Prior User Rights[*]

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Many inventions, great and small, are discovered independently at roughly the same time by two or more individuals or organizations. Famous examples include the light bulb (Edison and Swan), the telephone (Bell and Gray), and the integrated circuit (Kilby and Noyce). Such independent invention is common for minor technological improvements. How should property rights to an invention be defined and awarded in such cases?

Patent law has struggled with this question for many years. The basic rule in the U.S. is that the patent is awarded to the first firm to invent; later independent inventors come up empty-handed. However, this basic system can create some peculiar results.

Suppose that Firm A achieves an invention and files for a patent. Slightly later, but before the invention is made public, Firm B independently discovers the same invention. Firm A receives the patent and can even prevent Firm B from practicing its own invention. In legal terms, a party accused of patent infringement cannot defend itself by showing that it discovered the same invention independently. Would such an *independent invention defense* be desirable?

Alternatively, suppose that Firm A achieves an invention, but decides *not* to file for a patent, perhaps because Firm A does not believe this invention is sufficiently novel and non-obvious to be patentable. Instead, Firm A uses the invention internally in its own operations as a trade secret. Later, Firm B independently discovers the same invention and files for a patent. Under current U.S. patent law, Firm B is awarded the patent because Firm A kept its invention secret.

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Armed with its patent, Firm B usually can prevent Firm A from practicing the invention. In legal terms, Firm A, despite inventing and using the invention before Firm B obtained its patent, has no *prior user rights.* Would granting such rights be desirable?

Current U.S. law provides for very limited prior user rights. Prior user rights are awarded only for patents involving business methods and then only if the prior user reduced the invention to practice one year before the patent application was filed. These provisions were added to patent law in 1999, evidently in response to concerns that owners of patents on business methods would assert those patents opportunistically against prior users of those methods. European law provides far more generous prior user rights. Congress is currently considering legislation (H.R. 2795) that would greatly expand prior user rights in the U.S. by awarding them for all patents and requiring only that the prior user "commercially used, or made substantial preparations for commercial use of, the subject matter before the effective filing date of the claimed invention."

This paper explores the economic effects of awarding prior user rights. The analysis here abstracts away from the fine details of which party discovered the invention slightly before or after another, viewing slight differences in timing as essentially random. Formally, the various inventions are treated as simultaneous. With this abstraction, there is no fundamental difference between the independent invention defense and prior user rights.

For simplicity, suppose two firms are conducting R&D directed at the invention in question. The rules governing prior user rights only come into play if *both* firms successfully discover the invention. In that event, without prior user rights, each firm has a 50% chance of getting the patent, and thus a 50% chance of obtaining a monopoly over the patented invention. Denote the monopoly profits by π_M and total welfare under monopoly by W_M. In contrast, with prior user rights, both firms have the right to practice the invention, so a duopoly results. Denote the profits of the patent holder by π_P, the profits of the prior user by π_U, and total welfare with the resulting duopoly in the use of the invention by W_D. We assume that combined duopoly profits are less than monopoly profits; in the case where $\pi_P = \pi_U = \pi_D$, this becomes $2\pi_D < \pi_M$. We also assume that monopoly welfare is less than duopoly welfare, $W_M < W_D$. So, the *ex post* effects of prior user rights are straightforward. In the event that both firms discover the invention, prior user rights enhance competition, reduce joint profits, and increase total welfare.

What about the *ex ante* effects of awarding prior user rights? Awarding prior user rights reduces the return to achieving the invention. If the firms are making socially excessive R&D expenditures in the market equilibrium without prior user rights, as occurs in many models of patent races, then awarding those rights has favorable *ex ante* and *ex post* effects. This point is made in Maurer and Scotchmer (2002), using a static model with free entry in which each firm, by paying a fixed amount, can discover the invention with certainty. In their model, all R&D expenditures by multiple firms are entirely duplicative, and the market equilibrium involves excessive entry by rent-seeking firms. However, as we now show, the attractiveness of prior user rights extends well beyond situations in which equilibrium R&D expenditures are excessive.

I. R&D Expenditure Levels with Independent Projects

Suppose that two firms are engaged in R&D competition, with each firm choosing how much to spend on R&D. Greater expenditures increase the chance of success, subject to diminishing returns. The cost of achieving a success probability *p* is given by $C(p)$ with $C(0)=0$, $C'(p)>0$, and $C''(p)>0$. Success by one firm is independent of success by the other.

Two patent policy instruments are available: patent lifetime, *T*, and the strength of prior user rights. For simplicity, suppose there is no discounting, the invention is useful during the time period $[0,1]$, and the patent remains in force during the time period $[0,T]$. After the patent expires, the market is openly competitive, so the firms earn zero profits and welfare is W_C. Stronger prior user rights are modeled by an increase in the probability, α, that prior user rights will be granted in the event that both firms achieve the invention. Stronger prior user rights correspond to policy changes that lower the requirements necessary for such rights to be granted, as Congress is currently considering. In the event that both firms discover the invention, each firm receives a flow payoff of $\pi_B = \alpha[\pi_D] + (1-\alpha)[\pi_M / 2]$ during $[0,T]$, for a total payoff of $\pi_B T$. Flow welfare is $W_B = \alpha[W_D] + (1-\alpha)[W_M]$ during $[0,T]$ and W_C during $[T,1]$. If one firm is successful and the other firm is *not*, the successful firm's total payoff is $\pi_M T$, and total welfare is $W_M T + W_C(1-T)$.

A single firm whose rival's success rate is *q* chooses its own success rate *p* to maximize $\pi(p,q) = p(1-q)T\pi_M + pqT\pi_B - C(p)$. The first-order condition for this firm is given by $\pi_p(p,q) = T[(1-q)\pi_M + q\pi_B] - C'(p) = 0$. This game involves downward sloping best-response functions, since $\pi_{pq}(p,q) = T(-\pi_M + \pi_B) < 0$.

In the symmetric equilibrium, $\frac{C'(p)}{T\pi_M} = 1 - p(1 - \frac{\pi_B}{\pi_M})$. The equilibrium success rate is a function of the two policy parameters, T and α, which we write as $p(T,\alpha)$. Differentiating the above equation tells us that $p_\alpha(T,\alpha) < 0$, i.e., stronger prior user rights reduce the return to innovative efforts, and thus the probability of invention. Prior user rights raise *ex post* welfare by enhancing competition. Total welfare is given by

$$W(p,T,\alpha) = p^2[TW_B + (1-T)W_C] + 2p(1-p)[TW_M + (1-T)W_C] - 2C(p).$$

Theorem #1: Suppose that each firm chooses its R&D investment level, with greater investment increasing the chance of success, and with success at one firm independent of success at the other. Prior user rights are socially optimal if and only if the ratio of deadweight loss to profits is higher under monopoly than under duopoly.

Theorem #1 tells us that prior user rights are an attractive feature of the patent system so long as duopoly delivers returns to innovators more efficiently, in terms of the deadweight loss, than does monopoly. This finding fits very nicely with the deadweight loss to profit ratio test developed by Kaplow (1984) for patent and antitrust policy. Gilbert and Shapiro (1990) show that this condition holds if profits and welfare are concave in output. The Appendix shows how the ratio test in Theorem #1 changes if the patent holder also earns licensing revenues.

II. Diversification of Research Approaches

We now use the model from Dasgupta and Maskin (1987) to study how prior user rights affect firms' decisions to allocate their fixed research budgets across different R&D projects. Each of two firms can choose to adopt an approach that is less correlated with its rival, but doing so reduces its probability of success. Dasgupta and Maskin established conditions under which the market is biased towards overly correlated project choices, but did not study prior user rights.

The first firm selects a project $x \in [0,1/2]$ and the second firm selects a project $y \in [0,1/2]$. Higher values correspond to projects that are less likely to succeed: the probability of success for project z is $p(z)$, with $p(0) > 0$, $p'(z) < 0$, and $p''(z) < 0$. However, higher values of x and y correspond to research projects that are less correlated; the correlation between the two projects is given by $1-(x+y)$. For any given pair (x, y) we write the probability that both firms succeed as $B(x, y)$, and the probability that just the first firm succeeds as $A(x, y)$. We impose symmetry, so the probability that just the second firm succeeds is given by $A(y, x)$.

The first firm picks its project x to maximize $\Pi = A(x, y)\pi_M + B(x, y)\pi_B$, giving the first-order condition $A_x(x, y)\pi_M + B_x(x, y)\pi_B = 0$. Since $A(x, y) + B(x, y) = p(x)$, we know that $A_x(x, y) + B_x(x, y) = p'(x) < 0$. Substituting into the first-order condition, we have $A_x(x, y)\pi_M + [p'(x) - A_x(x, y)]\pi_B = 0$, which can we written as $A_x(x, y)[\pi_M - \pi_B] + p'(x) = 0$. Since $\pi_M > \pi_B$, if x is chosen optimally, we must have $A_x(x, y) > 0$ and $B_x(x, y) < 0$. Since $\partial^2\Pi / \partial x \partial \pi_B = B_x(x, y) < 0$, prior user rights, by reducing π_B, cause the first firm to increase x. Intuitively, prior user rights reduce the return if both firms are successful and thus cause each firm to select a less correlated research approach.

The symmetric equilibrium is characterized by the condition: $A_x(x, x)\pi_M + B_x(x, x)\pi_B = 0$. Welfare is given by $W(x, y, \alpha) = W_M(A(x, y) + A(y, x)) + W_B B(x, y)$. As usual, the direct effect of awarding stronger prior user rights is positive, so stronger prior user rights will raise welfare if their indirect effects are also favorable for welfare, which will be true if the market equilibrium is biased towards projects that are overly correlated.

Theorem #2: Suppose that each firm picks from a menu of R&D projects. Projects at one firm that are more likely to succeed are also more highly correlated with the other firm's projects. Strengthening prior user rights raises welfare if $\frac{\pi_B}{\pi_M} > \frac{W_B - W_M}{W_M}$.

For any given level of prior user rights, strengthening those rights raises social welfare if an individual firm is biased towards joint vs. sole discovery, in comparison with social welfare. The firm's tradeoff is reflected in the ratio π_B / π_M. The social tradeoff is reflected in the ratio

$(W_B - W_M)/W_M$. If $\pi_B / \pi_M > (W_B - W_M)/W_M$, then the market equilibrium is biased towards joint discovery, and prior user rights help correct for this bias. With no prior user rights $\alpha = 0$, $\pi_B / \pi_M = 1/2$ and $W_B = W_M$, so the inequality in Theorem #2 is satisfied:

Corollary #2A: At least some prior user rights are socially optimal.

Since π_B decreases with α and W_B increases with α, the inequality in Theorem #2 will be satisfied for all values of α if it is satisfied at $\alpha = 1$. Therefore, we also have:

Corollary #2B: Full prior user rights are socially optimal if $\frac{\pi_D}{\pi_M} > \frac{W_D - W_M}{W_M}$.

Cabral (1994) shows that this condition satisfied in Cournot duopoly with linear demand and constant marginal costs. However, with homogeneous products and Bertrand competition, we have $\pi_D = 0$ and $W_D > W_M$, so this inequality is not satisfied. If competition is sufficiently severe, each firm will see little value in being one of two inventors, even though there is a social benefit of having two rather than one inventor. Therefore, full prior user rights can cause the market to be biased towards projects that are less likely to succeed but less correlated. In that case, the indirect effect of stronger prior user rights on welfare can be adverse. Even in that case, however, full prior user rights may be optimal due to their favorable direct effect.

III. Allocation of R&D Budgets Across Markets

We now ask how prior user rights affect firms' decision to allocate their fixed R&D budgets across markets. Following Cabral (1994), suppose that each of two firms can allocate its R&D budget between a smaller market, in which innovation is easier, and a larger market in which innovation is harder. Success by one firm is independent of success by the other.

A firm that allocates a fraction *x* of its R&D budget to the smaller market will achieve the innovation in that market with probability $p(x)$, where $p'(x) > 0$ and $p''(x) < 0$. The larger, market involves a lower probability of success, for any given level of R&D expenditures, but a proportionately larger payoff. In particular, if a firm allocates a fraction $1 - x$ of its R&D budget to the larger market, it will achieve the innovation in this market with probability $p(1-x)/\sigma$, where $\sigma > 1$, but its payoff will be $\sigma\pi_M$ if the other firm fails to achieve the invention in this market and $\sigma\pi_B$ if the other firm succeeds in this market. The corresponding levels of welfare in the larger market are σW_M and σW_B.

Suppose that the other firm is expected to allocate a fraction y of its budget to the smaller market. Therefore, the other firm is expected to succeed in the smaller market with probability $f(y)$ and in the larger market with probability $f(1-y)/\sigma$. The payoff to the first firm of allocating a fraction x of its budget to the smaller market is given by

$$p(x)p(y)\pi_B + p(x)(1-p(y))\pi_M + \frac{p(1-x)}{\sigma}\frac{p(1-y)}{\sigma}\sigma\pi_B + \frac{p(1-x)}{\sigma}\frac{1-p(1-y)}{\sigma}\sigma\pi_M .$$

We study the symmetric Nash equilibrium in this R&D budget allocation game. Total welfare in a symmetric equilibrium is given by

$$W(x,\alpha) = p(x)^2 W_B + 2p(x)(1-p(x))W_M + (\frac{p(1-x)}{\sigma})^2\sigma W_B + 2\frac{p(1-x)}{\sigma}\frac{1-p(1-x)}{\sigma}\sigma W_M .$$

The Appendix shows that $\partial x/\partial\alpha < 0$, i.e., awarding stronger prior user rights causes the firms to shift R&D resources into the larger market. Since prior user rights only come into play if both firms succeed, stronger prior user rights tilt each firm towards the larger market, where discovery by its rival is less likely, so prior user rights are less likely to arise.

Therefore, awarding stronger prior user rights raises welfare if shifting the firms' R&D budgets towards the larger market increases welfare, i.e., if such rights correct for a pre-existing market bias against conducting R&D in larger markets where innovation is harder. Cabral (1994) proves the market is biased against R&D in the larger market if and only if $\frac{\pi_B}{\pi_M} > \frac{W_B - W_M}{W_M}$.

This is precisely the same condition that arose in Theorem #2 above, so we have:

Theorem #3: Suppose that each firm allocates its R&D budget between a smaller market and a larger market, in which innovation is more difficult. Stronger prior user rights cause the firms to shift their R&D budgets towards the larger market. Some prior user rights are always socially optimal. Full prior user rights are socially optimal if $\frac{\pi_D}{\pi_M} > \frac{W_D - W_M}{W_M}$.

The inequality in Theorem #3 is satisfied with Cournot duopoly, linear demand, and constant unit costs, and more generally if duopoly competition is not too sharp. The inequality in Theorem #3 is sufficient, but not necessary, for full prior user rights to be optimal.

IV. Concluding Remarks

When nearly simultaneous, independent invention occurs, awarding one inventor a patent and the other the right to use the invention has very attractive properties. Competition is enhanced, innovation is rewarded with relatively little deadweight loss, and the private and social incentives to be the sole vs. joint inventor are generally better aligned than in the absence of such rights.

The attractiveness of prior user rights is even stronger if we take account of the fact that a single patent lifetime is set for all industries and inventions, despite huge differences across inventions in their expected profit to cost ratios. Prior user rights *automatically* reduce the rewards precisely for those inventions with a high profit to cost ratio, since these are the inventions most likely to be discovered simultaneously. They also are the inventions that the patent system is most likely to over-reward. From a Bayesian perspective, the fact that an invention was discovered independently by two or more parties is evidence that the profit to cost for that invention was relatively high, so reducing the reward based on market power is attractive.

The appeal of prior user rights is especially great today given mounting evidence that the patent system is out of balance, as argued by the FTC (2003), the National Academies of Science (2003), Jaffe and Lerner (2004), Shapiro (2004), Lemley and Shapiro (2005), and Farrell and Shapiro (2005). Prior user rights can partially correct for problems caused when patents are issued for obvious or nearly obvious inventions, and for inventions that are not truly novel.

The main drawback associated with prior user rights is that they tend to encourage inventors to keep their inventions secret rather than disclosing them in patent applications. Denicolo and Franzoni (2004) develop a model in which a second party who duplicates and patents an invention that it knows had previously been discovered but kept secret should be granted the right to exclude the inventor from using its invention. However, the effectiveness of patent disclosures is in doubt, especially in industries where scientists and engineers are instructed not to read patents for fear of triggering additional liability for willful infringement. Plus, the current patent system rewards applicants who are most aggressive in seeking patents over those who simply use their own inventions internally as trade secrets. More generally, the effects of encouraging inventors to adopt trade secret vs. patent protection are not well understood. Further work is needed to compare the benefits of prior user rights, as described here, with any costs that result from inducing some inventors to seek trade secret rather than patent protection.

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Appendix

R&D Expenditures with Independent Outcomes

Discounting could easily be incorporated into this model by redefining T to represent the ratio of the value of an annuity that lasts for the lifetime of the patent to the value of a perpetuity.

A. Proof of Theorem #1

If the patent lifetime *T*, is set optimally, given α, we must have $\frac{dW}{dT} = \frac{\partial W}{\partial p}\frac{\partial p}{\partial T} + \frac{\partial W}{\partial T} = 0$, so $\frac{\partial W}{\partial p} = \frac{\partial W}{\partial T} / \frac{\partial p}{\partial T}$. The welfare impact of strengthening prior user rights is given by $\frac{dW}{d\alpha} = \frac{\partial W}{\partial p}\frac{\partial p}{\partial \alpha} + \frac{\partial W}{\partial \alpha}$. Substituting for $\partial W / \partial p$, we get $\left.\frac{dW}{d\alpha}\right|_{T=T*} = \frac{\partial p}{\partial \alpha}\frac{\partial W}{\partial T} / \frac{\partial p}{\partial T} + \frac{\partial W}{\partial \alpha}$, so $\left.\frac{dW}{d\alpha}\right|_{T=T*} > 0$ if and only if $\frac{\partial p}{\partial \alpha}\frac{\partial W}{\partial T} / \frac{\partial p}{\partial T} + \frac{\partial W}{\partial \alpha} > 0$. Since $\frac{\partial W}{\partial T} < 0$, we have $\left.\frac{dW}{d\alpha}\right|_{T=T*} > 0$ if and only if

$$\frac{\partial W}{\partial \alpha} / [-\frac{\partial W}{\partial T}] > [-\frac{\partial p}{\partial \alpha}] / \frac{\partial p}{\partial T}.$$

We now proceed to establish that this inequality is met.

The left-hand side of this inequality is easy to calculate. As noted above, $dW_B / d\alpha = W_D - W_M$, so $\frac{\partial W}{\partial \alpha} = p^2 T (W_D - W_M)$. From the definition of $W(p, T, \alpha)$ we also get $-\frac{\partial W}{\partial T} = p^2 (W_C - W_B) + 2p(1-p)(W_C - W_M)$. Therefore, we have

$$\frac{\partial W}{\partial \alpha} / [-\frac{\partial W}{\partial T}] = \frac{pT(W_D - W_M)}{p[W_C - W_B] + 2(1-p)[W_C - W_M]}.$$

We now look more closely at the $p(T,\alpha)$ function to obtain an expression for the right-hand side of above inequality .

Using the condition that defines the symmetric equilibrium level of *p*, we get

$\frac{\partial p}{\partial T} = \frac{(1-p)\pi_M + p\pi_B}{C''(p) + T(\pi_M - \pi_B)}$ and $-\frac{\partial p}{\partial \alpha} = \frac{pT(\pi_M / 2 - \pi_B)}{C''(p) + T(\pi_M - \pi_B)}$ so we have

$$[-\frac{\partial p}{\partial \alpha}] / \frac{\partial p}{\partial T} = \frac{pT(\frac{\pi_M}{2} - \pi_D)}{(1-p)\pi_M + p\pi_B}.$$

So, we have $\left.\frac{dW}{d\alpha}\right|_{T=T^*} > 0$ if and only if

$$\frac{(W_D - W_M)}{p[W_C - W_B] + 2(1-p)[W_C - W_M]} > \frac{\frac{\pi_M}{2} - \pi_D}{(1-p)\pi_M + p\pi_B}.$$

Substituting using $W_B = (1-\alpha)W_M + \alpha W_D$ and $\pi_B = (1-\alpha)\pi_M / 2 + \alpha\pi_D$, this becomes

$$\frac{(W_D - W_M)}{p[W_C - (1-\alpha)W_M - \alpha W_D] + 2(1-p)[W_C - W_M]} > \frac{\frac{\pi_M}{2} - \pi_D}{(1-p)\pi_M + p[(1-\alpha)\pi_M / 2 + \alpha\pi_D]}.$$

Collecting terms, this becomes

$$\frac{(W_D - W_M)}{(2-p)[W_C - W_M] - \alpha p[W_D - W_M]} > \frac{\pi_M - 2\pi_D}{(2-p)\pi_M - \alpha p[\pi_M - 2\pi_D]}.$$

Inverting both sides and simplifying gives

$$\frac{W_C - W_M}{W_D - W_M} < \frac{\pi_M}{\pi_M - 2\pi_D}.$$

Inverting again and simplifying gives $\frac{2\pi_D}{\pi_M} > \frac{W_C - W_D}{W_C - W_M}$. Defining the monopoly deadweight loss as $DWL_M = W_C - W_M$ and the duopoly deadweight loss as $DWL_D = W_C - W_D$, granting stronger prior user rights raises welfare if and only if $\frac{DWL_M}{\pi_M} > \frac{DWL_D}{2\pi_D}$, as asserted in the text.

B. Ratio of Profits to Deadweight Loss

Gilbert and Shapiro (1990) show that the ratio of deadweight loss to profits rises with price is profits and welfare are both concave in output. Here we establish an alternative sufficient condition. The material in this section was developed jointly with Joseph Farrell.

Call the demand function $X(p)$. Assume that output can be produced at constant marginal cost *c*. Denote by $L(p)$ the deadweight loss if the price is *p*. [For this subsection alone, *p* denotes price, not the probability of discovery.] Denote by $\Pi(p) = (p-c)X(p)$ the total profits if price is *p*. Under what circumstances is the ratio $L(p)/\Pi(p)$ increasing in price *p* in the range $c \le p \le p^M$, where p^M is the monopoly price?

The ratio $L(p)/\Pi(p)$ is increasing in *p* if and only if $L'(p)/\Pi'(p) > L(p)/\Pi(p)$. We look at each of these ratios in turn.

By definition, $L(p) = \int_c^p [X(t) - X(p)]dt$, so $L'(p) = (p-c)[-X'(p)]$.

$\Pi'(p) = (p-c)X'(p) + X(p) = X(p) - L'(p)$. Therefore, we get

$$\frac{\Pi'(p)}{L'(p)} = \frac{X(p) - L'(p)}{L'(p)} = -1 + \frac{X(p)}{-(p-c)X'(p)} = -1 + \frac{p}{p-c}\left[\frac{X(p)}{-pX'(p)}\right] = -1 + \frac{1}{mE(p)}$$, where

$m \equiv \frac{p-c}{p}$ is the Lerner Index and $E(p) \equiv -\frac{pX'(p)}{X(p)}$ is the absolute value of the elasticity of demand. Inverting this equation, we get $\frac{L'(p)}{\Pi'(p)} = \frac{mE(p)}{1 - mE(p)}$. Assuming that $\Pi'(p) > 0$ for $p < p^M$, we know that $mE(p) < 1$ in this range; only at $p = p^M$ do we get $mE(p) = 1$.

We now look at the first-order approximations to $L'(p)/\Pi'(p)$ and $L(p)/\Pi(p)$ for values of *p* near *c*. We express these in terms of *m*, which is zero at $p = c$. Using the above calculation, we have $\frac{L'(p)}{\Pi'(p)} \approx mE(c)$ for values of *p* near *c*. . From the definition of $L(p)$, for values of *p* near *c* we get the approximation $L(p) \approx \frac{1}{2}[p-c][X(c)-X(p)] \approx \frac{1}{2}[p-c][-(p-c)X'(c)]$. Some simple algebra shows that this expression is approximately equal to $\frac{1}{2}mE(c)\Pi(p)$. Therefore, for values of *p* near *c*, we have $\frac{L(p)}{\Pi(p)} \approx \frac{1}{2}mE(c)$. We have thus shown that in the neighborhood of $p = c$, the ratio $L'(p)/\Pi'(p)$ rises with *p* twice as rapidly as does the ratio $L(p)/\Pi(p)$. Both of these ratios approach zero as $p \to c$. This reflects the fact that the deadweight loss is second-order small in $p-c$ when price is near marginal cost.

Using $\frac{L'(p)}{\Pi'(p)} = \frac{mE(p)}{1-mE(p)}$, we know that $L'(p)/\Pi'(p)$ rises with *p* if $mE(p)$ rises with *p*, i.e. if $(\frac{p-c}{p})E(p)$ rises with *p*. Suppose that this condition is satisfied.

Now suppose that $d[L(p)/\Pi(p)]/dp = 0$ for some value of *p*, as it must if $L(p)/\Pi(p)$ is ever to decline with *p*, since $L(p)/\Pi(p)$ is increasing with *p* near $p = c$ (and we are assuming all functions are smooth). Call p_0 the lowest value of *p* at which $d[L(p)/\Pi(p)]/dp = 0$. So, for $p < p_0$, $L(p)/\Pi(p)$ is increasing, which we know requires that $L'(p)/\Pi'(p) > L(p)/\Pi(p)$. We must have $L(p)/\Pi(p) = L'(p)/\Pi'(p)$ at $p = p_0$. Since $L(p)/\Pi(p)$ is locally constant with respect to *p* at $p = p_0$, and since $L'(p)/\Pi'(p)$ is increasing in *p* (by assumption), this could only happen if $L'(p)/\Pi'(p)$ were *less* than $L(p)/\Pi(p)$ for values of *p* just below p_0. But this contradicts the fact that $L'(p)/\Pi'(p) > L(p)/\Pi(p)$ for $p < p_0$. We have therefore proven:

If $(\frac{p-c}{p})E(p)$ rises with *p*, then the ratio of deadweight loss to monopoly profits also rises with *p* for prices between marginal cost and the monopoly price.

C. Extensions to Theorem #1

1. Non-Essential Technology and Rent Shifting

The model just discussed assumes that a firm that is unsuccessful, or succeeds but lacks prior user rights, is excluded from the market and earns no profits. One extension would be to assume instead that each firm uses some older technology and is seeking to develop a new and improved technology, but that the new technology does not constitute a drastic innovation. In other words, if one firm has exclusive rights to the new technology, its rival using the older technology still imposes a competitive constraint (and may earn positive profits).

To study this case, we continue to normalize the profits and welfare to equal zero if neither firm achieves the invention, i.e., in the state (F, F). However, it is no longer true that a firm that fails earns this same amount if its rival succeeds. Rather, such a firm is worse off, since it is facing a stronger rival. So, we need to introduce a new variable R which measures the rents shifted away from the firm that is excluded from using the new technology and towards the patent holder. These shifted rents are relative to the status quo. (We further assume that both firms return to their baseline payoff level of zero after the patent expires, just as we assumed above the both firms earn the same level of profits (zero) if the new technology is not discovered and after the patent expires.) Each firm has an enhanced incentive to increase its success rate, which either captures an extra R in rents or prevents the other firm from doing so. The Appendix establishes:

Corollary #1A: If the ratio of deadweight loss to profits is higher under monopoly than under duopoly, then full prior user rights remain optimal if additional rents are shifted to the sole inventor from its rival whose R&D program was unsuccessful.

Proof: There is no change in π_B, which now equals $(1-\alpha)(\frac{\pi_M + R}{2} - \frac{R}{2}) + \alpha\pi_D$. Now the payoff function is given by $\pi(p,q) = T[p(1-q)(\pi_M + R) + pq\pi_B - (1-p)qR] - C(p)$, so the new first-order condition for this firm is given by $C'(p) = T[(1-q)\pi_M + q\pi_B + R]$. Repeating the steps from the proof of Theorem #1, we find that $[-\frac{\partial p}{\partial \alpha}]/\frac{\partial p}{\partial T} = \frac{pT(\frac{\pi_M}{2} - \pi_D)}{(1-p)\pi_M + p\pi_B + R}$. Since this ratio is declining in *R*, and since the proof of Theorem #1 relies on showing that this ratio is less than $\frac{\partial W}{\partial \alpha}/[-\frac{\partial W}{\partial T}]$, which does not contain *R*, the corollary is established.

2. Multiple Firms Engaging in R&D Competition

This model could be extended to include additional rivals conducting R&D in the same industry. In the natural extension model, any successful firm that does not receive the patent is granted prior user rights. If *m* of the firms are successful in their R&D programs, prior user rights would transform a monopoly into a *m*-firm oligopoly. The natural conjecture is that granting full prior user rights in that model is optimal so long as the ratio of deadweight loss to total profits is smaller for any *m*-firm oligopoly than for monopoly.

3. Licensing to Other Industries

The analysis is virtually unchanged if the patent holder has licensing opportunities in other industries. Suppose that the patent holder can earn licensing revenues of *L* by licensing the patent to third parties in other industries. Then we can write the payoff to being the sole discover of the invention as $\pi_M + L$, where *L* represents the licensing revenues to the patent holder. The expected payoff from being one of two discoverers of the invention is given by $\alpha[\pi_D + L/2] + (1-\alpha)[\pi_M/2 + L/2]$. Simplifying, this payoff equals $\pi_B + L/2$. So, the prospect of earning these licensing revenues increases the payoff from being the sole discoverer by *L* and increases the payoff from being one of two discoverers by $L/2$. In terms of our earlier notation, the monopoly profits rise by *L* as do the joint profits from dual discovery.

Suppose that welfare generated by the other industries from licensing rises by $L+E$, where *E* measures the external benefits to licensees and consumers from the licensing. Then, the problem with licensing is formally equivalent to the problem also solved if we replace π_M with $\pi_M + L$, $2\pi_D$ with $2\pi_D + L$, W_M with $W_M + L + E$, W_D with $W_D + L + E$, and W_C with $W_C + L + E + DWL_L$, where DWL_L is the deadweight loss associated with licensing in other industries, which is eliminated when the patent expires. Applying Theorem #1, we have:

Corollary #1B: If the patent holder licenses the patent for use in other industries, full prior user rights are socially optimal if

$$\frac{DWL_M + DWL_L}{\pi_M + L} > \frac{DWL_D + DWL_L}{2\pi_D + L}.$$

For patents that can be licensed to third parties in other industries, the relevant ratios of deadweight loss to profits need to be adjusted to include the deadweight loss and revenues associated with the licensing activities.

4. Licensing Between Duopolists

Another extension would consider licensing from the patent holder to the rival, if that firm did not enjoy prior user rights. Such licensing would be optimal, and predicted to occur, if the joint profits from licensing exceed the monopoly profits used so far in the analysis. This could occur if the second firm brings important assets to the market. For example, that firm might own useful manufacturing assets or control certain brands or distinct products. In that case, the payoffs without prior user rights are no longer $(\pi_M, 0)$ but rather (π_P, π_N), where π_P is the patent holder's profits and π_N measures the profits of the firm that did not obtain the patent. Katz and Shapiro (1985) provide conditions under which one duopolist will license to the other.

If the two firms would negotiate a patent license in the absence of prior user rights, prior user rights effectively replace the licensing agreement that would be negotiated *ex post* between the two firms with a royalty free license, but only in the event that both firms achieve the invention. As usual, prior user rights are attractive for consumers *ex post.* Theorem #1 suggests that prior user rights are optimal, so long as they lead to an outcome in which the ratio of deadweight loss to profits is less than in their absence, accounting for the negotiated licensing agreement.

5. Licensing to Other Firms in the Same Industry

Extending this model to include other firms in the same industry who will compete with or without the new technology is a more complex undertaking. Prior user rights, when they apply, create a second firm in the industry that can use the patented technology free of charge. The presence of such a firm will affect the incentives of the patent holder to license to the remaining rivals, and the willingness of these rivals to pay for a patent license. The analysis may vary depending upon the licenses studied: fixed fees, uniform running royalties, or two-part tariffs.

D. Uniqueness and Stability of the Symmetric Equilibrium

For ease of notation, we write $k = 1 - \frac{\pi_B}{\pi_M}$, so the first-order condition is $\frac{C'(p)}{T\pi_M} = 1 - kq$. Note that $1/2 \le k \le 1$; when $\alpha = 0$, $\pi_B = \pi_M / 2$ and $k = 1/2$, and when $\alpha = 1$, $\pi_B = \pi_D$, and $k = 1 - \pi_D / \pi_M$.

The first-order condition for the choice of *p* is given by $C'(p)/T\pi_M = 1 - kq$. The slope of the first firm's best response function is therefore given by $dp/dq = -kT\pi_M / C''(p)$. The symmetric equilibrium is stable if and only if the first firm's best-response schedule is steeper than the second firm's at that point. Since the payoffs are symmetric, this is true if and only if the absolute value of the slope of the *p* best-response curve is greater than unity at the symmetric equilibrium. So, we get stability of the symmetric equilibrium if and only if $kT\pi_M > C''(p)$ at the point where $C'(p)/T\pi_M = 1 - kp$. The necessary and sufficient condition for stability, $kT\pi_M > C''(p)$, can be written as $kpT\pi_M > pC''(p)$. From the first-order condition, we have $kpT\pi_M = T\pi_M - C'(p)$, so the stability condition can be written as $T\pi_M - C'(p) > pC''(p)$ or $T\pi_M > C'(p) + pC''(p) = C'(p)[1 + E]$ where $E \equiv pC''(p)/C'(p)$ is the elasticity of the cost function with respect to the success probability. Dividing this inequality by $T\pi_M$ gives $[C'(p)/T\pi_M][1 + E] < 1$. Finally, substituting using the first-order condition we get the necessary and sufficient condition for stability as $(1 - kp)(1 + E) < 1$.

We now provide a sufficient condition for the symmetric equilibrium to be the only equilibrium. The equation defining the symmetric equilibrium is $\frac{C'(p)}{T\pi_M} = 1 - kp$.

Suppose there were an asymmetric equilibrium with $p > q$. Then we must have $C'(p)/T\pi_M = 1 - kq$ and $C'(q)/T\pi_M = 1 - kp$. Taking ratios of these two first-order conditions, we would have $C'(p)(1 - kp) = C'(q)(1 - kq)$. There can be no such asymmetric equilibrium if the function $C'(p)(1 - kp)$ is monotonic in *p*. This expression is decreasing in *p* if and only if $pC''(p)/C'(p) < kp/(1 - kp)$, which we can write as $E(1 - kp) < kp$. This is the same as the stability condition, $(1 + E)(1 - kp) < 1$.

To illustrate using an example, suppose that $C(p)=[\gamma p+\beta p^2/2]T\pi_M$, so $C'(p)=[\gamma+\beta p]T\pi_M$ and $C''(p)=\beta T\pi_M$. Then the symmetric equilibrium level of *p* is given by $p^*=\frac{1-\gamma}{k+\beta}$. An interior equilibrium requires that $p^*>0$, so $\gamma<1$, and that $p^*<1$, so $\beta+\gamma>1-k$. The condition for stability is that $\beta<k$. So long as these three conditions are satisfied, we have a stable interior equilibrium.

Diversification of Research Approaches

We are interested in exploring the welfare effects of granting stronger prior user rights. Differentiating with respect to α, we get

$$\frac{dW(x,\alpha)}{d\alpha}=\frac{\partial W(x,\alpha)}{\partial x}\frac{dx}{d\alpha}+\frac{\partial W(x,\alpha)}{\partial\alpha}.$$

As usual, the direct effect of awarding stronger prior user rights is positive, since $\partial W/\partial\alpha=B(x,y)\partial W_B/\partial\alpha=B(x,y)(W_D-W_M)>0$. We know that $dx/d\alpha>0$, so a sufficient condition for stronger prior user rights to raise welfare is that $\partial W/\partial x>0$ at the equilibrium.

A. *Proof of Theorem #2*

Using the definition of *W*, we have $W(x,y,\alpha)=W_M(A(x,y)+A(y,x))+W_BB(x,y)$. Differentiating with respect to *x*, we have $W_x(x,y,\alpha)=W_M(A_x(x,y)+A_x(y,x))+W_BB_x(x,y)$. By symmetry, $A_x(y,x)=A_y(x,y)$. So $W_x(x,y,\alpha)=W_M(A_x(x,y)+A_y(x,y)+B_x(x,y))+(W_B-W_M)B_x(x,y)$. Evaluating this at a symmetric point where $x=y$ gives

$$W_x(x,x,\alpha)=W_M(A_x(x,x)+A_y(x,x)+B_x(x,x))+(W_B-W_M)B_x(x,x).$$

Since $A(x,y)+B(x,y)=p(x)$, we know that $A_y(x,y)+B_y(x,y)=0$. By symmetry, $B(x,y)=B(y,x)$, so $B_x(x,x)=B_y(x,x)$. Therefore we must have

$A_y(x,x)+B_y(x,x)=A_y(x,x)+B_x(x,x)$. Since the left-hand side of this expression is zero, the right-hand side must also equal zero, so we get

$$W_x(x,x,\alpha)=W_M A_x(x,x)+(W_B-W_M)B_x(x,x).$$

From the condition characterizing the symmetric equilibrium, $A_x(x,x)\pi_M+B_x(x,x)\pi_B=0$. Solving this for $B_x(x,x)$, substituting, and simplifying gives

$$W_x(x,x,\alpha)=W_M A_x(x,x)[1-\frac{W_B-W_M}{W_M}\frac{\pi_M}{\pi_B}]$$

at the symmetric equilibrium. Therefore, $W_x(x,x,\alpha)>0$ at the symmetric equilibrium if and only if $\frac{\pi_B}{\pi_M}>\frac{W_B-W_M}{W_M}$.

Proposition 3 in Dasgupta and Maskin (1987) provides conditions under which the market research portfolio consists of projects that are too highly correlated, so that $dx/d\alpha>0$ in my notation. However, they assume that welfare is the same whether one or both firms are successful: $W_B=W_M$ in my notation. This condition holds at $\alpha=0$, so Proposition 3 in Dasgupta and Maskin (1987), combined with the definition of prior user rights adopted in this paper, implies Corollary #2A, that some prior user rights are optimal. However, their analysis must be extended to study the effects of stronger prior user rights away from $\alpha=0$.

B. Second-Order Condition and Best-Response Functions

As calculated by Dasgupta and Maskin, using my notation,
$B(x,y)=(x+y)p(x)p(y)+[1-(x+y)](p(x)+p(y))/2$ and
$A(x,y)=[1+(x+y)]p(x)/2-[1-(x+y)]p(y)/2-(x+y)p(x)p(y)$.

The second-order condition for the first firm is $A_{xx}\pi_M+B_{xx}\pi_B<0$. A sufficient condition for this to hold (which is necessary if π_B is sufficiently small) is that $A_{xx}<0$. Direct calculations show that $A_{xx}(x,y)=p'(x)[1-p(x)-p(y)]+p''(x)[1+(x+y)(1-p(y))]/2$. This expression is negative if $p(x)$ and $p(y)$ are each no larger than one-half, which they must be if $p(0)\le 1/2$.

However, we could have if $p(x)+p(y)>1$ and if $p''(x)/p'(x)$ is small. In that case, the second-order condition is not satisfied, and the first firm should increase *x* to a higher level at which the first-order condition again holds to find the optimal level of *x*, avoiding a local minimum at a lower value of *x*.

The first-order condition for the first firm is $A_x(x,y)\pi_M + B_x(x,y)\pi_B = 0$. This firm's best-response function is downward sloping if $A_{xy}(x,y)\pi_M + B_{xy}(x,y)\pi_B < 0$, which we write as $\pi_M[A_{xy}(x,y)+B_{xy}(x,y)] - B_{xy}(x,y)[\pi_M - \pi_B] < 0$. Since $A(x,y)+B(x,y)=p(x)$, $A_y(x,y)+B_y(x,y)=0$, and $A_{xy}(x,y)+B_{xy}(x,y)=0$ as well, so this inequality is satisfied if and only if $B_{xy}(x,y)>0$. Since $B_{xy}(x,y) = p'(x)[p(y)-1/2] + p'(y)[p(x)-1/2] + (x+y)p'(x)p'(y)$, this inequality is satisfied so long as $p(x)$ and $p(y)$ are each no larger than one-half, which they must be if $p(0)\le 1/2$.

Allocation of R&D Budgets Across Markets

The welfare effect of strengthening prior user rights is given by

$$\frac{dW}{d\alpha} = \frac{\partial W}{\partial x}\frac{dx}{d\alpha} + \frac{\partial W}{\partial \alpha}.$$

As usual, we know that the $\partial W/\partial\alpha > 0$, because $\partial W_B/\partial\alpha = W_D - W_M > 0$.

We show here that each firm will shift away from the smaller market and towards the larger market as prior user rights are strengthened. Formally, we show that $\partial x/\partial\alpha < 0$. The first firm picks *x* to maximize $\pi(x,y,\alpha)$. Since $d\pi_B/d\alpha = \pi_D - \pi_M/2 < 0$, $\partial x/\partial\alpha < 0$ if and only if $\pi_x(x,y,\alpha)$ rises with π_B.

Differentiating $\pi(x,y,\alpha)$ with respect to π_B gives $p(x)p(y) + \sigma[p(1-x)/\sigma][p(1-y)/\sigma]$. Differentiating this with respect to *x* gives $p'(x)p(y) - p'(1-x)p(1-y)/\sigma$. This is positive if and only if $[p'(x)/p'(1-x)] > [p(1-y)/p(y)]/\sigma$. We now show that this expression is positive at the symmetric equilibrium, i.e., $\frac{p'(x)}{p'(1-x)} > \frac{p(1-x)}{p(x)}\frac{1}{\sigma}$ at the symmetric equilibrium.

In a symmetric equilibrium, Cabral shows (Equation A.4) that we must have

$\frac{p'(x)}{p'(1-x)} = \frac{\pi_M - (\pi_M - \pi_B)p(1-x)/\sigma}{\pi_M - (\pi_M - \pi_B)p(x)}$. So, we are attempting to show that

$\frac{\pi_M - (\pi_M - \pi_B)p(1-x)/\sigma}{\pi_M - (\pi_M - \pi_B)p(x)} > \frac{p(1-x)/\sigma}{p(x)}$. Cross-multiplying and simplifying, this is equivalent to $p(x) > p(1-x)/\sigma$, i.e., that the equilibrium probability of success is greater in the smaller market, a condition that Cabral establishes.