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**Tangential Friction in Nuclear Reactions
Studied with the Time-Dependent Hartree-Fock Model**

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Abstract:

The dissipation of orbital angular momentum is studied by analyzing TDHF calculations of the damped nuclear reaction $280 \text{ MeV } ^{40}\text{Ca} + ^{40}\text{Ca}$ in terms of the Nucleon Exchange Transport model. While the overall results of the TDHF calculation are well accounted for by the NET model, the more detailed behavior reveals evidence of a finite intrinsic nuclear relaxation time.

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About ten years ago, a classical theory was presented for the one-body dissipation in large-amplitude nuclear motion [1,2]. The resulting *wall* and *window formulas* [1] have a clear physical content and contain only well-known nuclear parameters. Due to the conceptual and formal simplicity, as well as the apparent success in practical applications, the classical one-body friction has been applied more widely than any other theory for investigations of nuclear reactions involving large-amplitude motion, such as fission, quasifission and damped reactions [3]. For damped reactions, an extension of the theory, nucleon exchange transport (NET) [4], describes also the time development of mean values and variances of observables other than the energy of the relative motion, especially the partition of mass and charge, and the three-dimensional angular momenta of the individual nuclides [5].

A self-consistent, quantal description of nuclear one-body dynamics is provided by the time-dependent Hartree-Fock (TDHF) model, which represents the system as a Slater determinant of single-particle wave functions evolving in the nuclear mean field. It reproduces the observed strong friction acting on the relative motion in damped nuclear reactions [6]. Detailed studies of TDHF results have shown that the Wigner distributions of the nucleons exhibit a strong similarity with classical phase space distributions for the same reaction [7,8].

The present letter compares TDHF to NET with respect to another property, namely the *tangential friction* in damped nuclear reactions. The tangential friction is well-suited for investigation by TDHF, since it is directly determined from the time development of the orbital angular momentum of the relative motion.

The study is carried out in several steps. First, the evolution of the angular momentum is determined as follows. The TDHF calculation is performed for the given (symmetric) reaction at a specified impact parameter, and at every time step the magnitude of the orbital angular momentum is inferred from the time dependence of the density distribution of nucleons. The density distribution is at each instant split in two halves by a planar window, which is perpendicular to the line joining the centers of mass of the two parts. This definition has the advantage that internal interactions within the two parts do not influence the relative motion.

The angular momentum dynamics displayed by the TDHF calculation will be compared to the NET results calculated by employing the same time-dependent one-body potential. From the TDHF results it is possible to extract the crucial quantity determining the nucleon-exchange friction, namely the rate of nucleon transfer between the two reacting nuclides, the transfer current N . This quantity is often expressed on the form $N = n_0 \sigma_{\text{eff}}$, where n_0 (≈ 0.25 nucleons per fm^2 per 10^{-22}s) is the one-sided flux in standard nuclear matter and σ_{eff} is the effective area of the "window" between the two parts of the dinuclear system.

Two alternative ways of calculating the effective window area are employed, both based on the proximity method. The first one is closely related to the Thomas-Fermi method applied in ref. [9]. In the Thomas-Fermi approximation, the local transfer flux $n(x, y)$ is proportional to the square of the local Fermi kinetic energy $T_F = \epsilon_F - V(x, y, z = 0)$. The total transfer current $N(t)$ can therefore be estimated as

$$N = n_0 \sigma_{\text{eff}} = n_0 \int_{\text{window}} dx dy \left(\frac{\epsilon_F - V(x, y, z = 0)}{\epsilon_F - V_0} \right)^2 \quad (1)$$

where V_0 is an appropriate average of the TDHF potential in the nuclear interior before the reaction. The integral in (1) is restricted to the classically allowed region of the window plane, $V \leq \epsilon_F$. The alternate estimate is based on the fact that $n(x, y)$ is also proportional to the four-thirds power of the local density $\rho(x, y, z = 0)$, in the Thomas-Fermi approximation. Thus

$$N = n_0 \sigma_{\text{eff}} = n_0 \int_{\text{window}} dx dy \left(\frac{\rho(x, y, z = 0)}{\rho_0} \right)^{4/3} \quad (2)$$

where $\rho(x, y, z = 0)$ is the TDHF density in the window plane, and $\rho_0 (\approx 0.17 \text{ nucleons per fm}^3)$ denotes the average density in the nuclear interior. Since the tails of the densities extend beyond the classically allowed region for nucleons, this second definition incorporates the effect of barrier penetration to some extent.

The window area and the center separation determine the time development of the orbital angular momentum according to NET [4,5]. For a symmetric reaction, the time development of the mean values of the angular momenta is governed by a single transport equation. As discussed in ref. [5], it is instructive to introduce the associated (time-dependent) relaxation time. For the present study it is most convenient to define

$$\tilde{t}_+ \equiv \frac{I_R}{I_0} \frac{I_+}{mNR^2} = \frac{I_R}{I_0} t_+ \quad (3)$$

where t_+ is the relaxation time defined in ref [5]. Here $I_+ = I_A + I_B$ is the sum of the moments of inertia for the individual nuclides; they are approximated by the rigid-body value for a spherical shape. $I_R(t)$ is the orbital moment of inertia determined by the actual value of the center separation $R(t)$, and $I_0(t) \equiv I_R + I_+$ denotes the total moment of inertia.

The equation for the evolution of the orbital angular momentum can then be written

$$\frac{dL}{dt} = \dot{L} = \frac{1}{\tilde{t}_+} \left(\frac{I_R}{I_0} J - L \right) \quad (4)$$

(Consistent with the absence of coordinate system fluctuations [5] in TDHF, terms arising from such fluctuations have been ignored.) Here J is the total angular momentum ($\equiv L(t = -\infty)$). The above equation (4) describes a simple relaxation

of the orbital angular momentum towards the rigid-rotation value $\frac{I_R}{I_0}J$. The analogous equations for the nuclear spins $S^A = S^B$ is superfluous because of angular-momentum conservation, $S^A + S^B + L = J$.

Figure 1 shows the result of the calculation for four different impact parameters in the reaction of 280 MeV $^{40}\text{Ca} + ^{40}\text{Ca}$. The computer code and the parameters for the density-dependent interaction are the same as those of ref. [10]. The impact parameters employed for the calculation in fig. 1 cover the cross section for damped reactions with large impact parameters, since $J = 75\hbar$ is just above the upper limit for fusion, and $J = 90\hbar$ leads to a very gentle reaction. (With the classical definition of the impact parameter in TDHF, $J/\hbar$ need not be an even integer number.)

Before discussing the content of fig. 1, we shall concentrate for a while on the more technical issue of how the results depend upon parameters and procedures. This is shown in fig. 2 for two values of impact parameter for the TDHF calculations. First of all, it is seen that the standard grid size of 1 fm is too crude for extracting a value of the orbital angular momentum to the accuracy needed for our investigation. Second, if the coordinate of the relative motion is required to be along the principal axis of minimum moment of inertia (as is standard in the computer program), internal vibrations in the two nuclei show up artificially in the relative motion. Especially, when the two nuclei are separated after the reaction itself, the orbital angular momentum acquires regular oscillations instead of approaching an asymptotic value. For the peripheral reaction, $J = 90\hbar$, the result is completely masked if one uses a too crude grid size, or an inconsistent prescription for defining the relative motion. On the other hand, during the more intimate reactions, the definition of the relative motion in terms of the inertia tensor gives almost the same result as our preferred prescription. Also, if the time evolution is followed for a while after the two nuclei have separated, one can average over the oscillation period to obtain the same result for the final orbital angular momentum as with our prescription. Concerning the NET calculation, one can read from fig. 1 that it only matters for the most peripheral collision which prescription (1) or (2) is used for the calculation of the window size.

As shown by fig. 1, The orbital angular momentum L begins to decrease strongly at the onset of the reaction, when the window opens. TDHF and NET agree fairly well during this stage of the reaction, although the NET curve has a small time delay relative to TDHF.

For the more central collisions, the TDHF curve of L continues to decrease into a pronounced and sharp minimum, whereas the NET curve flattens out, as determined by equation (4). We will discuss this minimum of the TDHF curve in terms of an *internal relaxation time* of the angular momentum within the nuclei.

To do this, eq. (4) is rewritten as an integro-differential equation,

$$\dot{S}^A = -\frac{1}{2}\dot{L} = \frac{1}{\tilde{t}_+} \left(\frac{I_A}{I_0} J - S^A \right) = \frac{1}{\tilde{t}_+} \frac{I_A}{I_0} J - \frac{1}{\tilde{t}_+} \int_{-\infty}^t \dot{S}^A(t') dt' \quad (5)$$

The time integral here is then modified by introducing an ansatz for the internal relaxation, governed by an internal relaxation time t_i , which measures the average delay for the influence of transferred nucleons on the exchange transport:

$$\dot{S}^A = \frac{1}{\tilde{t}_+} \frac{I_A}{I_0} J - \frac{1}{\tilde{t}_+} \int_{-\infty}^t \dot{S}^A(t') \left(1 - e^{-\frac{t-t'}{t_i}} \right) dt' \quad (6)$$

Within a pure one-body picture, a transferred nucleon must spend some time inside the receiving nucleus before it can affect the phase space distribution at the window (as for example displayed in phase-space diagrams in ref. [8]). Therefore, t_i should be of the order of the circumference of the nucleus divided by the Fermi velocity, i.e. in this case $t_i \approx 3 \cdot 10^{-22}$ s.

Ignoring the time dependence of $\tilde{t}_+$, eq. (6) can be differentiated once more to obtain a second order differential equation, equivalent to the equation for a damped harmonic oscillator. For $t_i > \frac{1}{4}\tilde{t}_+$, the solution displays an oscillatory behavior, the frequency of oscillation being $\omega = \sqrt{\frac{1}{\tilde{t}_+ t_i} - \frac{1}{4t_i^2}}$. With $t_i = 3 \cdot 10^{-22}$, oscillations will only be visible if the window radius is larger than ≈ 2 fm. (Such a value is required to obtain half an oscillation within $8 \cdot 10^{-22}$ s.) This gives a rough explanation why pronounced oscillations are observed in the TDHF calculations for $J = 75\hbar$ and $J = 82\hbar$, but not for the more peripheral reactions.

For the two most central trajectories the reaction time is much larger than a typical value of $\tilde{t}_+$. Consequently, NET predicts that the orbital angular momentum follows the value corresponding to rigid rotation in the later stage of the reaction. It is apparent that TDHF shows a more "elastic" behavior towards the end of the reaction. The largest deviation is found for $J = 75\hbar$, where the nuclei acquire only about $\frac{2}{3}$ of the value for rigid rotation, according to TDHF. This discrepancy may be caused by the fact that in a pure one-body picture some regions of phase space will not be accessible to the transfer process. For spherical nuclei, the classical orbits for independent particle motion are planar, and a rough estimate of the moment of inertia of the active parts of the phase space can be obtained by a phase space integration restricted to planes which intersect with the open window. This gives the result

$$I_{A(\text{active})} = I_{A(\text{rigid})} \frac{9}{8} \sin \theta_0 \left(1 - \frac{1}{9} \sin^2 \theta_0 \right) \quad (7)$$

where θ_0 is the opening angle of the window, $\sin \theta_0 \approx c_{\text{eff}}/2R$. During a reaction, the active part of phase space is continually redefined as the nuclides rotate around each other. In addition, deformations of the reacting nuclei will enlarge the active

phase space. Thus, only a crude feeling for the effect of the influence of the limitation of active orbitals can be gained by a simple modification of the moment of inertia. To do this, we introduce the value (7) for the moment of inertia both into $\tilde{t}_+$ and I_0 of equations (3) and (6), and to be definite, the largest value of c_{eff} during the reaction is used.

Results of such calculations with the integro-differential equation (6) are shown in figure 3 together with the TDHF and standard NET results. For the most peripheral reaction the time development is dominated by the term proportional to J of equations (4) or (6), and modifications of other terms only have minor influence on the results. For the more central reactions, the relaxation ansatz brings about a very good reproduction of the initial steep decrease of L seen in TDHF, almost all the way into the strong minimum. The value at the minimum is also reasonably well accounted for by the relaxation ansatz, which on the other hand gives a too broad and flat behavior of the curve around the minimum. The later oscillations of the TDHF curves are not reproduced by the relaxation ansatz with or without a scaling of the moment of inertia.

The minimum of the TDHF curves of L , related here to internal relaxation in the nuclei, is of relevance to real nuclei, because it occurs on a very short time scale. The "elastic" behavior of TDHF relative to rigid rotation at the later stages of some of the trajectories appears more artificial. It is expected that direct nucleon-nucleon collisions (ignored in TDHF) increase the rate of equilibration, and thereby help the motion approach rigid rotation. A rigid rotation value for the moment of inertia is well in accordance with experimental findings both for hot nuclei, and for rapidly rotating cool or cold nuclei.

An important part of understanding dissipative phenomena is to identify the most significant degrees of freedom. The transport equation (4) of NET incorporates explicitly the minimal set of dynamical variables for describing tangential friction. We have solved the NET equation for the mean orbital angular momentum in the environment provided by the TDHF evolution. The important quantities, namely R and σ_{eff} are quite well-determined. The overall agreement with TDHF shows that the main part of the angular momentum loss in TDHF can be interpreted as nucleon exchange window friction. The more detailed behavior of the TDHF results imply that additional variables play a role. In particular, the initial "undershoot" by L can be understood as a consequence of the finiteness of the relaxation time for the sharing of angular momentum within a single nucleus, and the deviations at the late stages may suggest that symmetries of the potential prevent some orbitals from participating in the exchange. The associated modifications introduced into the integro-differential equation effectively amounts to including a few more dynamical variables.

The type of study reported here is useful for clarifying the dissipative properties of nuclei. It appears particularly interesting to extend these considerations to asymmetric systems in order to separate sliding friction from rolling friction. This latter component is sensitive to the localization of the individual transfers and a calculation with quantal wave functions may therefore provide important new insights into the nucleon transfer mechanism in damped reactions.

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Figure 1. Time development of the orbital angular momentum L and effective window radius c_{eff} for four different values of the total angular momentum in the reaction 280 MeV $^{40}\text{Ca} + ^{40}\text{Ca}$. Heavy solid curves denote TDHF results. Light curves denote NET results, based upon a determination of the window area from TDHF results, using either the potential via eq. (1) (light dashed lines) or the density distribution via eq. (2) (light solid lines).

Figure 2. Time development of the orbital angular momentum L in TDHF obtained with different prescriptions and parameters. Heavy solid curves denote our standard calculation, dashed curves denote calculations with the cruder grid size 1 fm (the standard value is 0.75 fm), and dot-dashed curves are calculated with the prescription that the relative position vector $\vec{R}$ coincide with the major principal axis of the inertia tensor.

Figure 3. TDHF calculations of the orbital angular momentum L (heavy curves) compared to results of calculations with various modifications of NET. Light curves show the standard NET results. Dashed curves show solutions to the integro-differential equation (6) with the internal relaxation time t_i set equal to $3 \cdot 10^{-22}$ s. Furthermore, a change of the moment of inertia according to the expression (7) has been introduced into the integro-differential equation to obtain the dotted curves.

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