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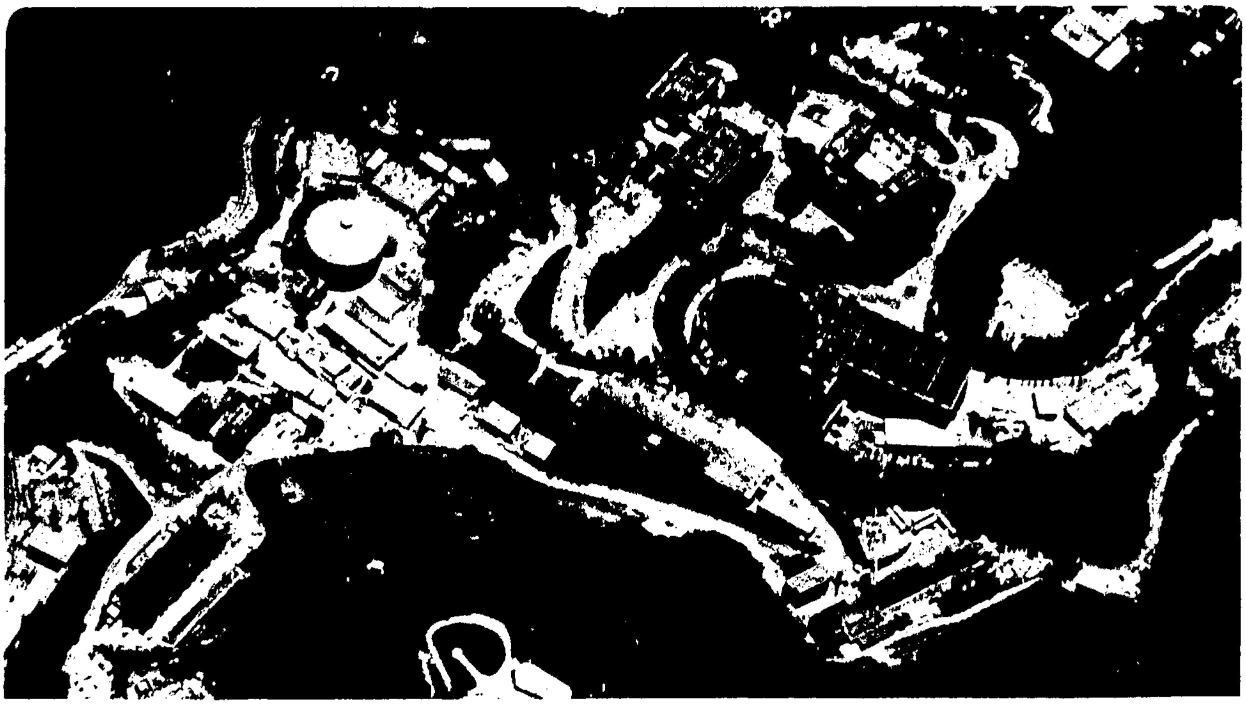

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WEAK SPECTRAL FUNCTIONS AND THEIR APPLICATION TO THE DECAY OF THE W-BOSON*

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ABSTRACT

The weak hadronic spectral function for an arbitrary number of flavors is derived. The method is applied to the hadronic decay width of the charged intermediate W-boson. For completeness, the oneparticle hadronic as well as the leptonic spectral functions are enumerated.

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1. INTRODUCTION

Since it is presently not possible to describe hadronization processes in a rigorous manner, spectral functions can serve as a useful and well defined tool for calculations involving hadronic currents.[1] This paper concentrates on the weak charged current $\langle h(p)|j_\sigma(0)|0\rangle$, where h(p) stands for an arbitrary hadronic state with total four-momentum p. The spectral function $\rho_{\sigma\tau}(k)$ can then be defined as

$$\rho_{\sigma\tau}(k) = \sum\!\!\!\!\!\!\int_{\begin{cases}\text{sum over}\\ \text{internal quantum numbers}\\ \text{integral over phase space}\end{cases}} \delta^4(k-p)\langle 0|j^\dagger_\sigma(0)|h(p)\rangle\langle h(p)|j_\tau(0)|0\rangle$$

$$= \left(\frac{k_\sigma k_\tau}{k^2} - g_{\sigma\tau}\right)\rho(k^2) + \frac{k_\sigma k_\tau}{k^2}\rho'(k^2) \qquad (1)$$

where the vector-and axialvector part $\rho(k^2)$ has been separated from the scalar function $\rho'(k^2)$. For the hadronic continuum, so far only contributions from three[2] or four[3] flavors have been included, leading to a haronic decay width of the W-boson with too few flavors switched on.

At the expected W-mass of approximately 80 [GeV] it is therefore necessary to apply spectral functions for five or more flavors.

2. SPECTRAL FUNCTIONS OF THE HADRONIC CONTINUUM

For the calculation of the spectral function it is assumed that $\rho'(k^2)$ is saturated by singleparticle contributions from the pion and other scalar states. This assumption is justified within the framework of the conserved vector current (CVC) hypothesis, since

$k_\sigma \rho^{\sigma\tau}(k) = 0$. Furthermore we decompose $\rho_{\sigma\tau}$ in (1) into a vector and an axialvector part.

$$\rho(k^2) = \rho_v(k^2) + \rho_a(k^2) \tag{2}$$

Assuming a generalized Weinberg sum rule for an arbitrary number of flavors (which has been rigorously proven up to SU(4))[4] the vector and axialvector parts can be identified.

$$\rho(k^2) = 2\rho_v(k^2) \tag{3}$$

$\rho_V(k^2)$ can be further decomposed into an isoscalar and an isovector part,

$$\rho(k^2) = 2\rho_v^{1}(k^2)[1+b(k^2)] \quad \text{with} \quad b(k^2) = \frac{\rho_V^0(k^2)}{\rho_V^1(k^2)} \tag{4}$$

The isovector part of the weak spectral function can be linked to the isovector part of the electromagnetic spectral function $\varepsilon_v^1(k^2)$ via CVC (From now on we drop the index v)

$$\rho(k^2) = 4\varepsilon^1(k^2)[1+b(k^2)] \tag{5}$$

In order to connect $\varepsilon^1(k^2)$ with the well known ratio $R(k^2) = \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)}$ the electromagnetic spectral function $\varepsilon(k^2)$ is decomposed as outlined above into the isovector and isoscalar part of the vector contribution

$$\varepsilon(k^2) = \varepsilon^1(k^2)[1+a(k^2)] \quad \text{with} \quad a(k^2) = \frac{\varepsilon^0(k^2)}{\varepsilon^1(k^2)} \tag{6}$$

Since $\varepsilon(k^2)$ can be expressed in terms of the $R(k^2)$-ratio (which can be seen by explicitly calculating the cross sections)

$$\varepsilon(k^2) = \frac{k^2}{12\pi^2} R(k^2) \tag{7}$$

one obtaine by inserting (6) and (7) into (5)

$$\rho(k^2) = \frac{k^2}{3\pi^2} R(k^2) \frac{1+b(k^2)}{1+a(k^2)} \tag{8}$$

The functions $a(k^2)$ and $b(k^2)$ can be inferred from the parton model: The cross-sections of the processes $\gamma \to q\bar{q}$ and $W \to qq'$ are proportional to the square roots of the charge matrices in flavor SU(N) space and can be decomposed into their isovector and isoscalar parts (the q's represent the wave functions of the quarks)

$$a(k^2) = \frac{\sum_{ij} |\bar{q}_i Q_{ij}^{em,0} q_j|^2}{\sum_{ij} |\bar{q}_i Q^{em,1} q_j|^2} \quad \text{and} \tag{9a}$$

$$b(k^2) = \frac{\sum_{ij} |\bar{q}_i Q^{W,0} q_j|^2}{\sum_{ij} |\bar{q}_i Q^{W,1} q_j|^2} \tag{9b}$$

These ratios are found by writing the matrices in a representation according to the parton picture. Omitting the mixing of the quarks, one obtains

$$a(n) = \frac{10}{9} n - 1 \qquad \text{and} \qquad b(n) = n - 1 \tag{10}$$

where n is the number of successive quark generations, equivalent to a flavor-SU(2n). Since the generation-dependence of the ratio in (8) containing $a(k^2)$ and $b(k^2)$ cancels the weak spectral function of the hadronic continuum can finally be written as

$$\rho_{\sigma\tau}(k) = \left(\frac{k_\sigma k_\tau}{k^2} - g_{\sigma\tau}\right) \frac{3}{10\pi^2} k^2 R(k^2) \tag{11}$$

3. SPECTRAL FUNCTIONS FOR SINGLE HADRON STATES AND LEPTONS

Just for completeness we enumerate the remaining weak spectral functions. According to their transformation under the Lorentz group the following ansätze for the currents of the singleparticle hadronic states can be made

$$\langle \text{scalar}, p | j_\sigma(x) | 0 \rangle = \frac{i}{(2\pi)^{3/2}} \frac{1}{\sqrt{2p_o}} f_s c_s p_\sigma e^{-ipx} \tag{12a}$$

$$\langle \text{vector,axialvector}, p | j_\sigma(x) | 0 \rangle = \frac{i}{(2\pi)^{3/2}} \frac{1}{\sqrt{2p_o}} f_{v,a} c_{v,a} \varepsilon_\sigma(p) e^{-ipx} \tag{12b}$$

where f_s, f_v and f_a stand for the coupling constants and the corresponding c's are the Cabbibo factors that indicate whether the matrix element is Cabbibo supressed ($\sin^2\theta_c \simeq 0.068$). Table 1 lists the

dominant hadronic states in the scalar ($J^P = 0^-$), vector (1^-) and axialvector (1^+) configuration [5].
It contains scalar and vector particles of the flavor SU(4) and the axialvector particles of SU(3). Other states have not yet been reported or if reported, have not been confirmed.

Particle/Quantum numbers (mass in [GeV])	I_3	S	C	J^P
$\pi^-(0.139)$	- 1			0^-
$K^-(0.493)_c$	- 1/2	- 1		0^-
$\rho^-(0.770)$	- 1			1^-
$K^{*-}(0.89)_c$	- 1/2	- 1		1^-
$A_1^-(1.10)$	- 1			1^+
$Q_1^-(1.28)_c$	- 1/2	- 1		1^+
$D^-(1.87)_c$	- 1/2	- 1	- 1	0^-
$D^*(2.01)_c$	- 1/2		- 1	1^-
$F^-(2.03)$	0	- 1	- 1	1^-
$F^{*-}(2.14)$	0	- 1	- 1	1^-

Dominant states in the 0^-, 1^+, 1^- - configuration Cabbibo supressed decay channels are indicated by the index c.

Table 1

The coupling constants f_π and f_ρ can be obtained by calculating the decays of some well established processes such as $\pi^- \to \mu^- \bar{\nu}_\mu$ that can be compared with experiment. From Weinbergs sum rule we conclude that $f_\rho = f_{A_1}$. Assuming flavor symmetry we finally obtain the value of all coupling constants contained in (12 a, b).[6]

$$f_\pi = f_K = f_D = f_F = 0.932\ m_\pi = 0.132\ [\mathrm{GeV}]$$

$$f_\rho = f_{K^*} = f_{D^*} = f_{F^*} = f_{A_1} = f_{Q_1} = \frac{m_\rho^2}{2\sqrt{\pi}} = 0.5\ [\mathrm{GeV}^2]$$

Insertion of (12) into the definition of the spectral function (1) gives

$$\rho_s(k^2) = 0\ , \quad \rho_s'(k^2) = f_\pi^2 c_{V,A}^2\, k^2\, \delta(k^2-m_s^2) \qquad (13a)$$

$$\rho_{V,A}(k^2) = f_\rho^2 c_{V,A}^2\, \delta(k^2-m_{V,A}^2), \quad \rho'_{V,A}(k^2) = 0 \qquad (13b)$$

Insertion of the leptonic matrix element $<1^-\bar{\nu}_1|j_\sigma(x)|0>$ into (1) leads to the leptonic spectral function

$$\rho(k^2) = \frac{1}{12\pi^2}\,(2k^2+m_1^2)(k^2-m_1^2)^2\, k^{-4}$$
$$\rho'(k^2) = \frac{1}{4\pi^2}\, m_1^2 (k^2-m_1^2)^2\, k^{-4} \qquad (14)$$

which for m_1=0 results in a pure vector contribution

$$\rho_{\sigma\tau}(k^2) = \left(\frac{k_\sigma k_\tau}{k^2} - g_{\sigma\tau}\right) \frac{k^2}{6\pi^2} \qquad (15)$$

4. THE DECAY WIDTH OF THE CHANGED INTERMEDIATE VECTOR BOSON

We are now prepared to apply the formalism to the decay of the W-boson. The contributing matrix element is,

$$T_{fi} = \frac{ig}{2\sqrt{2}} <h(p)|j_\sigma(0)|0> \varepsilon_w^\sigma(k) \qquad (16)$$

When we ask for the decay width $\Gamma(W \to \text{hadronic})$ only the contiuum contributes. After some calculations we find

$$\Gamma(W \to \text{hadr.cont.}) = \frac{3}{10\sqrt{2}\pi}\, G_F\, m_w^3\, R(m_w^2). \qquad (17)$$

The R-ratio depends on the number of flavors that are switched on. Assuming SU(6) at the mass of the W-boson and thereby including states with the (not yet reported) t-flavor we obtain for $G_F = 1.66\times 10^{-5}\ [\mathrm{GeV}]^{-2}$, m_w = 80[GeV] and $R(m_w^2) \approx 5$ (from the parton model),

$$\Gamma(W \to \text{hadron cont.}) \approx 2.1[\mathrm{GeV}] \qquad (18)$$

as compared to the W-decay width computed by using a diquark intermediate state[7] of $\Gamma(W \to qq') \approx 2.3$[GeV] for three generations of quarks.

5. CONCLUSION

Since the hadronic spectral functions of the weak charged currents can easily be written in terms of well-known quantities like couplings f_π, f_ρ and the R-ratio, they prove to be a helpful tool for investigations into the phenomonology of this sector of the standard model.

Given the measured value of R(k), the spectral function of the hadronic continuum includes corrections owing to quark hadronization.

When applied to the decay of the W-boson, however, the R-ratio has to be inferred from the parton model, leading to an agreement with the calculations involving diquark final states within 15%.

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REFERENCES

1. T. Das, V. S. Mather, S. Okubo, Phys. Rev. Lett. 19, 470(1967).
   Yung-Su Tsai, Phys. Rev. D4, 2821 (1971).
2. M. Pietschmann, "The Decay of the Changed and Neutral Intermediate Bosons" published in "Hadron Structure" 2nd Adriatic Meeting Dubrovnik (1976): ed. M. Martinis, N. Zorko.
3. M. Rupertsberger, K. Svozil, Acta Phys.Austr. 54, 255(1982).
4. C. Bernard, A. Ducan, T. LoSecco, S. Weinberg, Phys. Rev. D12, 792(1975).
5. Review of Particle Physics, Phys.Lett. 111B (1982).
6. M. K. Gaillard, B. W. Lee, J. L. Rosner, Rev. Mod. Phys. 47, 277.
7. J. Ellis, et al., Ann. Rev. Nucl. Part. Sci. 32,443 (1982).

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