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The Art of Seeing in Perspective:
How Medieval Optical Science Transformed into Renaissance Painting

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Introduction

The beginning of the fifteenth century initiated the use of single point perspective in painting and relief sculpture. It is an artistic change that is strongly associated with the Renaissance and is a defining element of Renaissance studies. While the Renaissance in Italy is a time of looking back to the ancient world for inspiration and knowledge, the development of the geometry of perspective was rooted in medieval science. Perspective was based on the geometry of optics, the study of sight and the behavior of light, which had been studied since ancient Greece. Even though the Greeks had theories on optics, many strides had happened in the field in the Middle Ages in the Islamic world and were brought into the west through England. The prevalent textbooks on optics in the fifteenth century were written in the thirteenth century and were very different from the theories of the ancient Greeks. There were no instructions on how the ancients painted in the lifelike manner described in the writings from the time, so the manner that the Renaissance painters used had to be developed from medieval scientific knowledge. While they may not have wanted to admit it, in order to paint as the ancients were supposed to have painted, the humanist artists of the early Renaissance had to use medieval science mixed with medieval painting traditions.

In this study, I will look at the medieval history of optics as it pertains to the knowledge of the fifteenth century artist and the persistence of scientific development by Arabs in the field of optics in a time when optics was not developing in the west. This applies to the assimilation of Greek sources into Islamic culture and translation of Greek

into Arabic, and then Arabic into Latin. There were important developments in optics in the Arabic-speaking world in the ninth and eleventh centuries. These important theories were then transferred into western culture in 13th century England, therefore giving these old theories a new kind of audience. While the thirteenth century was an important time for the dissemination of optical theories, it is primarily a time of assimilation and publication rather than the creating of new ground-breaking ideas. The extremely important part of these publications that included work highly derivative of the old Arabic treatises is that a different society was looking at it and could use it for different purposes. In the Islamic world, lifelike imagery was forbidden, but in the west, a society with a growing desire for naturalistic representation would use these optical theories to create the very lifelike images that were feared in the east.

I will also examine the question of why single point perspective developed in Florence at the beginning of the fifteenth century. Even though the science of optics had not changed in the west for two hundred years, it had not been used in such a way. Why in Florence and not anywhere else in Europe, and why the fifteenth century instead of earlier? What was it about the culture of Florence that encouraged this revolutionary use for optics geometry? Why was this such a desired result that it became popular and spread throughout Europe? The answers to these questions are to be found in the middle ages. While the people that were writing during the fifteenth century thought of themselves as bringing back antiquity, it was not antiquity in a pure sense, but one that was mixed with the medieval traditions of the centuries in between.

Finally, I will use two case studies in order to investigate the use of perspective in the early fifteenth century by two artists with different theoretical backgrounds and in two different mediums. One will be of two relief panels by Ghiberti, and the second will be a painting by Piero della Francesca. This will highlight the ways in which the geometry of perspective was changed in the service of composition and narration as well as the approach of artists with different mathematical and theoretical backgrounds. The two cases also show the use of perspective at different times in the fifteenth century.

The study of the beginnings of perspective has been varied. Some studies, such as those by Martin Kemp and Samuel Edgerton, focus on the mathematical and scientific development, particularly the geometric techniques of the painters. This is further developed with the study of why these techniques became popular and what they meant to those who used them and those who viewed them. Other studies, like the work of Michael Kubovy, focus on the role that human perception takes in the viewing and development of perspective. It would be a mistake to think of the perspective system as a static one. There was much development within the fifteenth century based on the goals and the skills of the artist. Both angles of study are large areas of expertise and strongly complement each other. However, they could also both be enhanced with further study of the medieval influence on the development of perspective which has been researched by Hans Belting.

The geometry used for the development of perspective in the Renaissance was derived from the geometry of the study of optics, or *perspectiva*, in the middle ages. The study of optics was the study in the way the eye sees and perceives. It was based on the

philosophies of the ancient Greeks, developed in the early Middle Ages in the Islamic world, and moved back into the western world in the thirteenth century in England through translated Arabic texts. It remained in this state throughout the Renaissance, and this was the basis of the optics that was used by the Italian Renaissance painters to create pictorial perspective. The study of medieval optical science is largely in the realm of the History of Science. David Lindberg has written many articles and books on the subject of medieval optics. He has translated the treatises of Roger Bacon and John Pecham and written extensively of their research and theories.¹ He acknowledges their heavy debt to the Arab scientists that came before them and that the contribution the English Franciscans made to the study of optics is largely that of bringing the Middle Eastern theories to the western world. The resulting influence of the treatises of Bacon and Pecham was wide and lasting throughout Western Europe, but since they were so heavily indebted to the Arab theories, it was truly a spreading of the older, eastern ideas.

A. I. Sabra writes on the Arab development in optical science in the medieval years.² He examines why and how the Arab world kept the Greek philosophical traditions alive while the west abandoned it. He sees it as an appropriation and naturalization of the Greek philosophies, transforming them into their own philosophy and into their own culture. The primary figure of Arab Medieval optics is that of Ibn Al-Haytham, or as he is known in the west, Alhazen. For simplicity sake, I will refer to him as Alhazen, which is the name he was known by in the west in the Middle Ages and the

¹ David C. Lindberg, *John Pecham and the Science of Optics* (Madison: University of Wisconsin Press, 1970) and Lindberg, *Roger Bacon and the Origins of Perspectiva in the Middle Ages* (Oxford: Clarendon Press, 1996).

² A. I. Sabra, *Optics Astronomy and Logic: Studies in Arabic Science and Philosophy* (Hampshire: Variorum, 1994).

Renaissance. He lived in the eleventh century and was the primary figure because of the key role he played in joining disparate theories together, and creating a theoretical underpinning that accounted for what we experience in everyday vision. He was himself indebted to some of the theories of his ninth-century predecessor, Al-Kindi. Al-Kindi and Alhazen had opposite ideas on the foundations of optics and how vision occurred, but Al-Kindi had some unrelated theories that led Alhazen to a working geometric template. The importance of Alhazen in the subsequent state of optical theory cannot be overstated as it was primarily his ideas that Roger Bacon and John Pecham so widely published and championed. It was these ideas that were present in Italy in the fifteenth century that influenced the way in which paintings would be drawn in perspective.

As the first person to record the method of painting in perspective, Alberti has a unique place in history. Recording painting methods in treatise form from which painters as well as non-painters could learn was not a common practice as painting was taught from master to pupil. The methods were trade secrets of the guild system and only taught to students in each school.³ Alberti himself was instead a ‘gentleman painter’ and was an educated man.⁴ *On Painting* was written for those who wanted to paint, but did not necessarily want to learn through the traditional workshop system. The new perspective system was novel in the way that it used higher learning and required a certain amount of education to understand. Alberti, by including perspective at the beginning of his book and dedicating so many pages to it, helped to raise the social standing of the artist by

³ John R. Spencer, introduction to *On Painting*, by Leon Battista Alberti (London: Routledge & Kegan Paul, 1956), 11.

⁴ *Ibid*, 15.

showing that it was an intellectual discipline, and that one could learn it without being part of the apprentice system. Conversely, in *On Painting*, Alberti repeatedly uses language that tries to simplify the method so that the non-educated could repeat it and copy it. While theoretical knowledge was required to develop it as a system and to make any personal changes in it, it was possible to simply copy the technique without knowing the mathematics behind it. In this way, Alberti allowed a broader audience to be able to do this increasingly popular technique without having cultivated the theoretical knowledge first. The style of Alberti's book was not common in painting but was more familiar to the format of medieval architectural books written for the uneducated, but skilled, masons working under the architect. Samuel Edgerton compares it to a geometry book, but since it was not concerned with theory, it would have been more like a geometry book used for architectural construction.⁵ Alberti showed both that the creator and developer of new geometric techniques in painting were akin to the level of the architect, who was considered the highest of intellectual artistic professionals, and also that the copying of the technique was like that of the masons doing a skillful job of cutting the stones to the precise measurements of the architect. Martin Kemp mentions the use of medieval optical theory as a part of the development of perspective when talking about Alberti's *On Painting*.⁶ He shows that Alberti's description of how to do geometric projection is the same as medieval theories of sight.

⁵ Samuel Y. Edgerton, *The Renaissance Rediscovery of Linear Perspective* (New York: Basic Books, 1975), 80.

⁶ Martin Kemp, *The Science of Art: Optical themes in western art from Brunelleschi to Seurat* (New Haven: Yale University Press, 1990), 22.

One of the extensive studies of the mathematics used in the development of perspective and the history of the developments in the science of perspective is Martin Kemp's *The Science of Art*. Kemp offers explanations of both the most widely used methods as well as lesser known methods and ideas. He attempts to follow routes of influence, both the probable and merely possible. His approach is based on written accounts and treatises and he focuses on the technical aspects, creating a historical account of the development and changes in the perspective systems. He looks at the situation widely, beyond Italy. When pictorial perspective became popular, it spread quickly to the north and had unique developments there that set it apart from the Italian practices.

Kemp talks about the treatise of Alhazen, as well as his western followers, and shows a part of the role that the knowledge in the middle ages of Alhazen's optical treatise had in the development of geometric projection for painters.⁷ Kemp shows that Renaissance perspective theory was not based directly on ancient texts and techniques, but was instead a mix of medieval optical theories and geometric techniques with ancient ideas. Samuel Edgerton goes further in this regard. He tries to trace the possible time and place that Alhazen's treatise was introduced into the educated circles of Florence, and therefore, how it could have directly influenced perspective's invention. He follows it from the English, through John Pecham and his treatise *Perspectiva Communis* of 1265, and then to Alberti.⁸ Edgerton asserts that the main source for this treatise was Alhazen, and secondarily Roger Bacon, who was also a promoter of Alhazen's ideas.

⁷ Kemp, 26.

⁸ Edgerton, *Renaissance Rediscovery*, 77.

Edgerton finds that it is likely that Pecham was Alberti's first introduction to optics.⁹ He then introduces the ancient sources with Ptolemy's cartographic treatise *Geographia* entering Florence at the same time.¹⁰ He traces the import of the Ptolemy treatise to Paolo dal Pozzo Toscanelli, who was a friend of both Brunelleschi and Alberti, and therefore conjectures that it was Toscanelli who helped Brunelleschi with the creation of pictorial perspective using cartographic methods of geometric projection in the treatise.¹¹ Vasari says that Toscanelli taught Brunelleschi geometry, so there is reason to think that he might have taught him this particular geometry as well.¹² What Edgerton primarily does in his book is find possible connections and confluences that could explain the unique appearance of pictorial perspective in fifteenth-century Florence when the mathematical structure was already known in different places at earlier times. He then directs these connections into a narrative that could explain the creation of the technique of perspective.

The role of Alhazen's treatise is thoroughly researched by Hans Belting. Belting looks at the development of geometry in the iconoclastic Arab world as a parallel to pictures in the Western world.¹³ The contemplation of geometry in the east was similar to the contemplation of religious art in the west. Belting finds that the concept of ordered space combined with the theories in Alhazen's treatise create the necessary concept of

⁹ Edgerton, *Renaissance Rediscovery*, 77.

¹⁰ *Ibid*, 93.

¹¹ *Ibid*, 123.

¹² Vasari, *Lives of the Painters, Sculptors and Architects*, trans. Gaston du C. de Vere (New York: Alfred A. Knopf, Inc, 1996), 328.

¹³ Hans Belting, "Perspective: Arab Mathematics and Renaissance Western Art," *European Review* 16 (2008): 187.

perspective.¹⁴ It was through the purely scientific concepts of optical theory that the ideas of orthogonals and a vanishing point came into existence. The skillful application of these theories into painting was Brunelleschi's achievement. Belting asserts that the Arab influence during the middle ages was a necessary and vital part of the invention of pictorial perspective.

In the introduction to Alberti's *On Painting*, John Spencer calls attention to the fact that in later years, Alberti's treatise has been taken to represent strict, immutable rules that constrict the development of invention.¹⁵ Alberti became associated with the rigid rules of academies, but Alberti did not intend his rules to be rigid and definite, as he only spends the first book explaining the mathematics, and the second and third books explaining other precepts of painting. Both Edgerton and Kemp discuss how different artists used slightly different methods of perspective in order to produce different results. Some of these methods we now strongly associate with one artist, such as the use of sfumato with Leonardo da Vinci. Kemp focuses on the developments that were made in order to try to provide new solutions to problems that persisted past the invention of single point perspective. For Donatello, communicative function took precedence over complete adherence to the geometry.¹⁶ For him, the use of perspective was more intuitive and to be used in order to strengthen the narrative, not to weaken it. Leonardo da Vinci continuously investigated and experimented with Brunelleschi's invention so as to bring it into a greater perfection.¹⁷ Leonardo's *Last Supper* illustrates how he would use

¹⁴ Belting, 189.

¹⁵ Spencer, 12.

¹⁶ Kemp, 40.

¹⁷ Ibid, 52.

perspective for pictorial suggestion rather than absolute definition in its many perspectival ambiguities and contradictions.¹⁸ (Fig. 1) While it is not geometrically correct, and therefore does not accurately represent a real room, it gives the appearance of optical legitimacy, and therefore shows how Leonardo chose to change the rules of perspective for compositional and perceptual effect. Edgerton finds that Alberti's distance point operation is the most consistently modified or ignored operation and that it was meant as a tool for painters rather than a rule.¹⁹ Michael Kubovy in *The Psychology of Perspective and Renaissance Art* illustrates the common intuitive fix to representing a sphere off-axis in perspective.²⁰ If the painter used the correct geometric projection, the off-axis sphere should always be represented as an ellipse, yet the viewer would not see it as a sphere, but as an ellipse, therefore, the painter represents it using the circle. This can clearly be seen in Raphael's *The School of Athens*. (Fig. 2, Fig. 3) This will be discussed more fully in Chapter 2. This illustrates the greater importance on the perception of the painting over the geometry of the perspective, and the willingness for the artist to change and mold the perspective system to suit their needs. Far from being immutable rules, the perspective system, as laid out by Alberti, provided a foundation so that the artist could grow from there. The good artist would then use this system to make appropriate changes and developments, while the poor artist would be left with mere copying of technique.

¹⁸ Kemp, 47.

¹⁹ Edgerton, *Renaissance Rediscovery*, 55.

²⁰ Michael Kubovy, *The Psychology of Perspective and Renaissance Art* (Cambridge: Cambridge University Press, 1986), 112-114.

Another element to consider of the new perspective style paintings of the fifteenth century is the religious aspect. Religious paintings were the most common theme at the time since the church was the most common patron. Religion was a large part of the lives of the people in the fifteenth century and piety played a key role in political and social standing. Alberti said that painting contains “a divine force that makes the dead seem alive” and that it is “most useful to piety and keeps our souls full of religion.”²¹ In such an atmosphere, the religious connotations of perspective could not be ignored. The bases for intellectual pursuits were rooted in theology, and as art was attempting to become a more intellectual profession, it too was based on theological concerns.

Science was not considered to be anathema to religion, but to be in the service of it. Proof in the perfection of nature was proof in the perfection of God as the creator. In the thirteenth century, Roger Bacon wrote a treatise for the pope including a section on optics in which he stated that geometric laws reflected God’s grace.²² The English Franciscan was concerned with optical science and the nature of light because of the divine nature of light in Christian theology. Light was how God spread his grace throughout the world, and so the manner in which he did that would be of a divine nature. Bacon suggested that painters should become skilled at geometry in order to better illustrate the geometric perfection of the works of God.²³ Order in nature showed moral order and the perfection of the creation of humans who were designed in God’s image.

²¹ Leon Battista Alberti, *On Painting*, trans. John R. Spencer (London: Routledge & Kegan Paul, 1956), 63.

²² Edgerton, *Renaissance Rediscovery*, 16.

²³ *Ibid*, 17.

Optics was both a branch of science and of faith that during the Renaissance became a part of the world of the artist.

Edgerton asserts that since realism at the time was as much metaphysical as physical, perspective was not for aesthetics, but for a sense of harmony with natural law, God's law.²⁴ The natural portrayed the divine, therefore the divine could be portrayed naturally. Therefore, the portrayal of religious scenes in proper geometric perspective and with natural proportions would increase the divine grace of the representation. While earlier medieval representation was concerned with portraying the soul and spirit of the figures, the fifteenth-century person saw nature as the perfect spiritual form since it was created by God in his image, and the greatest spirit could be shown as the closest to nature. Therefore the move toward greater naturalism was more a change in shape rather than in the spirit of the painting. Michael Baxandall calls it a "moralization of mathematics."²⁵ He says that conspicuous use of perspective was not just to show great skill, but to show the morality in the composition of the picture. While antique influences are clearly at work in the depiction, the humanist had to use a combined Christian theology and ancient philosophy to come to what they saw as "truth." The classical pagan forms of figures and nature were transformed into Christian forms representing the perfection of God and his divine plan. Geometric perspective was yet another explanation of the wisdom and perfection of God, and the representation of that natural perfection was to show the divine.

²⁴ Edgerton, *Renaissance Rediscovery*, 56.

²⁵ Michael Baxandall, *Painting and Experience in Fifteenth Century Italy*, 2nd ed. (Oxford: Oxford University Press, 1972), 103.

In *Painting and Experience in Fifteenth Century Italy*, Michael Baxandall investigates the socio-historical underpinnings of painting in the fifteenth century. He looks for the reasons why a painting was made and the reasons why it was made in that specific way, regarding both material and composition. He takes a good look at the economic realities of painting and how perspective fits into those economics. One of the ways he does this is by looking at painting commission contracts. Throughout the middle ages, it was largely the expense of the materials in a painting that made it worthy of honor. The more important figures or objects would be in gold leaf or in blue, which was a more expensive pigment.²⁶ Looking at the history, Baxandall finds that there was a general inclination towards a less ostentatious show of wealth in the fifteenth century, therefore the display of expensive materials in paintings declined.²⁷ For religious paintings, the patron would still want to show honor to those portrayed as well as piety and morality. Replacing the expensive material was expensive skill. The skillful portrayal of naturalistic figures and perspective was only accomplished by the best artists, and therefore the most expensive artists. The expense of skillful painting was well known, but considered more appropriate socially at that time than the presentation of wealth through materials. The role of the artist was changing at this point, becoming socially higher. As we have already seen how Alberti tried to raise the social standing of the artist, he also took a position on skill over material saying that for the portrayal of

²⁶ Baxandall, 11.

²⁷ Ibid, 14.

gold, “I should not wish gold to be used, for there is more admiration and praise for the painter who imitates the rays of gold with colours.”²⁸

Kubovy discusses how the portrayal of skill was used compositionally, as a method of getting the viewers attention to the right places. He uses Piero della Francesca’s *Flagellation of Christ* as an example of how perspective could be used for compositional purposes, rather than strict representation. (Fig. 14) Martin Kemp describes it as a “rational demonstration of ineffable presence.”²⁹ Kubovy shows how the most important action is related to the perspective, with the scourge being related to the orthogonals and that it passes through the vanishing point.³⁰ The orthogonals of the tile floor in Piero’s painting all point toward Christ, driving the attention to the narrative in the background where the action takes place instead of towards the foreground figures. As opposed to being a moment stopped in time, the perspective is used as a tool to create narrative within time.³¹ In a similar way Leonardo’s *Last Supper* uses “abnormalities” of the perspective to affect the way in which the painting is viewed and understood while still maintaining the immediate illusion of true perspective.

In order to understand Early Renaissance paintings and the circumstances around which they were made, Baxandall wants to be able to understand the people who were involved and their motivations through their socio-historical background. To try to understand what he calls the “period eye,” he looks into specifics of the period. He examines the specific way descriptive words would have been understood when used to

²⁸ Alberti, 85.

²⁹ Kemp, 32.

³⁰ Kubovy, 3.

³¹ Ibid, 2.

describe painting. It is only through this minute investigation that he finds that anyone from modern times can attempt to understand the meaning the artwork would have had for the contemporary viewer. He finds, for example, that there was a language of gesture that was used by the preacher in sermons that found its way into the portrayal of religious figures.³² Since religious paintings were made for the church and they would have been seen in the same venue as the preacher, the relationship makes sense. The figures were also often Jesus or saints, who would have been regarded as examples and teachers of the faith, much in the way that the preacher would have been seen. Only by attempting to translate these gestural meanings can we see the figures as they were meant to be seen and not put a modern interpretation on the composition. Baxandall concludes that in order to understand the picture, one must learn how to read it through the eyes of the culture in which it was made.³³ (This is true of different time periods as it is for different geographical areas, and looking at a painting in this way can also tell us about the culture in the same way that the culture can tell us about the painting.)

The “period eye” is not the only subjective part of viewing perspective. Any visual tool or illusion integrates psychological perception. The way in which people view the world is based on how we learned as infants to interpret the shapes around us. E.H. Gombrich has investigated the way in which people perceive the visual world around them in *Art and Illusion*, and Michael Kubovy has applied Gombrich’s findings to the world of the Italian Renaissance in *The Psychology of Perspective and Renaissance Art*. They delve deeper, past the period eye of Baxandall, to look at the unconscious

³² Baxandall, 61.

³³ Ibid, 152.

interpretation of color and line that occurs inside the brain. Gombrich starts with the question of artistic style and why it exists.³⁴ If artists are always trying to portray the truth of nature, than it must be the perception of nature that changes. Nature as represented in perspective is not just drawn by nature, but like all other styles, is based on traditional forms and the psychology of the time and the artist.³⁵ He looks into the very fundamentals of perception, such as how we perceive distance versus size, and color versus shade. The interpretation of the world through our window is learned, and therefore is subject to change in different cultures. He finds that the artist can never disregard traditional starting points, that it is always a case of starting with a generic form and than changing it in order to match what is perceived.³⁶ However, that which is perceived can change with time and culture. Gombrich relies on psychological experiments on the nature of perception and finds that when trying to discern an object in life, the eye and mind first try to match it to something familiar, something that is expected in that place.³⁷ In that way, a piece of paper floating in the sky can at first look like a bird or plane much further away. Only when the behavior of the object makes the interpretation impossible will the perception change to accommodate the new information.³⁸ Similarly, familiar forms can also be projected onto objects, such as the ability to see faces in the simplest arrangement of objects that do not truly resemble a

³⁴ E. H. Gombrich, *Art and Illusion: A Study in the Psychology of Pictorial Representation*, 2nd ed. (New York: Pantheon Books, 1965), 3.

³⁵ *Ibid*, 24.

³⁶ *Ibid*, 77.

³⁷ *Ibid*, 181-183.

³⁸ *Ibid*, 272.

face.³⁹ Expectation and schema are important aspects of how we view an image.

Baxandall must have understood this as he tried to understand what the expectations of the contemporary viewers were. All seeing is interpretation, but it is based on what we know to be possible and likely, and this is how illusion is possible. It plays on our natural tendency to assume the most likely situation for what we are seeing, that a house should be house sized and not a model.⁴⁰

Kubovy takes the findings of Gombrich's psychological experiments and thought experiments and applies them to Baxandall's imperative to understand the contemporary viewpoint that results in a discussion of the role of perception on the development of pictorial perspective. Kubovy finds different roles for perspective in the fifteenth century and therefore different styles to fit the different perceptual roles.⁴¹ In this way he also finds that the tool of perspective was not followed unwaveringly, but changed according to the practitioner and the intended viewer. Changes in the use of perspective rules were only possible if the perception of perspective remained intact. This was possible by what Kubovy calls, the "robustness of perspective." This refers to the ability of the perspective system to keep working even when the viewer is not at the precise viewing location required by geometry.⁴² In the method of geometric projection, the perspective should only remain undistorted when viewed from one particular position. The fact that viewers can walk around in front of paintings that use perspective and still experience the painting without distortion is a result of our perception rather than the technique of the painting.

³⁹ Gombrich, 105.

⁴⁰ Ibid, 301.

⁴¹ Kubovy, 6.

⁴² Ibid, 54.

Our own brains take into account our position and correct for it when looking at a painting. This is only possible when the eye can recognize the angle of the surface of the painting, and therefore only works when the art is not intended to be purely illusionistic space.⁴³ This discovery of the robust nature of our perception of geometric projection allowed painters more leeway in how they presented the world in painting. It is also for this reason that the situation of spheres being represented as circles against the suggested geometry occurs, as mentioned earlier. The eye makes allowances for the surface of the painting, and therefore does not see the geometrically correct ellipses as spheres.

Brunelleschi's original method of demonstration of perspective suggests that he was not aware of this robustness. His first experiment involved a painting of the Florence Baptistery as seen from the doors to the Cathedral. He showed it using a "peepshow" technique, putting a viewing hole in the vanishing point of the painting and then having the viewer look through the back of the painting through the hole into a mirror held at arm's length in front of the painting. (Fig. 4) The peepshow system allowed Brunelleschi to always keep the viewer in the exact location demanded by the geometry of the painting. It also resulted in a complete illusion by using the mirror and hole. The mirror took away the ability of the eye to perceive the picture plane and looking through the hole took away any outside influences that would alert the eye to the reality of what was being shown. This resulted in an astonishing display for those people at the door of the Cathedral. When Brunelleschi showed his painting, they would have believed they were seeing the real thing.

⁴³ Kubovy, 55.

In the second demonstration, Brunelleschi used a larger perspective painting in the Palazzo Vecchio without the peepshow technique. (Fig. 5) It was simply a perspective painting placed in the location of the real buildings. Kubovy tries to figure out why Brunelleschi abandoned his peepshow technique. It is suggested that Brunelleschi did not use the peepshow in the second demonstration because of practicalities; the painting was larger than the previous one and could not be held in one hand while a mirror was held in the other.⁴⁴ However, it seems that Brunelleschi could have painted the panel smaller and had a smaller viewing angle if he had wanted to continue the peepshow method. Kubovy suggests that he changed the method because he realized that it was not necessary for the viewer to be in the precise viewing location in order to get the experience of perspective.⁴⁵ The absence of complete illusion in the second demonstration also took away the more entertainment aspects of the technique and allowed the focus to be on the method. The laws of perspective were still yet to be completely figured out, which led to the experimentation of multiple artists throughout the Renaissance as well as the different styles of perspective used in Italy and the north.

⁴⁴ Kubovy, 127.

⁴⁵ Ibid, 128.

Chapter 1: The Medieval Roots of Perspective

The optical theories that mathematicians and philosophers read in the fifteenth century had been long developed through the centuries and had their genesis in the Ancient Greek world. It was a long circuitous route, however, from Ancient Greece to Renaissance Italy. The texts and prevailing theories in the fifteenth century were much the same since the thirteenth century when the Englishmen led the way in new optical theories. Roger Bacon and John Pecham made large progress for western philosophy in bringing the ancient to the medieval world. However, they were not without strong influence from the theories of the Arab world from the twelfth century by Alhazen, and he from the ninth-century Arab, Al-Kindi. Without Alhazen, the English philosophers would not have made the great strides that they did, and without Al-Kindi, Alhazen would not have been able to synthesize the earlier ancient philosophies into a unified, workable idea. In addition, it was the Arabs that kept the Greek works from disappearing in the middle ages and had an early period of translating and working the Greek texts in the ninth century. The Arabs, in turn, did not invent their philosophies out of nothing, but based them largely on the works of the Ancient Greeks, Aristotle and Euclid in particular. While the fifteenth-century Italian humanist proudly studied the classics and strove to put all things in his life in the way of the ancients, it would not have been possible to study optics, and therefore *perspectiva*, without the route through the near east and then England.

The study of optics, or the study of how the eye sees, had struggled with the same arguments since antiquity. The prevailing view in the thirteenth through the fifteenth centuries was by no means the only view and the points of contention had their roots in the arguments of Ancient Greece. For this reason, one must look at the early disagreements in the philosophy of optics among the ancient philosophers.

Brunelleschi is credited as the inventor of linear perspective in the early fifteenth century, but how did he create it and why? He was not primarily a painter, yet he painted two panels in linear perspective in order to demonstrate this new method of naturalistic painting. He painted the first panel as a peep-show technique that required the viewer to look through a small hole in the back of the panel and view the painting in a mirror held in front of the viewer. He gave this demonstration in the entrance of the unfinished Duomo in Florence, facing the Baptistery, which was the exact subject of the painting. This enabled Brunelleschi to show his astonished viewers the comparison of his panel and the reality in front of them.

What was the motivation behind using geometry in order to present a painting as through a window, and why would he desire to portray his painting in such a way? Why did he choose the geometry of optics and how did he learn it? Vasari says that the philosopher and humanist Paolo dal Pozzo Toscanelli taught Brunelleschi the mathematics required to create his new perspective scheme.⁴⁶ Toscanelli was a friend to both Brunelleschi and Alberti.⁴⁷ So where did Toscanelli learn this mathematics that would be applied to painting in such a novel way? He was highly educated and wrote

⁴⁶ Vasari, 328.

⁴⁷ Edgerton, *Renaissance Rediscovery*, 61.

books on perspective himself.⁴⁸ At the beginning of the fifteenth century, John Pecham's textbook on *perspectiva* was the standard in universities throughout Europe.⁴⁹ Even though it was written nearly 200 years earlier, it was still the standard in optical textbooks. Therefore, what Brunelleschi would have learned from Toscanelli would likely have been from Pecham's work. Other artists who worked on perspective after Brunelleschi's original demonstrations would also have been learning from the same material. Alberti, Ghiberti, and Piero della Francesca were all working on methods of linear perspective. While we speak of one method, at the time there were several methods employed, sometimes in the same painting. Ghiberti changed and experimented with his perspective techniques, and he used several sources for his methods.⁵⁰ Therefore, in order to understand the methods of the early Renaissance perspective painters, we must understand the science on which their methods were based.

The texts that these fifteenth-century painters were reading and discussing were primarily from 13th century English Franciscans. The three main enduring sources were Roger Bacon, John Pecham, and Witelo. They were writing and publishing at nearly the same time in the 1260s and 1270s. First came the work of Roger Bacon, then John Pecham, and lastly, Witelo.⁵¹ While Bacon wrote first, he was not widely published until later. Bacon and Pecham were personally acquainted, but Witelo most likely was not acquainted with Bacon. Witelo was, however, familiar with Bacon's writings.⁵² These three English theorists drew heavily from other writings, however. Of great influence on

⁴⁸ Edgerton, *Renaissance Rediscovery*, 62.

⁴⁹ Lindberg, *Roger Bacon*, xcvi.

⁵⁰ *Ibid.*

⁵¹ Lindberg, *Studies in the History of Medieval Optics* (London, Variorum Reprints, 1983), 66.

⁵² *Ibid.*, 76, 73.

all of them was Ibn Al-Haytham, commonly referred to as Alhazen, from the eleventh century. His writings had become known to Roger Bacon and were very influential on Bacon's theories. In order to understand the theories of the thirteenth century, one must first understand the revolutionary nature of Alhazen's theories.

Alhazen had created a revolutionary theory in optical science that combined all the previous competing theories of the ancient Greeks. He was determined to create a theory that unified all the ideas that had come before him and to show that all of them were right. In addition to the Greeks, he also studied the work of the ninth-century Arab before him, Al-Kindi. Al-Kindi had also studied the Greek theories of optics and had developed new ideas based on old theories. Without the work of Al-Kindi and his rejection of radiation in coherent form, Alhazen could not have made his leaps that so inspired Bacon, Pecham, and Witelo.

The basic philosophical theories of the transmission of light and shapes to the eyes were first developed by the Greeks. All later theories were predicated on these early Greek theories. The Greek ideas were subjected to different interpretations or combinations, but they stayed more or less the same until the modern era. Later optical philosophers were extremely reluctant to disregard the ideas of those Greeks held in such high esteem. To reject the established theories for completely new ones was beyond possibility, so medieval scientists tried to adapt the theories to account for observations.

While an exhaustive study of Greek thought on optics is not necessary for understanding later theories on the subject, the basic differences between theories was a prime point of argument and discussion for medieval optical philosophers. This came

from the premise that there must be contact between the object and the visual organ.⁵³ A large point of contention in medieval optical theory that was rooted in Greek theory and that was thought to be irreconcilable was the divide between extramission and intromission. These were the competing theories of whether visual rays left the eye and returned information to the eye about what it encountered, or whether visual rays radiated from the object and were accepted by the eye. There were problems with both theories and it was the basic point of contention for optics. A third option would be that there was an intervening medium through which contact was made. This middle ground leads to Aristotle and his theory of coherent forms.

Extramission was the theory of rays leaving the eye and going to the object and then relating that information back to the eye. Extramissionists included Ptolemy, Galen, and importantly, Euclid. In later years, it also included Al-Kindi and St. Augustine. An important precept of extramission theory was the Euclidean geometry that went along with it. Euclid was an extramissionist and developed a geometrical approach to the theory, creating a mathematical reasoning for the basis of the argument. The allure of the Euclidean approach is one reason why the extramission theory was so popular. The downside to the theory was in the physical reasoning. The method of how the information was related back to the eye was unclear. Intromission was a physicist's theory, describing the physical action, while extramission was a mathematician's theory, its strength coming from its geometry. An elegant mathematical model did not exist for intromission theory and this was seen as a handicap.

⁵³ Lindberg, *Studies*, 339.

Intromission was based on the theory of the atomists of Greece, like Epicurus, who explained that thin films of atoms left the objects in all directions and were received by the eye.⁵⁴ These atoms would maintain their configuration while moving through space. Intromission theory did not have many supporters because of the objections brought to it that seemed proved by our own everyday perception. Al-Kindi in the ninth century argued that intromission theory could not adequately explain the perception of size and shape.⁵⁵ For example, if a circle were to be perceived with its edge toward the viewer, the circle would send its image as a circle toward the viewer, however once the eye received the image, it would perceive the whole circle and not just the edge of that circle. Once in the eye, all forms would be perceived in their full roundness, which is not how we experience vision. This was a major problem with the theory of intromission. Another objection was in the perception of size. In order for the image of an object to enter the eye, it had to shrink from the original object based on the laws of perspective in order to account for visual perception, but once the image is at the eye, how can the eye know the true size of the object? Similar reasoning was used to show flaws in Aristotle's theory of coherent forms being transmitted through a medium between the object and the viewer. With these objections that so readily seemed to disprove intromission theory, extramission theory was in the dominant position in the middle ages. Alhazen was able to bring intromission theory out of these objections in the eleventh century. Interestingly, in order to do this, he used a theory of Al-Kindi's on a different subject.

⁵⁴ Lindberg, *Studies*, 339.

⁵⁵ *Ibid*, 344.

After the end of the ancient era, the Arab world became the inheritors of mathematical and scientific discourse. In the west, there were no great changes in optical science until the 13th century. A. I. Sabra discusses reasons why medieval Islam might have been a helpful force in the continuation of Greek learning. He speaks of the transmission of Greek learning to medieval Islam as appropriation rather than a passive reception.⁵⁶ What he emphasizes is that Islamic civilization was not merely a receiver, but they sought after and took Greek learning and made it their own, creating a new scientific legacy in the Islamic world.⁵⁷ Just as in the west, science had a rocky and uneven relationship with religion in the east. In much the same way that the Catholic Church would at one time support scientists and then turn and denounce them, Islamic institutions would also at times support and deny scientific discoveries and theories. Arithmetic, medicine, and astrology became useful importations to the religious leadership. The relationship of the sciences, both imported and Islamic, to religious institutions was complicated and not able to be covered in general statements. The question of whether the eventual decline of science in medieval Islam was solely due to the religious opposition remains open.⁵⁸

Sabra seeks to demonstrate that apart from the legacy of medieval Islamic science being a few people outside of the larger social and religious fabric, the continuation of Hellenistic thought was a part of the Islamic civilization.⁵⁹ The advanced institutions of learning in medieval Islam were *madrasas*, and they were only concerned with the

⁵⁶ Sabra, "Greek Science in Medieval Islam" in *Optics, Astronomy and Logic*, 225.

⁵⁷ *Ibid*, 226.

⁵⁸ *Ibid*, 232.

⁵⁹ *Ibid*, 229.

teaching the Islamic religion and the Arabic language.⁶⁰ The point of the *madrassa* was to be a theological institution with the study of law as its center, so the exclusion of science is understandable. However, because of this exclusion, scientific education had to lead a separate existence, leaving it in a precarious position. The exclusion of science from the *madrassa* can also be seen as a religious repudiation of science since it is the institution of the religion.⁶¹ However, scientific inquiry did not stop as a result of being excluded from the *madrassas*.

In some *madrassas* the sciences were included, but this was not a regular occurrence and was on account of the informal manner in which the curriculum was set.⁶² The teacher was often the greatest influence on the curriculum, teaching what was of interest to him rather than what was prescribed. Also, as mentioned earlier, certain sciences had become essential to the religion, such as astronomy and arithmetic. The official mathematician-astronomer of a mosque was responsible for producing the tables for the times of the daily prayers for his area.⁶³ The translated manuscripts of the Greek sciences were often in the libraries of *madrassas* and even copied there, ensuring its survival.⁶⁴ The process of incorporating the sciences into the religion was slow, but ended in the naturalization of the foreign sciences into the Islamic culture.

Sabra then suggests 4 stages of development and decline of the sciences in medieval Islam.⁶⁵ The first stage is the acquisition and translation of Greek philosophy

⁶⁰ Sabra, "Greek Science in Medieval Islam," 230.

⁶¹ Ibid, 233.

⁶² Ibid, 234.

⁶³ Ibid, 236.

⁶⁴ Ibid, 234.

⁶⁵ Ibid, 236.

and science. At this time, these foreign sciences slowly became a part of the local culture and religion. The second stage is the emergence of powerful Muslim thinkers who held a Hellenistic view of philosophy and science. It is at this stage that we see the emergence of Avicenna and Alhazen. The third stage is when the sciences have been completely assimilated into Islamic culture and the mathematicians and astronomers become a part of the mosque. Sabra attributes the subsequent decline in scientific thought and progress to the resurgence of the view that religious instruction alone is more important than all other forms of knowledge and science was once again excluded from the official religious institutions.⁶⁶ It is in the third stage, when men were both learned in the sciences and religious law, when the two were seen as compatible rather than antithetical, that the most important developments in optics and perspective came about in the east. Sabra posits that the decline came from this acceptance. As the sciences were accepted into the *madrastas*, the sciences were then assimilated into the religious way of thought, and were therefore limited by the institutions.⁶⁷ The sciences were no longer able to develop in their own manner as when they were excluded from the *madrastas*, but at this point were defined by these institutions in whatever way they chose to limit them.

It was in the early stages of development that we find Al-Kindi in the ninth century. Although he was an extramissionist, he had developed a theory separately that led Alhazen to a workable intromission theory. Al-Kindi posited that light radiation did not issue from an object or surface as a whole form as in Aristotle's coherent forms. Instead, he theorized that radiation was incoherent and radiated in all directions at once

⁶⁶ Sabra, "Greek Science in Medieval Islam," 239.

⁶⁷ Ibid, 240.

and independent of other points.⁶⁸ It was this theory of incoherent punctiform radiation that led Alhazen in the eleventh century to build a successful intromission theory.

Al-Kindi was a driving force in the appropriation of Hellenic thought into Islamic culture. He wrote against religious bigotry and the insularity of their intellectual institutions.⁶⁹ He developed and defended the theories of Euclid and Ptolemy and his book was widely influential in the east and later in the west through a Latin translation.⁷⁰ As a Muslim, his push for the Greek sciences was significant in the Islamic world in a time when most of the translators and proponents of Hellenistic thought were non-Muslims.⁷¹

Al-Kindi was an influence on the work of Alhazen in the eleventh century, but Alhazen gave the scientific community a more ground breaking theory. Alhazen did not so much create new ideas as join all the old ones together to create a unified whole. By taking Al-Kindi's theory of incoherent punctiform radiation, Alhazen was able to join the physics of intromission theory with the geometry of extramission theory. His goal was to satisfy the mathematical, physical, and physiological demands of every theory.⁷² He was aware that he was attempting something new and consciously sets out to combine the mathematical and the physical sciences.⁷³ The basis of his theory was the cone of vision that was used by the extramissionists. While the extramissionists said that the visual rays leave the eye in a cone, Alhazen said that the visual rays enter the eye in the shape of a

⁶⁸ Lindberg, "The Science of Optics" in *Studies in the History of Medieval Optics*, 345.

⁶⁹ Sabra, "Greek Science in Medieval Islam", 226.

⁷⁰ Lindberg, "The Science of Optics", 342.

⁷¹ Sabra, "Greek Science in Medieval Islam", 226.

⁷² Lindberg, "The Science of Optics", 343.

⁷³ Sabra, "The Physical and Mathematical" in *Optics, Astronomy, and Logic*, 1.

cone. The reason that he could concentrate on this cone of vision and ignore all surrounding paths to the eye was the fact that the eye is curved. He posited that the visual rays entering the eye at an angle not perpendicular to the convex surface of the eye would be refracted.⁷⁴ He further posited that those refracted rays were not seen by the eye. Therefore, the only rays that the eye sees are the ones in the precise area of the cone of vision used by Euclid. Since the only seen rays are in the cone, they maintain their arrangement and can account for the coherent image that is perceived.⁷⁵ It is this part of the theory that accounted for the perception of shape and perspective, thus eliminating some of the primary arguments against intromission. This development was influential throughout the centuries, and it is this cone of vision that shows up in Alberti's *On Painting* in the fifteenth century as a way to draw perspective.⁷⁶ He also eliminated the problem with the extramission theory of how the information returns to the eye by making it an intromission theory. In this way he joined all the theories and was able to say that none of the venerable ancient philosophers were wrong. He believed that each ancient philosopher was partially correct and the theories only incomplete by themselves.⁷⁷ While he considered his theory a combination of the doctrine of forms from Aristotle and the doctrine of visual rays from Euclid and Ptolemy, his theories also bring in the theories of the atomists.⁷⁸ While bringing in the doctrine of forms, his

⁷⁴ Lindberg, "The Science of Optics", 346.

⁷⁵ Ibid, 347.

⁷⁶ Kemp, *The Science of Art*, 22.

⁷⁷ Sabra, "The Physical and Mathematical" in *Optics, Astronomy, and Logic*, 19.

⁷⁸ Ibid, 15.

approach was mathematical and experimental in nature rather than philosophical. He subjected forms to geometrical analysis, which was not Aristotelian.⁷⁹

Alhazen believed that knowledge of truth was the way to find favor with God and that truth could only be found in rational doctrines.⁸⁰ He was led to this belief through his frustrations in studies of religious sciences and his readings of Aristotle.⁸¹ The importance of combining the intromission and extramission frameworks and creating a comprehensive optical theory that addressed every aspect cannot be overstated, as it is what leads to modern optics.⁸² Conflict with his theory was at a minimum because he agreed with everyone. He also filled the void of the physiological aspect in the work of Aristotle, Euclid, Ptolemy, and Al-Kindi.⁸³ However, Islamic law prohibited dissection, so Alhazen was dependent on the physiological descriptions of the Greeks, primarily Galen and Rufus of Ephesus.⁸⁴ Alhazen's comprehensive theory relies mostly on Ptolemy; however, it must be taken into account that the Arabic translation of Ptolemy's *Optics* was missing the first book and the end of the fifth book.⁸⁵

Within Alhazen's treatise were smaller developments that become more important in retrospect. One was the development of experiment as a tool for scientific thought, however it was used as a tool to prove theories rather than discovery of new ideas.⁸⁶ He not only joined optical theories together, but also joined mathematics with direct

⁷⁹ Sabra, "Form in Ibn Al-Haytham's Theory of Vision" in *Optics, Astronomy, and Logic*, 116-117.

⁸⁰ Sabra, "Ibn Al-Haytham" in *Optics, Astronomy, and Logic*, 190.

⁸¹ Ibid.

⁸² Lindberg, "Alhazen's Theory of Vision" in *Studies in the History of Medieval Optics*, 322.

⁸³ Ibid, 332.

⁸⁴ Ibid, 327.

⁸⁵ Sabra, "Criticisms of Ptolemy's Optics" in *Optics, Astronomy, and Logic*, 145.

⁸⁶ Sabra, "Ibn Al-Haytham", 191.

experience.⁸⁷ He never actually formulates a theory of experiment even though he introduces the concept of controlled experiment as distinct from the repetition of observations.⁸⁸ Alhazen wrote several treatises on many scientific topics, however, only *Optics* and *On Paraboloidal Burning Mirrors* were translated into Latin and were therefore available to Western scholars.⁸⁹ These translations, however, left much to be desired. They were missing passages and only paraphrased much of the Arabic. Despite these problems, it did convey the substance of Alhazen's ideas.⁹⁰

Alhazen was a major influence on the most prolific writers on optics in the 13th century. The treatises of Roger Bacon, John Pecham, and Witelo, which all became standard university textbooks, were all greatly influenced by the treatise of Alhazen.⁹¹ The first westerner to write about Alhazen's theories was Roger Bacon. Like Alhazen, Bacon had a religious view on the necessity for scientific research and progress. Bacon held to the Augustinian ideal of philosophy being in the service of religion.⁹² While the ideals of the Hellenistic world were often looked upon with suspicion of being pagan and therefore contrary to the service of religion, Bacon refused to see Classical philosophy as antagonistic to the Christian faith. He subscribed to the belief that God could be better understood through the study of his creations, which in turn were controlled through mathematics.⁹³ The closer that scientists and philosophers looked at nature, the more mathematical and perfect it was found to be. This mathematical perfection, instead of

⁸⁷ Kemp, *The Science of Art*, 26.

⁸⁸ Sabra, "The Physical and Mathematical", 11.

⁸⁹ Sabra, "Ibn Al-Haytham", 196.

⁹⁰ *Ibid*, 197.

⁹¹ Edgerton, *The Renaissance Rediscovery of Linear Perspective*, 73.

⁹² Lindberg, *Roger Bacon*, xxiii.

⁹³ *Ibid*, xliii.

being seen as contrary to Christian ideals, was seen as proof of God's direct hand in the creation of nature. To understand nature and mathematics was to understand God. Also, the study of light was directly related to the divine. Light was a metaphorical, and sometimes literal, presence of God. The relationship of light to the divine would have been on the mind of Roger Bacon, the Franciscan. He felt that geometrical optics, as the study of light, had an ability to illustrate spiritual truths.⁹⁴ Therefore, the study of optics, the way light is seen and experienced, would have been viewed as a religious philosophy. There was a kind of spiritual delight to be obtained from studying optics.⁹⁵ Theology, philosophy, and science went hand in hand. Even though it was a different religion, Bacon shared with Alhazen the belief that God could be better known through the discovery and understanding of the nature of optics.

Before Roger Bacon, the study of optical science was brought into greater prominence in the early thirteenth century west through Grosseteste (d.1253). Grosseteste, however, was not familiar with Alhazen's work, nor with Ptolemy's *Optica*.⁹⁶ Grosseteste called for a greater mathematization of optics, but did not propose a geometric system himself. He spoke of radiating forces in general, but had no new complete theory to revolutionize the system. He did revitalize the interest in the west, however. Grosseteste was an important predecessor to Bacon's work, having struggled with methodologies and experimental ideas before him.⁹⁷ Grosseteste himself had made

⁹⁴ Lindberg, *John Pecham*, 19.

⁹⁵ Kemp, *The Science of Art*, 26.

⁹⁶ Lindberg, *Roger Bacon*, xxxvii. This information on Grosseteste is largely taken from Lindberg's *Roger Bacon*.

⁹⁷ Lindberg, *Roger Bacon*, liv.

the connection between the radiation of visible light and divine illumination.⁹⁸

Grosseteste's interest in the subject helped to form the philosophical subject into a discipline in the west. The subject in the west consisted of fragmentary sources and not a unique science at this point. It consisted mainly of extramission theories in surveys rather than any new ideas.⁹⁹ There were many theological definitions and the lack of mathematical analysis, showing that they were mainly written for gentlemen scholars and not for scientific advancement. In either the late eleventh or early twelfth century, Alhazen's *De Aspectibus* was translated into Latin, changing the optical field.¹⁰⁰ The Greeks were also translated around this time, giving the forthcoming western optical scientists a good foundation that was not possible earlier.

Roger Bacon started his work in optics under these new conditions in the west. There was the translated works of the Greeks, the translated Arabic works of Alhazen and Al-Kindi, as well as the more recent work of Grosseteste. By being the first to publish works using Alhazen's theories, he became the first proponent of Alhazen in the west.¹⁰¹ This meant that he was a proponent of the intromission theory that Alhazen presents rather than the extramission theory that had been so prevalent in the medieval west. He constantly referred to Alhazen's work, such as presenting the mathematical solutions to the intromission theory and mixed the physical, mathematical, and physiological.¹⁰² His optical treatise contained many geometrical drawings in contrast to the earlier western

⁹⁸ Lindberg, *John Pecham*, 20.

⁹⁹ Lindberg, *Roger Bacon*, xxvii. This section comes mainly from Lindberg's *Roger Bacon*, pgs. xxvi-xxxii

¹⁰⁰ *Ibid*, xxxiii.

¹⁰¹ *Ibid*, xlvii.

¹⁰² Sabra. "Ibn Al-Haytham", 197.

writings on the subject.¹⁰³ However, he did not reduce the treatise to a purely mathematical nature; after presenting the geometrical analysis, he would proceed to the physical principles and the theological implications. While Bacon's work included much of Alhazen's theories, it also included material from Euclid, Ptolemy, Al-Kindi, Aristotle, Avicenna, St. Augustine, and Grosseteste. His own personal contribution was to combine this theoretical content and to turn it into a workable theory. Much in the same way that Alhazen combined all the theories available to him, Bacon combined the eastern, western, and ancient theories, mostly using Alhazen's method of combination.¹⁰⁴

Roger Bacon's treatise, unfortunately, was not widely distributed since it was held by the church and not approved. Therefore it was not widely read until much later, until after John Pecham had published his book.¹⁰⁵ Bacon chaffed under the restrictions that the Franciscan order had placed on scientific writing. While he repeatedly attempted to gain permission to copy and distribute his work by sending his manuscript to his superiors, he was repeatedly denied permission.¹⁰⁶ Bacon's work was circulated within the church, and therefore, other Franciscans, such as John Pecham had access to it.¹⁰⁷

Roger Bacon had a great influence on John Pecham, whose treatise on optics became the most widely circulated treatise on *perspectiva* in the middle ages. The purpose of Pecham's treatise was not to introduce new ideas, but to present the old theories in an easily understandable manner.¹⁰⁸ It was meant to be an elementary

¹⁰³ Lindberg, *Roger Bacon*, xlv- lii.

¹⁰⁴ Lindberg, *Roger Bacon*, Ch. 8.

¹⁰⁵ Lindberg, "Lines of Influence" in *Studies in the History of Medieval Optics*, 66.

¹⁰⁶ *Ibid*, 68.

¹⁰⁷ *Ibid*, 71.

¹⁰⁸ Lindberg, *John Pecham*, 20.

textbook on optical science, and in this way it was similar to the survey writings on *perspectiva* in the middle ages that were written for the gentlemen scholar. What makes Pecham's book important is the degree to which it was read. Because of the vast dissemination of Pecham's book, the theories that it explained became much more widely read. Pecham had access to Bacon's treatise, but also to the translation of Alhazen's work and the ancient Greeks. While Bacon was a source of great influence on Pecham, Alhazen was Pecham's chief source.¹⁰⁹ Since Bacon promoted Alhazen's ideas, it follows that an influence from Bacon was an influence from Alhazen. Pecham accomplished the task of presenting Alhazen's theories to the west and assimilating the ideas into western culture. Pecham followed Bacon and Grosseteste in combining divine light and the study of radiation of visible light, which also followed Alhazen's beliefs of religion and science being complimentary.

In format, Pecham more closely followed Alhazen than Bacon.¹¹⁰ He begins like Alhazen did with rays originating from the object, using the idea of Al-Kindi's that rays radiate in all directions, however, he also adds that there are rays from the eye, only they do not accomplish anything.¹¹¹ Pecham contends that the rays from the eye, unlike those of extramissionists, are not sufficient to accomplish the act of seeing. He does suppose that they moderate excessively bright lights, thus protecting the eye from damage and pain.¹¹² In this manner, Pecham presented a theory that could satisfy extramissionists as well as show the new and complete intromission theory of Alhazen.

¹⁰⁹ Lindberg, *John Pecham*, 24.

¹¹⁰ *Ibid*, 26.

¹¹¹ *Ibid*, 34.

¹¹² *Ibid*, 35.

Pecham's book was quite popular through the middle ages and into the Renaissance. It was the primary textbook on *perspectiva* and was part of the curriculum in many universities.¹¹³ It served as the basis for a number of commentaries on *perspectiva* in the fourteenth century and was the most widely used text from the fourteenth to the sixteenth centuries.¹¹⁴ Even on the tomb of Pope Sixtus IV that Antonio Pollaiuolo decorated with images of the Liberal Arts, in the representation of *perspectiva*, he chose phrases from Pecham's *Perspectiva Communis*.¹¹⁵ The work of Roger Bacon, John Pecham, and their successor, Witelo, and through them, Alhazen, became the standard in the study of *perspectiva* at universities.¹¹⁶ In 1338, Pecham became part of the curriculum at the Sorbonne along with the work of Alhazen. In 1325, Alhazen was put into the curriculum at Oxford, and in 1431, both Alhazen and Witelo were part of the curriculum there. During the late fourteenth century and early fifteenth century, these were still the primary theories and working models of optics. From these books and from those who had studied in the universities is where the painters in early fifteenth-century Italy who developed perspective in paintings would have learned about the way in which light behaves and how we perceive the world around us. When Alberti writes that the center ray of vision is the chief contributor to vision in his treatise *On Painting*, he is using an idea from Alhazen that only the center ray does not get refracted on the lens and goes into the eye in order to be interpreted.¹¹⁷ While Alhazen used the visual cone in order to try to reason why the eye does not receive multiple images from every direction

¹¹³ Lindberg, *John Pecham*, 30.

¹¹⁴ Lindberg, *Roger Bacon*, xcvi, and Lindberg, *John Pecham*, 32.

¹¹⁵ Lindberg, *John Pecham*, 32.

¹¹⁶ The data of the curriculum of universities comes from Lindberg, *Roger Bacon*, xcvi.

¹¹⁷ J. V. Field, *Piero della Francesca: A Mathematician's Art*, 37.

resulting in nonsense, Alberti used the idea as an aid in the representation of images as through a window. In his own treatise on mathematics, Piero della Francesca uses the wording of John Pecham in his discussion of the eye.¹¹⁸ Ghiberti was familiar with and used the works of Bacon, Pecham, Witelo, and Alhazen in his development of perspective in his works.¹¹⁹

¹¹⁸ Field, *Piero della Francesca*, 287.

¹¹⁹ Lindberg, *Roger Bacon*, xcvi.

Chapter 2: The Singularity of Fifteenth Century Florence

Two hundred years after the publication of John Pecham's text and Roger Bacon's treatise they were still the authoritative texts on the subject. There hadn't been any great changes in the field of optics, yet in Florence, the same geometry started appearing in paintings to show the world as the eye sees it. In the beginning of the fifteenth century, the new idea of painting a scene as if viewed through a window in the same space as the viewer became a popular and powerful tool. The likes of this method hadn't been seen since ancient times, and there were no surviving examples from antiquity for artists and mathematicians to see. E.H. Gombrich speaks of the attempts to match appearances in a naturalistic manner as "far from a natural procedure, they are the great exception."¹²⁰ So why did this idea take root and flower in Florence? I suggest that the answer can be found in a complicated amalgamation of the social and artistic world of fifteenth-century Florence and necessitates trying to put oneself in the mindset of a fifteenth century Florentine. Both Baxandall and Edgerton have tried to approach this by dissecting the concerns and knowledge of the average Florentine.

Before stepping into the role of a fifteenth-century Florentine, one should look at the bigger picture of perspective in its early development. We tend to think of perspective as a set system of rules laid down for all to follow faithfully; however, the reality was very different. In the early fifteenth century there were many perspective systems being experimented with and for different reasons following its first use by

¹²⁰ Gombrich, 118.

Brunelleschi in 1413. This was well enough recognized at the time so that Leonardo da Vinci even complained of the multiple systems of perspective found within some frescos.¹²¹ The reasons for using perspective systems that varied from the strict geometrical model were varied and had to do with painting being concerned with the composition and purpose of the final product over the use of mathematics in achieving that goal. It was used as a tool in a pragmatic manner in order to achieve the goal of the painting. As Gombrich attests, effective portrayal can only be achieved through “a constant willingness to correct and revise.”¹²² The *Holy Trinity* fresco by Masaccio (Fig. 6) is often considered the earliest example of single point perspective still extant. Its geometrical features have been well studied and consequently variances, or mathematical errors, have been found, yet these are not truly errors as it is thought that they were purposefully made in the name of visual effect, and they show how the artist’s use of judgment and creativity create a painting that looks mathematically correct to the naked eye. The coffers and ribs of the painted vault that appear to show the mathematical regularity are themselves not regular. The geometrical analysis of this painting by Martin Kemp supposes that the irregularities were not mistakes, but to improve subtle compositional issues that would have arisen had the changes not been made.¹²³ As J.V. Field says about the *Trinity* fresco, “Masaccio and Donatello were interested in a form of truth that was essentially visual rather than mathematical, though mathematics might be

¹²¹ J. V. Field, *The Invention of Infinity: Mathematics and Art in the Renaissance* (Oxford: Oxford University Press, 1997), 114-116.

¹²² Gombrich, 171.

¹²³ Kemp, *The Science of Art*, 17-18.

used in attaining it.”¹²⁴ Mathematics was a tool to accomplish this visual truth that perspective paintings were aiming toward, but perspective was not a slave to the geometrical nature of its use. Samuel Edgerton says that of all the insights that Alberti passed on to generations of painters, his perspective instructions were “the one most consistently modified or ignored altogether.”¹²⁵ He goes on to say that even though Alberti laid down specific rules for the use of perspective, he understood that “visual truth in a picture does not depend upon the correct perspective per se, but upon the basic moral and philosophical priorities of the civilization itself.”¹²⁶ This is a grander sweeping statement on the nature of visual truth than that of Field’s, however, they are essentially saying the same thing, that mathematics was merely the tool of the artist, including Alberti, and it was never meant to be used as a restraint to individual creativity. While we may look back at a single instance of the creation of painting using perspective, the panels of Brunelleschi, the people of the time saw perspective as several compatible systems being invented over time.¹²⁷ Even Vasari speaks of multiple perspectives and the origin of these systems was a loosely unified tradition rather than the idea of a single man.¹²⁸ If we follow Gombrich and assume that all seeing is a type of interpreting, then there is no unbiased eye, and therefore every artist is able to interpret the effects of perspective individually.¹²⁹

¹²⁴ Field, *The Invention of Infinity*, 61.

¹²⁵ Edgerton, *The Renaissance Rediscovery of Linear Perspective*, 55.

¹²⁶ *Ibid*, 59.

¹²⁷ James Elkins, *The Poetics of Perspective* (Ithaca: Cornell University Press, 1994), 8.

¹²⁸ Elkins, 46.

¹²⁹ Gombrich, 298. Gombrich discusses all seeing as interpreting. Perception begins with recognition of familiar forms in unfamiliar objects.

Why would the artist choose to use multiple perspectives within a single painting or purposefully overrule mathematical calculations after so carefully creating the perspective system? As in the previous example, changes in the geometrical structure are often made so that the image looks more correct to the viewer or to make the composition more pleasing to the eye. An example of this can be seen in the representation of the sphere. In correct perspective, a sphere represented anywhere except in the center of the painting would be represented as an ellipse, not a circle, however to the viewer, it would still look like an ellipse and not be perceived as a sphere.¹³⁰ This can readily be witnessed in Raphael's *The School of Athens* in a detail of the spheres that represent the heavens (fig. 2, 3). The spheres are painted as circles and are perceived by us, the viewers, as spheres, however they do not follow the geometrical rules of central projection. This is partly a result of our perception causing the 'robustness of perspective.' An image painted in perspective is designed so that there is a single point in which the viewer should stand in order to see the painting in correct perspective. However, in practice we find that a spectator can stand in almost any location and still perceive the painting as it is meant to be seen. This ability is thought to be based on the perception that it is an image and our brains making the appropriate adjustment.¹³¹ This is called the robustness of perspective, and it was not predicted by mathematical models. It is thought that the robust nature of perspective came as a surprise to Brunelleschi after he made his first demonstration piece and that is why he did not use a peep-show

¹³⁰ Kubovy, 112.

¹³¹ Ibid, 55.

technique for his second demonstration piece, having found that it was unnecessary.¹³² However, this ability to perceive the plane of the image and make adjustments in perception for our angle of viewing is also thought to be why we only perceive circles as representing spheres rather than the more mathematically correct ellipse. These changes to perspective systems, like the use of circles instead of ellipses, are often to correct for the problems of perception, making it a more complicated procedure than simply creating the geometry for a perspective system and drawing on the lines. While the people of the time may have been fascinated by geometry, they did not apply it rigidly, and artists often made geometry subordinate to the end composition and the effects of perception.¹³³

Another possible reason for changing the perspective system within a painting is compositional. Martin Kemp argues that Uccello used perspective as a narrative tool in his paintings.¹³⁴ In particular he looks at the damaged fresco of the *Nativity* that includes an underdrawing showing the geometry of the perspective system. The fresco includes two vanishing points, creating two different viewpoints that collide within the painting. (Fig.7) Kemp argues that this shows that Uccello intended the fresco to be viewed from two different positions, effectively changing the perspective for the viewer as he moved from one position to the next. This creates a narrative effect by forcing the aspect of time to be a part of the viewing of the painting as one shifts from the first perspective system to the second perspective system.¹³⁵ The aspect of time becomes a part of the interpretation of the composition. The perspective changed from focus on the nativity to

¹³² Kubovy, 128.

¹³³ Ibid, 171.

¹³⁴ Kemp, 37-38.

¹³⁵ The fresco is no longer in its original location and so the placement of the fresco cannot be taken into consideration.

focus on the shepherds at the side, thereby creating two narratives in a single image. Kemp also looks at Donatello's use of perspective in a narrative function. Donatello continually experimented with perspective for its communicative effects.¹³⁶ In his roundel *Assumption of St. John the Evangelist*, Donatello does not use the single point system that he has shown in previous works, but instead uses rhythmic forms and intuition to create a scene that can be read and perceived as existing in space. (Fig. 8) The composition, however, does not actually have a vanishing point consistent within the painting.

Perspective itself could also be used in a metaphorical way within the composition. The vanishing point could be a metaphor, symbolizing a part of the painting or leading us to a compositionally important part of the narrative.¹³⁷ Kubovy argues that artists even deliberately created a discrepancy between the actual point of view of the spectator and the point of view from which the painting was meant to be felt to be viewed.¹³⁸ In this way, the artist was not restricted by the positioning of the painting, and the artist could place the hypothetical viewer in a position more advantageous to the composition. Since the eye is naturally brought toward the vanishing point, it lends itself to being the point for the most important figure, whether spiritually important or in a narrative sense. This type of use of perspective as an object could be a function of the idea that in the Renaissance, the concept of space didn't exist as it does for us now, and they did not even have a term for the concept.¹³⁹ According to Elkins,

¹³⁶ Kemp, 40-41.

¹³⁷ Elkins discusses the use of perspective itself as a metaphor in Elkins, *Poetics of Perspective*, 16-17.

¹³⁸ Kubovy, 16.

¹³⁹ Elkins, 14.

the artists of the Renaissance were object-oriented rather than space-oriented and designed the compositions around particular objects rather than around the space. The notion of space was defined as extension from Aristotle and was not considered to be a separate entity unto itself.¹⁴⁰ Part of this was the lack of the idea of infinity in space at this time. The introduction of the vanishing point into painting created the beginnings of the sense of infinity. This was not simply a new technique, but a new intellectual concept.

These new ideas were cutting-edge advanced philosophical and mathematical arguments. The traditional education of a painter, however, did not include higher education. Medieval painters were taught through apprenticeship and often did not have an education that would allow for development of new intellectual concepts. The painters who are associated with the new methods of the early Renaissance, however, had generally been educated in one way or another, through abacus schools that taught basic vernacular literary skills and mathematics for merchants.¹⁴¹ The beginning of the social elevation of the painter, leading to the acknowledgement of the artist as an individual with a name and intellectual insight is seen at this same time that education begins to be important for the creation of works of art. Most artists could not read Latin when they began their apprenticeships. Cennino Cennini, in his handbook at the end of the fourteenth century, did not specify any knowledge of mathematics or letters to be learned during apprenticeship, only certain qualities of character, such as enthusiasm and

¹⁴⁰ Field, *Piero della Francesca*, 95.

¹⁴¹ Francis Ames-Lewis, *The Intellectual Life of the Early Renaissance Artist*, (New Haven: Yale University Press, 2000), 19.

exaltation were required to study painting.¹⁴² During the fifteenth century, however, there was an increase in the requirement of mathematics for the purpose of perspective and proportion that is seen in various painter's writings on the qualities necessary for a painter's education, such as Ghiberti and Leonardo, as well as a change in the intellectual atmosphere within the workshop toward a more demanding program of study.¹⁴³ Alberti was a prime proponent of the higher status of the artist. From a wealthy family, he became an artist through humanist interest and study rather than the traditional apprentice system. He insisted that a great painter should be "a good man and learned in liberal arts," implying the social skills and education of the higher social class rather than of the merchant and artisan classes.¹⁴⁴ Alberti's book itself was a way for the educated man to learn the art of painting without depending solely on the medieval tradition of the workshop system.

These new systems of perspective started in Florence instead of other areas of Italy or the rest of Europe. As Gombrich says in reference to Dürer's woodcut of the rhinoceros, "The familiar will always remain the likely starting point for the rendering of the unfamiliar."¹⁴⁵ Taking the familiar as a starting point would suggest that there was something in the geometrization of space that was collectively familiar to the people of Florence, for the artists were painting for individual patrons and also the public. The details of a painting were worked out beforehand with the patrons, therefore we cannot assume that the growth and popularity of perspective paintings was simply due to the

¹⁴² Cennino Cennini, *The Craftsman's Handbook*, trans. Daniel V. Thompson Jr. (New York: Dover Publications, Inc., 1954), 3.

¹⁴³ Ames-Lewis, 30-31, 36.

¹⁴⁴ Alberti, 89.

¹⁴⁵ Gombrich, 82.

artistic community, as patrons would also have been largely involved.¹⁴⁶ So what was familiar about the system to Florentines that would have made perspective paintings appealing?

Edgerton attributes the creation of pictorial perspective in Florence to a number of factors, some unique to Florence, and others just a confluence of ideas that came from many sources. The city-state was small enough to encourage communication between different social groups and for unorthodox opinions to be given tolerance.¹⁴⁷ This interaction would have been favored by the open Florentine society that encouraged ingenuity and unconventionality. The republican nature of the city also encouraged the idea that a citizen could make his fortune through flexibility, intelligence, moral integrity, and work. Leonard Goldstein brings the idea of a cultural reason even further by arguing that it was Florence's capitalism that allowed linear perspective to develop there.¹⁴⁸ Marvin Trachtenberg also discusses the effect that the aggressive form of capitalism in early Renaissance Florence had on the social stratification and subsistence level of the lower classes of Florentines.¹⁴⁹ However, an important social change that had been happening in Florence leading up to the fifteenth century was the mathematization and geometrization of all walks of life. Mathematics was becoming a part of not only the university, specialists, and professionals, but also of the broader population. It became a

¹⁴⁶ Baxandall, 6-8. Baxandall looks at the specifics of contracts drawn up between patron and artist and finds that all the minutiae were included, leaving very little to change by the artist.

¹⁴⁷ Edgerton, *Renaissance Rediscovery*, 32-33.

¹⁴⁸ Goldstein argues that linear perspective is simply one representation of the emerging state of modern capitalism in *The Social and Cultural Roots of Linear Perspective* (Minneapolis: MEP Publications, 1998).

¹⁴⁹ Marvin Trachtenberg, *Dominion of the Eye: Urbanism, Art, and Power in Early Modern Florence* (Cambridge: Cambridge University Press, 1997), 3-5.

“common language” between the people of different social standings.¹⁵⁰ Merchants, bankers, artisans, and intellectuals all needed to be able to understand basic mathematics in order to conduct daily business and it was increasingly taught in the *abacchi* schools for the merchant class. A case in point is Piero della Francesca who wrote a mathematics textbook for an abacus school before he became a painter.¹⁵¹ Florentines were using double entry accounting, creating maritime charts, and creating quantifiable amounts from the previously unquantifiable.¹⁵² Trachtenberg investigates the rise of the city piazza in fourteenth-century Florence with respect to the growth of urban planning before the fifteenth century. He finds that Florentines preferred straight and wide streets and tended to straighten their older winding streets, creating a system of more regularized streets than any other comparably sized medieval city.¹⁵³ The tendency to regularize the space around them and the habit of reducing all things to definable quanta might have formed a prerequisite for quantizing the “empty space” around visible objects.

Edgerton links this increased geometric description of space with the advancement of cartography.¹⁵⁴ He establishes that Ptolemy’s *Geographia* made its first appearance in Florence around 1400 through a group organized for the purpose of studying Greek.¹⁵⁵ This group consisted of prominent Florentines who imported Greek books primarily to translate them, thus leading to the translation and study of Greek texts

¹⁵⁰ Edgerton, *Renaissance Rediscovery*, 36.

¹⁵¹ Field, *Piero della Francesca*, Chapter 1 discusses Piero’s abacus book.

¹⁵² Alfred W. Crosby, *The Measure of Reality: Quantification and Western Society 1250-1600* (Cambridge: Cambridge University Press, 1997), 19. Crosby goes through a number of trends in the quantization of life in the centuries immediately preceding the creation of linear perspective.

¹⁵³ Trachtenberg, 153.

¹⁵⁴ Edgerton devotes two chapters in *The Renaissance Rediscovery of Linear Perspective* to the development of cartography and its similarities to the use of single point perspective.

¹⁵⁵ Edgerton, *Renaissance Rediscovery*, 93.

that had not previously been in the west. Ptolemy's book laid out a system of mapping the world. As a center for trade, and also exploration, Florence had an industry of mapping with 'portolan' charts, which were grid systems that were more accurate for determining direction and bearing, yet less accurate for determining distance.¹⁵⁶ Edgerton parallels the similarity of the old mapping system of having direction and geographical features without distance to the style of Giotto, showing the objects and the direction in which they are located, but not the distance between the objects that would create a sense of space. Since Florence already had a mapping tradition and an interest in bettering its trade abilities, a better mapping system was eagerly embraced. None of this had anything to do with art at this point; however, when looking at the third cartographic method that Ptolemy describes, it is indeed this same geometry used for single point perspective.¹⁵⁷ However, the question remains, did cartography have an immediate effect on the artists looking for a method of spatial representation, or did they simply both come from the same source, the study of optics? The third cartographic method, after all, is designed so that the globe is positioned in front of the viewer and the latitudinal lines are parallel to each other. (Fig. 9) This method becomes an optics geometric problem, laying a curved object onto a flat plane with minimal distortions, in a similar manner to translating a three dimensional round object, like a person or round building, onto the flat plane of the picture plane. The fact that this method was not the Ptolemaic method actually used in navigational practice makes the idea that this was a direct influence very unlikely and instead it is simply a visual system that was established from the same

¹⁵⁶ Edgerton, *The Renaissance Rediscovery of Linear Perspective*, 97.

¹⁵⁷ *Ibid*, 104.

roots.¹⁵⁸ However, the very idea of cartography is the ordering of the known world, the geometrization of empty space. The increase of cartographic activity at the same time and place as the advent of single point perspective seems to be pertinent to the general mindset of the learned population. The use of the grid to order space was already applied for town layout and farmland, as well as the system of accounting that has already been mentioned as a symptom of the general need to order and quantize life in Florence. The Ptolemaic method of cartography with its use of a grid and specific unchanging distances for all geographic features is another occurrence of this predilection to order space as well as harboring a parallel development of the art trends at the same time.¹⁵⁹ Florentines also started using grids to transfer smaller designs to walls for frescos as well as grid framework for the perspective of body parts, a curved surface.¹⁶⁰ While cartography shares many things in common with the development of perspective, it seems less to be a path to perspective and more of a field on a parallel line, developing in concert with perspective based on similar geometric problems and the new desires of the Florentine populace to organize space in this new way.

The Florentine preoccupation with optics was not limited to the geometric organization of the world. There was also a fascination with mirrors and the optics of reflection and refraction. Alberti mentions mirrors and windows several times in *On Painting* and the medieval optical treatises, both the earlier Arabic and the 13th century western treatises, had sections devoted to reflection and refraction laws. The medieval

¹⁵⁸ Edgerton mentions at the end of his analysis that the third Ptolemaic method seemed to be too complicated in practice for most cartographers and that it was never actually used, 105.

¹⁵⁹ Edgerton, *Renaissance Rediscovery*, 114.

¹⁶⁰ *Ibid*, 115.

treatises, however, treated reflection and refraction as different subjects than the study of geometric projection of how the eye sees. The Renaissance humanists studied mirrors in concert with single point perspective. The laws of angles in reflection and refraction were well known and studied, contributing to the geometrical nature of the behavior of light. The fact that the angle of incidence is equal to the angle of reflection was a demonstration of light's predictable and knowable behavior. The interest in mirrors for artists was not entirely new in the fifteenth century, as it is reported that Giotto was to have painted with the aid of mirrors.¹⁶¹ Perhaps this can be seen in Giotto's illusionistic attempts at trompe l'oeil chapels in the Arena Chapel (fig10). They are meant to be seen at an oblique angle and to create an illusion of space, maybe the earliest example of an intention of illusionistic deception.¹⁶² While the use of mirrors was not new, the appearance of good quality flat mirrors in Europe was still relatively new.¹⁶³ While introduced in the thirteenth century, it was not a common household tool, but an object of scientific interest. Mirrors captivated poets, painters, mathematicians and those studying optics. Alberti, as a humanist scholar, referred to mirrors several times in *On Painting*, and Brunelleschi used a mirror in his first perspective demonstration to increase the illusionistic qualities of the demonstration. In Book Two of *On Painting*, Alberti waxes poetic on the mirror saying, "A good judge for you to know is the mirror. I do not know why painted things have so much grace in the mirror. It is marvelous how every weakness in a painting is so manifestly deformed in the mirror. Therefore things taken

¹⁶¹ Edgerton, *Renaissance Rediscovery*, 134.

¹⁶² Thomas Puttfarcken, *The Discovery of Pictorial Composition: Theories of Visual Order in Painting, 1400-1800* (New Haven: Yale University Press), 81.

¹⁶³ Edgerton, *Renaissance Rediscovery*, 134.

from nature are corrected with a mirror.”¹⁶⁴ In this quote, Alberti describes the mirror as the ultimate judge, showing the weaknesses in that which is reflected. While describing it as a tool for the artist to use, he also gives it qualities of improvement, giving grace to painted things. The mirror can ‘see better’ than the painter, making it clear to the painter where improvement must be made and where the painting is satisfactory. Alberti endows the mirror with abilities beyond nature, revealing its place as a corrective object, reflecting nature in its ideal form. He also describes the eye as a “living mirror”, thereby giving grace to the eye as well.¹⁶⁵ Alberti, as a humanistic scholar, was reading medieval optical treatises, which were themselves full of mirror references. Avicenna in his advocacy of intromission argued that the eye was the receiver of information because it acted like a mirror, receiving and reflecting the image to the soul, rather than transmitting.¹⁶⁶ Edgerton points out the similarity of this description by Avicenna and Alberti’s description of the “living mirror” of the eye, yet repurposing it for the use of painting.¹⁶⁷

Furthermore, Alberti mentions that Narcissus is the mythological inventor of painting, whose story has him lost in the reflection of himself in water. He then says, “What else can you call painting but a similar embracing with art of what is presented on the surface of the water in the fountain?”¹⁶⁸ Narcissus’ love for his own reflection becomes a metaphor for the love of art. Alberti uses not only the mirror, but the window as a tool in painting technique. While the mirror, according to Alberti, can change or

¹⁶⁴ Alberti, 83

¹⁶⁵ Ibid, 47.

¹⁶⁶ Edgerton, *Renaissance Rediscovery*, 72-73.

¹⁶⁷ Ibid, 72.

¹⁶⁸ Alberti, 64.

improve the image, the window is meant to represent the world as we see it. Alberti describes a veil, finely woven with large threads, spread across the plane of vision so that the artist can more readily divide the world he sees into a fixed and divided format. He encourages the artist to use the veil as a tool to study the world through so that it can be represented “in good likeness.”¹⁶⁹ The use of the veil and the window were tools to represent the likeness of the world as it is seen, while the mirror was a greater object and tool showing the idealistic image of the world, an extension of perfection that is created by God.

Brunelleschi was probably taught optics by Toscanelli, who is the likely author of a treatise, *Della prospettiva*, in which he shows a keen interest in mirrors.¹⁷⁰ In Brunelleschi’s first demonstration of perspective, he used a peep-show system set up with a mirror. This use of the mirror increased the illusionism of the painting by taking away the eye’s ability to recognize a plane on which the image lies.¹⁷¹ It also creates the need to have the viewing of the image strictly limited, which is why there is a fixed hole to limit the point at which the viewer stands. The use of the mirror makes the image more than just a painting as the world looks through the window or the veil, but makes it more perfect than that, creating a new reality for the viewer to enter. The increased ability of illusionism through the use of the mirror, as opposed to the second demonstration that did not use the peep-show technique, only increases the perceived ability of the mirror to show something more than reality.

¹⁶⁹ Alberti, 69. The veil can also refer to the authentic image of Christ in the veil of Veronica which was a central story in Christianity.

¹⁷⁰ Edgerton, *Renaissance Rediscovery*, 134.

¹⁷¹ Kubovy, 47.

With the creation of single point perspective comes the beginning of the vanishing point and orthogonal lines. These are indicative of the notion of infinity, which was not a notion present in medieval mathematics. Built into the mathematics of the day was an avoidance of infinity. While today the definition of a line or a plane is infinite and only a line segment has an end and beginning, in the early fifteenth century, a line had a beginning and end and was not considered to be infinite.¹⁷² This lends itself directly to the problem of orthogonals. While we can understand how all non-parallel lines will eventually intersect when on the same plane, in the fifteenth century this was not an obvious idea. The idea of infinity and its existence was directly related to religion and God, and philosophers had been struggling with the idea of infinity since ancient times. Aristotle found infinity to be inconceivable and impossible, which flew in the face of Christian thought that nothing was impossible or out of reach of God.¹⁷³ Since God was omnipotent and infinite, he could not be constrained by anything and the heavens were therefore infinite. Yet an Aristotelian view of the universe for astronomy and mathematics was used. This led to inconsistencies and claims of heresy as the Aristotelian view of the universe was finite and therefore limited God. If God and heaven were the place of infinity, humans and the earthly realm were finite, being lesser than heaven. One might conclude that the creation of orthogonals as infinitely long lines into space brought the infinity of heaven down into the earthly realm. Panofsky relates the realization of infinity in paintings with a philosophical idea of an infinite experiential world, thus making the meaning of the use of perspective in a painting more than relating

¹⁷² Field, *Piero della Francesca*, 96.

¹⁷³ Panofsky, *Perspective as Symbolic Form*, 65.

the world as through an Albertian window.¹⁷⁴ This relates back to the use of perspective as a compositional tool. If the viewer in the fifteenth century recognized the importance of the representation of infinity, the vanishing point becomes a symbolic position for any object. This brings added meaning to objects or figures placed in the central position, which is both a natural focus for the eye as well as the location of the representation of infinity, a divine attribute.

The metaphorical aspects used in art in the fifteenth century were meant to be seen and appreciated by the viewer. The fifteenth-century Florentine was expected to be able to understand the symbolic potential and skill involved in painting a picture in perspective. More than simply understanding the skill, the viewer was meant to comprehend the placement of the art as well as its place in his own society. John Shearman gives a compelling example of the public responses to the statues placed in the Piazza della Signoria and their specific placement in the square.¹⁷⁵ In this analysis, Shearman examines the care that the artist put into the relationship of the statue in relation to the surroundings, purposefully having the subject look at one or another statue as well as the intricate interpretations of the public based on these small nuances. He examines poems written by citizens after the statues are unveiled, revealing the astute nature of viewing in fifteenth-century Florence. In numerous examples throughout the book, Shearman shows the manner in which Florentines viewed their art, painting and architecture as well as sculpture, as a larger part of their society and city, and the

¹⁷⁴ Panofsky, *Perspective as Symbolic Form*, 65-66.

¹⁷⁵ John Shearman, *Only Connect: Art and Spectator in the Italian Renaissance* (Princeton: Princeton University Press, 1992), 55-58.

involved role that they played both in the act of viewing as well as on the artist's intentions while creating the work of art. (A good review was just as important to the artist then as it is to artists in the 21st century.) An example is the already discussed *Nativity* by Uccello that uses two different viewpoints for the two scenes in the same panel. (Fig.7) These two different viewpoints encourage the viewer to change position and create a narrative.¹⁷⁶ This shows Uccello's attention to the spectator's role in the painting. He created the composition in such a way as to exert subtle control over the way in which it could be viewed and the behavior of the viewer. The spectator's experience in viewing the painting was a major concern of the artist in composing the image in the fifteenth century. At the end of *On Painting*, Alberti discusses the importance of listening to critique of your own paintings.¹⁷⁷ He says to "hear everyone" and "the work of the painter attempts to be pleasing to the multitude." This indicates that art criticism was an expected part of creating art in the fifteenth century, and therefore the artist would always be affected by the anticipated response to his work.

The artist's concern with the spectator is not limited to the reaction and criticism of the painting. The composition was meant to be understood as a skillful and valuable representation of the subject. Baxandall examined contracts for paintings in the fifteenth century and found that while demands for luxurious materials decreased, requests for skill increased in the early fifteenth century.¹⁷⁸ This is interpreted as a result of a larger societal shift from ostentation to a more subtle expression of wealth.¹⁷⁹ The contracts for

¹⁷⁶ Kemp, 38.

¹⁷⁷ Alberti, 97.

¹⁷⁸ Baxandall, 14.

¹⁷⁹ Ibid, 15.

paintings showed less gold and expensive blue pigment to be used in the painting since those would be seen as ostentatious and out of fashion. Baxandall shows that the contracts would express desire for different kinds of skill to be established in the painting. He would find contracts that specified that the master and not the apprentices would do the work, sometimes only for the most important figures.¹⁸⁰ The master painter doing the work implies a greater skill level. Also a contract may specify that the background was to be a landscape or architectural feature, requiring greater skill than the previously used gold backgrounds.¹⁸¹ Displayed skill in painting was known to be expensive and so these paintings continued to be a sign of wealth and social status even without the expensive gold and blue, however it was done in a more socially acceptable and fashionable manner than with expensive materials.

The spectator would have been an astute viewer, recognizing not only the skill and expense involved in the production, but also the themes and stories in the painting. Shearman examines the detailed responses to the specific placement of the statues in the Piazza della Signoria and the contemporary interpretations of the subtle nuances of the sculptures written in verse, showing that the spectator was expected to be knowledgeable in the modern arts. They were made for an increasingly humanistic society that valued education in classical studies along with a variety of arts, having elevated the arts from crafts to scholarly pursuits. Michael Kubovy says in regard to perspective that esoteric references were common in the Renaissance and were made because the audience

¹⁸⁰ Baxandall, 21-23.

¹⁸¹ Ibid, 18.

appreciated subtle references to perspective.¹⁸² In addition, Kubovy quotes Welliver, who says, “no people had ever been so given to communication by parable and riddle as the Florentines.”¹⁸³ If this is so, then it implies a level of artistic sophistication to be present in the average spectator. Florence would have provided a well-informed group of spectators. As discussed earlier, the basic merchant education at an abacus school would have included a significant amount of arithmetic as well as geometry (useful for finding volumes of containers among other things), which were the same basis that the painters used in order to draw in perspective.¹⁸⁴ It is for this secondary level of schooling that Piero della Francesca wrote his mathematics textbook. Florence would have had a great number of citizens that were educated with this mathematical background, and they would have been able to understand the mathematics as well as the poetics involved in the perspective of a painting or the proportions of a sculpture. As Edgerton says, there was an “an interest among Florentines not only in the moral but the practical application of arithmetic and geometry in all walks of the city’s life. Mathematics was becoming a kind of social lingua franca, linking upper and lower classes, creating a bond, among humanist intellectuals, bankers, artisans, and shopkeepers.”¹⁸⁵

In addition to the growing appearance of mathematics in paintings was also the movement toward what might be called greater naturalism. For Alberti, nature was homogenous and knowable through observation.¹⁸⁶ Before the Renaissance, naturalism had to take a secondary position behind the narrative function as Field shows in Giotto’s

¹⁸² Kubovy, 16.

¹⁸³ Kubovy, 16.

¹⁸⁴ Baxandall, 86.

¹⁸⁵ Edgerton, 36.

¹⁸⁶ Spencer, 18.

*Joachim and Anna meet at the Golden Gate.*¹⁸⁷ (Fig. 11) In this painting, Giotto has made the gate much smaller in proportion to the figures than it would have been in life in order to make the scene readable as a story. If the gate were in proportion to the figures, we would not be able to see it clearly and would not have a clear idea of where the scene takes place. We see the primacy of narration over naturalism in many of Giotto's paintings. While he clearly strived for greater naturalism as compared to painters before him and was also very adept as seen in the illusionistic chapel paintings (fig. 10), he continued to sacrifice naturalistic proportions for narrative conventions. However, naturalism is what we call the view of the world as through the window, another perspective may be an experiential view of the natural world, not the world as it is seen from a single view point, but how it is experienced through space and time. As Edgerton notes, the tendency to depict the world as it is experienced rather than how it is seen seems to be more innate for the artist as it is more prevalent throughout history.¹⁸⁸ Giotto's propensity to become more naturalistic than his predecessors shows a change in the cultural view toward the experience of viewing a painting and the idea of what naturalism is. His paintings show an increase in the 'view through a window' approach to the world and his maintaining certain aspects of non-naturalistic proportion would not have been viewed as an inability to illusionistically portray the world, but as a better representation of the entire narrative being portrayed.

While naturalism was increasingly made possible through the application of mathematics and proportion in painting, there had to be the desire to create a naturalistic

¹⁸⁷ Field, *The Invention of Infinity*, 11.

¹⁸⁸ Edgerton, *Renaissance Rediscovery*, 14.

image in the first place. This desire can be found in both the copying of ancient examples and in the medieval tradition of the path to God being found in nature. Written accounts of ancient paintings extolled the realistic qualities of the lost images. The humanist tendency to mimic the ancient ways would have led towards a similar increase in naturalism in painting. The goal in painting became to represent the world as we see it in nature. When giving instruction on how to draw the figure, Alberti recommends to first place the bones and muscles, then to complete the nude and finally cover it with drapery.¹⁸⁹ Through doing this, the painter will be able to depict the figure in the most natural way. Alberti then says that serious painter “will put as much study and work into remembering what they take from nature as they do in discovering it.”¹⁹⁰ Through this he is insuring that the painter makes a good study of nature and then brings that knowledge into the studio and does not leave it outside. Observation of nature is then combined with copying and invention to create what he called ‘good art.’

The study of nature in the middle ages was seen as a study of the nature of God. St. Francis of Assisi taught that since God was the creator and ruler of the natural world, the study of the natural world would lead the soul to God.¹⁹¹ Roger Bacon, a Franciscan himself, in his *Opus Majus*, a treatise written for the pope, expressed the desire that the geometric laws of optics would show how God’s grace spread throughout the universe.¹⁹² Bacon did his optical work while in the service of the church and submitted most of his work to the pope. While chaffing under the restrictions imposed on the Franciscans in

¹⁸⁹ Alberti, 73.

¹⁹⁰ Ibid, 73.

¹⁹¹ Field, *The Invention of Infinity*, 9.

¹⁹² Edgerton, *Renaissance Rediscovery*, 16.

matters of research and publication, he was trying to show a spiritual reason for the study of optics and geometry in order to loosen up his restrictions. As Otto von Simson says in relation to Gothic cathedral planners, mathematics was “considered the link between God and the world, the magical tool that would unlock the secrets of both.”¹⁹³ Bacon wanted to be able to publish his work and continue his research, believing it to be the work of God. Bacon also wished that painters would become proficient in geometry in order to help get across the spiritual message of paintings.¹⁹⁴ He felt that this would allow them to more appropriately illustrate the works of God. While not suggesting anything that could be construed as a perspectival system at this early time, he bemoans the lack of any fixed geometrical order in paintings.¹⁹⁵ It seems as if Bacon desired to have religious paintings presented as if they were happening directly in front of him. Edgerton refers to *Idiota* written around 1450 by Nicolaus Cusanus, a theologian, humanist, and friend of Toscanelli, in which he refers to God’s ideal form as inherent in geometry and His infinite goodness expressed by the straight line.¹⁹⁶ This sort of reasoning characteristic of humanism was a way of continuing the studies of the pagan ancients and combining such scholarship with Christian ideals. Baxandall discusses the moralization of mathematics in the fifteenth century in Italy as well.¹⁹⁷ In this way, scholars would feel free to study and laud the philosophies and sciences of the ancients while avoiding their association with paganism.

¹⁹³ Otto von Simson, *The Gothic Cathedral* (New York: Princeton University Press, 1988), 27.

¹⁹⁴ Edgerton, *Renaissance Rediscovery*, 16-17.

¹⁹⁵ *Ibid*, 18.

¹⁹⁶ *Ibid*, 37.

¹⁹⁷ Baxandall discusses the moralization of mathematics and perspective in *Painting and Experience in Fifteenth Century Italy*, 103-108, ending with the use of perspective as a religious visual metaphor being rather speculative.

Throughout the middle ages, God was considered to be the inherent in light and illumination. Light mysticism based on Pseudo-Dionysius's writings from the sixth century taught that light, "the purest substance, is the most important trace that leads us back to God."¹⁹⁸ Through this, the more that an object contains light, the more that it resembles God. The study of light was therefore the study of the nature of God. This was the basis of the reasoning in the development of Gothic Cathedral style with illumination becoming a major part of the design.¹⁹⁹ In the fifteenth century, it manifested in the use of optics in painting. The use of perspective could be seen as a representation of the behavior of light in the painting, and therefore the presence of God in the scene.

The religious imperative dictating the moral rationalization of the geometrization of space in a painting relates to the general requirement of having religious morality, or at least moralizing content, portrayed in the painting. In his treatise, Alberti said that paintings contain a divine force that make the dead seem alive, and that paintings are most useful to piety and keep our souls full of religion.²⁰⁰ It seems that the religious rationalization of the mathematical perspective system was part of the larger moralization of the entire painting and profession of painting. Baxandall outlines the guidelines to the functional requirements of a religious painting, since they were used as "vivid stimuli to meditation on the bible and the lives of the saints."²⁰¹ He says that the painting was required to clearly represent the story as well as to use full emotional resources and be

¹⁹⁸ William R. Cook and Ronald B. Herzman, *The Medieval World View: An Introduction* (Oxford: Oxford University Press, 2004), 127.

¹⁹⁹ Von Simson, 50-55.

²⁰⁰ Alberti, 63.

²⁰¹ Baxandall, 41.

memorable in order to be a satisfactory aid to devotion. Edgerton makes the connection between the new perspective paintings and the religious use of the construction of perspective by noting that all the new perspectival constructions were of religious subjects.²⁰² He goes on to say that this shows a harmonious relationship between mathematics and God's will, and between moral order and human perfection.

Another quality of Florence that encouraged the innovation potential in the arts was the relative openness of the society. The city itself encouraged individualism and humanist thought.²⁰³ Traditionally, the personality of the Renaissance Florentine is described as someone who strives for individuality and being exceptional, and thrives on inventing, and initiating.²⁰⁴ And while this idea of a society of exceptional inventors is a romantic notion, there is a grain of truth in it. Florence was more tolerant of the advancement of individuals through ingenuity and skill than other cities with more established oligarchic systems. This does not preclude the practical rule of Florence through wealthy families. After the plague swept through Italy in the fourteenth century, there were fewer families of power to contend with, allowing those surviving families to become more important. The political nature of Florence was one where the powerful families sought to have their will done, but not to overtly declare rulership as Florence was still a republic. Families, notably the Medici, showed their power and wealth by patronizing artists who would portray the family in an allegorical manner. Through the patronage system, artists would be favored by a family and would become an important

²⁰² Edgerton, *Renaissance Rediscovery*, 24,

²⁰³ *Ibid*, 33.

²⁰⁴ J. Lucas-Dubreton, *Daily Life in Florence*, trans. A. Lytton Sells (New York: The Macmillan Company, 1961), 35-36.

member of society through that association. The system of patronage was an elaborate web of obligation and loyalty.²⁰⁵ In this way, those from the artisan class could rise to greater social prominence, but not on their own as the romantic notion of Florence suggests, but with the help of patronage from prominent families that would help their “friends” rise in fortune. The system rewarded those that showed their talents and skills and therefore encouraged the atmosphere of openness between different elements of society.

Part of this open society was the cosmopolitan flavor of this trade city. Being an important trade city, Florence would have goods and people from many different parts of the world going through it every day. The dissemination of optics was a fruit of this trade since the people had access to books and ideas from other cultures to enhance their own. By being exposed to the products of different parts of Europe and the Mediterranean, the people of Florence were more open to the ideas of different cultures. Florence became a place ripe for the development of a Brunelleschi who had already shown his ingenuity through his engineering inventions.

How did Brunelleschi come about developing a painting system when he himself was primarily an architect? Reports on his demonstration panels do not record a precise reason why he did it.²⁰⁶ One major influence, however, is said by Vasari, to be Toscanelli, and this is repeated by many scholars since, including both Edgerton and Heinrich Klotz.²⁰⁷ Toscanelli is supposed to have been the one who taught optics to

²⁰⁵ Ronald Weissman, “Taking Patronage Seriously,” in *Patronage, Art, and Society in Renaissance Italy*, ed. F.W. Kent and Patricia Simons (Oxford: Clarendon Press, 1987), 43-44.

²⁰⁶ Edgerton, *Renaissance Rediscovery*, 125-126.

²⁰⁷ Edgerton, *Renaissance Rediscovery*, 62, Klotz, 150.

Brunelleschi so that he was able to create the system of single point perspective in the first place. Brunelleschi himself had not studied optics at a university, and so the knowledge of optics was likely passed onto him through Toscanelli who had read Alhazen and Bacon.²⁰⁸ Edgerton also emphasizes Toscanelli's interest in Ptolemaic cartographic methods.²⁰⁹ Toscanelli was most known for his work in geography and cartography, and therefore it is likely that the connection between cartographic methods, optical geometry, and the art of Brunelleschi come to connection with Toscanelli.²¹⁰ The influence of Toscanelli also brings in a Paduan connection into Florence.²¹¹

Brunelleschi, as an architect and engineer, decided to develop a mathematical painting system. Since Brunelleschi started in the goldsmith trade and had a background in relief sculpture, he had an expert knowledge of composition and pictorial space and was knowledgeable about the current trends. His loss in the contest for designing the relief sculptures for the baptistery doors to Ghiberti is said to have been on account of the differences in composition rather than a technical preference. As an architect, Brunelleschi would have had to draw designs for his buildings in a readable manner rather than for compositional aesthetics. Field suggests that it is for the readability of his designs that he developed perspectival drawing.²¹² By using perspective, he would have been able to present the drawing of a building to the patron with the proportions that would be seen when it was finished. It is this invariance that would have been attractive

²⁰⁸ Edgerton, *Renaissance Rediscovery*, 121.

²⁰⁹ Ibid, 121.

²¹⁰ Ibid, 121-123. Toscanelli was in contact with the Portuguese government and Christopher Columbus himself about his upcoming voyage and geometrical grid systems.

²¹¹ Heinrich Klotz, *Filippo Brunelleschi: The Early Works and the Medieval Tradition* (London: Academy Editions, 1990), 150.

²¹² Field, *The Invention of Infinity*, 21.

to the architect and engineer trying to sell his designs. As Edgerton points out, Brunelleschi presented these demonstrations as feats of technology while Alberti presented them later as compositional tools.²¹³ For Brunelleschi, the demonstration of linear perspective is more akin to his many engineering inventions that were tools to help with the greater production of the dome of the Florence Cathedral. Painting in linear perspective was another invention to help with the larger project. As Field mentions, Manetti, in his Brunelleschi biography, refers to the method of linear perspective as a ‘regola’ which implies that it was a mathematical solution.²¹⁴

For the first demonstration panel, Brunelleschi used a peep-show method that fixed the eye of the viewer in the correct mathematical position in order to create a striking illusion. However, for the second panel he had only the panel held out away from the viewer without the box limiting the view or the use of the mirror. In his writings, Leonardo da Vinci expressed the view that, theoretically, perspective would only be perceived as correct if the spectator is in the exact correct mathematical viewing point.²¹⁵ This was correct mathematically, and was predicted by the optical diagrams, however, when Brunelleschi painted his demonstration panel, he would have observed that perspective is perceived as being ‘correct’ from multiple positions in front of a painting. This ‘robustness’ of perspective seems to have come as a surprise to Brunelleschi and other optical scientists and mathematicians at the time.²¹⁶ The second demonstration panel does not fix the viewer’s eye, implying that the observed

²¹³ Edgerton, *Renaissance Rediscovery*, 40.

²¹⁴ Field, *Piero*, 33.

²¹⁵ Kubovy, 52.

²¹⁶ *Ibid*, 52-64. Kubovy thoroughly investigates the robustness of perspective. Field, *Piero*, 34. Robustness of perspective comes as a surprise to Brunelleschi.

'robustness' of perspective made it unnecessary for Brunelleschi's purposes to create a complete illusion as accomplished in the first demonstration panel. The second panel, again of architectural features, was able to show the viewer the observed proportions and view of the real building in front of him. This panel demonstrated that Brunelleschi could draw a design of a building that truly showed how it would appear to the viewer on the street. Brunelleschi did not write about the use of his invention for the larger artistic world; he kept the knowledge of perspective, like his other inventions, in his own mind. Other artists, friends and competitors of Brunelleschi, began using it for compositional purposes and Alberti wrote instructions on how artists should use it to create good art. There is no evidence to show that Brunelleschi thought of his invention in this manner. Instead, it was an engineering tool, much like his many other inventions, that was based on medieval science and mathematics with possible influences from cartographic methods. Once in the hands of artists, it became an artistic compositional tool that was useful for describing the world in the humanist age. It was helpful for relating the spectator more directly with the image, for imitating the descriptions of the ancient paintings, and for the representation of the perfection of nature, and therefore God, through mathematical perfection.

Chapter 3: Lorenzo Ghiberti and Piero della Francesca

While Brunelleschi is credited with the invention and first use of single point perspective, we do not have his demonstration panels nor other examples by him. The early examples of perspective were made by contemporaries of Brunelleschi. Some of these artists were friends, like Donatello and Masaccio, able to discuss perspective personally with Brunelleschi. Alberti helped to spread the invention to other artists who were not in Brunelleschi's circle. In this way, the system spread through the artistic community so that it was being used in early years by Ghiberti, who was a competitor rather than a friend to Brunelleschi.

While Brunelleschi did not win the famous competition for the baptistery door panels in 1401, his invention of single point perspective had a major place in the resulting doors. Ghiberti won the competition to create the bronze reliefs for the baptistery doors and it took 21 years to complete them. They showed scenes from the New Testament (as opposed to the Old Testament which was originally planned). In 1425 Ghiberti received the commission for the second set of bronze doors, now known as the Gates of Paradise, that portray scenes from the Old Testament. These doors took 27 years to complete. Through these years we can see the development of Ghiberti's portrayal of space and use of perspective as a compositional tool in the early formative years of perspective. Ghiberti changed his approach to space and the figures in that space in every panel,

which shows his desire to be innovative and perfect his own style.²¹⁷ Ghiberti was not afraid to combine the developments of other artists into his own work to create something new, much in the same way that Renaissance painters produced paintings based on emulation of antique principles that would create a new work that was different from ancient works. Ghiberti would combine the developments of Donatello and Masaccio with the theory of Brunelleschi and Alberti.²¹⁸ Based on his writing in his treatise, *Commentaries*, Ghiberti was familiar with the optical theories of Bacon, Pecham, Witelo, and Alhazen, with his most important source being Alhazen.²¹⁹ His principle source was Alhazen which he likely knew through an Italian translation from the fourteenth century.²²⁰ He was able to combine all these theories into his own style. Krautheimer finds that Ghiberti had a weak hold on the scientific basis of spatial representation, based on the “transitoriness” of Ghiberti’s perspective attempts.²²¹ Field finds that it is clear from the doors that Ghiberti used a mathematical perspective tool and from his writings that he was familiar with several perspective texts.²²² The fact that his writings were primarily a presentation of optical theories already known and his constant experimentation shows that Ghiberti was less of a theorist on perspective while understanding it well. He was able to use the theories of others to create his compositions, but he did not develop a theory of perspective all his own. It was in the middle of this development that Ghiberti designed and executed the baptistery doors.

²¹⁷ Amy R. Bloch, “The Evolution of Lorenzo Ghiberti’s Approach to the Narrative Relief” in *Depth of Field: Relief Sculpture in Renaissance Italy* (Oxford: Peter Lang AG, 2007), 128.

²¹⁸ Bloch, 136.

²¹⁹ Lindberg, *Roger Bacon*, xcvi.

²²⁰ Bloch, 136.

²²¹ Richard Krautheimer, *Lorenzo Ghiberti* (Princeton: Princeton University Press, 1970), xxii.

²²² Field, *Invention of Infinity*, 17.

When he was designing the *Life of Isaac* panel, Alberti was writing his treatise *On Painting* that explicitly gave instruction on how to produce a painting in Brunelleschian perspective.²²³

The popularity of perspective paintings and reliefs seems to be related to the increasing desire to reproduce nature in its mathematical perfection. The artwork that is composed so that it looks to be in front of the viewer through a window, as if the spectator is looking at the event, increases the emotional connection that the spectator has with that artwork. The presentation of figures and crowds in a measurable space, such as an architectural space, was considered essential to the reproduction of nature.²²⁴ The panels in the east baptistery doors show the dramatic changes in Ghiberti's approach to space and the presentation of figures in that space.²²⁵

Each panel narrates a story, sometimes more than one story in a panel, that all are key events in the life of an Old Testament figure. This can be seen very clearly in the *Isaac* panel. (Fig. 12) The multiple scenes from different times are formed to present a clear narrative of the story of the life Isaac. It is this clear narration that is the concern for Ghiberti.²²⁶ The panel is structured using single point perspective in an architectural setting that organizes the figures. The figures are arranged in small groups around the foreground and serve to encourage the viewer to move the eye around the composition to different parts of the story. The figure groupings move toward the right and become further into the background and up the side of the panel to the top right corner, which

²²³ Krautheimer, 251.

²²⁴ Krautheimer, 234.

²²⁵ Bloch, 126.

²²⁶ Ibid, 134.

shows the angel coming in to stop the sacrifice of Isaac. In Ghiberti's third book of his *Commentaries*, he says that movement is comprehended through perceiving the figure in two different positions at two different times.²²⁷ It is this process of experiencing that time has passed while perceiving two different parts of the story that was meant to convey movement in the panel and thereby actively engage the viewer. The figures that are farther from the center represent a later time. The center would be the first place the viewer would look, along the Albertian 'principle ray', a theory that supported by the optical theory of Alhazen which suggested that the center would have been comprehended with the most clarity.²²⁸ The eye would then move along the figure groupings increasingly farther from the center to give a narration of the life of Isaac.

The architectural background serves as a setting and an organizational element, but also a display of the space within which the figures play out the narration. It is an open loggia with three rows of arches advancing into the background of the space, forming a square filled with open arches. The arches that proceed into the center of the relief contain the point in which the vanishing point would be, were there more of a background. The space behind the arched loggia is left empty in the center row while the row on the left is blocked by drapery and the arches on the right have figures in the middle ground taking up the space. The arches recess into infinity, thereby also extending toward the viewer and including the viewer's space as an extension of its own. The color of the bronze doors has the added association with the gold backgrounds of medieval religious paintings. The gold backgrounds represented the heavenly world of

²²⁷ Ibid, 145.

²²⁸ Bloch, 141.

the holy figures, and these figures were both in the heavenly gold as well as the infinite mathematical world of the viewer. By presenting the scenes in an architectural space that followed the same mathematical rules of nature that existed in the viewer's world, the spectator is linked more fully to the scene, reducing the emotional distance from the panel.²²⁹

The panel showing *King Solomon and the Queen of Sheba* is done in Ghiberti's later style of narration. (Fig. 13) This is clearly presenting a single scene rather than a narration of multiple events. The central building is placed in the center of the composition as opposed to the *Isaac* panel which had the architectural structure slightly to the left, leaving landscape space to the right. The building again has three rows of open arches, but this time the central row of arches rise higher than the arches on either side, emphasizing the central line of the relief. The space to the left and right of the central building contains other buildings and the background through the arches is also full of architectural features. There are no longer flat spaces without detail in the background of the relief. While there is still a central architectural element in perspective, it no longer organizes the space, but is more of a backdrop.²³⁰ The figures are not inside the architecture but stand before it as if on a stage. Therefore the figures are not organized by the architecture but are instead ornamented by the architectural background. The crowd is composed of anonymous figures without separate stories of their own. The groupings of the figures are all arranged so as to give emphasis to the central figures in the primary central position while also looking at the event or reacting

²²⁹ Ibid, 135.

²³⁰ Bloch, 132.

to it, much in the way that the viewer would be looking at or reacting to the portrayal of this event. The spectator is now represented as a spectator of the event itself, and the viewer can now be emotionally connected to the event and has no barrier between the viewing space and the represented space. The figures at the front and sides even reach and extend beyond the plane, taking away the border between reality and illusion and creating a fictive extended space just out of sight. This later style shows an increase in Ghiberti's awareness of including and integrating the viewer physically through the figures extended into the viewer's space, as well as emotionally.²³¹

Ghiberti's use of perspective was a tool in accomplishing his goals of a narrative approach. As his ideas about how best to portray an event changed, he used the theories of optics and perspective to best show the event. Using the theory of the cone of vision, the central position was considered the most easily perceived position by the medieval scientists who followed Alhazen. When putting these optical theories into a compositional format, Alberti gave this central position a primary meaning in a painting in his treatise.²³² It is this compositional rule that had been written out by Alberti that Ghiberti is following by placing his most important figures in the center of the relief in the *Sheba* panel. In the earlier *Isaac* panel, he uses the central point as the first position in the narrative since the optical theory says that this would be the first point that the viewer focuses on. As the viewer looks around the composition deliberately, the point of focus changes. The composition of the panel is constructed so as to direct the motion of the eye in chronological order of the individual scenes, thereby introducing the element

²³¹ Ibid, 148.

²³² Alberti, 48.

of time and telling the entire story of Isaac's life and not just one important element. The representation of the Sacrifice of Isaac is in fact in a very small space at the top of the panel while one might expect it to be the most important part of the narrative. Instead of having the Sacrifice be placed in the center as an important scene, it is placed on the edge in the chronological spiral that is constructed, putting the last scene chronologically in the last place that would theoretically be looked at.

As a contrast to Ghiberti, who was neither a mathematician nor established an original theory of his own, Piero della Francesca started as a mathematician and became a painter later. Not surprisingly, Piero's paintings in perspective showed an unusually correct mathematical construction, while still allowing for changes on account of the perception of the painting as a whole. Piero della Francesca's painting *The Flagellation of Christ* is considered one of the more mathematically correct perspective paintings, and because of this it is an unusual example for its time.²³³ Before Piero was a painter, he wrote a textbook for an abacus school on mathematics for future merchants. His textbook, however, included some things that were unusual for this type of book, such as a section on three-dimensional geometry.²³⁴ The mathematics tradition in abacus schools was based on algebra and very little geometry. It was more heavily dependent on the Islamic tradition than the mathematics taught at universities, which relied more on the Greek methods of geometry.²³⁵ On account of this, the individuals with more education learned a debased version of Greek mathematics while the merchant class learned highly

²³³ Field, *Piero*, 174.

²³⁴ *Ibid*, 31.

²³⁵ Field, *Piero*, 32.

sophisticated Islamic mathematics, which was useful in developing new methods, including linear perspective. Piero's inclusion of three dimensional geometry in his textbook links rather nicely with his spatial relationships in his paintings.²³⁶

Piero's treatise on painting, *De prospectiva pingendi*, specifically concentrated on perspective and proportions only, even specifically identifying this as his concern in the introduction.²³⁷ In the treatise he uses instruction for geometric diagrams and mathematical terms, writing more in the style of Euclid than of Alberti. Piero does not go into specifics of the physical or physiological aspects of optics, and it is not even clear if he subscribed to intromission or extramission; however, Judith Field finds that some of the language that he uses implies extramission more than intromission even though intromission would have been a popular theory by this time through the dissemination of Pecham and Witelo's texts on the subject. Since Alhazen had applied the extramission geometry of Euclid to the intromission theory, subscribing to one or the other system did not affect the mathematics. Piero begins his treatise with defining the mathematical terms that he will use throughout the text, followed by theorems. This placing of theorems at the outset instead of moving directly into examples is different from the abacus style book and closer to the Euclidean style of treatise.²³⁸ He then continues with drawing instructions for different problems. Field compares this usage of instructions to be copied to the earlier manuals of drawings used in workshops.²³⁹ The text as a whole is extremely mathematical in nature and includes drawings and diagrams as opposed to

²³⁶ Ibid, 31.

²³⁷ The information about Piero della Francesca's treatise comes from Field, *Piero della Francesca*, 130-132.

²³⁸ Field, *Piero*, 136.

²³⁹ Field, *Piero*, 136.

Alberti's book which is more about painting as a whole with perspective construction merely a part of it.

Piero's treatise on painting also includes a defense of perspective in painting. This statement generally says that painting is a demonstration "of surfaces and bodies degraded or magnified on the limit [the picture plane], placed like real things seen by the eye as subtending different angles on the said limit..."²⁴⁰ This definition of painting seems to imply that Piero saw painting as a means of representing the world completely naturalistically. However, a look at his paintings allows us to see that composition and emotional impact would overrule his adherence to the perfection of ideal nature. Perhaps it is only that in the context of the treatise, which specifically addresses the mathematical aspects of painting, this was the part of painting which he was teaching. In any case, he takes for granted the fact that good painting is meant to be a representation of natural appearances, like Pliny does.²⁴¹

As mentioned earlier, *The Flagellation of Christ*, (fig. 14) is considered unusual because of the mathematical correctness of the painting. Many of the individual elements seem to come directly from Piero's treatise.²⁴² Despite this, it still takes liberties in the name of composition. The painting's composition is divided in two halves with three figures in the foreground on the right side. Behind the three men are a domestic building and tower on the far right with a tree. On the left side is the scene of the Flagellation in the middle ground. The wall of the building behind the flagellation blocks any

²⁴⁰ Translation by Judith Field in *Piero*, 163.

²⁴¹ Field, *Piero*, 163.

²⁴² *Ibid*, 178.

background from this half. The division between the two sides is a row of columns on a white stripe progressing toward the viewer on a red tile floor. The divided composition of the painting is strongly defined by this line. There have been many reconstructions of the space represented in the painting since the 1950's, including a full model that was built in 1992 by Philip Steadman for British television.²⁴³ (Fig. 15) This model allows us to easily see the few discrepancies, notably the building that is at the right side of the painting in the background behind the three figures in the foreground. This building turns out to be strangely tall and thin and was likely constructed in this way to frame the figures. If it were painted shorter, the roof line would awkwardly cut through the figures and not be as recognizable either. Another change that was found during reconstruction was that the prominent white line that extends into the foreground is thinner in the foreground than it would have been in the middle ground based on the column bases with which it aligns.²⁴⁴ This line is important compositionally because it is the link between the viewer's space and the fictive space of the painting. It is what leads the eye directly into the painting from the plane, which is referred to as the 'limit' by Piero in his treatise. This line brings the viewer's eye first to the three figures in the foreground by having the feet of the leftmost figure on the line. It then proceeds as an orthogonal back into the plane, stopping just short of the vanishing point at a wall thereby bringing the eye to the scene of the flagellation. The changing width of the line can only be on account of composition. It is not apparent by looking at the painting that the line is abnormally thin, however if the thickness of the line were widened, it would affect how it interacts with

²⁴³ Field, *Piero*, 177.

²⁴⁴ *Ibid*, 176.

the figure's feet. If the figure were moved to accommodate the line, the entire framing of the figures would be changed. The line was made thinner than mathematically correct so that the composition would draw attention to the two scenes without awkwardly cutting at the feet of the forward figure. Piero apparently thought that changing the composition around the line would have been enough of a problem that instead he changed the thickness of the line.

This white line also brings attention to the scene of the flagellation which is occurring in the middle ground. Unlike Ghiberti who placed the important figures or scene directly on the centric point, Piero only guides us close to the scene. The vanishing point of the painting would be on the wall, just to the left of the molding, at about the height of the hip of the man with the whip.²⁴⁵ While not being in the obvious position of importance, the viewer is still led to the man with the whip, and therefore to the scene of flagellation all together. While moving from the front of the picture plane to the back, the eye is also moving from the right to the left. Piero has used his own technique to control the movement of narration from one scene to another, but has used orthogonals and proximity to make the movement natural and controlled. He chooses a composition more visually interesting than a central placement of figures that also confuses the viewer as to what is more important in the painting. While there is an order in which one sees the scenes, there is not a primary importance placed on either, which leads to a more ambiguous and open narrative.

²⁴⁵ Field, *Piero*, 176.

These examples of Piero's *The Flagellation of Christ*, and Ghiberti's panels of *The Life of Isaac* and *King Solomon and the Queen of Sheba*, show the differences in approach for the early use of perspective in the fifteenth century. Piero came from a more mathematical background and created his perspective paintings in a stricter mathematical manner; Ghiberti experimented and searched in other's theories for a more effective way to represent scenes. Both men were looking for the same ideal, a good composition and representation of nature, which affected their decisions and results, however, their primary means of achieving those goals were different. Both of these illustrate the changes in the approach to perspective from Brunelleschi's use of it as an engineering tool and mathematical solution. It is referred to as Brunelleschi's 'invention' and that seems to be how he thought of it, as an invention that did not need to lead to philosophy. Once it came into the hands of his artistic colleagues, it was changed into an artistic theory that could improve 'good painting' and the representation of both nature and the divine.

Conclusion

Brunelleschi ushered in one of the defining features of Renaissance painting with his invention and demonstration of single point perspective. While painting in perspective is one of the defining features of the Renaissance, it was based on science largely of the Middle Ages. It was not designed on knowledge from the ancient world, even though it was thought that the ancients used some form of perspective to create lifelike paintings. Also, it does not seem to have been motivated by a desire to emulate the lost paintings of antiquity, but as a useful tool to help Brunelleschi in his architectural career. It was after the first demonstrations that the method was integrated into the burgeoning field of art theory.

When other artists saw what Brunelleschi had created, they used it in the pictorial fields of painting and relief sculpture to enhance their ability to create images that looked lifelike, and therefore closer to nature and God. The humanist artists creating paintings in the new classical style were searching for something that would enhance the appearance of the image as being of nature, both because it was what the ancients were supposed to have done as well as the theological argument of perfect nature showing the perfection of God. Alberti took this new style and related it as “good painting” because of its relationship to the idea of ancient paintings. The association with the classical world was developed after the demonstration panels. The classical association of naturalistic painting was accepted because of the medieval arguments of the relationship between God, science, and nature. Giotto had attempted more and more naturalistic paintings before Brunelleschi invented single point perspective, showing the growing desire for a

‘correct’ representation of nature as we perceive it. Perspective did not create this desire, but was a symptom of the growing interest in representing images in such a way.

The social atmosphere of Florence at the beginning of the fifteenth century allowed for the development and flourishing of perspective. The republican nature of the city-state allowed for someone like Brunelleschi to have the freedom to invent and try new things and promoted the exchange of ideas. The merchant culture of the city and general education of merchants gave the people a prior expectation of the incorporation of mathematics into daily life. It also gave an expectation of the mixing of information from different cultures. This allowed for the scientists and humanists of Florence to accept readily the works of Alhazen and Al-Kindi even though they were not western theories. Florence was full of intelligent and educated artists and humanists because of these aspects of social life. The city allowed the people to grow and develop a new use for optical theories that had been widely spread throughout Europe for two hundred years. It was that Brunelleschi decided to apply these theories directly to the painted panel that created single point perspective. It was his fellow artists that created a method and theory out of it, and the popularity of the images that made it a lasting phenomenon.

While the optics had been around for centuries, the Islamic world where it started prohibited representational art. The English, where it was developed for the western world, did not have the desire to create as naturalistic paintings as seen in the fourteenth century Italian art. It was the Italians with their move toward naturalistic painting that saw something useful in optics for the artistic community. It was also in Florence where

there was a more liberal and animated exchange of ideas that the artists were able to develop and take hold of this idea.

Ghiberti shows this taking hold of perspective in his early development of perspective for narrative schemes in his Gates of Paradise doors on the Florence Baptistery. On account of being designed and created over the early years of perspective while Ghiberti was developing his own approach, it accords us a view into the artistic development in Florence at this time. Piero della Francesca gives us the mathematical approach to perspective in his mathematical treatise on the subject. His relatively strict adherence to the geometric proscriptions only brings his variations into greater light. Where he decides to alter the perspective shows that even the most mathematical of artists used perspective in the service of composition as opposed to being a “slave to the geometry.” His concessions to composition over geometry are a clear indication of the mutability and personal preferences in the use of perspective.

The transition of *perspectiva* from the medieval science of vision into the Renaissance method of naturalistic representation of space took the mathematical system to an unlikely home in the arts. Through the middle ages, the study of optics was kept alive through the work of Islamic scientists, who would not have dared use these geometric theories for life-like images. In the hands of the fifteenth-century Italian humanist, optical geometry was transformed to be in the service of the ambition to emulate the ancients. While the aim was to recreate the life-like naturalism of the paintings of antiquity, the resulting use of perspective in Renaissance Italy owes much to medieval science in general and middle eastern medieval science in particular.

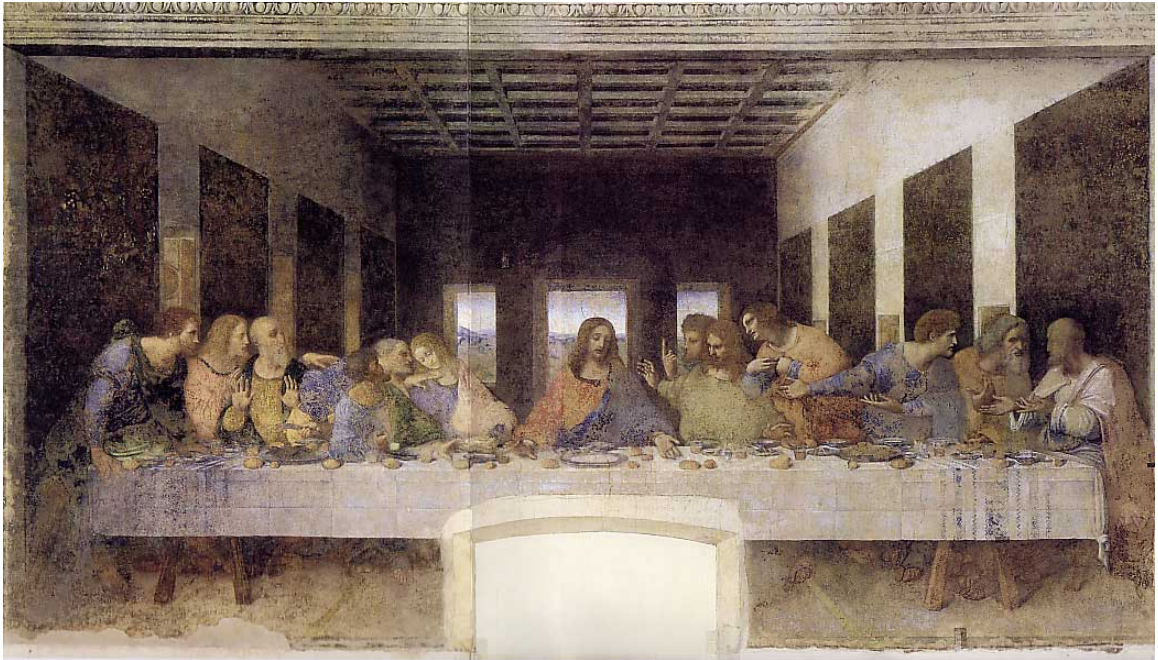


Fig. 1. Leonardo da Vinci. *The Last Supper*. 1498. Milan.

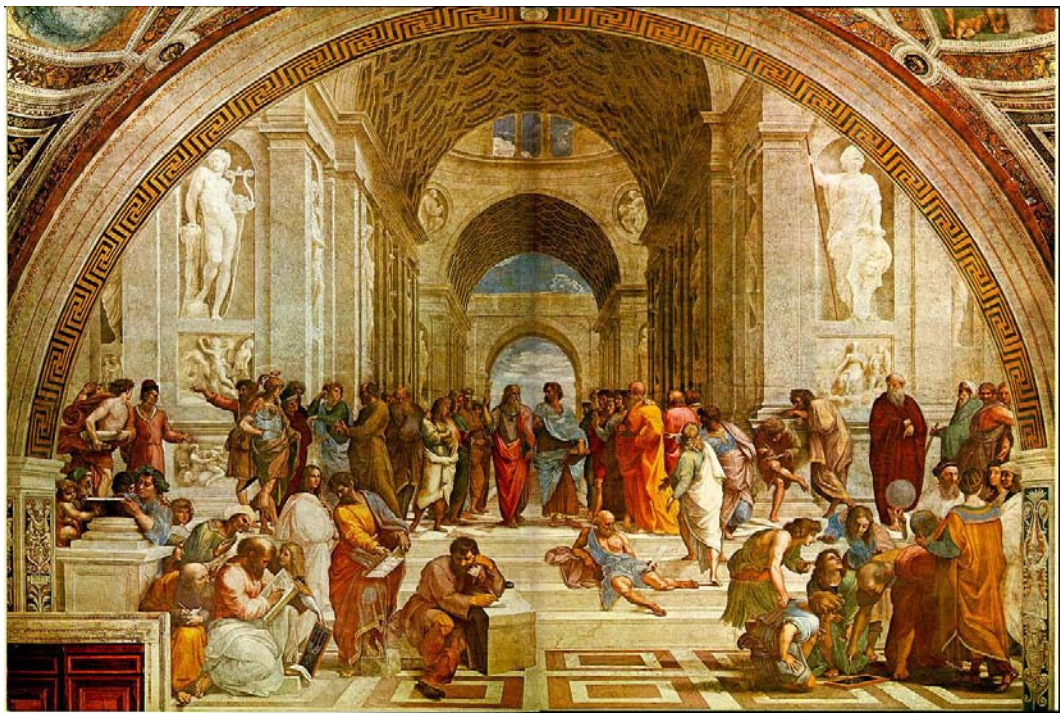


Fig. 2. Raphael. *The School of Athens*. 1510. Vatican Palace.



Fig. 3. Detail of *The School of Athens* showing spheres painted as circles.

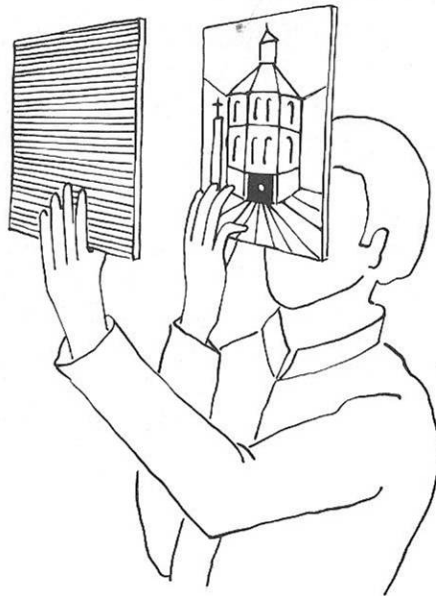


Fig. 4. Samuel Edgerton's reconstruction of Brunelleschi's first perspective demonstration.

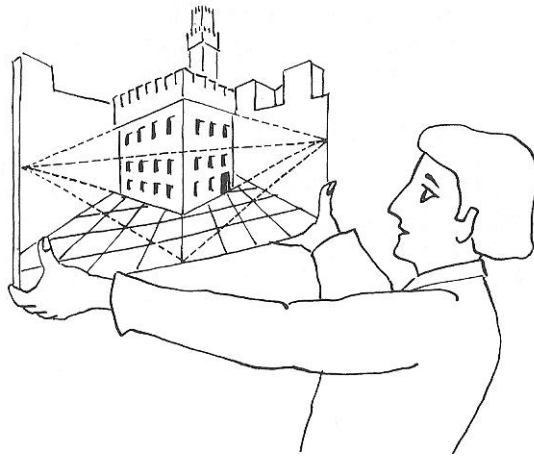


Fig. 5. Samuel Edgerton's reconstruction of Brunelleschi's second demonstration.

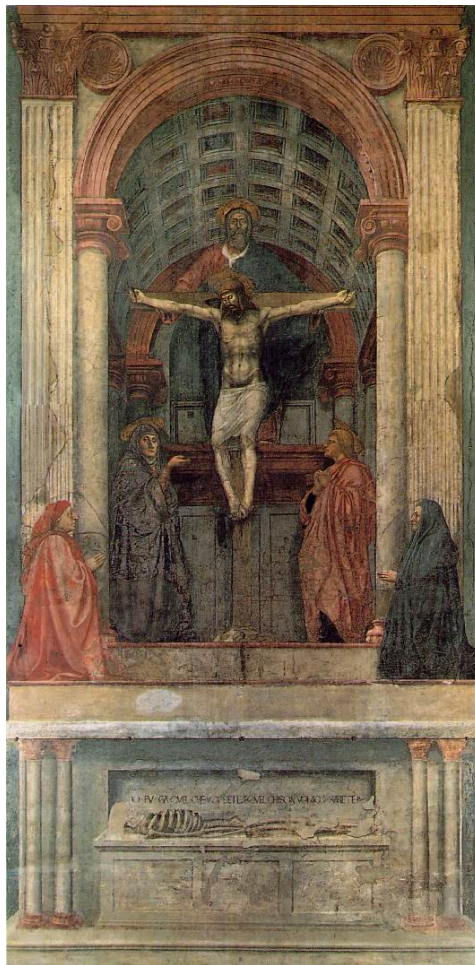
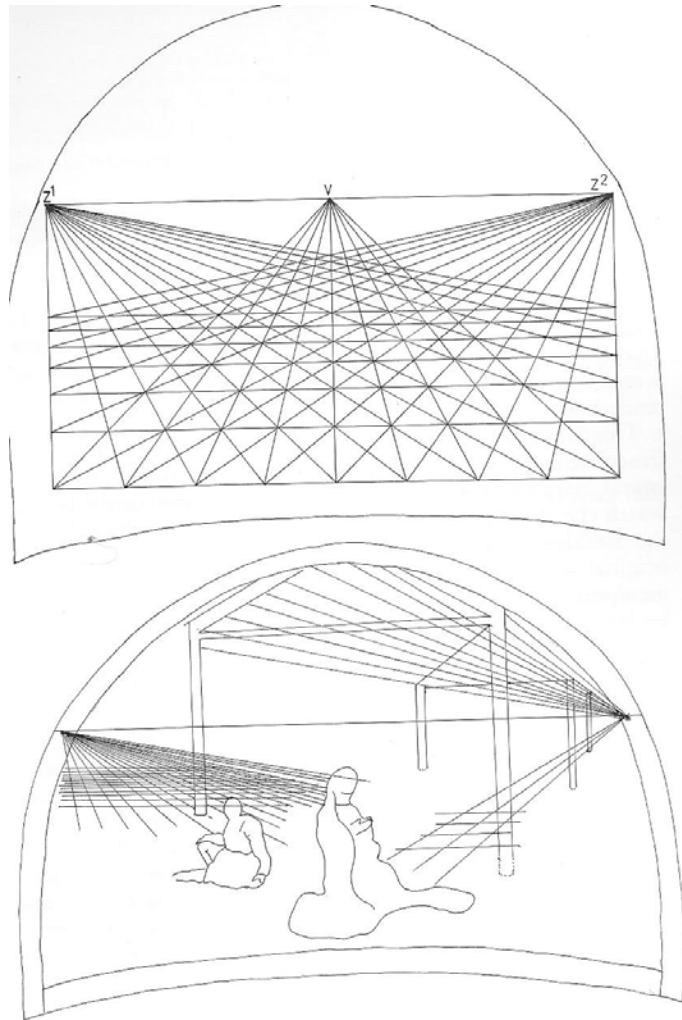


Fig. 6. Masaccio. *Holy Trinity*. 1427. Florence.



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Fig. 7. Reconstruction Diagram by Martin Kemp of the perspective geometry of the underdrawing of Uccello's *Nativity*.



Fig. 8. Donatello. *Assumption of St. John the Evangelist*. 1430. Florence.

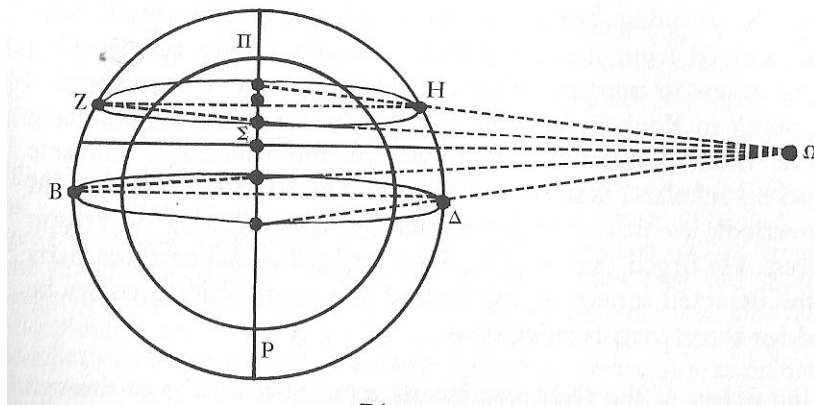


Fig. 9. Ptolemy's Third Cartographic method as illustrated by Samuel Edgerton.

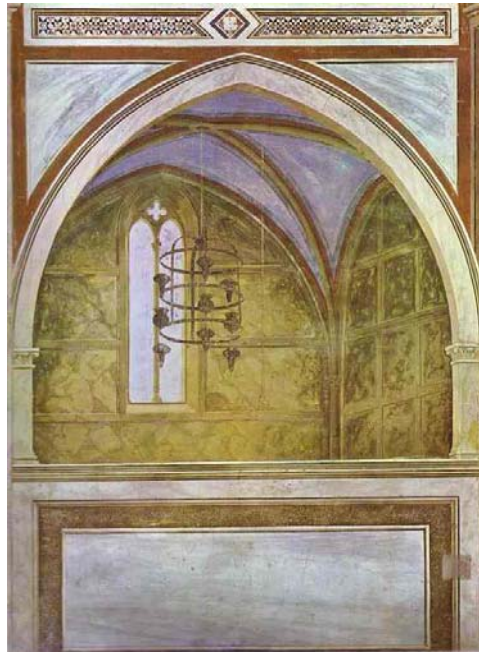


Fig. 10. Giotto. One of the two *Coretti* in the Arena Chapel. 1305. Padua.



Fig. 11. Giotto. *Joachim and Anna meet at the Golden Gate*. 1305. Padua.



Fig. 12. Ghiberti. *Life of Isaac*. Gates of Paradise. Florence.



Fig. 13. Ghiberti. *King Solomon and the Queen of Sheba*. Gates of Paradise. Florence.



Fig. 14. Piero della Francesca. *The Flagellation of Christ*. 1455-1460. Urbino.

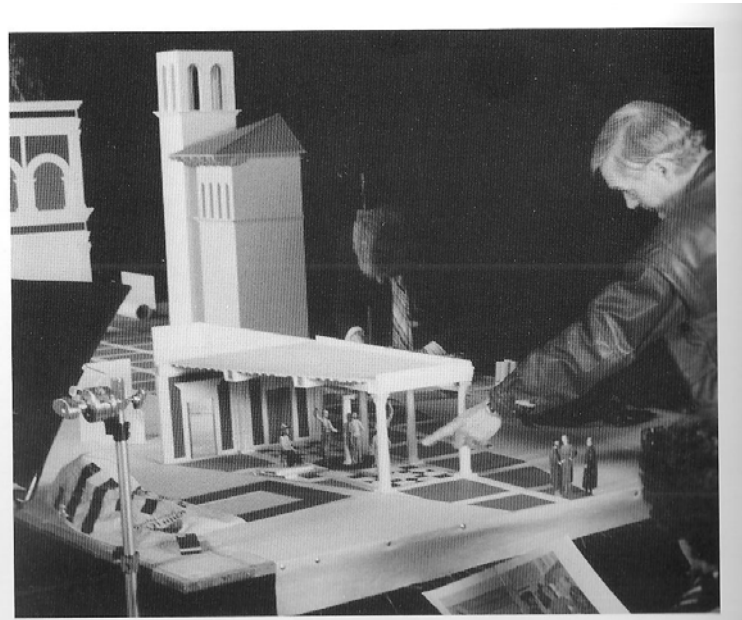


Fig. 15. Model of Piero della Francesca's *The Flagellation of Christ* made by Philip Steadman in 1992 for a BBC television program.

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