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## Research Article

# Application of Nondimensional Dynamic Influence Function Method for Eigenmode Analysis of Two-Dimensional Acoustic Cavities

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This paper establishes an improved NDIF method for the eigenvalue extraction of two-dimensional acoustic cavities with arbitrary shapes. The NDIF method, which was introduced by the authors in 1999, gives highly accurate eigenvalues despite employing a small number of nodes. However, it needs the inefficient procedure of calculating the singularity of a system matrix in the frequency range of interest for extracting eigenvalues and mode shapes. The paper proposes a practical approach for overcoming the inefficient procedure by making the final system matrix equation of the NDIF method into a form of algebraic eigenvalue problem. The solution quality of the proposed method is investigated by obtaining the eigenvalues and mode shapes of a circular, a rectangular, and an arbitrarily shaped cavity.

## 1. Introduction

The authors developed the nondimensional dynamic influence function method (NDIF method) for extracting highly accurate eigenvalues and eigenmodes of arbitrarily shaped membranes and acoustic cavities [1, 2]. Later, the authors extended the NDIF method to membranes with high concavity [3] and plates with various boundary conditions [4–7]. In the NDIF method, as in the boundary element method (BEM) [8], a field problem is solved on its boundary along which nodes are distributed. The distinct feature of the NDIF method is related to the fact that no interpolation functions between the nodes are required, so that the basic collocation method is employed to satisfy a given boundary condition. This approach enables us to reduce a large amount of numerical calculation induced due to the interpolation functions and, as a result, to obtain highly accurate eigenvalues.

On the other hand, the weak point of the NDIF method is that its final system matrix depends on a frequency parameter, unlike in the finite element method (FEM) [9]. In

general, the final system matrix equation of FEM has a form of algebraic eigenvalue problem [10] and as the result its system matrices are independent of the frequency parameter. Recently, to overcome this weak point for the NDIF method, the authors employed a modified approach of expanding the nondimensional dynamic influence function in a Taylor series for free vibration analysis of membranes with arbitrary shapes [11]. In this paper, the modified approach [11] is extended for eigenmode analysis of two-dimensional acoustic cavities with general shapes.

Common methods for extracting eigenvalues of an arbitrarily shaped acoustic cavity are the finite element method and the boundary element method [8, 9]. It is well known that BEM has the advantage of discretizing only the boundary of the domain of interest unlike FEM. However, there was the limitation that system matrices involved in BEM depend on a frequency parameter before the innovative work of Nardini and Brebbia [12]. In 1982, Nardini and Brebbia succeeded in formulating a final system matrix equation in BEM as a form of algebraic eigenvalue problem and

opened new horizons in the BEM research [12]. Since then, BEM researches have focused on improving the accuracy of eigenvalues. Kirkup and Amini introduced a practical way of reducing the nonlinear eigenvalue problem to a standard generalized eigenvalue problem through a polynomial approximation [13]. Ali et al. presented a historical and critical review of BEM in acoustic eigenvalue analysis [14]. Provatidis tested different types of basis functions for more accurate eigenvalues of two-dimensional acoustic cavities using the dual reciprocity/boundary element technique [15]. Recently, Wang et al. investigated approximation functions such as RBF (radial basis functions) and TPS (thin plate spline functions) in the dual reciprocity BEM for accurate acoustic eigenvalue analysis [16]. Gao et al. presented accurate solutions for eigenvalue analysis of three-dimensional acoustic cavities using BEM with the block Sakurai-Sugiura method [17].

Many researchers have studied new numerical methods for more accurate eigenvalue analysis than FEM and BEM. For instance, the NDIF method [1–7], which was developed by the authors, offers much more accurate eigenvalues than FEM. For acoustic cavities with simple shapes having no exact solution, a great deal of analytical or semianalytical research has been performed to increase the accuracy of eigenvalues and eigenmodes. Amir and Starobinski studied a method for calculating the eigenmodes of two-dimensional cavities having two axes of symmetry by computing wave propagation in waveguides of arbitrarily changing cross section [18]. Willatzen and Voon solved quasianalytically a triaxial ellipsoidal acoustic cavity with walls using the Frobenius power-series expansion method [19]. Koch computed acoustic resonances in rectangular two-dimensional deep shallow open cavities [20]. Lee presented a semianalytical approach to solve the eigenproblem of an acoustic cavity with multiple elliptical boundaries by using the collocation multipole method [21]. Although analytical and semianalytical methods such as abovementioned methods [18–21] give highly accurate solutions, there is the limitation that they are not applicable to arbitrarily shaped acoustic cavities. In this paper, a simple and practical approach, which is applicable to arbitrary shapes and offers a highly accurate solution, is proposed by extending the authors' previous research [11].

## 2. Theoretical Formulation

### 2.1. Review of the Nondimensional Dynamic Influence Method.

The original NDIF method [2] for acoustic eigenproblems is reexamined before the development of an improved theoretical formulation. As shown in Figure 1, imagine a waveform that spreads circularly outward from the center point  $\mathbf{r}_0$  in an infinite acoustic field. Since the field is infinite in extent, the waveform will depend on the scalar distance from the center point  $\mathbf{r}_0$  to the field point  $\mathbf{r}$ ,  $r = |\mathbf{r} - \mathbf{r}_0|$ . The wave equation in this case reduces to

$$\frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial p}{\partial r} \right) = \frac{1}{c^2} \frac{\partial^2 p}{\partial t^2}, \quad (1)$$

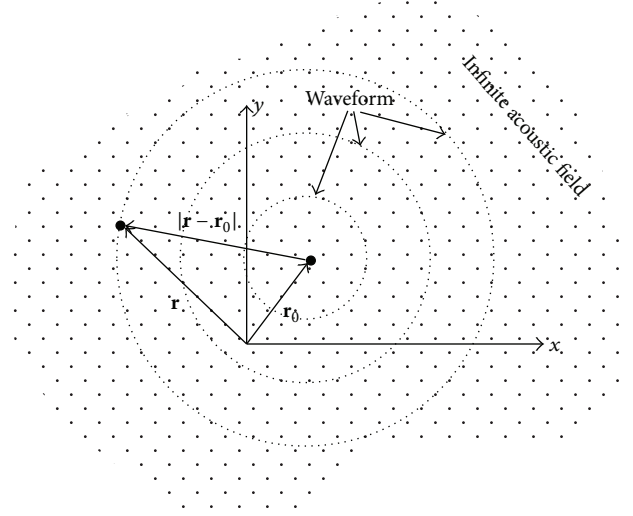


FIGURE 1: Waveform that spreads circularly outward from the center  $\mathbf{r}_0$  in an infinite acoustic field.

where  $p = p(r, t)$  is sound pressure and  $c$  is the speed of sound. In the case of harmonic problems with a time-dependent term  $e^{j\omega t}$ , the relation  $p = P(r) e^{j\omega t}$  leads (1) to

$$\frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial P}{\partial r} \right) + k^2 P = 0, \quad (2)$$

where  $k = \omega/c$  ( $\omega$  is the angular frequency).

If the physical consideration that sound pressure is *bounded* at the center point  $\mathbf{r}_0$  ( $r = 0$ ) is given, a unique solution of (2) is the Bessel function of the first kind of order zero,  $J_0(kr)$ , which is finite everywhere including the center point. Note that the Bessel function of the second kind and order zero  $Y_0(kr)$  is discarded since it is infinite at the center point ( $r = 0$ ) although it satisfies (2). Now,  $J_0(kr)$  is termed a nondimensional dynamic influence function in a two-dimensional infinite acoustic field. This function physically represents the *pressure* at a field point due to *unit pressure* at the center point of a wave that spreads circularly in the infinite region. Note also that the argument of  $J_0(kr)$  used in this study is dimensionless and that the nondimensional dynamic influence function satisfies the homogeneous Helmholtz equation:

$$\nabla^2 P + k^2 P = 0, \quad (3)$$

which is a governing differential equation of eigenvalue problems.

In an infinite acoustic field shown in Figure 2,  $N$  nodes are distributed along the fictitious contour (the dotted line) of which the shape is exactly the same as the boundary of the cavity of interest. Next, we consider that  $N$  waves, of

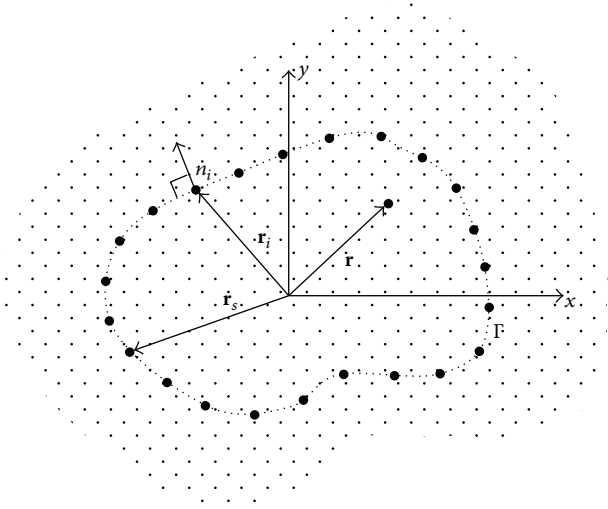


FIGURE 2: Two-dimensional acoustic cavity depicted by the dotted line in an infinite acoustic field.

which the forms are given by the nondimensional dynamic influence functions, spread circularly from each of the nodes. Then, the pressure at field point  $\mathbf{r}$  can be obtained by linearly superposing the nondimensional dynamic influence functions:

$$P(\mathbf{r}) = \sum_{s=1}^N A_s J_0(k|\mathbf{r} - \mathbf{r}_s|), \quad (4)$$

which also satisfies the Helmholtz equation (3) because each of the nondimensional dynamic influence functions does. Thus (4) can be employed as a trial solution for solving the eigenfield of the finite-sized cavity represented by the dotted line in Figure 2.

The unknown coefficients  $A_1 \sim A_N$  involved in the trial function are determined by applying a given boundary condition to the function. If a rigid-wall boundary condition is given, the boundary condition defined continuously along the boundary  $\Gamma$  is discretized so as to be satisfied only at previously located nodes to obtain the trial function; that is,

$$\frac{\partial P(\mathbf{r}_i)}{\partial n_i} = 0, \quad i = 1, 2, \dots, N, \quad (5)$$

where  $n_i$  denotes the normal direction from the boundary at  $\mathbf{r} = \mathbf{r}_i$ , as shown in Figure 2. Substituting the eigensolution (4) into the discrete boundary condition (5) gives

$$\sum_{s=1}^N A_s \frac{\partial}{\partial n_i} J_0(k|\mathbf{r}_i - \mathbf{r}_s|) = 0, \quad i = 1, 2, \dots, N. \quad (6)$$

Equation (6) can be written into the system matrix equation:

$$\mathbf{SM}(k) \mathbf{A} = \mathbf{0}, \quad (7)$$

where the elements of the system matrix  $\mathbf{SM}(k)$  of order  $N \times N$  are given by

$$SM_{is} = \frac{\partial}{\partial n_i} J_0(k|\mathbf{r}_i - \mathbf{r}_s|), \quad (8)$$

and the elements of the column vector  $\mathbf{A}$  of order  $N \times 1$  correspond to the unknown coefficients  $A_1 \sim A_N$ .

It may be seen in (7) and (8) that the elements of the system matrix  $\mathbf{SM}(k)$  depend on the frequency parameter  $k$ . As a result, the inefficient procedure of searching the frequency parameter that makes the system matrix singular by sweeping the frequency parameter in the range of interest is required to extract eigenvalues in the NDIF method.

**2.2. Improved Formulation of the NDIF Method.** First, (6) is rewritten as

$$\sum_{s=1}^N A_s k J_1(k|\mathbf{r}_i - \mathbf{r}_s|) \frac{\partial}{\partial n_i} (|\mathbf{r}_i - \mathbf{r}_s|) = 0, \quad (9)$$

$$i = 1, 2, \dots, N.$$

The Bessel function of the first kind of order 1  $J_1$  in (9) is expanded in a Taylor series [22] as follows:

$$J_1(k|\mathbf{r}_i - \mathbf{r}_s|) \approx \sum_{j=0}^M \frac{(-1)^j (k|\mathbf{r}_i - \mathbf{r}_s|/2)^{1+2j}}{\Gamma(j+1)\Gamma(j+2)}, \quad (10)$$

where  $M$  denotes the number of terms of the series and  $\Gamma(j+1)$  and  $\Gamma(j+2)$  represent the Gamma functions. Substituting (10) into (9) yields

$$\sum_{s=1}^N A_s k \left( \sum_{j=0}^M \frac{(-1)^j (k|\mathbf{r}_i - \mathbf{r}_s|/2)^{1+2j}}{\Gamma(j+1)\Gamma(j+2)} \right) \frac{\partial}{\partial n_i} |\mathbf{r}_i - \mathbf{r}_s| = 0, \quad (11)$$

$$i = 1, 2, \dots, N.$$

As the first step to extract a system matrix equation having a form of algebraic eigenvalue problem, (11) is rearranged in

$$\sum_{j=0}^M k^{2(j+1)} \sum_{s=1}^N A_s \frac{(-1)^j (|\mathbf{r}_i - \mathbf{r}_s|/2)^{1+2j}}{\Gamma(j+1)\Gamma(j+2)} \frac{\partial}{\partial n_i} |\mathbf{r}_i - \mathbf{r}_s| = 0, \quad (12)$$

$$i = 1, 2, \dots, N.$$

For simplicity, (12) is rewritten in

$$\sum_{j=0}^M \Lambda^{(j+1)} \sum_{s=1}^N A_s \psi_{is}^{(j)} = 0, \quad i = 1, 2, \dots, N, \quad (13)$$

where  $\Lambda = k^2$  and  $\psi_{is}^{(j)}$  is given by

$$\psi_{is}^{(j)} = \frac{(-1)^j (|\mathbf{r}_i - \mathbf{r}_s|/2)^{1+2j}}{\Gamma(j+1)\Gamma(j+2)} \frac{\partial}{\partial n_i} |\mathbf{r}_i - \mathbf{r}_s|. \quad (14)$$

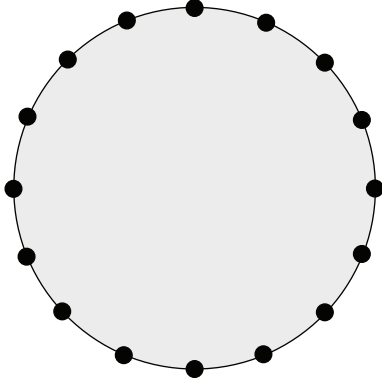


FIGURE 3: Circular acoustic cavity discretized by 16 boundary nodes.

Next, (13) is rewritten as a form of polynomial equation with respect to  $\Lambda$  by removing the first summation as follows:

$$\Lambda \sum_{s=1}^N A_s \psi_{is}^{(0)} + \Lambda^2 \sum_{s=1}^N A_s \psi_{is}^{(1)} + \dots + \Lambda^{M+1} \sum_{s=1}^N A_s \psi_{is}^{(M)} = 0, \quad i = 1, 2, \dots, N. \quad (15)$$

Resolving (15) in factors yields

$$\Lambda \left[ \Lambda^0 \sum_{s=1}^N A_s \psi_{is}^{(0)} + \Lambda^1 \sum_{s=1}^N A_s \psi_{is}^{(1)} + \dots + \Lambda^M \sum_{s=1}^N A_s \psi_{is}^{(M)} \right] = 0, \quad i = 1, 2, \dots, N, \quad (16)$$

which is divided into

$$\Lambda = 0, \quad (17)$$

$$\Lambda^0 \sum_{s=1}^N A_s \psi_{is}^{(0)} + \Lambda^1 \sum_{s=1}^N A_s \psi_{is}^{(1)} + \dots + \Lambda^M \sum_{s=1}^N A_s \psi_{is}^{(M)} = 0, \quad i = 1, 2, \dots, N. \quad (18)$$

Equation (17) denotes that the first eigenvalue is equal to zero for an acoustic cavity with a rigid-wall boundary condition. The higher eigenvalues may be obtained by changing (18) into a form of algebraic eigenvalue problem. For this purpose, (18) is first expressed in the simple matrix equation:

$$\Lambda^0 \Psi_0 \mathbf{A} + \Lambda^1 \Psi_1 \mathbf{A} + \dots + \Lambda^M \Psi_M \mathbf{A} = \mathbf{0}, \quad (19)$$

where the elements of matrix  $\Psi_j$  are given, using (14), by

$$\Psi_j(i, s) = \psi_{is}^{(j)} = \frac{(-1)^j (|\mathbf{r}_i - \mathbf{r}_s|/2)^{1+2j}}{\Gamma(j+1)\Gamma(j+2)} \frac{\partial}{\partial n_i} |\mathbf{r}_i - \mathbf{r}_s|. \quad (20)$$

Equation (19), which is called the higher order polynomial eigenvalue problem [24], may again be changed into the algebraic eigenvalue problem [10] as follows:

$$\mathbf{SM}_L \mathbf{B} = \Lambda \mathbf{SM}_R \mathbf{B}, \quad (21)$$

where the system metrics  $\mathbf{SM}_L$  and  $\mathbf{SM}_R$  are given, using the diagonal matrix  $\mathbf{I}$ , by

$$\mathbf{SM}_L = \begin{bmatrix} \mathbf{0} & \mathbf{I} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{I} & \cdots & \mathbf{0} \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ \vdots & \vdots & \vdots & \ddots & \mathbf{I} \\ -\Psi_0 & -\Psi_1 & -\Psi_2 & \cdots & -\Psi_{M-1} \end{bmatrix}, \quad (22)$$

$$\mathbf{SM}_R = \begin{bmatrix} \mathbf{I} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{I} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \ddots & \ddots & \vdots \\ \vdots & \vdots & \ddots & \mathbf{I} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & \Psi_M \end{bmatrix}$$

and the vector  $\mathbf{B}$  is given by

$$\mathbf{B} = \{\mathbf{A}^T \quad \Lambda \mathbf{A}^T \quad \Lambda^2 \mathbf{A}^T \quad \cdots \quad \Lambda^{M-1} \mathbf{A}^T\}. \quad (23)$$

Note that the newly obtained final system matrices  $\mathbf{SM}_L$  and  $\mathbf{SM}_R$  are independent of the frequency parameter unlike  $\mathbf{SM}(k)$  in (7). As a result, eigenvalues can simply be extracted from (21) without the inefficient procedure required in the original NDIF method. On the other hand, the  $i$ th mode shape can be obtained by plotting (4) where the unknown coefficients  $A_1 \sim A_N$  are given by the elements of  $\mathbf{A}^T$  in (23) for the  $i$ th eigenvalue.

### 3. Verification Examples

The validity and accuracy of the proposed method are shown in numerical tests of circular, rectangular, and arbitrarily shaped acoustic cavities.

**3.1. Circular Acoustic Cavity.** The proposed method is first applied to a circular acoustic cavity of unit radius where the exact solution [23] is known. As shown in Figure 3, the boundary of the circular cavity is discretized with 16 nodes for the proposed method. Eigenvalues obtained by the proposed method using  $M = 10$ ,  $M = 15$ ,  $M = 20$ , and  $M = 25$  are presented in Table 1, which also shows the eigenvalues given by the exact method [23], NDIF method [2], and FEM (ANSYS). In Table 1, it may be said that the eigenvalues by the proposed method in the case of  $M = 20$ , which coincide with those by the NDIF method [2], converge rapidly and accurately to those by the exact method [23]. Furthermore, it should be noted in Table 1 that the proposed method using

TABLE 1: Eigenvalues of the circular cavity obtained by the proposed method, the exact method, the NDIF method, and FEM (parenthesized values denote errors (%) with respect to the exact method).

Number	Proposed method (16 nodes)				Exact method [23]	NDIF method [2] (16 nodes)	FEM (2042 nodes)
	$M = 10$	$M = 15$	$M = 20$	$M = 25$			
1	1.8412 (0.000)	1.8412 (0.000)	1.8412 (0.000)	1.8412 (0.000)	1.8412	1.8412 (0.000)	1.8419 (0.038)
2	3.0542 (0.000)	3.0542 (0.000)	3.0542 (0.000)	3.0542 (0.000)	3.0542	3.0542 (0.000)	3.0564 (0.072)
3	3.8317 (0.000)	3.8317 (0.000)	3.8317 (0.000)	3.8317 (0.000)	3.8317	3.8317 (0.000)	3.8374 (0.149)
4	4.2012 (0.000)	4.2012 (0.000)	4.2012 (0.000)	4.2012 (0.000)	4.2012	4.2012 (0.000)	4.2059 (0.112)
5	5.3176 (0.000)	5.3176 (0.000)	5.3176 (0.000)	5.3176 (0.000)	5.3176	5.3176 (0.000)	5.3261 (0.160)
6	5.3313 (0.002)	5.3313 (0.002)	5.3314 (0.000)	5.3314 (0.000)	5.3314	5.3314 (0.000)	5.3427 (0.212)

TABLE 2: Eigenvalues of the rectangular cavity obtained by the proposed method, the exact method, the NDIF method, and FEM (parenthesized values denote errors (%) with respect to the exact method).

Number	Proposed method (24 nodes, $M = 20$ )	Exact method [23]	NDIF method [2] (24 nodes)	FEM	
				1813 nodes	2500 nodes
1	2.618 (0.00)	2.618	2.618 (0.00)	2.618 (0.00)	2.618 (0.00)
2	3.490 (0.00)	3.491	3.491 (0.00)	3.492 (0.03)	3.492 (0.03)
3	4.363 (0.00)	4.363	4.363 (0.00)	4.364 (0.02)	4.364 (0.02)
4	5.236 (0.00)	5.236	5.236 (0.00)	5.240 (0.08)	5.238 (0.04)
5	6.293 (0.00)	6.293	6.293 (0.00)	6.297 (0.06)	6.295 (0.03)
6	6.982 (0.03)	6.981	6.981 (0.00)	6.990 (0.13)	6.989 (0.11)

only 16 nodes yields more accurate eigenvalues than FEM (ANSYS) using 2042 nodes.

In addition, mode shapes produced by the proposed method using 16 nodes for  $M = 20$  are presented in Figure 4 and they agree well with those given by the exact method [23], which are omitted in the paper. Note that white regions in the mode shapes are nodal lines, at which the pressure has a minimum value.

On the other hand, the accuracy of an eigenvalue obtained by the proposed method can be verified by plotting its mode shape. If the plotted mode shape does not satisfy exactly the given boundary condition (the rigid-wall boundary condition), it may be said that the eigenvalue is not accurate and larger number of nodes and series functions are required to improve its accuracy.

**3.2. Rectangular Acoustic Cavity.** In this section, a rectangular acoustic cavity with dimensions 1.2 m  $\times$  0.9 m is discretized with 24 nodes as shown in Figure 5, where the location and the corresponding normal directions are illustrated. Since the rectangular cavity has 4 corners unlike the circular cavity, the normal directions at the corners are approximately determined by the sum of the two normal vectors for the edges adjacent to each corner.

In Table 2, eigenvalues obtained by the proposed method are compared with those computed by the exact method [23], the NDIF method, and FEM (ANSYS). It may be said that the

proposed method using 24 nodes for  $M = 20$  gives accurate eigenvalues within 0.03% error. However, it is noted that the eigenvalues by FEM using 2500 nodes have much larger errors than those by the proposed method. On the other hand, the reason that the sixth eigenvalue by the proposed method has some error unlike that by the NDIF method is that the Bessel function (10) is approximately expanded in a Taylor series.

Figure 6 shows mode shapes obtained by the proposed method, which agree well with those by the exact method [23], which are omitted in the paper.

**3.3. Arbitrarily Shaped Acoustic Cavity.** An arbitrarily shaped cavity whose boundary is composed of a semicircle of unit radius and two equilateral edges  $\sqrt{2}$  m in length is shown in Figure 7 where the normal directions at the 3 corners are approximately determined as illustrated in the rectangular cavity. Eigenvalues obtained by the proposed method, NDIF method [2], and FEM (ANSYS) are summarized in Table 3. Since the current cavity has no exact solution, errors of the proposed method with respect to FEM using 1571 nodes are calculated in Table 3 where it may be observed that the proposed method has very small errors within 0.3%. It may also be said in Table 3 that the proposed method always yields lower eigenvalues than FEM and, as a result, it gives very accurate results because it is well known that exact results exist below FEM results.

TABLE 3: Eigenvalues of the arbitrarily shaped cavity obtained by the proposed method, the NDIF method, and FEM (parenthesized values denote errors (%) with respect to FEM using 1571 nodes).

Number	Proposed method (16 nodes, $M = 20$ )	NDIF method [2] (16 nodes)	FEM		
			719 nodes	1088 nodes	1571 nodes
1	1.958 (0.10)	1.958 (0.10)	1.961	1.960	1.960
2	2.025 (0.05)	2.025 (0.05)	2.027	2.026	2.026
3	3.082 (0.16)	3.082 (0.16)	3.090	3.088	3.087
4	3.633 (0.30)	3.633 (0.30)	3.649	3.646	3.644
5	3.996 (0.25)	3.996 (0.25)	4.014	4.009	4.006
6	4.578 (0.26)	4.578 (0.26)	4.600	4.594	4.590

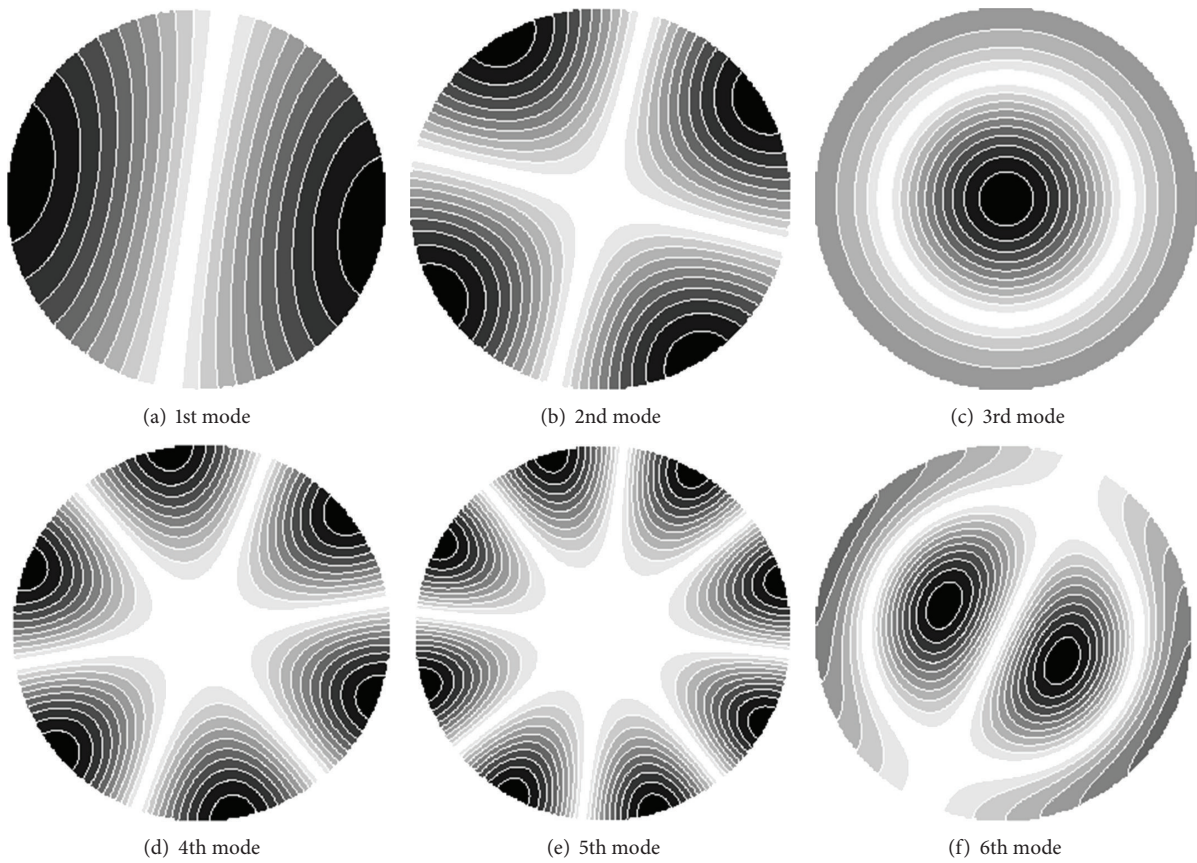


FIGURE 4: Mode shapes produced of the circular cavity by the proposed method using 16 nodes for  $M = 20$ .

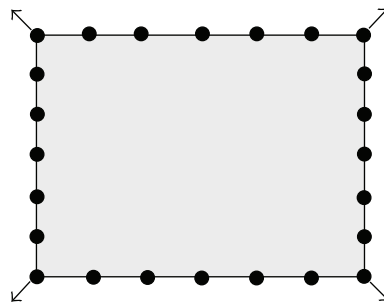


FIGURE 5: Rectangular acoustic cavity discretized by 24 boundary nodes (the 4 arrows denote the normal directions of the corner nodes).

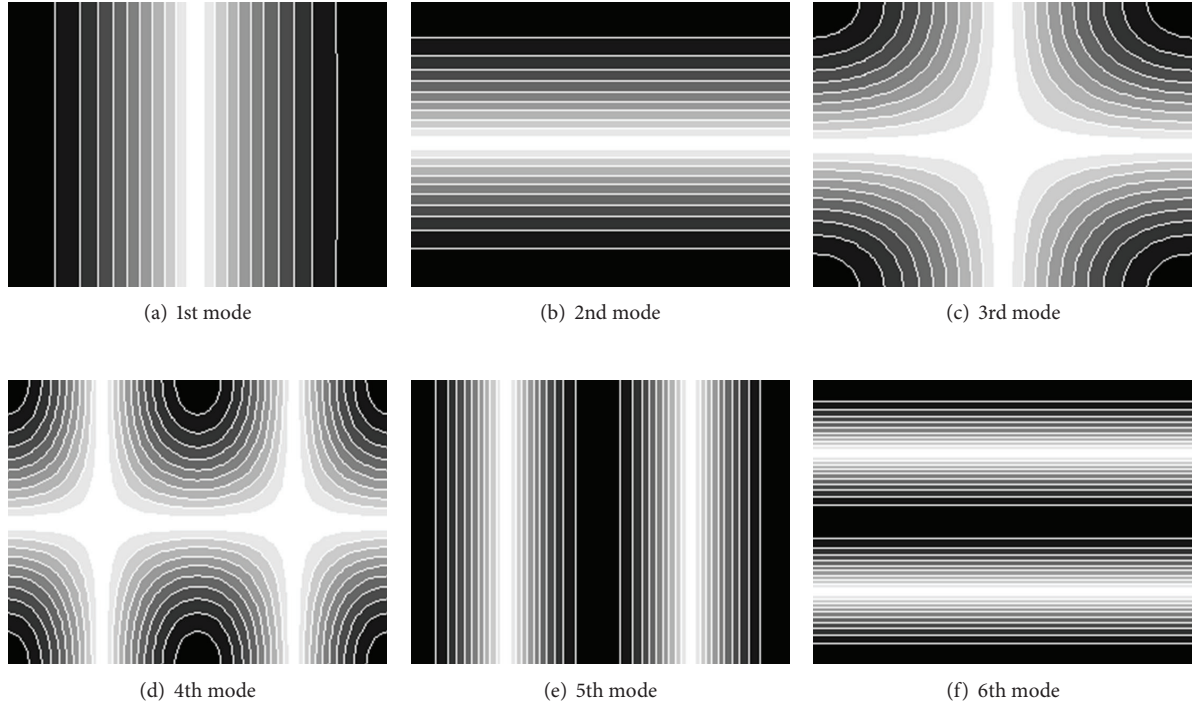


FIGURE 6: Mode shapes of the rectangular cavity obtained by the proposed method using 24 nodes for  $M = 20$ .

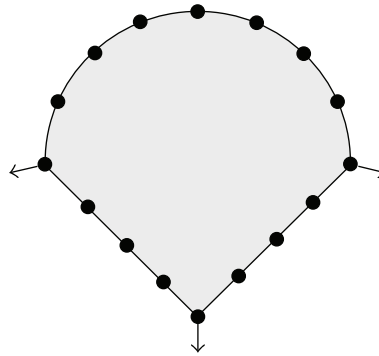


FIGURE 7: Arbitrarily shaped acoustic cavity discretized by 16 boundary nodes (the 3 arrows denote the normal directions of the corner nodes).

On the other hand, Figure 8 shows mode shapes obtained by the proposed method, which are in good agreement with those by FEM (ANSYS), which are shown in Figure 9 [2].

**4. Conclusion**

An improved NDIF method is proposed to more efficiently extract eigenvalues and mode shapes of arbitrarily shaped acoustic cavities. It is revealed that the proposed method

yields highly accurate eigenvalues, which converge to the exact solution, and it gives much more accurate eigenvalues than FEM using a large number of nodes thanks to its concise formulation. It is expected that the method presented in the paper can be extended to accurately analyze multiply connected two-dimensional cavities and three-dimensional cavities. Note that the NDIF method does not give accurate results for concave membranes and acoustic cavities [3]. To overcome this problem, a subdomain method of dividing the concave region of interest into several convex regions will be developed in future research.



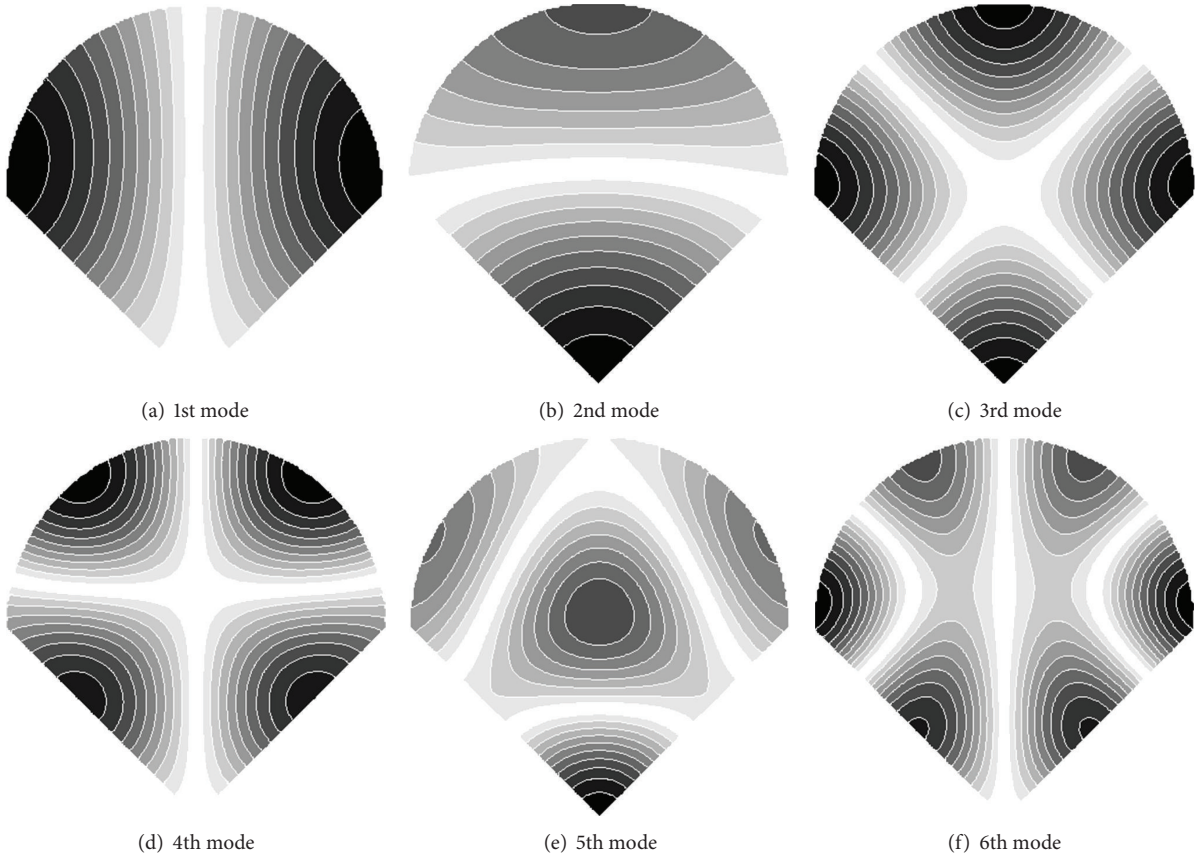


FIGURE 8: Mode shapes of the arbitrarily shaped cavity obtained by the proposed method for using 16 nodes for  $M = 20$ .

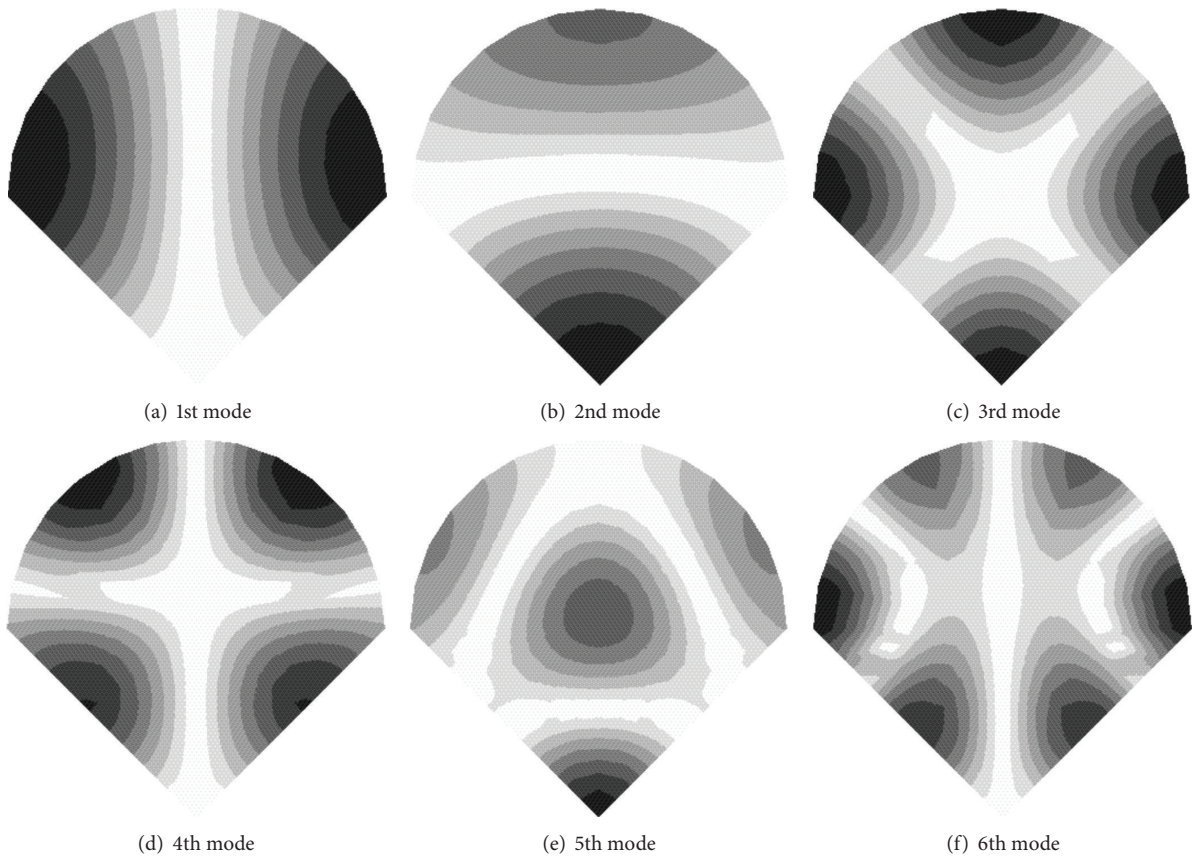


FIGURE 9: Mode shapes of the arbitrarily shaped cavity obtained by FEM [2].

## Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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