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ABSTRACT

A discussion is presented on the effect of porosity on strength of brittle ceramics. The theoretical approaches to predict the effect of porosity on strength are compared with experimental observations. The discrepancies between the various theories and experiment are explained in terms of the relative size of pore to the size of the Griffith flaws. The effect of porosity on strength is divided into three regions. In region I the pore size is much greater than the Griffith flaw. In this region macroscopic strength exhibits a precipitous decrease on addition of a single pore, the decrease corresponding to the maximum stress concentration at the pore. In region II the pore is of approximately the same size as the flaw, strength exhibiting a precipitous decrease in strength but not to a value predicted on the basis of stress concentrations. In region III the pore size is much smaller than the flaw size, the effect of porosity being independent on stress concentrations but a function of the stress level within the material only.

A quantitative estimate is presented for the effect of porosity on uniaxial strength. It is suggested that the effect of porosity on strength should be at least as great as the effect of porosity on

Young's modulus of elasticity. A porosity clustering factor (λ) is introduced to take into account that fracture may occur in a region of the ceramic body where locally the value of porosity exceeds the average porosity.

I. INTRODUCTION

In the production of bodies composed of brittle refractory ceramics at the temperatures usually employed, complete densification generally is not achieved. Of extreme technological importance is the large effect of the resulting porosity on the mechanical properties of the final ceramic body.

Previous approaches[1,2] used to theoretically predict the effect of porosity on tensile strength have been based on the effective decrease in cross-sectional area of the ceramic body due to the presence of the pores. Knudsen[1] considered the net cross-sectional area of hypothetical porous bodies obtained by sintering together spherical particles in various types of packing. Brown et al.[2] used the same approach and predicted the strength of matrices containing pores of various shape. Based on the continuum mechanics approach that a material will fail when a certain stress level is reached, the effect of porosity on strength can also be predicted by calculating the stress concentrations around pores of various geometries.[3-6]

In general, the "cross-sectional area" approach predicts a smooth monotonic decrease in strength from the value of zero-porosity strength, with increasing porosity. The "stress concentration approach", however, predicts an instantaneous decrease in strength upon introduction of the first pore in the body, no matter how small, as stress concentration values are independent of pore size. The stress concentration approach predicts a relative reduction in strength equal to the maximum stress concentration factor for the pore shape and stress condition.

Experimentally,[1,7-10] the change in tensile strength of polycrystalline ceramics with increasing porosity generally is thought to show a smooth, but rapid, decrease from the strength of the zero-porosity material, in qualitative agreement with the prediction based on the cross-sectional area approach. The present writers,[11] however, obtained data for the effect of spherical porosity on the tensile strength of a glass in qualitative and quantitative agreement with the predictions based on stress concentrations.

A discrepancy, therefore, exists between the theoretical approaches as well as an apparent discrepancy between theory and experimental observations. Also, variations appear to exist in the manner in which porosity affects tensile strength. It is the purpose of this paper to present an alternative approach to the prediction of the effect of porosity on strength in order to clarify the theoretical approaches and to explain the apparent experimental discrepancies.

II. DISCUSSION

Among the numerous approaches[12] to predict the strength of a material under various stress conditions is the "maximum stress theory". In this theory the material is considered to be completely homogeneous throughout the body being tested and is assumed to fail when a predetermined level of stress is reached anywhere within the specimen. For the prediction of the effect of porosity on strength, both the "stress concentration" approach as well as the "cross-sectional area" approach are based on this criterion. The stress concentration approach computes the maximum stress near a pore and predicts that when this computed stress reaches the value of the strength of the nonporous material, failure will occur. In effect, the cross-sectional area approach uses the same criterion and computes an average stress concentration for the remaining material while ignoring the local stress concentrations near the pores.

In predicting the effect of porosity on strength, it should be realized that the materials with which the investigator is concerned generally are practical engineering materials, the actual strength of which is a few orders of magnitude less than the theoretical strength calculated on the basis of interatomic forces.[13] This discrepancy has been attributed by Griffith[14] to the existence of microscopic cracks or flaws contained within the material or on the material surface. The existence of these flaws has been verified by numerous investigators, excellent reviews recently having been presented by Ernsberger[15] and Philips.[16]

The writers suggest that a proper estimate of the effect of porosity on the tensile strength of an industrial brittle ceramic should be based on an estimate of the effect of the pores on the Griffith flaws. The effect of a pore on a Griffith flaw should be a function of the relative size of the pore as compared with the size of the flaw.

An approximate value for the flaw size can be calculated from the Griffith relation for the strength (S_o) of a uniformly stressed brittle material containing a flaw of length d, given by

$$S_o \approx \left(\gamma \frac{E}{d}\right)^{1/2}, \qquad (1)$$

where γ is the surface energy of the material, E is Young's modulus of elasticity, and d is the flaw size.

On the basis of Eq. (1) for a high strength dense alumina, Passmore et al.[10] found a flaw size approximately equal to the grain size. A similar flaw size was also found by Guard and Romo.[17] The microstructure of the alumina investigated by Passmore et al.[10] revealed a pore size at least one order of magnitude smaller than the grain size, i.e., the pore size was considerably smaller than the flaw size. For the low strength glass investigated by the present writers[11] a flaw size of approximately 40μ was determined, whereas the pore size measured approximately 60μ, i.e., the pore size was of the same order as the flaw size. Therefore, it appears that large variations occur for the ratio of flaw size to pore size.

Examination of the theoretical solutions for the stress concentrations near pores shows that for all practical purposes the volume of material subjected to the stress concentrations extend over a distance of the order of the diameter of the pore. As a consequence, for a pore size much smaller than the flaw size only a small segment of the flaw is subjected to the stress concentrations due to the presence of the pores.

It appears reasonable to assume that the Griffith criterion (Eq. 1) is valid only when a major part of the flaw is located in material subjected to the stress, S_o. When only a small segment of the flaw is subjected to the stress concentrations, the stress concentrations no longer affect strength. Failure is governed entirely by the increase in the stress within the material of a porous body.

On this basis, the authors should like to suggest that the effect of porosity on strength, i.e., the Griffith flaws, can be divided into three regions. A sufficiently high flaw density is assumed.

In region I the pore size is substantially larger than the flaw size such that a flaw lies entirely in material stressed to the maximum value of stress concentration. Engineering structures where drilled holes, grooves, etc., represent the porosity fall in this region. Here the stress concentration approach can be applied successfully, the structure failing when the maximum stress concentration exceeds the strength of the nonporous material. In this region the effect of porosity on tensile strength will exhibit an instantaneous decrease in strength upon introduction of the first pore in the body. The decrease in strength will correspond to the maximum stress concentration factor. The recent results obtained by the present writers[11] for the biaxial tensile strength of a glass containing artificial spherical pores appear to be representative of the effect of porosity on tensile strength in region I.

In region II the flaw size is of the order of the pore size such that only a segment of the flaw is subjected to the stress concentration. The effect of porosity on strength in this region will exhibit a precipitous decrease in strength upon introduction of the first pore but not to a value corresponding to the calculated maximum stress concentration. The recent results for the uniaxial tensile strength[11] of a glass containing spherical pores appear to fall in this region.

In region III the pore is considerably smaller than the Griffith flaw. The flaw will be completely unaffected by the stress concentrations near the pores. Strength should exhibit a monotonic decrease with increasing porosity, without the precipitous decrease in strength characteristic of regions I and II. The effect of porosity on high strength polycrystalline industrial ceramics should fall in this region, as well as many high strength ceramics investigated in the laboratory. It is in this region where the "cross-sectional area" theories of Knudsen[1] and Brown et al.[2] appear to be most applicable. As these theories are not restricted to pore size, they should be applicable to engineering structures as well as brittle ceramics containing small pores. However, care should be taken in predicting the effect of porosity on strength using these theories in regions where stress concentrations are the governing factor. In particular, at low porosities the cross-sectional area approach may overestimate strength by a considerable amount.

Once the effect of stress concentration on the tensile strength has taken place in regions I and II, further changes in strength with increasing porosity are governed by the increase in stress within the material of the porous body, due to the decrease in material available to carry the applied load. As the stress distribution around pores of a given shape is independent of the pore size, it is suggested that the relative increase in stress in the remaining material upon an increase in porosity is identical for all regions.

An approximate estimate of the increase in stress level within the material of the porous body with increasing porosity can be obtained by considering the average strain of the porous body under conditions of an applied load. The discussion will apply to uniaxial tensile loading and will be confined to small values of porosity such that all porosity effects can be expressed in terms of linear equations and all second order effects can be neglected. In this manner the average deformation of the porous body can be obtained from the macroscopic value of Young's modulus of elasticity, which can be expressed by

$$E = E_o (1 - \alpha_E P) , \tag{2}$$

where E and E_o are Young's modulus of the porous and nonporous body, respectively, P is the volume fraction porosity, and α_E is a constant.

Under conditions of an applied uniaxial load (σ_o), the average strain (ϵ) in the material must equal

$$\epsilon = \frac{\sigma_o}{E_o (1 - \alpha_E P)} . \tag{3}$$

As Young's modulus of the material in the porous body remains unchanged, the average stress (σ) in the material must equal

$$\sigma = \frac{\sigma_o}{(1 - \alpha_E P)} . \tag{4}$$

As conditions of compatibility of stress and strain must be satisfied throughout the porous body, on the basis of Eq. (4), it is suggested that the stress distributions for a porous body of finite size can be obtained by multiplying the stress in a body of infinite extent[3-6] by the factor $1/(1 - \alpha_E P)$.

The experimental results for the effect of porosity on strength can be expressed by

$$S = \frac{S_o}{K} (1 - \alpha_S P), \tag{5}$$

where S and S_o are the tensile strength of the porous and nonporous body, respectively, α_S is a constant, and K is a constant which ranges from the value of unity for region III to the maximum value of stress concentration factor in region I.

On the assumption that the porous body will fail when the stress in the material reaches the value of S_o/K (i.e., $\sigma = S_o/K$), comparison of Eqs. (4) and (5) result in the equality

$$\alpha_S = \alpha_E \; . \tag{6}$$

Equality (6) then suggests that the relative effect of porosity on uniaxial tensile strength should equal the relative effect of porosity on Young's modulus of elasticity. This conclusion can also be obtained by considerations of the concept of the existence of a fraction of "stress-free" material within the porous body, proposed elsewhere.[18] Equality (6) is also supported by the derivations by Gurney[19] who calculated a stress concentration factor for the effect of holes on the strength of a flaw-free glass given by

$$\text{stress concentration factor} = \frac{\Delta E/E}{\Delta V/V} \tag{7}$$

which using Eq. (2) also leads to the equality shown in Eq. (6).

Equality (6) is supported for the effect of spherical porosity on the uniaxial strength of a glass[11] with $\alpha_S \approx 2$, which is in good agreement with the theoretical[20-22] value of $\alpha_E \approx 2$. Many experimental results, however, show the effect of porosity on uniaxial tensile strength of polycrystalline ceramics to be somewhat greater than the relative effect on Young's modulus. A reasonable explanation can be based on the realization that within the porous body local variations in porosity will occur. Fracture is more likely to be initiated in material near regions of higher porosities than in material near regions of average or lower porosity. A porosity clustering factor (λ) can be introduced which describes the ratio of the maximum value of local porosity (P_{loc}) to the average porosity (P) defined by

$$\lambda = P_{loc}/P \,. \tag{8}$$

For a better estimate for the effect of porosity on uniaxial strength, the quantity (P) in Eqs. (3), (4), and (5) should be replaced by $P_{loc} = \lambda P$ with the result that

$$\alpha_S = \lambda \alpha_E \tag{9}$$

which upon substitution in Eq. (5) results in

$$S = \frac{S_o}{K} (1 - \lambda \alpha_E P) \,. \tag{10}$$

A recent compilation[18] of values of (α_E) and (α_S) for polycrystalline alumina prepared by various techniques suggests a porosity clustering factor $\lambda \approx 1.5$, i.e., the maximum local porosity (P_{loc}) lies approximately 50% above the value of average porosity, which cannot be considered unreasonable. The recent results of Fryxell and Chandler[9] for the mechanical properties of beryllium oxide with the quantity (α_S) on the average only slightly greater than (α_E) suggest a highly uniform distribution of porosity in the beryllium oxide investigated. The rather high value of α_S (≈ 12) as found by Passmore et al.[10] for hot-pressed alumina compared to the theoretical[20-22] value of α_E (≈ 2)

for the observed spherical porosity suggests a clustering factor $\lambda \approx 6$ which infers a pore distribution which is rather nonuniform.

As an example of Eq. (10), Fig. 1 illustrates the effect of spherical porosity with a porosity clustering factor $\lambda = 1.5$ on the uniaxial tensile strength of a brittle ceramic material, in all three regions for the effect of porosity on strength. Under these conditions the maximum stress concentration factor, K, is equal to 2 and $\alpha_E \approx 2$.[20-22] In Fig. 1 the curve for region II is drawn arbitrarily between the curves for regions I and III. It is suggested that equivalent curves for polycrystalline ceramics may fall well below the curves in Fig. 1 due to higher values of K, α_E, and λ.

Equality (6) should be strictly valid only for the effect of porosity on uniaxial tensile strength. For the effect of spherical porosity on a uniform biaxial tensile strength of a glass[11] the present authors found a value of $\alpha_S \approx 1$. This suggests that the effect of porosity on biaxial tensile strength of polycrystalline ceramics may be less severe than the effect of porosity on uniaxial tensile strength. At present, experimental data are not available which could substantiate this hypothesis.

Although the present discussion has limited itself to the effect of stress inhomogeneities due to porosity, the same arguments can be applied to matrices containing any kind of elastic discontinuity giving rise to stress inhomogeneities. For instance, the recent experimental results for uniaxial and biaxial strength of a glass matrix containing spherical alumina dispersions[11] previously interpreted in terms of volume of material under stress can also be interpreted in terms of ratio of flaw size to size (or volume) over which the stress inhomogeneity acts.

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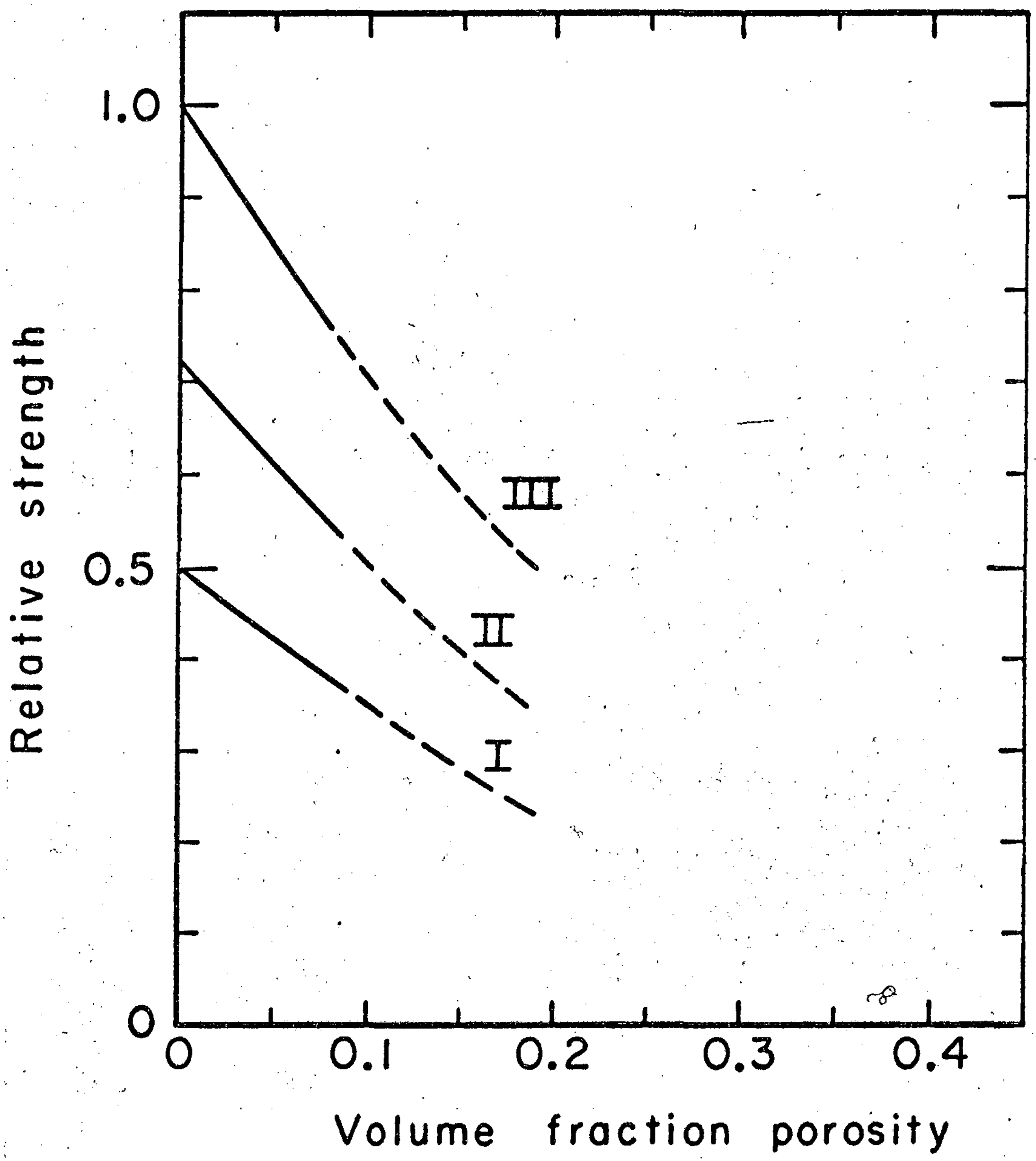

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Fig. 1. Effect of spherical porosity on uniaxial strength of a brittle ceramic.