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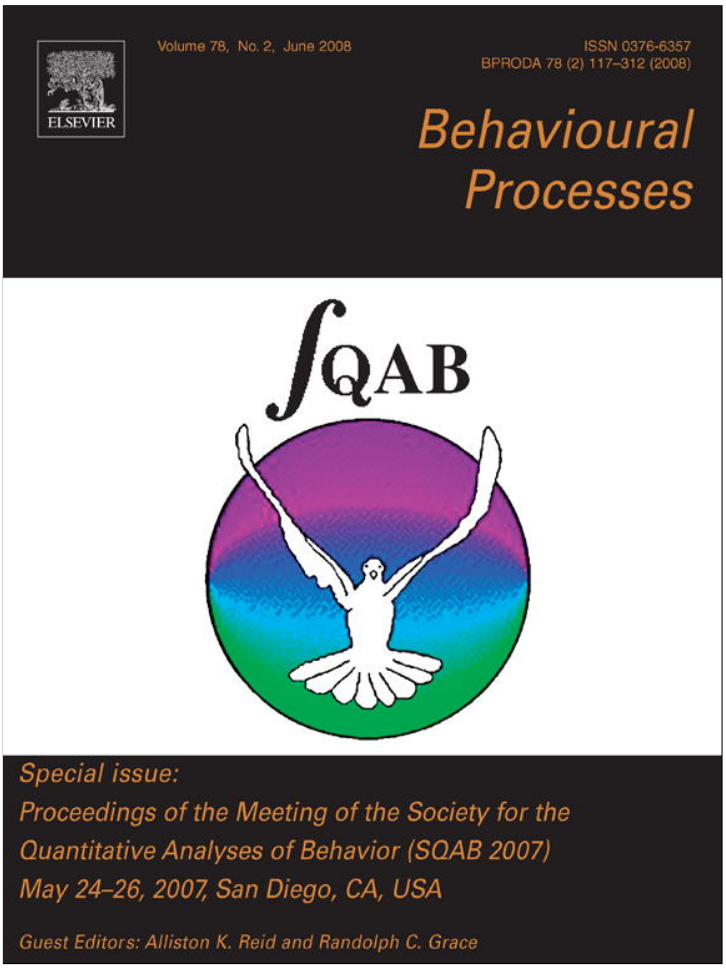

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Short communication

Comparing the generalized matching law and contingency discriminability model as accounts of concurrent schedule performance using residual meta-analysis☆

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Abstract

To compare the generalized matching law (Baum, W.M., 1974b. On two types of deviation from the matching law: bias and undermatching. J. Exp. Anal. Behav. 22, 231–242) and contingency discriminability model (Davison, M., Jenkins, P.E., 1985. Stimulus discriminability, contingency discriminability, and schedule performance. Anim. Learn. Behav. 13, 77–84) as accounts of concurrent schedule performance, we conducted a residual meta-analysis of response- and time-allocation data from 20 studies (n's = 886 and 774, respectively). Both models were fitted to the individual-subject data from each study, and residuals were obtained. Polynomial regressions were then performed on the pooled residuals to determine whether systematic trends were present as a function of predicted values. For the contingency discriminability model, the cubic coefficients were positive and statistically significant for both response- and time-allocation data. By contrast, no statistically significant systematic trend was obtained in the residuals for the generalized matching law. These results suggest that the relationship between log response allocation and log reinforcer allocation does not deviate significantly from linearity over an approximate range of +1.25 to −1.25 log units, consistent with the generalized matching law. Although qualitative criteria are also important in comparing models of behavioral phenomena, residual meta-analysis provides a powerful quantitative methodology for model selection and should prove useful in future research.

Keywords: Choice; Concurrent schedules; Generalized matching law; Contingency discriminability model; Meta-analysis; Key peck; Pigeons

1. Introduction

Response allocation under concurrent variable-interval (VI) VI schedules is typically well described by the generalized matching law (GML; Baum, 1974b, 1979):

$$\frac{B_1}{B_2} = b\left(\frac{R_1}{R_2}\right)^a, \tag{1}$$

or alternatively,

$$\log\left(\frac{B_1}{B_2}\right) = a\log\left(\frac{R_1}{R_2}\right) + \log b \tag{2}$$

In Eqs. (1) and (2), B indicates responses or time allocated and R represents reinforcers obtained, subscripted for alternatives one and two, and the parameters a and b represent sensitivity and bias, respectively. According to Eq. (2), there is a linear relationship between relative response or time allocation and the reinforcer ratio in logarithmic terms.

Davison and Jenkins (1985) proposed an alternative model for concurrent schedule performance, the contingency discriminability model (CDM). According to their model, if the two alternatives of a concurrent schedule are imperfectly discriminated by the subject, then some proportion of the reinforcers that are obtained from one alternative will strengthen responding at

☆ This research was presented at the 4th ABA International Conference (Sydney, August 2007). The residual meta-analysis methodology was first described in a presentation given as part of a *festschrift* for Tony Nevin, held at the University of New Hampshire in May 2004. His beautiful paper on invariance (Nevin, 1984) was an inspiration for the ideas developed here.

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the other. The CDM takes the form:

$$\frac{B_1}{B_2} = b\frac{R_1 - pR_1 + pR_2}{R_2 - pR_2 + pR_1}. \quad (3)$$

B, R and b are the same as in the GML. The parameter p is a measure of proportional confusion (i.e., the inverse of discriminability), and varies from 0 (implying strict matching) to 0.5 (complete insensitivity to the reinforcer ratio). Thus, the CDM explains the common finding of undermatching – that is, $a < 1.0$ in Eq. (2) (Baum, 1979) – as the result of the subject's failure to discriminate the choice alternatives.

There is considerable disagreement as to whether the GML or CDM provides a better model for behavior allocation under concurrent VI VI schedules (Baum et al., 1999; Davison and Jones, 1995). The question has been difficult to resolve empirically, because over the range of reinforcer ratios typically studied in concurrent schedules (e.g., 10:1 to 1:10), the two models make very similar predictions. Thus, it is unlikely that the GML and CDM can be differentiated by comparing the proportion of variance that each model accounts for (r^2). In fact, the performance of the two models tends to be very similar when values of r^2 are compared (Baum et al., 1999; Davison and Jones, 1995).

However, the models' predictions are not identical, even within the 2-log unit range of reinforcer ratios that is typical in the literature. Baum et al. (1999) noted that if either the GML or CDM was the 'true' model, then specific patterns should be found in the residuals of the other model. To illustrate, the upper left panel of Fig. 1 shows log behavior ratios generated by the GML across the range of log reinforcer ratios between −1 and +1 (symbols), and the predictions of the CDM when fitted to these data (the line). Conversely, the lower left panel shows log behavior ratios generated by the CDM across the same range of reinforcer ratios, and the predictions of the GML. Although each model accounts for over 99.9% of the variance in the data generated by the other model, there are systematic deviations in the fits. The right-hand panels show the residuals (i.e., obtained values minus predicted values) for the fits of each model, plotted as a function of the predicted values. For the residuals of the CDM when fitted to data generated by the GML (upper right panel), the pattern can be described as a third-order polynomial with a positive cubic coefficient and a negative linear coefficient. By contrast, the residuals of the GML when fitted to data generated by the CDM are described by a third-order polynomial with negative cubic and positive linear coefficients.

Although the models' predictions can be distinguished, it is important to recognize that the residuals are very small – less than 0.1% of the variance in the data. At more extreme reinforcer ratios (e.g., 100:1), the differences between the models' predictions are more apparent. Experiments by Davison and Jones (1995) and Baum et al. (1999) have tested performance of the

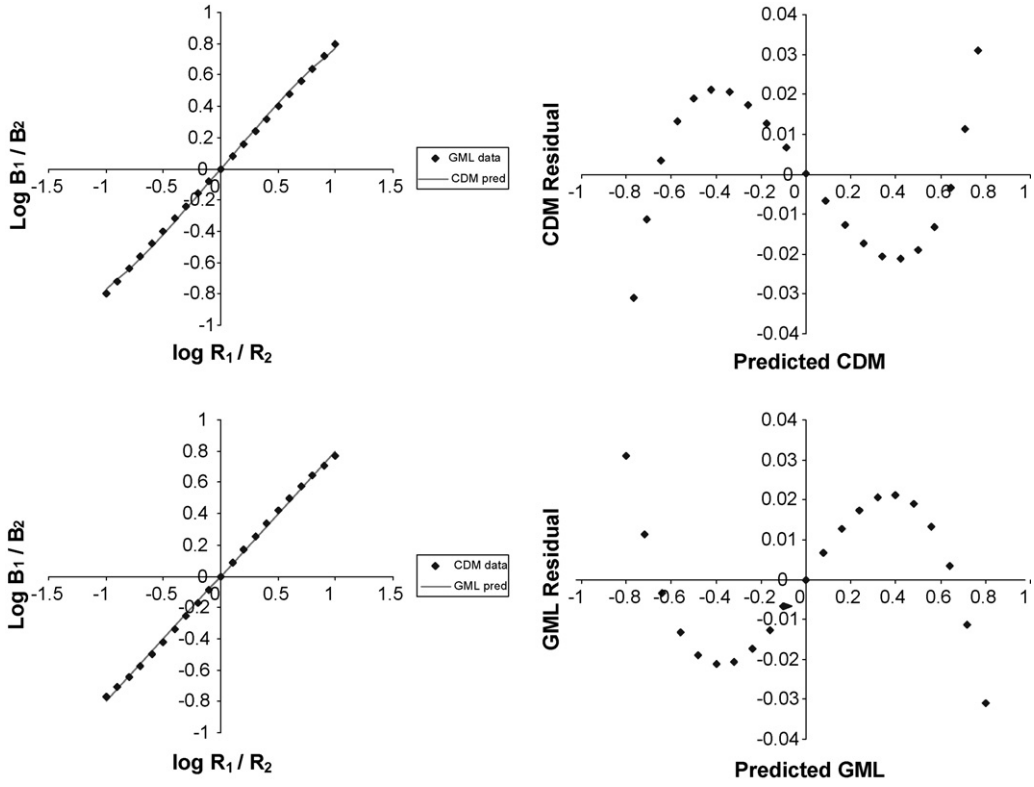

Fig. 1. The upper left panel shows log response allocation (filled symbols) predicted by the generalized matching law for a range of reinforcer ratios from 1:10 to 10:1, assuming $a = 0.80$ and $\log b = 0$ in Eq. (2). The line shows the best fitting predictions of the contingency discriminability model (Eq. (3)), for which a logarithmic transformation of Eq. (3) with $p_r = 0.067$ and $\log b = 0$ accounted for more than 99.9% of the variance. The residuals from the fits of the contingency discriminability model are shown in the upper right panel. The bottom panels show the corresponding situation in which log response allocation data were generated by the contingency discriminability model with $p_r = 0.067$ and fitted by the generalized matching law, which with $a = 0.80$ accounted for more than 99.9% of the variance. The residuals from the fit of the generalized matching law are shown in the lower right panel.

CDM and GML at extreme reinforcer ratios. The results of these experiments are considered further in the discussion.

Here we pursue an alternate strategy to compare the GML and CDM as accounts of concurrent VI VI performance. Because the models' residuals are expected to be small within the range of reinforcer ratios examined in Fig. 1, there is little chance that any individual study will provide convincing evidence that would support one model or the other. However, by aggregating results across individual studies we can increase the statistical power to detect a systematic pattern like those displayed in Fig. 1, even within the range of reinforcer ratios in Fig. 1. This methodology might be called residual meta-analysis, because similar to the techniques for aggregating and analyzing effect sizes, which are widely used for literature review and synthesis in the social sciences (i.e., meta-analysis; Rosenthal and DiMatteo, 2001), statistical power is increased by combining results across studies. Our goal in the present study was to determine if this strategy could be used with published studies of concurrent performance to detect evidence of deviations from predictions within the range of reinforcer ratios that is typically studied.

2. Method and materials

We conducted a literature search to identify possible studies for inclusion in the residual meta-analysis. To be eligible for inclusion, a study had to report results from an experiment using a two-alternative concurrent VI VI schedule and at least one data series with a minimum of four different conditions in which relative reinforcer rate was varied, and in which all other variables (e.g., changeover requirements, stimulus conditions) remained constant. Only steady-state data were included. We excluded a number of studies because of deviations from absolutely standard concurrent VI VI (e.g., qualitatively or quantitatively different reinforcers, constraints on opportunity to respond, additional contingencies in the procedure, and many others). Some studies included multiple data series, which met our criteria (e.g., Alsop and Elliffe, 1988). In order to make the model comparison fair, studies which reported strong overmatching, such as those using fixed-ratio changeover requirements (Pliskoff et al., 1978) were omitted. Although the CDM (Eq. (3)) can be combined with other models to predict overmatching (see Davison and McCarthy, 1994), this requires further parameters that would naturally be expected to increase variance accounted for (Pitt et al., 2002). Without this addition, Eq. (3) cannot predict overmatching.

An initial analysis showed that, except for some conditions in Davison and Jones (1995) and Baum et al. (1999), all obtained log reinforcer ratios fell between −1.65 and +1.25. Accordingly, we included in our analysis all studies that had at least four conditions within the range, symmetrical around zero, of −1.25 to +1.25. A total of 20 studies (including Davison and Jones, 1995, and Baum et al., 1999) were identified that met these criteria, providing a total of 886 data points for response allocation and 774 data points for time allocation. Table 1 provides a summary of relevant features of the studies included.

Parameter values for both models were estimated for individual subjects for each data series within each study. Logarithmic versions of both models (i.e., Eq. (2) for the GML; a logarithmic transformation of Eq. (3) for the CDM) were applied to both response- and time-allocation data. In all cases, parameters were estimated that maximized the variance accounted for by the model in question. The estimation procedure was constrained so that values for p (Eq. (3)) could not be less than zero.

Table 1
Summary of the relevant features of the studies included in the residual meta-analysis

Study	Species	Number of subjects	Number of data series	Number of data points
Alsop and Elliffe (1988)	Pigeons	6	36	191
Baum (1972, unpublished)	Pigeons	3	3	47[a]
Baum (1974a)	Pigeons	20	1	5[a,c]
Baum (1976)	Rats	5	5	57[a]
Baum and Rachlin (1969)	Pigeons	6	6	111[b]
Baum et al. (1999)	Pigeons	4	4	16[a]
Belke and Beliveau (2001)	Rats	6	6	24
Catania (1963)	Pigeons	3	3	24
Davison (1983)	Pigeons	6	6	28
Davison (1991)	Pigeons	5	5	25
Davison and Hogsden (1984)	Pigeons	6	6	34
Davison and Jones (1995)	Pigeons	6	6	30
Davison and Kerr (1989)	Pigeons	6	6	35
Elliffe and Alsop (1996)	Pigeons	6	40	212
Grace and McLean (Unpublished)	Pigeons	4	8	40
Graft et al. (1977)	Rats	5	1	9[a,c]
Leigland (1979)	Pigeons	2 Groups	2	10[c]
Lobb and Davison (1975)	Pigeons	5	1	10[c]
McSweeney (1975)	Pigeons	4	4	16[a]
McLean and Blampied (2001)	Pigeons	8	32	73[a]

[a] Only response data from this study was available.
[b] Only time data from this study was available.
[c] Only aggregated group data was available.

Residuals were calculated for both models as the obtained minus predicted log response ratios. To test for systematic deviations in the models' predictions, polynomial regressions were conducted in which the residuals were regressed on the predicted values. However, because both models incorporate bias in exactly the same way (i.e., assume that the effects of bias and reinforcer allocation are additive in logarithmic terms), estimated bias values were subtracted from the predicted choice values prior to running the polynomial regressions. This was done because bias is otherwise an additional source of variance in the data that could potentially obscure a systematic pattern such as those shown in Fig. 1. Polynomial regressions were then conducted in which each model's residuals were regressed on predicted log response and time ratios. Linear, quadratic and cubic terms were included in all models.

3. Results

A comparison of the average r^2 for all series revealed very similar values for the two models. There was a small, statistically significant advantage for the GML with response allocation data (M's = 0.949 and 0.944 for GML and CDM, respectively; $t(163) = 3.40$, $p < 0.001$), and with time allocation data (M's = 0.963 and 0.957; $t(130) = 3.86$, $p < 0.001$). Although this shows that the GML accounted for slightly more variance in the data than the CDM, the difference does not necessarily mean that the GML is the superior model. Goodness-of-fit measures such as r^2 are influenced by model flexibility, which depends on functional form and thus can differ even for cases in which number of parameters for the competing models is the same (Pitt et al., 2002). Thus, no inference should be drawn regarding the relative merits of the GML and CDM based on the r^2 values.

Comparison of the models' sensitivity parameters showed that for the GML, estimated a values were overall greater for time ($M = 0.81$) than response allocation ($M = 0.72$), $t(293) = -4.18$, $p < 0.001$. This is consistent with Baum's (1979) review, which found that sensitivity of time allocation to relative reinforcement was greater than that for response allocation. Similarly, for the CDM, estimated proportional confusion parameter values were greater for response ($M = 0.11$) than time allocation ($M = 0.08$), $t(293) = 4.21$, $p < 0.001$.

The major question in the present study was whether systematic patterns in the residuals would be obtained consistent with the predictions of GML or CDM. Table 2 shows the results of the polynomial regressions, in which response- and time-allocation residuals for both models were regressed on the predicted log response ratios (corrected for bias). For the CDM, the regressions were significant for both response- and time-allocation residuals. In both cases, the cubic coefficients were positive and significant, as predicted by the GML. For the GML's residuals, the analysis was not significant for either response residuals or time residuals (p's > 0.30 and 0.09, respectively). The absolute values of the cubic coefficients were 0.014 and 0.013 for response- and time-allocation, respectively; less than one fifth of the values found for CDM (0.072 and 0.090). The coefficients for the linear components were in the predicted directions (negative for CDM, positive for GML) but did not reach significance in any case. For the CDM, the quadratic component was also significant in time-allocation residuals. We consider the interpretation of this quadratic component in Section 4.

Table 2
Results of polynomial regressions performed on the CDM and GML residuals

Dependent variable	Predictor variables	B	S.E. of B	R^2	F	d.f.
CDM residuals						
Responses	Linear	−0.0231	0.0127			
	Quadratic	0.0246	0.0142			
	Cubic	0.0719**	0.0252			
				0.0146**	4.3672	3, 882
CDM residuals						
Time	Linear	−0.0141	0.0166			
	Quadratic	0.0558***	0.0160			
	Cubic	0.0901***	0.0254			
				0.0508***	13.7300	3, 770
GML residuals						
Responses	Linear	0.0078	0.0119			
	Quadratic	0.0212	0.0126			
	Cubic	−0.0136	0.0198			
				0.0041	1.2036	3, 882
GML residuals						
Time	Linear	0.0148	0.0139			
	Quadratic	0.0297*	0.0129			
	Cubic	−0.0131	0.0173			
				0.0081	2.0975	3, 770

Dependent variables were residuals from either the CDM or GML, computed using log response or time ratios. Independent variables were log predicted values (bias corrected; 'linear'), plus squares and cubes of log predicted values ('quadratic' and 'cubic'). B is the unstandardised regression coefficient, and R^2 is the proportion of variance accounted for. *$p < 0.05$, **$p < 0.01$, ***$p < 0.001$.

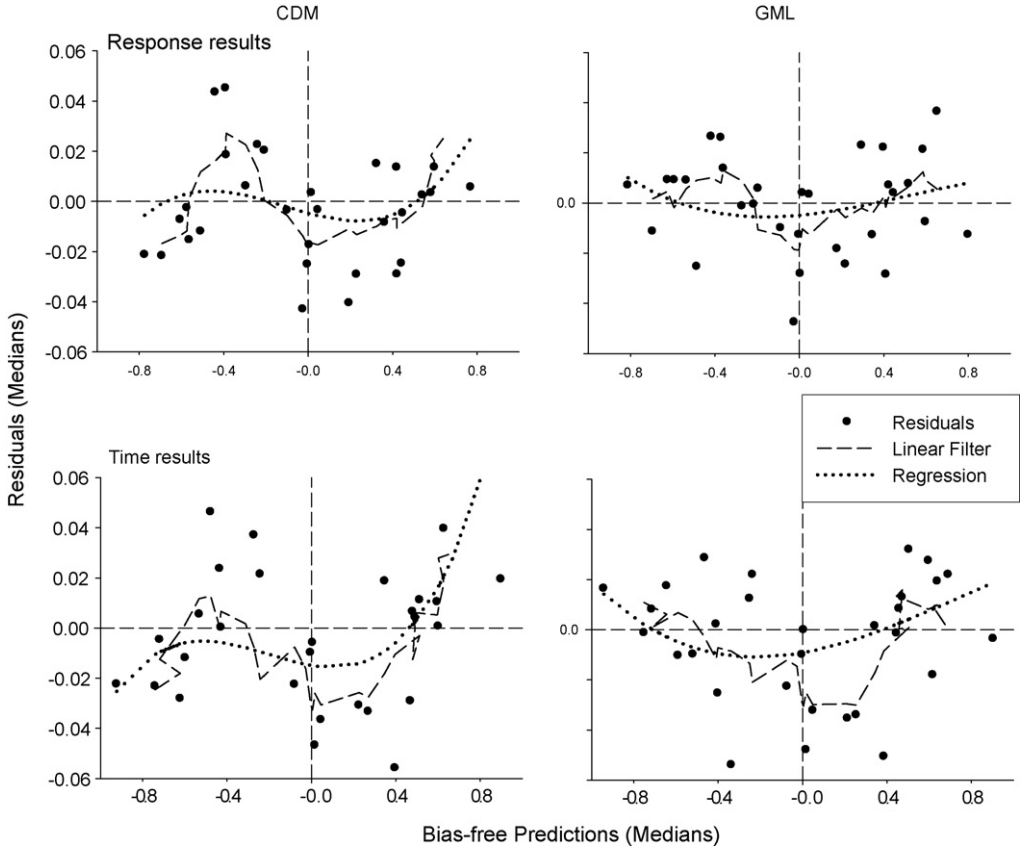

Fig. 2. Median residuals in narrow ranges of the log reinforcer ratio scale, plotted as a function of medians of bias-corrected log predicted response or time ratios. Medians are shown for the contingency discriminability model (left panels) and generalized matching law (right panels). Response-allocation data are shown in the upper panels, and time-allocation data in the lower panels. The dashed line gives a linear filter of average 5 for median residuals, and the dotted line plots the polynomial regression equations reported in Table 2. Note that neither regression analysis was significant for the GML.

To examine the cubic trends found in the regression analyses more directly, we divided the range of reinforcer ratios into 31 adjacent subintervals, each containing approximately 32 data points. Exactly equal numbers of data points per subinterval was not possible because of ties. We then took the median of residuals, and the median of predictions, in each subinterval. Medians (as opposed to the averages inherent in regression analysis) were used to prevent occasional large residual values from having a disproportionate influence on the results. Fig. 2 shows median residuals for responses and time, plotted against median predicted log response or time ratios, for each model. A linear filter (the moving average of five medians; Tryon, 1983) is plotted to clarify the trends in residuals, and for comparison, the predictions of the polynomial regressions from Table 2 are also shown.

Fig. 2 confirms the main results from the regression analyses. Residuals for the CDM show a clear sigmoidal pattern (positive cubic component) for both response and time data. Those for the GML do not show the negative cubic pattern that would appear if CDM was the true model, and if anything, the linear filtered medians sometimes suggest a weak positive cubic pattern. The GML's residuals for response- and time-allocation data also suggest a quadratic (u-shaped) trend, although the regression equations were not significant in either case. Predictions from the polynomial regression equations (dotted lines) clearly capture the main trends that are evident in the residuals.

A large proportion of the data points in the current sample were contributed by two large studies from the same laboratory (Alsop and Elliffe, 1988; Elliffe and Alsop, 1996; see Table 1). It is important to determine whether the results in Table 2 might have been produced by idiosyncratic features of these studies. Thus, we repeated the polynomial regressions, omitting the data from Alsop and Elliffe (1988) and Elliffe and Alsop (1996). Because of the smaller sample size, statistical power was reduced and so the key question was whether coefficients that were similar in sign and magnitude would be obtained. This question was answered in the affirmative. For the CDM, the coefficients (linear and cubic) were −0.032 and 0.071 for response allocation and 0.015 and 0.073 for time allocation, respectively. For the GML, the corresponding values were 0.002 and 0.000 for response allocation, and 0.024 and −0.019 for time allocation. Examination of scatterplots showed that the patterns were similar to those in Fig. 2. This suggests that the Alsop and Elliffe (1988) and Elliffe and Alsop (1996) data sets were not idiosyncratic and that the results in Table 2 cannot be attributed solely to the influence of these studies.

4. Discussion

The major result of the present study was that the residuals from fits of the contingency discriminability model to both response- and time-allocation data from the present sample of concurrent schedule studies showed the deviations that would be expected if the generalized matching law was the 'true' model. A sigmoidal pattern in the CDM residuals could be described by a third-order polynomial with a significant positive cubic coefficient (see Figs. 1 and 2). By contrast, polynomial regression analysis of GML residuals was not significant (R^2), and plots of the residuals did not suggest the negative cubic pattern that would be expected if the CDM was the 'true' model.

In contrast to the GML, which asserts a linear relationship between log response ratios and log reinforcer ratios, the CDM predicts that the relationship is sigmoidal. The fitted line in the upper left panel of Fig. 1 confirms the slight nonlinearity in the predictions of the CDM. According to the CDM, sensitivity of log response to log reinforcer allocation (measured as the first derivative of the function) is lower at relatively extreme reinforcer ratios, whereas according to the GML sensitivity is constant at all reinforcer ratios. The obtained pattern in the CDM residuals is consistent with an underlying linear relationship in the data over the range of reinforcer ratios used across the studies in the sample (approximately 18:1–1:18). The systematic residuals obtained for the CDM provide evidence that the relationship is not characterized by a sigmoidal function over this range.

There were also significant quadratic components in the regression analyses of CDM residuals, and for the GML, similar patterns were evident in Fig. 2 (although they were not significant in the regression analyses). Repeating the multiple regression analysis with this quadratic term omitted resulted in only one minor change in the outcome: The linear component for CDM (response residuals) became non-significant. The quadratic trend is difficult to interpret, because its sign in any particular data set depends on which alternative is assigned to the numerator and which to the denominator of response and reinforcer ratios. Because this assignment is essentially arbitrary, we were surprised to find positive quadratic trends in these analyses. Because the majority of studies included in our sample used the two-key procedure, and authors generally assign the manipulandum on the subject's left to the numerator and that on the right to the denominator, it may be that this pattern is somehow related to position preferences. However, these quadratic patterns are not predicted by either the GML or the CDM, and we do not consider them further here.

A number of criteria have previously been proposed to help assess the merits of a model as a theoretical explanation of data, and choose between competing models (Pitt et al., 2002). These include qualitative as well as quantitative criteria. Although this paper is concerned with a quantitative method of model selection, it is worth noting that, with regards to the qualitative criteria, the CDM does have an advantage over the GML. The proportional confusion parameter in the CDM has a clearer interpretation than the sensitivity parameter in the GML (Davison and Jones, 1995). The GML is not explicit about the mechanisms responsible for variations in the value of a (although Baum, 1974b, 1979, did speculate about factors that may affect it). The CDM, on the contrary, is clear about the mechanism: contingency discriminability. Where choice alternatives cannot be distinguished perfectly by the subject, or where delays or other events occur between an instance of reinforcement and the response that produced it, reinforcers obtained for making one response might strengthen the alternative response.

In considering the differences between their results and those of Davison and Jones (1995), Baum et al. (1999) noted that the CDM "may apply to situations in which the concurrent alternatives are imperfectly discriminated" (p. 371). Davison and Jones (1995) arranged the alternative responses according to the method used by Findley (1958), whereby responses to a changeover key changed the schedule and stimulus associated with a main key. The stimuli associated with the different schedules differed only in brightness, making them more confusable than the color stimuli often used with this arrangement. It is possible that the alternatives in the Findley (1958) procedure may generally be less well discriminated by subjects than those in a two-key procedure in which the schedules are associated with separate manipulanda. However, we could not test this hypothesis with the present data set, which contained only three studies that used the Findley's (1958) switching-key procedure, contributing just six data series in total (Catania, 1963; Lobb and Davison, 1975; Leigland, 1979). Future research could extend the present study by comparing studies, which have used the switching-key procedure with those that have used two-key procedures. If Baum et al.'s (1999) suggestion is correct, the data from switching-key procedures might show residual patterns that are more consistent with the CDM, even when readily discriminable stimuli signal different components of the concurrent schedule.

Although only the CDM and GML were considered explicitly here, the evidence for a strictly linear relationship between log response and log reinforcer allocation may be relevant for some other accounts of choice. For example, Baum et al. (1999) proposed a model for choice based on foraging theory. According to their 'fix and sample' model, organisms generally allocate most of their responses to the richer alternative, only occasionally switching to the lean alternative. Baum et al. showed that the fix-and-sample model provided an explanation for undermatching as a bias towards the lean alternative, which implies that the residuals for the generalized matching law should have a negative linear component (see Baum et al's Fig. 7). Because we failed to find evidence of a negative linear component, our results pose a challenge for this model. However, Baum (2002) reanalyzed Alsop and Elliffe's (1988) data and found evidence of the fix-and-sample pattern, so the ultimate status of this account awaits more comprehensive study.

We refer to the methodology used here as residual meta-analysis because, similar to the procedures that have been developed to combine and analyze effect sizes from individual studies (Rosenthal and DiMatteo, 2001), statistical power is greatly increased by pooling results across studies. However, unlike traditional meta-analysis in the social sciences, which is generally concerned with estimating effect sizes and identifying

moderator variables which influence effect size, our goal here was model selection (Cutting, 2000; Pitt et al., 2002) – that is, making a decision about which of several candidate models was most likely to be the 'true' model. The utility of the technique is illustrated by our finding credible evidence in favor of the GML, even though the differences between the fits of the GML and CDM to individual data sets are so small as to make it virtually impossible to distinguish between them by goodness-of-fit within any individual study. Just as meta-analysis has become the *de facto* method for literature reviews in the social sciences, allowing reliable conclusions to be reached based on a body of empirical studies, residual meta-analysis may provide a useful method for choosing between quantitative models of behavior. The necessary requirements for such an analysis include (1) a body of empirical studies in which one (or more) variables have been manipulated parametrically; (2) raw data available that are quantitative, and measured on a common scale across studies; and (3) two or more competing models. Other research areas for which residual meta-analysis might be useful include temporal discounting (Green and Myerson, 2004), choice in concurrent-chain schedules (e.g., Grace, 1994; Mazur, 2001), and response rate under VI schedules (Dallery et al., 2000; Herrnstein, 1970; McLean, 2006).

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