LBL-20092
c.2

Lawrence Berkeley Laboratory
UNIVERSITY OF CALIFORNIA

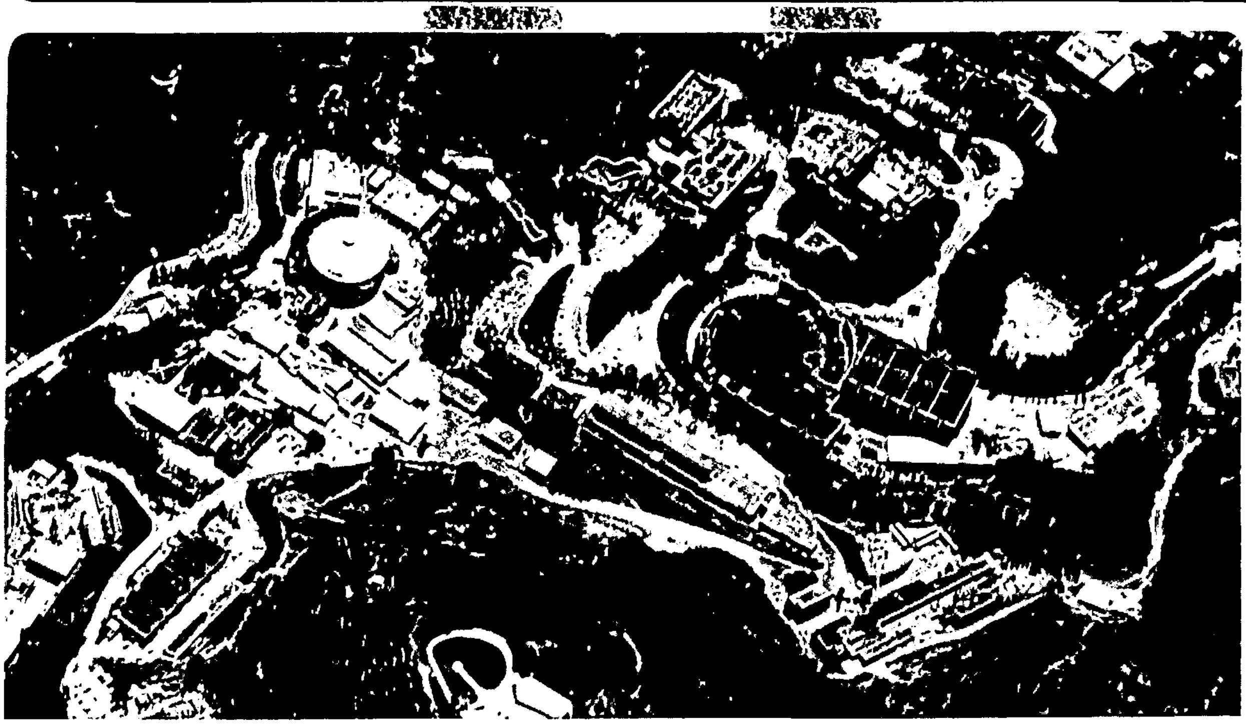
Physics Division

Invited talk presented at the 6th Workshop on
Grand Unification, Minneapolis, MN,
April 18-20, 1985

SUPERGRAVITY AND INFLATION

P. Binétruy

August 1985



Prepared for the U.S. Department of Energy under Contract DE-AC03-76SF00098

August 1985 LBL-20092

Supergravity and Inflation*

Pierre Binétruy**

Lawrence Berkeley Laboratory
University of California
Berkeley, California 94720

* This work was supported by the Director, Office of Energy Research, Office of High Energy and Nuclear Physics, Division of High Energy Physics of the U. S. Department of Energy under Contract DE-AC03-76SF00098, and is an invited talk at the 6th Workshop on Grand Unification, Minneapolis, April 18-20, 1985.

** On leave of absence from LAPP, Annecy-le-Vieux, France.

In the past few years, inflation has become increasingly connected with supersymmetric models. Although there is no fundamental reason for that - the basic ingredient for inflation, at least in the way it has been described so far , is a scalar field whose potential has certain definite properties - the number of arguments in favor is rather compelling and almost all inflationary scenarios which have a claim for success incorporate super-symmetry. The number of problems to which an inflationary scenario is supposed to give an answer - in particular the spectrum and the amplitude of density fluctuations, the gravitino and the monopole problems - as well the number of problems it must avoid - insufficient baryon number generation (in connection with reheating) and entropy crisis - give a set of constraints tight enough to require a good deal of fine tuning in the choice of parameters. We are still left however with a rather large set of models, which reflects the freedom in the choice of the underlying supersymmetric theory. It seems that we have reached a point where the only way to sort out which is the right model for inflation will be to make some progress in the understanding of the exact content of the supersymmetric theory that describes our particle physics world. For example, such issues as the mass of the gravitino, the particles it can decay into, which scalar field is responsible for inflation - the so-called inflaton -, how it couples to the matter fields, how it relates to supersymmetry-breaking are questions that a unique supersymmetric model would answer in an unambiguous way. In that respect, the models inspired by superstring theories that have recently appeared might represent a progress in our understanding of inflation, if they incorporate it.

In Section 1, I review the reasons that led to study supersymmetric models in the context of inflation, by setting up the constraints that candidates to an inflationary scenario must satisfy. Section 2 then raises the question whether the groundstate of the new scalar field that we have introduced- i.e. the inflaton - breaks supersymmetry. This is discussed in connection with the so-called thermal constraint. I take the opportunity to discuss some problems about the study of thermal fluctuations that have received a lot of attention recently. Section 3 then reviews the different models available and the way they address those issues. A discussion of baryon number generation and of the gravitino problem follows in Section 4.

1. Why Supersymmetry?

The standard scenario for inflation[1,2] - which, for historical reasons, is often referred to as "new inflation "- requires the presence of a scalar field, from now on called the inflaton and noted ϕ. The scalar potential has, in the direction of that field, a plateau followed by a dip towards the groundstate σ (see Fig. 1). One assumes that the cosmological constant is zero at the ground state. For reasons to be determined, at some early stage of the evolution of the universe, the inflaton field is held at the top of the plateau ($\phi = 0$). If that region is flat enough, once the inflaton field starts evolving classically in the potential, the time τ necessary to cross the plateau will be large on the time scale of the evolution of the universe. That evolution is governed by the well-known equation (neglecting the curvature term)

$$H^2 = (\dot{R}/R)^2 = (\dot{T}/T)^2 = \rho/3M^2 \qquad (1)$$

where R is the cosmic scale factor that enters in the Robertson-Walker metric, T is the temperature, and M is the reduced Planck mass ($M = M_P/\sqrt{8\pi} \simeq 2.4 \times 10^{18}$ GeV). ρ, the energy density of the universe is given, at early times when all matter is relativistic, by

$$\rho = V(\phi) + \frac{1}{2}\dot{\phi}^2 + \frac{1}{2} R^{-2}(\vec{\nabla}\phi)^2 + \rho_r \qquad (2)$$
$$\rho_r = \frac{\pi^2}{30} N T^4 \qquad N = N_b + \frac{7}{8} N_f$$

where $N_{b,f}$ is the effective number of bosonic (fermionic) degrees of freedom. At high enough temperatures, the radiation term dominates the energy density and the universe is radiation-dominated. Soon, when temperature drops, ρ becomes dominated by the energy stored in the vacuum V_0 and this lasts for the time τ that the inflaton field rolls down the plateau. Plugging $\rho = V_0$ into (1) gives the exponential behavior for the cosmic scale factor that is characteristic of inflation

$$R = R_0 \, e^{\frac{V_0}{3M} t} \qquad (3)$$

At the end of this de Sitter phase (H = cst), R will have undergone $N = \tau\, V_0/(3M)$ e-foldings. In order that inflation solve the horizon and flatness problems, we must require[1] $N \geq 65$, which sets a limit on the height of the plateau (V_0) as well as on its slope (via τ).

Once the inflation field reaches the end of the plateau, it falls down to the minimum σ and starts oscillating around it. The energy density of the universe is then dominated by the density ρ_ϕ of these coherent oscillations(the $\dot{\phi}^2$ term in (2) starts playing a role). For example[3], if the potential is quadratic around the ground state, the coherent oscillations will behave like nonrelativistic matter and the universe will expand like a matter-dominated one. Because of that expansion, oscillations are damped. Eventually after a time $t \sim \Gamma_\phi^{-1}$, the inflation field decays into ordinary matter (Γ_ϕ is the corresponding decay rate). This is what is known as reheating although the term is somewhat misleading: no reheating occurs, strictly speaking, because the matter which was present initially has been diluted away by inflation; on the other hand, the decay products of the inflaton are produced at a temperature T_{RH} which is given by (2)

$$\rho_r = \frac{\pi^2}{30} N T_{RH}^4 = \rho_\phi (t = \Gamma_\phi^{-1}) \qquad (4)$$

Of course all fields that can be produced through the decay of ϕ (or the subsequent interactions between its decay products) are present, even those whose mass is greater than T_{RH}, that would long have decoupled if the temperature of universe had decreased smoothly. We will come back to these matters in Section 4.

Before reviewing the candidates to a successful inflationary scenario, let me quantify the previous analysis to emphasize the role of the two basic parameters: the energy stored in the vacuum at the origin-V_0- and the groundstate value σ. In order to do that, I will take the simplest possible form for the potential (for a more general and more elaborate analysis, see Steinhardt and Turner[4]):

$$V(\phi) = V_0 \left[1 - 4 \left(\frac{\phi}{\sigma} \right)^3 + 3 \left(\frac{\phi}{\sigma} \right)^4 \right] \qquad (5)$$

The fact that there are no linear and quadratic terms assures that the potential is flat at the origin ($V'(0) = V''(0) = 0$). Using the classical equation of motion for the ϕ field,

$$\ddot{\phi} + 3H\dot{\phi} = -V'(\phi) \qquad (6)$$

and neglecting the $\ddot{\phi}$ term throughout the slow-rollover, we can easily compute the number of e-foldings.

$$N = \int_{\phi_0}^{\phi_e} H\, dt = -\int_{\phi_0}^{\phi_e} 3H^2 \frac{d\phi}{V'(\phi)} \qquad (7)$$

In this formula ϕ_0 is the initial value for ϕ; because of quantum fluctuations in the de Sitter phase, ϕ_0 is non-zero. Since the only relevant scale in a de Sitter phase is the Hubble parameter we will simply take[5] $\phi_0 \simeq H = (V_0/3M^2)^{1/2}$. ϕ_e is the value of ϕ at the time t_e when the exponential expansion ends: in other words, $\ddot{\phi}(t_e) \simeq 3H\,\dot{\phi}(t_e)$, which gives, using $3H\dot{\phi}(t_e) = -V'(\phi_e)$*

$$|V''(\phi_e)| \simeq 9H^2 \quad \text{or} \quad \phi_e \simeq \frac{1}{8}\frac{\sigma^3}{M^2} \qquad (8)$$

We can then approximate N by evaluating the integrand in (7) for ϕ near the origin

$$N \simeq -\int_{\phi_0} 3H^2 \frac{d\phi}{V_0(-12\phi^2/\sigma^3)} \simeq \frac{H^2\sigma^3}{4V_0}\frac{1}{\phi_0} = \frac{1}{4\sqrt{3}}\frac{\sigma^3}{MV_0^{1/2}} \qquad (9)$$

One of the major successes of the inflation scenario is to explain the origin of fluctuations in the universe.[6] They start as quantum fluctuations in the de Sitter phase. As inflation goes on, they grow until at time t_i they become bigger than the horizon radius H^{-1} (constant in a de Sitter phase). From then on, their evolution is purely mechanical because the horizon screens dynamics. Exponential inflation ends, the

---

* Note that we can treat the motion of the inflaton field classically only if $\phi_e >> \phi_0$ or $\sigma^3/V_0^{1/2}M >> 1$.

universe becomes, say, radiation-dominated and the horizon radius, now $H^{-1}(t) = 2t$, grows faster than the fluctuations; eventually at time t_f, fluctuations reenter the horizon. Note that t_i and t_f depend on the scale of the fluctuation that we are considering; on the other hand, $(\delta\rho/\rho)(t_f)$ does not and this is a success of inflation: inflation predicts a scale invariant Harrison-Zel'dovich spectrum[7]. The reason is that the time translation invariance of the de Sitter phase insures the scale independence of $(\delta\rho/\rho)(t_i)$ and hence of $(\delta\rho/\rho)(t_f)$ since no dynamics is acting between t_i and t_f. What about the amplitude $\delta\rho/\rho\ (t_f)$? Since we want to account for the size of all density fluctuations in the universe, it is rather strictly constrained. On one hand[8], on the scale of galaxies ($10^{15}\ M_O$), we can expect at most a factor 10^5 growth between t_f and the present time, for which it is established on firm grounds that $\delta\rho/\rho \sim 1$. On the other hand[9], on the scale of the background radiation ($10^{19} M_O$), the observed anisotropy $\delta T/T \simeq 1/2(\delta\rho/\rho)(t_f)$ is smaller than 10^{-4}. We must therefore have

$$\delta\rho/\rho\ (t_f) \simeq 10^{-4} \tag{10}$$

What does the theory predict? The magic formula is[6] (see Ref. 10 for caveats)

$$\frac{\delta\rho}{\rho}(t_f) = \frac{H\,\delta\varphi(t_i)}{|\dot{\varphi}(t_i)|} \tag{11}$$

where $\delta\phi(t_i)$ is the fluctuation at the time of crossing the horizon in de Sitter phase, roughly H. I will crudely neglect the evolution between t_i and t_e and write $\dot\phi(t_i) \simeq \dot\phi(t_e) \simeq -V'(\phi_e)/(3H)$ to obtain, using (8),

$$\delta\rho/\rho\,(t_f) \simeq 3H^3/|V'(\phi_e)| = \frac{16}{3\sqrt{3}}\, V_0^{1/2} M/\sigma^3 \tag{12}$$

To recapitulate before starting our hunt for candidates to an inflationary scenario, we have the following constraints (cf eq. (9, 12))

$$N \simeq O(1)\,\frac{\sigma^3}{MV_0^{1/2}} \gtrsim 65 \tag{13a}$$

$$\frac{\delta\rho}{\rho}(t_f) \simeq O(1)\,\frac{MV_0^{1/2}}{\sigma^3} \simeq 10^{-5} \text{ to } 10^{-4} \tag{13b}$$

Note that basically one parameter appears, namely $I = \sigma^3/(MV_0^{1/2})$, which must be greater than one if our approach makes sense (see footnote to Eq. 8) - and that the two constraints are compatible.

The models originally proposed[2] to realize this "new inflation" scenario were Coleman-Weinberg[11] potentials in grand unified theories. Typically, the potential reads

$$V(\phi) = B\phi^4\left(\ln \phi^2/\sigma^2 - 1/2\right) + 1/2\, B\sigma^4 \tag{14}$$

where

$$B = \frac{5625}{64}\,\alpha_{GUT}^2 \sim 4\times 10^{-2}\,, \quad \sigma = M_{GUT} \sim 10^{15}\,\mathrm{GeV} \tag{15}$$

Using our toy model parametrization ($V_0 = 1/2\, B\sigma^4$), we get

$$\frac{\sigma^3}{MV_0^{1/2}} \simeq B^{-1/2}\,\frac{M_{GUT}}{M} \sim 5\times 10^{-3} \tag{16}$$

which violates the constraints (13). A little caution is needed here. First of all, our toy model parametrization does not really apply here. Furthermore, the fact that $\sigma^3/(MV_0^{1/2}) << 1$ shows us that a classical estimation of N is invalid. More precisely, we can parametrize the Coleman-Weinberg potential near $\phi = 0$ as (we cut off the argument of the logarithm at $\alpha_{GUT}\phi^2 = H^2$):

$$\begin{gathered} V(\phi) = V_0 - \tfrac{1}{4}\lambda\phi^4 \\ V_0 = \tfrac{1}{2}B\sigma^4\,, \quad \lambda \sim 4B\left[-\ln\frac{H^2}{\alpha_{GUT}\sigma^2} + \frac{1}{2}\right] \sim 10^2 B \sim 4 \end{gathered} \tag{17}$$

With this form, one finds, along the same lines as before, that $\phi_e \simeq H\sqrt{3/\lambda} \simeq H$. One has therefore to include gravitational effects in the determination of N, which gives[5]

$$N \sim O(1)\ \lambda^{-1/2} \tag{18}$$

As for the scale of density perturbations, we obtain from (12)(| $V'(\phi_e) \sim H^3/\sqrt{\lambda}$)

$$\frac{\delta\rho}{\rho}(t_f) \sim O(10)\ \lambda^{1/2} \tag{19}$$

where I have included the numerical factor that a less crude estimation would yield.[6] It is therefore now $\lambda^{-1/2}$ which plays the role of the parameter *I* and it is clear that the value of λ is such that we cannot satisfy the constraints (13).

The solution to those constraints is clear: we have to increase σ (and since we had $\sigma = M_{GUT}$, the next step seems to be $\sigma = M$), or decrease V_0 or act on both.

A first possibility adopted by Shafi and Vilenkin[12] is to take a scalar field with a potential given by (17) where $\lambda \sim 10^{-10}$. The field has to be a gauge singlet because gauge radiative corrections would induce self-couplings of a size much bigger than λ. But where does this very weakly coupled singlet field come from?

A second line of approach is to remark[5, 13] that fermions contribute with the opposite sign to B and including them would decrease B and therefore increase *I* as desired. Indeed, in a supersymmetric theory with unbroken supersymmetry,

$$B \propto \Sigma\ (m_B^4 - m_F^4) = 0 \tag{20}$$

and if supersymmetry is broken at a scale μ,

$$B \sim \Sigma\ \frac{m_B^2 - m_F^2}{M^2} \sim \left(\frac{\mu}{M}\right)^4 \tag{21}$$

In order to satisfy the constraints (13), we need to decrease B (or λ) by some 12 orders of magnitude:

$$\frac{\mu}{M} \sim 10^{-3} \text{ to } 10^{-4} \tag{22}$$

This yields a gravitino mass

$$m_{3/2} = \frac{\mu^2}{M} \sim 10^{10} \text{ to } 10^{12}\ \text{GeV} \tag{23}$$

This is the idea of supersymmetric inflation which was first proposed for global supersymmetry[13] and then extended to local supersymmetry.[14]

So much for motivations. From now on, we couple the theory to N = 1 supergravity. The inflaton field is a gauge singlet and its potential has the standard form[15]

$$V(\phi) = M^4 e^{\mathcal{G}} \left[\mathcal{G}_{,\varphi}\, \mathcal{G}^{-1}_{,\varphi\varphi^*}\, \mathcal{G}_{,\varphi^*} - 3 \right] \tag{24}$$

where $\mathcal{G}$ is the Kähler potential (for the time being, we take a flat Kähler metric)

$$\mathcal{G} = \frac{1}{M^2}|\phi|^2 + \log\frac{|f(\phi)|^2}{M^6}\ , \quad \mathcal{G}_{,\varphi} = \frac{d\mathcal{G}}{d\phi} \tag{25}$$

$f(\phi)$ is the superpotential. We will assume that, apart from the Planck scale M, there is basically one (overall) scale present in $f(\phi)$:

$$f(\phi) = \mu^2 M \sum_{n=0}^{\infty} a_n (\phi/M)^n \tag{26}$$

where the a_n are of order 1. This gives a minimum σ of order M, which was one of our goals.

Moreover, we put the inflaton field in a hidden sector,[16] which means that the superpotential describing the interactions of the inflaton field and of the ordinary matter fields y_i reads:

$$G(\varphi, y_i) = f(\phi) + F(y_i) \qquad (27)$$

This ensures that the inflaton field is weakly coupled ($\sim \mu^2/M^2$) to ordinary matter. Since the self-coupling λ for the inflaton is of order μ^4/M^4(cf eqs (24-26)), for

$$\mu/M \sim 10^{-3} \text{ to } 10^{-4} \qquad (28)$$

the hidden sector provides us with the very weakly coupled field that was searched for in an analysis a la Shafi-Vilenkin. We now discuss in some detail the issue of supersymmetry -breaking.

2. Supersymmetry breaking and the inflaton field. The initial conditions.

We now have in our theory two singlet chiral fields: one will provide the goldstino field and is necessary to break supersymmetry via the superHiggs effect, the other one takes care of inflation through its scalar component. It is certainly tempting to identify them. And there is indeed a good reason to do so, apart from the aesthetical or economical ones.

Remember that we need to explain why the inflaton field is initially at the top of the plateau. In the case of the Coleman-Weinberg potentials that we were considering earlier, it was easy to do so: at high temperatures, the grand unified symmetry is restored, therefore the global minimum is at $\phi = 0$; when the universe cools down, the inflaton

quite naturally finds itself in position to slowly roll down to its zero-temperature groundstate $\phi = \sigma$. There is no such symmetry in the supersymmetric case since the inflaton is a gauge singlet.

We therefore have to impose that the potential have an absolute minimum at $\phi = 0$, at high temperatures. This is known as the <u>thermal constraint</u>[17] and it has some interesting consequences on supersymmetry breaking. Before coming to that, I want to stress the following point. We have implicitly assumed that the inflaton is in thermal equilibrium with the matter at $T \sim M_P$. If it was not so, thermal equilibrium between inflaton and matter would be restored only at very low temperatures[18] because the inflaton field is so weakly coupled, and considering thermal corrections to the potential would not make sense. Ovrut and Steinhardt[19] stressed that as the temperature of the universe reaches the Planck scale M_P, the number of particle per horizon becomes less than one (the particle number goes as $(T/M)^3$ and the horizon radius as $(M/T)^2$) which makes it "difficult" to talk about thermal equilibium. This only means that we do not know what is going on in the Planck era and if we want to have a thermal equilibium at $T \sim M_P$, we have to include it in our assumptions.

We will study the behavior of the potential at high temperatures by computing the effective potential in the one-loop approximation.[20-24] It turns out that one piece of this effective potential is proportional to the total number N of chiral fields (for a minimal low energy supersymmetric model N = 50). In the leading N approximation, the temperature corrections to the potential $V(\phi)$ of Eq. (24) read simply:[23,24]

$$\Delta V_T(\phi) = \frac{T^2}{12M^2} N \left[V(\phi) + e^{\mathcal{G}} + o(1/N) \right] \qquad (29)$$

Using this form, it is straightforward to show a rather strong result on supersymmetry breaking:[24] *if the thermal constraint is satisfied, the absolute minimum ϕ of the superpotential must be supersymmetry-breaking*. Indeed, let us suppose for a moment that it is not, i.e. $m_{3/2}^2 = M^2 e^{\mathcal{G}}|_\sigma = 0$, then since $V(\sigma) = 0$, we have $\Delta V_T(\sigma) = 0$. On the other hand, $\Delta V_T(\phi) > 0$ for $\phi \neq \sigma$. Therefore σ is the absolute minimum even at high temperature, which contradicts the thermal constraint. One can show[24] that the result still holds when we generalize our study to include: i) terms non-leading in N, ii) partial supersymmetry-breaking in the ordinary sector. On the other hand, two key assumptions are the facts that we considered that the *inflaton sector consists only of one field* (ϕ) and that we used a flat Kähler metric (eq. 25) (see below). To summarize, we have the following result: with only one field in the inflaton sector, it is not possible to conserve supersymmetry in that sector and to satisfy the thermal constraint. It is possible to go further and to quantify that result. We show on Fig. 2 the behavior of the temperature corrections for a very simple example of potential. In order to satisfy the thermal constraint, we need $e^{\mathcal{G}} M^4/\mu^4 \geq 4$. This result proves to be general[24] Therefore in order to satisfy the thermal constraint, we need

$$m_{3/2} > 2\,\mu^2/M \qquad (30)$$

which gives, using (28)

$$m_{3/2} \sim 10^{10} \text{ to } 10^{12}\,\text{GeV} \qquad (31)$$

It is not a surprise that we obtain again the gross estimate which prompted our analysis (eqs. 22, 23). Note also that we will get such a range for $m_{3/2}$ whenever we break supersymmetry in the inflaton sector- independently of the thermal constraint.

Ovrut and Steinhardt[17] were actually the first to stress the connection between the thermal constraint and supersymmertry breaking by proving the following theorem: a supersymmetry-conserving real minimum σ with $V(\sigma) = 0$, which satisfies the thermal constraint and gives enough inflation, is always separated, along the real axis, from $\phi = 0$ by another minimum σ' for which $V(\sigma') < 0$.

Now, a constraint such as (31) is a real embarrassment if we want the theory to account also for the low energy phenomenology, in particular the breaking of $SU(2) \times U(1)$. The point is that, quite generally when we couple gauge theories to supergravity, scalars acquire tree level masses of the order of the gravitino mass. In order to see that, let us generalize the potential of Eq. 24 to include one low energy scalar y - the superpotential is given by (27)-

$$V(\phi, y) = e^{(|\phi|^2+|y|^2)/M^2}\left[\left|\frac{\partial f}{\partial \phi} + \frac{\phi^*}{M^2}G\right|^2 + \left|\frac{\partial F}{\partial y} + \frac{y^*}{M^2}G\right|^2 - \frac{3}{M^2}|G|^2\right] \qquad (32)$$

When we shift the inflaton field, we obtain in particular mass terms for the scalar y:

$$\mathcal{L}_m = |y|^2 \frac{1}{M^4} e^{\sigma^2/M^2} |f(\sigma)|^2 = m_{3/2}^2 |y|^2 \qquad (33)$$

This is innocuous for most of the scalar fields, but certainly not for the Higgs doublets of the Weinberg-Salam model.[25] Such a term would induce a breaking of the gauge-symmetry at the scale $m_{3/2}$. Therefore requiring that we can describe successfully the low energy phenomenology seems to require

$$m_{3/2} \simeq M_W \qquad (34)$$

We will spend most of the next section trying to reconcile (31) with (34).

Let me finish this section by adding some important comments to the preceding discussion of thermal effects.

To determine the conditions for inflation, we have relied - through the thermal constraint - upon the effective potential at the one-loop level. As Mazenko, Unruh and Wald[26] have strongly emphasized, we must be very cautious in using such a tool. Their argument goes as follows.[26] First of all, the effective potential $V_T(\phi)$ describes the properties of the spatially averaged value $<\phi>_{sp}$ of ϕ. In particular, the fact that as the temperature increases, $V_T(\phi)$ becomes more convex around $\phi = 0$ only tells us that the fluctuations in $<\phi>_{sp}$ become small, not that the local fluctuations in ϕ become small. Actually, locally the latter are of order T and therefore become increasingly large. This suggests that some domains can form where $\phi \simeq \sigma$, at $T > T_c$, where T_c is the temperature at which the potential energy starts to dominate the energy density of the universe. If such a thing happens, this is bad news for inflation. One could think that still some domains will form in which $\phi = 0$; these domains would inflate and eventually reach a size that could easily encompass our universe. Unfortunately, once we allow for the presence of $\phi = \sigma$ domains, the domains where $\phi \simeq 0$ become very scarce. The reason is that they have to be extremely big in order to survive the pressure exerted on their boundary by the true vacuum domain. Typically, their radius ρ has to be bigger than the Hubble radius H^{-1} in order that expansion overcomes the fact that their boundary is contracting inward at the speed of light.* Since for $T > T_c$,

the universe is radiation dominated $\rho \sim H^{-1} \sim M/T^2$. On the other hand, the size of the $\phi = \sigma$ domains should just be $\xi \sim T^{-1}$, since they originate from thermal fluctuations. Therefore, when we approach the critical temperature, the number of $\phi = \sigma$ domains overcomes the number of $\phi = 0$ domains by a factor

$$\left(\frac{\xi}{\rho}\right)^{-3} \sim \left(\frac{M}{T_c}\right)^3$$

In our case, $T_c = V_0^{1/4} = \mu$ which gives a very large factor (10^9 to 10^{12} from Eq. 28). The occurrence of domains with $\phi = 0$ is therefore very improbable.

Is this the ungraceful exit of inflation? Not quite. First, let me point out that our use of the effective potential was a perfectly valid one. We were looking for the conditions for inflation. Certainly, one condition to fulfill is to require the existence of a metastable state with non-zero vacuum energy V_0 (i.e. the thermal constraint). As commented by Mazenko, Unruh and Wald,[26] the effective potential is a

---

* Taking into account the contraction of the boundary, volume of the domain is $V(t) = \rho(t)^3$ with $\rho(t) = R(t)(\rho_0/R(t_0) - \int_{t_0}^{t} dt'/R(t'))$ where ρ_0 is the radius at $t = t_0$. Requiring that $V(t) > 0$ gives $\rho(t) > R(t)/R(t) = H^{-1}(t)$.

perfectly valid tool to do so. Of course, once we have that metastable state, it does not mean that inflation occurs: this is the whole point of their discussion. Secondly, their analysis is incomplete because it assumed that all domains occur with an equal probability. This assumption is model dependent. Remember that at T_c, the fluctuations in the field ϕ are of order T_c, therefore if $T_c << \sigma$, or more precisely if T_c is of the order of ϕ_e or smaller, (ϕ_e is the value of ϕ when the universe leaves the exponential expansion era, cf eqs. (7,8)), basically all domains present at T_c will lead to inflation. Since $\phi_e \sim \sigma^3/M^2$ (eq. 8), we conclude that this is not verified for Coleman-Weinbergy potentials ($T_c \sim M_{GUT}$, $\sigma \sim M_{GUT}$) but that it is so in the case of supersymmertric models ($T_c \sim \mu$, $\sigma \sim M$). This result is substantiated by the analysis of Albrecht and Brandenberger.[27] They showed that the potential term in the energy density of the universe (eq. 2) starts to dominate when T drops below T_c and that the time Δt necessary to recreate domains where $\phi = \sigma$ (through the interaction terms of the potential) is longer than the duration of inflation τ. Actually expressing the requirement $\Delta t > \tau$ for a general potential $V(\phi) = \Sigma\ \lambda_n \phi^n$ yields the condition[27,28]

$$\sum_n \frac{\sqrt{n}\,|\lambda_n|}{(2\pi)^n}\, T_c^{n-4} < \frac{H}{T_c} \tag{35}$$

In the case of our toy model (eq. 5), the condition on λ_3 reads

$$|\lambda_3| \ll H \quad \text{or} \quad \frac{M V_0^{1/2}}{\sigma^3} \ll 1 \tag{36}$$

This is nothing else than the condition (13) that we found necessary to impose to obtain enough inflation and the right amplitude for density fluctuations. Conditions on higher order couplings (λ_4) from (35) are less stringent. Similar results have been obtained by Guth and Pi[29]

who found that sufficient inflation occurs if the inflaton field is coupled weakly enough ($I^{-1} = \lambda^{1/2} << 1$).

The picture now seems clear enough. Let me bring some confusion how-ever by adding the following remark, due to Linde.[30] At large enough temperatures ($T >> T_c$), domains where $\phi >> T_c$ are created by thermal fluctuations. Naively, when the temperature decreases ($T < \phi$) the field rolls down the temperature dependent potential in order that at T_c all domains have $\phi \le T_c$. This is not so because the inflaton field is too weakly coupled to be in thermal equilibium. More quantitatively, if we come back to our toy langrangian, the temperature correction reads

$$\Delta V_T \sim T^2 \frac{d^2V}{d\phi^2} \sim V_0 \frac{\phi T^2}{\sigma^3} \tag{37}$$

and the motion of the ϕ field in the potential is given by (6)

$$\ddot{\phi} + 3H\dot{\phi} \simeq -\frac{V_0 T^2}{\sigma^3}\ , \quad H \sim \frac{T^2}{M}\ , \quad t \sim \frac{M}{T^2} \tag{38}$$

which gives

$$\phi \propto -\frac{V_0 M}{\sigma^3}\, t \tag{39}$$

Therefore if we consider the largest possible fluctuation $\phi \sim M$ (created at $T \sim M$), it will take a time σ^3/V_0 to go back to zero. But the age of the universe at T_c is M/T_c^2. Thus if

$$\frac{\sigma^3}{V_0} \gg \frac{M}{T_c^2} \quad \text{or} \quad \frac{\sigma^3}{M V_0^{1/2}} \gg 1 \tag{40}$$

the field will never come back to the minimum and the high-temperature configuration will be "frozen". But eq. (40) is nothing else than condition (13). This seems to indicate that for all the models that we might consider all possible configurations (even $\phi >> T_c$) are allowed at $T = T_c$. It led Linde[30] to describe supersymmetric inflation in the context of chaotic inflation.[31] According to this hypothesis, the initial distribution of the field ϕ is a chaotic one where all values are equally probable. Of course, we are back to the problem raised earlier: why should our universe be in one of the $\phi = 0$ domains that were so rare at T_c? Moreover, we have heavily relied upon the effective potential to derive (40) and although this certainly points towards a problem, a more elaborate analysis is needed to determine quantitatively in which cases the standard scenario does not work. In particular, gravitational interactions might drive the inflaton field more quickly to its minimum.[19]

3. The models.

We now describe some of the models, emphasizing the way each of them solves the problem of the discrepancy between the constraints (31) from the amplitude of density fluctuations and (34) from the low-energy spectrum of scalar masses.

3.1. Models where supersymmetry is not broken in the inflaton sector.[14,18,32] If supersymmetry is not broken in the inflaton sector, then $m_{3/2}$ is not related to the parameter μ (Eq. 30) which was fixed by the constraints (13). For example, Holman, Ramond and Ross used the very simple superpotential[18]

$$f(\phi) = \frac{\mu^2}{M}(\phi-\sigma)^2 \tag{41}$$

and showed that it satisfies all the basic requirements (note that $f(\sigma) = 0$: supersymmetry is conserved at the minimum).

Of course, the thermal constraint is violated. One way out of it[18] is to introduce a second field ψ in the inflaton sector (remember that to obtain the result of last section, we assumed that there is only one field in that sector). For example, a superpotential for the fields ϕ, ψ of the form[18,33]

$$h(\phi,\psi) = f(\phi) + \psi^2 g(\phi) \tag{42}$$

where $f(\phi)$ is given by (26) will give the right behavior at high temperature (minimum at $\phi = 0$), if we choose correctly the couplings in $g(\phi)$. This means in particular that some of the couplings in $g(\phi)$ have to be of order N (otherwise, the argument using the leading N temperature correction (29) would still be valid), which is not very natural. Moreover, it is somewhat unsatisfactory to introduce this ad hoc field ψ whose only purpose is to satisfy the thermal constraint.

One issue that is raised in models where the inflation field ϕ is different from the field z responsible for supersymmetry-breaking is the so-called *entropy crisis*.[34] Let us consider the field z; it is clear from our discussion of temperature corrections, that its groundstate at high temperature lies away from the minimum of the $T = 0$ potential. Therefore, when temperature decreases, the z field will oscillate around its minimum and these coherent oscillations will release entropy when the field decays. (This is very similar to our discussion of reheating at the end of inflation but remember that now $z \neq \phi$.) Moreover even if inflation washed away the energy stored in the oscillations the minimum of the z field would be displaced by its coupling to the inflaton ϕ and oscillations would resume at the end of inflation. Now, the mass m_z of the z field is of the order of $m_{3/2}$, which we want to be light ($O(M_W)$, cf eq. (34)); if we place the z field in a hidden sector, (as for a Polonyi field[35]) its decay rate is

$$\Gamma_z \sim \frac{m_z^3}{M^2} \sim \frac{m_{3/2}^3}{M^2} \sim \frac{M_w^3}{M^2} \tag{43}$$

Therefore z is very long-lived and entropy will be released at a very late stage of the evolution of the universe, thus diluting away any baryon number abundance (This is similar to the gravitino problem discussed in Sect. 4.1). The solution[32] is to break supersymmetry a la O'Raifeartaigh[36]. In this case

$$m_z \sim \alpha_\lambda \sqrt{m_{3/2} M}\ ,\ \Gamma_z = \frac{\alpha_\lambda^3}{(4\pi)^2} m_z \sim \frac{\alpha_\lambda^4}{(4\pi)^2}\sqrt{m_{3/2}M} \sim \frac{\alpha_\lambda^4}{(4\pi)^2}\sqrt{M_w M} \tag{44}$$

where α_λ is a coupling in the superpotential ($\alpha_\lambda \sim 10^{-4}$ typically). The lifetime of the z field is therefore much shorter, and one can find a range of parameters for which there is no entropy crisis.[32]

3.2 Models obtained by perturbing a supersymmetry-conserving potential.[33]

The idea here is to start with a potential whose absolute minimum is supersymmetry-conserving and to see under which conditions a perturbation can make this minimum supersymmetry-breaking. Let ε be the per-turbation parameter. It is easy to deduce from the form of the potential (eqs. 24, 25), that a necessary condition in order that supersymmetry be broken when $\varepsilon \neq 0$, is

$$\frac{\partial^2 f}{\partial \phi^2}(\sigma)\bigg|_{\varepsilon=0} = 0 \tag{45}$$

or equivalently.

$$\frac{d^2V}{d\phi^2}(\sigma)\bigg|_{\varepsilon=0} = \frac{d^3V}{d\phi^3}(\sigma)\bigg|_{\varepsilon=0} = 0 \tag{46}$$

This somewhat fixes the potential we are starting with. In particular, one deduces from (45) that the mass of the inflaton field is small (actually, it is of order $\varepsilon^{1/2}$) which gives a small decay rate $\Gamma_\phi \sim m_\phi^{\ 3}$ and therefore a low reheating temperature.

What is ε? In Ref. 33, it was assumed that $(-\varepsilon)$ is the slope of the potential at the origin. It is usually assumed to be zero but no symmetry argument requires that. But it certainly has to be small ($\varepsilon \leq 10^{-6}$) in order that sufficient inflation occurs.

The gravitino mass is at least of order ε since the model at $\varepsilon = 0$ conserves supersymmetry. Actually, the global minimum (in the complex plane) yields

$$m_{3/2} \sim \frac{\mu^2}{M}\,\varepsilon^{3/2} \tag{47}$$

which easily gives $m_{3/2} \sim M_w$ when one uses the constraints on μ (Eq. 28) and ε. One can show that this result is stable under radiative corrections. Of course, the fact that we can obtain such a low mass for $m_{3/2}$ tells us that the thermal constraint is violated (the $\varepsilon = 0$ model clearly violates it, since it is supersymmetry conserving) and we have to introduce a second field to restore it, as in the previous models.

3.3 Inflaton sector with two fields.[37]

If we have to introduce two fields in the inflaton sector, why not introduce them from the beginning? Taking that point of view, Ovrut and Steinhardt[37] were able to use a method that gives very naturally a low supersymmetry-breaking scale. The method can be illustrated on the following example.[19,38] Let us consider the superpotential

$$f(\phi, \psi) = -\mu^2\phi + \frac{1}{2}\phi^2\psi \tag{48}$$

and study the minimum of the potential in global supersymmetry, for the time being. It is given by

$$\partial h/\partial\phi = -\mu^2 + \phi\psi = 0$$
$$\partial h/\partial\psi = \frac{1}{2}\phi^2 = 0 \tag{49}$$

which gives $\phi = \mu^2/\psi$ and $\psi \to \infty$. If we turn gravity on (local supersymmetry), the Planck mass acts as a cutoff and the minimum is at

$$\langle\psi\rangle \sim M \qquad \langle\phi\rangle \sim \mu^2/M$$
$$\langle D_\phi h\rangle \sim 0 \qquad \langle D_\psi h\rangle \sim (\mu^2/M)^2 \tag{50}$$

We conclude therefore that the gravitino mass is

$$m_{3/2} \sim \frac{|\langle D_\psi h\rangle|}{M} \sim \frac{\mu^4}{M^3} \tag{51}$$

From the constraint $\mu \sim 10^{-3}$ to 10^{-4} (Eq. 28), we see that we fall precisely in the range $m_{3/2} \sim M_W$. Ovrut and Steinhardt applied this method to the inflaton sector.[37] The potential they used is sketched on Fig. 3. Starting at the origin, one evolves first down a plateau in the ϕ direction; when ϕ reaches the saddle point σ, one falls down rapidly to the super-symmetry-breaking minimum described above ($\phi \sim \mu^2/M$, $\psi \sim M$). The computation of the temperature corrections shows that they stabilize the minimum at the origin (one can note indeed that the superpotential used by Ovrut and Steinhardt[37] is basically of the generic form (42)). Therefore the thermal constraint is satisfied.

The study of reheating and of the gravitino problem requires special treatment in this kind of model because the presence of two fields creates some anharmonicities in the potential near the true groundstate. In particular, one has to check that there is no entropy crisis in the direction orthogonal to the direction of supersymmetry-breaking.[37]

3.4 No-scale models.

There are some cases where the constraint (34) can be evaded. This is when the scalar sector respects some global symmetry.[39] To each unbroken generator corresponds a massless Goldstone boson. Usually, the gauge sector does not respect that symmetry, which therefore becomes approximate. The Goldstone bosons turn out to be pseudo-goldstone bosons and acquire a mass through radiative corrections. The interesting point for us is that in that case, the tree level supersymmetry-breaking mass term is zero for those pseudo Goldstone bosons. If the doublets that break $SU(2) \times U(1)$ are among these fields, their mass is no longer directly related to $m_{3/2}$, which can therefore take any value (at least as far as this problem of $SU(2) \times U(1)$ breaking is concerned).

No-scale models provide a nice example of how this works. They were introduced[40] as a way to obtain a vanishing cosmological constant at tree level without unnatural fine tuning. To start with the simplest version, let z be a singlet field. If we drop our assumption of a flat Kähler metric and use as a Kähler potential, instead of (25),

$$G = -3 \ln \frac{z+z^*}{M} \tag{52}$$

we obtain from eq. 24 a potential that is identically zero. Therefore z, and hence $m_{3/2} = Me^{g/2}$ remains undertermined at the tree level. The connection with the SU(1,1) invariance

$$z \rightarrow \frac{\alpha z + i\beta}{i\gamma z + \delta} \qquad \alpha\delta + \beta\gamma = 1 \tag{53}$$

was soon realized[40,41]:it is that non-compact symmetry that assures the flat potential to be zero.

If we now include matter fields (some of them getting a vev at the GUT scale), the generalization of (52) is[42]

$$\begin{aligned} \mathcal{G} &= G + \ln \frac{|F(y_i)|^2}{M^6} \\ G &= -3\ln\left(\frac{z+z^*}{M} - \frac{1}{3}\frac{y_i y_i^*}{M^2}\right), F(y_i) = c + d_{ijk}\, y_i y_j y_k \end{aligned} \tag{54}$$

The invariance is now SU(n, 1)/SU(n) $\times$ U(1) and the potential reads

$$V = e^{\frac{2}{3}G} \left| \frac{\partial F}{\partial y_i} \right|^2 + \text{D-terms} \tag{55}$$

The dependence of the scalar potential in z (through G) and therefore in $m_{3/2}$ is spurious because the fields y_i are not normalized properly. In terms of the normalized fields $y_i = e^{<G>/6} y_i$,

$$V = \left| \frac{\partial F}{\partial Y_i} \right|^2 + \text{D-terms} \ , \ F = c + d_{ijk} Y_i Y_j Y_k \tag{56}$$

There, once again the potential is flat in the direction of supersymmetry-breaking (z) and no mass term of order $m_{3/2}$ appears (compare with eq. 33). In those models, supersymmetry-breaking comes from the gaugino masses m_v and the scalar masses are of the order of m_v. Therefore our previous constraint (34) transforms into $m_v \simeq M_W$. On the other hand, the ratio $\xi = m_{3/2}/m_v$ is determined

dynamically by studying the radiative corrections that raise the degeneracy of the potential and determine z. This ratio ξ is model dependent and can yield gravitino masses $m_{3/2} \sim M_W$[42], $m_{3/2} \sim M_P$[43], $m_{3/2} << M_W$.[44]

When inflation is discussed in the context of these models,[45-47] the inflaton field ϕ is taken to be among the fields y_i. The fact that the potential (56) is always positive definite shows us that the theorem of Ovrut and Steinhardt[17] discussed above does not apply here: even if the groundstate for ϕ conserves supersymmetry, there cannot be a lower minimum elsewhere. Similarly, our result on the relation between thermal constraint and supersymmetry-breaking does not apply here because we have relaxed one of our hypothesis: a flat Kähler metric. In the leading N approximation, however, one obtains instead[24,47] of (29)

$$\Delta V_T = \frac{T^2}{18M^2} N \left[V(\phi) + O(1/N) \right] \tag{57}$$

where V is given by (55) (F-term); therefore, the thermal constraint is violated in that approximation: the minimum is unchanged at high temperature. One has also to be careful about the entropy crisis in those models since ϕ is distinct from z, the field responsible for supersymmetry-breaking. Let me mention finally that chaotic inflation scenarios can be realized in the same context.[48]

3.5 Superstring models

Recently, no-scale models have received some special attention because they apparently[49] emerge from the reduction[50] of ten dimensional superstring theories[51] to four dimensions. Of course, the underlying superstring theory brings some new constraints to the model, in particular to its particle content. Let me therefore review which are the candidates for the inflaton field in that context.[52] First of all, the sector of the gauge singlet scalar fields is very poor. It consists of the dilaton, another scalar field coming from the 10-dimensional metric and two pseudoscalars (axions). These four fields make up two complex scalars S and T in terms of which the Kähler potential reads:[49]

$$\begin{aligned} \mathcal{G} &= -\ln \frac{S+S^*}{M} + G + \ln \frac{|F(y_i)|^2}{M^6} \\ G &= -3 \ln \left(\frac{T+T^*}{M} - \frac{1}{3} \frac{y_i y_i^*}{M} \right) \end{aligned} \qquad (58)$$

where the y_i are the gauge non-singlet scalar fields. A comparison with eq. 54 shows that the scalar T plays the role of the field z that breaks supersymmetry. Similarly, the vacuum expectation value for $T + T^*$ remains undetermined at tree level, and so is the scale of super-symmetry-breaking. On the other hand, the presence of the S field in (58) has some important consequences. In order to describe the fields y_i, let us consider the $E_8 \times E_8'$ model,[51] compactified on a Calabi-Yau manifold of SU(3) holonomy.[50] Once one identifies one of the SU(3) subgroups of E_8 with the holonomy group, the gauge group becomes $E_6 \times E_8'$. Moreover if the manifold is not simply connected,[50,53] the grand unified group E_6 is broken at tree level to a low energy gauge group K. The smallest realistic (i.e. including $SU(3)_c$ and Weinberg-Salam gauge group) such group turns out to be $K_0 = SU(3) \times SU(2) \times U(1) \times U(1)$. The number of generations N_G is fixed by the topology of the manifold. On the other hand, the assumption of an SU(3) holonomy group tells us that scalar fields are in $\underline{27}$ representations of E_6. We therefore have N_G families of $\underline{27}$ plus some self-conjugate part of $\underline{27} + \overline{\underline{27}}$. We are looking for fields singlet under $SU(3)_c \times SU(2)_L \times U(1)_Y$ to play the role of the inflaton. It is easy to check that only two such fields exist for each $\underline{27}$.

Let me call them N_1 and N_2. The restrictions on the superpotential as well as the invariance under the group K forbid any interaction term involving only N_1 and N_2. We therefore need the presence of their counterpart in the $\overline{\underline{27}}$, $\overline{N}_1$ or $\overline{N}_2$. One can check[53] that such a field is present only when K is of rank 6 (therefore not in the minimal case K_0). Moreover, the scalar fields N_1 and N_2 are precisely the ones that will break the extra U(1) groups (at least one) at an intermediate scale.[54] One has to make sure that this does not interfere with inflation. Therefore, the use of one of the y fields for inflation is rather constrained by superstring theories.[52] There are some other differences with the previous no scale models. In particular, the mass of the gravitino seems to be constrained to be close to the Planck scale.[55] The source for supersymmetry breaking is the condensation of the gauginos of the E_8' sector.[56] In the usual no-scale models,[41,42] the scale of supersymmetry-breaking (i.e. $m_{3/2}$) is determined by the radiative corrections due to the light fields (y). Here, it turns out that the heavy gauge-singlet sector (S, T) already fixes that scale at the one-loop level and drives it to the Planck mass.[55] With such a high gravitino mass, one clearly avoids the gravitino problems discussed in the next section.

4. The problems of reheating.

As we mentioned in Section 1, at the end of inflation, the inflaton field falls down to the minimum σ and starts oscillating around it. Since the energy density of the universe is dominated by these coherent oscillations, the universe expands which redshifts away the energy stored in the oscillations. Eventually at time $t \sim \Gamma_\phi^{-1}$, the inflaton field decays into ordinary matter. Since we have placed the inflaton field in a hidden sector, it can only interact gravitationally with the rest of the matter (i.e. its coupling is of order M^{-1}) and, simply on dimensional grounds (or from Eq. 27 and 32), its decay rate is given by

$$\Gamma_\phi = m_\phi^3 / M^2 \tag{59}$$

Since V is of order μ^4, the mass of the inflaton field m_ϕ is generally of order μ^2/M. Now from Eq. 4, it is straightforward to compute T_{RH}

$$T_{RH} \sim \left[\rho_\phi(t=\Gamma_\phi^{-1})\right]^{1/4} \sim \left[\frac{M^2}{(\Gamma_\phi^{-1})^2}\right]^{1/4} \sim (\Gamma_\phi M)^{1/2} \tag{60}$$

where we have neglected the contribution to the energy density of the matter that is created during the oscillating phase, and we have used the fact that $\rho \sim M^2/t^2$. Therefore, typically

$$T_{RH} \sim \left(\frac{m_\phi^3}{M}\right)^{1/2} \sim \frac{\mu^3}{M^2} \sim 10^6 \text{ to } 10^9\,\text{GeV} \tag{61}$$

We see that a general property of supersymmetric inflation is a low reheating temperature. Of course, there are variations from model to model. For example, for the model of subsection 3.2, T_{RH} is even lower[33] ($T_{RH} = \mu^3/M^2\, \varepsilon^{3/4}$). On the other hand, in the model with two fields,[37] one of them, ϕ, is very heavy ($m_\phi \sim M$) and although oscillations proceed in the ψ direction (see Fig. 3), anharmonicities of the potential around the minimum convert a fraction of the oscillation energy to the ϕ direction[37] ($T_{RH} = \mu^2/M$). Anyway, the low reheating temperature (60) requires that some special attention be addressed to the questions of baryon number generation and gravitino production.

4.1. Baryon number generation.

With such a low T_{RH}, it is hopeless to create by thermal equilibrium processes the color triplet Higgs H^c that we need to generate baryon number in supersymmetric theories (the non(?)-observation of proton decay sets their mass m_{HC} to be bigger than, say, 10^{15}GeV). For this

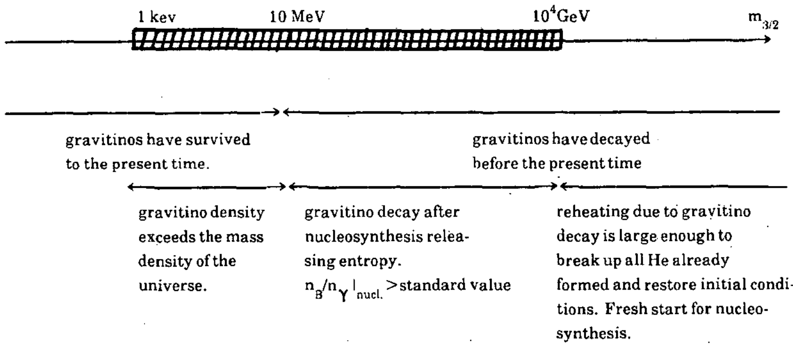
reason, one cannot reproduce the standard scenario for baryon number generation. On the other hand, the mass of the inflaton is bigger than T_{RH}; (see Ref. 57 for a review) $m_\phi \sim \mu^2/M$. Therefore, if $m_\phi > m_{HC}$, the color triplets are produced, and they are produced very far from equilibrium (since $T_{RH} << m_\phi$), which is a bonus for baryogenesis.[57] In this case,[58] if we assume that the inflaton decays predominantly into the heaviest particles (H^c), we obtain typically for the baryon to photon ratio[58,4,18]

$$\frac{n_B}{n_\gamma} \sim \frac{\delta B\, n_\phi}{T_{RH}^3} \sim \frac{\delta B\, \rho_\phi / m_\phi}{T_{RH}^3} \sim \delta B \frac{T_{RH}}{m_\phi} \sim \delta B \frac{\mu}{M} \tag{62}$$

where δB is the baryon asymmetry produced per decay of H^c($\delta B << 1$). In most models, this is enough to reproduce the observed value ($\sim 10^{-10}$). On the other hand, if $m_\phi < m_{HC}$, there is no way to produce the color triplet Higgs and we have to advocate some low temperature non-standard scenarios for baryogenesis.[59]

4.2 The gravitino problem

Let me first summarize the situation of the gravitino problem in the standard scenario[60] by means of the following table:



We therefore have the following bound for the mass of the gravitino:

$$m_{3/2} < 1\ \text{keV} \quad \text{or} \quad m_{3/2} > 10^4\ \text{GeV} \tag{63}$$

It was first thought that inflaton could "save the gravitino"[61] by diluting away the abundance of primordial inflatons but it was soon realized that in an inflationary scenario there are new sources of gravitinos. They are:

i) production by direct decay of the inflaton field

This happens whenever $m_\phi > m_{3/2}$. Note however that in our general analysis $m_\phi \sim m_{3/2} \sim \mu^2/M$; on the other hand, when $m_{3/2} \sim M_w$, we have $m_\phi >> T_{RH} >> m_{3/2}$ (cf eq. 61). The problem is the following[19]: (see Fig. 4) at T_{RH}, gravitinos and photons (radiation) are produced in roughly equal amounts. Since both of them are relativistic, their energy densities behave the same way until $T_{NR} \sim m_{3/2}$ where the gravitino becomes non-relativistic: $\rho_{3/2}(T_{NR}) = \rho_\gamma(T_{NR})$. From then on, $\rho_{3/2}$ scales as T^3 (non-relativistic matter) whereas $\rho_\gamma \sim T^4$. Therefore at $t_D \sim \Gamma_{3/2}^{-1}$, or $T_D \sim (\Gamma_{3/2}M)^{1/2}$ when the gravitino decays ($\Gamma_{3/2}$ is the gravitino decay constant and since the gravitino couples only gravitationally to the matter, $\Gamma_{3/2} \sim m_{3/2}^3/M^2$ - cf eq. 59 - which gives $T_D \sim m_{3/2}^{3/2}/M^{1/2}$)

$$\frac{\rho_{3/2}(T_D)}{\rho_\gamma(T_D)} \sim \frac{\rho_{3/2}(T_{NR})}{\rho_\gamma(T_{NR})}\left(\frac{T_{NR}}{T_D}\right) \sim \frac{T_{NR}}{T_D} \tag{64}$$

The energy stored in the gravitino ($\rho_{3/2}$) is released in the universe as entropy. We therefore have to make the ratio T_{NR}/T_D as small as possible in order not to destroy the successes of the standard big bang scenario.

Ovrut and Steinhardt noticed in that respect that T_{NR} is actually bigger than $m_{3/2}$. The reason is that the gravitinos are produced at T_{RH} with a momentum $|p_{3/2}| \sim m_\phi$ which is subsequently simply reshifted because the gravitinos couple too weakly to the rest of the matter. Therefore the gravitino becomes non-relativistic at T_{NR} determined by

$$m_{3/2} \sim |\vec{p}_{3/2}(T_{NR})| \sim |\vec{p}_{3/2}(T_{RH})| \frac{T_{NR}}{T_{RH}} \sim m_\phi \frac{T_{NR}}{T_{RH}} \tag{65}$$

which gives

$$T_{NR} \sim \frac{T_{RH}}{m_\phi} m_{3/2} \ll m_{3/2} \tag{66}$$

and therefore lowers the ratio T_{NR}/T_D.

ii) production by thermal equilibrium processes.

The presence of such gravitinos provides a very definite constraint on the reheating temperature T_{RH} because their number is proportional to T_{RH}.[62-64] In order to see that, let us suppose that gravitinos $\tilde{G}$ are produced in processes such as $XY \to Z\tilde{G}$. The evolution of the number density of gravitinos $m_{3/2}$ is governed by[65]

$$\frac{dn_{3/2}}{dt} - 3\frac{\dot{T}}{T} n_{3/2} = \sum_{X,Y,Z} \sigma_{XY\to Z\tilde{G}}\, n_X n_Y \tag{67}$$

where I have assumed that the number densities for X, Y species $n_{x,y}$ are much larger than $n_{3/2}$ to start with (any primordial $n_{3/2}$ has been washed away by inflation). Assuming that $n_X \sim n_Y \sim n_\gamma = 2\zeta(3)T^2/\pi^2$, and introducing the relative abundance $Y_{3/2} = n_{3/2}/n_\gamma$ we can write (67) as

$$\frac{dY_{3/2}}{dt} = \sigma_{tot}\, n_\gamma \qquad \sigma_{tot} = \sum_{X,Y,Z} \sigma_{XY \to Z\tilde{G}} \tag{68}$$

and using $t = (32\,\pi^2 N/90)^{-1/2} M/T^2$, (N is defined in Eq. 2), we obtain

$$\frac{dY_{3/2}}{dt} = -\frac{K}{M} \tag{69}$$

where K is constant given by ($\sigma_{tot} \sim \alpha M^{-2}$)[64]

$$K = M^2\, \frac{4\zeta(3)}{\pi^2} \left(\frac{90}{32\pi^2 N}\right)^{1/2} \sigma_{tot} \simeq 10^{-1} \tag{70}$$

The solution of (69), with the boundary condition $Y_{3/2}(T_{RH}) = 0$ is obviously for $T << T_{RH}$

$$Y_{3/2}(T) = K\, \frac{T_{RH}}{M} \tag{71}$$

Therefore any bond on $Y_{3/2}$ will translate into a bound on the reheating temperature. In particular, nucleosynthesis provides a limit on $Y_{3/2}$ (a gravitino of mass $m_{3/2} = M_w$ has not decayed at nucleosynthesis; see table above): too large a gravitino abundance would spoil the successes of the standard analysis by increasing the rate of expansion of the universe or the baryon-to-entropy ratio at the time of nucleosynthesis. It turns out however that the most stringent bounds come from the $\bar{p}$-dissociation[63,66] or the photodissociation[67] of ^{4}He. Let me illustrate the method in the case of the $\bar{p}$-dissociation. The annihilation of even a small fraction of ^{4}He through ($\bar{p}\,^4He \to {}^3He$ + anything) or ($\bar{p}\,^4He \to {}^3H$ $\to$ anything) could very well account for the total amount of ^{3}He observed in the universe: $X_{^3He} < 7 \times 10^{-5}$(tritium ^{3}H subsequently decays into ^{3}He). Actually, the amount of ^{3}He produced in these reactions is given by

$$n_{^3He} = n_{^4He}\, \frac{n_{\bar{B}}}{n_B}\, f_{^3He} \tag{72}$$

where[66]

$$f_{^3He} = \frac{\sigma(\bar{p}\,^4He \to {}^3He \text{ or } {}^3H + \text{anything})}{\sigma(\bar{p}\,^4He)_{tot}} = 0.237 \pm 0.014 \tag{73}$$

This gives a lower bound on the abundance of antibaryon

$$\frac{n_{\bar{B}}}{n_\gamma} < \frac{4}{3}\, \frac{X_{^3He}}{X_{^4He}}\, \frac{n_B}{n_\gamma}\, f^{-1}_{^3He} \tag{74}$$

where X denotes the mass concentration of an element ($X_{^4He} \simeq 0.25$). Assuming for simplification that there is one $p\bar{p}$ pair per gravitino decay, this yields

$$Y_{3/2} < \frac{4}{3}\, \frac{X_{^3He}}{X_{^4He}}\, \frac{n_B}{n_\gamma}\, f^{-1}_{^3He} \sim 10^{-12} \tag{75}$$

and we obtain from (70) and (71)[63,66]

$$T_{RH} \lesssim 10^8 \text{ GeV} \tag{76}$$

The same sort of constraint[67] is obtained by studying the photodissociation of ^{4}He: $\gamma + {}^4He \to n + {}^3He,\ p + {}^3H,\ p + n + D$ (see Ref. 67 for a more complete analysis than the one presented here). It is one of the successes of supersymmetric inflation to predict such a low reheating temperature (compare with eq. 61)

To conclude, supersymmetric inflation is characterized by a large value of the parameter $I = \sigma^3/(MV_0^{1/2})$, which assures enough inflation and the right amplitude for density fluctuations. In most models, the reheating temperature is low ($\sim 10^8$ GeV), which might be a problem for baryogenesis but is a necessary condition to avoid the gravitino problem. The discrepancy between the scale μ present in the inflaton sector and the scale M_w of $SU(2) \times U(1)$ breaking might pose problem when one relates the two of them to the supersymmetry-breaking scale ($m_{3/2}$) but there are ways to solve that contradiction. Finally, progress in the predictive power of supersymmetric theories as well as in the quantum mechanical analysis of inflation - how do fluctuations really behave? - should help in the near future to discreminate between the different models.

Acknowledgments

References

1. Guth, A., Phys. Rev. D23, 347 (1981).
2. Linde, A. D., Phys. Lett. 108B, 389 (1982).
   Albrecht, A., and Steinhardt, P. J., Phys. Lett. 140B, 44 (1984).
3. Turner, M., S., Phys. Rev. D28, 1243 (1983).
4. Steinhardt, P. J., and Turner, M. S., Phys. Rev. D29, 2162 (1984).
5. Vilenkin, A., and Ford, L., Phys. Rev. D26, 1231 (1982).
   Linde, A. D., Phys. Lett. 116B, 335 (1982).
6. Hawking, S. W., Phys. Lett. 115B, 295 (1982).
   Starobinsky, A. A., Phys. Lett. 117B, 175 (1982).
   Guth, A., and Pi, S. Y., Phys. Rev. Lett. 49, 1110 (1982).
   Bardeen, J. M., Steinhardt, P. J., and Turner, M. S., Phys. Rev. D28, 679 (1983).

7. Zel'dovich, Ya. B., Mon Not. Roy. Astron .Soc. 160, 1P (1972).
   Harrison, E. R., Phys. Rev. D1, 2726 (1970).
8. Silk, J., Elementary particles and the large-scale structure of the universe - To appear in the Proceedings of the 1st ESO-CERN Symposium - November 1983.
9. Sachs, R. K., and Wolfe, A. M. - Ap. J. 147, 73 (1967).
10. Brandenberger, R. H., - Rev. Mod. Phys. 57, 1 (1985).
11. Coleman, S., and Weinberg, E., Phys. Rev. D7, 1888 (1973).
12. Shafi, Q., and Vilenkin, A., - Phys. Rev. Lett. 52, 691 (1984).
13. Ellis, J., Nanopoulos, D. V., Olive, K. A., and Tamvakis, K., - Phys. Lett. 118B, 335 (1982); Nucl. Phys. B221, 524 (1983).
14. Nanopoulos, D. V., Olive, K. A., Srednicki, M., and Tamvakis, K., Phys. Lett. 123B, 41 (1983) - Nanopoulos, D. V., Olive, K. A., and Srednicki, M., Phys. Lett. 127B, 30 (1983).
15. Cremmer, E., et al., Phys. Lett. 79B, 231 (1978); Nucl. Phys. B147, 105 (1979); Phys. Lett. 116B, 231 (1982);Nucl. Phys. B212, 413 (1983).
    Witten, E., and Bagger, J., Phys. Lett. 115B, 202 (1982); *ibid* 118B, 103 (1982) - Bagger, J., Nucl. Phys. B211, 302 (1983).
16. Chamseddine, A. M., Arnowitt, R., and Nath, P., - Phys. Rev. Lett. 49, 970 (1982). Barbieri, R., Ferrara, S., and Savoy, C., Phys. Lett. 119B, 343 (1982). Cremmer, E., Fayet, P., and Girardello, L., Phys. Lett. 122B, 41 (1983).
17. Ovrut, B. A., and Steinhardt, P. J., - Phys. Lett. 133B, 161 (1983).
18. Holman, R., Ramond, P., and Ross, G. G., - Phys. Lett. 137B, 343 (1984).

19. Ovrut, B. A., and Steinhardt, P. J., Talk at Inner Space/Outer Space Conference - Fermilab - May 2-5, 1984 - University of Pennsylvania preprint UPR-0266T (July 1984).
20. Bernard, C. W., Phys. Rev. D9, 3312 (1974).
Dolan, L., and Jackiw, R., - Phys. Rev. D9, 3320 (1979).
Weinberg, S., Phys. Rev. D9, 3357 (1974)
21. Girardello, L. Grisaru, M. T., and Salomonson, P., Nucl. Phys. B178, 331 (1981).
22. Gelmini, G. B. Nanopoulos, D. V., Olive, K. A., - Phys. Lett. 131B, 53 (1983).
23. Olive, K. A., and Srednicki, M. A., - Phys. Lett. 148B, 437 (1984).
24. Binétruy, P., and Gaillard, M. K., - Nucl. Phys. B254, 388 (1985); Phys. Rev. D32 , n^{o}4 (Aug. 1985, in press).
25. Ellis, J., Nanopoulos, D. V., - Phys. Lett. 116B, 133 (1982).
26. Mazenko, G. F., Unruh, W. G., and Wald, R. M., - Phys. Rev. D31, 273 (1985).
27. Albrecht, A., and Brandenberger, R. - "On the realization of new inflation" Santa Barbara preprint NSF-ITP-84-146 (1984).
28. Jensen, L. G. and Olive, K. A. - "More on the realization of new inflation" Fermilab preprint FERMILAB-Pub-85/71-A (1985).
29. Guth, A. H., and Pi, S. Y., - "The quantum mechanics of the scalar field in the new inflationary universe" - Preprnt CTP-1246 (1985).
30. Linde, A. D., - Phys. Lett. 132B, 317 (1983).
31. Linde, A. D., - Phys. Lett. 129B, 177 (1983). Goughlan, G. D., et al., Phys. Lett. 149B, 44 (1984).
32. Dine, M., Fischler, W., and Nemeschansky, D., Phys. Lett. 136B, 169 (1984).

33. Binétruy, P., and Mahajan, S., - "Models for inflation with a low supersymmetry-breaking scale" - Santa Barbara preprint NSF-ITP-84-161, LBL-18566 (Rev. May 1985) (to be published in Nucl. Phys.).
34. Coughlan, G. D., et al., Phys. Lett. 131B, 59 (1983).
35. Polonyi, J., Budapest preprint KFKI-1977-93, unpublished.
36. O'Raifeartaigh, L., Nucl . Phys. B96, 331 (1975).
37. Ovrut, B. A.,, and Steinhardt, P. J., Phys. Rev. Lett. 53, 732 (1984); Phys. Rev. D30, 2061 (1984); Phys. Lett.. 147B, 263 (1984).
38. Oakley, C., and Ross, G., Phys. Lett. 125B, 59 (1983); Ibáñez, L., and G. Ross, Phys. Lett. 131, 335 (1983); Ovrut, B., and S. Raby, Phys. Lett. 134B, 51 (1984).
39. Gaillard, M.K., et al., Phys. Lett. 122B, 355 (1983).
40. Cremmer, E., Ferrara, S., Kounnas, C., and Nanopoulos, D. V., Phys. Lett. 133B, 61 (1983).
41. Ellis, J., Kounnas, C., and Nanopoulos, D. V., Nucl. Phys. B241, 406 (1984).
42. Ellis, J., Kounnas, C., and Nanopoulos, D. V., Nucl. Phys. B247, 373 (1983).
43. Ellis, J., Kounnas, C., and Nanopoulos, D. V., Phys. Lett. 143B, 410 (1984).
44. Ellis, J., Enqvist, K., and Nanopoulos, D. V., Phys. Lett. 147B, 99 (1984).
45. Gelmini, G. B., Kounnas, C. and Nanopoulos, D., Nucl. Phys. B250, 177(1985); Kounnas, C., and Quiros, M., Phys. Lett. 151B, 189(1985).

46. Enqvist, K., and Nanopoulos, D. V., Phys. Lett. 142B, 349 (1984), Nucl. Phys. B252, 508 (1985); Enqvist, K. et al., CERN preprint TH.4027(1984).

47 Ellis, J., et al., Phys. Lett. 152B, 175(1985); .Enqvist, K. et al., Phys. Lett. 152B, 181(1985).

48. Goncharov, A. S, and Linde, A. D. - Class Quantum Grav. 1, L75 (1984).

49. Witten, E., Phys. Lett. 155B, 151 (1985).

50. Candelas, P. et al., UCSB preprint NSF-ITP-84-170 (1984).

51. Green, M. B., and Schwarz, J. H., Phys. Lett. 149B, 117 (1984); Gross, D. J. et al., Phys. Rev. Lett. 54, 502 (1984), Princeton University preprint (January 1985).

52. Binétruy, P. and Gaillard, M. K., "The inflaton field in superstring theories", LBL preprint to appear.

53. Witten, E., "Symmetry breaking patterns in superstring models" Princeton preprint (February 1985).

54. Dine, M. et al., "Superstring model building" Princeton preprint (February 1985);
Mangano, M., "Low energy aspects of superstring theories", Princeton preprint (april 1985);
Binétruy, P., Dawson, S., Hinchliffe, I., and Sher, M., LBL preprint (August 1985).

55. Binétruy, P. and Gaillard, M. K., preprint LBL-19972, UCB-PTH-85/31 (1985).

56. Dine, M., et al., Phys. Lett. 156B, 55 (1985).

57. Kolb, E. W., Turner, M. S. - Ann. Rev. Nucl. Part. Sci, 33, 645 (1983).

58. Dolgov, A. D., and Linde, A. D., Phys Lett. 116B, 329 (1982); Abbott, L, Farhi, E. and Wise, M. B., Phys. Lett. 117B, 29 (1982).

59. Masiero, A., and Mohapatra, R. N., Phys. Lett. 103B, 343 (1981); 103B, 343 (1981); Masiero, A. and Senjanovic, G., Phys. Lett. 108B, 191 (1982); Masiero, A., and Yanagida, T., Phys. Lett. 112B, 336 (1982); Masiero, A., Nieves, J. F., and Yanagida, T., Phys. Lett.116B, 11 (1982); Claudson, M., Hall, L. J., and Hinchliffe, I., Nucl. Phys. B241, 309 (1984).

60. Pagels, H., and Primack, J., Phys. Rev. Lett. 48, 223 (1982). Weinberg, S., Phys. Rev. Lett. 48, 1303 (1982).

61. Ellis, J., Linde., A. D. and Nanopoulos, D. V., Phys. Lett. 118B, 59 (1982).

62. Nanopoulos, D. V., Olive, K. A., and Srednicki, M., Phys. Lett. 127B, 30 (1983).

63. Khlopov, M. Yu. and Linde, A. D., Phys. Lett. 138B, 265 (1984).

64. Ellis, J., Kim, J. E., and Nanopoulos, D. V., Phys. Lett. 145B, 181 (1984).

65. Lee, B. W., and Weinberg, S., Phys. Rev. Lett. 39, 165 (1977).

66. Falomkin, I. V., et al., Nuovo. Cim. 79A, 193 (1984).
Batusov, Yu. A., et al., Lett. al Nuvo. Cim. 41, 223 (1984).

67. Ellis, J., Nanopoulos, D. V., and Sarkar, S., CERN preprint TH.4057 (1984).

Figure Captions

Fig. 1. Schematic form of a scalar potential leading to inflation.

Fig. 2. Effective potential $V_T(\phi)/\mu^4$ at $T = M$ for a simple shape of the potential at $T = 0$ (dashed line) and different values of $e^g M^4/\mu^4|_\sigma$: 0 (a), 1(b), 4(c), 9(d); x is ϕ/M. (See Ref. 24.)

Fig. 3. Schematic evolution in the ϕ-ψ plane for the model of Ovrut and Steinhardt[37]. The shape of the potential in the first phase of the evolution (inflation) is given on the left and the shape of the potential in the second phase is given on the top, versus ψ.

Fig. 4. Comparative evolution of the energy density of gravitinos G and radiation (γ) after reheating.

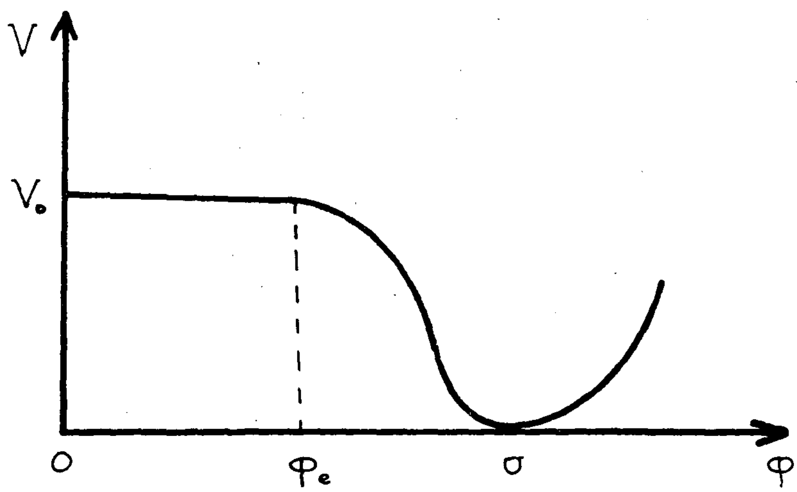


FIGURE 1

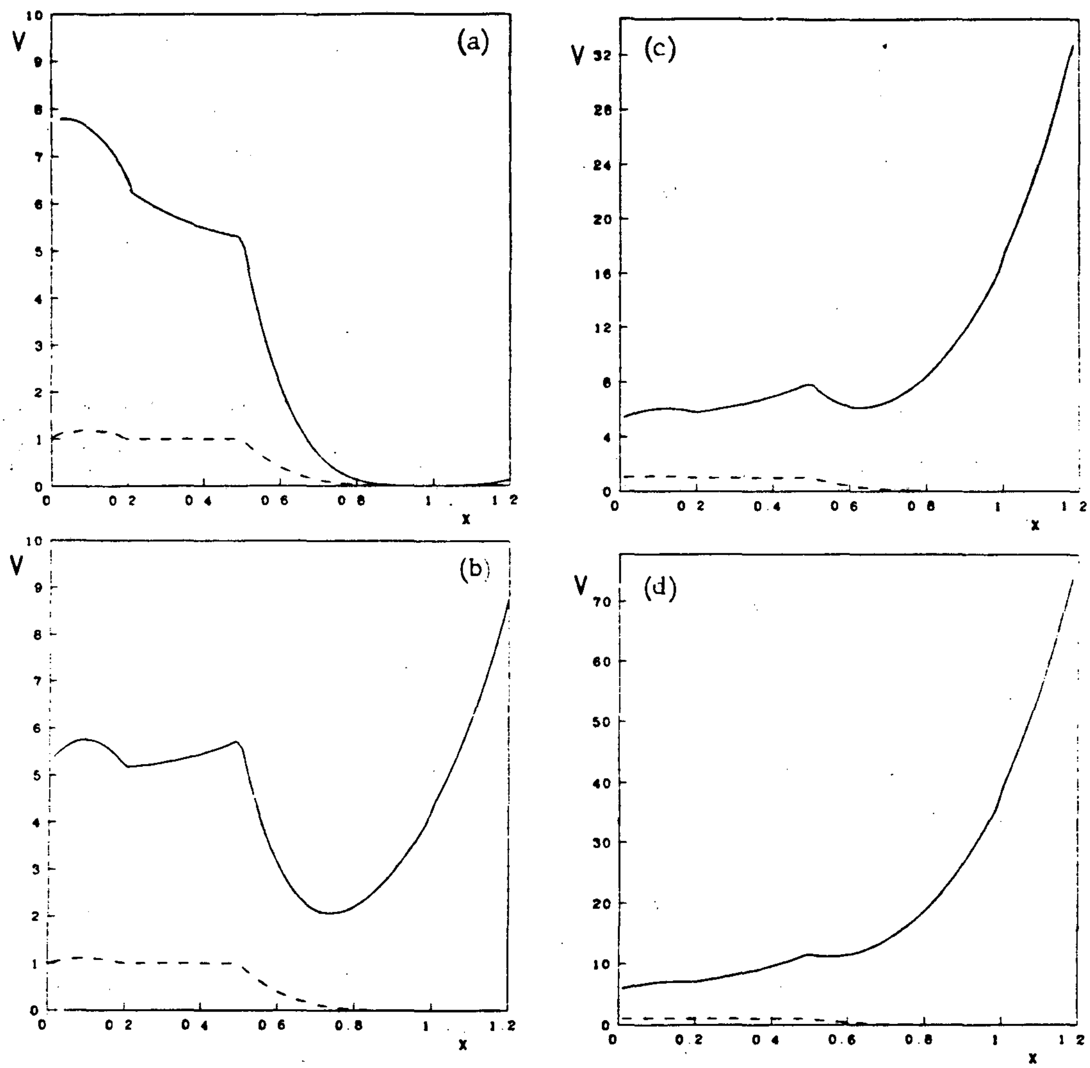


FIGURE 2

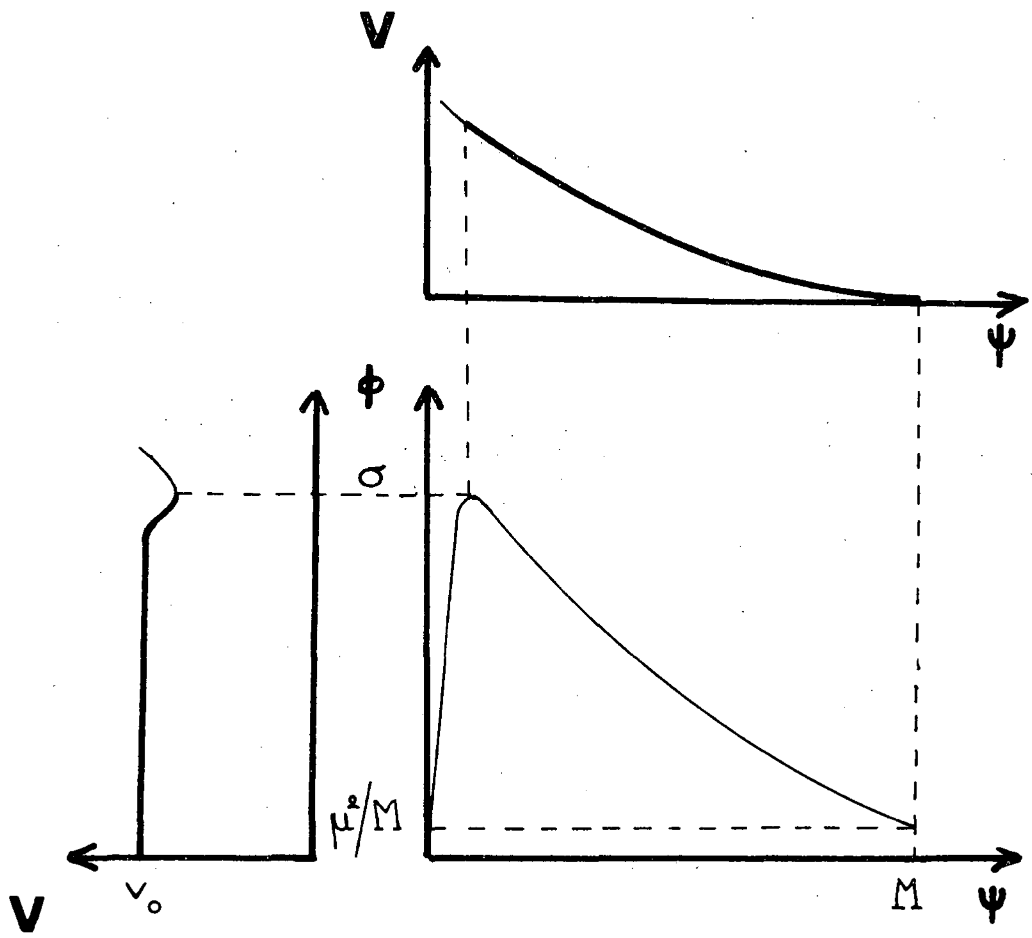


FIGURE 3

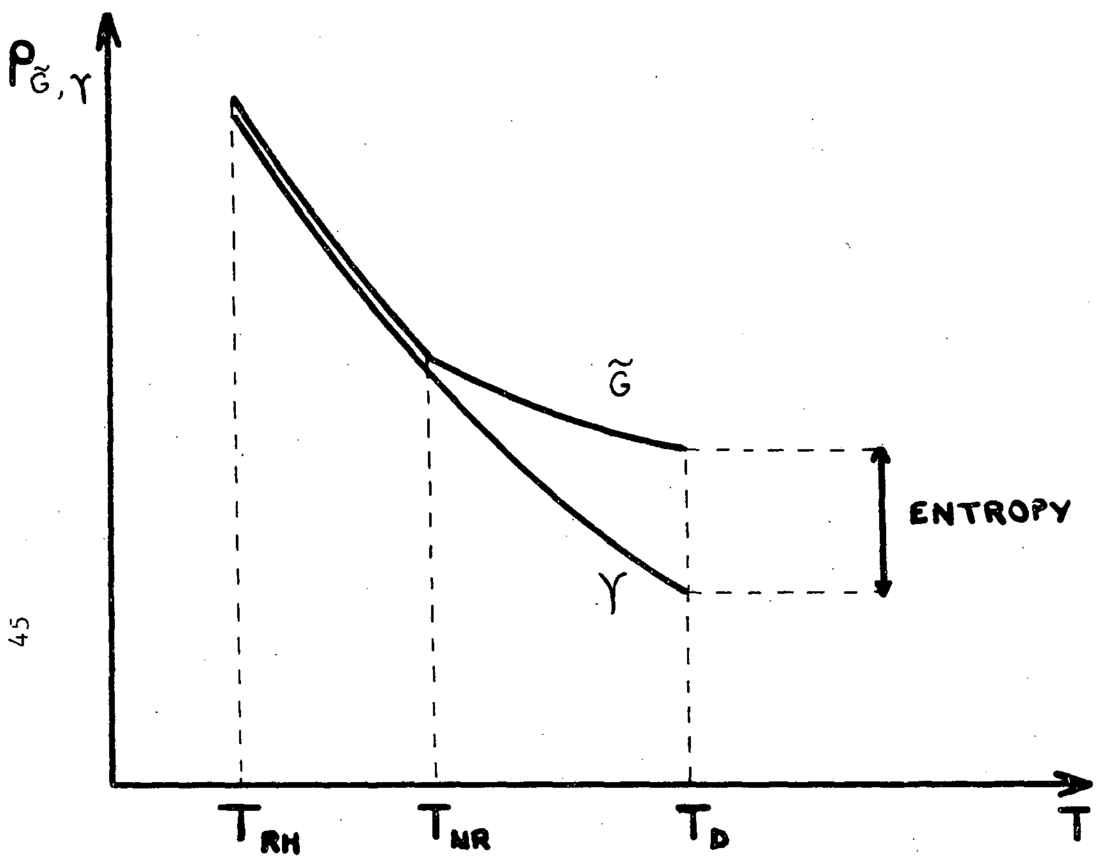


FIGURE 4