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Authors

Zahner, William

Velazquez, Griselda

Moschkovich, Judit

et al.

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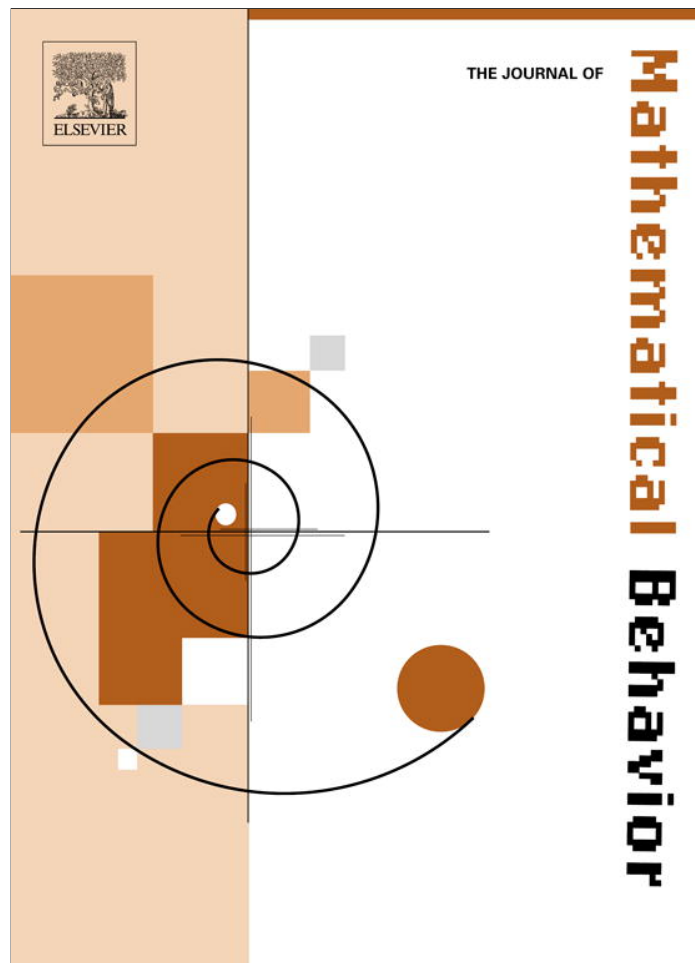
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Mathematics teaching practices with technology that support conceptual understanding for Latino/a students

William Zahner^{a,*}, Griselda Velazquez^b, Judit Moschkovich^b, Phil Vahey^c,
Teresa Lara-Meloy^c

^a Boston University School of Education, 2 Silber Way, Boston, MA 02215, United States

^b Education Department, University of California, Santa Cruz, 1156 High Street, Santa Cruz, CA 95064, United States

^c SRI International, 333 Ravenswood Avenue, Menlo Park, CA 94025-3453, United States

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ABSTRACT

We analyze how three seventh grade mathematics teachers from a majority Latino/a, linguistically diverse region of Texas taught the same lesson on interpreting graphs of motion as part of the Scaling Up SimCalc study (Roschelle et al., 2010). The students of two of the teachers made strong learning gains as measured by a curriculum-aligned assessment, while the students of the third teacher were less successful. To investigate these different outcomes, we compare the teaching practices in each classroom, focusing on the teachers' use of class time and instructional format, their use of mathematical discourse practices in whole-class discussions, and their responses to student contributions. We show that the more successful teachers allowed time for students to use the curriculum and software and discuss it with peers, that they used formal mathematical discourse along with less formal language, and that they responded to student errors using higher-level moves. We conclude by discussing implications for teachers and mathematics educators, with special attention to issues related to the mathematics education of Latinos/as.

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1. Introduction

Due to persistent patterns of inequitable outcomes, the state of education for Latinos/as in the United States has been characterized as one of crisis (Gándara & Contreras, 2009). The mathematics achievement of Latinos/as is one central area of concern. When examining the mathematics education of Latinos/as, issues of language are consistently near the forefront of the discussion (Khisty, 1995; Moschkovich, 1999). Nationwide, 10% of students in the US are classified as English Learners, and in states like Texas and California, English Learners make up 15–25% of the student population, with much higher concentrations in particular schools and districts (California Department of Education, 2008; Texas Education Agency, 2011). Not all students who are English Learners are Latino/a, and not all Latinos/as are English Learners. However, of English Learners in the US, over 70% are Latinos/as whose first language is Spanish (National Center for Educational Statistics, 2011). Moreover, many US-born Latino/a students who speak English fluently still participate in linguistically diverse communities, and their opportunities to learn are shaped by folk theories about language, and teaching language minority students (Khisty, 1995; Moschkovich, 1999).

At the same time that the mathematics education of Latinos/as has become a major issue in the US, mathematics educators have highlighted the centrality of language and classroom discourse as mediators of student learning (Forman, 1996;

* Corresponding author. Tel.: +1 617 358 5330.
E-mail address: wzahner@bu.edu (W. Zahner).

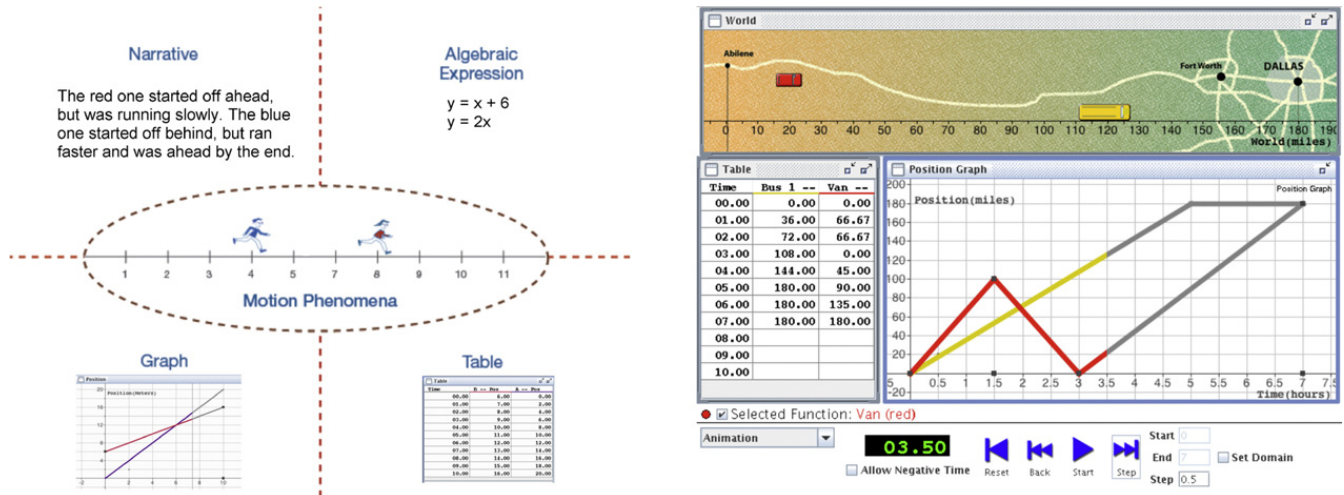


Fig. 1. SimCalc linked representations (left); SimCalc MathWorlds® activity screenshot (right).

Moschkovich, 2007, 2010; O'Connor, 1998, 2001). In this perspective, learning mathematics is inextricably tied with learning to participate in mathematical discourse practices. The ascendance of this perspective is evidenced in both the research literature (e.g., Moschkovich, 2010; Stein, Engle, Smith, & Hughes, 2008), as well as in the proliferation of professional development materials focused on developing students' participation in classroom mathematical discussions (e.g., Chapin, O'Connor, & Anderson, 2009; Smith & Stein, 2011). These demographic and curricular trends present a challenge: how do educators use language and develop the use of mathematical discourse practices in classrooms with students from linguistically diverse communities?

In this paper, we examine the teaching practices of three seventh grade mathematics teachers in classrooms with majority Latino/a¹ students located in communities characterized by linguistically diversity. All three teachers taught the same lesson on interpreting piecewise linear functions representing motion using SimCalc MathWorlds® and associated curriculum materials. We analyze two instances of mathematics teaching practices and classroom discussions that supported the development of Latino/a students' conceptual understanding. A third teacher's class provides a contrast case, where his commonsense instructional decisions may have inadvertently constrained the mathematics learning of his students.

These three cases illustrate generalizable teaching practices that can promote or constrain students' conceptual understanding of important mathematical concepts. Moreover, these cases were intentionally drawn from a linguistically diverse area of Texas in order to illuminate how teaching practices with technology might support the development of conceptual understanding for all students, and particularly Latino/a students from linguistically diverse communities. The findings from this study are informative for mathematics educators generally, and also for the growing numbers of mathematics educators who work in linguistically diverse settings.

1.1. Background on Scaling Up SimCalc Study

This investigation grew out of Scaling Up SimCalc, a large-scale, delayed treatment randomized controlled experiment that examined whether positive results with SimCalc could be observed at scale. For more than fifteen years, the SimCalc project has aimed to ensure that all learners have access to complex and important mathematical ideas (Kaput, 1994; Roschelle, Kaput, & Stroup, 2000). To achieve this goal, SimCalc software and curricula place motion phenomena at the center of learning (see Fig. 1), which enables students to build on existing competencies as they develop fluency using mathematics.

In the first year of Scaling Up SimCalc, 95 seventh grade teachers (48 treatment classes and 47 delayed treatment or control classes) across varying regions in Texas participated. Both groups received professional development on teaching rate and proportional reasoning. The treatment teachers implemented a SimCalc-based three-week replacement unit on proportional reasoning and rates, while the control group taught their usual lessons on these topics. Student learning was assessed with a test administered at the start and end of the teachers' units on rate and proportion. The assessment included procedural proportional reasoning problems that typically appear in Texas's seventh-grade state mathematics assessments. It also included conceptually-focused questions extending beyond the standards, and which form the basis of more advanced mathematics.

The experiment showed a large and significant main effect on student learning gains across the pre- and post-assessments for the SimCalc group, with an effect size of 0.8 (Roschelle et al., 2010). This effect was robust across a diverse set of student

¹ We use the term "Latino/a" to be consistent with current usage, although "Hispanic" is the term commonly used in Texas to designate people of Latin American—usually Mexican—descent.

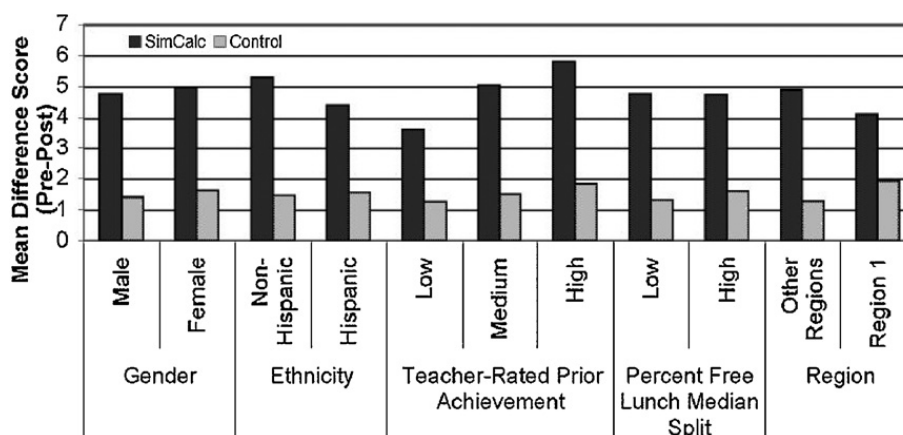


Fig. 2. Mean student-learning gains by subpopulation group.

demographics including gender, ethnicity, teacher-rated prior achievement, and SES (measured by the percentage of the campus eligible for free or reduced price lunch, see Fig. 2). Moreover, much of the advantage for SimCalc was documented on the conceptually focused questions. Consistent with data from other regions of Texas, the majority Latino/a students in the Rio Grande Valley who used SimCalc had greater learning gains than students in the control condition, and these learning gains included improvement on conceptual test content as well as procedural problems. The Rio Grande Valley is labeled Region 1 in Fig. 2.

In this analysis, we use the SimCalc study as a site to investigate teaching practices with technology that supported the conceptual learning of Latinos/as. We base this analysis in the Scaling Up SimClac study for three reasons. First, SimCalc is grounded in a rich theoretical and empirical base of research, so it provides an excellent proxy for representation-rich curriculum and technology more generally. Second, a wide array of qualitative data, including videotaped lessons, field notes, and interviews with the teachers, was collected during the study. Finally, the study included a large number of Latino/a participants (about half of the 1621 total students) from linguistically diverse areas of Texas, so the data from this study can illuminate student learning in classrooms with Latinos/as from linguistically diverse communities. For this comparative analysis, we focus on Latino/a students in the study from the Rio Grande Valley (Region One in Texas's state education system), a linguistically diverse region of Texas bordering Mexico.

1.2. Question for this study

Our central goal is to examine how three seventh grade mathematics teachers engaged in teaching practices that supported the development of their Latino/a students' conceptual understanding. We follow up a previous correlational study of classroom discourse in SimCalc classes (Pierson, 2008) by providing more detailed descriptions of teaching practices that were productive (or not) for developing students' conceptual understanding. The research questions guiding this investigation are:

1. How did the three teachers structure the same lesson on interpreting graphs of motion, and what were the affordances of different ways of structuring the lesson?
2. How did the three teachers use mathematical discourse practices and whole class discussions to promote students' conceptual understanding in their linguistically diverse classes?

2. Framework and literature

2.1. Theoretical framework

We take a socio-constructivist theoretical approach, which assumes that students learn mathematics and develop conceptual understanding through interacting with material and technological artifacts while participating in discourses (Forman, 1996; Kaput, 1991; Moschkovich, 2004). In this approach, mathematical discourse is multi-modal and multi-semiotic (O'Halloran, 2000; Schleppegrell, 2007), and it includes not only talk, but also other communicative tools such as written inscriptions or computer animations. While we focus primarily on verbal communication, this theoretical orientation is reflected in our data selection and analysis. For example, to capture the multi-semiotic character of mathematical discourse, we used transcripts of classroom talk in conjunction with video recordings to analyze classroom discussions.

2.2. Framing the learning of Latino/a students

Framing research on Latino/a students is complex due to the diversity of Latinos/as in the US, and to the lack of reliable data about Latino/a students. The Latino/a student population in the US is heterogeneous—in some geographical areas this population includes students from many countries who speak many languages (Moschkovich, 1999). Within our relatively constrained population of students from three seventh grade classrooms in the Rio Grand Valley, almost all students were labeled “Hispanic” and had roots in the US-Mexico border region. We do not have an exact account of how many students in these classrooms were learning English, and the publicly available data about the number of bilingual students and English Learners in the schools appears unreliable.² However, from observation notes and analysis of classroom discourse, we note that students in these classrooms, like students in this geographic area in general, varied in how they used Spanish and English along a spectrum from monolingual to bilingual modes. We also know that the Rio Grande Valley is a linguistically diverse region where both Spanish and English are commonly used (United States Census Bureau, 2008). In light of the school-level data, Census data, and researcher observations, we characterize the three classes in this study as “linguistically diverse,” and we examine the practices of these three teachers to investigate teaching practices that supported learning conceptual mathematics for Latino/a students from a region characterized by linguistic diversity.

2.3. Conceptual understanding and the role of classroom discussions in linguistically diverse mathematics classrooms

The development of students' conceptual understanding (Hiebert & Carpenter, 1992) is one central goal of mathematics instruction. Hiebert and Grouws's review (2007) found that mathematics teaching that promotes students' conceptual understanding has two central features: 1) teachers and students explicitly attend to mathematical concepts, and 2) students wrestle with and make connections among important mathematical ideas. All teachers face a considerable challenge in balancing both of these features in their teaching, and prior research reveals that teachers in schools with high numbers of Latinos/as and language minority students often focus on procedural content (Gándara & Contreras, 2009). In contrast, recommendations for effective environments for students from non-dominant linguistic backgrounds emphasize that instruction should provide “abundant and diverse opportunities for speaking, listening, reading, and writing” and “encourage students to take risks, construct meaning, and seek reinterpretations of knowledge” (Garcia & Gonzalez, 1995, p. 424).

One way to engage students in attending to concepts and making connections between important ideas is through engaging mathematical discussions (O'Connor, 2001; O'Connor & Michaels, 1993; Stein et al., 2008). As students participate in classroom mathematical discussions, they learn to communicate mathematically by making conjectures, presenting explanations, constructing arguments, and so on. These communicative practices shape student thinking and reasoning. For example, strategic teacher discourse moves such as revoicing and building on student contributions have been shown to engender student learning in discourse-intensive classrooms (O'Connor & Michaels, 1993).

However, the use of classroom mathematical discussions is in tension with commonsense, or “folk theory,” strategies for teaching mathematics in linguistically diverse settings where some students may be learning the language of instruction. These strategies often reduce the role of language in mathematics classrooms to addressing vocabulary through direct instruction, using realia, rehearsal, drill and practice, or memorization (Moschkovich, 1999). In contrast to these strategies, research shows that academic discourse is more than vocabulary, and that mathematical discourse is most successfully learned through participating in instructional environments that are language-rich, actively involve students in using language, require both receptive and expressive understanding, and require students to use words in multiple ways over extended periods of time (Blachowicz & Fisher, 2000).

In the particular case of mathematics, bilingual students and students from linguistically diverse backgrounds need opportunities to learn the language of mathematics by participating in mathematical discourse practices such as describing patterns, making generalizations, and using representations to support mathematical claims (Moschkovich, 2002, 2007). For example, Moschkovich (2002) illustrates how bilingual students drew upon multiple resources, including their first language, multiple semiotic tools, and metaphors from everyday talk to create an argument comparing the slope of two linear functions. In a similar vein, Khisty and Chval (2002) showed how a successful teacher in a linguistically diverse fifth grade classroom engaged her students in high levels of mathematical reasoning by using a mixture of everyday and academic language as well as challenging problems that required the students to talk through complicated calculations. These studies and others (e.g., Brenner, 1994) show how discussions can be a very useful tool for developing mathematical understanding among linguistically diverse groups of students.

² Students' language proficiency was not formally assessed in the Scaling Up SimCalc study. Defining which students are bilingual or English Learners is a complex, and contested, task (see, for example, Solano-Flores, 2008). We claim the publicly available data are unreliable because the reported number of English Learners and bilingual students in each school varies depending on the source. For example, the U.S. Department of Education database indicates there are no English Learners at one of the focal schools, while the Texas Education Agency reports 21% of the students are enrolled in bilingual education or English as a Second Language courses. Due to these inconsistencies we refer to these schools as linguistically diverse, reflecting the overall population profile from the US Census Bureau. The linguistic diversity of the classes is also evident in the field notes and video records.

Table 1
Summary data for each teacher.

	Teacher N	Teacher E	Teacher M
Mean class gain score ^a	2.00	5.26	5.63
Intellectual work ^b	0.183	0.409	0.333
Responsiveness ^c	0.691	0.967	0.846
Percent Latino students in class	95%	96%	100%
School free/reduced lunch (% in school)	93.9%	94.3%	84.3%
Pages of workbook completed (out of 59)	54	51	53
Days on unit	15	11	12

^a The gain scores were computed based on pre- and post-assessments administered as part of the Scaling Up SimCalc Study (Roschelle et al., 2010). We focus on the sample of 13 teachers in Pierson (2008) out of the 48 treatment teachers (mean = 5.087, SD = 2.052).

^b Intellectual Work was a measure created by Pierson (2008) for a sample of 13 teachers out of the 48 treatment teachers (mean = 0.3302, SD = 0.0823).

^c Responsiveness was a measure created by Pierson (2008) for a sample of 13 teachers out of the 48 treatment teachers (mean = 0.9188, SD = 0.181).

2.4. Content focus: rates and functional representations

The seventh grade unit used in the Scaling Up SimCalc study focused on rates, proportionality, and coordinating multiple representations of functional relationships involving change. The unit was developed based on an extensive body of mathematics education research related to student learning of rate and proportionality, and how students interpret graphical representations of functional relationships (Leinhardt, Zaslavsky, & Stein, 1990; Roschelle et al., 2000). The focal lesson that we analyze here involved interpreting graphs of piecewise linear functions showing distance over time. Prior research has shown that interpreting horizontal segments and segments with negative slopes can be particularly challenging for many students (Leinhardt et al., 1990; Robutti & Ferrara, 2002).

2.5. Examining mathematics teaching practices

There are a wide variety of frameworks for analyzing mathematics teachers, and teaching. For example, some researchers examine the teachers' mathematical knowledge (e.g., Hill, Rowan, & Ball, 2005), or how teachers implement reform-based mathematics tasks (e.g., Henningsen & Stein, 1997). Some analyses from the larger Scaling Up SimCalc study collected data and built on these frameworks (e.g., Roschelle et al., 2010; Sechtman, Roschelle, Haertel, & Knudsen, 2010). For this paper we focus on how three teachers set up and taught the same lesson to meet the needs of their students. In light of this focus, we build on the work of mathematics education researchers who have examined classroom discourse and lesson structure, with an emphasis on work that has examined the mathematics learning of linguistically diverse students (Brenner, 1994; Khisty, 1995; Khisty & Chval, 2002; Moschkovich, 1999). From these researchers, we focus on how the teachers created opportunities for linguistically diverse students to engage in mathematical reasoning, and provided (or restricted) access to conceptually focused mathematics. Also, because we are using a comparative design, we draw on work by Stigler, Fernandez, and Yoshida (1996) and Khisty and Chval (2002) who used comparative analyses to highlight differences in mathematics lessons across contexts. In particular, we adopt from Stigler et al. (1996) a focus on the teachers' use of class time and their response to student errors.

3. Data and methods

3.1. The teachers and students

The primary data for this study are video recordings of three teachers (who we call Teacher N, Teacher E, and Teacher M) teaching the same lesson from the Scaling Up SimCalc seventh grade curriculum. A total of thirteen treatment group teachers from across Texas were video recorded teaching the same lesson. We selected the recordings of Teachers N, E, and M for careful analysis because all three were from the Rio Grande Valley and had similar student demographics. Over 95% of the students in each class were Latino/a, and in each of the three schools, over 85% of the students were eligible for free or reduced-price lunch. As noted above, the school level data about the presence of bilingual students and English Learners appears unreliable. However, census data indicate that a language other than English was spoken in over 70% of the households in the region (United States Census Bureau, 2008), and the observations and field notes provide further evidence of linguistic diversity in these classrooms.

In addition to the demographics, we selected these three classes because they show a contrast in terms of student gains on the curriculum-aligned assessment. Students in the classes of Teachers E and M had gains between the pre- and post-assessment that were greater than the mean gains for all classes in the study (5.26 and 5.63 respectively), while students in the third class (Teacher N's class) had a mean gain of 2.00, which was lower than the mean gain for all classes in the study (see Table 1). The two teachers with higher student gain scores also had high scores on measures of discourse developed by Pierson (2008): "intellectual work" and "responsiveness." Table 1 contains a summary of these data.

We include Teacher N in this analysis because many of his instructional practices aligned with common sense ideas for teaching linguistically diverse students. For example, we show in Section 4.3 that Teacher N used mathematically accurate

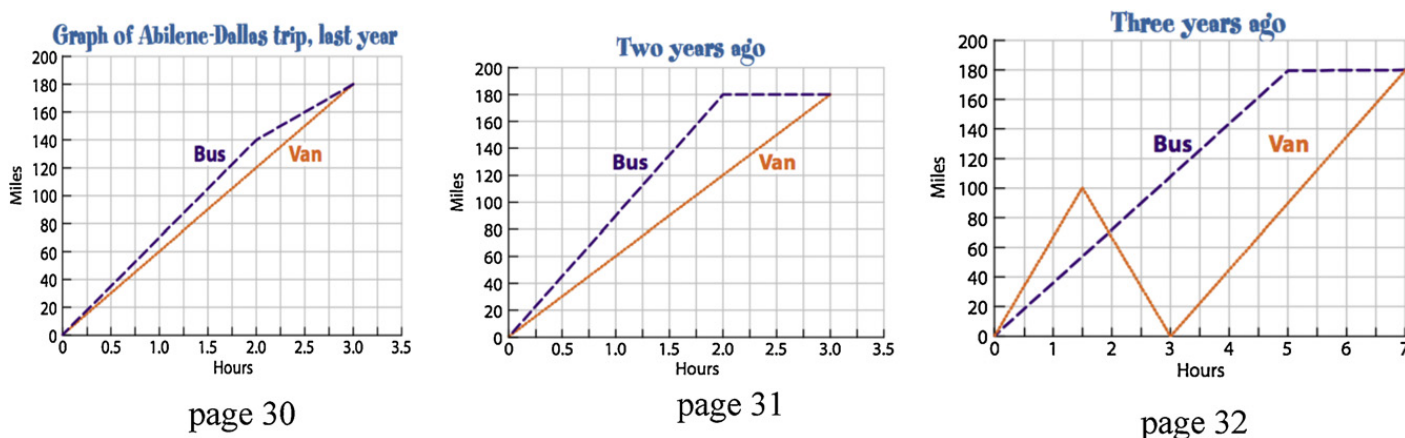


Fig. 3. The three graphs discussed in *On the Road*.

vocabulary to describe a graph of motion. Modeling the use of correct terminology is a commonsense way to help students who may be learning the language of instruction as well as mathematical concepts. However, we show that this instructional decision may have limited Teacher N's students' opportunities to learn because, although he used correct terminology, he did not provide space for his students to construct stories about motion on their own. Our hope is that these contrasting cases can be illuminating for both teachers and researchers.

3.2. The focal lesson: *On the Road*

On the Road consisted of four pages (pp. 29–32) from a 59-page student workbook that was used along with the SimCalc software for the treatment portion of the study. The lesson focused on interpreting graphs of piecewise linear functions showing distance over time. Each graph showed two linear functions: one line represented the motion of a bus, while another line represented the motion of a van, each making the same trip. The lesson started with a map that showed a geographical representation of a trip. This map was followed by three distance-time graphs showing the bus and van making the 180-mile trip (see Fig. 3). The three graphs were progressively more complex, and for each graph the students were asked to predict what happened (e.g., What did the van do? What did the bus do?), run a SimCalc simulation to check their predictions, and then write a summary or a story about the trip aligning with the graph. All three graphs showed two piecewise linear functions graphed on the same axes, and the last two graphs included segments with zero and negative slopes. This was a pivotal lesson in the curriculum because *On the Road* contained a review of the major concepts, and it was the first instance in the unit where the students worked with piecewise functions, line segments with a slope of zero, and line segments with negative slope.

3.3. Data and analysis

The data for this study are video and audio records of the three teachers' activity during a single 45–50-min lesson as the teacher taught all or most of *On the Road*. Each lesson was recorded with a camera focused on the teacher and the projection screen or whiteboard. All three teachers taught this lesson in a computer laboratory, so the students were sitting at computers or at a table in the middle of the laboratory. The video recordings were transcribed to record public talk during whole class discussions, and throughout the analysis we referred to both the transcripts and the video recordings to understand the lessons. We focus on the teachers' talk, and the teachers' and students' use of the technology and curriculum resources. Our data do not let us investigate aspects of these classrooms that took an extended time to develop, such as teacher beliefs or the development of classroom norms. We also could not focus on the students' talk with peers because it was not fully captured in the video records.

We first partitioned the videos into segments based on which questions in the unit the students worked on, and shifts in class format (e.g., transitioning between whole class teacher-led discussion, small group discussions, and individual work time). While the curriculum materials influenced the partitioning, the segments were primarily determined by the particular interactions within each class. Since the teachers put different emphases on different parts of the lesson, the lengths of each segment and the total number of segments varied by teacher.

We then conducted several analyses to describe teaching practices for each teacher and to compare practices across the three teachers. First, following Stigler et al. (1996), we analyzed how the teachers used class time and structured their class period while teaching the same lesson. Next, we examined how each teacher engaged in mathematical discourse practices that support students' conceptual learning in mathematics (Moschkovich, 2007). Specifically, we examined how the teachers introduced and used content-specific academic language, how they conducted students' sharing of stories and justifications, and how each teacher responded to student errors.

We present several excerpts of classroom interaction to illustrate how these teachers led mathematical discussions. We also show that these excerpts are representative of each teacher's practices for the entirety of the recorded lesson by providing summaries of certain teaching practices across all whole-class discussions in each class. To make direct comparisons across the three teachers, we also compare excerpts from the equivalent segments, that is, segments where the teachers and students were discussing the same problem in the workbook. This side-by-side comparison illustrates differences in how these teachers confronted similar content-related issues that arose in the lesson.

4. Findings

4.1. Overview of lessons—similarities and differences

The primary similarity among all three classes is that the sequence of questions discussed in each teacher's class was similar. This similarity was not surprising since, by design, the teachers were all teaching the same lesson from the same curriculum unit. All three teachers taught all or part of *On the Road* during a 45–50-min class period and all three teachers discussed the three graphs on pages 30–32 of the workbook (see Fig. 3). A second similarity is that all three teachers allocated some time for whole class discussion, and some time for the students to work alone or in groups while using computers running the SimCalc simulation/software.

The two major differences in how each teacher used a single class period to teach this lesson were 1) the pace of the lessons (i.e., how much content from the workbook each teacher attempted to cover), and 2) the students' access to the computers running SimCalc. In terms of pacing, Teacher N moved the fastest and he discussed all three graphs from *On the Road* in about 30 min. He then had his students complete the next section in the workbook called *Road Trip Records* during the same class period. Teacher M taught only *On the Road*, and she attempted to complete just that section during the single class meeting. Finally, Teacher E moved the slowest, and she only discussed part of *On the Road*, skipping the preliminary discussion of the map representation.

In terms of using the SimCalc software, the students in Teacher N's class did not have access to computers while discussing the three graphs from *On the Road*. Instead, Teacher N ran the simulations on his computer while his students sat at a table in the middle of a computer lab with their workbooks. In contrast, in Teacher E and Teacher M's classes, the students were seated at computers running the SimCalc simulations throughout the whole class period, and they also had access to their workbooks with the discussion questions.

4.2. Instructional format and sequencing

Following Stigler et al. (1996), we analyzed how the teachers structured their lessons, with a focus on how public, teacher led-discussions were alternated with time for the students to work individually or in groups. Stigler et al. used this focus on lesson structure to compare how Japanese teachers and their US counterparts created opportunities for their students to think about mathematics and construct meanings. Researchers who focus on the development of mathematical discourse practices among linguistically diverse students have emphasized that bilingual students and students learning the language of instruction need opportunities to practice using the discourse of school mathematics (Moschkovich, 2002). However, some teachers of students with varied language backgrounds focus on modeling the correct use of mathematical terminology without creating opportunities for students to practice using mathematical discourse (Moschkovich, 1999). Therefore, we use an analysis similar to Stigler et al.'s to examine how these three teachers created opportunities for their students to think and to practice using mathematical language.

We define "public teacher-led discussions" as all time periods in the lesson when all students were expected to listen to the same speaker, whether it was the teacher lecturing, or other students speaking. We found suggestive differences in how the three teachers divided class time between the instructional formats of whole-class teacher led discussions and individual/group work with the software. These differences in the use of class time created different opportunities for the students to talk and refine their ideas while using the dynamically linked representations in the software.

As shown in Fig. 4, Teacher N presented the four problems of *On The Road* as a continuous block of public teacher-led discussion lasting 31:25. For the final block of class time, Teacher N's students worked individually at their computers to complete the next section in their workbooks. Teacher E broke her lesson into segments, and she alternated public teacher-led discussions with time for her students to run the simulation at their computers or work with partners developing a story for each graph. For example Teacher E introduced the graph on page 31 ("Two Years Ago") in two minutes of public discussion where she asked the students to focus on the distance covered by each vehicle and to explain the meaning of the horizontal line in the graph. Next, Teacher E allowed approximately 1:30 for the students to run the simulation on their computers to check their predictions. This individual work time was then followed by more whole-class discussion about the graph. Like Teacher E, Teacher M also alternated whole class teacher-led public discussion and time for her students to work individually or in pairs at their computers, but she used more and shorter alternations between these two instructional formats.

The fact that Teacher N completed more pages from the workbook in the same amount of time as the other two teachers indicates that for this lesson, Teacher N moved faster through the content than did Teacher E and Teacher M. Looking

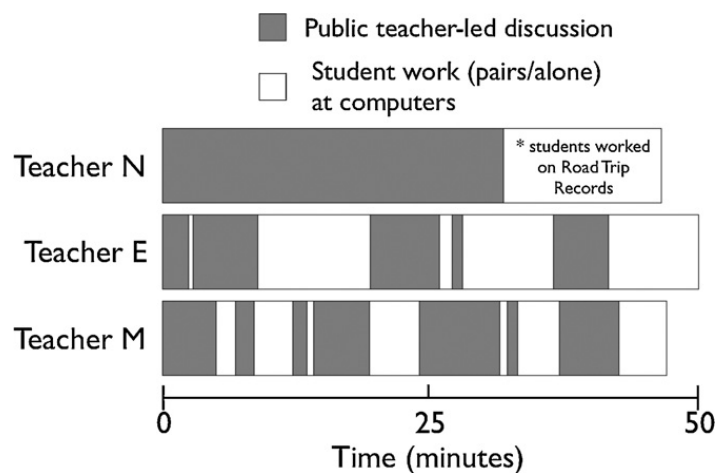


Fig. 4. A graphical comparison of the three teachers' lessons for *On the Road*.

at Fig. 4, it appears that the primary reason for this difference is because Teachers E and M provided time for their students to work with the software and talk to classmates as they created stories to go with the graphs. Removing the white space in Fig. 4, all three teachers spent about the same amount of time discussing *On the Road* in whole-class format. This difference suggests that for these students, opportunities to talk and use the software may have shaped their learning of the content.

4.3. Mathematical discourse practices

Mathematical discourse practices include a variety of ways of talking mathematically that are distinct from other ways of talking. The use of precise, succinct, and elegant explanations is one example of a discourse practice valued in academic and school mathematics (Moschkovich, 2007). Other discourse practices valued in mathematics classrooms are explaining, justifying, and describing solutions. In the following sections we analyze the teachers' use of mathematical discourse practices by revealing similarities and differences in 1) how these three teachers used content-specific vocabulary and 2) how they structured the sharing of stories about the graphs.

4.3.1. Use of content-specific vocabulary

Developing fluency with content-specific vocabulary is a necessary, but not sufficient, element of developing mathematical discourse practices and learning to use the academic language of mathematics (Moschkovich, 2002). None of the teachers used a vocabulary list or other explicit vocabulary instruction methods during their teaching of *On the Road*. Instead, following the SimCalc curriculum guide, they led a discussion of the graphs, and each teacher introduced content-specific vocabulary (e.g., terms like *speed*, *rate*, *slope*, *horizontal*, etc.) during the discussion. Looking simply at word use, all three teachers used nearly the same set of content-related vocabulary during the lesson. The similarities in the teachers' use of vocabulary makes sense: the three teachers were teaching the same lesson from the same curriculum, and all three teachers had participated in the same professional development. The observation that all three teachers used a similar set of terminology in this lesson, but had different student outcomes on the curriculum aligned assessment, reiterates the notion that learning conceptually demanding mathematics requires more than being exposed to technical vocabulary.

While the three teachers used a similar set of content-specific vocabulary during the focal lesson, our analysis revealed one potentially important difference in *how* the teachers introduced new content-related terms. Teachers E and M, whose students had relatively high gain scores, both used informal language as a resource to introduce an important concept, and then used the formal vocabulary term after the idea had been introduced into the discussion. In contrast, Teacher N supplied the formal terminology in talk that appeared less carefully structured to bridge informal and formal ways of reasoning about graphs of motion. We examine two excerpts of classroom talk in detail to highlight this difference.

In Excerpt 1 below, we see how Teacher E introduced the terms *speed* and *constant speed* into the whole-class discussion of the graph "Last Year" (see Fig. 3 for the graph). One of the mathematically salient issues that arises when discussing this graph is that the bus traveled at two speeds—70 mi/h for the first two hours, and 40 mi/h for the final hour—but arrived at the same time as the van, which traveled at a constant rate of 60 mi/h. In Excerpt 1, Teacher E focused the speed of the bus as she introduced the conceptually-laden term *constant speed*. Prior to the start of this excerpt, one of the students made the observation that the bus was "going faster" at the beginning and then "slowed down."

Excerpt 1. Teacher E introduces the phrase “constant speed”³

36. T: How fast was the bus going?
 37. S: One second.
 38. T: Ok we're going hours and miles. Miles and hours.
 39. S: Sixty miles per hour.
 40. T: Ok how did you determine that?
 41. Same S: Divided the one hundred and eighty miles by three hours.
 42. S: Yeah that's what I got.
 43. T: Ok if
 I divide the final distance divided by the final time. Ok that's gonna give me a speed. You're right.
But you told me first he was going fast and then he slowed down. So was this a constant speed?
 44. Mult SS: No!
 45. T: No. Ok so I need to determine the speed to see how fast he was going before he started
 slowing down. Ok?

This excerpt illustrates how Teacher E used student contributions to introduce technical terminology. She first asked how fast the bus was traveling, and one student answered with the bus's average speed (i.e., total distance divided by total time). Next, Teacher E asked the students to reconcile the single speed of sixty miles per hour with the already-stated observation that the bus “was going fast and then he slowed down” (see the underlined part of line 43 in Excerpt 1). With this move, Teacher E used less formal terminology (even referring to the bus as *he*) and related her question to the students' previous contributions before she introduced the more technical term “constant speed.” The contrastive phrases “going fast” and “slowing down” helped set the stage as Teacher E introduced the more technical term “constant speed.”

In Excerpt 2, we see how Teacher N introduced “constant speed” during a whole-class discussion of the same graph. This discussion appears less focused because Teacher N was asking questions about both the van's motion and the bus's motion at the same time.

Excerpt 2. Teacher N introduces the phrase “constant speed”

73. T: Now, Vin ((student name)), you were telling me something very (early). So, the bus was what?
 74. Vin: The bus was winning, but the-
 75. T: The bus was winning, right? So what happened? The first what? Can you see here? This one's
 what? ((Points to the first segment for the bus)) It's steeper than this one, right?
 76. Vin: Yeah.
 77. S: The first two hours.
 78. T: The first two hours. The bus was what?
 79. Vin: Then it goes (ahead), then-
 80. T: Guys. Let me back (right) here. Three hours, right?
 81. S: Yeah.
 82. T: For both. What about the miles? How many miles?
 83. S: One hundred eighty
 84. T: For both, they run ((pause))
 85. S: One hundred and eighty
 86. T: They travel one hundred and eighty. Right? Yes or no? You see that, Jay?
So, one hundred and eighty miles with ((pause)) three hours, right? Now, the problem is that,
 who has the constant speed or constant rate?
 87. S: The ((inaudible word)).
 88. T: The bus or the van.
 89. S: The van. The van.
 90. T: Guys-
 91. S: 'Cuz it goes straight.
 92. T: Very good, sir. It goes straight. So this one is going straight. Remember yesterday, the slope. It
 goes in a straight line. What about this one?
 ((gestures arm in an arc.))
 93. S: How many speeds do you have in this one?
 94. T: One hundred and forty
 95. S: No, how many speeds?
 96. T: Two
 97. S: Two

Comparing Excerpts 1 and 2, both Teachers E and N combined informal and formal mathematical vocabulary as they introduced *constant speed* in these excerpts, but they did so in different ways. First, Teacher N compared the motion of the bus and the motion of the van, while Teacher E focused only on the speed of the bus. Second, Teacher N's use of “constant speed” in line 86 of Excerpt 2 presumed that one vehicle had a constant speed and one did not, while teacher E asked whether the bus traveled at a constant speed.⁴ While both teachers used informal terminology to talk about the different speeds, their emphasis was different. Teacher N focused on which vehicle was “winning” (Excerpt 2 lines 74 and 75) while Teacher E explicitly focused on the fact that the bus was “going fast and then he slowed down” (Excerpt 1, line 43) to compare the

³ In each excerpt, T is the Teacher and S is for student. If a student is named, then we use the student's name throughout the excerpt (actual names were replaced with pseudonyms). Because the camera was focused on the teacher, it was not always clear which students participated in the discussions. When we can identify that the same student participated in multiple exchanges we used “Same S” for the speaker. Uncertain transcription is indicated with single parentheses. Comments within the transcripts are in double parentheses (()). Line numbers in the transcripts come from the original transcripts.

⁴ We are aware of the technical issue that both vehicles traveled at constant rates. We and the teachers interpret the bus's piece-wise function as indicating that it did not maintain a constant speed for the entire three-hour trip, in contrast with the van.

velocity of the bus in the first two hours and the bus's velocity in the final hour. Teacher N's metaphor of winning, which entails both characters, may have detracted from the goal of comparing the two slopes in the bus's graph, which he seemed to want his students to see by line 97 of Excerpt 2.

4.3.2. Use of stories and justifications

One recurring question in the workbook was to write a story or a paragraph to describe the motion of the bus and the van for each graph (see Fig. 3). All three teachers asked their students to answer these questions, but the way that they set up and facilitated the creation and sharing of stories differed. Teacher N largely wrote the stories for his students, talking through the key details in a block of whole class discussion. Meanwhile, both Teachers E and M allowed time for their students to create stories while working independently with SimCalc, and then facilitated a whole class share-out. To illustrate this difference we focus on how each teacher shared stories about the graph "Last Year" (the same graph discussed in Excerpts 1 and 2).

In creating a story to go along with the graph "Last Year," Teacher N talked through each part of the graph during a period of whole class discussion. Continuing the pattern of talk in Excerpt 2, he solicited student's input using several "fill in the blank" questions. However, Teacher N largely ignored student contributions when they did not match his story. Ultimately, he generated the story for the students and wrote a summary in a projected workbook. By the end of the 1:15 segment, Teacher N had written "Van \Rightarrow constant speed of 60 mph. Bus \Rightarrow First two hours at 70 mph last hour at 40 mph" in the projected workbook. In addition to doing much of the intellectual work of creating a story, Teacher N consistently ignored incorrect student input while responding to correct student contributions. In Section 4.4 below we examine his responses to incorrect student contributions in more detail.

Teacher E structured the writing and sharing of stories very differently. She started with a whole-class introduction to the graph (as shown in Excerpt 1 above), gave her students time to work independently and with partners to generate a story, and then finally she called on individual students to share their story with the class. During the time period that students developed their stories, Teacher E circulated among the groups and helped students refine their stories. One interesting detail in the way Teacher E structured the whole-class sharing is that she provided little explicit feedback on the quality of the students' stories, even though some students provided more complete explanations. It is possible, however, that she did provide this feedback during the small group work time.

Finally, Teacher M's approach to the students' stories for the graph was different from both Teacher N's approach and Teacher E's approach. Teacher M first introduced the graph; then she allowed the students to run the simulation independently. Finally, she gave her students time to write the story individually or in pairs. One difference between Teacher E and Teacher M's sharing is that Teacher M pressed the students who shared stories to reconcile the details of their stories with the mathematical features of the graph.

In the following excerpt, Teacher M called on a student to share his story and demanded that he justify how his unusual story about a monster eating the bus driver could align with the graph showing the bus's distance over time. Teacher M's choice to focus on the mathematical issue in this student's fantastical story, rather than the questionable content about shooting the monster with an emergency shotgun(!), also shows how she maintained a focus on the mathematical issues at play. Teacher M thus used two mathematical discourse practices that are important when leading whole-class discussions, a) requiring a justification for a claim and b) focusing on the mathematical features of a "real-life" situation (while ignoring distracting details like monsters).

Excerpt 3. Teacher M responds to a student's story for the graph last year

221. T: Everybody listen to David's story. Go ahead David.
222. David: Alright. The bus driver is going seventy miles per hour. But then a monster got in the way and the bus driver had to slow down to forty miles per hour. And, then he shot him with an emergency shotgun, ((student laughter)) but not before the monster ate him. But at the end they played in memory of him. But I had to take over and drive the bus-
223. T: ((interrupting)) But the bus ((pause)) was already there, if I'm interpreting correctly, you already said that they got there and that's when the monster attacked him?
224. David: No! ((inaudible words))
225. T: Read it again. Go ahead.
226. David: The bus was going seventy miles per hour. But then all of a sudden, a monster gets in the way grrrrr ((he makes a monster growl)).
227. T: And it slowed down?
228. David: Yeah. And the bus, it slowed down, he got scared, and took the emergency shotgun and shot him but not not-
229. T: While he was driving?
230. David: Yeah, while he was- like this ((pretends to shoot a gun, students laugh)).
231. T: Okay. That's kind of scary.
232. David: But not before the monster ate him. So I had to take over and drive the bus. And the game is played in memory of him ((inaudible, sounds like he reads an imagined name and date of birth and death for the bus driver)).
233. T: But okay, in the real world, okay, if they eat something, how is it staying constant? ((gestures to the graph projected on the board)) How does it stay constant? ((points to the graph of the bus)) Forty miles per hour. Unless you were there while he was being eaten, and you put your foot, and you stayed steady. Understand?

234. David: Yeah
 235. T: So maybe if you would have arrived in Dallas ((points to David)) and then the monster could have eaten the bus driver.

The way that these teachers structured the sharing of stories illuminates the tension at the heart of this analysis: some teachers of students from varied language backgrounds are tempted to “teach by telling” and to model the use of correct terminology. This was the approach used by Teacher N. In contrast, prior research suggests that students in linguistically diverse classes also need opportunities to practice using the language of mathematics and to receive feedback (Khisty, 1995; Khisty & Chval, 2002; Moschkovich, 1999, 2002). This was the approach taken by Teachers E and M. Both teachers allowed time for their students to work in small groups, use the simulation, and create stories. The then provided feedback, either individually (as Teacher E did) or in a whole-class discussion (as Teacher M did).

4.4. Responsiveness in detail: how did each teacher react to student errors?

In a prior study based on videos of 13 teachers teaching *On the Road*, Pierson (2008) defined a measure of the level of responsiveness in the teachers’ talk. This measure captures the extent to which teachers take up and respond to student contributions. Pierson documented different levels of responsiveness to student contributions among the 13 teachers, and she correlated teacher responsiveness with student scores on the assessment. In this study we follow up on Pierson’s quantitative analysis of responsiveness by further investigating how each teacher responded to one of the most challenging moments in a class discussion: student errors.

How teachers ought to react to students’ errors has been a focus of work on facilitating classroom discussions (Smith & Stein, 2011). In their international comparison of mathematics teaching, Stigler et al. (1996) found that American and Japanese mathematics teachers oriented differently to student errors. While errors are usually ignored or downplayed in American classrooms, in Japanese classrooms, student errors were often treated as opportunities for investigation and public discussion. Stigler and colleagues note that errors can be useful sources of discussion.

In these three cases of teachers teaching the same lesson, we have an opportunity to investigate how each teacher responded to similar errors during a whole class discussion. This analysis helps illuminate how the teachers used their students’ contributions as a resource (or not) for furthering mathematical discussions. Prior work on the use of mathematical discourse in classrooms with linguistically diverse groups of students has examined how teachers can build on students’ contributions, even when the grammatical form of the student contribution is nonstandard (Moschkovich, 1999). Here we extend this type of analysis, but we use the comparative design to shift our focus to how each teacher used the mathematical contributions of students in his or her classroom with a linguistically diverse group of students.

We illustrate response to student errors by examining how each teacher introduced the graph “Two Years Ago” from page 32 of the student workbook. To interpret this graph and predict what happened (question 3a in the workbook), the students had to identify the meaning of the horizontal segment in the graph. This was the first time horizontal segments in distance–time graphs were introduced in the seventh grade SimCalc unit. Previous research has documented that students often struggle to interpret horizontal segments in distance time graphs, especially the first time they see such a segment (Leinhardt et al., 1990; Robutti & Ferrara, 2002). One common erroneous interpretation is that the horizontal segment represents constant motion rather than no movement. Therefore, it is not surprising that students in all three classes needed support to correctly interpret the horizontal segment.

In each of the three classes, the teachers followed the workbook and teacher’s guide and they introduced this graph with the question, “Predict from the graph: Which vehicle arrived first? How long did it take each vehicle to make the trip?” The teachers’ guide indicated that the teachers should ask students what the “flat line” in the graph meant and elicit from the students the response that the bus stayed in the same place for an hour. Each of the three teachers followed these guidelines, but they differed in the way they responded to and took up students’ incorrect contributions.

4.4.1. Teacher N’s response to student errors on question 3a

Teacher N started his whole class discussion of the graph “Two Years Ago” by asking his students to predict what happened based on the graph. Two students focused on which vehicle “won,” but they gave contradictory answers. Teacher N responded to inaccurate student contributions during this discussion in two different ways. He either (a) ignored incorrect answers and attended to correct responses (notice, for example, how he attended only to Ramon’s response in line 206), or (b) he offered immediate negative evaluations after a student contribution that he interpreted as inaccurate (as in line 246). In the following four excerpts, lines marked with the arrow symbol (→) are instances of inaccurate student contributions.

Excerpt 4. Teacher N selects the correct responses

203. T: This one, guys, this one is two years ago ((pointing at a graph)). That (last) one was last year. This one is two years ago. This is what happened, ok? Let me, can you predict something that is gonna that, what is gonna happen in that one, guys?
204. S: → They’re gonna tie. They tied.
205. Ramon: No, the bus is gonna win.
206. T: Listen up. I want to take a look to this graph and then tell me why, ok? So, Ramon, you’re telling me that the bus is gonna win. OK? Why do you say that? ((Another Student raises his hand)) Wait, sir. It’s gonna be your turn next.

- ...
244. T: ((Lines 207–244 omitted for space limitations. During the omitted discussion the teacher uses several questions to establish that the bus arrived at 180 miles after two hours))
Guys, if the bus is taking two hours to get there– Look ((at the point where the bus hits one hundred eighty miles)). Only two hours. This ((points to the horizontal segment of graph)) line right here is very important. That means that this guy's is already here and then–
245. S:→ He's taking a shortcut.
246. T: No. No. He stopped. He stopped here. That's why you have a horizontal line. If he slowed down, it's gonna continue with a line less steeper. Make sense? So, in this case, it's a horizontal line. That means that this guy is already ((pause))
247. S: He already stopped

The form of Teacher N's questions and responses to students follows the traditional template of classroom discourse where the teacher asks known answer questions, solicits student responses, and then evaluates the students' correctness (Cazden, 2001; Mehan, 1979). Moreover, Teacher N's follow up moves generally did not build upon, or directly respond to, students' incorrect responses.

4.4.2. Teacher E's response to student errors on question 3a

Teacher E conducted a very different whole class discussion as she introduced the graph “Two Years Ago.” First, she solicited multiple student contributions, and she repeated all student responses, correct as well as incorrect (see lines 146, 151, and 153 below). In the traditional mode of classroom discourse, repeating a student's answer is one way that teachers signal that a response is incorrect (Cazden, 2001). However, Teacher E did not appear to be doing this. Instead, Teacher E asked other students to agree or disagree with their classmates' claims, a move that promotes the development of classroom discourse community (O'Connor, 2001). This discourse pattern forced the students to participate in deciding which responses were correct or not, rather than solely relying on the authority of the teacher. Excerpt 5 illustrates this pattern.

Excerpt 5. Teacher E repeats incorrect and correct student contributions

144. T: Yes. ok. This happened two years ago. Let's see if we can answer the first question. Which vehicle arrived first? Raise your hand if you think you know the answer. Who arrived first?
Andrea?
145. Andrea: → Um ((Multiple Students in the background saying “Miss, miss, miss!”)) the van.
146. T: The van arrived first?
147. Mult SS: No.
148. S: No the bus did.
149. S: The bus.
150. S: → At the same time.
151. T: At the same time ok. You change your mind. At the same time. They both arrived at the same time? Javier?
152. Javier: No.
153. T: No?

Teacher E used these contributions as a launching point for further discussion. For example, in lines 158 through 172 (see below in Excerpt 6), Teacher E responded to Andrea's incorrect suggestion in line 145 that the van arrived first by referencing the graph and asking guiding questions. In Excerpt 6, we see Teacher E focusing on the fact that the bus arrived in two hours and the van arrived in three hours. After establishing this fact through an extended discussion, one of Teacher E's students volunteered that the horizontal segment represented that the bus had stopped.

Excerpt 6. Teacher E follows up on incorrect student contribution

158. T: Ok. Let's see. Our destination was still the same trip. How many miles did the other trip take?
One hundred eighty right? Ok so the bus is this line. He's at one hundred eighty at what time?
159. S'S: Two hours.
160. T: At the two hours. Ok. How long did it take the van to cover the one hundred eighty miles?
161. Mult SS: Three hours.
162. T: Three hours. Ok. So let's see who arrived first again?
163. Mult SS: The bus.
164. T: The bus ok. ((T writes in the projected workbook)). The bus arrived first. Let's just add in that it took them two hours to get there. Ok it took two hours. ((T says the following aloud as she writes it)) The van arrived an hour later. Very good. How long was the trip for the van?
165. S: Three hours.
166. T: Three hours. Ok so let me just put it there in parenthesis. So I know it took the van three hours.
Good. Looking at this graph what do you think this line going across for the bus means? What do you think that means?
167. S: It stopped.
168. T: That it what?
169. Same S: It stopped. It waited.
170. T: That it stopped very good. It waited for who?
171. S: The van.
172. T: For the van to get there. Ok even though he got there at 2 hours he still waited for the van for that extra hour. So this is representing time is still going on. It goes from two to three. This is still the one hour. But he didn't go anywhere. He just waited there. Ok.

4.4.3. Teacher M's response to student errors on question 3a

Finally, Teacher M responded in a wide variety of ways to her students' inaccurate responses about the graph "Two Years Ago." Like Teacher E, Teacher M also repeated some inaccurate interpretations and asked for more suggestions from other students (see line 269 of Excerpt 7). However, Teacher M also repeated the question (lines 289), used direct contradiction (line 287), and, as Teacher N did, ignored some inaccurate answers while repeating correct answers (lines 278 and 281). In this sense, Teacher M's whole class discussion of the graph "Two Years Ago" appeared more similar to Teacher N's discussion than it was to Teacher E's.

Excerpt 7. Teacher M responds in multiple ways to student errors

261.	T:	Alright, here we go. Page thirty one. "The trip from Abilene to Dallas two years ago went fairly well." Okay, do not log in yet, just look at the graph on your booklet. "Predict from the graph, which vehicle arrived first and how long did it take each vehicle to take the trip?" So, so let's see. Hector. Who arrived first?
262.	Hector:	The bus.
263.	Other SS:	The bus.
264.	T:	The bus. Okay. What does that flat line segment mean?
265.	S: →	A turn
266.	Hector: →	the distance from well
267.	S: →	they turned.
268.	Hector:	No.
269.	T:	Okay, someone said he turned, any more ideas?
270.	S:	Stop.
271.	T:	Ok someone said it stopped or he stopped. Any more ideas?
272.	S:	((makes inaudible comment.))
273.	T:	Okay, don't get in ((open the simulation)), but I'm going to put it up here so you can see.
274.		((teacher projects simulation in front of class))
275.	T:	Alright up here guys. I know some of you can't see, but stand up. Here is what we're talking about. ((Points to flat segment of graph.)) Alexandra and Lina. Lina. How many miles did the bus go in this straight line? ((no answer)). How many miles?
276.	S: →	One.
277.	Mult SS:	None.
278.	T:	None. Right, none, the number zero. Okay? How many hours did he ((the bus)) go no miles?
279.	S:	one hour.
280.	Other S: →	two hours
281.	T:	One hour. So if I wanted to find the speed of the bus, what was the speed of the bus for that segment?
282.	Mult SS:	Zero.
283.	T:	Zero right. Because zero divided by one is zero. So if he went zero miles per hour what does that mean?
284.	S:	He stopped.
285.	T:	That he stopped, right? And what is the slope for zero miles per hour.
286.	David: →	Nothing.
287.	T:	Not nothing, but zero. So kind of make yourselves a note there that this line, ((points to flat segment on graph projected on board)) did go zero miles per hour, but it also has a zero slope, okay. Zero slope and all horizontal lines are going to have zero slope. Okay, remember that. Alright. How long did the bus take to get there?
288.	Mult SS: →	Three hours.
289.	T:	Look at it again.

One interesting note about Excerpt 7 is that Teacher M used the slope calculation as a resource to argue that horizontal lines indicate stopping. In line 287 she repeated that observation and generalized the observation to all horizontal lines.

4.4.4. A wider look at teacher corrections

All three teachers' whole class teacher-led discussions roughly followed the triadic "*Initiation, Response, Evaluation/Follow-up*" pattern that Cazden claims is the default for classroom discourse (Cazden, 2001). But that does not mean that the teachers' talk during whole-class teacher-led discussions was the same. Rather, *how* the teachers responded to student contributions in the Evaluation/Follow up slot differed in several ways. Based on our analysis of the teachers' responses to student errors we saw five primary types of responses to a student's inaccurate answer.

1. Direct contradiction
2. Ignore error and select correct answer (if multiple answers are given)
3. Repeat the question
4. Repeat the inaccurate contribution and solicit further student feedback
5. Ask a clarifying question or give a hint

We classified the first three responses as "low-level" responses to students' errors because they do not require the students to reason through the question. Responses one and two terminate the IRE/F sequence. Response three keeps the sequence alive, but also provides immediate negative evaluation of the prior statement (Mehan, 1979). Repeating inaccurate responses and asking clarifying questions (four and five on the list) keeps the sequence going and engages the students in

Table 2

Summary of each teacher's responses to incorrect student contributions.

	Teacher N	Teacher E	Teacher M
Total incorrect student contributions taken up by teacher	30	13	41
Low level responses			
Direct contradiction/negative evaluation	5	0	3
Ignore and select correct	8	3	7
Repeat question	7	0	6
Subtotal of low-level (%)	20 (67%)	3 (23%)	16 (40%)
High-level responses			
Repeat inaccurate response and solicit more answers	1	8	5
Ask refining question/give hint	9	2	20
Subtotal of high-level (%)	10 (33%)	10 (77%)	25 (60%)

thinking through the problem. Therefore, we classified these as high-level responses to student errors. We are not proposing that these are the only high level responses—pressing for reasoning, or asking another student to respond to an incorrect contribution are two other possible high-level follow up moves (Chapin et al., 2009). However, we do not include those moves in this analysis because none of the teachers used them.

Looking across the whole data set of three transcripts, we identified all inaccurate student contributions in each class's whole-class teacher-led discussions, and we classified the teachers' response to each of these inaccurate responses. Table 2 contains a summary of this analysis.

As seen in Table 2, both Teacher N and Teacher M were most likely to give hints or ask refining questions after an inaccurate student contribution, though Teacher M was more likely to do this (49% of her responses to inaccurate contributions) than was Teacher N (30%). In contrast, Teacher E's most frequent move of repeating inaccurate contributions (over 60% of her responses) stands out among all three teachers. Repeating inaccurate answers and soliciting more feedback may have been one way that Teacher E raised the level of cognitive demand in comparison with the other two teachers. Of course, this discourse move did not stand alone, it was part of a repertoire of strategies for leading whole-class discussions used by Teacher E that also included alternating between whole-class and group format, providing time for the students to use the software and talk to peers, using informal language and formal vocabulary together, and selecting student responses for whole class sharing.

5. Discussion

In this paper, we have analyzed how three teachers enacted the same lesson using SimCalc technology in different ways. Two of the teachers were relatively successful in promoting students' learning of conceptually focused mathematics as measured by curriculum-aligned assessments, while one of the teachers was less successful. Following a model similar to Stigler et al.'s (1996) comparative study of mathematics teaching, we selected particular elements of the teachers' practices for detailed analysis. The major differences we examined were the teachers' pacing and lesson structure, the teachers' use of mathematical discourse practices, and the teachers' responses to student errors. This list of observed differences is not intended to be an exhaustive list of all of the differences between these teachers, but the distinctions we have documented can serve as a starting point for a discussion about teaching conceptually demanding mathematics to linguistically diverse groups of students.

Three major differences that we documented were:

1. The more successful teachers alternated whole-class and individual/group work time and they moved through the lesson more slowly, covering less content and generally creating more space for students to talk and reason through the problems. The two more successful teachers also gave their students access to the SimCalc simulations throughout the lesson.
2. The two more successful teachers introduced new content-specific vocabulary when needed, and built on student contributions to introduce new terminology. They also allowed time for the students to construct stories about each graph. One of the two more successful teachers also focused to the mathematical relationships in the students' stories and pressed for explanation from the students.
3. The two successful teachers were more likely to use higher-level responses to student errors such as repeating inaccurate responses and soliciting more answers, or asking refining questions.

While Teacher E and Teacher M both achieved relatively high student gain scores on the SimCalc assessment, their teaching was different in important ways. For example, Teacher E selected students to share stories about the graphs, while Teacher M called on student volunteers without knowing the content that they would share in advance (which led to some unexpected stories as seen in Excerpt 3).

Our analysis reveals some parallels and tensions between recommendations for teaching cognitively demanding conceptually focused mathematics lessons and recommendations for teaching mathematics to linguistically diverse students. For example, one recommendation for implementing reform mathematics tasks is to maintain a high level of cognitive demand

by requiring students to make sense of the mathematics and engage in non-routine problem solving (Henningsen & Stein, 1997). Likewise, one research-based recommendation for teaching English Learners is that students need the opportunities to engage in content and learn the language of the discipline by using it for authentic communicative purposes (Blachowicz & Fisher, 2000; Moschkovich, 1999). In the data presented here, we saw that both Teachers E and M provided time for their students to engage in the content, produce stories about motion, and use the available resources (both human and technological) to refine their understandings of the mathematics of change. That is, their teaching practices overlapped both sets of recommendations for teaching mathematics.

There are also some tensions between the recommendations for teaching conceptually focused mathematics and commonsense ideas about how to accommodate linguistically diverse students. For example, some teachers attempt to minimize the linguistic demands on their students by adjusting their speech and modeling the correct use of technical terminology (Moschkovich, 1999). However, the effect of reducing the linguistic demand too much may be that the mathematical cognitive demand is reduced as well. This tension is highlighted the way that Teacher N reduced the cognitive demand of the story generation tasks by talking through the stories himself in a whole class discussion. His pedagogical choice reduced the demands of the task, and also removed an opportunity for his students to use (and build) academic language through creating and sharing stories. Meanwhile, Teachers E and M both allowed their students time to work independently or with a classmate to develop a story (with the support of the SimCalc simulation), and then these teachers asked students to share the stories they had generated in a whole class setting. These examples illustrate how the two issues—cognitive demand and opportunities to use and produce domain-specific language—are intimately related.

The results of Scaling Up SimCalc indicate that the benefits of representation-rich curriculum can be experienced by all students at scale. The teaching practices highlighted here illustrate some of the components of the SimCalc curriculum that may have engendered students' conceptual development in a linguistically diverse setting. By focusing on the learning of Latino/a students from a linguistically diverse region of Texas, we have attempted to highlight the overlap between "best practices" for using mathematical discussions with technology in general, and "best practices" for teaching linguistically diverse groups of students and students learning the language of instruction.

6. Conclusion

In this paper we have contrasted the teaching of three teachers who used SimCalc to teach students in majority Latino/a classrooms in a linguistically diverse region in Texas. We showed that the two more successful teachers moved more slowly through the content while allowing time for their students to interact with the software and their peers. They also introduced technical vocabulary after discussing concepts with less formal language, and they used higher-level responses to student errors.

Our analysis shows that while some teaching practices were common across the three teachers, other teaching practices differed substantially. Since some of the teaching practices enacted by the two more successful teachers differed (e.g., how the teachers responded to student errors and solicited student's stories about graphs of motion), we are not proposing that one single aspect of the teachers' practices can account for the differences in student learning outcomes. The analyses and examples show that there is no single way to enact best practices and no simple formula for teaching conceptually demanding mathematics in linguistically diverse settings. Instead, there are multiple ways that teachers can instantiate best practices so that all students can learn conceptually demanding mathematics.

The findings presented here and in the larger study (Roschelle et al., 2010) illustrate the potential for using research-based technology to teach conceptually demanding mathematics to Latino/a students from linguistically diverse communities. This finding paints a hopeful picture for scaling up mathematics instruction focused on conceptual understanding for all students.

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