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Preservice Secondary Science Teachers' Implementation of an NGSS Practice: Using Mathematics and Computational Thinking

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ABSTRACT

We investigated six preservice secondary science teachers' implementation of reform-based science, in particular, their teaching of the *Next Generation Science Standards'* (*NGSS*) science and engineering practice of *using mathematics and computational thinking*. A modified version of the Task Analysis Guide in Science served as our conceptual framework: It assesses both the *integration* of practices and content (i.e., the kind of thinking required), and the *cognitive demand* of tasks (i.e., the level of thinking required) in teachers' lessons. We used this framework to qualitatively analyze our preservice teacher participants' edTPA (teacher performance assessment) lessons—including their written commentaries, video-recorded lesson excerpts, and student work samples—for their implementation of the *NGSS using mathematics and computational thinking* practice. We examined (1) the integration of the mathematical content and practices outlined in the *Common Core State Standards for Mathematics* within the target *NGSS* practice, and (2) the cognitive demand of the mathematics in relation to science and mathematical practices. We found that four of our six preservice teachers implemented lessons that were integrated and cognitively demanding: These participants used the mathematics to move students' understanding of the science phenomena forward. However, the other two participants implemented lessons that integrated mathematical content and practices but were low in cognitive demand. We conclude with implications for how teacher education programs can better support preservice teachers' implementation of lessons that are both integrated *and* cognitively demanding so as to promote students' mathematical reasoning and scientific sensemaking.

KEYWORDS
Secondary science teaching; preservice science teachers; *NGSS* science and engineering practices; *Common Core* mathematical practices

Recent science education reform documents in the United States call for teachers to facilitate all students' learning of science in ways that are engaging, authentic, and relevant to their lives (National Research Council [NRC], 2012; NGSS Lead States, 2013). Instead of reading about science in books or memorizing the steps of the scientific method, teachers are now expected to encourage their students to learn science by actually *doing science* (Furtak, 2017). More specifically, the *NGSS* identifies eight science and engineering practices that should be *integrated* with disciplinary core ideas and crosscutting concepts to engage students in *cognitively demanding* work where they reason about and make sense of phenomena (NGSS Lead States, 2013); creating science classrooms that resemble

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professional science communities promotes students' engagement in science and engineering practices and content to build deep and complex understanding (Windschitl et al., 2018). This move from presenting science knowledge as fixed and factual to one that is constructed, integrated, and thus cognitively demanding makes the processes of teaching and learning more challenging for both teachers and their students (Reiser, 2013; Sandoval et al., 2016). Preparing preservice secondary science teachers to adequately support their students in learning science as envisioned in these current reform documents should be a priority for all teacher education programs.

One of the eight *NGSS* practices, *using mathematics and computational thinking*, emphasizes mathematics and computation as fundamental tools for representing physical variables and their relationships when engaged in science and engineering (NGSS Lead States, 2013). Mathematics and computation as tools are used for tasks ranging from constructing simulations, to making quantitative predictions, to statistically analyzing data, to recognizing, expressing, and applying quantitative relationships. Although all eight practices are interrelated and integral to reform-based science instruction, this particular practice is rarely investigated (Weintrop et al., 2016).

To help fill a gap in the literature, then, we explored how preservice secondary science teachers (PSTs) implemented tasks that facilitated students' use of the *NGSS* science and engineering practice of *using mathematics and computational thinking*. We investigated how participants implemented this practice in their edTPA (a teacher performance-based assessment) lesson series: how they engaged students in *using mathematics and computational thinking* both by integrating this practice with mathematical practices and content specified in the *Common Core State Standards for Mathematics* (*CCSS-M*; National Governors Association Center for Best Practices & Council of Chief State School Officers [CCSO], 2010), and by creating cognitively demanding tasks that connected this practice to other *NGSS* and *CCSS-M* practices. To do so, we used a modified version of the Task Analysis Guide in Science (TAGS; Tekkumru-Kisa et al., 2017). The modified TAGS-M framework allowed us to assess PSTs' lessons based on the kinds of epistemic and cognitive opportunities they provided for student reasoning and sensemaking—to determine the extent to which these lessons engaged students in learning both science and mathematical practices and content. More precisely, PST participants' lessons were examined along two dimensions: the *integration/isolation* of mathematical content and practices (i.e., the kind of thinking required of students), and the *cognitive demand* of the science and mathematical practices included (i.e., the level of thinking required of students). Our analysis was guided by the following research question: How did preservice secondary science teacher participants implement the *NGSS* practice of *using mathematics and computational thinking* to engage their students in science and mathematical reasoning and sensemaking? More specifically, we asked: (1) To what extent did participants integrate mathematical content and practices in their lesson series? (2) To what extent did their lessons provide opportunities for students to engage in cognitively demanding science and mathematical work?

Conceptual framework

Our conceptual frame extends the two-dimensional TAGS framework proposed by Tekkumru-Kisa and colleagues (Tekkumru-Kisa et al., 2017; Tekkumru-Kisa et al., 2015) to include mathematics as well as science. The original framework enables researchers to track the *NGSS* science and engineering practices and content included in tasks so as to evaluate both the kind (i.e., integrated or isolated) and level (i.e., cognitive demand) of student reasoning and sensemaking required. More concretely, one dimension of this framework examines whether a task is integrated or isolated across the content and practices of the discipline; the other determines the extent to which a task promotes authentic disciplinary thinking, or is cognitively demanding. We extended the TAGS framework to create a TAGS-M framework to better understand participants' implementation of the practice *using mathematics and computational thinking*. We considered both the integrated/isolated nature of the mathematical content in relation to the mathematical practices and the cognitive demand of the mathematics within the science tasks. In other words, we used the TAGS-M framework to investigate both the science and mathematical content and practices included, and the cognitive demand delineated in participants' implementation of the *NGSS* practice of *using mathematics and computational thinking*.

The *NGSS* practice of using mathematics and computational thinking

As stated above, we investigated PSTs' implementation of one of the eight *NGSS* science and engineering practices: *using mathematics and computational thinking* (NGSS Lead States, 2013). This practice presents mathematics as a tool that is central to science and engineering—that helps to explain the natural and designed world (Hoda Wilkerson & Fenwick, 2017). The practice of using mathematics is considered tightly tied to another of the *NGSS* practices, that of *developing and using models*: Mathematical relationships are often powerful ways to represent, share, and test models for how and why a phenomenon happens. When engaged in mathematical work in a science classroom, students are expected to examine patterns in a system, describe the relationships between variables using the language and symbols of mathematics, and test or adjust these mathematical relationships as more data are collected.

The *NGSS* also encourages students to engage in computational thinking by using and developing simulations of natural and designed systems, and by working with data to organize, search, and create sequences of steps called algorithms (Hoda Wilkerson & Fenwick, 2017). As when engaged in mathematical work, computational thinking most often immerses students in simulations around mathematical models. Such simulations are often used to see if a model of a phenomenon makes sense by comparing the outcomes with what is known about the real world. The PSTs we investigated more often engaged their students in using mathematics than in computational thinking.

The integration of the CCSS-M mathematical content and practices

Our modified TAGS-M framework considers the ways in which the mathematical and computational work specified in the *NGSS* practice of *using mathematics and computational thinking* can be informed by the mathematical content and practices outlined in the *CCSS-M* (CCSO, 2010). For the first dimension of the TAGS-M framework, we used the *CCSS-M* to determine the integration/isolation, or kind of thinking, required of students in the lesson. Key *CCSS-M* mathematical content relevant to reform-based high school science classrooms includes algebra, functions, and modeling. To elaborate, algebra content standards ask students to create and use algebraic expressions and equations that describe number relationships between variables. The function content standards encourage students to graph and interpret functional relationships that arise in applications in terms of their context, such as force versus time graphs when conducting experiments about impulse or momentum. When engaged with algebra and functions content, students also need to address modeling content standards: Students choose and use appropriate mathematics and statistics to display and analyze empirical results, to understand them better, and to communicate them in a concise manner to an audience. The authors of the *CCSS-M* underscored that modeling is best understood not as isolated content but as related to other content standards.

The eight mathematical practices (MPs) included in the *CCSS-M* describe the types of mathematical expertise that students should develop as they learn mathematical content. See Table 1 below for a list of these eight practices, their definitions, and possible linkages to the *NGSS* practices. As one example, in *modeling with mathematics*, students apply the mathematics they know to solve problems encountered in everyday life, society, and the workplace. They might use geometry to solve a design problem or a function to describe how one quantity of interest depends on another. It is important to repeat that the practice of mathematical modeling is also a set of content standards at the high school level. As a second example, in *making sense of problems and persevering in solving them*, students look closely at the mathematics to discern a possible solution pathway. They explain to themselves the meaning of a problem, look for entry points, make conjectures about the form and meaning of the solution, consider analogous problems, monitor and evaluate their progress, and change course if necessary. Students would need to draw on both of these *CCSS-M* mathematical practices, as well as others, to fully engage in the level of mathematical work called for in the *NGSS* practice of *using mathematics and computational thinking*.

The cognitive demand of the mathematics within science lessons

The second dimension of the TAGS-M framework attends to the cognitive demand of a lesson. It characterizes the degree of thinking and reasoning required for students to carry out tasks, or the level of thinking required of them (Tekkumru-Kisa et al., 2017). As Tekkumru-Kisa et al. argued, “Existing activities may appear to be aligned with conceptual shifts in the *NGSS* because they are ‘hands on’ but often miss the mark because they do not address building and testing explanatory ideas” (p. 5). Cognitively demanding lessons in science include intellectually challenging tasks that “prompt students to engage in disciplinary practices that deepen their understanding of the world through reasoning and advance students’ thinking by inviting them to link observable phenomena and theoretical ideas” (p. 4).

To elaborate, within the context of reform-based instruction, PSTs have the important role of encouraging students to think deeply about scientific concepts and ideas (Roth & Givvin, 2008). Previous studies of inquiry-based instruction have found that active student thinking (i.e., thinking creatively and/or building on prior knowledge) connects to improved student learning (Boaler & Staples, 2008; Minner et al., 2010). As a result, students in classrooms that are aligned with the new science standards should be expected to participate in intellectually complex and deep thinking and reasoning related to scientific big ideas so as to grapple with “substantive relationships between concepts in the form of scientific models that help learners understand, explain, and predict a variety of important phenomena in the nature world” (Windschitl et al., 2018, p. 182). Such complex and deep thinking often involves using mathematics Lessons grounded in practices and big ideas are

Table 1. The eight CCSS-M mathematical practices and hypothesized relationships to NGSS from the literature.

Mathematical Practice	Definition	Links to *NGSS* Practices
MP 1. Make sense of problems and persevere in solving them	Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals.	**SEP 1**. Asking question and defining problems **SEP 2**. Developing and using models **SEP 3**. Planning and carrying out investigations **SEP 5**. Using mathematics and computational thinking
MP 2. Reason abstractly and quantitatively	Mathematically proficient students make sense of quantities and their relationships in problem situations.	**SEP 2**. Developing and using models **SEP 3**. Planning and carrying out investigations **SEP 5**. Using mathematics and computational thinking
MP 3. Construct viable arguments and critique the reasoning of others	Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.	**SEP 5**. Using mathematics and computational thinking **SEP 6**. Constructing explanations and designing solutions. **SEP 7**. Engaging in argument from evidence **SEP 8**. Obtaining, evaluating and communicating information
MP 4. Model with mathematics	Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.	**SEP 2**. Developing and using models **SEP 3**. Planning and carrying out investigations **SEP 5**. Using mathematics and computational thinking
MP 5. Use appropriate tools strategically	Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software.	**SEP 2**. Developing and using models **SEP 3**. Planning and carrying out investigations **SEP 4**. Analyzing and interpreting data **SEP 5**. Using mathematics and computational thinking
MP 6. Attend to precision	Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning (e.g.: units, symbols).	**SEP 3**. Planning and carrying out investigations **SEP 5**. Using mathematics and computational thinking **SEP 8**. Obtaining, evaluating and communicating information
MP 7. Look for and make use of structure	Mathematically proficient students look closely to discern a pattern or structure.	**SEP 4**. Analyzing and interpreting data **SEP 5**. Using mathematics and computational thinking **SEP 6**. Constructing explanations and designing solutions **SEP 7**. Engaging in argument from evidence
MP 8. Look for and express regularity in repeated reasoning	As students work to solve a problem, they maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.	**SEP 5**. Using math and computational thinking **SEP 6**. Constructing explanations and designing solutions

In creating this table, we drew from Mayes and Koballa (2012) work.

essential to promoting rigorous learning in science and in particular, when grounded in the practice of using mathematics and computational thinking, require students to participate in complex and deep thinking about mathematics in order to arrive at those big scientific ideas. Hence, by identifying patterns in a system, measuring those patterns quantitatively, and using the language and symbols of mathematics to describe the physical or causal relationships between quantities, students must engage in appropriate mathematics and science content and practices to increasingly and effectively enhance their understanding of natural phenomena (Hoda Wilkerson & Fenwick, 2017).

In the context of this study, we understand high-level, cognitively demanding tasks to offer substantial opportunities for students to reason as they engage in the *NGSS* practice of *using mathematics and computational thinking* in interaction with other *NGSS* practices and *CCSS-M* practices so as to make sense of both science and mathematics ideas (Stein et al., 2009). Such high cognitively demanding tasks give students opportunities to use mathematics for the purpose of developing deeper levels of understanding of scientific phenomena—to engage with the conceptual mathematical ideas when analyzing data collected, to represent and find relationships between variables, and to better understand mathematical and scientific ideas in the context of developing, testing, arguing, and explaining models. In contrast, low-level tasks offer very limited opportunities for student reasoning by prompting students to recall, remember, check, define, or replicate prior scientific facts and concepts. Students are asked to memorize a given formula, to produce tables or graphs with little analysis of their meaning or connections to larger scientific ideas, and/or to focus on producing correct answers to word problems rather than on developing mathematical and science understanding. As a result, low cognitively demanding tasks deliver minimal opportunities for students to participate in thinking and sensemaking in regard to science content and/or practices.

Review of relevant literature

While there are a number of studies on how mathematics teachers have implemented the *CCSS-M* mathematical practices (e.g., Davis et al., 2013; Selling, 2016), we found little research on science teachers' understanding and/or implementation of the *NGSS* practice of *using mathematics and computational thinking*. A few studies focus on what PSTs learn as a result of courses that target some aspect of this particular practice; these studies report mixed success in helping PSTs understand and implement mathematics and computational thinking. As one example, Menon and Devadas (2019) developed a lesson series on green energy in a nanoscience context to help secondary PSTs understand how the *NGSS* and its three dimensions can be applied in chemistry. The authors found that PSTs were able to identify a number of the *NGSS* practices used in these lessons in addition to a number of the crosscutting concepts. However, PSTs failed to identify the practice of *using mathematics*—engaging in mathematical thinking when using real wooden-cube models to calculate surface-to-volume ratios in differently sized nanoparticles. In a second study, Zha et al. (2020) implemented a flipped learning module on block programming in their educational technology course to teach K-8 PSTs computational thinking ideas and integration. The authors found that although PSTs were able to improve their understanding of computational thinking, they did not have enough time to create a specific application in coding, including the development of lesson ideas.

A handful of other studies examine preservice and practicing teachers' implementation of the practice of *using mathematics and computational thinking* as one of the eight *NGSS* practices implemented; as with those studies reviewed above, these studies report mixed success in implementing this practice. In one study, Smith and Nadelson (2017) investigated three inservice elementary school teachers' implementation of all eight *NGSS* practices. While researchers found that most of the *NGSS* practices were present in these teachers' science instruction, the practice of *using mathematics and computational thinking* was one of the least likely to be included. (For similar findings, see Kang et al., 2019.) In a second study, Brownstein and Horvath (2016) examined 10 PSTs' edTPA portfolios and the artifacts therein for evidence of these beginning teachers' enactment of the *NGSS* practices. Researchers found that while there was evidence of *using mathematics and computational thinking* in a majority of portfolios, two-thirds of all instances were implemented by only three physical science teachers. Brownstein and Horvath argued that it was unclear if the type of mathematics occurring in these instances was the deep thinking argued for in this practice.

Our study serves to inform science teacher preparation centered on the *NGSS* practices. We extend the latter group of studies by focusing specifically on the strengths and limitations of preservice secondary science teachers' implementation of *using mathematics and computational thinking* in terms of both content-practice integration and cognitive demand in the context of their edTPA lesson series. Our research also builds from the former group of studies by providing additional suggestions for ways to strengthen PSTs' understanding and implementation of *using mathematics and computational thinking* during their teacher education experiences.

Method

Teacher education program and participants

Data were collected from two cohorts (2015–2016, 2016–2017) of preservice secondary science teachers enrolled in a small, 13-month, post-baccalaureate teacher education program in California. All preservice secondary science teachers completed the same three science methods courses taught by two instructors during their program. Because the science cohorts were small, methods courses specific to physics, chemistry, or biology were not offered. Participants included 16 of the 17 PSTs who had received a Noyce scholarship and, thus, had committed to teaching for two years in a high-needs school district; the 17th declined to participate. We examined all 16 participants' edTPA portfolios and selected all of those who provided their students substantive opportunities to engage in the practice of *using mathematics and computational thinking*. While many PST participants included tasks that required simple calculations, only six of the 16—four preservice physics/engineering teachers and two preservice chemistry teachers—engaged their students in mathematics that went beyond arithmetic and multiplication. As such, we focused our investigation on those six participants who provided substantive opportunities for their students to engage in mathematics. Table 2 lists the six focal PSTs and the topics of the lesson series they implemented for their edTPA.

Table 2. PST participants and their edTPA lesson series.

Preservice Science Teacher	Haylee	Lucas	Thatcher[a]	Zeke	Sidney[b]	Noah
edTPA Science Discipline(s)	Physics/Engineering				Chemistry	
edTPA School Context	Foothill High	Foothill High	Coastal High	Coastal High	Foothill High	Foothill High
edTPA Science Topic(s)	Torque and rotational equilibrium to inform engineering design project	Compound gears and transmissions to inform engineering design project	Conservation of momentum		Gas laws	

[a]Although their topic was the same, Thatcher and Zeke taught different lesson series to similar classes of students at Coastal High. [b] Sidney and Noah implemented the same lesson series distinctively to different audiences at Foothill High, the former to an honors class and the latter to a college preparatory class.

Data

We collected and analyzed our six focal PSTs' edTPA portfolios. The edTPA is a national performance assessment used for initial teacher certification in a number of U.S. states, including California. It focuses on a two-to-five-day lesson series and consists of three sections: planning, instruction, and assessment. Each section includes a typed, single-spaced written commentary of approximately six pages in length. PSTs also submit lesson plans, two video excerpts of instruction (a combined three to 20 minutes), and three student work samples.

Analytic process

We conducted a qualitative comparative case study (Merriam & Tisdell, 2016) because our goals were to understand each of our participant's use of mathematics in implementing reform-based science instruction and to identify strengths and limitations across these cases. To answer the two parts of our research question, we began by transcribing the video excerpts of instruction. We then qualitatively analyzed the written commentaries, lesson plans, and video transcripts using four cycles of analysis and a different set of a priori or emergent codes for each (Saldaña, 2016). For each cycle, the research team first identified and defined the codes collectively. Next, team members coded the data individually and met as a group to discuss codes assigned and to reconcile coding differences. The final coding represented group consensus.

More specifically, for our first cycle of analysis, we coded the data for all instances of the *NGSS* science and engineering practice of *using mathematics and computational thinking*. Researchers determined what counted as an instance of this practice independent of PST participants' labels. Indeed, most PSTs failed to explicitly identify students' use of mathematics in their commentaries and lesson plans. For our second analytic cycle, for each of these instances, we coded for the *CCSS-M* mathematical content standards and eight mathematical practice standards. We cross-checked participants' edTPA lesson plans and commentaries with the transcripts of their video clips to ensure appropriate documentation of content and practices implemented. We used these codes to begin to understand how integrated/isolated the implementation of the *using mathematics and computational thinking* practice was. As one example, the algebra content standard code *create equations in two or more variables to represent relationships between quantities* was used when students derived an equation to describe a mathematical relationship, satisfying rotational equilibrium, among the forces, masses and acceleration of objects they were

Table 3. Modified TAGS-M framework.

Quadrant	Type of Sensemaking	Description
I	Integrated and High Cognitive Demand	Students were asked to engage in significant mathematical thinking within scientific sensemaking. One or more mathematical practices were integrated with mathematical content in such a way that mathematical reasoning led to greater scientific understanding of the phenomenon under investigation.
II	Isolated and High Cognitive Demand	Students were positioned to engage in high-level cognitive processes for understanding. However, either mathematical content or mathematical practices were not implemented in such a way as to increase students' scientific sensemaking through mathematical reasoning.
III	Isolated and Low Cognitive Demand	Students were asked to focus on either the mathematical content or the mathematical practices. The mathematical thinking requirement was also low; mathematical reasoning and greater scientific sensemaking did not occur.
IV	Integrated and Low Cognitive Demand	Students were asked to engage with superficial mathematical content within the context of one or more mathematical practices. The mathematical thinking requirement was low; mathematical reasoning and greater scientific sensemaking did not occur.

to place on a hanging mobile. As a second example, the mathematical practice code *model with mathematics* was applied when students used the area under a graph of the relationship between force applied over time to understand momentum. During our third analytic cycle, we coded for the remaining *NGSS* practices (e.g., *developing and using models, engaging in argument*), again determining what counted as instances of these practices independent of PST participants' labels. We did so to begin to determine the cognitive demand of the mathematics implemented in service of helping students understand the science. Finally, in our fourth cycle of analysis, we used the TAGS-M framework to assess the mathematics and science included in each PST's lesson series: We determined (1) the integration/isolation of the mathematical content and practices implemented, and (2) the level of cognitive demand required of students to engage in the mathematical and science practices. We then placed each participant's lesson series in one of four quadrants (see Table 3).

Findings

As stated above, we placed each PST's lesson series in one of the four quadrants of our modified TAGS-M framework. We found that four PSTs' lessons could be placed in Quadrant I: These lessons both integrated mathematical content and practices, and engaged students in cognitively demanding tasks so as to move students' scientific sensemaking forward. In contrast, we found that two PSTs' lessons could be thought of as integrated in mathematical content and practices but low in cognitive demand: They were placed in Quadrant IV of our modified TAGS-M framework (see again Table 3). Table 4 provides a summary of our findings.

Finding set 1: integration of content and practices when implementing using mathematics and computational thinking

As introduced above, we found that four of our PST participants (i.e., Haylee, Lucas, Thatcher, and Zeke) implemented different lesson series that fell within Quadrant I of our modified TAGS-M framework, while two participants (i.e., Sidney and Noah) implemented the same lesson series that fell within Quadrant IV. Still, all six integrated mathematical content and mathematical practices as they engaged students in *using mathematics*

Table 4. Opportunities PSTs provided their students to participate in integrated, cognitively demanding mathematical and scientific sensemaking.

Preservice Science Teacher		Haylee	Lucas	Thatcher	Zeke	Sidney	Noah
TAGS-M Framework		Quadrant I				Quadrant IV	
Integration of Content and Practices	*CCSS-M Mathematical Content*						
	Content standard(s) *(The * relates mathematical content to mathematical modeling, which is considered both content and practice)*	Create equations to represent relationships between quantities (HSA.CED.A.2*); Interpret expressions in terms of context (HSA.SSE.A.1*).	Create equations to represent relationships between quantities (HSA.CED.A.2*).	Graph equations on coordinate axes with labels and scales (HSA.CED.A.2*).	Graph equations on coordinate axes with labels and scales (HSA.CED.A.2*); Interpret functions that arise in applications in terms of the context (HSF.IF.B.4*).	Create equations to represent relationships between quantities; Graph equations on coordinate axes with labels and scales (HSA.CED.A.2*).	
	CCSS-M Mathematical Practices						
	MP1. Make sense of problems and persevere in solving them.	▨	▨	▨	▨	▨	
	MP2. Reason abstractly and quantitatively.						▨
	MP3. Construct viable arguments and critique the reasoning of others.	▨	▨	▨		▨	
	MP4. Model with mathematics.	▨	▨	▨	▨	▨	▨
	MP5. Use appropriate tools strategically.				▨	▨	
	MP6. Attend to precision.	▨					
	MP7. Look for and make use of structure.	▨					
	MP8. Use and express regularity in repeated reasoning.	▨			▨		
Cognitive Demand	*Mathematical Emphasis (purpose of the math within the science)*	Creation of mathematical models of scientific phenomena				Data for constructing solutions to problems	Language for displaying results

Content standard HSA.CED.A.2* has two parts: Some PSTs implemented only one part and some, both parts. A shaded cell means students engaged in this practice during a participant's lesson series.

and computational thinking. Below, we discuss the integration found within their lesson series by science topic (see again Table 2).

As one example, both Haylee and Lucas implemented their edTPA lesson series in physics classrooms at the same engineering academy at Foothill High School. In this engineering setting, students worked on long-term engineering design projects as they moved through computer science, physics, engineering, and art classroom spaces. While Haylee focused on torque and Lucas explored gears, both Haylee and Lucas' lesson series were intended to inform the engineering project in which their students were engaged, used an *NGSS* performance expectation related to energy (HS-PS3-3), and foregrounded the similar *CCSS-M* mathematical content in algebra and modeling: Create and/or look for structure in equations to represent relationships between quantities (HSA.CED.A.2* for both, HSA.SSE.A.1* for Haylee only).

More specifically, in Haylee's edTPA lesson series, she asked her 9th grade students to explore torque and rotational equilibrium, to create a mathematical model to describe these constructs in order to inform their design of a hanging mobile, and to test their model using different data points. Students began by examining mobiles made of metal and glass beads that hung around the classroom, mobiles that were designed and built by previous cohorts of students, to identify factors that they thought influenced their balance. In an exploratory but structured investigation, the students were then challenged to create the mathematical equation to describe that balance. Students made observations of different balance scenarios, identifying patterns of how mass and placement affected balance, and predicting the locations of masses to make the beam balance. Further, students "refine[d] their patterns to create a mathematical model to describe the relationship between mass and distance from the fulcrum" [Haylee, Planning Commentary]. Indeed, "this process of identifying patterns, predicting, developing a mathematical model, and refining it help[ed] students to learn scientific practices through inquiry and develop and use a model that describes the scientific concepts of rotational equilibrium and torque" [Haylee, Planning Commentary].

As Haylee's students worked with the algebra and modeling content of creating, using, and interpreting equations, they were also engaged with mathematical practices. As they collected data to look for patterns and relationships between the variables that they investigated, they were engaged in the *CCSS-M* mathematical practice *look for and make use of structure*. And as they used this understanding of structure in the data to develop an equation for balancing their elements on beams, they made use of another *CCSS-M* practice, *express regularity in repeated reasoning*. Throughout their mathematical work, students used the *CCSS-M* practice of *make sense of problems* to further their understanding of torque and how it applied to their long-term engineering project of balancing the beams and elements of their hanging mobiles. This last practice is illustrated in the exchange between Haylee and two students below.

Haylee: [To student group] So how did you account for those two different masses in your model?

Student 1: We just added them to the mass on each side.

Student 2: We did mass times distance from the center.

Haylee: From the center? So what's the equal sign in this case?

Student 1: The fulcrum is the point of balance so is the equal sign.

Haylee: Yeah, it's balanced, right?

Student 2: We were able to use the model we developed to find the mass of this [points to a test item for the mobile].

Haylee: And how was the comparison between the mass calculated with your equation and from an actual scale weighing?

Student 1: Well, 5.5% error. It was worse the first time. [Haylee, Video Excerpt]

We note that Haylee engaged her students in the two additional *CCSS-M* practices of *model with mathematics* and *attend to precision* as well.

As a second example, Thatcher and Zeke's lesson series related to a unit on forces and interactions in their physics classes at Coastal High School. The two lesson series had the same purpose: for students to understand conservation of momentum. Indeed, as explicitly indicated in the *NGSS* performance expectation HS-PS2-2, students were expected to use mathematical representations to support their claims that the total momentum of a system of objects is conserved when there is no net force on the system. However, although the purpose of Thatcher and Zeke's lesson series was the same, the topics, activities, and assignments they implemented were different. These two PSTs' lesson series also differed in the ways in which they emphasized *using mathematics and computational thinking* and the mathematical content and practices used; these differences are discussed further below.

More specifically, Thatcher foregrounded the importance of using mathematics to understand the phenomena of collisions and explosions in relation to the law of conservation of momentum:

> [S]tudents will observe phenomena, make predictions, collect data and construct arguments surrounding the phenomena of inelastic collisions and explosions. Through analysis of collected data students recognize patterns to conclude that momentum is conserved for a system on which no external force is exerted. [Thatcher, Planning Commentary]

Students worked with a *CCSS-M* mathematical content standard related to algebra and modeling (HSA.CED.A.2*), engaging in opportunities to use technology to graphically display and analyze empirical results. They also engaged in six mathematical practices.

In Thatcher's lessons, students constructed mathematical representations from data they themselves collected: Students "create[d] Mass, Velocity and Momentum (MVP) charts as a mathematical model to make predictions about these physical interactions" [Thatcher, Planning Commentary]. As students created and tested their model against real and simulated collision data, they were given opportunities to engage in the *CCSS-M* mathematical practices of *make sense of problems* and *model with mathematics* when working on mathematics problems. The students then created and interpreted position versus time graphs to "analyze velocity and mass data and look for patterns in the total momentum of the objects involved" [Thatcher, Planning Commentary]. As they did so, they were engaged in two other *CCSS-M* practices: *use appropriate mathematical tools strategically* (i.e., the force sensors attached to computers), and *use regularity in repeated reasoning*. (Thatched also engaged his students in an additional practice, *attend to precision*.)

Zeke's lessons on the conservation of momentum, in comparison to Thatcher's, took more of an engineering design focus. Students carried out an investigation to discover how bumpers and crumple zones affect collisions so as to evaluate the effectiveness of different bumper designs:

> Students worked in groups to design and build bumpers on metal electrical boxes that served as models of cars. Each group then conducts a crash test in which they attached their car to a swing apparatus and swung it onto a force sensor which was connected to a computer running Logger Pro. The force sensor measured the force applied to the car and sent data to the computer which plotted the data in a graph of force vs. time. Students used the graphs and software to record the maximum force that their car applied to the force sensor during impact and the total time the collision took. [Zeke, Instructional Commentary]

Students worked not only with the *CCSS-M* mathematical content standard related to algebra and modeling (HSA.CED.A.2*) that Thatcher's students did, but with a function and modeling content standard as well (HSF.IF.B.4*). They also engaged in four mathematical practices.

In Zeke's lesson series, students first used mathematics as they collected crash data using an electronic force sensor to display and compare results from iterations of their designs. As they did so, students engaged in the *CCSS-M* mathematical practices *model with mathematics* and *use appropriate mathematical tools strategically*. During the next lesson in the series, Zeke asked students to analyze and compare each other's force versus time graphs to find the optimal bumper design; he engaged students in the *CCSS-M* mathematical content standard of interpreting functions. In this case, the student groups were tasked with constructing arguments on how to interpret momentum and impulse from their graphs and how these could be used to select the best bumper design. This task gave students the opportunity to engage in the *CCSS-M* practice of *construct viable arguments and critique the reasoning of others*. (Zeke also had his students engage in the *CCSS-M* practice of *make sense of problems*.) In sum, across both Zeke's and Thatcher's lessons, students were expected to engage in mathematical sensemaking in ways that enhanced their scientific understanding of the conservation of momentum.

As our third example, Sidney and Noah implemented their edTPA lesson series in chemistry classrooms at Foothill High. Each taught the same sequence of chemistry lessons, using identical materials, within a unit on Earth's systems. However, Sidney taught an honors chemistry course while Noah taught a college preparatory one. The purpose of the lesson series was for students to use the gas laws to understand how wind occurs in a qualitative climate change model. Although not explicitly included in the *NGSS* performance expectation for their lessons (HS-ESS2-4), students were asked to engage in algebra and modeling content both by graphing functional relationships between scientific variables and writing equations to describe these relationships (HSA.CED.A.2*).

In Sidney and Noah's introductory lesson, students carried out a series of investigations across lab stations to explore and then describe the relationship between pairs of variables associated with the properties of gases (i.e., pressure, temperature, volume, and number of moles of gas). Sidney discussed these investigations in detail in her edTPA planning commentary:

> In each lab station they explore a different relationship and are asked to form a hypothesis of how the two properties are related to one another, draw their idea, test their idea, draw the

> outcome of the experiment, then write a math equation and a graph to explain the outcome of the experiment. [Sidney, Planning Commentary]

We again remind readers that Sidney and Noah taught the same series of lessons. In these lessons, students were expected to sketch a graph of the relationship between pairs of variables (e.g., volume, number of moles of gas, temperature) and to write down the standard form of the mathematics equation for each graph. At one station, for example, students explored the relationship between pressure and volume using a large bottle filled with water and a small dropper floating inside. Students found that when they pressed on the bottle from the sides, the dropper sank to the bottom. Students then drew a diagram or model with particles to try to understand the variables that were being manipulated and the relationship between them (i.e., that as pressure increases, volume decreases). With this understanding, students modeled this relationship graphically with pressure on the x-axis and volume on the y-axis to illustrate the direction of the relationship between pressure and volume. Finally, students selected the appropriate equation, either a direct equation, such as $P = \mathrm{kV}$ (i.e., pressure and volume are directly proportional), or an inverse equation, $P = \mathrm{k/V}$ (i.e., pressure and volume are inversely proportional). As stated by Sidney, the students were "writing an equation and sketching a graph to explain the outcome of the experiment" [Sidney, Instructional Commentary].

These sketches and equations in standard form were intended to provide students with different ways of viewing and remembering their qualitative results. Noah reiterated to several student groups who asked for clarification: "The goal is to kind of be able to visualize what's going on" [Noah, Video Excerpt]. Further, Sidney included the terms *direct relationship* and *inverse relationship* in a list of vocabulary words that "students must know and be able to apply" [Sidney, Lesson Plans]. As such, students used mathematical relationships to describe a phenomenon by reasoning abstractly about the implied quantitative relationship they had observed, thereby integrating the algebra and modeling mathematical content with the *CCSS-M* mathematical practices *reason abstractly and quantitatively*, and *modeling*. However, while the content of the mathematics, using standard forms of an equation for direct and inverse relationships between variables, was integrated with the mathematical practices of *reason abstractly and quantitatively* and *model with mathematics*, the mathematical activity as intended was deemed low in cognitive demand. This last point is discussed further below.

Finding set 2: cognitive demand of tasks when implementing using mathematics and computational thinking

To determine additional ways the mathematics implemented supported or constrained students' engagement in scientific sensemaking—to better understand the cognitive demand of PSTs' lessons as described in the second dimension of the TAGS-M framework—we examined PSTs' implementation of *using mathematics and computational thinking* in relation to the other *NGSS* science and engineering practices and the *CCSS-M* mathematical practices. In other words, we identified differences in the cognitive demand of lessons by the ways in which opportunities were or were not afforded students to participate in other science and engineering and mathematical practices in combination with *using mathematics and computational thinking*. We found three different ways in which the tasks that participants implemented engaged their students in *using mathematics and computational thinking*: Three PSTs asked their students to create mathematical models of scientific phenomena; one, to use mathematics as data for constructing solutions to problems; and two, to use mathematics as a language for displaying results. The lessons of the former four PSTs were considered high in cognitive demand while the lessons of the latter two were deemed low in cognitive demand.

Using mathematics to create mathematical models of scientific phenomena

Haylee, Lucas, and Thatcher implemented *using mathematics and computational thinking* in intersection with three other *NGSS* practices: *developing and using models, planning and carrying out investigations*, and *engaging in argument from evidence*. The intersection of these four practices was an integral part of the models their respective students developed. As such, it was as a result of using mathematics to collect quantitative data for and argue about models that their tasks could be considered cognitively demanding.

As one example, Haylee's lesson series on torque and rotational equilibrium provided students the opportunity to engage in the *NGSS* practices of *using mathematics and computational thinking* and *developing and using models* in interaction with the *CCSS-M* mathematical practices of *make sense of problems and persevere in solving them, model with mathematics*, and *use and express regularity in repeated reasoning*:

> Students take their data and the patterns they have observed in them and start to develop a mathematical model that explains their observations and enables them to predict future rotational equilibrium scenarios. The students explain to me what they have noticed so far … [Haylee, Instructional Commentary]

Haylee's lessons also engaged students in the *NGSS* practice of *arguing from evidence* and the *CCSS-M* practice of *construct viable arguments and critique the reasoning of others* based on their evidence:

> Once students have developed a model that they think fits their data, they use it to determine the mass of an object with undisclosed mass. To determine how well their model is after they have a prediction, they compare this to the actual mass of the object and calculate the percent error to determine if their model is "good enough." Even though some students know what torque is from AP [Advanced Placement] Physics they will get to see what it is like constructing an explanation from identifying patterns in mathematical data and deriving the equation. [Haylee, Instructional Commentary]

Engagement in these practices in these ways ensured that the cognitive demand of the mathematics in the lessons was high. In Haylee's lesson series, rather than being given a torque formula to verify, students derived the formula from their own data.

Using mathematics as data for constructing solutions to engineering problems

In comparison to Haylee, Lucas, and Thatcher's lessons, Zeke's lesson series focused less on modeling and more on engineering design. As introduced above, Zeke asked students to investigate if and how "a bumper or crumple zone can reduce the force on a car during a collision by increasing the time the collision takes, but that it cannot change the total impulse the car experiences during the collision" [Zeke, Planning Commentary]. In his commentaries, Zeke specifically mentioned the *NGSS* practices of *defining problems* and *constructing solutions* in addition to *using mathematics and* *computational thinking, planning and carrying out investigations*, and *analyzing and interpreting data*. He did not explicitly discuss *developing and using models* or *engaging in argument from evidence* as the above three participants did, even though the tasks his students engaged in clearly gave them these opportunities. For instance, students used force versus time graphs as models to interpret the effectiveness of their bumper car designs. As expected, he also did not explicitly mention any of the *CCSS-M* mathematical practices.

The mathematics in Zeke's lessons played a critical role when students analyzed their experimental data to construct mathematical explanations for impulse and then used that understanding to evaluate the effectiveness of bumper designs.

> At this station, students are presented with three force vs time graphs for three different bumper designs and the students are asked to rank their effectiveness and justify their ranking. Here, students will need to look at patterns in the given data to support their explanation of how a bumper works and what makes one bumper more or less effective than another. [Zeke, Instructional Commentary]

It was because students were asked to analyze and argue about these data so as to design a solution that Zeke's lessons could be considered cognitively demanding.

To elaborate, Zeke's lessons involved students in collecting and analyzing data to use a mathematical model to construct an argument about a phenomenon. Students were asked not only to apply but to *construct* criteria for determining mathematically which of their designed bumpers was most effective. This involved analyzing data in the form of force versus time graphs, graphs that they generated during their engineering design work, and constructing ways of determining impulse from such data. As such, students had opportunities to engage in the *NGSS* practices of *using mathematics, modeling, and engaging in argument from evidence* along with the *CCSS-M* practices of *make sense of problems and persevere in solving them, construct viable arguments and critique the reasoning of others*, and *model with mathematics*. Engagement in these practices allowed students to explore and understand the nature of the mathematical relationships across force, time, and impulse.

Using mathematics to display results

The use of mathematics in Sidney and Noah's lessons had a third emphasis, quite different from the four discussed above: using mathematical language to display results from investigations. As Noah explained:

> Students will move through stations that will further demonstrate the relationship between the gas properties of pressure, volume, temperature and number of moles. At each station students will be required to illustrate what is happening at a particle level. Students will be carrying out investigations by manipulating a variable in order to see what happens to another variable when all other variables are held constant. Using their observations and their illustrations, students will be asked to come up with a way to graph the relationship between the two variables being manipulated. Students will ultimately determine the mathematical relationship between variables using their graphs, observations and illustrations. [Noah, Planning Commentary]

In Sidney and Noah's lessons, students used mathematics to display results; they did not derive or explore the mathematical formulas that they were working with from data. These lessons partially supported only one other *CCSS-M* mathematical practice besides the *NGSS* and *CCSS-M* practice of *modeling: reason abstractly and quantitatively*. To elaborate, the specific mathematics students used in these chemistry lessons about gas laws was to represent their understanding of the relationship between the two variables with which they had experimented in graphical and algebraic form; this allowed students to *model with mathematics*. This modeling task also involved having students abstract variables out of their context and represent them symbolically and graphically. The opportunity to do so corresponded to part of *reason abstractly and quantitatively*.

However, neither Sidney nor Noah's lessons allowed for mathematical sensemaking about the nature of the relationships between variables as a result of completing these stations. Rather than the stations providing students opportunities to make connections between the formulas being used and the underlying mathematical meaning of these formulas, the focus was on producing predefined formulas. Ironically, although the mathematics was prescribed, there was still evidence that students were unsure of the mathematics they were expected to use. For example, students in both Sidney and Noah's classrooms had questions about why the function representing the relationship between two variables that varied directly was a straight line. The following excerpt was taken from Sidney's classroom:

Student 1: How do we know if the direct relationship is linear or exponential?

Sidney: Direct or indirect is what you want to know? Direct is when if you increase one the other one also increases.

Student 2: [in another group] I need help with the math on this one.

Sidney: Are they direct or indirect?

Student 2: Direct.

Sidney: Ok, we had a direct equation before how was that? So same idea.

Student 2: So like T? And there is a k. What is k again?

Sidney: K is just a constant.

Student 2: So the moles were M. So would it be k times something?

Sidney: So what is the something?

Student 2: I don't understand any of it.

As such, in Sidney and Noah's classrooms, the use of the mathematics did little to enhance their students' understanding of the gas laws.

Summarizing the cognitive demand in PSTs' lessons

In summary, across our six PST participants, within the context of the *NGSS* practice *using mathematics and computational thinking*, their implementation of the *NGSS* science and engineering practice of *developing and using models* could be understood to overlap with the *CCSS-M* mathematical practice of *model with mathematics*. However, as presented above, the purposes for modeling, the ways in which mathematics models were used, and if students constructed them from their own data differed across PST participants. Indeed, we identified three different ways PSTs emphasized *using mathematics and computational* *thinking* in relationship to *modeling* and other *NGSS* and *CCSS-M* practices. For Haylee, Lucas, and Thatcher, as well as for Zeke, the mathematical model in question and the scientific model were one and the same. For Sidney and Noah, in contrast, the use of the mathematical model would later support the creation of a pictorial scientific model. As such, the preservice chemistry teachers did not have their students use mathematical modeling to understand a phenomenon, rather, they had them use graphing techniques and equations as forms of display to help them remember these relationships when later asked to describe wind in the context of climate change. This latter use of a mathematical model did not give students opportunities to participate in other *CCSS-M* practices associated with mathematical sensemaking and reasoning, such as *make sense of problems and persevere in solving them, construct viable arguments and critique the reasoning of others*, or *use and express regularity in mathematical reasoning.*

Discussion

Few studies focus on preservice secondary science teachers' implementation of the *NGSS* practice of *using mathematics and computational thinking* (e.g., Brownstein & Horvath, 2016). None of the studies to date, to our knowledge, interrogate the integration and cognitive demand of the mathematics that is used within reform-based science instruction. Hence, the six cases presented in this paper give examples of both opportunities and missed opportunities to implement integrated and challenging science and mathematical content and practices. The four PSTs who taught physics/engineering designed and implemented lessons that better supported students' understandings of mathematical content and practices when compared to the two participants who taught chemistry. In Sidney and Noah's lessons, students were constrained in their use of *NGSS* and *CCSS-M* practices, which both lowered the cognitive demand of the tasks and interfered with their use of mathematics to deepen their understanding of the scientific phenomenon under investigation. Said another way, in our examination of the edTPA lesson series implemented by six preservice secondary science teachers, we found that the four preservice physics teachers gave their students more and richer opportunities to participate both in this *NGSS* practice and in related *NGSS* and *CCSS-M* practices and content as compared to the two preservice chemistry teacher participants. As such, our findings build on those of Brownstein and Horvath (2016): They found that teachers teaching physical science lessons implemented *using mathematics and computational thinking* to a greater extent than other types of science teachers.

Similar to the obvious convergence of disciplinary practices hypothesized by Lee et al. (2013), our results also provide empirical evidence for the overlap between the *NGSS* practices of *using mathematics and computational thinking* and *developing and using models* with the *CCSS-M* practice of *model with mathematics*. That is, reform-based science lessons that give students opportunities to develop and use mathematical models are natural candidates for integrating science and mathematical content and practices.

Further, when considering the cognitive demand of such lessons, our empirical work resonates with the conceptual argument made by Mayes and Koballa (2012) that the practices of *planning and carrying out investigations* as well as *analyzing and interpreting data* can mediate the relationship between *using mathematics and computational thinking* and *developing and using models*: Our study provides empirical evidence to support Mayes and Koballa's conceptual claims. In fact, we extend their argument by emphasizing that the creation of mathematical models through the analysis and interpretation of empirical data, collected as part of an investigation, provides opportunities for students to make sense of the mathematics that they are using and hence can help to ensure that tasks are cognitively demanding. This is a crucial point that other scholars have not emphasized.

To elaborate, we found that those lessons in which the purpose was either the creation of a mathematical model of a scientific phenomenon using student-generated data or the use of mathematical data to design a solution gave students authentic opportunities to participate in the *CCSS-M* practices of *make sense of and persevere in problem solving* and *construct viable arguments and critique the reasoning of others*. Again, this was the case for Haylee, Lucas, Thatcher, and Zeke's students but not so for the students of Sidney or Noah. In other words, the four preservice physics teacher participants provided their students opportunities to participate in integrated mathematical practices and content at a level of high cognitive demand. The two preservice chemistry teachers did not provide their students with similar opportunities, even though the topic of their lessons and the investigations their students completed could have been (re)fashioned to be both integrated and cognitively demanding. Indeed, the two chemistry PSTs included the similar mathematical content to the physics PSTs (i.e., mathematical content standard HSA.CED.A.2*).

Given our findings, we recommend preservice secondary science teachers be given explicit opportunities to gain the knowledge and skills needed to effectively implement the *NGSS* practice of *using mathematics and computational thinking* in their lessons—both as a tool to enhance students' thinking about the important science concepts that they support and as an integral part of the science and mathematical understanding that students are asked to actively construct. For example, in a methods course, PSTs might spend a week learning about each of the eight *NGSS* science and engineering practices, considering practical applications of their implementation within high cognitive demand science lessons, and trying out such implementation in their placements. Specifically when discussing *using mathematics and computational thinking*, PSTs could review the high school mathematical content and practices in *CCSS-M* that are relevant to the science phenomena in their unit under study; for this particular practice, knowledge of and ways to implement the *CCSS-M* mathematical content and practices naturally embedded within it should be emphasized. PSTs could then share their experiences and reflections with each other in the following class.

Without such focused development and opportunities to learn, some PSTs will miss opportunities to effectively implement *using mathematics and computational thinking*. Although we found that the majority of our participants were able to successfully engage students in *using mathematics and computational thinking* in ways that supported their understanding, there is still a need for teacher educators to more clearly articulate for preservice secondary science teachers how to appropriately support their students' reasoning and sensemaking across both science and mathematics—how to engage their students in the mathematics so as to move their understanding of the science forward.

Conclusion

We acknowledge that there is still much work to be done to understand the multiple intersections of the *NGSS* and *CCSS-M* practices and content in the context of preservice science teacher preparation. We discuss three limitations and related areas of future research here. One limitation is that our data consisted solely of edTPA portfolios (e.g., commentaries, lesson plans, and video clips) from our participants. As such, future studies should include additional data, including interviews with PSTs, additional classroom observations during their student teaching, and follow-up visits during their first year of teaching. Such data collection would enable detection of changes in these beginning teachers' understanding and implementation of the *NGSS* and *CCSS-M* content and practices over time and across topics. A second limitation and area for future research is the size of our sample. We focused our investigation on six PSTs. Even though this sample size is appropriate for a qualitative study, future research should attempt to gather data from a larger number and wider range of participants so as to include PSTs with life science credentials and to conduct quantitative analyses. A larger sample would also allow investigation into similarities and differences between physics and chemistry teachers. Because our two chemistry PSTs, Sidney and Noah, took the same methods courses as their physics colleagues and taught the same chemistry lesson series, we are unable to determine if differences in how they addressed mathematics in comparison to their physics PST colleagues were an artifact of their own implementation or a reflection of differences between the disciplines of chemistry and physics. A third limitation involves the scope of our study: Our analysis emphasized the first half of the *NGSS* practice *using mathematics and computational thinking* because our PST participants provided more opportunities to engage in mathematics than in computational thinking. As such, future studies should investigate engagement in computational thinking (e.g., using and creating original simulations of natural systems) in addition to mathematical thinking.

In closing, the recently adopted *NGSS* expects secondary science teachers to engage their students in rich, complex practices and content so as to construct deep and integrated understanding (NGSS Lead States, 2013). *Using mathematics and computational thinking* is a practice that can help students draw connections across the disciplines of science, engineering, and mathematics. Our paper provides rich descriptions of how preservice secondary science teachers should (and should not) integrate this practice with other science and engineering practices, mathematical practices, and science and mathematical content to move toward the goal of engaging students in reasoning and sensemaking in science classrooms.

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