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THE IMPORTANCE OF BEING TOPOLOGICALLY EXCITED

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ABSTRACT

We identify a class of Euclidean configurations which appear to be dominant in the functional integral of the CP^{N-1} models. These configurations are point-like topological excitations, and they may be viewed as constituents of instantons, although they are defined independently of instantons through a continuum duality transformation. We show not only that these configurations survive as $N \to \infty$, but that in the plasma phase they are responsible for the effects encountered within the 1/N expansion — confinement, θ-dependence, and dynamical mass generation.

INTRODUCTION

This is a report of work done with K. Bardakci and H. Neuberger, much of which is contained in a paper entitled "Dominant Euclidean Configurations for All N".[1]

We want to re-emphasize the usefulness and importance of quasiclassical methods in the study of quantum field theories. The essential feature of this approach is, of course, to identify field configurations which dominate the Euclidean functional integral. On the other hand, the 1/N expansion has received much attention recently, and its relation to the quasiclassical approximation needs clarification.[2] We will show that in the two-dimensional CP^{N-1} models, point-like topological excitations (vortices, merons, instanton quarks) are important in the functional integral; that they survive as $N \to \infty$; and that in the plasma phase they are responsible for the results seen in the 1/N expansion, namely confinement, θ-dependence, and dynamical mass generation.

The CP^{N-1} model is defined by the following action for N complex fields z_α,

$$S = \frac{1}{g^2}\int d^2x \; (D_\mu z_\alpha)^*(D_\mu z_\alpha) \; , \; z^*_\alpha z_\alpha = 1 \; , \; \alpha = 1 \; , \; \ldots N \; , \qquad (1)$$

where $D_\mu = \partial_\mu + iA_\mu$, and $A_\mu = iz^*_\alpha \partial_\mu z_\alpha$. The models are considered a good laboratory for QCD since they are asymptotically free, conformally invariant, and topologically non-trivial. They are also 1/N expandable (but note N here is the number of flavors, not color),

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and the results are those mentioned above.

Classical solutions, i.e. instantons, are a very restrictive set of configurations and so by themselves are not important statistically; they also violate cluster decomposition. One needs more entropy. So far, one has been able to mix up instantons and anti-instantons only in the dilute gas approximation. However, recent calculations of the Gaussian fluctuations around the exact multi-instanton solutions have shown that the instanton quarks (merons) are liberated from their bound state, the instanton, and form a plasma, which is very different from the dilute gas. We shall identify a class of topological excitations which have sufficient entropy to be statistically significant and which satisfy cluster decomposition. Furthermore, although they are defined independently of instantons, they may be viewed as constituents of instantons, and indeed, the instanton configurations form a subset of these configurations.

TOPOLOGICAL EXCITATIONS IN CP^{N-1}

To find configurations with sufficient entropy to be statistically significant, a change of variables is useful. Let $z_\alpha = e^{i\theta\alpha}\rho_\alpha$; $\sum_{\alpha=1}^{N} \rho_\alpha^2 = 1$. Then

$$S = \sum_\alpha \int (\partial_\mu \rho_2)^2 + \sum_\alpha \int (\partial_\mu \theta_\alpha)^2 \rho_\alpha^2 - \sum_{\alpha\beta} \int \partial_\mu \theta_\alpha \partial_\mu \theta_\beta \rho_\alpha^2 \rho_\beta^2 \; . \tag{2}$$

Topological excitations are seen to exist since $\epsilon_{\mu\nu}\partial_\mu\partial_\nu\theta_\alpha$ does not have to be zero but may equal $2\pi \sum_i q_i^\alpha \delta^2(x-x_i^\alpha) \equiv J_\alpha(x)$. We now proceed by a continuum duality transformation to constrain the functional integral to a well-defined configuration and then sum over all possible configurations. The resulting field theory is

$$\mathcal{Z}_\theta = \int \left[\frac{\Pi d\rho_\alpha}{\rho_\alpha}\right]\left[\delta\left(\sum_\alpha \rho_\alpha^2 - 1\right)\right][\Pi dB_\alpha]$$
$$\left[\delta\left(\sum_\alpha B_\alpha - \frac{1}{2\pi} g^2\theta\right)\right] \exp\left\{-\frac{1}{g^2}\left[\sum_\alpha \int (\partial_\mu \rho_\alpha)^2 + \right.\right. \tag{3}$$
$$\left.\left. + \frac{1}{4}\sum_\alpha \int \frac{1}{\rho_\alpha^2}(\partial_\mu B_\alpha)^2\right] + \lambda \sum_\alpha \int \cos\left(\frac{2\pi}{g^2} B_\alpha - \theta\rho_\alpha^2\right)\right\} ,$$

where we have incorporated the constraints $\epsilon_{\mu\nu}\partial_\mu\partial_\nu\theta_\alpha = J_\alpha$ via

Lagrange multipliers B_α, and then integrated out the θ_α and summed over all J_α's. The term $\lambda\cos\frac{2\pi}{g^2}B_\alpha$ describes a Coulomb gas of the topological excitations or vortices, and since λ is the fugacity and has dimensions of mass^2, dynamical mass generation has occurred. This system of Sine-Gordon fields is complicated by being coupled to the ρ_α fields which describe the effects of the "dialectric medium". But it is just this complication that provides the large amount of entropy in these configurations.

It has been argued that instantons cannot be important in the $N\to\infty$ limit since their contribution goes like e^{-cN} and hence disappear. Without necessarily agreeing with this argument, it is plausible that if instanton quarks are liberated to form a plasma, then the action per quark is reduced by N, so that their contribution survives in the $N\to\infty$ limit. This is confirmed by an examination of (3), since if we make the following rescalings: $g^2N=\kappa$, $\theta=\bar{\theta}N$, $\rho_\alpha=g\bar{\rho}_\alpha$, $B_\alpha=g^2(\bar{B}_\alpha+\frac{\bar{\theta}}{2\pi})$; and exponentiate the δ-function constraints we find

$$\mathcal{Z}_\theta = \int [du]\,[dv]\,\exp\left\{N\left[\ln I\,(u,v,\bar{\theta}) - \frac{i}{\kappa}\int d^2x\,u\right]\right\},$$

$$I = \int\left[\frac{d\bar{\rho}}{\bar{\rho}}\right][d\bar{B}]\,\exp\left\{\int d^2x\left[-(\partial_\mu\bar{\rho})^2 - \frac{1}{4}\frac{1}{\bar{\rho}^2}(\partial_\mu\bar{B})^2 + \lambda\cos\left[2\pi\bar{B}-\bar{\theta}\,(\kappa\bar{\rho}^2-1)\right] + iu\bar{\rho}^2 + iv\bar{B}\right]\right\}. \tag{4}$$

Hence we see that factorization has taken place so that the topological excitations survive in the large N limit.

If we consider the criterion for confinement $\mathcal{Z}_\theta\frac{d^2\mathcal{Z}_\theta}{d\theta^2}\neq 0$, and isolate the following term in the large N limit,

$$\frac{1}{V}\left\{\frac{\partial^2}{\partial\bar{\theta}^2}\ln I\right\}_{\bar{\theta}=0} = -\lambda\left\langle(\kappa\bar{\rho}^2-1)\cos\left[2\pi\bar{B}(0)\right]\right\rangle, \tag{5}$$

we see that the topological charge density (and so the Wilson loop) is proportional to λ. Hence if topological excitations are projected out of the functional integral by setting $\lambda=0$, the Wilson loop vanishes and there would be no θ-dependence and no confinement. Furthermore, the gas of topological excitations must be in the plasma phase since otherwise $\langle\cos(2\pi\bar{B})\rangle=0$ due to long-range correlations in the dipole phase.

Whether the approach taken here in the CP^{N-1} model can be generalized to QCD depends on finding a parametrization of the degrees of freedom for which the techniques used here would be appropriate. This is currently under investigation.

REFERENCES

1. K. Bardakci, D.G. Caldi and H. Neuberger, Nucl. Phys. B, to be published.
2. See ref. 1 for extensive references.