Title
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Key words: quench protection, superconducting magnets

ABSTRACT

This report demonstrates the role of quench back in the quench protection of high current density superconducting solenoid magnets with well-coupled shorted secondary circuits. The phenomenon of "quench back" can be used to greatly reduce the size of an external quench protection resistor or even to eliminate the need for an external quench protection system altogether. A comparison is made with conventional magnet quench protection systems with and without a closely coupled secondary circuit.
The Lawrence Berkeley Laboratory (LBL) has been building large superconducting solenoid magnets for high energy physics applications for some years. These magnets have required a minimum amount of material between the inside and outside surfaces of the solenoid. In this application, conventional cryostable magnets are not appropriate. The approach developed at LBL uses high current density superconducting coils which are closely coupled inductively to one or more secondary circuits.  

Using superconductors at high current densities \( (j > 2 \times 10^8 \text{ Am}^{-2}) \) requires an understanding of the quench process. When a secondary circuit is involved, an integral part of the quench process is quench back between the secondary circuit and the superconducting coil. Quench back is defined as the process by which a secondary circuit drives the superconducting coil (the primary circuit) normal faster than quench propagation would permit. Quench back becomes a key element to being able to safely quench large high current density superconducting solenoids which have no external quench protection system.

The quench process in a one-dimensional superconductor (with no transverse heat transfer) can be characterized by the one-dimensional equation:

\[
C \frac{\partial T}{\partial t} = \rho j^2 + \frac{\partial}{\partial x} \left( k \frac{\partial T}{\partial x} \right) \tag{1}
\]

where \( C \) is the specific heat per unit volume \( (\text{Jm}^{-3]\text{K}^{-1}) \); \( T \) is temperature \( (\text{K}) \); \( t \) is time \( (\text{s}) \); \( \rho \) is the electrical resistivity of the wire \( (\text{ohm-m}) \); \( j \) is the current density \( (\text{Am}^{-2}) \); \( x \) is the dimension along the wire \( (\text{m}) \); and \( k \) is the thermal conductivity of the wire \( (\text{Wm}^{-1}\text{K}^{-1}) \). \( C, \rho, \) and \( k \) are nonlinear functions of temperature. Equation (1) can
be rearranged into the form

\[
\frac{\partial F}{\partial t} = j^2 + \frac{1}{\rho} \frac{\partial}{\partial x} \left( \alpha \rho \frac{\partial F}{\partial x} \right)
\]  

(2)

where \( F \) is defined as

\[
F(T) = \int_0^T \frac{C}{\rho} dT
\]  

(3a)

and \( \alpha \) the thermal diffusivity is

\[
\alpha = k/C
\]  

(3b)

Equation (2), which is nonlinear, can be simplified if certain assumptions are made. The quench process can be divided into three regions; the first two are not important in this analysis. The third is when \( \alpha \) is small. (This region occurs at temperatures above 30 K.) When \( \alpha \) is neglected, (2) takes the following approximate form:

\[
\frac{\partial F}{\partial t} = j^2
\]  

(4)

Equation (4) is the so-called "burnout" equation, which is commonly used in the electronics industry. This equation was applied to superconductors by Maddock and James. When (4) is integrated at the superconductor hot spot (the point of origin of the quench), it takes the following form:

\[
F(T_M) = \int_0^{T_M} \frac{C(T)}{\rho(T)} dT = \frac{r+1}{r} j^2 \int_0^\infty \Xi(t)^2 dt
\]  

(5)
where $T_M$ is the superconductor hot spot temperature (K), and $r$ is ratio of matrix material volume to superconductor volume. $C$ and $\rho$ are applied to the superconductor matrix material; $j_0$ is the current density in the superconductor at the start of the quench; and

$$E(t) = i(t)/i_0$$

where $i(t)$ is the coil current during the quench and $i_0$ is the starting current in the coil.

For a superconducting magnet to quench safely, $T_M$ must be 400 K or less. For a typical copper-based high current density superconductor, this means that $F(T_M)$ must be $1.7 \times 10^{17} \text{ A}^2 \text{m}^{-4} \text{s}$ or less. (In this case, $j$ and $j_0$ are defined as the current density in the superconductor plus matrix material.) Figure 1 shows $F(T_M)$ as a function of $T_M$ for coppers and aluminums of various residual resistance ratios (RRR).

Equation 5 is a statement of the conditions needed to have safe quenching of a superconducting magnet. This equation can be applied to magnets with or without a secondary circuit and it can be applied whether or not quench back is present. This report presents the equations for safe quenching of superconducting magnets with or without a well coupled shorted secondary circuit. The use of the secondary circuit promotes quench back in superconducting coils which are not well cooled. Quench back can aid conventional quench protection systems as well as make it possible for large superconducting magnets to be quenched without an external quench protection system.
Fig. 1  Peak hot spot temperature $T_M$ as a function of $F(T_M)$ for various coppers and aluminums.

\[ F(T) = \int_{T=0}^{T_{\text{HOT}}} \frac{C(T)}{\rho(T)} dT = \frac{r+1}{r} \int j^2(t) dt \quad (A^2 m^{-4} s) \]
As a review, quench protection of large superconducting magnets without well coupled shorted secondary circuits is presented.\(^7,9\) In general, cryostable magnets where it is not desirable to dump the magnet energy into the helium bath would not use a shorted secondary circuit. On the other hand, the use of a shorted secondary circuit permits one to operate larger superconducting magnets above the cryostable current density limit \((J > 10^8 \text{ Am}^{-2})\). The price one pays for this circuit is that most of the stored magnetic energy will be dumped into the coil and the secondary circuit where it must be removed by the helium refrigeration. The shorted secondary circuit can enhance quench protection, by permitting certain novel quench protection schemes to be used, even when quench back is not induced.\(^4,9\) The conditions for fail safe quench are presented. The use of quench back to promote safe quenching without an external quench protection system is discussed.

**Safe quenching of magnets without a secondary circuit.**

It is possible to design a magnet and an accompanying quench protection circuit that is safe against burnout. The most pessimistic assumption one can make is to assume that the superconducting coil has zero resistance (the hot spot occurs in an infinitely small piece of conductor). Thus, the coil energy must be removed by the quench protection circuit shown in Fig. 2a. The current decay is represented by:

\[
i = i_0 e^{-t/\tau_1}
\]  

\((6)\)
a) Magnet Coil without the Secondary Circuit

b) Magnet Coil with the Secondary Circuit

Fig. 2 Electrical circuit diagrams of superconducting magnets with and without shorted secondary circuits.
where \( \tau_1 \), the time constant of the coil circuit (s), is \( L_1/R_{ex} \) with \( L_1 \) defined as the coil inductance (H) and \( R_{ex} \) as a constant external resistance. (In Fig. 2a, \( R_{ex} \) is defined as \( R_1 \).)

\[
F(T_M) = \frac{r^+1}{r} j_0^2 \left( \frac{\tau_1}{2} + t_{50} \right)
\]  
(7)

where \( j_0 \) is the starting current density in the superconducting matrix and \( t_{50} \) is the time required to detect the quench and switch on the resistor. If a constant resistance is used, the value of this resistance \( R_{ex} \) is:

\[
R_{ex} = \frac{j_0^2}{2F(T_M)} \frac{r^+1}{r} L_1
\]
(8)

where \( L_1 \) is the coil inductance, \( j_0 \) is the starting current density in the matrix, \( F(T_M) \) is found in Fig. 1, and \( r \) is the normal metal to superconductor volume ratio. The maximum voltage developed across the coil electrical leads (neglecting transients associated with switch opening) is:

\[
V_0 = i_0 R_{ex}
\]
(9)

where \( i_0 \) is the starting current.

In general, one wants to limit \( V_0 \) to some reasonable value. In most superconducting magnets, the product of \( V_0 \) and \( i_0 \) has been limited to around \( 10^6 \) W. (The product of \( V_0 \) and \( i_0 \) can be greater when the magnet current \( i_0 \) is above 3000 A.) Thus one can define a limit over which one should not operate a magnet. From
Fig. 1, one can see that $F(T_M)$ for a copper-based superconducting magnet is about $10^{17} \text{A}^2\text{m}^{-4}\text{s}$. Thus, the operating limits for superconducting magnets should be set so that

$$E_0j_0^2 = V_0i_0 F(T_M) \frac{r+1}{r} = 10^{23} \text{J}\text{A}^2\text{m}^{-4}$$  \hspace{1cm} (10)

where $E_0$ is the magnet stored energy (J); $j_0$ is the superconducting matrix current density (A/m$^2$); $V_0$ is the maximum voltage across the quench protection resistor (V); $i_0$ is the starting current (A); and $F(T_M)$ is found in Fig. 1.

In order to make substantial changes in the $E_0j_0^2$ limit given in (10), one must either change the type of external resistor used or employ a secondary circuit. The thyrite resistor or varistor is an external resistor with nonlinear resistance. The resistance is low at high current and high at low current. The resistance of a varistor is characterized by

$$R = R_0(\Xi)^{b-1}$$  \hspace{1cm} (11)

with $b$ on the order of 0.2 to 0.25. The starting resistance of the varistor is $R_0$. The value of $F(T_M)$ in a circuit using a varistor is:

$$F(T_M) = \frac{r+1}{r} j_0^2 \left( \frac{i_1}{3-b} + t_{S0} \right)$$  \hspace{1cm} (12)

where $i_1 = L_1/R_0$. As $b > 0$, the varistor circuit reduces $F(T_M)$ to two-thirds of the value that would be obtained using a constant
resistance $R_{ex} = R_0$. This is not an impressive gain. It should be pointed out that varistor resistors are not good at absorbing and dissipating a lot of thermal energy, and they are expensive.

Safe quenching of magnets with a secondary circuit but no quench back

A closely, inductively coupled low resistance secondary circuit permits one to increase the superconductor current density $j_0$ for a given stored magnetic energy $E_0$. The $E_0 j_0^2$ limit is no longer a function of quench protection parameters when a secondary circuit is used. (The $E_0 j_0^2$ limit is a function of such things as maximum allowable strain in the superconductor). 8

The well-coupled low resistance secondary circuit will affect the quench process in the following ways:

(1) The secondary circuit behaves as a shorted secondary, which causes a shift in current away from the coil to itself.

(2) The secondary circuit will absorb a substantial amount of the magnet stored energy during the quench process.

(3) Since the time constant for magnetic flux decay is long compared to the time constant for the initial coil current decay, the transient voltages in the magnet coil system are reduced.

(4) A secondary circuit will greatly improve the performance of the varistor resistor and will permit other unusual types of external quench protection systems to be used (e.g. the center tap discharge method 4).
(5) The secondary circuit causes portions of the coil that otherwise would not do so to go normal by ordinary quench propagation. This phenomenon is called "quench back."

The secondary circuit must be closely coupled, which means that 
\[ \epsilon \leq 0.05 \] where \( \epsilon = (1 - \frac{M^2}{L_1L_2}) \), where \( M \) is the mutual inductance between the superconducting coil, which has a self-inductance \( L_1 \), and the secondary circuit, which has a self-inductance \( L_2 \). When an ordinary resistor is used for quench protection, the coil circuit time constant \( \tau_1 \) is defined as \( \frac{L_1}{R_{ex}} \) (\( R_1 \) in Fig. 2b) and the secondary circuit time constant \( \tau_2 \) is defined as \( \frac{L_2}{R_2} \).

When \( \tau_1 \) and \( \tau_2 \), the coil and secondary circuit time constants, respectively, are constant, the current \( i \) in the coil has the following time relationship:

\[
i = \frac{i_0}{\tau_L - \tau_S} \left( (\tau_1 - \tau_S)e^{-t/\tau_L} + (\tau_L - \tau_1)e^{-t/\tau_S} \right)
\] (13)

where, when \( \epsilon \) is small,

\[
\tau_1 = \frac{L_1}{R_{ex}}
\] (14a)

\[
\tau_2 = \frac{L_2}{R_2}
\] (14b)

\[
\tau_L = \tau_1 + \tau_2
\] (14c)

\[
\tau_S = \frac{\epsilon \tau_1 \tau_2}{1 + \tau_1 \tau_2}
\] (14d)

\[
\epsilon = 1 - \frac{M^2}{L_1L_2}
\] (14e)
When $\tau_1$ and $\tau_2$ are constant, the value of $F(T_M)$ takes the following approximate form (no quench back is assumed):

$$F(T_M) \approx r^2 \left[ \frac{\tau_1^2}{2(\tau_1 + \tau_2)} + \frac{\tau_s}{2} + t_{so} \right].$$

A comparison of (7) and (15) shows that a low-resistance, well-coupled shorted secondary circuit will reduce $F(T_M)$ and the hot spot temperature considerably. Maddox and James pointed out that if the secondary and primary circuits are made from the same material, the secondary circuit should be combined with the primary circuit (the secondary circuit material becomes part of the primary circuit stabilized material) to yield a somewhat lower value of $T_M$ (when quench back is not present). If one makes the primary circuit from a copper-based superconductor and the secondary circuit from aluminum, the hot spot temperature $T_M$ will be lower for a given superconductor plus secondary circuit mass when a well coupled secondary circuit is used. (Note: when the aluminum is combined with the primary circuit material, $F(T_M)$ will be lower for a given value of $T_M$, which is undesirable.) The use of an aluminum secondary circuit permits one to reduce the overall magnet mass for a given hot spot temperature $T_M$ even when quench back does not occur.

If the external resistor is replaced by a varistor resistor, a dramatic reduction in $T_M$ results (even when there is no quench back present). The method for calculating $T_M$ in this case is given in Refs. 9 and 10. The varistor resistor required for protection is very small when the secondary circuit is present. The coil energy
is deposited in the coil and the secondary circuit rather than in the external resistor. Thyrite resistors are quite expensive, so it is fortunate that only a small amount of the thyrite material (similar to ferrite) is required. The options for external quench protection systems will be compared later.

Before leaving the varistor resistor, we should point out its major disadvantage. When a magnet quenches through a varistor, the voltage across the terminals remains high for some time during a quench. If the terminals are in the cryostat vacuum, the magnet may heat up, releasing frozen gases during this high voltage period. This hazard must be considered when designing coils protected by varistor resistors.

*The quench back time required for fail-safe quenching*

If the whole superconducting coil turns normal fast enough, the superconductor hot spot temperature $T_M$ will be below its maximum allowable limit (say 400 K). Thus, fail-safe quenching without an external resistor can be achieved. If one can control the quench back time, then one can make the whole coil turn normal fast enough to allow for fail-safe quenching without an external quench protection system.

The quench back time required for fail-safe quenching, $t_{QBR}$, is a function of the maximum allowable hot spot temperature $T_M$, the normal metal to superconductor volume ratio $r$, the starting current density $j_0$ over the whole superconductor plus matrix, and the temperature, $T_{S1}$, that the superconductor would achieve if the entire coil were to turn normal instantaneously.
Using adiabatic theory, one can rewrite (5) so that

\[
F(T_M) - F(T_{S1}) = \int_{T_{S1}}^{T_M} \frac{C(T)}{\rho(T)} \, dT = \frac{r+1}{r} r_0^2 \int_0^{t_{QBR}} \Xi(t)^2 \, dt \quad (16)
\]

where \( F(T) \) is defined by (3a) and \( \rho, C, \) and \( \Xi(t) \) are previously defined.

The only assumption which may be applied to (16) is that \( \Xi(t) = 1 \). This assumes that the current in the conductor does not change until after quench back occurs. (This assumption results in a minimum value for \( t_{QBR} \).) Using (16), one can estimate \( t_{QBR} \):

\[
t_{QBR} = \frac{r}{r+1} \left[ \frac{F(T_M) - F(T_{S1})}{r_0^2} \right] \quad (17)
\]

The temperature \( T_M \) can be arbitrarily set, but \( T_{S1} \) is a function of the material contained in the superconducting coil and in all of the coupled secondary circuits.

Using a more complete theory for quench evolution, \( T_{S1} \) can be found by solving the following integral equation for \( T_{S1} \):

\[
G_1(T_{S1}) = \int_0^{T_{S1}} \rho(T)C(T) \, dT = \frac{r}{r+1} \int_0^{\infty} E(t)^2 \, dt \quad (18)
\]

where \( \rho(T), r, \) and \( C(T) \) are previously defined for the superconductor matrix and \( E(t) \) is the electrical field in the superconducting material which has been turned normal. The value of \( E(t) \) is very much
a function of the various time constants of the secondary and primary circuits as the coil energy is deposited in those windings when the coil quenches.

There are equations similar to (18) that calculate $G(T)$ and $T_s$ for each of the active circuits in the magnet. At the end of the quench, each circuit will have a different temperature $T_s$, and the thermal energy deposited in all of the circuits must equal the initial energy stored in the magnet. For a magnet with only one well-coupled secondary circuit, the equation of thermal state at the end of the quench takes the following form:

$$E_0 = L_1 \frac{i_0^2}{2} = V_1[H_1(T_{S1}) - H_1(T_0)] + V_2[H_2(T_{S2}) - H_2(T_0)]$$  

where $L_1$ is the superconducting coil self inductance; $i_0$ is the starting current in the superconducting coil; $H_1(T_{S1})$ is the enthalpy per unit volume of the material in the coil circuit at a temperature $T_{S1}$; $H_2(T_{S2})$ is the enthalpy per unit volume in the secondary circuit at a temperature $T_{S2}$. $H_1(T_0)$ and $H_2(T_0)$ are the enthalpies per unit volume of the materials in the two circuits at the initial temperature $T_0$. $V_1$ and $V_2$ are the metal volumes in the two circuits. Calculations of $T_{S1}$ and $T_{S2}$ are not easy to make with only a hand calculator, so simplifying assumptions are often made.

These simplifications are: (1) the temperatures of the coil and secondary circuit are the same at the end of the quench and (2) the structural and insulating material does not absorb any of the energy during the quench. Making these assumptions is not totally satisfactory, but generally the resulting error in $T_{S1}$ is on the conservative side.
Once $T_{S1}$ has been determined, the value of $F(T_{S1})$ can be determined from Fig. 3 for various copper matrix superconductors. Figure 3 gives $F(T_{S1})$ as a function of $T_{S1}$ and the residual resistivity ratio RRR of the matrix material in the superconducting wire. Table 1 gives $F(T_M)$ for various RRR at values of $T_M = 300 \text{ K}$ and $T_M = 400 \text{ K}$. It is interesting to note that $F(T_M)-F(T_{S1})$ is nearly constant for all values of residual resistivity ratio from 10 to 1000 as long as $T_{S1} \geq 65 \text{ K}$. In large coils one should set the value of $T_{S1}$ to 100 K or less. If $T_M$ is set to 300 K and $T_{S1}$ is set to 100 K, $F(T_M)-F(T_{S1}) = 6.7 \times 10^{16} \text{ A}^2\text{m}^{-4}\text{s}$ for copper-based superconductors with matrix residual resistivity ratios of 10 to 1000. Increasing $T_M$ from 300 to 400 K will increase $F(T_M)-F(T_{S1})$ by about 25 percent.

The role of quench back in the quench protection of a magnet with an external resistor

Quench protection of a superconducting magnet with a secondary circuit is enhanced if the superconductor can be heated through quench back. The form of quench back developed here is thermal quench back which is caused by heating of the secondary circuit from currents transferred there. This heat is transferred to the superconductor through the electrical insulation between the primary and secondary circuits. (The development of the theory of quench back is described elsewhere.) When quench back is considered, the value of the external resistor $R_{ex}$ can be made smaller than one would calculate using (15).
Fig. 3 The value of $F(T_S)$ versus the temperature $T_S$ for coppers of various residual resistance ratios (RRR) and for aluminum (RRR = 25)
Table 1. Values of $F(T_M)$ for $T_M = 300$ K and 400 K for copper of various residual resistance ratios (RRR) and for aluminum at $RRR = 25$

<table>
<thead>
<tr>
<th>Material</th>
<th>RRR</th>
<th>$F(T_M)$ ($A^2 m^-4 s$)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$T_M = 300$ K</td>
</tr>
<tr>
<td>Copper</td>
<td>10</td>
<td>$10.5 \times 10^{16}$</td>
</tr>
<tr>
<td>Copper</td>
<td>30</td>
<td>$11.9 \times 10^{16}$</td>
</tr>
<tr>
<td>Copper</td>
<td>100</td>
<td>$14.5 \times 10^{16}$</td>
</tr>
<tr>
<td>Copper</td>
<td>140</td>
<td>$14.8 \times 10^{16}$</td>
</tr>
<tr>
<td>Copper</td>
<td>200</td>
<td>$15.4 \times 10^{16}$</td>
</tr>
<tr>
<td>Aluminium</td>
<td>25</td>
<td>$4.0 \times 10^{16}$</td>
</tr>
</tbody>
</table>
For a superconducting magnet with a single secondary circuit, using adiabatic theory, one can define a function $F_2^*$ which is a function of the integral of current density in the secondary circuit squared with time:

$$F_2^* = F_2(T_Q) - F_2(T_0) = \int_0^{t_Q} j_2(t)^2 \, dt$$

where

$$F(T_Q) - F_2(T_0) = \frac{\Delta H_2}{\rho_2}$$

and

$$j_2(t) = \frac{i_2(t)}{A_{c2}}$$

where $i_2$ is the current in the secondary circuit; $A_{c2}$ is the cross-sectional area of the secondary circuit; $\Delta H_2$ is the change in enthalpy per unit volume of the secondary circuit material from $T_0$ to $T_Q$; $\rho_2$ is the resistivity of the secondary circuit material ($\rho_2$ is a constant from 0 to 15 K); $T_0$ is the starting temperature of the secondary circuit; $T_Q$ is the temperature of the secondary circuit that drives the superconducting coil normal through quench back ($T_Q$ is between 10 and 15 K); and $t_Q$ is the time for the secondary to heat up from $T_0$ to $T_Q$ in order to induce quench back.

The current in the secondary circuit for a well-coupled system ($\varepsilon$ and $\tau_s$ are small) can be approximated by

$$i_2 = \frac{\tau_2}{\tau_1 + \tau_2} \frac{N_1}{N_2} i_0$$
where $\tau_1$ and $\tau_2$ are defined by (14a) and (14b); $N_1$ is the number of turns in the primary circuit (the superconducting coil); $N_2$ is the number of turns in the secondary circuit from which quench back is induced; and $i_0$ is the starting current in the superconducting coil.

Using (21), one can redefine (20) to the following form:

$$\frac{\Delta H_2}{\rho_2} = \int_0^{t_Q} \left[ \frac{\tau_2}{\tau_1 + \tau_2} \frac{N_1}{N_2} \frac{i_0}{A c_2} \right]^2 dt \quad (22)$$

Equation (22) can be solved for $t_Q$ if several assumptions are made as follows:

(1) The current in the primary circuit drops only a little before quench back occurs ($\tau_2 \ll \tau_1$).

(2) The energy dissipated in the coil and its secondary circuit is small compared to the total magnetic energy in the coil before quench back.

(3) The coupling between the coil circuit and the secondary circuit is good ($t_Q > \tau_s$).

(4) Quench propagation in the coil does not affect quench back (the coil resistance is zero; $R_1 = R_{ex}$).

When (22) is solved for $t_Q$ using these assumptions, the following solution evolves:

$$t_Q = \frac{\Delta H_2}{\rho_2} \left[ \frac{\tau_1}{\tau_2} \frac{N_2}{N_1} \frac{A c_2}{i_0} \right]^2 \quad (23)$$

Equation (23) can be solved for $R_{ex}$ directly in the following form:

$$R_{ex} = \left[ \frac{L_1}{\tau_2} \frac{N_2}{N_1} \frac{A c_2}{i_0} \right] \left[ \frac{\Delta H_2}{\rho_2 t_Q R} \right]^{1/2} \quad (24)$$
where \( L_1, \tau_2, N_2, N_1, A_{c2}, i_0, \Delta H_2, \) and \( \rho_2 \) have been previously defined. \( t_{QR} \) is a function of the quench back time required for fail-safe quenching, \( t_{QBR} \), defined by (17). \( t_{QR} \) is defined as follows:

\[
t_{QR} = t_{QBR} - \left( t_H + \frac{\tau_s}{2} + t_{SO} \right)
\]

where \( \tau_s \) is defined by (14d); \( t_{SO} \) is the time required to detect the quench and switch in the resistance \( R_{ex} \); and \( t_H \) is the time associated with heating the superconductor and the insulation between the superconductor and secondary circuit by heat conducted from the secondary circuit. \( t_H \) has a value of 0.03-0.04 seconds when the insulation thickness between the superconductor and the secondary circuit is 0.5 mm. When the insulation thickness is increased to 1.0 mm, \( t_H \) increases to 0.08 to 0.10 seconds.

Before using (24), one should make sure \( t_H \) is less than \( t_{QBR} \). One should also check \( \tau_s \). If \( R_{ex} \) is small (which means \( \tau_1 \) is large compared to \( \tau_2 \)), \( \tau_s = \epsilon \tau_2 \). One must check to see that the basic assumptions behind (24) are met before applying the equation. If they are not, one must use (23), substituting \( (\tau_1 + \tau_2)/\tau_2 \) for \( \tau_1/\tau_2 \). The full form for \( \tau_s \) must also be used. Equations (14a), (14d), and the modified form of (23) can be solved iteratively to yield a minimum value for \( R_{ex} \).

Table 2 presents the parameters of a magnet similar to the Time Projection Chamber (TPC) magnet built by LBL. It is assumed that
Table 2. Parameters of a superconducting magnet and its secondary circuit

<table>
<thead>
<tr>
<th>Magnet Parameters</th>
<th>Secondary Circuit Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>Self inductance, $L_1$</td>
<td>Circuit time constant, $L_2/R_2$</td>
</tr>
<tr>
<td>Number of turns, $N_1$</td>
<td>Number of turns</td>
</tr>
<tr>
<td>Starting current, $i_0$</td>
<td>Secondary circuit cross sectional area, $A_{c2}$</td>
</tr>
<tr>
<td>Starting current density, $j_0$</td>
<td>Enthalpy change from $T_0$ to $T_0$ ($T_0 = 15 K$)</td>
</tr>
<tr>
<td>Copper to superconductor ratio, $r$</td>
<td>Secondary circuit resistivity, $\rho_2$</td>
</tr>
<tr>
<td>Temperature, $T_{s1}$</td>
<td>$\varepsilon = 1 - \frac{M^2}{L_1 L_2}$</td>
</tr>
<tr>
<td>Hot spot temperature, $T_M$</td>
<td>Heat transfer period, $t_H$</td>
</tr>
<tr>
<td>$F(T_M)$ in matrix ($RRR = 100$)</td>
<td></td>
</tr>
<tr>
<td>Required quench back time, $t_{QBR}$</td>
<td></td>
</tr>
<tr>
<td>Switching time for resistor</td>
<td></td>
</tr>
</tbody>
</table>
a 3.3 m long, 9.5 mm thick aluminum bobbin is used as a quench back element. Table 3 compares the starting resistance for various external quench protection resistors $R_{ex}$ and the peak voltage across that resistor for various types of quench protection systems.

Table 3 shows that the addition of a secondary circuit reduces the values of an external ordinary quench protection resistor. The performance of a varistor resistor is enhanced by the secondary circuit. The reduction is even greater when quench back from the secondary circuit is considered. Quench back as an aid to quench protection using an external resistor is planned for the Collider Detector Facility (CDF) thin solenoid at the Fermi National Laboratory in the United States. Quench back, if properly controlled, can be used for safe quenching of magnets without an external resistor.

Controlled quench back as a method of safe quenching without an external quench protection system

The preceding section showed that thermal quench back can reduce the value of an external resistor used for quench protection of a superconducting magnet with a closely coupled secondary circuit. Reference 5 shows that quench back is an important element in the quench protection of two LBL test solenoids with closely coupled circuits and no external quench protection. Quench back was caused by heating from currents induced in the secondary circuit by the growing resistance of the coils as the normal region propagated within the superconducting coil.

If a superconducting magnet turns completely normal fast enough (in a time less than $t_{QBR}$), no external quench protection system is needed. If the growth of resistance can be controlled and if the current
Table 3. Resistance of an external quench protection resistor and its peak voltage for safe quenching of a magnet with parameters given in Table 2.

<table>
<thead>
<tr>
<th>Quench Protection System</th>
<th>$R_{ex}$</th>
<th>$V_{max}$&lt;sup&gt;c&lt;/sup&gt;</th>
</tr>
</thead>
<tbody>
<tr>
<td>Magnet coil without a secondary circuit and with a normal resistor (Eq. (8))</td>
<td>9.07</td>
<td>20860</td>
</tr>
<tr>
<td>Magnet coil without a secondary circuit but with a varistor resistor, $b = 0.2$ (Eq. (12))</td>
<td>6.48&lt;sup&gt;b&lt;/sup&gt;</td>
<td>14900</td>
</tr>
<tr>
<td>Magnet coil with a secondary circuit and a normal resistor, no quench back (Eq. (15))</td>
<td>2.51</td>
<td>5770</td>
</tr>
<tr>
<td>Magnet coil with a secondary circuit and a varistor resistor, $(b = 0.2)$, no quench back&lt;sup&gt;a&lt;/sup&gt;</td>
<td>0.37&lt;sup&gt;b&lt;/sup&gt;</td>
<td>850</td>
</tr>
<tr>
<td>Magnet coil with a secondary circuit and a normal resistor and quench back (Eq. (23))</td>
<td>0.29</td>
<td>670</td>
</tr>
</tbody>
</table>

<sup>a</sup> See Ref. 10 for the method of calculation

<sup>b</sup> Resistance of the varistor at 2300 A

<sup>c</sup> Excludes the transient voltages associated with the opening of the quantum protection switch
is transferred to the secondary circuit fast enough, fast quench back (where the whole superconducting coil turns normal) can occur such that $t_{QB}$, the quench back time, is less than $t_{QBR}$. This section presents the conditions favorable for fast quench and hence for fail-safe quenching without an external quench protection system.

The quench back time for a superconducting magnet with a closely coupled secondary circuit in thermal contact with the superconducting coil (e.g. the superconducting coil is wound on top of a secondary circuit) can be calculated with the following approximate expression:

$$t_{QB} = t_Q + t_H$$  \hspace{1cm} (26)

where $t_Q$ is the period associated with the shift of current from the superconducting coil to the secondary circuit and the heating of the secondary circuit to a temperature $T_Q$ (about 10 K), and $t_H$ is the period associated with heating the superconductor and the insulation between the superconductor and the secondary circuit by heat inducted from the secondary circuit (see (15) in Ref. 5). Note: A shorted secondary circuit which is bi-filar with the superconducting coil will be well coupled to it inductively and thermally, but such a circuit will slow down the normal region propagation within the superconducting coil. The secondary circuit should be placed to minimize $t_H$ yet the normal region propagation in the coil should not be affected.

$t_Q$ can be calculated using (20) where $j_2(t)$ is defined by (20b), and $i_2$ is defined by (21). Here $T_1$ is not a constant; instead, it starts out at infinity and gets smaller as the resistance of the coil grows. The theory for the growth of the coil resistance is discussed in some detail in Ref. 5. The important thing to understand is whether the quench is predominately three-dimensional,
two-dimensional, or one-dimensional. In Ref. 5, expressions for $t_Q$ are developed for thin coil superconducting solenoids (the coil thickness is small compared to the coil radius such that a three-dimensional quench changes to a two-dimensional quench quickly).

The approximate expressions for $t_Q$ developed in Ref. 5 are as follows:

If the normal region propagation within the coil is two-dimensional, the expression for $t_Q$ is:

$$t_Q = \left[ 5F_2^* \left( \frac{L_1 R_2 N_2 A_{c2}}{L_2 R_0 i_0 V_L^2} \right)^2 \right]^{1/5}$$  \hspace{1cm} (27)

where $F_2^*$ is determined by (20) and (20a) and Table 4. $L_1$, $L_2$, $N_1$, $N_2$, $R_2$, $i_0$, and $A_{c2}$ are previously defined. $V_L$ can be determined by measurement or by the method given in Ref. 14. (Measured normal region propagation velocities along the wire made by LBL are shown in Fig. 4.)

$a$ is the ratio of turn-to-turn normal region propagation velocity to the normal region velocity along the wire $V_L$. The normal region velocity ratio $a$ for a coil with rectangular conductors can be expressed as:

$$a = \left[ \frac{a_{k_i}^{n_{10}}}{L_{c}} \frac{a b'}{S b} \frac{r+1}{r} \right]^{1/2}$$ \hspace{1cm} (28a)

when $b'/b$ is greater than 0.5, and as:
Table 4. Enthalpy of various materials as a function of temperature from 4.5 to 15 K.

<table>
<thead>
<tr>
<th>Temperature</th>
<th>Aluminum&lt;sup&gt;a&lt;/sup&gt;</th>
<th>Copper&lt;sup&gt;a&lt;/sup&gt;</th>
<th>Nb-Ti&lt;sup&gt;a&lt;/sup&gt;</th>
<th>Epoxy&lt;sup&gt;b&lt;/sup&gt;</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.5</td>
<td>1620</td>
<td>1600</td>
<td>8800</td>
<td>9000</td>
</tr>
<tr>
<td>5.0</td>
<td>2100</td>
<td>2230</td>
<td>11300</td>
<td>13000</td>
</tr>
<tr>
<td>6.0</td>
<td>3270</td>
<td>3830</td>
<td>17200</td>
<td>24000</td>
</tr>
<tr>
<td>7.0</td>
<td>4860</td>
<td>6330</td>
<td>24900</td>
<td>40000</td>
</tr>
<tr>
<td>8.0</td>
<td>7020</td>
<td>9790</td>
<td>34700</td>
<td>62000</td>
</tr>
<tr>
<td>10.0</td>
<td>13200</td>
<td>21400</td>
<td>—</td>
<td>120000</td>
</tr>
<tr>
<td>12.0</td>
<td>22400</td>
<td>40100</td>
<td>—</td>
<td>200000</td>
</tr>
<tr>
<td>15.0</td>
<td>48600</td>
<td>95200</td>
<td>—</td>
<td>360000</td>
</tr>
</tbody>
</table>

<sup>a</sup>See Reference 10

<sup>b</sup>Estimate
Fig. 4  Measured normal region propagation velocities along the superconducting wire $V_L$ as a function of superconductor overall current density $J_0$. 
\[ \alpha = 0.7 \left( \frac{\rho_i k_i}{L T_c} \frac{a + r + 1}{S r} \right)^{1/2} \]  

(28b)

when \( b'/b \) is less than 0.5. \( k_i \) is the thermal conductivity of the insulation between turns; \( \rho_i \) is the resistivity of the matrix material in the superconductor; \( r \) is the normal metal (matrix material) to superconductor volume ratio; \( L \) is the Lorentz number \( (L = 2.45 \times 10^{-8} \text{mW/K}^{-2}) \); and \( T_c \) is the critical temperature of the superconductor \( (\text{use } T_c = 7 \text{ K}) \).

The values of \( a, b, b', \) and \( S \) are given as follows: \( a \) is the width of the conductor in the turn-to-turn propagation direction; \( b \) is the thickness of the conductor; \( b' \) is the width of the flat face at the edge of the conductor \((b = b' \text{ when there are no rounded corners on the rectangular conductor})\); and \( S \) is the thickness of the insulation between turns.

For thin superconducting coils with round conductors, the ratio of turn-to-turn to longitudinal normal region propagation velocity \( \alpha \) takes the same form as (28b) except that \( a \) is defined as the diameter of the round conductor.

The resistance function \( R_{02} \), which has the units of \( \Omega \text{m}^{-2} \), takes the following form for a thin solenoid magnet:

\[ R_{02} = \frac{N_1 \rho_1}{\ell_1 A c_1} \left( \frac{r+1}{r} \right) \]  

(27a)

where \( N_1, \rho_1, \) and \( r \) are previously defined; \( \ell_1 \) is the length of the solenoid \((27a) \) assumes that the quench propagates from the end of the solenoids; if the quench propagates from the center of the
solenoid, \( R_{02} \) is doubled); and \( A_{c1} \) is the cross-sectional area of the superconductor (matrix material plus superconductor).

If the normal region propagation within the coil is one-dimensional, the approximate solutions for \( t_Q \) are:

\[
t_Q = \frac{t_1}{2} + \left[ 3F^* \left( \frac{L_1 R_2 N_2 A_{c2}}{L_2 R_{01} N_1 A_{c1} V_L} \right)^2 \right]^{1/3}
\]

when \( t^* = t_1 \) (one-dimensional propagation is from turn to turn, as would be the case in a long solenoid), where \( t^* \) is the shorter of two times \( t_1 \) or \( t_2 \), which are defined as follows:

\[
t_1 = \frac{\pi a_1}{V_L}
\]

and

\[
t_2 = \frac{\xi_1}{\alpha V_L}
\]

where \( a_1 \) is the radius of the solenoid; \( \xi_1 \), \( \alpha \), and \( V_L \) have been previously defined.

The resistance function \( R_{01} \), which has the units \( \Omega m^{-1} \), is defined for the case where \( t^* = t_1 \) as follows:

\[
R_{01} = \frac{2 \pi a_1}{\xi_1} \frac{N_1 \rho_1}{A_{c1}} \frac{r+1}{r}
\]

where \( a_1 \), \( \xi_1 \), \( N_1 \), \( \rho_1 \), \( A_{c1} \), and \( r \) have been defined previously. A quench started at the center of the coil will have double the value given by (29a).
The second form of one-dimensional propagation occurs when $t^* = t_2$ (one-dimensional propagation along the wire, in a short solenoid); $t_Q$ has the following approximation for this case:

$$t_Q = \frac{t_2}{2} + \left[3F_2^*(\frac{L_1 R_2 N_2 A_{c2}}{L_2 R_0 N_1 V_1 L_1})^2\right]^{1/3}$$

(31)

where $R_{01}$ takes the form:

$$R_{01} = \frac{2N_1 \rho_1}{A_{c1}} \frac{r+1}{r}$$

(31a)

where $N_1$, $\rho_1$, $r$ and $A_{c1}$ are previously defined.

Equations (27), (29), and (31) can be used to define $t_Q$ for a thin solenoid, but a better understanding of the quench process requires further modification of these equations. If one assumes that a thin solenoid is well coupled to its shorted secondary circuit ($\varepsilon$ is small), $L_1$, $L_2$, and $R_2$ are defined as follows:

$$L_1 = \frac{\mu_0 \pi a_1^2 N_1^2}{\ell_1} g_1(a_1, \xi_1)$$

(32a)

$$L_2 = \frac{\mu_0 \pi a_2^2 N_2^2}{\ell_2} g_2(a_2, \xi_2)$$

(32b)

$$R_2 = \frac{\rho_2 2\pi a_2^2 N_2}{A_{c2}}$$

(32c)

where $a_1$ is the average radius of the superconducting coil; $a_2$ is the average radius of the secondary circuit coil; $\ell_1$ is the length
of the superconducting coil; \( l_2 \) is the length of the secondary circuit coil; \( N_1 \) is the number of turns in the superconducting coil; \( N_2 \) is the number of turns in the shorted secondary; \( \rho_2 \) is the resistivity of the secondary circuit material; and \( A_{c2} \) is the cross-sectional area of the secondary circuit. \( g_1(a_1, l_1) \) and \( g_2(a_2, l_2) \) are geometric factors for the solenoid. (\( g_1 \) and \( g_2 \) are one when the solenoid is very long or bounded by unsaturated iron poles.) \( \mu_0 \) is the magnetic permeability of vacuum \( (\mu_0 = 4\pi \times 10^{-7} \text{ Hm}^{-1}) \).

Since the coupling between the primary and secondary circuit is good, \( a_1 = a_2 \) and \( l_1 = l_2 \). Therefore, the geometric factors \( g_1 \) and \( g_2 \) are to first order, equal to one another. Thus (30a) and (32b) can be used for \( L_1 \) and \( L_2 \) in the general case with no iron.

The value of \( F_2^* \) is given by (20) and (20a). \( F_2^* \) is a function of nothing but \( \Delta H_2 \) and \( \rho_2 \). The turn-to-turn to longitudinal quench velocity ratio \( \alpha \) is defined by (28a) and (28b), and \( R_{02} \) and \( R_{01} \) by (27a), (29a) and (31a). When one makes the appropriate substitutions into (27), (29), and (31), one gets the following forms:

\[
\begin{align*}
  t_Q & = \left[ \frac{200 \Delta H_2 \rho_2}{\rho_1} \frac{L}{T} \frac{a_1^2 l_1^2}{V_L^4} \frac{2}{J_0^2} \left( \frac{r}{r+1} \right)^3 \right]^{1/5} \\
  \text{for a two-dimensional quench in a thin solenoid from (27);}
\end{align*}
\]

\[
\begin{align*}
  t_Q & = \frac{t_1}{2} + \left[ \frac{3 \Delta H_2 \rho_2}{\rho_1^3} \frac{L}{T} \frac{c_1^2}{V_L^2} \frac{2}{J_0^2} \left( \frac{r}{r+1} \right)^3 \right]^{1/3} \\
  \text{and (33)}
\end{align*}
\]
when $t^* = t_1$, the one-dimensional quench from (30); and

$$t_Q = \frac{t_2}{2} + \left[ \frac{30\Delta H_2 \rho_2 a_1^2}{\rho_1^2 V_L^2 j_0^2} \left( \frac{r}{r+1} \right)^2 \right]^{1/3}$$  \hfill (35)$$

when $t^* = t_2$, the one-dimensional quench from (31). Note that $\Delta H_2$, $\rho_2$, $L$, $T_c$, $a_1$, $\rho_1$, $r$, $V_L$, $j_0$, $t_1$ and $t_2$ have been previously defined. $\Gamma$ is the turn-to-turn to longitudinal propagation function, defined as:

$$\Gamma = k_i (a/S)(b'/b)$$  \hfill (36a)$$

for rectangular conductors with $b'/b$ greater than 0.5 (see (28a)) and

$$\Gamma = 0.5 k_i (a/S)$$  \hfill (36b)$$

for rectangular conductors with $b'/b$ less than 0.5. (Equation (36b) can be applied to round conductors if $a$ is defined as the conductor diameter.)

We find from (33), (34), and (35) that quench back in a solenoid is a function of several variables: (1) $\rho_2$, the resistivity of the secondary material; (2) $\Delta H_2$, the enthalpy change in the secondary needed to induce quench back; (3) $\rho_1$, the resistivity of the matrix material in the superconductor; (4) $r$, the matrix material to superconductor volume ratio in the conductor; (5) $j_0$, the starting current density in the conductor, and (6) $V_L$, the quench velocity along the wire at a current density $j_0$. ($V_L$ is a function of $j_0$ as well; from Fig. 4, one can use $V_L = 2.65 \times 10^{-15} j_0^{1.8}$. See Ref. 16
for Karlsruhe-measured data where $V_L$ has a $j_0^2$ relationship.) The quench back time $t_Q$ is also a function of various coil geometric factors and, where applicable, it is also a function of $\Gamma$, the constant which determines the turn-to-turn quench propagation.

From (17), one can see that $t_{QBR}$ is also a function of $r$ and $j_0$. The reduction of $t_Q$ which results from increasing $j_0$ and decreasing $r$ is more than compensated for by a reduction in $t_{QBR}$. Thus, from a practical standpoint, the only functions which can make a change in $t_Q$ without changing $t_{QBR}$ in a given superconducting magnet are $p_2$, $\Delta H_2$, and $\rho_1$.

Changes in $\rho_2$ and $\Delta H_2$ go together; $\rho_2$ and $\Delta H_2$ are functions of the type, not the amount, of material used in the secondary circuit. The amount of material in the secondary circuit or circuits does affect $t_{QBR}$ but not $t_Q$. (The secondary circuit absorbs energy from the magnetic field; therefore, $T_{S1}$ is reduced when the amount of material in the secondary circuit increases.)

One can use two secondary circuits to promote quench back and absorb energy from the magnetic field. One secondary circuit can induce quench back in a magnet (this circuit needs very little material and should be very closely coupled to the superconducting coil), while a second circuit absorbs a substantial portion of the magnet stored energy (which increases $t_{QBR}$). Such a pair of secondary circuits has been used in the TPC magnet.12,16,17

The parameter with the largest effect on quench back time $t_Q$ is $\rho_1$, the resistivity of the superconductor matrix material. If one
wants to reduce $t_{QB}$ by reducing $t_Q$, one can increase $\rho_1$. This has its price, however, in reduced superconductor adiabatic and dynamic stability. Up to a point, one can compensate for this reduced stability by reducing the superconductor filament size.\textsuperscript{18} When $\rho_1$ is increased substantially, stability must be considered. Unfortunately there are no direct experimental measurements of the effect of $\rho_1$ on $t_{QB}$.

The theory for calculating quench back time $t_{QB}$ and the required quench back time for fail-safe quenching $t_{QBR}$ has a number of assumptions, and there is not much experimental data on the effects of changes in parameters such as $\Delta H_2$, $\rho_2$, and $\rho_1$. The theory appears to be conservative when compared with experimental data taken at LBL. For example, the A and B solenoid experimental data described in Ref. 5 showed that one can predict the quench back time $t_{QB}$. One can also predict $t_{QBR}$ and compare it with measured $t_{QB}$ data.

Figure 5 shows the calculated values for $t_{QB}$ and $t_{QBR}$ in the B solenoid built by LBL.\textsuperscript{19} The measured quench back time $t_{QB}$ is shown. The open square and triangle shows that the calculated value of the hot spot temperature $T_M$ (based on measurements of the integral of $j^2 \, dt$) is less than 300 K. The filled square and triangle show a calculated value of $T_M$ greater than 300 K. In reality the hot spot temperature in B coil did not become higher than 300–400 K because the insulation around the superconductor absorbed alot of the heat generated within the superconductor. The coil was safely quenched even when the current density in the conductor was as high as $1.25 \times 10^9 \, \text{Am}^{-2}$ during a run at Sanida National Laboratory.\textsuperscript{20}
Fig. 5  Calculated values of $t_{QB}$ and $t_{QBR}$ compared with measured quench back time in the B solenoid (see text) as a function of start current $i_0$ in the solenoid
Fail safe quenching using controlled quench back is suited to indirect cooled solenoids (cooled with two phase helium in tubes near the coil) similar to those built by LBL. The higher the magnet stored energy, the better the coupling between the primary and secondary circuit must be. Controlled quench back should extend the range of self protected superconducting magnet considerably once the needed experimental work as been done.
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