Title
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Publication Date
2004-10-01

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Abstract. It has been suggested that a change of nuclear shape may be described in terms of a phase transition and that specific nuclei may lie close to the critical point of the transition. Analytical descriptions of such critical-point nuclei have been introduced recently and they are described briefly. The results of extensive searches for possible examples of critical-point behavior are presented. Alternative pictures, such as describing bands in the candidate nuclei using simple ΔK=0 and ΔK=2 rotational-coupling models, are discussed, and the limitations of the different approaches highlighted. A possible critical-point description of the transition from a vibrational to rotational pairing phase is suggested.

INTRODUCTION

Notable benchmarks of collective nuclear behavior are the harmonic vibrator [1], the symmetrically deformed rotor [2], and the triaxially soft rotor [3]. They correspond to limits of the interacting boson model (IBM) and an algebraic description of the nature of the transition between these limits has been developed in direct analogy with classical phase transitions [4]. Recently, it has been suggested that a useful approach is to find an analytic approximation of the critical point of the shape change as a new benchmark against which nuclear properties can be compared [5,6].

The critical-point description of the transition from a symmetrically deformed rotor to a spherical harmonic vibrator, denoted as X(5), involves the solution of the Bohr collective Hamiltonian with a potential that is decoupled into two components – an infinite square well potential in the quadrupole deformation parameter, β, and a harmonic potential for the triaxiality deformation parameter, γ [6]. Similarly, the critical-point description of the transition from a triaxially soft rotor to a spherical harmonic vibrator, denoted as E(5), requires the solution of the Bohr Hamiltonian with an infinite square well potential depending only on β [5]. IBM calculations indicate that these potentials approximate the "true" potentials found at the critical point of the shape change. The analytic solutions of the resulting equations are related to Bessel functions and involve only two free parameters used to fix the scales of the energies and transition strengths.

Such descriptions have obvious limitations. The infinite square well potentials used are unphysical. Microscopic ingredients, such as pairing, are ignored. Despite this, several examples of both X(5) [7,8] and E(5) [9] critical-point nuclei have been suggested. In this contribution the results of searches for other possible examples of critical-point nuclei are presented. Alternative pictures, such as describing bands in the candidate nuclei using simple ΔK=0 and ΔK=2 rotational-coupling models, are then discussed, and the limitations of the different approaches highlighted. Finally, a possible critical-point description of the transition from a vibrational to rotational pairing phase is suggested.

SEARCHING FOR CRITICAL POINT NUCLEI

If the critical-point descriptions are to be taken as new benchmarks for describing shape transitional behavior, then it is important to find nuclei that follow closely the predicted behavior. In this section, the results of searches for X(5) and E(5) behavior are presented [10,11].

Searching for X(5) Nuclei

The ENSDF data file [12] was searched for examples of even-even nuclei, with A>60, Z>30, which display the predicted characteristics of the X(5) critical-point description [10]. The experimental signatures of X(5) behavior that were used in the search were the following:

a) The energies of the yrast states, $E(I_1^+)$, should show characteristic ratios lying between those of a vibrator and a rotor (for example, $E(4_1^+)/E(2_1^+)\approx 2.91$, for an X(5) nucleus).
b) The strength of transitions between yrast states as reflected in the B(E2;I→I-2) values should increase with angular momentum I at a rate intermediate between the values for a vibrator and a rotor.
c) The position of the first excited collective 0_2^+ state is approximately 5.67 times the energy of the 2_1^+ level.
d) The non-yrast states based on the 0_2^+ state have larger energy spacings than the yrast sequence.
e) The B(E2;I→I-2) values for intrasequence transitions should be lower for the non-yrast sequence relative to those of the yrast sequence.
f) The intersequence B(E2) values should show a characteristic pattern.

As a first step, the $E(4_1^+)/E(2_1^+)$ ratio was used to select possible X(5) nuclei. For an axially symmetric rotor this ratio should be $E(4_1^+)/E(2_1^+)=3.33$, an harmonic vibrator has $E(4_1^+)/E(2_1^+)=2.0$, while an X(5) nucleus should have $E(4_1^+)/E(2_1^+)=2.91$. Figure 1 shows a histogram of all the $E(4_1^+)/E(2_1^+)$ values of nuclei included in the search. An interesting, and unexplained, feature is the broad peak found for $E(4_1^+)/E(2_1^+)\sim 2.3$, which appears to arise mainly from nuclei with A<150. All even-even nuclei with A>60, Z>30 and $2.71<E(4_1^+)/E(2_1^+)<3.11$ were identified, yielding a total of 35 candidate X(5) nuclei.

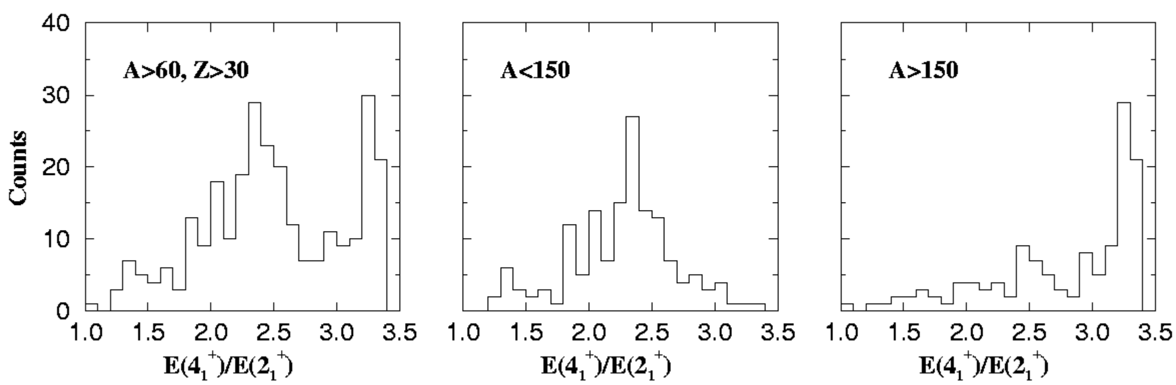

FIGURE 1. Histograms of all $E(4_1^+)/E(2_1^+)$ values for nuclei with A>60, Z>60 (left panel). Cuts on this histogram for A<150 (middle panel) and A>150 (right panel) are also shown.

Of these 35 candidates, reliable lifetime measurements up to at least the $I^\pi=8_1^+$ of the yrast sequence were known for fifteen nuclei. In Figure 2 the energies of the yrast sequences (normalized to the energy of their respective 2_1^+ levels) in these nuclei are compared with the expected behavior of an harmonic vibrator, an axially deformed rotor, and the X(5) predictions. In Figure 3 the B(E2;I→I-2) reduced transition strength (normalized to their respective $B(E2;2_1^+\rightarrow 0_1^+)$ values) are shown and again compared with the expected behavior for an harmonic vibrator, an axially deformed rotor, and the X(5) prediction.

Clearly there are many examples of nuclei with yrast energies that closely follow the X(5) prediction. However, most of these can be excluded on the basis of their deduced yrast B(E2;I→I-2) values. Indeed, from the available data, the only nuclei that remain candidates are ^{126}Ba, ^{130}Ce, and the N=90 isotones from Nd (Z=60) to Er (Z=68). For this subset of nuclei, the properties of the excited states, and the transitions from them, can be examined in more detail. For ^{126}Ba and ^{130}Ce the positions of the 0_2^+ levels are significantly lower than the X(5) prediction and little further information is known about states in the relevant excited sequences. For the N=90 isotones, the positions of the 0_2^+ levels are close to the X(5) prediction of $E(0_2^+)\sim 5.67E(2_1^+)$ but the energy spacings of sates in the excited sequence are much lower than predicted. In addition, a significant drop in the excited intrasequence B(E2;I→I-2) values, predicted by the X(5) model, is not seen in the current data for the N=90 isotones. To summarize, the X(5) picture can be applied to a limited number of transitional nuclei, where it is able to reproduce properties of the yrast states. It fails when confronted with data on excited states in those nuclei, but this is no surprise for such a simple model.

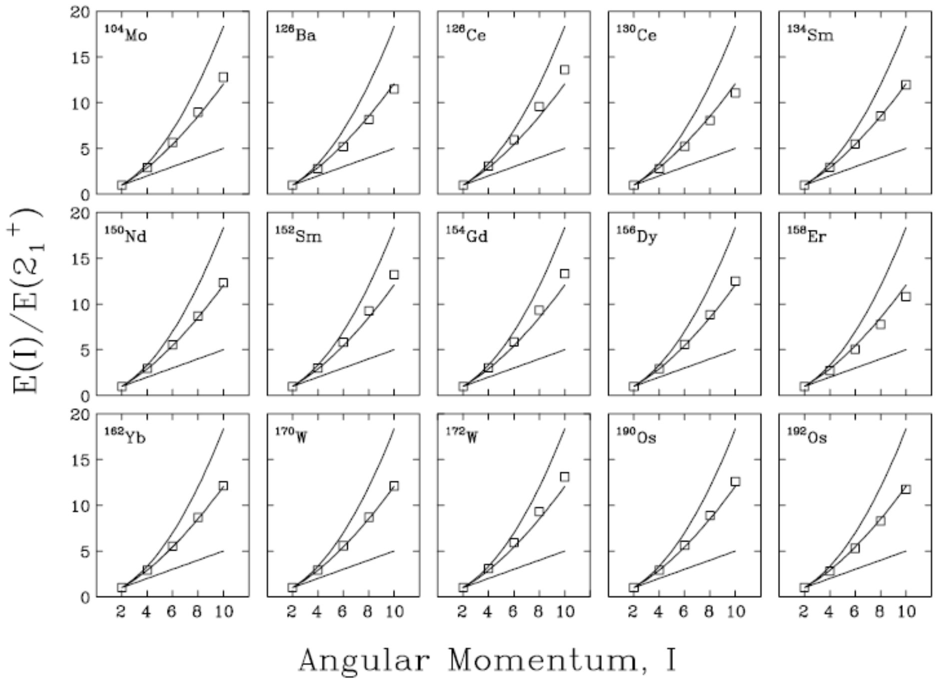

FIGURE 2. Plots of the normalized energies for the yrast sequences of fifteen candidate nuclei for X(5) behavior. The relevant nucleus is indicated in the top left of each panel and the experimental data are plotted with open squares. For comparison the expected energies for a harmonic vibrator (lowest solid line), an axially deformed rotor (highest solid line), and an X(5) critical-point nucleus (intermediate solid line) are also shown.

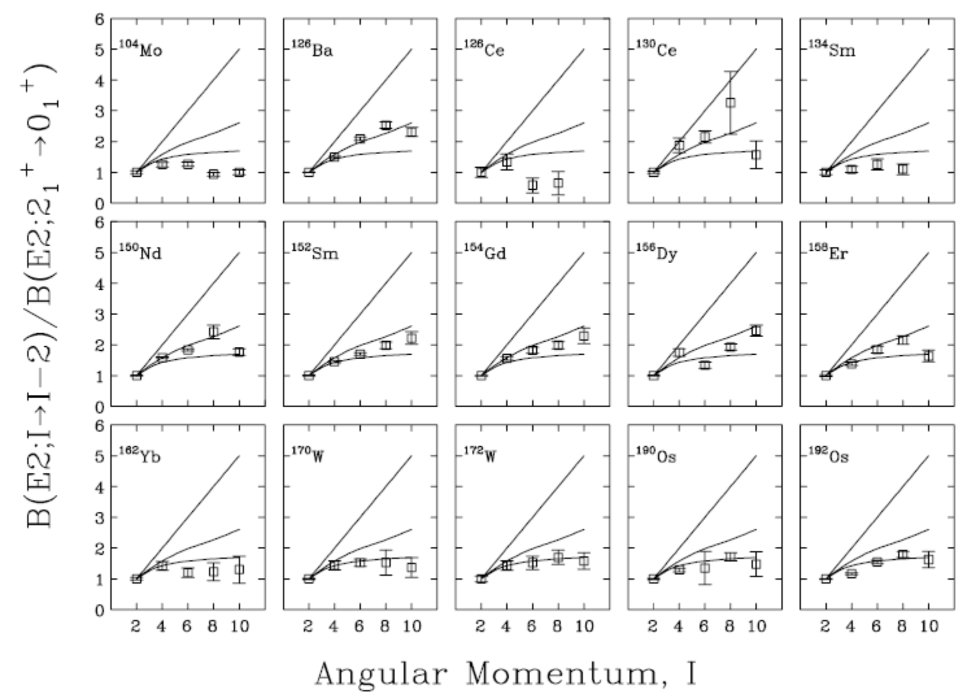

FIGURE 3. Plots of the normalized B(E2;I→I-2) values of transitions in the yrast sequences of the fifteen candidate nuclei for X(5) behavior. The relevant nucleus is indicated in the top left of each panel and the experimental data are plotted with open squares. For comparison the expected energies for a harmonic vibrator (highest solid line), an axially deformed rotor (lowest solid line), and an X(5) critical-point nucleus (intermediate solid line) are also shown.

Searching for E(5) Nuclei

A similar search of the ENSDF data file [12] was performed to find examples of even-even nuclei, with A>60, Z>30, which display the predicted characteristics of the E(5) critical-point description [11]. The key properties of the E(5) description that were used in the search can be summarized as follows:

a) The energy ratio $E(4_1^+)/E(2_1^+)$ should be ≈2.20.
b) The $B(E2;4_1^+\rightarrow 2_1^+)$ should be ≈1.5 times the $B(E2;2_1^+\rightarrow 0_1^+)$ value.
c) There should be two excited 0^+ states lying at approximately 3-4 times the energy of the 2_1^+ state.

It is the properties of these excited 0^+ states that are key to identifying possible E(5) behavior. Eigenfunctions from the E(5) solution can be characterized by two quantum numbers (ξ, τ) related to the zeros of Bessel functions as described in [5]. The ξ quantum number labels major families of E(5) levels, while τ labels the phononlike levels within a given ξ family. The first excited 0^+ state is predicted to be in the ξ=2 family and is labeled in shorthand as 0_ξ^+. The second excited 0^+ state is predicted to belong to the three phononlike multiplet of the ξ=1 family and is labeled 0_τ^+. While the ordering of these two 0^+ states is sensitive to effects such as the finite boson number, their decays are reflective of the E(5) symmetry properties. The decay of the 0_τ^+ should reflect its multiphonon structure. There is an allowed E2 transition to the 2_2^+ level, but no allowed transition to the 2_1^+ level. The 0_ξ^+ state is predicted to have an allowed transition to the 2_1^+ level with a strength of ≈0.5 the $B(E2;2_1^+\rightarrow 0_1^+)$ value.

The details of the search can be found in [11], but only six E(5) candidate nuclei were found, namely ^{102}Pd, ^{106,108}Cd, ^{124}Te, ^{128}Xe, and ^{134}Ba. It is interesting to note that these nuclei are all in regions expected to display transitional behavior from spherical vibration to gamma-soft rotation. The closest agreement between experimental data and the predictions of E(5) is for ^{128}Xe and the previously suggested example of ^{134}Ba [9]. This is illustrated in Figure 4 which compares the E(5) predictions with the experimental data on these two candidate nuclei.

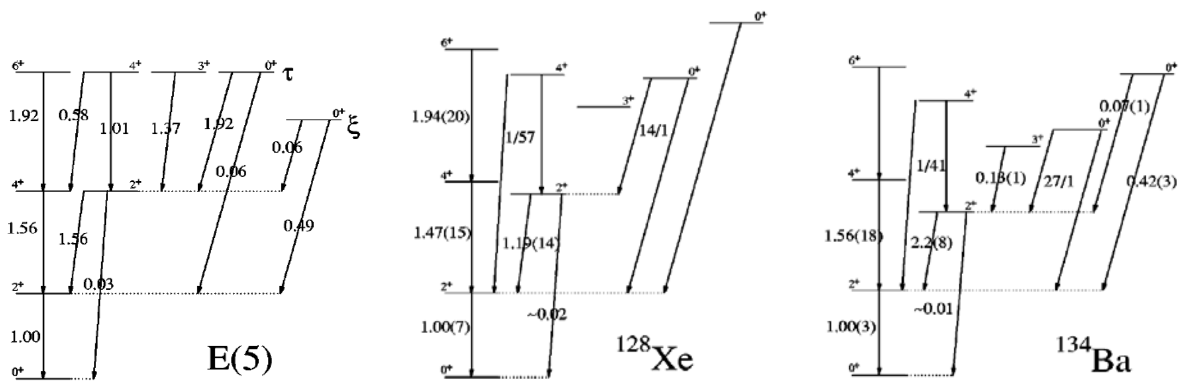

FIGURE 4. Level scheme calculated for the E(5) symmetry (left, the 0_τ^+ and 0_ξ^+ levels are marked with a bold τ and ξ, respectively), and empirical schemes for ^{128}Xe (middle), and ^{134}Ba (right). The excitation energies of the states are normalized to the energy of the 2_1^+ level in each case. The numbers indicate E2 transition strengths, normalized to the $B(E2;2_1^+\rightarrow 0_1^+)$ value. Relevant branching ratios are indicated by two numbers separated by a slash.

ALTERNATIVE DESCRIPTIONS

In this section, an alternative description of the states in ^{152}Sm, the first nucleus proposed as an example of X(5) behavior [7], is presented in terms of ΔK=0 [13] and ΔK=2 [14] couplings between rotational bands. This is important for highlighting the limitations of both the new X(5) critical-point description and the "traditional" interpretations of structures in transitional nuclei. Such investigations may point the way to the modifications that must be made to the alternative models in order to achieve a better description of transitional nuclei. Indeed, some simple modifications to the X(5) picture have been investigated [15,16] and have had some success in describing phenomena such as the evolution of the excited 0_2^+ states in the rare-earth nuclei [16].

First, by assuming the structure based on the 0_2^+ state in ^{152}Sm is a rotational band with a deformation very similar to the ground-state band, the prescription discussed by Bohr and Mottelson [17], for describing the effects from mixing the two bands by an effective $\Delta K=0$ coupling, can be followed. In particular this mixing will affect the intersequence E2 transition strengths such that they obey the relationship:

$$B(E2;I_i\rightarrow I_f)=\langle I_i020|I_f0\rangle^2\{M_1+M_2[I_i(I_i+1)-I_f(I_f+1)]\}^2.$$

M_1 is the intrinsic matrix element for the transition while M_2 is the contribution to the transition matrix element attributed to the $\Delta K=0$ coupling and is related to the mixing amplitude. Figure 5 presents the experimental data in ^{152}Sm and the fact that they are consistent with the generalized intensity rule given above is clear from the fact that a straight line can be fitted through the experimental points. Deviations from the straight line may indicate the presence of effects that are not included, such as, unequal quadrupole moments for the two bands or multiple band mixing. Also shown in the figure are the predictions of the X(5) model (normalized to the $B(E2;2_1^+\rightarrow 0_1^+)$ value). It is interesting to note that these values can be scaled by a constant factor of 1.7 to reproduce the data well and this same scaling factor succeeds in reproducing the data on intersequence transitions of the other N=90 X(5) candidates.

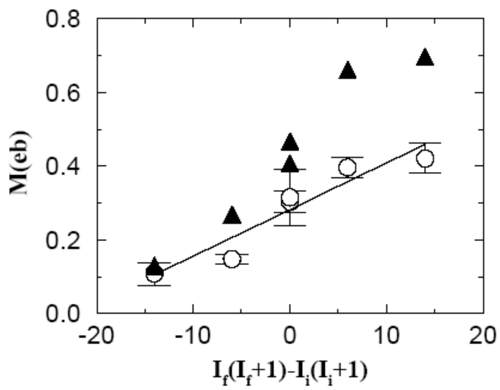

FIGURE 5. Plots of $M=[B(E2;I_i\rightarrow I_f)/\langle I_i020|I_f0\rangle^2]^{1/2}$ as functions of $I_f(I_f+1)-I_i(I_i+1)$. The experimental data are shown as open circles. The linear fit to the data is shown by the straight line. The X(5) predictions are given by closed triangles.

A similar analysis can be performed on the quasi-γ band in ^{152}Sm [14]. There are a full set of predictions for the behavior of an X(5) nucleus in the γ degree of freedom [18]. As before, by assuming the quasi-γ band has a deformation similar to the ground-state band, the effects from mixing with the ground-state band by an effective $\Delta K=2$ coupling can be studied. Again, the mixing will affect the intersequence E2 strengths which will now obey the relationship:

$$B(E2;I_i\rightarrow I_f)=2\langle I_i22\text{-}2|I_f0\rangle^2\{M_1+M_2[I_i(I_i+1)-I_f(I_f+1)]\}^2.$$

M_2 is the contribution to the transition matrix element attributed to the $\Delta K=2$ coupling and is related to the mixing amplitude. Using this relationship one can analyze both absolute intersequence transition strengths and branching ratios of transitions from the quasi-γ band to the ground-state band. For instance, Figure 6 shows the ratio of E2 intensities of transitions from the same even-spin state in the γ-band to two different even-spin states in the yrast band, yielding a ratio of the matrix elements involved. The accuracy of $\Delta K=2$ mixing in reproducing the data is clear. A detailed comparison of the mixing picture and the X(5) model in the γ degree of freedom is underway [14].

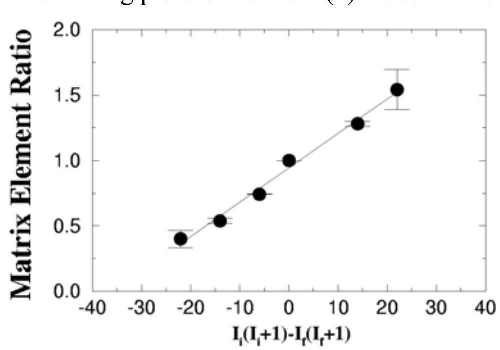

FIGURE 6. Plot of ratios of intersequence matrix elements for transitions from the quasi-γ band to the ground state band in ^{152}Sm as a function of $I_f(I_f+1)-I_i(I_i+1)$. The experimental values are given by closed circles and the straight line fit for the generalized intensity rules is given by the solid line.

A CRITICAL-POINT DESCRIPTION IN THE PAIRING PHASE

It may be possible to apply the idea of critical-point approximations to other transitional systems. Of particular interest is the transition from pairing vibrations to pairing rotations as seen in the isotopes of Sn and Pb [19]. Bes and collaborators developed a collective pairing Hamiltonian in direct analogy with the Bohr collective Hamiltonian [20]. It may be possible to use a simple approximation of the potential of the deformation of the pair field at the critical-point to gain insight into the transition from one pairing regime to the other. Presumably, separation energies and pair transfer strengths can be calculated and tested against the predictions of such a picture [21].

ACKNOWLEDGMENTS

This work was supported by the U.S. DOE under contract No. DE-AC03-76SF00098.

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