences in expected profit and project duration between first-best (FB), optimal linear (OL) and fixed price (FP) contracts across all 81 scenarios

7 Discussion, conclusions, and further research

In this paper we used a moral hazard model with a principal and two agents, working in sequence or parallel, to study the value of incentive contracts in project management. We derived closed-form solutions for the first-best contract under full information and the optimal linear and fixed-price contracts under asymmetric information for the serial case, and confirm that the incentive contract results in shorter expected project duration and higher expected profit for the principal than the fixed-price contract does. However, if the agents have high costs of effort, the task durations are highly variable, the agents are highly risk averse, or the duration reduction per unit of effort is low, then the difference between the incentive and fixed-price contracts is minimal.

For the parallel case, we derived optimality conditions for the first-best and the optimal linear contracts, and show how the optimal effort levels vary with key problem parameters. We found that the optimal contract in the parallel case sometimes behaves in unexpected ways. We obtained closed-form solutions for the parallel case with symmetric agents and tasks, and performed numerical experiments to illustrate the behavior of the various contracts in the serial and the parallel cases.

We discussed the basic findings of our paper with project managers working in software development, aerospace, construction and engineering contracting firms. Some of the results were consistent with their experience. One of the project managers mentioned that “...incentive contracts help us to complete projects in shorter time...” consistent with our model. Another project manager noted that “...if the project scope is not clear, incentive contracts are indeed little or no better than fixed-price contracts...”. Our model suggests that when variability in activity durations is high, incentive and fixed price contracts give similar results. In another case we were told that “...we like to keep the risky projects in-house so we can have more control...”. Our model suggests that when variability in the project duration is higher, the information asymmetry becomes more costly to the project manager.

When we posed the research questions stated at the beginning of the paper to the practitioners, they mostly did not know what to do or would not do anything. With the results obtained here, we can now answer them as follows:

- If the duration of wing development becomes more uncertain, then the project manager should induce the wing manufacturer to exert less effort and the engine manufacturer to exert more if wing development is expected to be the bottleneck. If engine development is expected to be the bottleneck, then the opposite recommendation holds.
- If the wing manufacturer becomes more risk averse, then the project manager should induce both wing and engine manufacturers to exert less effort.
- If it becomes more costly to accelerate wing development, then the project manager should induce both wing and engine manufacturers to exert less effort.

This paper provides the beginning of a framework to integrate contracting and asymmetric information into a project management context. The simple and highly stylized cases we study here are first steps towards modeling more complicated incentive contracting issues in this area. There are several natural extensions to the work presented here. For example, sometimes the

principal can observe both duration and quality of tasks and contracts on both of them. Alternatively, the breakdown of the total project into sub-projects could be endogenous; the principal would then determine how to split a project into pieces by considering the characteristics of the available agents building on the work by Gutierrez and Paul (2000). Another possibility is to let the principal decide the degree to crash when the tasks are performed in series or partly in parallel, building on the stream of work starting from Krishnan et al. (1997) and Loch and Terwiesch (1998). Other extensions are the case where the task durations are correlated, or where the principal can re-negotiate the contract with other agents after observing the outcome of the first task.

Appendix

Proof of Proposition 1

Substituting $R(d)$, $W_i(d_i)$, and $C_i(e_i)$, agent i 's problem can be expressed as:

$$\begin{aligned} \max P_{FB,i} &= r_0 - r_1 d_{0i} + r_1 d_{1i} e_i - (w_{0i} - w_{1i} d_{0i} + w_{1i} d_{1i} e_i) \\ \text{s.t. } w_{0i} - w_{1i} d_{0i} + w_{1i} d_{1i} e_i - \frac{1}{2} c_i e_i^2 - \frac{1}{2} k_i (w_{1i} \sigma_i)^2 &\geq \underline{\pi}_i \quad (\text{IR}) \end{aligned}$$

At optimality, the IR condition is binding. If it were not, we could replace w_{0i} with $w'_{0i} < w_{0i}$. Since the objective function is decreasing in w_{0i} , w'_{0i} improves the objective function. After substituting the expected payment into the objective function, the problem reduces to:

$$\max_{e_i} P_{FB,i} = r_0 - \underline{\pi}_i - r_1 d_{0i} + r_1 d_{1i} e_i - \frac{1}{2} c_i e_i^2 - \frac{1}{2} k_i (w_{1i} \sigma_i)^2.$$

This is concave in e_i , so we can use first-order conditions: $\frac{\partial P_{FB,i}}{\partial e_i} = r_1 d_1 - c_i e_i = 0$, which yields the first-best effort level, $e_i^{FB} = \frac{r_1 d_1}{c_i}$. Substituting this into the objective function and maximizing with respect to w_{1i} yields the desired contract. ■

Proof of Proposition 2

Agent i will choose e_i^* to maximize his utility, which is the same as maximizing $E[W_i(d_i(e_i))] - \frac{1}{2}c_i e_i^2 - \frac{1}{2}k_i(w_{1i}\sigma_i)^2 = w_{0i} - w_{1i}d_{0i} + w_{1i}d_{1i}e_i - \frac{1}{2}c_i e_i^2 - \frac{1}{2}k_i(w_{1i}\sigma_i)^2$. This is concave in e_i ; the first-order condition is $w_{1i}d_{1i} - c_i e_i^* = 0$ which gives $e_i^* = \frac{w_{1i}d_{1i}}{c_i}$. After substituting e_i^* into the original problem we have:

$$\begin{aligned} \max_{w_{0i}, w_{1i}} P_{OL,i} &= r_0 - r_1 d_{0i} + r_1 d_{1i} \frac{w_{1i} d_{1i}}{c_i} - w_{0i} + w_{1i} d_{0i} - w_{1i} d_{1i} \frac{w_{1i} d_{1i}}{c_i} \\ \text{s.t.} \quad w_{0i} - w_{1i} d_{0i} + w_{1i} d_{1i} \frac{w_{1i} d_{1i}}{c_i} - \frac{1}{2} c_i \left(\frac{w_{1i} d_{1i}}{c_i} \right)^2 - \frac{1}{2} k_i (w_{1i} \sigma_i)^2 &\geq \underline{\pi}_i \end{aligned}$$

At optimality, the individual rationality constraint is again binding, and the objective is decreasing in w_{0i} , so the problem is equivalent to

$$\max_{w_{1i}} P_{OL,i} = r_0 - r_1 d_{0i} - \underline{\pi}_i + r_1 \frac{w_{1i} (d_{1i})^2}{c_i} - \frac{(w_{1i} d_{1i})^2}{2c_i} \left(1 + c_i k_i \left(\frac{\sigma_i}{d_{1i}} \right)^2 \right)$$

This is concave in w_{1i} , so the first-order condition is necessary and sufficient. ■

Proof of Proposition 4

The IR constraints are binding. Using Lemma 3 for the expected value of the maximum of two normally distributed random variables, the problem becomes:

$$\begin{aligned} \max_{e_1, e_2} P_{FB} &= r - r_1 [(d_{01} - d_{11}e_1)\Phi(\theta) + (d_{02} - d_{12}e_2)\Phi(-\theta) + (\sigma_1^2 + \sigma_2^2)\phi(\theta)] \\ &\quad - \underline{\pi}_1 - \frac{1}{2}c_1 e_1^2 - \frac{1}{2}k_1 w_{11}^2 \sigma_1^2 - \underline{\pi}_2 - \frac{1}{2}c_2 e_2^2 - \frac{1}{2}k_2 w_{12}^2 \sigma_2^2 \end{aligned}$$

where $\theta = \frac{d_{01} - d_{11}e_1 - d_{02} + d_{12}e_2}{\sigma_1^2 + \sigma_2^2}$. We first need to prove that P_{FB} is concave in e_1 and e_2 . After simplification, the first-order derivatives are:

$$\frac{\partial P_{FB}}{\partial e_1} = r_1 d_{11} \Phi(\theta) - c_1 e_1 \quad \text{and} \quad \frac{\partial P_{FB}}{\partial e_2} = r_1 d_{12} \Phi(-\theta) - c_2 e_2.$$

After simplifications, the second-order derivatives are given by:

$$\begin{aligned}\frac{\partial^2 P_{FB}}{\partial(e_1)^2} &= -\frac{r_1(d_{11})^2}{\sigma_1^2 + \sigma_2^2}\phi(\theta) - c_1 < 0 \\ \frac{\partial^2 P_{FB}}{\partial(e_2)^2} &= -\frac{r_1(d_{12})^2}{\sigma_1^2 + \sigma_2^2}\phi(\theta) - c_2 < 0 \\ \frac{\partial^2 P_{FB}}{\partial e_1 \partial e_2} &= \frac{r_1 d_{11} d_{12}}{\sigma_1^2 + \sigma_2^2}\phi(\theta) > 0\end{aligned}$$

Let $H(P_{FB})$ be the Hessian of P_{FB} . The first principal minor of $H(P_{FB})$ is negative and the determinant of $H(P_{FB})$ is positive. So $H(P_{FB})$ is negative definite and P_{FB} is jointly concave in e_1 and e_2 . The first order conditions in the Proposition are therefore sufficient.

e_1^{FB} and e_2^{FB} are not functions of w_{11} or w_{12} . After substituting e_1^{FB} and e_2^{FB} in P_{FB} , we have:

$$\max_{w_{11}, w_{12}} P_{FB} = \text{Constant} - \frac{1}{2}k_1 w_{11}^2 \sigma_1^2 - \frac{1}{2}k_2 w_{12}^2 \sigma_2^2$$

At optimum, therefore, $w_{11}^{FB} = w_{12}^{FB} = 0$; the IR constraint gives $w_{0i}^{FB} = \underline{\pi}_i + \frac{1}{2}c_i(e_i^{FB})^2$. ■

Proof of Proposition 6

The principal's problem is given by:

$$\begin{aligned}\max P_{OL} &= E[R(\max(d_1(e_1), d_2(e_2)))] - E[W_1(d_1(e_1))] - E[W_2(d_2(e_2))] \\ \text{s.t. } E[W_i(d_i(e_i))] &- \frac{1}{2}c_i e_i^2 - \frac{1}{2}k_i \text{Var}(W_i(d_i(e_i))) \geq \underline{\pi}_i \quad i = 1, 2 \\ E[W_i(d_i(e_i))] &- \frac{1}{2}c_i e_i^2 - \frac{1}{2}k_i \text{Var}(W_i(d_i(e_i))) \geq \\ E[W_i(d_i(e'_i))] &- \frac{1}{2}c_i (e'_i)^2 - \frac{1}{2}k_i \text{Var}(W_i(d_i(e'_i))) \quad \text{for all } e'_i \quad i = 1, 2\end{aligned}$$

To maximize their utility, the agents solve the same problem as in the serial case, so they will choose $e_i^* = \frac{w_{1i} d_{1i}}{c_i}$. At the optimal solution, the individual rationality constraints are binding:

$$E[W_i(d_i(e_i^*))] - \frac{1}{2}c_i (e_i^*)^2 - \frac{1}{2}k_i \text{Var}(W_i(d_i(e_i^*))) = \underline{\pi}_i \quad i = 1, 2$$

As before, $W_i(d_i(e_i))$ is normally distributed with mean $w_{0i} - w_{1i}d_{0i} - w_{1i}d_{1i}e_i$ and variance $(w_{1i})^2\sigma_i^2$. From Lemma 3, the expected revenue is given by:

$$E[\max(d_1(e_1^*), d_2(e_2^*))] = r_0 - r_1[(d_{01} - d_{11}e_1)\Phi(\theta) + (d_{02} - d_{12}e_2)\Phi(-\theta) + (\sigma_1^2 + \sigma_2^2)\phi(\theta)]$$

where $\theta = \frac{d_{01} - e_1d_{11} - d_{02} + e_2d_{12}}{\sigma_1^2 + \sigma_2^2}$. So the principal's problem can be written as:

$$\begin{aligned} \max P_{OL} = & r_0 - r_1(d_{01} - e_1d_{11})\Phi(\theta) - r_1(d_{02} - e_2d_{12})\Phi(-\theta) - r_1(\sigma_1^2 + \sigma_2^2)\phi(\theta) \\ & - \underline{u}_1 - \frac{1}{2}c_1e_1^2 - \frac{1}{2}k_1\left(\frac{e_1c_1\sigma_1}{d_{11}}\right)^2 - \underline{u}_2 - \frac{1}{2}c_2e_2^2 - \frac{1}{2}k_2\left(\frac{e_2c_2\sigma_2}{d_{12}}\right)^2 \end{aligned}$$

We first prove that P_{OL} is jointly concave in w_{11} and w_{12} . After simplifications, the first-order derivatives are:

$$\begin{aligned} \frac{\partial P_{OL}}{\partial e_1} &= r_1d_{11}\Phi(\theta) - e_1c_1 - k_1e_1\left(\frac{c_1\sigma_1}{d_{11}}\right)^2 \\ \frac{\partial P_{OL}}{\partial e_2} &= r_1d_{12}\Phi(-\theta) - e_2c_2 - k_2e_2\left(\frac{c_2\sigma_2}{d_{12}}\right)^2 \end{aligned}$$

After simplifications, the second-order derivatives are given by:

$$\begin{aligned} \frac{\partial^2 P_{OL}}{\partial (e_1)^2} &= -\frac{r_1d_{11}^2}{\sigma_1^2 + \sigma_2^2}\phi(\theta) - c_1 - \frac{k_1c_1^2\sigma_1^2}{d_{11}^2} < 0 \\ \frac{\partial^2 P_{OL}}{\partial (e_2)^2} &= -\frac{r_1d_{12}^2}{\sigma_1^2 + \sigma_2^2}\phi(\theta) - c_2 - \frac{k_2c_2^2\sigma_2^2}{d_{12}^2} < 0 \\ \frac{\partial^2 P_{OL}}{\partial e_1\partial e_2} &= \frac{r_1d_{11}d_{12}}{\sigma_1^2 + \sigma_2^2}\phi(\theta) > 0 \\ \text{where } \theta &= \frac{d_{01} - e_1d_{11} - d_{02} + e_2d_{12}}{\sigma_1^2 + \sigma_2^2} \end{aligned}$$

Let $H(P_{OL})$ be the Hessian of P_{OL} . The first principal minor of $H(P_{OL})$ is negative and the determinant of $H(P_{OL})$ is positive. So $H(P_{OL})$ is negative definite. P_{OL} is jointly concave in e_1 and e_2 . The first-order conditions in the Proposition are therefore sufficient. ■

Proof of Corollary 8

In this symmetric case, the first-best effort levels must satisfy: $e_1^{FB} = \frac{r_1 d_1 \Phi(\frac{d_1(e_2^{FB} - e_1^{FB})}{2\sigma^2})}{c}$ and $e_2^{FB} = \frac{r_1 d_1 \Phi(\frac{d_1(e_1^{FB} - e_2^{FB})}{2\sigma^2})}{c}$. We first show, by contradiction, that the effort levels must also be symmetric. Assume that $e_1^{FB} \neq e_2^{FB}$. So we should either have $e_1^{FB} > e_2^{FB}$ or $e_1^{FB} < e_2^{FB}$. Consider the case that $e_1^{FB} > e_2^{FB}$; then $\theta < 0$ which implies that $\Phi(\theta) < 1/2$. In this case we have $e_1^{FB} < \frac{r_1 d_1 (1/2)}{c}$ and $e_2^{FB} > \frac{r_1 d_1 (1/2)}{c}$, which implies $e_2^{FB} > e_1^{FB}$, in contradiction with our assumption. The other case proceeds analogously. Hence, $e_1^{FB} = e_2^{FB}$; the corollary now follows immediately from the first-order conditions. The optimal linear contract parameters must satisfy: $w_{11}^* = \frac{r_1 \Phi(\frac{(d_1)^2 (w_{12}^* - w_{11}^*)}{2c\sigma^2})}{(1+ck(\frac{\sigma}{d_1})^2)}$ and $w_{12}^* = \frac{r_1 \Phi(\frac{(d_1)^2 (w_{11}^* - w_{12}^*)}{2c\sigma^2})}{(1+ck(\frac{\sigma}{d_1})^2)}$. The proof is analogous to that for the first-best symmetric case. ■

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