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Leaky Modes on a Grounded Wire-Medium Slab

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Abstract — Leaky modes supported by a grounded wire-medium slab are studied here in detail for the first time. Such leaky modes are responsible for highly-directive radiation when the structure is excited by a dipole source. The analysis is carried out by means of a rigorous full-wave numerical approach based on the method-of-moments and also by using an approximate homogenized model. The use of a homogenized model that properly accounts for anisotropy and spatial dispersion leads to a remarkable and surprising result: the wavenumber of the leaky mode is independent of the azimuth angle of propagation on the surface. This result, which has not previously been observed in any other leaky-wave structure, allows for the creation of narrow conical beams that are azimuthally symmetric, as well as symmetric broadside beams. This conclusion is confirmed by the full-wave method-of-moments analysis.

Index Terms — Anisotropic media, dispersion, leaky modes, metamaterials, spatial dispersion, wire medium.

I. INTRODUCTION AND BACKGROUND

Among the variety of artificial media belonging to the class of metamaterials, the *wire medium* has received considerable attention, e.g., as a constituent of single- and double-negative materials with a negative effective permittivity. The simplest (1-D) realization of the wire medium consists of a periodic arrangement of thin perfectly-conducting cylinders (wires), infinitely long and parallel, embedded inside a homogeneous host dielectric medium.

The 1D wire medium has been known since the 1950s to be described in the long-wavelength regime by a scalar permittivity with a plasma-like dispersion behavior for waves having the electric field polarized along the axis of the wires [1]. More recently, a grounded wire-medium slab configuration as shown in Fig. 1 has been studied, in which a finite number of periodic layers of wires are placed above a perfectly-conducting ground plane (the y axis is chosen parallel to the wires and the z axis is normal to the air-slab interface). The structure is excited by a simple electric dipole source inside the wire-medium slab, oriented in the y direction. For frequencies slightly above the plasma frequency, where the relative permittivity is positive but small, it has been shown that the structure is capable of producing narrow-beam radiation patterns [2]. Depending on the frequency, the beam may be either a pencil beam at broadside, or a conical beam at a particular scan angle θ_0 . It was shown in [2] that this directive radiation is due to the excitation of a leaky mode supported by the structure.

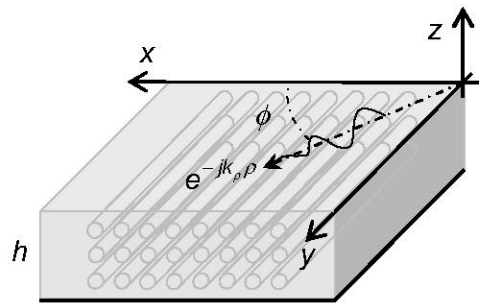


Fig. 1. Grounded wire-medium slab and relevant geometrical and physical parameters.

One remarkable observation, reported here for the first time, is that the beamwidth and beam angle of the conical beam produced by this structure is always independent of the azimuth angle (the angle ϕ in spherical coordinates) for any scan angle θ_0 , a feature that has not previously been observed with any other two-dimensional leaky-wave antenna. At first glance this result seems quite counter-intuitive, since the wire medium structure is certainly highly anisotropic. However, a careful analysis of the homogenized structure (discussed in the next section) reveals that the dominant leaky mode that is supported by the grounded wire-medium slab has a complex wavenumber that is *independent of the azimuth angle*. This feature of the wave propagation is itself quite remarkable, since the wires are oriented in a fixed direction (the axes are parallel to the y direction), and hence a wave propagating outward in a radial direction from the dipole source certainly sees a different orientation of the wires relative to the angle of propagation as ϕ changes.

Interestingly, if the wire medium is replaced by an isotropic dielectric-layered medium (e.g., a dielectric-layered leaky-wave antenna) then two leaky modes are responsible for the radiation, with a TM_z leaky mode determining the E-plane pattern and a TE_z leaky mode determining the H-plane pattern. The difference in the wavenumbers of the two leaky modes results in a beam that is not independent of the angle ϕ for a scanned beam, and the variation of the wavenumber with angle ϕ increases as the scan angle θ_0 increases. For the wire-medium structure, the uniaxial nature of the medium allows only a single TM_z leaky mode to propagate (when the host medium is air), and remarkably, the natural spatial dispersion of the medium is such the wavenumber for this leaky mode is independent of ϕ for any scan angle θ_0 .

An approximate homogenized model of the wire medium is first adopted in the next section in order to explain this phenomenon, based on a previously proposed anisotropic and spatially dispersive permittivity dyadic [3]. The dispersion equation for the leaky mode is obtained by enforcing the transverse resonance equation in the appropriate transverse equivalent network (TEN) associated with the leaky mode. A full-wave method-of-moments simulation of the actual periodic grounded wire-medium slab is then presented in order to validate the results obtained with the approximate homogenized model.

II. HOMOGENIZED MODEL AND FULL-WAVE ANALYSIS

A. Homogenized Model and Transverse Resonance Equation

The wire medium is anisotropic for electromagnetic waves with an arbitrary polarization, requiring the use of a uniaxial permittivity dyadic with optical axis directed parallel to the wire (y) axis when it is homogenized at wavelengths sufficiently larger than the wire spacing. Actually, as shown in [3], not only anisotropy but also spatial dispersion needs to be included in a homogenized description of the medium, even for large wavelengths. The effective permittivity dyadic thus depends on both frequency and wavenumber and it reads

$$\boldsymbol{\epsilon}_t = \epsilon_{th} \left[x_0 x_0 + \left(1 - \frac{k_p^2}{\epsilon_{th} k_0^2 - k_y^2} \right) y_0 y_0 + z_0 z_0 \right] \quad (1)$$

where ϵ_{th} is the relative permittivity of the host medium, k_0 is the free-space wavenumber, k_p is the plasma wavenumber, and k_y is the wavenumber along the wire axis.

The field of a leaky mode propagating at an angle ϕ has an exponential dependence on the radial coordinate ρ , with complex wavenumber $k_\rho = \beta - j\alpha$. A transverse equivalent network (TEN) can be derived by decomposing the modal field in both the air and the slab regions into TE_y and TM_y components (see Fig. 2, where the transmission lines 1 and 3 represent TE_y waves, whereas lines 2 and 4 represent TM_y waves). The relevant transmission-line parameters are:

$$\begin{aligned} k_{z1} &= k_{z2} = \sqrt{k_0^2 - k_\rho^2}, \\ k_{z3} &= \sqrt{\epsilon_{th} k_0^2 - k_\rho^2}, & k_{z4} &= \sqrt{\epsilon_{th} k_0^2 - k_p^2 - k_\rho^2} \\ Z_{c1} &= \eta_0 k_0 \frac{k_{z1}}{k_0^2 - k_y^2}, & Z_{c2} &= \frac{\eta_0}{k_0} \frac{k_0^2 - k_y^2}{k_{z2}} \\ Z_{c3} &= \eta_0 k_0 \frac{k_{z3}}{\epsilon_{th} k_0^2 - k_y^2}, & Z_{c4} &= \frac{\eta_0}{k_0} \frac{\epsilon_{th} k_0^2 - k_y^2}{k_{z4}} \end{aligned} \quad (2)$$

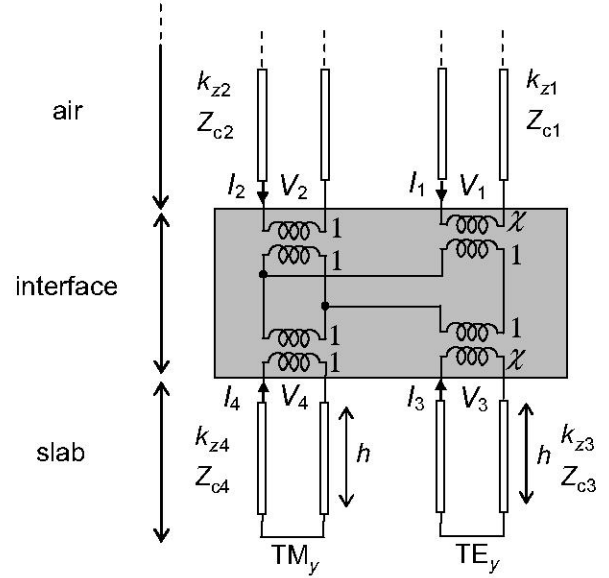


Fig. 2. Transverse Equivalent Network (TEN) for the modal analysis of the homogenized grounded wire-medium slab.

where η_0 is the free-space characteristic impedance, $k_x = k_\rho \cos \phi$, and $k_y = k_\rho \sin \phi$. By enforcing the continuity of the tangential components of the electric and magnetic fields at the air-slab interface, the interface is found to be represented by a four-port network (see Fig. 2), that couples the TE_y and TM_y transmission lines through ideal transformers with turns ratio

$$\chi = \frac{(\epsilon_{th} - 1)k_x k_y}{(k_0^2 - k_y^2)(\epsilon_{th} k_0^2 - k_y^2)}. \quad (3)$$

It can be noted that no TE_y/TM_y coupling occurs ($\chi = 0$) when $\epsilon_{th} = 1$ (wires in air) or when $k_x = 0$ or $k_y = 0$ (modal propagation along the principal directions $\phi = 90^\circ$ or $\phi = 0^\circ$).

The dispersion equation is obtained by enforcing the condition of resonance for the TEN in Fig. 2. The result is

$$\begin{aligned} & [Z_{c1} + jZ_{c3} \tan(k_{z3}h)][Z_{c2} + jZ_{c4} \tan(k_{z4}h)] + \\ & + \chi^2 Z_{c2} Z_{c4} = 0. \end{aligned} \quad (4)$$

When $\chi = 0$ modes are purely TE_y (ordinary polarization) or TM_y (extraordinary polarization) and (4) factors into the product of their respective dispersion equations; when $\chi \neq 0$ the modes are hybrid.

A remarkable consequence of (4) is that, when $\epsilon_{th} = 1$, no TE_y modes may exist (since the ordinary polarization does not interact with the wires and ‘sees’ only the host medium), whereas the dispersion equation for the extraordinary TM_y modes becomes

III. NUMERICAL RESULTS

$$k_{z4} + jk_{z2} \tan(k_{z4}h) = 0, \quad (5)$$

which turns out to be *independent* of the propagation angle ϕ (and equal to the dispersion equation for TE_z modes of an ordinary isotropic slab with $\epsilon_r = \epsilon_{\text{th}} - k_p^2/k_0^2$). This is quite unexpected, considering the apparent directionality of the structure, but this property will be confirmed by full-wave simulations.

B. Full-Wave Moment-Method Approach

A rigorous numerical analysis of the actual periodic wire-medium slab has been carried out through the method of moments (MoM) in the spatial domain for the case of wires in air ($\epsilon_{\text{th}} = 1$). In this case TE_y and TM_y polarizations can be separately treated (only the latter is of interest, as explained above), and the periodicity of the structure allows for the consideration of one spatial period (unit cell) only. This requires the use of a periodic free-space Green's function, which has been accelerated using the Ewald method [4].

The uniformity of the background medium, the exponential dependence of the modal field on the x and y coordinates, and the invariance of the structure along the wire axis reduces the problem to a two-dimensional one, in which the unknown is the longitudinal current on the perimeter of the wires within a unit cell. A further simplification follows by realizing that the Electric Field Integral Equation (EFIE) for wave propagation at an arbitrary angle ϕ is equivalent to that at angle of $\phi = 0^\circ$ (propagation orthogonal to the wires, i.e., no variation of currents or fields with y) provided the free-space wavenumber k_0 is replaced by the scaled wavenumber $k'_0 = \sqrt{k_0^2 - k_y^2}$, with propagation along x accounted for by the phasing used in the periodic Green's function that is employed in the periodic MoM.

Having thus reduced the problem to a two-dimensional one in the xz plane, the cross section of each circular cylinder in this plane has been approximated with a regular M -edge polygon ($M = 16$ in all the simulations), and the relevant EFIE has been discretized adopting piecewise-constant subsectional basis functions and a Galerkin testing procedure. The complex wavenumber of each leaky mode supported by the grounded wire-medium slab has then been determined by searching for the zeros of the determinant of the coefficient matrix of the linear system resulting from the discretization process. Because the period is usually much smaller than a wavelength, the leaky mode has a fundamental Floquet wave that is improper (exponentially increasing in the vertical z direction), while all other Floquet waves are proper (exponentially decreasing in the z direction), and the periodic Green's function is calculated accordingly.

In this Section we illustrate the numerical results for the propagation wavenumber of the dominant leaky mode obtained with both the approximate (homogenized) and the rigorous (MoM) models described in the previous section. In particular, a grounded wire-medium slab with wire radius $a = 0.5$ mm, wire spacing in both the x and z directions of $d = 20$ mm, and $N = 6$ layers of cylinders has been considered, with the ground plane at a distance equal to $d/2$ from the center of the cylinders in the bottom layer. The plasma wavenumber k_p in the homogenized model is calculated as [5]

$$k_p = \frac{2\pi}{d} \frac{1}{\sqrt{2\pi \left(\ln \frac{d}{2\pi a} + 0.5275 \right)}} \quad (6)$$

whereas an equivalent slab thickness $h = Nd$ is adopted, in order to approximately take into account the presence of fringing fields above and below the top and bottom wire layers, respectively.

In order to point out the role of spatial dispersion in determining the propagation properties of the dominant leaky mode, Fig. 3 shows the phase and attenuation constants for propagation in the planes $\phi = 0^\circ$, $\phi = 45^\circ$, and $\phi = 90^\circ$, calculated from the homogenized model by neglecting the spatial dispersion in the permittivity dyadic of the homogenized medium (i.e., by letting $k_y = 0$ in (1)).

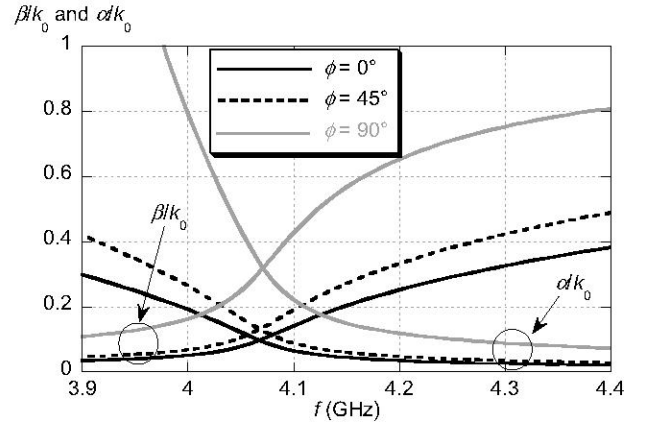


Fig. 3. Dispersion curves for the dominant TM_y leaky mode of the grounded wire-medium slab in Fig. 1, propagating in three different planes, calculated with the homogenized model without taking into account spatial dispersion. Parameters: $a = 0.5$ mm, $d = 20$ mm, $\epsilon_{\text{th}} = 1$, $N = 6$.

It can be seen that the results are strongly dependent on the propagation angle. However, the correct dispersion curves, calculated by taking into account the spatial dispersion of the homogeneous medium, are instead independent of the propagation angle ϕ and coincide with the case $\phi = 0^\circ$ in Fig. 3. It can be concluded that the presence of the spatial

dispersion in the homogenized model is the crucial element that results in an *omnidirectional* modal propagation. When a dominant leaky mode is excited, such omnidirectional propagation gives rise to an omnidirectional pencil-beam radiation at broadside (recently reported in [2]) at the frequency for which $\beta = \alpha$, and a conical beam having an omnidirectional beam angle and beamwidth, for $\beta > \alpha$.

When the host medium of the wires is different from air, i.e., $\epsilon_{\text{rh}} \neq 1$, an angular dependence of the modal wavenumber can be expected from (4). However, such dependence is a mild one for small or moderate values of the relative permittivity of the host medium. This can be observed in Fig. 4, where the normalized phase and attenuation constants are reported as a function of the propagation angle ϕ for two structures with $\epsilon_{\text{rh}} = 4$ and $\epsilon_{\text{rh}} = 9$ at the frequencies for which $\beta = \alpha$ at $\phi = 90^\circ$ (other parameters as in Fig. 3).

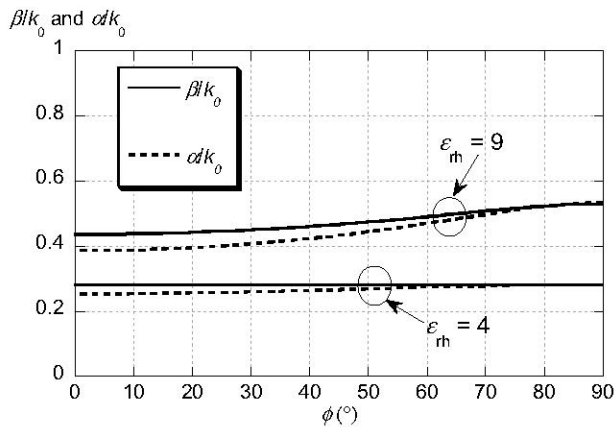


Fig. 4. Normalized phase (β/k_0) and attenuation (α/k_0) constants for the dominant leaky mode supported by two different grounded wire-medium slabs as functions of the propagation angle ϕ , calculated with the homogenized anisotropic spatially-dispersive model. Parameters: $h = 120$ mm, $k_p = 8.126$ rad/m, $\epsilon_{\text{rh}} = 4$ (with $f = 2.024$ GHz) and $\epsilon_{\text{rh}} = 9$ (with $f = 1.337$ GHz).

The omnidirectionality of modal propagation when $\epsilon_{\text{rh}} = 1$ is confirmed by the full-wave results obtained with the moment-method analysis described in the previous section, and reported in Fig. 5. In particular, the $\phi = 0^\circ$, $\phi = 45^\circ$, and $\phi = 90^\circ$ cases have been considered again, and it can be seen that the phase and the attenuation constants of the dominant leaky mode for the different angles of propagation are almost superimposed over the frequency range shown.

The dispersion curve obtained with the homogenized model is also reported in Fig. 5. A frequency shift can be observed with respect to the full-wave results, which could in part be due to the choice of the slab thickness h [6].

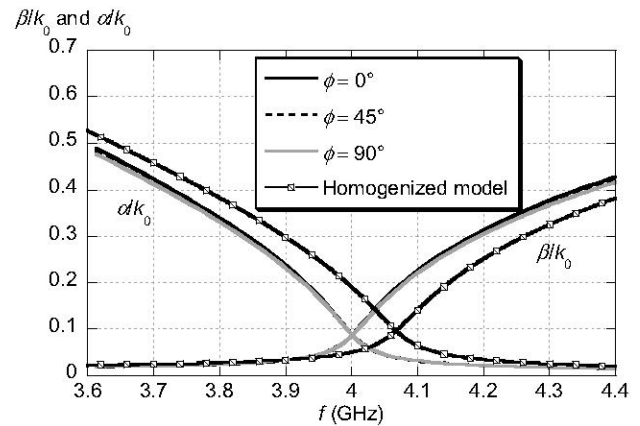


Fig. 5. Comparison of the dispersion curves for the dominant TM_y leaky mode supported by the grounded wire-medium slab with parameters as in Fig. 3, propagating in three different planes, calculated with the full-wave MoM approach and with the homogenized anisotropic spatially-dispersive model.

IV. CONCLUSION

A study of leaky-mode propagation on a grounded wire-medium slab has been presented. The analysis has been carried out first by means of an approximate homogenized model, which takes into account both anisotropy and spatial dispersion, and then by means of a rigorous moment-method solution. The homogenized model predicts the interesting feature of *omnidirectional* modal propagation when the host medium of the wires is air; i.e., the wavenumber of the leaky mode is independent of the azimuthal propagation angle, in spite of the apparent directionality of the structure. This leads to an interesting application, namely the creation of azimuthally-symmetric radiation patterns.

REFERENCES

- [1] J. Brown, "Artificial dielectrics having refractive indices less than unity," *Proc. IEE*, vol. 100, pp. 51-62, May 1953.
- [2] P. Burghignoli, G. Lovat, F. Capolino, D. R. Jackson, and D. R. Wilton, "3D directive radiation from a horizontal dipole embedded in a homogenized grounded wire-medium slab," *Digest 2006 IEEE AP-S Intl. Symp. Antennas Propagat.*, Albuquerque (NM), 9-14 July 2006, pp. 2989 - 2992.
- [3] P. A. Belov, R. Marqués, S. I. Maslovski, M. Silveirinha, C. R. Simovski, and S. A. Tretyakov, "Strong spatial dispersion in wire media in the very large wavelength limit," *Phys. Rev. B*, vol. 67, pp. 113103-1 - 113103-4, 2003.
- [4] F. Capolino, D. R. Wilton, and W. A. Johnson, "Efficient calculation of the 2-D Green's function for 1-D periodic structures using the Ewald method," *IEEE Trans. Antennas Propagat.*, vol. 53, pp. 2977-2984, Sept. 2005.
- [5] P. A. Belov, S. A. Tretyakov, and A. I. Viitanen, "Dispersion and reflection properties of artificial media formed by regular lattices of ideally conducting wires," *J. Electromag. Waves Appl.*, vol. 16, pp. 1153-1170, Sept. 2002.
- [6] G. Guida, D. Maystre, G. Tayeb, and P. Vincent, "Mean-field theory of two-dimensional metallic photonic crystals," *J. Opt. Soc. Am. B*, vol. 15, pp. 2308-2315, 1998.