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This note compares published theoretical expressions[1,2,3] for the elastic moduli of two-phase systems with experimental results obtained for a sodium borosilicate glass containing dispersions of angular alumina particles. The glass used was of the same composition as the D glass (16% Na_2O, 14% B_2O_3, 70% SiO_2) used in previous investigations.[4,5] The system D glass-Al_2O_3 was selected because of the nearly identical coefficients of thermal expansion of the components and the high degree of bonding between them.[4] The system D glass-Al_2O_3 is characterized in that the dispersed phase has elastic moduli considerably higher than those of the matrix. One-quarter inch thick disks were prepared by vacuum hot-pressing intimate mixtures of the components in powder form in 2 in. diam graphite dies at 725°C for 10 min. Glass particle size was approximately 3 to 5μ. The alumina (crushed sapphire) had a particle size of approximately 50μ. Figure 1 shows the microstructure of a glass-alumina composite containing 30 volume percent alumina. Two specimens measuring 1-3/4 by 1/4 by 1/4 in. were cut from the hot-pressed disks

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by means of a high precision diamond saw. No further surface treatment was employed. Young's modulus was determined by a flexural resonance technique,[6] thereby yielding four data points per composition. Calculations were made by means of tables compiled for this purpose.[7] For the calculations Poisson's ratio (ν) was assumed to vary as calculated from Hashin and Shtrikman's equation[2] for the lower bound of the bulk and shear moduli of a matrix containing dispersions of arbitrary geometry. The shear modulus could not be determined because of the frequency limitations of the instrument employed* (> 27,000 cps). Table I gives the experimental results. Young's modulus for the glass was found to be 805 kilobars, which is slightly higher than the value obtained by extrapolating to zero porosity data for Young's modulus of specimens containing spherical pores.[5] This discrepancy probably can be attributed to differences in thermal history. To facilitate comparison between theory and experiment, Table II lists values for 10, 20, 30, 40, and 50 volume percent calculated from the theoretical expressions. For completeness, Hashin's results for spherical inclusion are also included. Poisson's ratio of the glass was taken as 0.194.[5] Elastic[8] property values for the alumina were shear modulus: 1635 kilobars† and $\nu = 0.257$. Young's modulus for the predictions of Hashin and Shtrikman[3] and Hashin[2] were calculated from the corresponding equations for the bulk and shear modulus. Figure 2 presents the combined theoretical and experimental results. The apparent decrease in Young's modulus at approximately

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*Magnatest, Magnaflux Corporation, Chicago, Illinois.

†Mean value of three values reported.

50 volume percent is probably due to a lack of complete densification due to alumina particle-to-particle contact. In general, the experimental results agree very well with Hashin and Shtrikman's lower bound for arbitrary phase geometry, which coincides with Hashin's approximate expression for spherical phase geometry.

It is of interest to note that for the system zirconium carbide-graphite,[9] which consists of a matrix containing a dispersed phase with a much lower Young's modulus, Young's modulus for the composite appears to be given best by Hashin's lower bound for spherical phase geometry. The lower bound for arbitrary phase geometry of Hashin and Shtrikman[10] when used to calculate Young's modulus for the zirconium carbide-graphite system gives values approximately 25 percent of those experimentally observed.

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Table I. Observed effect of alumina dispersions on Young's modulus of a sodium borosilicate glass

Volume percent Al_2O_3	Young's modulus (kilobars)
10	924 ± 4[*]
20	1071 ± 5
30	1228 ± 5
40	1447 ± 14
45	1585 ± 10
50	1678 ± 13
55	1372 ± 13[†]

[*] Standard deviation.

[†] Approximately 9.6% porosity.

Table II. Calculated effect of alumina dispersions on Young's modulus of a sodium borosilicate glass (E_o = 805 kilobars)

Volume percent Al_2O_3	Spherical phase geometry[*]			Arbitrary phase geometry[†]		Paul[‡]		
	Upper bound	Lower bound	Approximate	Upper bound	Lower bound	Upper bound	Lower bound	Cubical inclusion
10	925	912	920	1011	920	1137	870	1031
20	1069	1027	1057	1235	1057	1470	960	1225
30	1238	1163	1209	1479	1209	1802	1053	1428
40	1434	1330	1397	1748	1397	2135	1185	1641
50	1665	1542	1620	2043	1620	2467	1345	1891

[*] Footnote 2.

[†] Footnote 3.

[‡] Footnote 1.

FIGURE LEGENDS

Figure 1. Microstructure of a sodium borosilicate glass containing 30 volume percent alumina (crushed sapphire).

Figure 2. Experimental and theoretical results for Young's modulus of a sodium borosilicate glass matrix as a function of the volume percent of alumina particles.

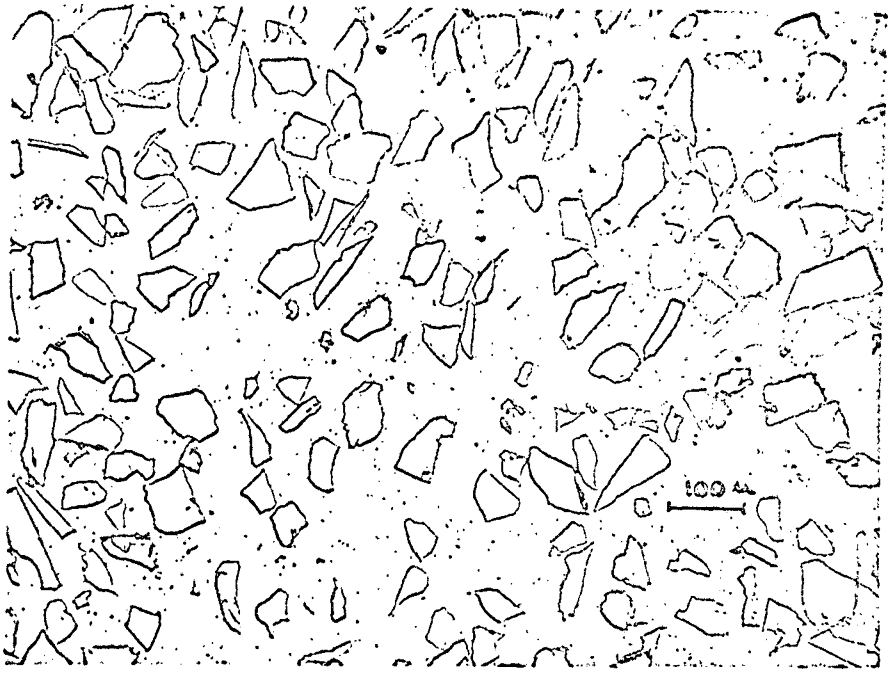

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Fig. 1

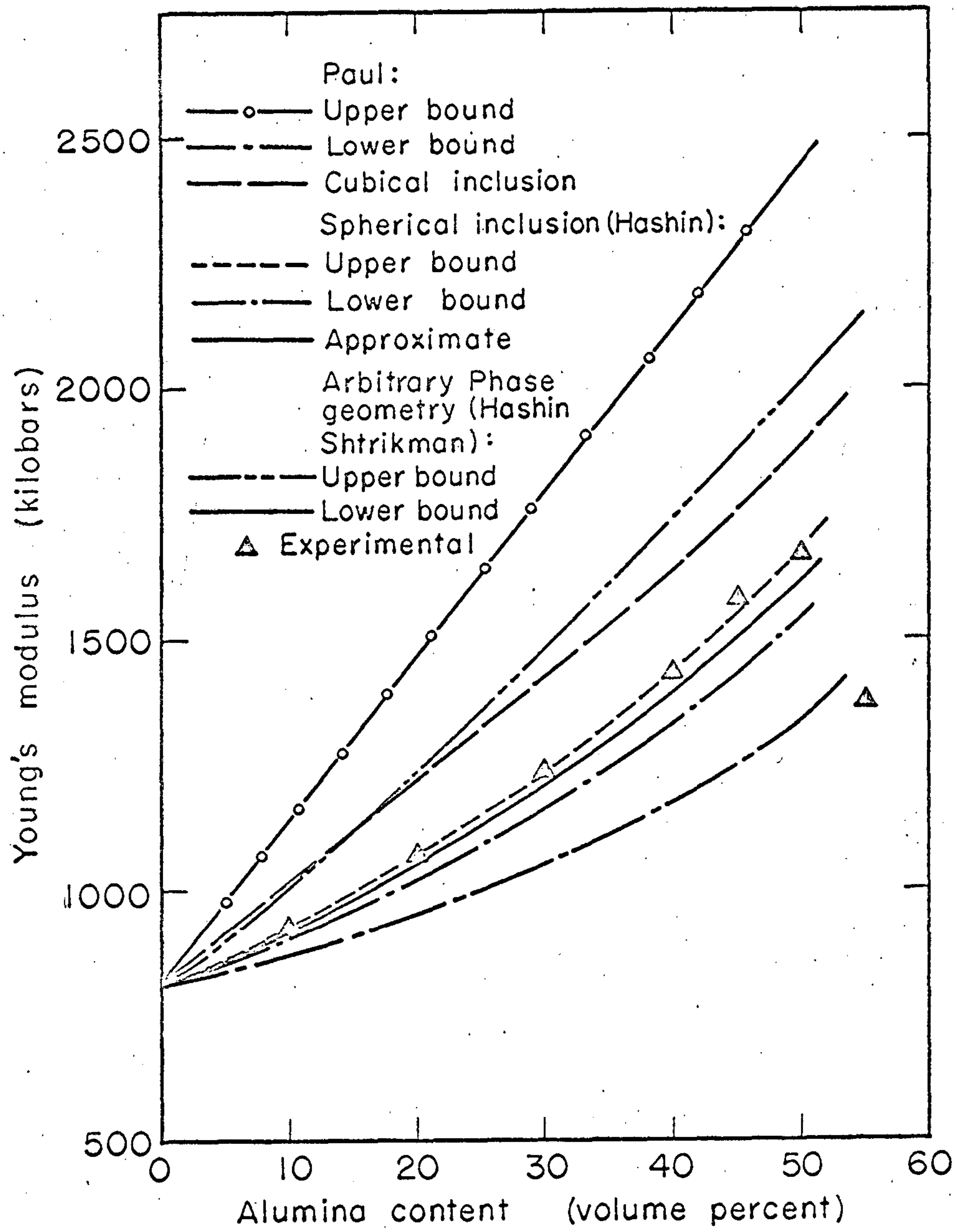

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Fig. 2