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M Theory Fivebrane and SQCD*

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1. Introduction

A low energy effective theory of parallel D(irichlet) branes is a gauge theory with sixteen supercharges, but one can consider a web of brane to realize situations with reduced number of supersymmetry [1]. In this talk, I will discuss four-dimensional theories with $N = 1$ and 2 supersymmetry (i.e. four and eight supercharges). In the case of theories with $N = 2$ supersymmetry, the exact description of the Coulomb branch is given by reinterpreting the web of branes as a configuration of a single fivebrane in the IIA theory [2,3]. Recently we studied the case with $N = 1$ supersymmetry, and found that description in term of the fivebrane in M Theory captures strong coupling dynamics of the $N = 1$ gauge theory in four dimensions [4]. In particular, we found that the configuration of the fivebrane geometrically encodes information on the Affleck-Dine-Seiberg superpotential and the structure of the quantum moduli space of vacua. Simultaneously to our work, the case without matter was studied in [5]. A related work also appeared in [6].

2. Geometric Engineering of $N = 2$ Gauge Theory

2.1. Web of Branes

Let me first describe how to construct $N = 2$ gauge theory from a web of branes in the type IIA theory. We first separate the ten dimensions into

six and four. The $\mathbf{R}^{3,1}$ part of the spacetime is parallel to the worldvolume of all the branes, and therefore it is where the gauge theory is realized. We then tie together the branes in the six dimensions as shown in figure 1 so that the desired field content is realized.

To realize the theory with $SU(N_c)$ gauge group, we consider N_c D4 branes suspended between two parallel NS5 branes. The open string attached to the D4 branes gives rise to the gauge boson of $SU(N_c)$. We then introduce N_f D6 branes perpendicular to the D4 and NS5 branes, and the open string going between the D4 and D6 branes give N_f matters in the fundamental representations, which I will call quarks. The moduli space of $N = 2$ vacua consists of the Coulomb and Higgs branches (and also their mixed branches). The Coulomb branch is parametrized by the vacuum expectation value (vev) of the adjoint scalar field in the $N = 2$ vector multiplet while the Higgs branch is parametrized by the squarks. In the IIA picture, they are described by locations of the D4 branes. The motion of the D4 branes along the NS5 brane parametrizes the Coulomb branch. To go from the Coulomb to the Higgs branch, one has to move the D4 brane to the D6 brane, break the D4 brane on the D6 brane, and then move a segment of the D4 brane along the D6 brane. Note that in order to do this, we have to have at least two D6 branes and they have to be located at the same point in the x^4, x^5 direction (see figure 1). This corresponds to the field theory fact that the Higgs branch requires at least two massless matters.
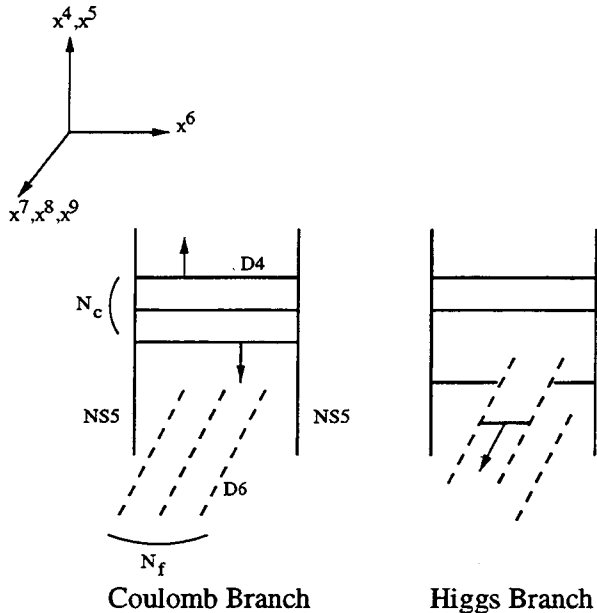
Figure 1. Coulomb and Higgs branches in the IIA picture

2.2. Lifting to M Theory

We can lift this configuration to M Theory [3]. As it will become clear later, this will enable us to incorporate nonperturbative gauge theory dynamics in the brane picture. The D4 and NS5 branes of the IIA theory come from the fivebrane of M theory. When the fivebrane is wrapped on the S^1 of the eleventh dimension, it becomes the D4 brane. When it is not wrapped, it gives the NS5 brane. On the other hand, D6 brane localized at points in the (x^4, x^5, x^6) plane is interpreted as the Kaluza-Klein monopole and is given by the Taub-NUT metric

$$ds^2 = V(x)dx^i dx^i + V(x)^{-1}(dx^{10} + A_i(x)dx^i)^2$$

$$V(x) = 1 + \sum_n \frac{R}{|x - x_n|}, \quad \nabla V = \nabla \times A, \quad (1)$$

where the D6 branes are located at $x^i = x_n^i$ ($i = 4, 5, 6$) and x^{10} is the eleventh dimensional coordinate with the periodicity $x^{10} \simeq x^{10} + 2\pi R$

[7].

Let us now discuss the configuration of the fivebrane. As in the IIA picture, we have to keep four dimensions of the six-dimensional worldvolume of the fivebrane to be flat $\mathbf{R}^{3,1}$. In order to preserve the $N = 2$ supersymmetry in four dimensions, the remaining two dimensional part has to be embedded holomorphically in the Taub-NUT geometry and is localized in the x^7, x^8, x^9 directions. (If the fivebrane is also extended in those directions, it will have generically smaller supersymmetry, i.e. $N = 1$ in four dimensions. I will discuss more about it later.) So let us introduce complex coordinates,

$$v = x^4 + ix^5, \quad t = \exp(-\frac{x^6 + ix^{10}}{R}). \quad (2)$$

Now we can transform the IIA picture into the M theory picture. The fivebrane must have two asymptotic regions corresponding to the two infinite NS 5-branes. In the middle, there are N_c branches of the brane corresponding to the D4 branes. We also need N_f D6 branes, which are now represented by the Taub-NUT geometry (1). It was shown in [3] that such a fivebrane configuration is unique and is the same as the Seiberg-Witten curve [8,9],

$$t^2 - B(v)t + \Lambda_{N=2}^{2N_c - N_f} v^{N_f} = 0, \quad (3)$$

where $B(v) = v^{N_c} + \cdots$ is a polynomial of degree N_c in v and depends on the Coulomb branch moduli of the theory.

2.3. Why does this work?

Since the eleven-dimensional supergravity is supposed to describe the low energy dynamics of the strong coupling limit of the type IIA theory, it is reasonable to expect that the reinterpriting the web of branes in the IIA theory as a configuration of the single fivebrane captures the nonperturbative physics of the gauge theory, and in fact we saw in the above that it is the case for the $N = 2$ gauge theory. This statement needs some clarification and it is useful to understand exactly how it works.

In the IIA picture, the gauge coupling constant λ is related to the string coupling constant g as

$$\lambda^2 = \frac{gl_s}{L} \quad (4)$$

where l_s is the string scale set by the string tension and L is the distance between the NS 5-branes. The L factor appears in the formula since the gauge theory lives on the D4 brane world-volume, which is $\mathbf{R}^{3,1}$ times the line segment in the x^6 direction. If $1/L$ is larger than the typical gauge theory scale, we can ignore the Kaluza-Klein type excitation in the x^6 direction and the low energy theory is on $\mathbf{R}^{3,1}$. The volume L then renormalizes the gauge coupling constant as in (4). Since the radius of the eleventh dimensional S^1 is $R = gl_s$, we can also express λ as

$$\lambda^2 = \frac{R}{L}. \tag{5}$$

To relate the low energy theory on the D4 branes to the standard gauge theory in four dimensions, we have to make some limit.
(1) We need to take the string coupling constant g to be small in order the theory on the brane to decouple from physics of the bulk ten or eleven dimensions. In particular, we should be able to ignore gravitational effects in the bulk in order to concentrate on physics on the brane. This is possible if we send $g \to 0$, or equivalently $R \ll l_s$.
(2) On the other hand, we want λ to be finite in order to be able to observe interesting strong coupling physics of the gauge theory.
(3) These conditions require $L \ll l_s$. In fact this is also required in order to be able to ignore the Kaluza-Klein type mode on the brane, as I mentioned in the above.

So we need to take the limit, $R, L \ll l_s$, while keeping $\lambda^2 = R/L$ finite.

On the other hand, the eleven-dimensional supergravity gives a good description in the limit $l_p \ll R, L$ where $l_p = g^{1/3} l_s$ is the eleven-dimensional Planck length, and this means $l_s \ll R, L$, which is complete opposite of the above limit.

Nevertheless the M theory fivebrane gives the correct description of the $N = 2$ Coulomb branch, as we saw in section 2.2. The reason for this is the same as the one used to derive exact results on Calabi-Yau compactification [10,11,2], i.e. the decoupling of the vector and hypermultiplet fields. In the present case, parameters characterizing the size of the brane are in hypermultiplets of

the four-dimensional theory while its shape is determined by vector multiplet fields. Since the Coulomb branch of the four-dimensional gauge theory is parametrized by the vector multiplet fields, it should depend on R and L only through the ratio L/R. This means that, although we have to take both R and L to be small in order to describe the gauge theory in four dimensions, since the theory depends only on their ratio, the same result is obtained by taking them to be large as far as we keep the ratio fixed. This is why we can trust the eleven-dimensional supergravity to give the correct low energy description.

2.4. Relation to Calabi-Yau Compactification

It has been known earlier in the context of the Calabi-Yau compactification [11] that the exact solution of the $N = 2$ gauge theory arises from the classical geometry of string compactification if one takes the field theory limit in such a way that gravity is turned off while the gauge theory scale is held fixed (this involves taking the singular limit of the Calabi-Yau manifold). It was then found in [2] that such a Calabi-Yau manifold in the IIB case is T-dual to a configuration of the NS 5-brane in the IIA theory, which is exactly given by (3). The T-duality of a singular Calabi-Yau manifold and a NS 5-brane configuration was pointed out in [12] and was further clarified recently in [13].

In the following, we will use the M theory five-brane to describe the gauge theory with $N = 1$ supersymmetry. We will see the strong coupling physics of gauge theory is geometrically engineered in the $N = 1$ case. It is reasonable to expect that such a fivebrane configuration is related to the F-theory compactification [14,15] on a Calabi-Yau four-fold [2]. It would be very interesting to reinterpret the results of our work from the point of view of the F-theory.

3. Rotation to N=1

3.1. IIA Picture

Now we break the $N = 2$ supersymmetry to $N = 1$ by adding a mass μ for the adjoint chiral

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[2] I would like to thank Cumrun Vafa for discussion on this.

multiplet in the $N = 2$ vector multiplet. Since

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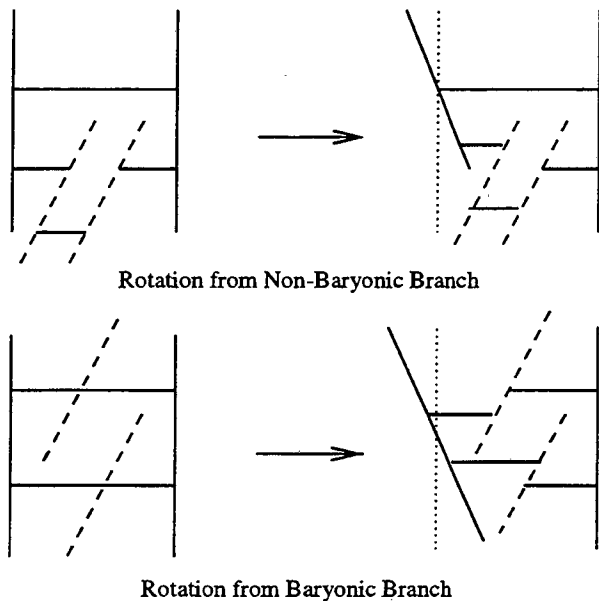
Rotation from Baryonic Branch

Figure 2. Rotation of the left NS 5-brane in the IIA picture

the adjoint scalar becomes massive, most of the Coulomb branch is lifted except at points on the roots of the Higgs branches [16] and some other exceptional points in the Coulomb branch [4]. Let us first view this procedure in the IIA picture. It was pointed out by Barbon [17] that adding the adjoint mass is the same as changing the relative orientation of the two NS5 branes. Let us see how it looks like in the examples of $SU(2)$ with 2 flavors. If the two D4 branes are at generic positions, we cannot rotate the NS5 branes without breaking the supersymmetry completely. In this case, there are two situations in which the rotation is possible, as shown in figure 2. One is at the non-baryonic branch root. Note that the location of one of the D4 brane in figure 2 is fixed to be at the axis of the rotation. This

means that we have to start at particular points on the non-baryonic root. In general, these are points where there are maximum number of mutually local monopoles. Another possibility is at the baryonic branch root, where all the D4 branes are fixed on the D6 branes from the begining. We can then break the D4 branes on the D6 branes, as shown in figure 2, and rotate the configuration. In the limit when the two NS5 branes make the right angle, we recover the configuration introduced in [18] to study $N = 1$ SQCD.

3.2. M Theory Picture

Now we come to one of the main points of this talk that is to reinterpret this in the M theory picture. Let me first describe it using the same example of $SU(2)$ gauge theory with 2 flavors. *spin-up (data)*

In the M theory description, the fivebrane at the rotable point on the non-baryonic branch root is given by the equation

$$t^2 - v^2 t + \Lambda^2 v^2 = 0. \tag{6}$$

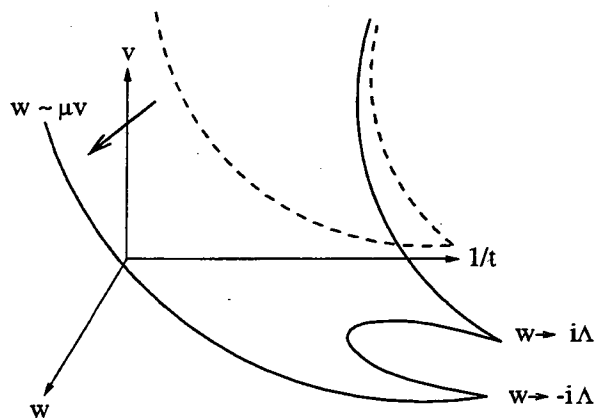The corresponding curve is drawn in figure 3, in the dotted line in the $v - t$ plane. The two

Figure 3. Brane Rotation from Non-Barynonic Branch

branches extending to infinities correspond to the

two NS5 branes. Therefore the brane rotation that I described in the previous subsection should correspond to pulling one of the branches out of the $v - t$ plane and toward the $w = x^6 + ix^7$ direction. We have found a unique way to do this, and it is described by the following equations in the $v - t - w$ plane.

$$wv = \mu^{-1}(w^2 + \mu^2\Lambda^2)$$
$$t = \mu^{-2}(w^2 + \mu^2\Lambda^2). \tag{7}$$

One can easily check that this solves (6) for any value of w. The corresponding curve is drawn in the solid line in figure 3. We note that the left branch is now extended into $w = \mu v$ direction, where μ is the adjoint mass while the right branch asymptotically approaches the original curve. We also note that there are two additional asymptotic regions, with different values of w. This will become important later when we compare this brane configuration with field theory results.

At the baryonic branch root, the equation factorized into two components,

$$(t - v^2)(t - \Lambda^2) = 0, \tag{8}$$

and this means that the curve factorizes into two components as shown in the dotted line in figure 4. We can then easily rotate the curve as

(1) $t = v^2$, $w = \mu v$

(2) $t = \Lambda^2$, $w = 0$,

and the corresponding curve is depicted in the solid line in figure 4.

Let us generalize this to $SU(N_c)$ gauge theory with N_f quarks, for arbitrary N_c and N_f. First I want to point out that the brane is rotatable if and only if the configuration is birationally equivalent to $\mathbf{CP}^1$. To show this, it is convenient to regard w of the rotated curve as a function along the original curve. This is always possible for small values of μ and it can be shown to be true for any μ by using the R symmetry (see [4]). The brane configuration after the rotation then can be regarded as a graph of w. We note that w is infinite only at one point, corresponding to the left NS5 brane extending to the $w = \mu v$ direction. Since $1/v$ is a good coordinate on that branch, it should be regarded as a simple pole.
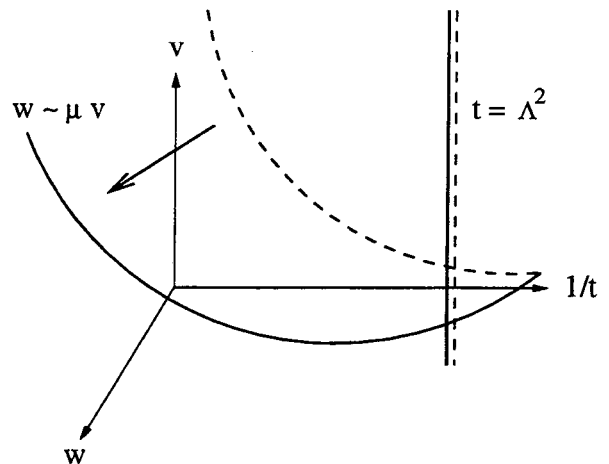

Figure 4. Brane Rotation from Baryonic Branch

In order to have a holomorphic function w with a simple pole only at one point, the original curve must have been equivalent to $\mathbf{CP}^1$. I used the word "birationally" in the above since the original curve may have two or more points coinciding. Such points are separated after the rotation since any two points must have different values of w. That the original curve has to be genus 0 is consistent with what we expect from the field theory analysis [8,16]. The Seiberg-Witten curve of the $N = 2$ theory that survives the adjoint mass perturbation has exactly this property.

This observation also makes it possible to find the brane configuration explicitly, for any N_c and N_f, after the rotation. This is because w as a function with a simple pole, is unique up to $SL(2,\mathbf{Z})$ on $\mathbf{CP}^1$, and $SL(2,\mathbf{Z})$ is fixed by the asymptotic conditions on the curve.

If one starts with the r-th non-baryonic branch root $r = 1, ..., [N_f/2]$, the rotated curve is given explicitly by[3]

$$vw = \mu^{-1}(w - w_+)(w - w_-)$$
$$t = \mu^{-N_c}w^{N_c-N_f}(w - w_+)^r(w - w_-)^{N_f-r}, \tag{9}$$

[3]For $N_f \geq N_c$, $r = N_f - N_c$ has to be excluded from rotatable curves [4].

where

$$w_{\pm} = \left[(-1)^{N_c+r_{\pm}} \left(\frac{N_c - r_-}{N_c - r_+} \right)^{N_c-r_{\pm}} \right]^{\frac{1}{2N_c - N_f}} \mu\Lambda, (10)$$

and $r_+ = r$, $r_- = N + f - r$. It turned out there is another rotatable curve which is obtained by setting $r = 0$ in the above, and there is a corresponding point in the Coulomb branch which survives the perturbation by the adjoint mass term. This point is is not attached to any Higgs branch and therefore is not in the list of [16] where they studied points on the Higgs branch roots.

As before, the left branch of the curve extends to $w = \mu v$ direction, while the right branch approaches the original curve. There are additional two asymptotic regions, with fixed values of $w = w_+$ and w_-. These correspond to locations of D4 branes in the IIA picture.

The rotation from the baryonic branch can be easily carried out since the curve factorizes there. The result of the rotation is

(1) $t = v^{N_c}$, $w = \mu v$

(2) $t = \Lambda^{2N_c-N_f} v^{N_f-N_c}$, $w = 0$. (11)

4. Comparison with Field Theory Results

4.1. Affleck-Dine-Seiberg Superpotential

Now I would like to compare these with field theory results. When $N_c > N_f$, it is known that the $N = 1$ theory dynamically generates the Affleck-Dine-Seiberg superpotential. At $N_c - 1 = N_f$, it is generated by the instanton effects and for $N_c - 1 > N_f$ by some strong coupling effects.

Let us start with the $N = 2$ theory and add the adjoint mass μ. Before we turn on the adjoint mass, the superpotential of the $N = 2$ theory is $\text{tr}Q\Phi\tilde{Q}$ where Φ_{ij} is the adjoint field and Q_{ia}, $\tilde{Q}_{ia}$ are quarks ($i, j = 1, ..., N_f$, $a, b = 1, ..., N_c$). If μ is much larger that the scale Λ of the $N = 2$ theory, we can integrate out Φ and generate the superpotential term

$$\frac{1}{2\mu} \left[\text{tr}M^2 - \frac{1}{N_c}(\text{tr}M)^2 \right], \quad (12)$$

where $M_{ij} = (\tilde{Q}Q)_{ij}$ is the meson made out of the quarks. In the $\mu \to \infty$ limit, we obtain the

$N = 1$ SQCD where we know that the Affleck-Dine-Seiberg potential is generated. For large μ, therefore, we expect that the effective superpotential is given by

$$W_{eff} = (N_c - N_f) \left(\frac{\Lambda_{N=1}^{3N_c-N_f}}{\det M} \right)^{\frac{1}{N_c-N_f}} +$$
$$+ \frac{1}{2\mu} \left(\text{tr}M^2 - \frac{1}{N_c}(\text{tr}M)^2 \right), \quad (13)$$

where $\Lambda_{N=1}$ is the scale of the $N = 1$ theory set by the renormalization group matching condition

$$\Lambda_{N=1}^{3N_c-N_f} = \mu^{N_c} \Lambda^{2N_c-N_f}. \quad (14)$$

In fact, by using the standard holomorphy argument [20], one can prove that the superpotential W_{eff} is exact for any μ.

In the $\mu \to \infty$ limit, the $N = 1$ theory has no vacua since W_{eff} has no minima (the vacuum runs away). The $\frac{1}{\mu}$ term in W_{eff} stabilizes the vacua for finite μ, so we can analyse its structure. By extrematizing W_{eff}, we can determine vev of the meson field M_{ij}. One can show [4] that M solving

$$\frac{\partial W_{eff}}{\partial M_{ij}} = 0 \quad (15)$$

is diagonalizable by a unitary matrix, and have two different eigenvalues, m_+ and m_-. Let r be the number of m_+ ($r = 0, ..., [N_f/2]$). The eigenvalues are then give by

$$m_{\pm} = \left[(-1)^{r_{\pm}} \left(\frac{N_c - r_-}{N_c - r_+} \right)^{N_c-r_{\pm}} \right]^{\frac{1}{2N_c-N_f}} \mu\Lambda. (16)$$

As one can see by comparing with (10), the eigenvalues $m_{\pm}$ of the meson field M are equal to the asymptotic locations $w_{\pm}$ of the branches of the rotated fivebrane, upto a trivial overall phase factor independent of r.

This means that the information on the Affleck-Dine-Seiberg potential is *geometrically encoded* in the fivebrane configuration. In the IIA picture, the vev of the meson field is specified by the locations of the D4 branes along the D6 branes. In the M theory picture, these are exactly the locations of the fivebrane branches characterized by $w_{\pm}$.

Not only their values agree with the field theory computation of M in the above, but their multiplicities match (the fivebrane is wrapping the two asymptotic directions r and $(N_f - r)$ times respectively, and these numbers agree with the multiplicities of the eigenvalues of M).

So far we have discussed the case with massless quarks. We can add massed to the quarks and perform the same analysis. We found a complete agreement between the field theory analysis and the brane configuration.

4.2. Quamtum Moduli Space

For $N_c \leq N_f$, there are stable vacua in the $\mu \to \infty$ limit and their moduli space is of interest [20]. Classically the moduli space is parametrized by gauge invariant polynomials of the squarks obeying the constraints. When $N_c = N_f$, the moduli space is deformed by the quamtum corrections. On the other hand, for $N_c < N_f$, the quamtum moduli space at $\mu \to \infty$ is the same as the classical one. For finite μ, however interesting quamtum effects appear even for $N_c < N_f$. In the paper [4], we have shown that one can read off the vevs of the gauge invariant polynomials from the configuration of the fivebrane and found complete agreement with the field theory predictions. For more detail, I would like to refer the reader to the original paper.

5. Conclusion

We have found that the classical eleven-dimensional supergravity can be used to study strong coupling dynamics of $N = 1$ SQCD in four dimensions. Nonperturbative corrections to the superpotential and the moduli space structure are geometrically encoded in the configuration of the fivebrane. First of all, this result provides a very strong evidence for the conjecture that the strong coupling limit of the IIA string theory is the eleven-dimensional theory. This geometrical description may also give us new insights into nonperturbative effects in the gauge theory in four dimensions.

I should point out that the dynamical information on the gauge theory that we have extracted from the fivebrane is holomorphic in nature, i.e.

the holomorphic superpotential and the holomorphic property of the moduli space. For these, an argument similar to that given in section 2.3 guarantees that the classical supergravity computation captures the corect gauge theory dynamics in four dimensions. To study non-holomorphic aspects of the gauge theory, one needs to control possible corrections to the classical fivebrane description. There are potentially two sources of corrections. The classial supergravity approximation may fail when the curvature induced by the fivebrane becomes strong compared to the Planck scale. Other corrections may arise if Kaluza-Klein modes on the curve Σ or bulk degrees of freedom do not decouple from the four-dimensional gauge theory. It would be important to identify the region of validity of the supergravity description. Related issues were discussed in Maldacena's talk at this conference [21].

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