Forward Bias, Uncovered Interest Parity and Related Puzzles*

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Abstract

Uncovered interest parity is widely used in open economy macroeconomics. But when exchange rates are flexible the evidence rejects UIP and implies forward bias. There are many suggested explanations for this failure of UIP and forward bias, but none are widely accepted, at least partially because none appear to explain the related puzzles discussed below. This article shows how sterilized "leaning against the wind" and a combination of the inflationary and liquidity effects associated with open market operations can explain forward bias and the failure of UIP even when expectations are rational. They also appear to be able to explain the related puzzles.

Keywords
Risk Premiums, Rational Expectations, Uncovered Interest Parity, Intervention, Covered Interest Parity, Forward Bias, Liquidity Effects

1. Introduction

As [1] points out, uncovered interest parity (UIP) is one of three key international financial relations used repeatedly in open-economy macroeconomics. But the evidence rejects UIP and implies forward bias. With flexible rates, standard test equations that should produce coefficients of 1.0 routinely produce negative coefficients, some of which are significant. For a discussion of that evidence see [2].

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# Author’s Note: Unlike most economists, I am a Logical Positivist. As such I assume that all theories are false, but that some are less false than others. I offer this explanation for forward bias and the failure of UIP not as something that is or could be “true”, but as something that I believe could be less false than any of the current alternatives.

But as Section 2.3 points out, those negative coefficients disappear under the gold standard and with pegged exchange rates. This article shows for the first time how sterilized “leaning against the wind” and/or a combination of liquidity and inflationary effects associated with open market operations due to macro-economic stabilization produce negative coefficients under flexible exchange rates.

The importance of forward bias and the failure of UIP have prompted many attempts to explain them, but none has distinguished between flexible and managed exchange rates. Two of the most widely cited explanations are risk premiums and the failure of Rational Expectations. For a discussion of those two alternatives see [2].

Less frequently cited alternatives include the following: using interest rates to stabilize exchange rates [5], a tradeoff between interest rate and exchange rate stability [6], uncertain monetary policy [7], nonlinearities [8], fads and fashion [9], perpetual learning [10], adverse selection [11], career risk [12], deep habits [13], infrequent portfolio decisions [14], carry and momentum trading [15], and over confidence [16]. A recent book, [17], uses an inter-temporal version of UIP with systematic errors in expectations to explain the apparent failure of UIP. But the puzzle as to why the two theories fail so badly under flexible rates, but not managed rates, remains.

All these explanations, including the one proposed here, were developed to explain forward bias and/or the failure of UIP. As a result, that bias and/or failure does not provide empirical support for any of the explanations. There are, however, several related puzzles. They are 1) the Carry Trade Puzzle, 2) the Commodity Puzzle, 3) the Development Puzzle, 4) the Inflation and Outlier Puzzles, 5) the Maturity Puzzle and 6) the Time Dependency Puzzle. Also explaining these puzzles would support explanations for forward bias and the failure of UIP. For one theory to explain all these puzzles would be a major contribution to open-economy macroeconomics.

Section 2 reviews forward bias and the failure of UIP under flexible rates. It then reviews the evidence for managed exchange rates. Section 3 discusses Covered Interest Parity (CIP) because CIP plays an important role in forward bias and the failure of UIP. Section 4 discusses the basic ideas behind the model and some of its important assumptions. Section 5 describes the model and its important implications. Section 6 discusses how well the model, and the ideas behind it, explains the related puzzles. Section 7 is a brief summary.

2. Primary Puzzles

This section begins with the two primary puzzles: forward bias and the failure of UIP under flexible exchange rates. It then reviews the relevant evidence for managed exchange rates.

2.1. Uncovered Interest Parity

Discussions of UIP often take the economic theory behind it for granted, but the
theory behind UIP is not obvious. There are at least two different approaches.

Equation (1) describes UIP. The expected change in the exchange rate equals an appropriate interest rate differential.

\[ E(\Delta s_{t+k} | I_t) = i_t - i_t^* \]  

where \( s_t \) is the log of the domestic price of foreign exchange, \( E(x_{t+k} | I_t) \) is the conditional expectation of \( x_{t+k} \) based on the information set \( I_t \) available at \( t \). \( i_t \) and \( i_t^* \) are risk-free domestic and foreign interest rates with the same maturity as \( s_{t+k} \).

Expectations are “rational” in the sense of Muth [18], when \( I_t \) contains all of the current information in the model.

One approach to UIP begins with CIP and assumes that speculation eliminates expected speculative returns by eliminating the difference between \( E(s_{t+k} | I_t) \) and the log of the appropriate current forward rate denoted \( f_t \). Together they imply Equation (1). This approach ignores transaction costs and risk premiums, and seems to implicitly assume that expectations are rational.

Given the large body of evidence supporting CIP, when UIP fails, it is natural to question the assumption that \( E(s_{t+k} | I_t) = f_t \). That question appears to be the source of the idea that risk premiums “cause” UIP to fail. Less than Rational Expectations provides another potential explanation for the failure of this version of UIP.

A second approach uses the Fisher equation, an expectations version of purchasing power parity and the simplifying assumption that real interest rates are equal.\(^3\) When nominal interest rates equal real rates plus expected rates of inflation, \( i_t - i_t^* \) equals the appropriate difference in expected rates of inflation plus the difference in real interest rates. With real differentials zero, the Fisher equation and an expectations version of PPP imply Equation (1) because \( E(\Delta s_{t+k} | I_t) \) and \( i_t - i_t^* \) both depend on expected inflation. Like the first approach this one ignores transaction costs and risk premiums, and seems to implicitly assume rational expectations.

Under both approaches, Equation (2) is the standard test equation where expectations are rational, \( e_{t+k} \) has a zero mean, is uncorrelated and is orthogonal to \( i_t - i_t^* \).

\[ \Delta s_{t+k} = a + b(i_t - i_t^*) + e_{t+k} \]  

UIP implies that estimates of \( b \), denoted \( \hat{b} \), should equal 1.0. With flexible exchange rates, \( \hat{b} \) are routinely negative, and often negative and significant. But as the review of the relevant evidence in Section 2.3 shows, \( \hat{b} \) are seldom negative, and hardly ever negative and significant, with managed exchange rates.

### 2.2. Forward-Bias

The modern forward-bias puzzle begins with [21] who splits \( f_t \) into \( E(s_{t+1} | I_t) \) and a “premium” denoted \( p_t \).

\(^2\)This is the approach used in [19].

\(^3\)This appears to be the approach used in [20].
Although subsequent literature almost universally calls \( p_t \) a risk premium, [21] does not. He first mentions \( p_t \) in his Abstract without any mention of a “risk premium”. He then points out that Equation (3) is no more than a particular definition of the premium component of the forward rate. To give this equation economic content, a model that describes the determination of \( p_t \) is required. Equation 12 provides that economic content.

Although almost all the relevant literature refers to \( p_t \) as a risk premium, from this point on, \( p_t \) is the expected return to speculation. If \( E(s_{t+1} | I_t) \) is less than \( f_t \), a speculator expects to make a profit by selling the currency forward at \( f_t \) and then buying the currency in the future to cover the forward sale at the lower \( E(s_{t+1} | I_t) \).

As [2] points out on page 678, given the analysis of risk-premium models in general or partial equilibrium, it is hard to explain excess returns in forward foreign exchange by an appeal to risk premiums; either \( \varphi \), the coefficient of relative risk aversion, must be incredibly large, or else the conditional covariance of consumption must be incredibly high.

Rather than concentrating on risk premiums in the equilibrium condition that risk premiums equal expected speculative returns, Section 4 concentrates on showing how sterilized intervention and a combination of inflationary and liquidity effects produce expected speculative returns. Those expected returns produce forward bias and risk premiums even when expectations are rational.4

Whatever the interpretation of \( p_t \), Equation (3) implies Equation (4).

\[
f_t - s_t = E(\Delta s_{t+1} | I_t) + p_t
\]

Assuming Rational Expectations and rearranging Equation (4) produces Equation (5).

\[
\Delta s_{t+1} = (f_t - s_t) - p_t + \varepsilon_{t+1}
\]

Omitting \( p_t \) produces the “Fama equation”.

\[
\Delta s_{t+1} = \alpha + \beta(f_t - s_t) + \xi_{t+1}
\]

where \( \hat{\beta} \) is the estimated \( \beta \).

\( \hat{\beta} \) between countries with flexible rates are routinely negative and often significant. For some recent evidence see Table 2 in [22]. As the next subsection shows, when rates are not flexible, that is not the case.

2.3. Flexible versus Managed

Most discussions of forward bias and the failure of UIP refer to the negative and often significant estimates of \( \hat{b} \) and \( \hat{\beta} \) under the current float. But there also are estimates under the gold standard, when rates were flexible during the early 1920s and under pegged exchange rates, all between developed countries. In addition there are estimates between developed and emerging countries under both

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4The combination of forward bias and rational expectations raises an interesting question about the meaning of an “efficient” market.
“managed” and flexible rates.5

[23] estimates \( b \) between the U.S. and U.K. from 1888 to 1905, the classical gold standard. His \( \hat{b} \) are about 0.5. [24] estimates \( \beta \) between the U.K. and U.S. from December 1921 to May 1925 when rates were flexible. Their \( \hat{\beta} \) are negative and significant.6

[25] pools their daily data and uses SUR to estimate \( b \) under both pegged and flexible exchange rates. Their pegged rates are from the Exchange Rate Mechanism of the European Monetary System from March 1979 to March 1994 where Germany is the home country. Their flexible rates use Australia, Canada, France, Germany, Japan, Switzerland and the U.K. from 1981 to October 1994 where the U.S. is the home country. Using flexible rates, a common intercept and a three month interval their \( \hat{b} \) is \(-0.04\). With specific intercepts it is \(-0.88\). Both \( \hat{b} \) are significantly less than zero.

Using pegged rates, a common intercept and a three month interval their \( \hat{b} \) is 0.54. With specific intercepts it is 0.60. Both \( \hat{b} \) are significantly greater than zero but significantly less than one. Using one month rather than three month intervals produces similar results. These estimates are designed to avoid the peso problem. When uncorrected for peso problems estimates are closer to, but still significantly greater than zero.

[1] estimates \( b \) using annual data for France and the U.K. versus the U.S. for about the 200 years before 1999. Their rolling regressions tell an interesting story. \( \hat{b} \) are generally positive up to the general adoption of flexible rates in the 1970s, they then turn negative until the 1990s where they begin to turn positive again. But this turn to positive appears to be only temporary. Using data from 1987 to 2006, [26] finds that all four of their flexible currencies versus the U.S. dollar have negative \( \hat{b} \) and that two are significant.

[27] and [28] provide \( \hat{\beta} \) between developed countries during the current float with the U.S the home country and between the U.S. and emerging countries with both flexible and managed rates. Whether country pairs are developed or developed and emerging, when rates are flexible \( \hat{\beta} \) are usually negative and at times significant. When rates are managed, \( \hat{\beta} \) are generally positive and not significantly different from zero. (Note that these estimates are not corrected for peso problems.)

This overview of the relevant evidence suggests the following: systematically negative and occasionally negative and significant \( \hat{b} \) and/or \( \hat{\beta} \) are restricted almost exclusively to flexible exchange rates. Whether country pairs are developed or developed versus emerging does not seem to make a difference.7

5“Managed” covers a wide range of regimes from currency boards to crawling pegs. Particularly with emerging countries, currencies can be flexible de jure but not necessarily de facto because of trade and/or capital controls.

6This interval is a bit unusual because the U.K. was deliberately deflating in order to restore the pre WWI value of sterling versus the U.S. dollar, which had maintained the pre WWI gold content of the dollar.

7This result suggests that the Development Puzzle discussed below is not really about development, but rather about managed versus flexible rates with developing countries usually having managed rates and developed countries usually having flexible.
3. CIP

Covered Interest Parity is based on effective arbitrage and is one of the few theories in open economy macroeconomics for which there is convincing empirical support.\textsuperscript{8} [30] probably provides the best available analysis of Covered Interest Parity.\textsuperscript{9} They say that one can safely assume that CIP holds for daily and lower frequency data. Given that strong support, any explanation for the failure of UIP or forward bias that is not consistent with CIP is suspect. Section 4 assumes CIP.

Since, as is typical, [30] uses maturities of one year or less from developed countries with flexible rates, strictly speaking their conclusions only hold under those conditions. Whether or not CIP holds under other conditions is an issue that needs to be resolved.\textsuperscript{10}

Equation (7) describes the theory of Covered Interest Parity.

\[
 f_t - s_t - (i_t - i^*_t) = d_t
\]  

(7)

where forward premiums and interest rate differentials have the same maturity.

For good data deviations $d_t$ should reflect primarily transaction costs, particularly bid-ask spreads. To simplify the discussion, it ignores $d_t$ and assumes that CIP holds exactly.

CIP implies that the failure of UIP and forward bias are two sides of the same coin. With $d_t$ zero, Equation (7) implies (8).

\[
 f_t = s_t + (i_t - i^*_t)
\]  

(8)

Subtracting an appropriate $E(s_{t+1} | I_t)$ from both sides of Equation (8) produces Equation (9).

\[
 f_t - E(s_{t+1} | I_t) = (i_t - i^*_t) - [E(s_{t+1} | I_t) - s_t]
\]  

(9)

The forward bias $f_t - E(s_{t+1} | I_t)$ equals the deviation from UIP $(i_t - i^*_t) - [E(s_{t+1} | I_t) - s_t]$.

4. Background

Subsection 4.1 explains how a combination of liquidity and inflationary effects can produce negative $\hat{b}$ and $\hat{\beta}$. Subsection 4.2 explains how sterilized foreign exchange intervention can produce negative $\hat{b}$ and $\hat{\beta}$. Both sections assume that central banks have the freedom to try to stabilize the macro economy provided by flexible exchange rates. These ideas are not model specific. They should hold in a wide range of reasonable models. One objective of the model described in Table 1 is to show that the failure of UIP and forward bias can be consistent with rational expectations.

Subsection 4.3 takes up the issue of sticky versus flexible prices. Subsection 4.4 takes up the issue of how to model foreign exchange markets.

\textsuperscript{8}Several papers claim that CIP fails after 2008. For a review of that work and an evaluation of CIP after the Great Recession see [29].

\textsuperscript{9}For additional work on CIP see [31] and the work cited there.

\textsuperscript{10}[32] finds unexploited deviations from long-term CIP. [33] finds violations of long-term CIP, but only for emerging markets. [34] seems to suggest substantial deviations from CIP for emerging markets.
Table 1. The model.

<table>
<thead>
<tr>
<th>Expression</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_t - s_t = i_t - i^*_t = \Delta\bar{r}$</td>
<td>Covered Interest Parity. (I)</td>
</tr>
<tr>
<td>$s_t = (\lambda-1)\bar{P}_t + \bar{P}_t - ZX\Delta\lambda_t + z_t$</td>
<td>Flexible spot rate. (II)</td>
</tr>
<tr>
<td>$f_t = \lambda\bar{P}_t + \bar{P}_t - ZX\Delta\lambda_t + z_t$</td>
<td>Flexible forward rate. (III)</td>
</tr>
<tr>
<td>$e_t = Du_t - FX\Delta\lambda_t + v_t$</td>
<td>Changes in the monetary base. (IV)</td>
</tr>
<tr>
<td>$\bar{r} = E\left(\Delta\bar{P}_t</td>
<td>t \right) + \bar{r} + \Lambda e_t - HX\Delta\lambda_t$</td>
</tr>
<tr>
<td>$E\left(\bar{P}_t</td>
<td>t \right) = C(Du_t - FX\Delta\lambda_t + v_t)$</td>
</tr>
<tr>
<td>$\Delta\bar{P}<em>t = C(Du_t - FX\Delta\lambda_t + v_t) + x</em>{t, t}$</td>
<td>Actual inflation. (VII)</td>
</tr>
</tbody>
</table>

Definitions:
- $\bar{P}_t$: Price level differential in logs.
- $\bar{r}_t$: Real interest rate differential.
- $\bar{r}$: Nominal interest rate differential.
- $u_t$: Actual minus natural rate of unemployment.

Shocks and restrictions:
- $v_t = V\nu_t + v_t$, $u_t = U\nu_t + \omega_t$, $\bar{r} = K\nu_t + y_t$, $z_t = z_{t-1} + \epsilon_t$.
- $C$, $D$, $\Lambda$, $\alpha$ and $b$ are all $\geq 0$ while $V$, $U$, $R$ and $\epsilon$ are all $\geq 0$ but less than 1.0.
- Random variables $\nu_t$, $x_t$, $y_t$, $v_t$ and $z_t$ have zero means, zero initial values, are uncorrelated and orthogonal.
- $1 \geq F \geq 0$, $Z = \alpha(1+F)X$ and $H = h(1-F)$.

4.1. Liquidity and Inflationary Effects

Keynesian models generally ignore the inflationary effects of expansionary monetary policies. Many non-Keynesian models ignore the liquidity effect of those same policies. Combining liquidity and inflationary effect produces an explanation for forward bias and the failure of UIP.

Inflationary effects of expansionary monetary policies produce depreciation. Liquidity effects can produce negative interest rate differentials. The resulting positive $\Delta s(t)$ and negative $i_t - i^*_t$ imply negative $\hat{b}$ and $\hat{\beta}$. This reasoning is consistent with a wide range of macro models.

There is substantial evidence that monetary policy is a source of forward bias and the failure of UIP. A large literature uses vector auto-regression to analyze how shocks to monetary policy affect UIP. It includes [35]-[43]. Most articles claim that policy shocks create at least temporary deviations from UIP.

4.2. Leaning against the Wind

Central banks lean against the wind when they buy a foreign currency as the price falls and sell as the price rises. They do so to reduce the short-run volatility in exchange rates while at the same time allowing fundamental forces to determine exchange rates in the long-run.

Central banks sterilize to prevent their intervention from affecting the monetary base in ways that would conflict with other policy objectives. They sterilize their intervention by acquiring short-term securities as they sell foreign exchange, which leaves the monetary base unchanged. Referring to the era of flexible exchange rates, [44] reports that almost 90% percent of central banks some-
times or always lean against the wind and that 40% fully sterilize while only 30% never sterilize.

When two plausible conditions hold, sterilized leaning against the wind produces negative $\hat{b}$ and $\hat{\beta}$. First “leaning against the wind” introduces inertia into changes in exchange rates and second that sterilized and unsterilized intervention affect nominal interest rate differentials differently.

There is empirical support for the first assumption. [45] finds that sterilized intervention introduces inertia into changes in exchange rates. Although she attributes it to a signaling effect, in her review of the early literature [46] refers to a similar temporary effect. In a review of the later literature [47] also mentions a similar temporary effect. More recently [48] finds evidence of inertia. [49] probably provides the best evidence because it is able to avoid simultaneity by assuming that, in the absence of intervention, exchange rates are random walks; an assumption that [50] supports.

The second assumption holds when domestic and foreign assets are not perfect substitutes and central banks have a desired level of foreign exchange that they want to hold at the foreign central bank. In that case, fully sterilized intervention leaves both the domestic and the foreign monetary base unchanged. But the change in the relative stock of short-term assets alters relative interest rates.\footnote{Suppose the Fed buys pound sterling and sells an equivalent amount of U.S. Treasury bills. The stock of T bills held by the public increases, tending to increase U.S. T bill rates. To maintain the desired level of sterling deposits at the Bank of England, the Bank, acting on instructions from the Fed, buys an equivalent amount of U.K. bills. The stock of U.K. bills held by the public declines, tending to lower U.K. T bill rates. The net result is a rise in U.S. minus U.K. T bill rates.}

There is evidence that intervention produces negative $\hat{b}$. [51], [52] and [53] all find that intervening in foreign exchange markets causes UIP to fail.

4.3. Sticky Prices

One key issue in macroeconomics is sticky versus flexible commodity prices. This article assumes that commodity prices are both sticky and flexible. It depends on the commodity market: auction versus wholesale and retail. It also depends on the monetary regime: stable versus unstable.

4.3.1. Market

Under stable monetary regimes, as is widely recognized, information and transaction costs produce sticky wholesale and retail prices. Similar information and transaction costs make arbitrage difficult in wholesale markets and impossible in retail markets. One cannot buy grapes at 99 cents per pound at one grocery store and walk across the street and sell them for $2.00 a pound at another grocery store. At the wholesale level if, after accounting for the exchange rate, Kleenex prices are much higher in Japan than the United States, marketing contracts prevent one from buying Kleenex wholesale in the U.S. and selling them in Japan.

On the other hand, prices in auction markets like those for many agricultural
products, metals and petroleum products are highly flexible because the cost per dollar traded is much lower when trading a “homogenous” product by the truck or shipload rather than by the ounce or pound.

Recognizing both “sticky” and “flexible” commodity prices raises issues widely ignored in the debate over purchasing power parity or PPP. Almost all empirical work uses retail prices where arbitrage is impossible and the law of one price or LOP does not apply. But as [57] points out, arbitrage and the LOP are the basic building blocks of PPP. As a result, there is a disconnect between testing and theory. Testing PPP uses primarily retail prices where arbitrage is not possible, while theory assumes effective arbitrage.

This article assumes that arbitrage is effective in all auction markets and that expectations are rational in those markets. Of course neither assumption holds in wholesale and retail markets. That assumption affects the way this article treats PPP.

4.3.2. Regime
There should be no disagreement over the issue of “sticky” wholesale and retail prices under stable monetary regimes. Sticky prices are an implication of basic microeconomics. See for example [60]. Holding prices constant has benefits. For example it increases demand by reducing price uncertainty. It also has costs, e.g., larger inventories. In general the solution to this optimization problem will be neither perfectly rigid nor perfectly flexible prices. That is sticky prices.

The same reasoning implies that wholesale and retail prices should be more flexible under both inflation and deflation because both impose additional costs. Inflation increases the cost of holding prices constant by further increasing the need for inventories. Holding prices constant with deflation reduces quantity demanded.

As far as I am aware, there is no convincing evidence one way or the other as to whether prices are more flexible with inflation than with comparable deflation. But there is impressive evidence that prices become very flexible under very unstable monetary regimes, i.e., hyperinflation. See for example the German hyperinflation described by [61] and the Austrian hyperinflation described by [62], both in the early 1920s.

The French experience in the early 1920s is more interesting because it contains moderate inflation and deflation. See [63]. There are only wholesale price indexes for that experience because it was before the development of consumer price indexes. But it is clear that prices were flexible in both directions. To the best of my knowledge, no one has analyzed that experience to determine whether or not prices were more flexible upward than downward.

The model described in Table 1 assumes a stable monetary regime because

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12Like PPP, the LOP is based on effective arbitrage. The mistaken belief that commodity arbitrage and the LOP fail is based on articles like [54], [55] and [56]. But they do not really test either commodity arbitrage or the LOP because they all use prices from retail markets where arbitrage is impossible.

13For some evidence that arbitrage is effective in commodity markets, see [58] and [59].
almost all the evidence for forward bias and the failure of UIP comes from stable monetary regimes. That assumption does not mean that the model would not apply to less stable regimes.

4.4. Stock versus Flow

Since the late 1970s most open economy macro models have assumed an asset approach to exchange rates in which asset market equilibrium determines exchange rates. That approach has been a total failure. I know of no one who claims that we can predict, or even “postdict”, the behavior of exchange rates, particularly the short-run behavior, any better today than we could before the asset approach. For that reason, the model in Table 1 assumes a “flow” model in which a flow demand for imports, supply of exports, international investments and portfolio adjustment interact to determine the exchange rate.

5. Model

The formal model begins with Equation (I) in Table 1 that describes CIP. Like the rest of the model it ignores transaction costs. Equation (II) describes a flow market for spot foreign exchange, e.g., the dollar price of sterling, where exchange rates are flexible. Equation (III) describes the corresponding forward market. As required by CIP, subtracting (II) from (III) produces the short-term nominal interest rate differential \( \tau \) on the right-hand side. With the log of the exchange rate \( s(t) \) exogenous in Equation (I), which is often an implicit assumption in the relevant literature, \( \lambda - 1 \) is zero in Equation (II).

The primary role of Equation (III) is to show that the model is consistent with CIP. Assuming that \( \lambda - 1 \) is zero simplifies the later derivations of \( \hat{b} \) and makes them more consistent with the relevant literature without distorting the main implications of the model.

With \( \lambda - 1 \) zero, three factors determine spot exchange rates: central bank intervention captured by \( ZXs_n \), exogenous shocks captured by \( z_t \) and relative price levels captured by \( \bar{P}_r \). For simplicity, the foreign price level is assumed to be constant, which implies that foreign expected inflation is zero.

\( ZXs_n \) describes the most common form of intervention when exchange rates are flexible, leaning against the wind. Central banks lean against the wind by selling a currency as its price rises and buying as its price falls.

\( X \) describes the Fed’s response to a given \( \Delta s \). The larger \( X \) the more the Fed leans against the wind. \( Z \) describes how that intervention affects \( \Delta s \). A positive \( Z \) introduces positive autocorrelation in \( \Delta s \).

Both \( Z \) and the resulting change in the monetary base depend on sterilization.

The Fed sterilizes its intervention by selling (buying) assets like U.S. T bills as it

14 Almost any market can be modeled in terms of stocks or flows. For example the market for cars is usually modeled as the daily, weekly or monthly flow demand for and supply of cars. But it also can be modeled as an existing stock of cars and a demand for that stock with the resulting stock equilibrium determining the price and that price determining the rate of output.

15[64] tests the two approaches and concludes that the evidence on balance rejects an asset approach.
buys (sells) sterling. With complete sterilization, intervention does not affect the monetary base and $F$ in Equation (IV) is zero. With no sterilization, $F$ is one and each dollar’s worth of sterling that the Fed sells reduces the domestic monetary base by one dollar. There is a general consensus that sterilization reduces $Z$. There is a less general consensus that $Z$ is positive even when sterilization is complete. For the reasons discussed in Section 4.2, $Z$ is positive even when $F$ is zero.

Selling spot sterling reduces the Fed’s holdings of sterling deposits at the Bank of England. To restore those deposits to their desired level, at the direction of the Fed, the Bank of England sells U.K. assets like T bills that it holds in the Fed’s account at the Bank. Those sales prevent the intervention from affecting the monetary base in the U.K. Buying sterling as its price falls does the opposite.

As a result, when a central bank fully sterilizes, intervention does not affect either the domestic or the foreign monetary base. But purchases (sales) of domestic short-term assets tend to lower (raise) domestic short-term interest rates while sales (purchases) of foreign short-term assets tend to raise (lower) foreign interest rates. As a result, when leaning against the wind causes the domestic central bank to sell foreign exchange, full sterilization causes $\tilde{T}$ to fall.

If a central bank does not sterilize, then the same sale of foreign exchange reduces the domestic monetary base, which tends to restrict domestic short-term credit and increase domestic short-term interest rates. That sale also increases the foreign monetary base, which tends to increase foreign short-term credit and lower foreign short-term interest rates. As a result, unsterilized intervention tends to increase $\tilde{T}$.

$\tilde{P}_t$ in Equation II captures Purchasing Power Parity. $\tilde{P}_t$ uses a ratio of auction prices where the weights are identical. It does not use retail prices because arbitrage is not possible in retail markets. There are other reasons for using auction rather than retail prices. First, at the retail level the distinction between traded and non-traded goods is an illusion. At the retail level all goods are effectively non-traded. No one buys shoes at Marks & Spencer in London and sells them to Macy’s in New York. On the other hand, all goods with auction markets are traded. Second, it does not seem appropriate to use retail prices, which are not driven by expectations, to explain auction exchange rates, which are driven by expectations.

If both central banks target price levels, they stabilize $\tilde{P}_t$ and the exchange rate. In that case exchange rates would be stationary as they were under the true gold standard where gold flows stabilized relative price levels.

If either central bank targets inflation, price ratios tend to have unit roots because at least one central bank allows changes in its price level to accumulate. Since Taylor rules target inflation, not price levels, they would imply that $\tilde{P}_t$ have unit roots. Here exchange rates are flexible and $\tilde{P}_t$ is roughly a random walk.
\(\varepsilon_t\) in Equation (IV) describes changes in the domestic monetary base.\(^{16}\) Most central banks use a short-term interest rate like the Fed funds rate as their primary policy tool. With a positive \(D\), when unemployment is above the natural rate and \(u_t\) is positive, the Fed lowers the Fed funds rate. To make that lower rate effective the Fed acquires short-term assets, which increases the reserves of commercial banks and the monetary base. When \(u_t\) is negative, it does the opposite. The question is whether or not other short-term interest rates fully respond to changes in the Fed funds rate.

In Table 1 responses to \(u_t\) are symmetric, but only for simplicity. I know of no central bank that responds symmetrically to high and low unemployment.

For simplicity, the model uses unemployment to illustrate how macro-stabilization produces the inflationary and liquidity effects that produce negative \(\hat{b}\) and \(\hat{\beta}\). Stabilizing prices has similar effects. Suppose the Fed raises the Fed funds rate to dampen inflation. To make the higher rate effective, it reduces its portfolio, which reduces the domestic monetary base and the legal reserves of depository institutions. Liquidity effects raise other short-term interest rates relative to what they would have been. Deflationary effects lower them and the exchange rate relative to what they would have been. UIP fails because \(\tau_t\) is larger than \(\Delta s_t\).

A positive \(X\) implies intervention. With full sterilization, intervention does not affect the monetary base. With less than full sterilization, selling sterling reduces the monetary base and buying increases it. \(v_t\) captures random changes in the base due to things like bad weather.

Equation (V) describes the nominal interest rate differential \(\tau_t\) for short-term financial assets like three month Treasury bills. The Fed responds to a positive \(u_t\) by lowering the Fed funds rate. How lowering the Fed funds rate affects \(\tau_t\) depends on the inflationary and liquidity effects of the expansionary open market operation used to reduce the Fed funds rate. \(C\) describes the inflationary effects and \(\Lambda\) describes the liquidity effects. If \(C - \Lambda\) is negative expansionary open market operations lower \(\tau_t\) and there is a net liquidity effect.

When \(\Lambda\) is zero and there is no intervention, the Fisher equation holds. In that case, the model implies that increasing the monetary base by \(x\%\) percent per year increases prices by \(x\%\) per year. With foreign prices constant for simplicity, the exchange rate rises by \(x\%\) per year. With foreign expected inflation zero for simplicity, \(\tau_t\) and \(\Delta s_t\), both positive, \(\hat{b}\) is positive. If \(\tau_t\) is constant, \(\hat{b}\) is one. This link between expansionary open market operations, inflation, interest rate differentials and depreciation seems to be the basic idea behind UIP.

Liquidity effects weaken that link. Even with \(\tau_t\) constant, a liquidity effect, i.e., a positive \(\Lambda\), implies that \(\tau_t\) is less than \(x\%\), which implies that UIP fails in the sense that \(\hat{b}\) is less than one. If there is a net liquidity effect, then \(\tau_t\) and \(\hat{b}\) are negative.

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\(^{16}\)The precise nature of \(\varepsilon_t\) is not critical. It could be some other appropriate monetary aggregate.
A combination of liquidity and inflationary effects help explain negative \( \hat{b} \) under flexible rates, but not other regimes, because central banks have more freedom to engage in the kind of open market operations that produce liquidity effects under flexible rates than under other regimes. Sterilized leaning against the wind also helps explain negative \( \hat{b} \) under flexible rates, but not other regimes. Under gold and pegged rates central banks are committed to defending the official exchange rate, not moderating short-run movements in the rate. They also have little interest in sterilization because it weakens the effects of their intervention.

Equation (VI) describes expected inflation. How strongly \( E(\Delta \bar{P} \mid I) \) responds to changes in the monetary base depends on the inflationary effects of open market operations denoted \( C \). \( C \) depends on the time horizon and monetary regime. The shorter the time horizon the smaller is \( C \). A substantial increase in the monetary base today would not cause much of a price increase tomorrow, but it should within a year. In a highly stable monetary regime like most of post WWII Germany, one would expect \( C \) to be relatively small. With hyperinflation as in Germany during the early 1920s or the moderately unstable monetary conditions in France at about the same time, one would expect \( C \) to be larger.

Equation (VII) describes actual inflation \( \Delta \bar{P} \). It equals expected inflation plus \( x_{t+1} \), as required by the assumption of Rational Expectations.

5.1. \( \hat{b} \)

Appendix I shows the solution for \( \hat{b} \) using the full model. It is complex. Part of that complexity is the result of three things that most other explanations for the failure of UIP ignore: (1) that \( (\lambda - 1) \) might not be zero, (2) that several different kinds of shocks affect \( \hat{b} \) and (3) the possibility that those “shocks” might not be white noise. But most of the complexity is the result of including intervention.

5.1.1. No Intervention

Equation (10) shows the solution for \( \hat{b} \) in a “stripped down” version of the model where \( (\lambda - 1)\), \( V\), \( U \) and \( R \) are all zero and there is no intervention. That is when spot rates are “exogenous” in the CIP equation, all shocks except \( z_t \) are white noise and there is no intervention. These are common implicit assumptions in most of the relevant models and simplify comparing this model to others.

\[
\hat{b}_1 = \{ C(\lambda - 1) \left[ D^2 \sigma^2 + \sigma^2 \right] \} / \left[ \left( \sigma^2 \right)^2 \right] \] (10)

If there is no inflationary effect and \( C \) is zero, \( \hat{b}_1 \) is zero. If there is no liquidity effect and \( \lambda \) is zero, \( \hat{b}_1 \) goes to one as \( \sigma^2 \) goes to zero. If \( \lambda - C \) is negative and \( C \) is positive, \( \hat{b}_1 \) is negative. A negative \( \hat{b}_1 \) requires both an inflationary and net liquidity effect.

The idea behind Equation (10) is simple and does not depend on the specifics
of this model. Consider an expansionary open market operation. With no liquidity effect, let it produce actual inflation, expected inflation, a positive interest rate differential and depreciation.  is positive. As long as they are equal,  goes to one as  goes to zero because the Fisher equation holds. If liquidity effects partially offset expected inflation, the nominal differential is smaller than the depreciation and UIP fails in the sense that  is less than one. If there is a net liquidity effect,  is negative because there is depreciation and the interest differential is negative.

5.1.2. Intervention
Central banks in developed countries with flexible rates routinely lean against the wind and sterilize their intervention. Countries with managed rates have less incentive to do so because they are usually more concerned with longer run movements in their exchange rate. Even if they did lean against the wind, countries with managed rates would have little incentive to sterilize.

Sterilized leaning against the wind with flexible exchange rates can reinforce the combination of liquidity and inflationary effects that explain why  are negative when rates are flexible.

Equation (11) adds intervention to Equation (10).

\[
\hat{b}_2 = \left[ \Pi C (C - \Lambda) + (\Pi C) \Omega \left( \frac{\Phi}{1 - \Phi^2} \right) \right] \left[ D^2 \sigma^2 + \sigma^2 \right] + \Pi^2 \Omega \left[ \frac{\Phi}{1 - \Phi^2} \right] \left[ \sigma^2 + \sigma^2 \right]
\]

(11)

where  equals  and  equals .

 is positive and probably close to one because  and  are both positive and probably much less than one. As a result, the contribution of intervention to the sign of  depends primarily on the signs of  and  when they have the same sign, leaning against the wind contributes to a positive  . With opposite signs it contributes to a negative  . If sterilization is incomplete, their signs are difficult to determine. But if sterilization is complete and  is zero, they have opposite signs and leaning against the wind contributes to a negative  .

To see how sterilized intervention reduces  , consider how a central bank responds to a positive  . It sells foreign exchange to moderate the rise in the exchange rate. To sterilize that sale it acquires domestic short-term assets. To rebuild its holdings of foreign exchange, it sells foreign T bills.

Sterilized leaning against the wind contributes to negative  when two things happen: First sterilization produces negative  . Second leaning against the wind produces positive \( \Delta s_{t+1} \).

5.2. Risk Premiums
There are no risk premiums in Table 1. Ignoring transaction costs, equilibrium

\[\text{The following discussion assumes an initial equilibrium where both } \Delta s \text{ and } \Gamma \text{ are zero.}\]
implies that expected speculative returns equal risk premiums. That equilibrium implies that exogenous increases in risk premiums increase equilibrium expected returns by reducing the incentive to speculate. It also implies that exogenous increases in those expected returns increase risk premiums by increasing the incentive to speculate. But that equilibrium condition says nothing about “causation”; it only says that the two must be numerically equal. As far as I am aware, no one has explained how risk premiums create expected speculative returns. On the other hand it is easy to see how expected speculative returns create risk premiums.

Consider the following mental experiment: Start with both sides of that equilibrium condition equal to zero with no one speculating in either currency. First consider the effects of an exogenous reduction in perceived risk. There may be indirect effects, but there are no direct effects. No one buys speculative assets because of the reduced perceived risks because there is no expected return.

Now, starting with the same initial equilibrium, consider the effects of an open market operation whose liquidity and inflationary effects create expected speculative returns. Those returns induce speculators to take uncovered positions. Those uncovered positions create risk premiums.

This article concentrates on explaining expected speculative returns. Risk premiums are important, but for simplicity this article ignores them. Modeling the dynamic adjustment process between risk premiums and expected speculative returns that produces negative $\hat{b}$ and $\hat{\beta}$ is beyond the objectives of this article, which are to show how sterilized leaning against the wind and a combination of inflationary and liquidity effects can produce the expected speculative returns that cause UIP to fail and produce forward bias.

5.3. Forward Bias

CIP implies that $\hat{b}$ and $\hat{\beta}$ are two sides of the same coin. So Equation 10 describes how a combination of liquidity and inflationary effects produces negative $\hat{\beta}$ while Equation (11) describes how sterilized leaning against the wind contributes to negative $\hat{\beta}$. They do not provide the economic content that Fama’s $\rho$ requires.

The model in Table 1 provides that content. Equation (12) describes Fama’s premium when there is no intervention, $(\lambda - 1)$ and $\sigma^2$ are zero and “shocks” are white noise.18

$$f_t = E(s_{t+1} | I_t) - \Lambda(Du_t + Y_t)$$

(12)

This $\rho$ is not a risk premium because there are no risk premiums in this model. It represents an expected speculative return. In spite of Rational Expectations, liquidity effects drive a rational wedge between forward rates and expected future spot rates. That wedge creates a rational expected speculative return.

To see the economics behind Equation (12) start with all actual and expected

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18 Appendix II describes the full solution for $\rho$. 

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changes zero. Then a positive \( v \) creates one percent actual inflation, expected inflation and depreciation. First consider the case where \( \Lambda \) is zero and the Fisher equation holds. With \( \sigma^2 \) zero, \( E(\Delta s_{t+1}/I_t) \), \( \Delta s_{t+1} \), \( \bar{\ell}_t \) and \( \ell_t - s_t \) are all one percent. There is no forward bias, \( E(s_{t+1}/I_t) \) equals \( \ell_t \).

Now consider the same case where \( \Lambda \) is positive. A positive \( \Lambda \) violates the Fisher equation and reduces both \( \bar{\ell}_t \) and \( \ell_t - s_t \), but it does not directly affect \( E(\Delta s_{t+1}/I_t) \) or \( \Delta s_{t+1} \). As a result, even though expectations are rational, \( \ell_t \) is less than \( E(s_{t+1}/I_t) \) and there is forward bias.

### 6. Related Puzzles

Explaining many different phenomena with a single theory is a major objective of science. A Unified Field Theory is the holy grail in physics.

This article and all of the other explanations listed in the introduction were designed to explain forward bias and/or the failure of UIP. As a result, that evidence does not provide empirical support for any of the explanations. Support requires something new; something that the theory was not designed to explain.

Neither this article nor any of the other explanations in the Introduction were developed to explain the related puzzles listed there. If any of these explanations, including this one, could solve those related puzzles it would provide strong support for that explanation and also be a major contribution to open economy macroeconomics.

This section shows that leaning against the wind and/or a combination of inflationary and liquidity effects can explain the related puzzles. Whether or not other explanations can do as well I leave up to the supporters of those explanations.

Discussions of the related puzzles are brief because a thorough discussion of each puzzle would be as long as this article.

#### 6.1. Carry Trade

The “Carry Trade” refers to borrowing where international interest rates are “low” and lending where they are “high” without cover, which appears to produce profit with little risk. Those profits suggest that, for at least some trades, expected speculative returns exceed risk premiums. For some recent articles on the carry trade, see [65], [66] and [22].

[65] shows how diversification increases the returns to the Carry Trade. [66] stresses the problems associated with trying to explain this puzzle with risk premiums and concludes that the related limits to speculation explanation does not provide a complete explanation. [22] attributes carry-trade profits to “low volatility” and claims that they decline or disappear with “high volatility”.

Sterilized leaning against the wind and a combination of liquidity and inflationary effects explain the Carry Trade. They create the expected speculative returns that drive the Carry Trade. Banks and other institutions with relatively low transaction costs take risky positions based on rational expected returns. Those
risky positions require risk premiums.

6.2. Commodities

One would expect Fama’s premium to be as valid for commodity markets as for foreign exchange markets. His premium can refer to the price of wool as well as the price of foreign exchange. Equation (5) would appear to be as relevant for wool, or any other commodity with forward markets, as it is for foreign-exchange markets.

Given the importance of the bias in foreign-exchange markets, looking for the same bias in commodity markets would seem an obvious and important thing to do. To the best of my knowledge there have been only two attempts to do so: [67] and [68]. Using futures indexes, [68] finds positive $\hat{\beta}$ for commodities. Using individual futures prices, [67] finds mostly positive $\hat{\beta}$ for commodities. Mostly negative $\hat{\beta}$ for flexible exchange rates and mostly positive $\hat{\beta}$ for commodities is the Commodity Puzzle.

Sterilized leaning against the wind and a combination of liquidity and inflationary effects explain the Commodity Puzzle. They produce forward bias in foreign exchange markets. However neither directly affects auction commodity markets. As a result, there is no systematic forward bias in auction commodity markets because, unlike foreign exchange markets, liquidity effects do not drive a rational wedge between forward prices and expected future prices.

6.3. Development

[27] was the first to suggest that the forward-bias puzzle is confined largely to developed countries. Later [28] estimated $\beta$ between the U.S. and 21 developed countries and 14 emerging countries. Their average $\hat{\beta}$ between developed countries and the U.S. is $-4.3$. Their average $\hat{\beta}$ between emerging countries and the U.S. is 0.003. That difference illustrates what has been called the Development Puzzle.

Section 2.3 points out that, whether countries are developed or emerging, $\hat{\beta}$ tend to be negative when exchange rates are flexible and zero or positive when managed. The “Development Puzzle” therefore appears to be due to the fact that most emerging countries have managed rates while almost all developed countries have flexible rates.

By explaining why $\hat{\beta}$ tend to be negative when exchange rates are flexible, sterilized leaning against the wind and a combination of inflationary and liquidity effects explain this puzzle.

6.4. Inflation and Outliers

The Inflation Puzzle is that $\hat{b}$ increases with inflation. The Outlier Puzzle is that there are positive (negative) $\hat{b}$ or $\hat{\beta}$ with outlier (non-outlier) interest.

[19] Even developed countries intervene directly or indirectly in basic commodities. Agricultural price supports and export subsidies are examples. But that is not the same as leaning against the wind in the international auction markets where those products are traded.
rate differentials and/or forward premiums. See for example [8] and [15]. Both puzzles are probably the result of outlier inflation.

[17] does not suggest an explanation for the Outlier Puzzle. He concludes his discussion by pointing out that the reason why non-outlier IDs yield negative estimates of $\hat{\beta}$ appears to be a key toward solving the UIP puzzle. (Note that his $\beta$ is my $b$.) Regarding the Inflation Puzzle, [17] points out that no one has developed a UIP framework that generates a positive relation between estimates of $\beta$ and the inflation rate.

A combination of liquidity and inflationary effects provides a framework for inflation and outliers. With low inflation, liquidity effects tend to dominate inflationary effects. The resulting negative $C-\Lambda$ produces negative $\hat{\beta}$ and $\hat{b}$. As inflation increases and produces outliers $C-\Lambda$ increases, which increases $\hat{\beta}$ and $\hat{b}$.

6.5. Maturity

Using developed countries with flexible exchange rates, [69], [70] and [71] find that $\hat{b}$ are usually negative for maturities of one year or less but that $\hat{b}$ are usually positive for over one year. More recently [1] finds a similar pattern using about 200 years of annual data. Negative $\hat{b}$ for short maturities and positive $\hat{b}$ for long maturities is the conventional Maturity Puzzle.\footnote{Using pound sterling and the euro as home currencies, [72] still finds negative $\hat{\beta}$ at 12 month maturities.}

[69] does not suggest any solution for this puzzle. [70] points out that none of the standard explanations for the UIP puzzle—risk premiums, expectational errors, or peso problems—appear at first glance to offer an explanation for why the result should be so different. They then go on to develop an extension of [5] to explain this puzzle. [71] refers to several possible solutions including the extension of a “preferred habitat” explanation and differing expectations similar to those in [17].

Sterilized leaning against the wind and a combination of liquidity and inflationary effects explain this puzzle. As the maturity of $\tau$ increases $C-\Lambda$ increases as liquidity effects fade and inflationary effects increase. An open market operation or intervention that would produce a positive $\Delta s_{n+1}$ and negative $\tau$ at short maturities produces a positive $\Delta s_{n+1}$ and a positive $\tau$ at long maturities.\footnote{Larger deviations from CIP at longer maturities also can help explain the Maturity Puzzles. For some evidence of such deviations see [33].}

6.6. Time Dependency

Between developed countries under the current float, $\hat{\beta}$ and $\hat{b}$ vary widely over time. For examples of this time dependency for $\hat{b}$, see [73]. For examples of this dependency for $\hat{\beta}$, see [74].

Using rolling regressions, [1] estimates $\hat{b}$ using over two hundred years of data. Before the early 1970s when most rates were managed, $\hat{b}$ are relatively stable, usually positive and often not significantly different from zero. After al-
most all rates for developed countries become flexible in the early 1970s, \( \hat{\beta} \) become unstable, consistently negative and at times negative and significant. The Time Dependency Puzzle therefore is why are \( \hat{\beta} \) and \( \hat{\beta} \) so time dependent when rates are flexible but not when they are managed?

Sterilized leaning against the wind and a combination of liquidity and inflationary effects can explain why \( \hat{\beta} \) and \( \hat{\beta} \) are negative when rates are flexible. Variations in sterilization, intervention and the open market operations that produce inflation and liquidity effects can explain why \( \hat{\beta} \) and \( \hat{\beta} \) are so time dependent under flexible exchange rates.

For simplicity, the model in Table 1 assumes that sterilization \( (F) \) and the intensity of intervention \( (X) \) and how intervention affects exchange rates \( (Z) \) are all constant, but they are not. For some central banks there will be times when it is convenient not to sterilize or to sterilize only partially. Foreign exchange markets have periods of stability and volatility. Central banks are more likely to intervene when rates are volatile than when they are stable. \( Z \) is likely to change as market conditions including volatility change. For all these reasons the extent of sterilization and intervention are likely to change over time, causing \( \hat{\beta} \) and \( \hat{\beta} \) to vary over time. Since leaning against the wind with full sterilization is a characteristic of flexible rather than managed rates, it will cause \( \hat{\beta} \) and \( \hat{\beta} \) to vary more over time when rates are flexible than when they are managed.

For simplicity the model assumes that central banks respond symmetrically to macroeconomic shocks like unemployment. But central banks do not respond as aggressively to low inflation as to high inflation or to low unemployment as to high unemployment. As inflation and unemployment vary over the business cycle, the kind of open market operations that produce liquidity and inflationary effects will vary. These variations, and the variations in the inflationary and liquidity effects themselves, can cause \( \hat{\beta} \) and \( \hat{\beta} \) to vary over time when rates are flexible. When exchange rates are managed, central banks do not have the kind of freedom to macro-stabilize that they have under flexible rates.

### 7. Summary

This article shows how sterilized leaning against the wind and a combination of liquidity and inflationary effects of open market operations responding to macroeconomic stabilization policies can cause forward bias and the failure of UIP under flexible exchange rates. It also raises the possibility that they can explain several related puzzles. If they do explain forward bias, the failure of UIP and related puzzles, it would be a major contribution to macroeconomics.

### Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

### References

[1] Lothian, J. and Wu, L. (2011) Uncovered Interest-Rate Parity over the Past Two


Appendix I

\[ \hat{b} = \left[ \Pi C(C - \Lambda) \frac{1}{(1 - \Phi U)} + (\Lambda - 1)(C - \Lambda)^2 \Pi \left\{ U + \left[ \frac{(\Phi - 1)}{(1 - \Phi U)} \right] \right\} \right. \\
+ (\Pi C)^2 \Omega \left\{ U + \Phi \right\} \left[ \frac{(1 - \Phi U)}{(1 - \Phi^2)} \right] \right. \\
+ (\lambda - 1)(C - \Lambda) \Pi^2 \Omega C \left[ U^2 + \frac{(1 + \Phi)}{(1 - \Phi U)} \right] \left[ \frac{(1 - \Phi U)}{(1 + \Phi)} \right] \\
+ \Omega C \Pi^2 (\lambda - 1)(C - \Lambda) \left\{ 1 - \frac{(1 - \Phi U)}{(1 + \Phi)} \right\} \left[ \frac{(1 - \Phi U)}{(1 + \Phi)} \right] + \Pi^2 \Omega (\lambda - 1)^2 (C - \Lambda)^2 \\
\times \left[ 1 - \frac{(1 - \Phi U)}{(1 + \Phi)} \right] \left[ \frac{(1 - \Phi U)}{(1 + \Phi)} \right] \right] D^2 \sigma_r^2 \\
+ \left[ (\lambda - 1) \Pi \left\{ \frac{R}{(1 - \Phi U)} \right\} \right. \\
+ \Pi^2 \Omega (\lambda - 1)^2 \left[ 1 - \frac{(1 - \Phi U)}{(1 + \Phi)} \right] \left[ \frac{(1 - \Phi U)}{(1 + \Phi)} \right] \right. \\
+ \left[ \Pi C(C - \Lambda) \frac{1}{(1 - \Phi V)} + (\lambda - 1)(C - \Lambda)^2 \Pi \left\{ V + \left[ \frac{(\Phi - 1)}{(1 - \Phi V)} \right] \right\} \right. \\
\left. + (\Pi C)^2 \Omega \left\{ V + \Phi \right\} \left[ \frac{(1 - \Phi V)}{(1 - \Phi^2)} \right] \right. \\
+ (\lambda - 1)(C - \Lambda) \Pi^2 \Omega C \left[ V^2 + \frac{(1 + \Phi)}{(1 - \Phi V)} \right] \left[ \frac{(1 - \Phi V)}{(1 + \Phi)} \right] \\
+ \Pi^2 \Omega \left\{ \frac{1}{(1 - \Phi^2)} \right\} \left[ \frac{(1 - \Phi V)}{(1 + \Phi)} \right] \left[ \frac{(1 - \Phi V)}{(1 + \Phi)} \right] \right. \\
+ \left[ 1 + 2 \Omega \Pi (\lambda - 1) \frac{(1 - R)}{(1 - \Phi R)} \right] \\
+ \left[ \frac{(1 + \Phi R - \Phi R)}{(1 - \Phi V)} \right] \left[ \frac{(1 - \Phi R)}{(1 - \Phi^2)} \right] \right. \\
+ \left[ (C - \Lambda)^2 + 2 \Omega \Pi C(C - \Lambda) \left\{ U + \Phi \right\} \right. \\
+ 2 \Omega \Pi (\lambda - 1)(C - \Lambda)^2 \left\{ 1 - \frac{(1 - \Phi U)}{(1 - \Phi V)} \right\} \\
+ \left( \Omega \Pi C \right)^2 \left\{ 1 + \Phi \right\} \left[ \frac{(1 - \Phi U)}{(1 - \Phi^2)} \right] \right. \\
\left. + 2 \Omega \Pi (\lambda - 1)(C - \Lambda) \left\{ \frac{(\Phi + U - \Phi U - 1)}{(1 - \Phi U)} \right\} \left[ \frac{(1 - \Phi U)}{(1 - \Phi^2)} \right] \right. \\
\left. + 2 \Omega \Pi \left(\lambda - 1\right)^2 (C - \Lambda)^2 \left\{ 1 + \frac{(\Phi U - \Phi V - U)}{(1 - \Phi U)} \right\} \left[ \frac{(1 - \Phi U)}{(1 - \Phi^2)} \right] \right. \\
\left. + \left[ (C - \Lambda)^2 + 2 \Omega \Pi C(C - \Lambda) \left\{ V + \frac{(1 - \Phi V)}{(1 - \Phi^2)} \right\} \right. \\
+ 2 \Omega \Pi (\lambda - 1)(C - \Lambda)^2 \left\{ 1 - \frac{(1 - \Phi V)}{(1 - \Phi V)} \right\} \\
+ \left( \Omega \Pi C \right)^2 \left\{ 1 + \Phi V \right\} \left[ \frac{(1 - \Phi V)}{(1 - \Phi^2)} \right] \right. \\
\left. + 2 \Omega \Pi (\lambda - 1)(C - \Lambda) \left\{ \frac{(\Phi V - \Phi V - 1)}{(1 - \Phi V)} \right\} \left[ \frac{(1 - \Phi V)}{(1 - \Phi^2)} \right] \right. \\
\left. + 2 \Omega \Pi \left(\lambda - 1\right)^2 (C - \Lambda)^2 \left\{ 1 + \frac{(\Phi V - \Phi V - 1)}{(1 - \Phi V)} \right\} \left[ \frac{(1 - \Phi V)}{(1 - \Phi^2)} \right] \right. \\
\left. + \left( \Omega \Pi \right)^2 \left\{ \frac{1}{(1 - \Phi^2)} \right\} \left[ \sigma_r^2 + \sigma_v^2 \right] \right. \]

where \( \Pi \) equals \( \frac{1}{1 + ZX - (\lambda - 1)\Omega} \), \( \Omega \) equals \( \left[ (\Lambda - C) F - H \right] X \), and \( \Phi \) equals \( \left[ ZX - CFX - (\lambda - 1)\Omega \right] \left[ 1 + ZX - (\lambda - 1)\Omega \right] \).
Appendix II

\[ p_t = \bar{p}_t - (\lambda - 1) \Pi E \left( \sum_{i=0}^{\infty} \Phi^i \Delta u_{t+i} \right) + \Omega \Pi \left( \lambda - 1 \right) \left( \sum_{i=0}^{\infty} \Phi^i \Delta \bar{u}_{t+i} \right) \]

\[ - \Pi C D E \left( \sum_{i=0}^{\infty} \Phi^i u_{t+i} \right) + (C - \Lambda) D u_t + \Omega \Pi C D \left( \sum_{i=0}^{\infty} \Phi^i u_{t+i} \right) \]

\[ - (\lambda - 1) (C - \Lambda) \Pi D E \left( \sum_{i=0}^{\infty} \Phi^i u_{t+i} \right) \]

\[ + \Omega (\lambda - 1) (C - \Lambda) \Pi D \left( \sum_{i=0}^{\infty} \Phi^i u_{t+i} \right) - \Pi CE \left( \sum_{i=0}^{\infty} \Phi^i v_{t+i} \right) \]

\[ + \Omega \Pi C \left( \sum_{i=0}^{\infty} \Phi^i v_{t+i} \right) + (C - \Lambda) v_t - (\lambda - 1) (C - \Lambda) \Pi E \left( \sum_{i=0}^{\infty} \Phi^i V_{t+i} \right) \]

\[ + \Omega (\lambda - 1) (C - \Lambda) \Pi \left( \sum_{i=0}^{\infty} \Phi^i V_{t+i} \right) - \Pi E \left( \sum_{i=0}^{\infty} \Phi^i x_{t+i} \right) \]

\[ + \Omega \Pi \left( \sum_{i=0}^{\infty} \Phi^i x_{t+i} \right) - \Pi E \left( \sum_{i=0}^{\infty} \Phi^i e_{t+i} \right) + \Omega \Pi \left( \sum_{i=0}^{\infty} \Phi^i e_{t+i} \right) \]