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DECK MODEL FOR TRIPLE-REGGE COUPLINGS

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DECK MODEL FOR TRIPLE-REGGE COUPLINGS*

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ABSTRACT

Inclusive cross sections in the triple-Regge region are calculated using a version of the ABFST multiperipheral model that amounts to an extension of the Deck model. Even though almost no parameters are available for adjustment, presently available moderate-energy data are described in a qualitatively satisfactory manner. Predictions are made for future experiments at higher energies, and the various triple-Regge couplings are extracted from the model. It is found that vertices corresponding to high-lying trajectories are suppressed. In particular the dimensionless parameter η_{PPP} that characterizes the triple-pomeron vertex turns out to be $\lesssim 10^{-3}$.

I. INTRODUCTION

This work constitutes a semiquantitative test of certain theoretical ideas relevant to the triple-Pomeranchon vertex, an entity to be defined below. In this introduction we will briefly explain the experimental significance of triple-Regge expansions and then review the theoretical developments that motivated our effort.

Consider the inclusive experiment $a + b \to c + X$; defining $s = (p_a + p_b)^2$, $s' = (p_a + p_b - p_c)^2$, $t = (p_a - p_c)^2$. We will focus on events where particle c emerges as a fast leading particle separated from the rest of the secondaries by a large rapidity gap. We require in addition that the missing mass $\sqrt{s'}$ be large. In terms of the invariants defined above, such conditions mean (see Appendix A)

$$t \text{ fixed and small}, \left(\frac{s}{s'}\right) \gg 1 \ , \quad s' \gg 1 \ .$$

In such a region of phase-space the following expansion is expected to be useful:

$$\frac{d^2\sigma}{ds'dt} = \frac{1}{16\pi s^2} \sum_{ij,k} \beta_{a\bar{c}}^{\ i}(t)\ \zeta_i(t)\ \zeta_j^*(t)\ \beta_{\bar{c}a}^{\ j}(t) \left(\frac{s}{s'}\right)^{\alpha_i(t)+\alpha_j(t)}$$

$$\times\ g_{ij,k}(t)\ s'^{\alpha_k(0)}\ \beta_{b\bar{b}}^{\ k}(0)\ . \qquad (I.1)$$

In the above, the $\beta_{a\bar{c}}^{\ i}(t)$ denote the factorized couplings of trajectory α_i to the $a\bar{c}$ system, the $\zeta_i(t)$ are signature factors and the $g_{ij,k}(t)$ will be referred to as triple-Regge vertices.

The expansion reflects the view that it is possible to explain most of the physics in terms of a set of discrete powers with factorizable coefficients. We will also assume that the trajectories are the familiar ones from two-body phenomenology, i.e., we will consider a Pomeranchon trajectory (P) with $\alpha_P(0) \approx 1$, and a set of secondary trajectories (R) with $\alpha_R(0) \approx 0.5$. In the limit $\left(\frac{s}{s'}\right) \to \infty$, $s' \to \infty$, only the triple-Pomeranchon contribution survives, and the magnitude of $g_{PP,P}$ controls the rate at which events of the above type are observed. (We have tacitly assumed that particle c has the same internal quantum numbers as particle a.) Let us review the reasons that make $g_{PP,P}$ an interesting and controversial object.

Expressions like (I.1) were first written down by researchers studying multi-Regge models.[2] Following Mueller's observation[21] that inclusive cross sections could be related to discontinuities of certain connected parts, most of the consequences of multiperipheral models were understood to follow from Mueller's generalized Regge analysis. In particular, De Tar et al.[1] identified the variable that has to be made asymptotic in a forward 3-3 connected part in order to obtain the triple-Regge expansion.

To discuss the special problems presented by the triple-Pomeranchon vertex, it is worthwhile to outline the basic steps that most physicists would follow to arrive at an expansion like (I.1).

We have already explained that the kinematic condition $\left(\frac{s}{s'}\right) \gg 1$, t fixed and small, implies a large rapidity gap between the fast leading particle and the rest of the secondaries (see Fig. 1a). Regge behavior is associated with large rapidity differences and implies that the matrix element will exhibit a behavior $(s/s')^{\alpha(t)}$, the quantum numbers associated with α being determined by those of a and c.

If Regge behavior is supplemented by good factorization properties, the amplitude will exhibit a form (see Fig. 9)

$$A_{ab\to c+n} \propto \sum_i \beta_{a\bar{c}}^{\;i}(t)\, \zeta_i(t) \left(\frac{s}{s'}\right)^{\alpha_i(t)} F_{ib}(s',t,\cdots) \qquad (I.2)$$

where F_{ib} is a function that characterizes the particles in the right-hand cluster. Squaring, summing over different numbers of particles and introducing appropriate flux and phase-space factors, we find

$$\frac{d^2\sigma}{ds'dt} \propto \frac{1}{s^2} \sum_{i,j} \beta_{a\bar{c}}^{\;i}(t)\, \zeta_i(t)\, \zeta_j^{*}(t)\, \beta_{\bar{c}a}^{\;j}(t) \left(\frac{s}{s'}\right)^{\alpha_i(t)+\alpha_j(t)}$$

$$\times\; B_{ib\to jb}(s',t) \qquad (I.3)$$

$B_{ib\to jb}(s',t)$ can be thought of as the imaginary part of the forward amplitude for the scattering of reggeon i of mass t from particle b to produce a reggeon j of mass t and particle b.

Now assume that the reggeon-particle amplitude reggeizes like a regular particle-particle amplitude when the energy goes to infinity

$$B_{ib\to jb}(s',t) \propto \sum_k g_{ij,k}(t)\, s'^{\,\alpha_k(0)}\, \beta_k^{\,b\bar{b}}(0) \quad . \qquad (I.4)$$

Substituting (I.4) into (I.3) yields the desired expansion. The three ingredients that entered into this heuristic derivation were: Regge-behavior associated with large rapidity gaps, factorization and the assumption that reggeon-particle amplitudes behave just like particle-particle amplitudes.

Among the accepted Regge singularities, the Pomeranchon is placed by many physicists in a special category.[37] According to their way of looking at things, the Pomeranchon is not supposed to be associated with any peripheral exchange, as are other Regge trajectories, and therefore is not necessarily a pole with factorizable residue. This lack of factorization would prevent us from carrying through the "derivation" of the asymptotic expansion that we outlined above. Once the Pomeranchon has been given special status it is possible to attribute to it special properties, for example, its decoupling from inelastic states. The meaning of the "inelastic" decoupling is not without ambiguities. In the context of the present discussion, it means that triple-Regge couplings of the form $g_{PP,k}$ could be identically zero.[22] This would imply no observed events in the region of phase space where $(s/s') \gg 1$, $s' \gg 1$, t fixed and small.

A theoretical argument to be presented later implies however that there will be some effective nonvanishing $g_{PP,P}$, irrespective of the factorization properties of the Pomeranchon singularity. This suggests that the decoupling of the Pomeranchon from inelastic states is an inconsistent assumption.

We review now what serious work has been done in studying the "inelastic" couplings of the Pomeranchon, assuming that it is a factorizable pole.

The first hard result which indicated the need for care in handling the inelastic couplings of the Pomeron was obtained by Bali, Chew, and Pignotti.[39] Using a multi-Regge model that allowed multiple couplings of the Pomeron, they observed that, if the P were a fixed pole at $J = 1$, partial cross sections involving production of more than four particles would violate the Froissart bound[23] for total

cross sections. This paper, however, tended to reinforce the belief that the P was not different from the other trajectories since the authors showed that, if the P has a nonvanishing slope, i.e., it is a moving pole, then the violation disappeared. It was then pointed out by Finkelstein and Kajantie[24] in a careful multi-Regge calculation, that even though a moving P prevents the partial cross sections from violating the Froissart bound, their sum, the total cross section, still violates the bound if $\alpha_p(0) = 1$. In other words, there is an incompatibility between constant asymptotic total cross sections $\left(\alpha_p(0) = 1\right)$ and multiple couplings of the Pomeron.

Since the original work of Finkelstein and Kajantie, the effect has been rediscovered many times. For instance, it shows up in all calculations with multiperipheral models.

A particularly interesting version of the result was found by Abarbanel, Chew, Goldberger, and Saunders.[10] They obtained a relation between the intercept of the Pomeron $\alpha_p(0)$ and an almost dimensionless parameter $\eta = [g^2_{PP,P}(0)]/[16\pi\, 2\alpha'_P(0)]$. Their result is $1 - \alpha_p(0) > \eta$, and was obtained by considering double diffractive dissociation into high masses and noticing that such events give a contribution to the total cross section that increases asymptotically if $\alpha_p(0) = 1$. This is obviously incompatible with a constant asymptotic total cross section.

Since the publication of Ref. 10, efforts have revolved around the alternatives $\alpha_P(0) = 1$ $g_{PP,P}(0) = 0$, or $\alpha_P(0) < 1$ $g_{PP,P}(0) \neq 0$. Asymptotically constant total cross sections are esthetically appealing. In an effort to preserve them, independent arguments have been sought to prove that $g_{PP,P}(0) = 0$.

Chang et al.[25] studied the spin properties of the forward Pomeron-particle amplitude (referred to above as B) and noticed that if $\alpha_p(0) = 1$ then the amplitude is at a wrong signature nonsense point. To eliminate the spurious pole one requires that the residue, which is proportional to $g_{PP,P}(0)$, vanish. Their argument has the additional feature that even if one chooses to have $\alpha_p(0) < 1$, $g_{PP,P}$ has to be small because the wrong-signature zero is somewhere in the neighborhood. To support their claim, Chang et al. studied a planar ladder model and found indeed the desired zero. De Tar and Weis[26] showed that in a naive dual resonance model in which $\alpha_p(0)$ is arbitrarily set equal to one, the same wrong signature nonsense zero is obtained. It has already been pointed out by Mueller and Trueman[27] that if one enlarges the model of Chang et al. to include nonplanar diagrams, then it is possible to have a fixed pole that cancels the wrong-signature nonsense zero. Dual resonance models without the wrong-signature nonsense zero have been constructed by Virasoro.[28] In an effort to reinstate the zero, Abarbanel and Green[29] studied the connection between the discontinuity across the two-Pomeranchon cut of the Pomeranchon-particle amplitude and $g_{PP,P}$. Using a very plausible expression for such a discontinuity, they found that, at the branch-point, the discontinuity was proportional to the residue of the fixed pole mentioned above. They then invoked a result of Bronzan and Jones,[36] which states that, at the branch-point, the discontinuity has to vanish as a consequence of unitarity, to claim that the residue of the fixed pole vanishes and, therefore, we still have a zero of $g_{PP,P}$. One obvious difficulty with the above argument is that the authors have chosen to focus on the two-Pomeranchon cut in spite of the fact that if $\alpha_p(0) = 1$ then, at $t = 0$, all the other iterations

of the Pomeranchon pole are also at $J = 1$. Even accepting the approximation of keeping only the first iteration of the Pomeranchon, there are some difficulties. The Bronzan-Jones theorem may not be valid when one has a pole colliding with the branch point as will be the case at $t = 0$ if $\alpha_p(0) = 1$. It has been shown by Muzinich et al.[31] that the theorem can be used in this situation only if the absence of the fixed pole is assumed, i.e., the argument of Ref. 29 is circular. Recent work by Jones et al.[35] and Brower and Weis[38] has underlined dramatically the difficulties encountered by a Pomeranchon pole with $\alpha_p(0) = 1$. The authors of Ref. 35 show in a very general way, using energy-momentum conservation sum rules,[32] that $\alpha_p(0) = 1$ implies not only that $g_{PP,P}(0) = 0$ but also the vanishing of vertices of the form Pomeranchon-particle-Reggeon when the mass of the Pomeranchon is zero. This suggests that the P will not contribute at all to asymptotic total cross sections because the quantity that controls its contribution is the Pomeranchon-particle-antiparticle vertex, which can be obtained by analytically continuing the Pomeranchon-particle-Reggeon vertex to a physical value of the reggeon mass. Brower and Weis[38] study this continuation in detail and find that the vanishing of the Pomeranchon-particle-antiparticle vertex does in fact follow from that of the Pomeron-particle-Reggeon vertex.

In summary, there is by now serious evidence that if a simple Regge pole controls asymptotic behavior at $t = 0$, then its intercept, $\alpha_p(0)$, has to be less than one.

To conclude we outline an older argument due to Chew,[20] which motivated our work and which leads to the same conclusion as the more recent paper of Brower and Weis: A straightforward extension of the

Finkelstein-Kajantie analysis shows that the coupling of two Pomeranchons to a particle-antiparticle system ($V_{PP\pi\bar{\pi}}$) has to vanish when evaluated for zero masses of the Pomeranchon legs, if $\alpha_P(0) = 1$. This will be so for a continuous range of values of a momentum transfer u indicated in Fig. 1b. Since $V_{PP\pi\bar{\pi}}$ is expected to be an analytic function of u it will vanish everywhere in the complex u plane. This is impossible because we know that $V_{PP\pi\bar{\pi}}$ will have a pole at $u = m_\pi^2$. The presence of this pole can be established by considering the six-line connected part $A_{ab\to ab\pi\bar{\pi}}$. The pole factorization theorem,[33] a cornerstone of S-matrix theory, states that such a matrix element has a pole at $u = m_\pi^2$ (see Fig. 1c). The residue of the pole is $A_{a\pi\to a\pi}\cdot A_{b\bar{\pi}\to b\bar{\pi}}$, i.e., the product of two elastic amplitudes. Taking the limit $s_1, s_3 \to \infty$, $t_1, t_3 \to 0$ (see Fig. 1c), these elastic amplitudes are evaluated in the forward direction and at high energies. Hence they are dominated by Pomeranchon exchange. Making a double Regge expansion of the six-line connected part and comparing we find

$$(m_\pi^2 - u)A_{ab\to ab\pi\pi}\Big|_{u=m_\pi^2} = \beta_{a\bar{a}}^P(0)\left(\beta_{\pi\bar{\pi}}^P(0)\right)^2\beta_{b\bar{b}}^P(0)s_1^{\alpha_p(0)}s_3^{\alpha_p(0)}$$

$$= (u - m_\pi^2)\beta_{a\bar{a}}^P(0)\beta_{b\bar{b}}^P(0)V_{PP\pi\bar{\pi}}\, s_1^{\alpha_p(0)}\, s_2^{\alpha_p(0)}\Big|_{u=m_\pi^2}$$

therefore

$$\left(\beta_{\pi\bar{\pi}}^P(0)\right)^2 = (u - m_\pi^2)V_{PP\pi\bar{\pi}}\Big|_{u=m_\pi^2} .$$

Since $\beta_{\pi\bar{\pi}}^P(0)$ is the quantity responsible for the contribution of the P to total cross sections involving particle π, we have arrived at the result that either $V_{PP\pi\bar{\pi}}$ has a pole and therefore, by Finkelstein-Kajantie, $\alpha_p(0) < 1$ or the P decouples from total cross sections.

Chew's argument also suggests a way of estimating the various triple-Regge couplings. Let us focus on the pole in u corresponding to the pion. Because of the smallness of the pion mass, such a pole will be very close to a section of the physical region. In such a section it is reasonable to approximate (see Fig. 10):

$$A_{ab\to c\pi\pi b} \approx \frac{A_{a\pi\to c\pi}(s_1,t_1)\ A_{b\pi\to b\pi}(s_3,t_3)}{u - \mu_\pi^2} .$$

Always staying in the section of phase-space that is close to the pion pole, consider now the subsection where $s_1 \gg 1\ \text{GeV}^2$, $s_3 \gg 1\ \text{GeV}^2$. Because s_3 is large, the missing mass s' will be large. Note that the only dependence on s_3 in the matrix element occurs in $A_{b\pi\to b\pi}(s_3,t_3)$. Since we know that elastic cross sections at high energies are very slowly varying and nonvanishing, we are sure that we can produce a missing mass s' as large as we wish. In addition, the fact that s_1 is very large guarantees that we have a large rapidity gap (see Appendix B for more details on the kinematics). For large s_1 the amplitude $A_{a\pi\rightarrow c\pi}(s_1,t_1)$, which controls the s_1 dependence, will be Pomeranchon dominated; i.e., we have isolated a contribution to the triple-Pomeranchon vertex.

The same pion pole will also occur in connected parts for the production of more than four particles. In a similar way one expects a contribution to the triple-Pomeranchon from each of these.

In the following sections, we study these contributions quantitatively. We first derive (Sec. II) an expression for the inclusive cross section for $a + b \rightarrow c + X$ using the approximation (see Fig. 8)

$$A_{ab\rightarrow c+n} \approx \frac{A_{a\pi\rightarrow c\pi}\, A_{\pi b\rightarrow n}}{u - \mu_\pi^2} .$$

The result is identical to what one obtains in simple multiperipheral models.[3-5] Next we show that, in the limit $\left(\frac{s}{s'}\right) \gg 1$, $s' \gg 1$ the model generates a triple-Regge expansion with a nonvanishing triple-Pomeranchon vertex. We also discuss in Sec. II the form factors used to select the region of phase-space close to the pion pole. In Sec. III we test the reliability of the model by comparing its predictions with presently available data. The experimental results are only marginally within the triple-Regge region. The pion pole approximation, however, is valid also outside this very small region. We find the comparison with the data encouraging in that several features are reproduced reasonably well by the model. Among other favorable results, we find that events from the phase-space close to the pion pole can account for about half the empirically measured cross sections. Having gained some confidence in the model, we use it,

in Sec. III, to predict cross sections at energies accessible to the new generation of accelerators (CERN ISR, NAL). In Sec. IV we discuss in more detail the properties of the triple-Regge couplings that derive from the model. Section V contains conclusions.

II. THE MODEL

We are interested in calculating the double-differential cross section $(d^2\sigma)/(ds' d|t|)$ for the inclusive experiment

$$a + b \rightarrow c + X,$$

where

$$s' = (p_a + p_b - p_c)^2 ,$$

$$t = (p_a - p_c)^2 ;$$

we also define

$$s = (p_a + p_b)^2 .$$

The inclusive cross section may be obtained by summing over the exclusive ones. For the latter we use the approximation:

$$A_{ab\rightarrow c+n} \approx \frac{A_{a\pi\rightarrow c\pi} \, A_{\pi b\rightarrow n-1}}{(p_a - p_c - k)^2 - \mu_\pi^2}$$

(see Fig. 8a).

As explained in Appendix II, one obtains, after squaring the matrix element, summing over final states, and introducing the appropriate flux factor, the following expression for the double-differential cross section:

$$\frac{d^2\sigma}{ds' d|t|} = \frac{\sum_i}{2\lambda^{\frac{1}{2}}(s, m_a^2, m_b^2)} \int ds_3 \, dp_c \, dk \, d^4K \, \delta(K^2 - s_3)$$

$$\times \; 2\lambda^{\frac{1}{2}}(s_3, \mu_\pi^2, m_b^2) \, \sigma^{TOTAL}_{\pi_i b}(s_3) \, \frac{|A\pi_i a \rightarrow \pi_i c(s_1, t)|^2}{(\mu_\pi^2 - u)^2}$$

$$\times \; \delta(t' - t) \, \delta(s' - s'') \, \delta^4(p_a + p_b - p_c - k - K) \qquad \text{(II.1)}$$

$\sum_i$ indicates a sum over the three charge states of the pion,

$$dp_c = \frac{d^3 \underset{\sim}{p}_c}{2(\underset{\sim}{p}_c^2 + m_c^2)^{\frac{1}{2}}} \frac{1}{(2\pi)^3} \quad \text{and similarly for } dk$$

$$s_1 = (p_c + k)^2$$

$$t' = (p_a - p_c)^2$$

$$u = (p_a - p_c - k)^2$$

$$s'' = (p_a + p_b - p_c)^2$$

$$\lambda(x,y,z) = x^2 + y^2 + z^2 - 2xy - 2xz - 2yz .$$

Formula (II.1) is represented schematically in Fig. 2. Depending on the quantum numbers of particles a, b, and c, it may also be necessary to add contributions like the ones shown in Fig. 3. In our calculation of $p + p \to p + X$ (see Sec. III), such terms have to be included.

The reader will recognize in Fig. 2 an extended Deck model.[7] In previous applications of such a model the total cross section on the right was restricted to a given resonance (e.g., Δ or ρ), in our case we use the total cross section for any energy.

The same result obtains if one uses a multiperipheral model of the ABFST type.[3] It is interesting to note, however, that the result is really independent of the detailed structure of the multiperipheral chain. The only feature required is the capacity for generating a realistic total cross section. The details could be much more complicated than the simple model of Ref. 3.

If the limit $s' \to \infty$, $s/s' \to \infty$ is taken, one obtains,[10] assuming that high-energy scattering is well described by Regge poles (see Appendix II for details):

$$\frac{d^2\sigma}{ds'd|t|} = \frac{1}{16\pi s^2} \sum_{i,j,k} \beta_{\bar{a}ci}(t)\, \beta_{\bar{a}cj}(t)\, \zeta_i(t)\, \zeta_j^*(t)$$

$$\times \left(\frac{s}{s'}\right)^{\alpha_i(t)+\alpha_j(t)} s'^{\alpha_k(0)}\, g_{ij,k}(t)\, \beta_{b\bar{b}k}(0) \qquad \text{(II.2)}$$

where

$$g_{ij,k}(t) = \frac{3}{16\pi^3} \int_{-\infty}^{0} \frac{du}{(\mu_\pi^2 - u)^2} \left[\lambda^{\frac{1}{2}}(\mu_\pi^2,u,t)\right]^{\alpha_i(t)+\alpha_j(t)}$$

$$\times\ \beta_{\pi\pi i}(\mu_\pi^2,u,t)\, \beta_{\pi\pi j}(\mu_\pi^2,u,t)\, \beta_{\pi\pi k}(u,u,0) \cdot \int_0^{(u/t)^{\frac{1}{2}}e^{-q}} dx\, x^{\alpha_k(0)}$$

$$\times\ P_{\alpha_i(t)+\alpha_j(t)}(z)\ ; \qquad \text{(II.3)}$$

$$z = \frac{\operatorname{ch} q - (t/u)^{\frac{1}{2}} x}{\operatorname{sh} q} \qquad \operatorname{ch} q = \frac{\mu_\pi^2 - u - t}{2(ut)^{\frac{1}{2}}}\ ;$$

β denotes a factorized Regge residue, ζ a signature factor. In formula (3), the fact that one of the pion masses is not μ_π^2 has been exhibited explicitly. We will have more to say about this later.

The normalization of the residues is such that the contributions of a pole i to the (a,b) total cross section and to the elastic differential cross section are:

$$\sigma^{\text{Total}}_{i,ab}(s) = \frac{1}{\lambda^{\frac{1}{2}}(s,m_a^2,m_b^2)}\, \beta_{a\bar{a}i}(0)\, \beta_{b\bar{b}i}(0)\, s^{\alpha_i(0)}$$

$$\frac{d\sigma_{i,ab}(s,t)}{dt} = \frac{1}{16\pi\lambda(s,m_a^2,m_b^2)}\, |\beta_{aai}(t)\, \beta_{bbi}(t)\, \zeta_i(t)|^2 .$$

The factor 3 in formula (3) reflects the three charge states of the pion. It is correct if the trajectories i and j have the quantum numbers of the vacuum.[16]

In our calculation of $p + p \rightarrow p + X$ by means of formula (1), we have used the experimentally observed (π,p) total cross sections together with isospin invariance. For the elastic amplitude $A_{\pi N \rightarrow \pi N}$, we have adopted the following simplifying prescription,

$A_{\pi N \rightarrow \pi N}(s,t) = A_{\pi N \rightarrow \pi N}(s,0)\, e^{\gamma t}$. This form is fairly accurate for small t, even in the resonance region, when γ is taken to be $\approx 4(\mathrm{GeV}/c)^{-2}$. For $A_{\pi N \rightarrow \pi N}(s,0)$ we used the tabulation of Ref. 17.

Off-shell corrections to the πN elastic amplitude and πN total cross section have been introduced to provide the necessary cut-off in the integrals over the virtual pion mass. We have used a form factor of the type suggested by the ABFST integral equation:[3]

$$A_{\pi N}(s,t,u) = A_{\pi N}(s,t,u = \mu_\pi^{\,2}) \left[\frac{u_0}{u_0 - \frac{1}{2}(\mu_\pi^{\,2} + u - \frac{t}{2})}\right]^{(\bar{\alpha}+1)}$$

$$\sigma_{\pi N}(s,u) = \sigma_{\pi N}(s,u = \mu_\pi^{\,2}) \left[\frac{u_0}{u_0 - u}\right]^{\bar{\alpha}+1} .$$

The ABFST model suggests that the asymptotic behavior of the residue β_i of the pole i is $(-u)^{\alpha_i+1}$ as $-u \rightarrow \infty$. Since, for low energies, we do not use a Regge parametrization of σ_{Total} or of $A_{\pi N \rightarrow \pi N}$, we cannot incorporate this result in a simple manner. Motivated by the perturbative approach of Refs. (4) and (10) we have taken an average intercept $\bar{\alpha} = 0.7$.

The form of the off-shell corrections given above has been shown to provide an adequate fit to numerical solutions of the ABFST

equation even in the low-virtual-mass region if $u_0 \approx 1\ \mathrm{GeV}^2$.[15] We have experimented with other cutoff procedures, such as "reggeizing" the pion, and found that the results do not change significantly. Thus our model contains almost no free parameters.

III. RESULTS AT INTERMEDIATE AND HIGH ENERGIES

Figure 4 shows a comparison of the predictions of the model for the process $pp \to pX$ with the experimental results of Anderson et al.[6] The following aspects of the experimental data are satisfactorily reproduced:

(i) Energy dependence: one of the striking features of the data of Ref. 6 is a rapid decrease of the average cross section with increasing beam momentum. This effect, indicative of the weakness of diffractive excitation in this energy range, is well reproduced by the model.

(ii) Missing mass dependence: Given that we cannot expect to reproduce the full resonance structure, we consider our results satisfactory. The calculations yield bumps in the missing mass at the appropriate locations[11] but with insufficient strength.

(iii) Absolute normalization: This point is sensitive to the cutoff procedure adopted for the integration over the momentum transfer u in formulas (1) and (2). This is the problem of off mass-shell corrections mentioned in Sec. II. It may be seen from Fig. 4 that our normalization is better for high missing masses, where resonances are not expected to be important.

Another interesting result from our calculations is the importance, at intermediate energies and small t, of pion exchange, in the sense of Fig. 3b. For high missing masses, this mechanism accounts for about 50% of the cross section at $p_{lab} = 20$ GeV/c and $t \approx -0.04$ (GeV/c)^2. The importance of diagram 3b has also been recognized recently by other authors.[8,9]

Figure 5 exhibits the t dependence of the double differential cross section for a fixed value of the incident energy, at various values of the missing mass. Comparison with the data is again satisfactory.

The broken curve in Fig. 4 shows that a triple Regge expansion with the vertices predicted by the model (see Table I) provides a reasonably accurate approximation to the more exact calculation even at low energies.[19]

Figure 6 shows what we expect to observe at higher energies on the basis of this model. Dominance of the triple-Pomeranchon component would give a flat, energy-independent, curve in Fig. 6. The broken line at the bottom of the graph is what $g_{PP,P}$ alone contributes to the double differential cross section. It can be seen that, for the ISR energies, this contribution can amount to about 50% of the cross section in the appropriate missing mass range ($s' \sim 10$-30 GeV^2). Thus it may be possible to extract the value of $g_{PP,P}$ by means of a fit to the data.

Figure 7 shows a comparison of the recent results of the CERN-IHEP boson spectrometer[18] (continuous curve) with a triple-Regge expansion with the vertices predicted by our model. It can be seen that our expansion yields a very satisfactory missing mass dependence and energy dependence.

Figure 11 shows a comparison of the prediction of our triple-Regge expansion with recent experimental results on $p + p \to p + X$ at the CERN ISR. The following comments are in order: (i) The bump and dip structure in x is reasonably well reproduced; (ii) the absolute magnitude of the calculated cross section is about half the experimental value as long as $x \gtrsim 0.7$ where the model can be believed; (iii) an exception to (ii) is the very sharp peaking exhibited by the data when x approaches 1. If the experimental results continue to show such violent behavior, one would have to conclude that the lower bound to $g_{PP,P}$, $g_{PP,R}$, etc. provided by the model is far from what is observed in nature.

IV. TRIPLE-REGGE VERTICES

Table I exhibits the values of the various triple-Regge vertices obtained from formula (3). For the off-shell residues $\beta_{\pi\pi i}(u,u',t)$ we have used the following prescription suggested by the ABFST integral equation and already discussed in Sec. II:

$$\beta_{\pi\pi i}(u,\mu_\pi^{\ 2},t) = \beta_{\pi\pi i}(t)\left[\frac{u_0}{u_0 - \frac{1}{2}\left(\mu_\pi^{\ 2} + u - \frac{t}{2}\right)}\right]^{\alpha_i(t)+1} .$$

$$\beta_{\pi\pi i}(u,u,0) = \beta_{\pi\pi i}(0)\left[\frac{u_0}{u_0 - u}\right]^{\alpha_i(0)+1}$$

where $\beta_{\pi\pi i}(t)$ is the on-shell coupling of trajectory i to the $\pi\pi$ system obtained by using factorization and fits to high-energy data. (For more details see the footnote to Table I.)

According to Table I, at small t, all the couplings are within a factor 10. There is, however, a definite trend. The higher the intercept of trajectories $\underline{i}$ and $\underline{j}$, the smaller the coupling.

The rough equality of the vertices derives, in this model, from the comparable values of the couplings of the P and the P' to the $\pi\pi$ system, as extracted from total cross sections. The relation between the strength of the vertex and the height of the intercept can be understood most easily by writing formula (3) for $t = 0$

$$g_{ij,k}(0) \propto \frac{1}{16\pi^3}\frac{3}{\alpha_k(0)+1}\int_{-\infty}^{0} du(\mu_\pi^{\ 2} - u)^{\alpha_i(0)+\alpha_j(0)-2-\alpha_k(0)-1}$$

$$\times\ (-u)^{\alpha_k(0)+1}\ \beta_{\pi\pi i}(u,\mu_\pi^{\ 2},0)\ \beta_{\pi\pi j}(u,\mu_\pi^{\ 2},0)\ \beta_{\pi\pi k}(u,u,0) \qquad (4)$$

or, letting $\mu^2 \approx 0$,

$$g_{ij,k}(0) \propto \frac{1}{16\pi^3}\frac{3}{\alpha_k(0)+1}\int_{-\infty}^{0} du(-u)^{\alpha_i(0)+\alpha_j(0)-2}$$

$$\times\ B_{\pi\pi i}(0)\ \beta_{\pi\pi j}(0)\ \beta_{\pi\pi k}(0)\left(\frac{u_0}{u_0 - \frac{u}{2}}\right)^{\alpha_i(0)+\alpha_j(0)+2}$$

$$\times\left(\frac{u_0}{u_0 - u}\right)^{\alpha_k(0)+1} . \qquad (4')$$

In (4') we have exhibited the explicit form of the off-shell corrections.

The influence both of the trajectory intercept and of the magnitude of the couplings of the trajectories to the $\pi\pi$ system is exhibited in (4) and (4'). Note that we use the appropriate values for the trajectory intercepts in the form factors. Since we have singled out a specific set of trajectories we do not have to use an average intercept as we did in Sec. II.

The effect of the trajectory intercept can be traced back to kinematical limitations on the minimum momentum transfer u between the two blobs of Fig. 3 when s_1 and s_3 are large.

The reader may have noticed that the only secondary trajectory included in Table I is the P'. As already discussed in footnote 16, the ρ and A_2 trajectories are not important for the $p + p \to p + X$ and $\pi p \to X + p$ experiments in the kinematical regions that we have been considering. However, the absence of the ω coupling is more serious. So long as we restrict ourselves to π exchange, the ω decouples. This circumstance can be interpreted either as an interesting consequence of the π-exchange model or as a disturbing violation of the cherished, and empirically well supported, notion of exchange degeneracy. Note that, if degeneracy is assumed, the extraction of $g_{PP,P}$ from experimental fits becomes easier because nondiagonal terms of the form (PP',k) are approximately cancelled by terms with P' replaced by ω.

V. CONCLUSIONS

The existence of the pion pole in connected parts together with the pole factorization theorem implies that the triple-Pomeranchon vertex cannot vanish.[20] The question is then whether the contribution to the cross section from the region of phase space dominated by the pion pole is a significant fraction of the total. We believe that the results of Sec. III show that the pion pole has something to do with the observed cross sections. We have not attempted to fit the data but rather to show that the gross features could be reproduced with a very simple model. Section IV, however, shows that the triple-Pomeranchon vertex is so small as to be almost unobservable except at extremely high energies.

The dimensionless parameter[10] η_{PPP} that determines the displacement of $\alpha_P(0)$ from 1 is, assuming $\alpha'_P = 0.5\ (\mathrm{GeV}/c)^{-2}$

$$\eta_{PPP} = \frac{g^2_{PP,P}(0)}{16\pi\ 2\alpha'_P(0)} \approx 5 \times 10^{-4}.$$ [12]

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APPENDIX A.

We discuss here some kinematical details relevant to the experimental significance of the triple-Regge limit.

(a) Equivalence of small t and fast leading particle (assuming that a is the projectile)

$$t = (p_c - p_a)^2 = m_c^2 + m_a^2 - 2p_c \cdot p_a .$$

In the rest frame of a,

$$p_c \cdot p_a = 2m_a E_c$$

$$t = m_c^2 + m_a^2 - 2m_a E_c .$$

If t is to be bounded it is clear that E_c has to be bounded; i.e., particle c has a finite velocity in the rest frame of the projectile. It therefore moves very fast in the laboratory.

(b) To show that $\frac{s}{s'} \to \infty$ implies that the rapidity gap between the leading particle and any other secondary tends to infinity. The rapidity y_i of particle i with four-momentum $(E^i, \underline{p_\perp}^i, p_z^i)$ is

$$p_z^i = w_i \, \text{sh} \, y_i$$

$$E^i = w_i \, \text{ch} \, y_i ,$$ where ch and sh denoted hyperbolic functions.

$$w_i = (m_i^2 + p_{\perp i}^2)^{\frac{1}{2}}, \qquad p_{\perp i}^2 = p_{xi}^2 + p_{yi}^2 .$$

Define y_F by $K = \sum_i p^i$, where $\sum_i$ means sum over all secondaries but c;

$$K = [(s')^{\frac{1}{2}} \operatorname{ch} y_F ; \underset{\sim}{K}_\perp , (s')^{\frac{1}{2}} \operatorname{sh} y_F]$$

$$s = (p_c + K)^2 = m_c^{\ 2} + s' + 2p_c \cdot K$$

$$s = m_c^{\ 2} + s' + 2(s')^{\frac{1}{2}} w_c \operatorname{ch}(y_c - y_F) - 2p_{\perp c}^{\ 2} .$$

Dividing by s' and using the fact that $p_{\perp c}^{\ 2}$ is small because t is small, we conclude that

$$\frac{w_c}{(s')^{\frac{1}{2}}} \operatorname{ch}(y_c - y_F) \sim \frac{s}{s'} \gg 1 .$$

Therefore

$$w_c\, e^{y_c} \gg (s')^{\frac{1}{2}}\, e^{y_F} = \sum_i w_i\, e^{y_i} .$$

This implies $y_c \gg y_i$ for all i.

APPENDIX B.

We derive formula (II.1) and then take the limit $s \to \infty$, $\frac{s}{s'} \to \infty$ to obtain the triple-Regge expansion for $a + b \to c + X$:

$$\frac{d^2\sigma}{ds'dt} = \frac{\sum_n}{2\lambda^{\frac{1}{2}}(s, m_a^{\ 2}, m_b^{\ 2})} \int dp_c\, d\Phi_n (2\pi)^4\, \delta^4\left(p_a + p_b - p_c - \sum_{i=1}^{n} p_i\right)$$

$$\times\ \delta[s' - (p_a + p_b - p_c)^2]\ \delta[t - (p_a - p_c)^2]\, |A_{ab \to c+n}|^2 ,$$

where

$$d\Phi_n = \prod_{i=1}^{n} dp_i , \qquad dp_i = \frac{1}{(2\pi)^3} \frac{d^3 \underset{\sim}{p}_i}{(m_i^{\ 2} + \underset{\sim}{p}_i^{\ 2})^{\frac{1}{2}}}$$

and similarly for dp_c, $\lambda(x,y,z) = x^2 + y^2 + z^2 - 2xy - 2xz - 2yz$. Now use the approximation

$$A_{ab \to c+n} \approx \frac{A_{a\pi \to c\pi}\ A_{\pi b \to n-1}}{u - \mu_\pi^{\ 2}}$$

where $u = (p_a - p_c - k)^2$, k being the four momentum indicated in Fig. 8a ($k \equiv p_1$). Also introduce a dummy four momentum

$$K = \sum_{i=2}^{n} p_i$$

and a mass $s_3 = K^2$

$$\frac{d^2\sigma}{ds'dt} = \frac{1}{2\lambda^{\frac{1}{2}}(s,m_a{}^2,m_b{}^2)} \int dp_c \; dk \; \delta[s' - (p_a + p_b - p_c)^2]$$

$$\times \; \delta[t - (p_a - p_c)^2] \; d^4K \; \delta^4(p_a + p_b - k - p_c - K) \; ds_3 \; \delta(K^2 - s_3)$$

$$\times \; \frac{|A_{a\pi\to c\pi}(s_1,t)|^2}{(u - \mu_\pi{}^2)^2} \sum_{n-1} \int d\Phi_{n-1}(2\pi)^4 \; \delta^4\left(K - \sum_i^{n-1} p_i\right) |A_{\pi b\to n-1}|^2$$

$$s_1 = (p_c + k)^2 .$$

The summation in the third line can be identified with

$$2\lambda^{\frac{1}{2}}(s_3,m_b{}^2,m_\pi{}^2) \; \sigma_{\pi b}^{\text{Total}} = 2 \; \text{Im} \; A_{\pi b}^{\text{elastic}}(s_3,t = 0) .$$

Substituting this in the above expression and remembering to sum over the three charge states allowed for the pion pole, we obtain formula (1). The attentive reader may have noticed that we are neglecting interference terms. We now proceed to take the limit $s' \to \infty$, $\frac{s}{s'} \to \infty$ and obtain the triple-Regge expansion with explicit expressions for the triple-Regge vertices. The product $dp_c \; dk \; d^4K \; \delta(K^2 - s_3) \; \delta^4(p_a + p_b - p_c - k - K)$ is just a three-body phase space with total four-momentum $p_a + p_b$ and masses $m_c{}^2$, $m_\pi{}^2$, s_3 as shown in Fig. (8b). We will use BCP variables[39] to handle it. As shown in Ref. 39

$$dp_c \; dk \; d^4K \; \delta(K^2 - s_3) \; \delta^4(p_a + p_b - p_c - k - K)$$

$$= \frac{1}{(2\pi)^6} \; \frac{dt \; du \; d\omega \; d\nu \; d(\text{ch}\;\zeta_1) \; d(\text{ch}\;\zeta_2)}{16\;\lambda^{\frac{1}{2}}(s,m_a{}^2,m_b{}^2)}$$

$$\times \; \text{ch}\;q_0 \;\text{sh}\;q_1 \;\text{ch}\;q_2 \;\delta\left(\frac{s - m_a{}^2 - m_b{}^2}{2m_a m_b} - \text{ch}\;\eta\right)$$

where

$$\text{sh}\;q_0 = \frac{m_c{}^2 - m_a{}^2 - t}{2m_a(-t)^{\frac{1}{2}}} ,$$

$$\text{ch}\;q_1 = \frac{\mu_\pi{}^2 - t - u}{2(tu)^{\frac{1}{2}}} ,$$

$$\text{sh}\;q_2 = \frac{s_3 - u - m_b{}^2}{2(-u)^{\frac{1}{2}}\;m_b}$$

$$\text{ch}\;\eta = \text{ch}\;q_0\;\text{ch}\;q_2[\text{ch}\;q_1\;\text{ch}\;\zeta_1\;\text{ch}\;\zeta_2 + \cos\omega\;\text{sh}\;\zeta_1\;\text{sh}\;\zeta_2]$$

$$+ \;\text{ch}\;q_0\;\text{sh}\;q_1\;\text{sh}\;q_2\;\text{ch}\;\zeta_1 + \text{sh}\;q_0\;\text{sh}\;q_1\;\text{ch}\;q_2\;\text{ch}\;\zeta_2$$

$$+ \;\text{sh}\;q_0\;\text{ch}\;q_1\;\text{sh}\;q_2 .$$

The invariants s_1, s' of interest to us are given in terms of these variables by

$$s_1 = m_a^2 + u + 2m_a(-u)^{\frac{1}{2}} (\mathrm{ch}\, q_0\, \mathrm{sh}\, q_1\, \mathrm{ch}\, \zeta_1 + \mathrm{ch}\, q_1\, \mathrm{sh}\, q_0)$$

$$s' = m_b^2 + t + 2m_b(-t)^{\frac{1}{2}} (\mathrm{ch}\, q_2\, \mathrm{sh}\, q_1\, \mathrm{ch}\, \zeta_2 + \mathrm{ch}\, q_1\, \mathrm{sh}\, q_2).$$

Note that

$$\mathrm{ch}\, \eta = \mathrm{ch}\, q_0\, \mathrm{ch}\, q_2(\mathrm{ch}\, q_1\, \mathrm{ch}\, \zeta_1\, \mathrm{ch}\, \zeta_2 + \cos\omega\, \mathrm{sh}\, \zeta_1\, \mathrm{sh}\, \zeta_2)$$

$$+ \mathrm{ch}\, q_0\, \mathrm{sh}\, q_1\, \mathrm{sh}\, q_2\, \mathrm{ch}\, \zeta_1 + \mathrm{sh}\, q_0 \left(\frac{s' - m_b^2 - t}{2m_b(-t)^{\frac{1}{2}}}\right) .$$

Dividing the above equation by s' and taking the limit $s \to \infty$, $s' \to \infty$ we find that in order to have $\frac{s}{s'} \gg 1$, we need to have $\mathrm{ch}\, \zeta_1 \gg 1$. We do not assume that $\mathrm{ch}\, \zeta_2 \gg 1$ since experience with pion exchange indicates that it is important when the "boost" related to it is not very big (we have in mind here the successes of pion-exchange models at intermediate energies). This implies that $s' \gg 1$ will result from $\mathrm{ch}\, q_2 \gg 1$; i.e., most of the produced mass comes from the $\sigma^{\mathrm{Total}}(s_3)$ "blob." Taking advantage of the fact that $\mathrm{ch}\, \zeta_1 \gg 1$, $\mathrm{ch}\, q_2 \gg 1$ (i.e., $\mathrm{ch}\, \zeta_1 \approx \mathrm{sh}\, \zeta_1$, $\mathrm{ch}\, q_2 = \mathrm{sh}\, q_2$) we obtain

$$\frac{s}{2m_a m_b} \approx \mathrm{ch}\, \zeta_1\, \mathrm{ch}\, q_0\, \mathrm{sh}\, q_2[\mathrm{ch}\, q_1\, \mathrm{ch}\, \zeta_2 + \cos\omega\, \mathrm{sh}\, \zeta_2 + \mathrm{sh}\, q_1]$$

$$+ \mathrm{sh}\, q_0 \frac{s'}{2m_b(-t)^{\frac{1}{2}}}$$

$$\mathrm{sh}\, q_2 \approx \frac{s_3}{2m_b(-u)^{\frac{1}{2}}}$$

$$s' \approx 2m_b(-t)^{\frac{1}{2}}\, \mathrm{sh}\, q_2(\mathrm{sh}\, q_1\, \mathrm{ch}\, \zeta_2 + \mathrm{ch}\, q_1)$$

$$s_1 \approx 2m_a(-u)^{\frac{1}{2}}\, \mathrm{ch}\, q_0\, \mathrm{sh}\, q_1\, \mathrm{ch}\, \zeta_1 .$$

We now eliminate $\mathrm{ch}\, \zeta_1, \mathrm{ch}\, \zeta_2$ and express s_1 in terms of s, s', s_3, u and t;

$$s_1 \approx \frac{\lambda^{\frac{1}{2}}(\mu^2,u,t)}{\left(\frac{\mathrm{ch}\, q_1 - \left(\frac{t}{u}\right)^{\frac{1}{2}} x}{\mathrm{sh}\, q_1}\right) + \left[\left(\frac{\mathrm{ch}\, q_1 - \left(\frac{t}{u}\right)^{\frac{1}{2}} x}{\mathrm{sh}\, q_1}\right)^2 - 1\right]^{\frac{1}{2}} \cos\omega} \cdot \frac{s}{s'}$$

with $x = \frac{s_3}{s'}$. Phase-space, plus flux-factor, plus integration over $d\,\mathrm{ch}\, \zeta_1$, $d\,\mathrm{ch}\, \zeta_2$, $d\nu$ (assuming spin averages to eliminate all dependence on ν) yields a factor

$$\frac{1}{(2\pi)^5} \frac{1}{32\lambda(s,m_a^2,m_b^2)} \frac{1}{s'}$$

$$\times \frac{1}{\left(\frac{\mathrm{ch}\, q_1 - \left(\frac{t}{u}\right)^{\frac{1}{2}} x}{\mathrm{sh}\, q_1}\right) + \left[\left(\frac{\mathrm{ch}\, q_1 - \left(\frac{t}{u}\right)^{\frac{1}{2}} x}{\mathrm{sh}\, q_1}\right)^2 - 1\right]^{\frac{1}{2}} \cos\omega}$$

in the integrand.

Now we use the high-energy behavior of $|A_{a\pi\to c\pi}(s,t)|^2$ and $\mathrm{Im}\, A_{\pi b\to \pi b}(s_3,0)$

$$|A_{a\pi\to c\pi}(s,t)|^2 \approx \sum_{i,j} \beta_{a\bar{c}}^{\ i}(t)\, \beta_{a\bar{c}}^{\ j}(t)\, \zeta_i(t)\, \zeta_j^{*}(t)\, s_1^{\alpha_i(t)+\alpha_j(t)}$$

$$\mathrm{Im}\, A_{\pi b}(s_3,0) \approx \sum_k \beta_{\pi\pi}^{\ k}(0)\, \beta_{b\bar{b}}^{\ k}(0)\, s_3^{\alpha_k(0)} .$$

Putting everything together and changing the integration over ds_3 to $s'dx$ we find

$$\frac{d^2\sigma}{ds'dt} = \frac{1}{(2\pi)^5} \frac{\sum_{ijk} \beta_{a\bar{c}}^{\ i}(t)\, \beta_{a\bar{c}}^{\ j}(t)\, \zeta_1(t)\, \zeta_j(t)}{32\lambda(s,m_a^{\,2},m_b^{\,2})} \beta_{\pi\pi}^{\ k}(0)\, \beta_{b\bar{b}}^{\ k}(0)$$

$$\times\ s'^{\alpha_k(0)} \int_{-\infty}^{0} du \frac{[\lambda^{\frac{1}{2}}(\mu^2,u,t)]^{\alpha_i(t)+\alpha_j(t)}}{(\mu_\pi^{\,2}-u)^2} \int_0^{(\frac{u}{t})^{\frac{1}{2}} e^{-q_1}} dx\, x^{\alpha_k(0)}$$

$$\times\ 2\int_0^{2\pi} \frac{d\omega}{\left\{\left(\frac{\mathrm{ch}\, q_1 - \left(\frac{t}{u}\right)^{\frac{1}{2}} x}{\mathrm{sh}\, q_1}\right) + \left[\left(\frac{\mathrm{ch}\, q_1 - \left(\frac{t}{u}\right)^{\frac{1}{2}} x}{\mathrm{sh}\, q_1}\right)^2 - 1\right]^{\frac{1}{2}} \cos\omega\right\}^{\alpha_i(t)+\alpha_j(t)+1}} .$$

To obtain the upper limit of integration on x, we used the fact that the maximum value of s_3 is limited by the requirement that $\mathrm{ch}\,\zeta_2 \geq 1$.

Recalling the identity:

$$P_\lambda(z) = \frac{1}{\pi}\int_0^{\pi} \frac{d\phi}{[z + (z^2-1)^{\frac{1}{2}} \cos\phi]^{\lambda+1}}$$

we arrive at the triple-Regge expansion (II.2) with the corresponding expression (II.3) for the $g_{ij,k}$.

FOOTNOTES AND REFERENCES

* This work was supported by the U. S. Atomic Energy Commission.

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have used the variable $\nu = s' - m_b^2 - t$ instead of s' in formula (2). This is consistent with what is usually done when extrapolating asymptotic expansions for two-body reactions to low values of the energy. When $s' >> m_b + t$ the variables coincide. I am indebted to Stephen Ellis and Tony Sanda for suggesting the use of this variable.

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Table I. Triple-Regge vertices.[a]

t $(GeV/c)^2$	PP,P	PP',P	P'P',P	PP,P'	PP',P'	P'P',P'
0	0.142	0.299	0.745	0.236	0.488	1.21
-0.05	0.135	0.293	0.734	0.244	0.523	1.31
-0.10	0.127	0.285	0.734	0.247	0.547	1.40
-0.15	0.118	0.275	0.733	0.246	0.562	1.48
-0.20	0.109	0.263	0.731	0.240	0.569	1.55
-0.25	0.099	0.251	0.726	0.231	0.567	1.61
-0.30	0.091	0.237	0.719	0.219	0.559	1.65
-0.35	0.082	0.223	0.709	0.206	0.546	1.68

[a] Values of the triple-Regge vertices as a function of t, assuming that $\beta_P(t)$ varies as $e^{1.5t}$ and that $\beta_{P'}(t)$ is a constant. The magnitude of the coupling of the P to the $\pi\pi$ system has been obtained via factorization assuming that the asymptotic πN and NN cross sections are 23 mb and 38 mb respectively. The P' was assumed to couple to the $\pi\pi$ system with the same strength as the P. This is empirically well supported (see for example, Ref. 14). The couplings given above are to be used with and expansion like Eq. (II.2). The scale parameter in the form factor has been taken to be $u_0 = 1\ GeV^2$.

FIGURE CAPTIONS

Fig. 1(a). Schematic description of the kinematics;

(b). The vertex $V_{PP\to\pi\bar{\pi}}$;

(c). The π pole in the connected part for $ab \to ab\pi\bar{\pi}$.

Fig. 2. Schematic representation of formula (II.1). If the detected particle is a pion other diagrams having a pion coming not from the leftmost blob may have to be considered. The contribution of these other terms is not important very close to the kinematical boundary.

Fig. 3. Other terms that may have to be included depending on the quantum numbers of particles a, b, and c.

Fig. 4. Comparison of our results with the data of Anderson et al. The circles correspond to the experimental values, the smooth curve is our calculation and the broken line is what a triple-Regge expansion with the couplings of Table I predicts. In 4 (a-d) we have plotted the same quantity as have the authors of Ref. 6 for "fixed" t and varying energy (see the relevant footnotes in Ref. 6). "t" = $-0.042\ (GeV/c)^2$.

Fig. 5. Comparison of our results with the t dependence of the data of Ref. 6 at three different values of the missing mass: (a), $(s')^{\frac{1}{2}} = 1.4$ GeV; (b), $(s')^{\frac{1}{2}} = 1.7$ GeV; (c), $(s')^{\frac{1}{2}} = 1.9$ GeV. The normalization of our results has been adjusted to coincide with the data at the lowest value of t.

Fig. 6. Expectations at higher energies. We plot $\ln[s'(d^2\sigma)/(ds'dt)]$ versus $\ln s'$ at $t = -0.04\ (GeV/c)^2$. The triple-Pomeranchon component, if sufficiently strong, would be evident as an almost flat, energy independent, section of the curve. The flat broken line at the bottom of the graph is the contribution of the triple Pomeron. The curvature at small s' is due to the use of the variable $\nu = s' - m_b^2 - t$ instead of s'. See footnote 19. Continuous lines are the result of calculating with the complete model; broken lines correspond to the triple-Regge expansions. The reaction is $p + p \rightarrow p + X$.

Fig. 7. The experimental results of the CERN-IHEP boson missing-mass spectrometer (continuous curves) compared with the predictions of a triple-Regge expansion with the vertices given in Table I. We have taken $t = -0.25\ (GeV/c)^2$. The experimental $|t|$ varies between $0.17\ (GeV/c)^2$ and $0.35\ (GeV/c)^2$.

Fig. 8(a). Schematic representation of the matrix element;

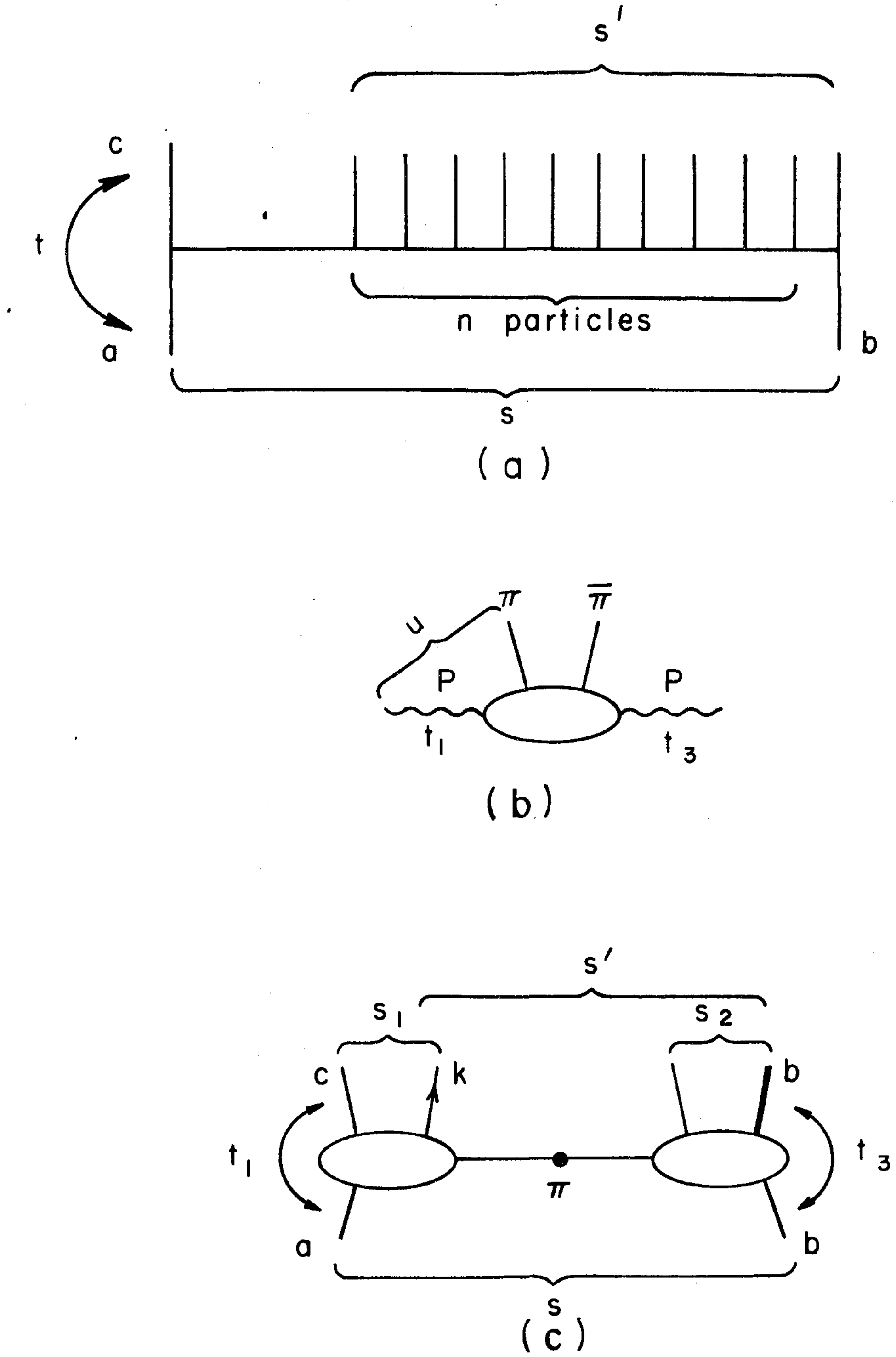
(b). The relevant kinematic variables.

Fig. 9. Schematic representation of the arguments leading to a triple-Regge expansion.

Fig. 10. Pole approximation for the matrix element $A_{ab\rightarrow c\pi\pi b}$.

Fig. 11. Comparison of the predictions of our triple-Regge expansion with recent experimental results on $p + p \rightarrow p + X$ at fixed angle. The data was taken from J. C. Sens, Invited paper presented at the Fourth International Conference on High

Energy Collisions, Oxford, U.K., 1972. The sum of all triple-Regge contributions is given by the continuous line. ○, □, and △ show partial contributions.



XBL727-3427

Fig. 1.

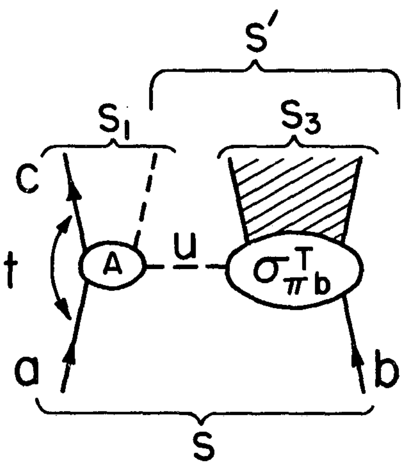


Fig. 2

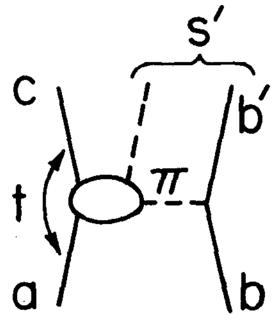


Fig.3a

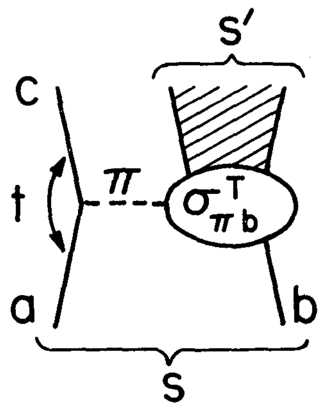
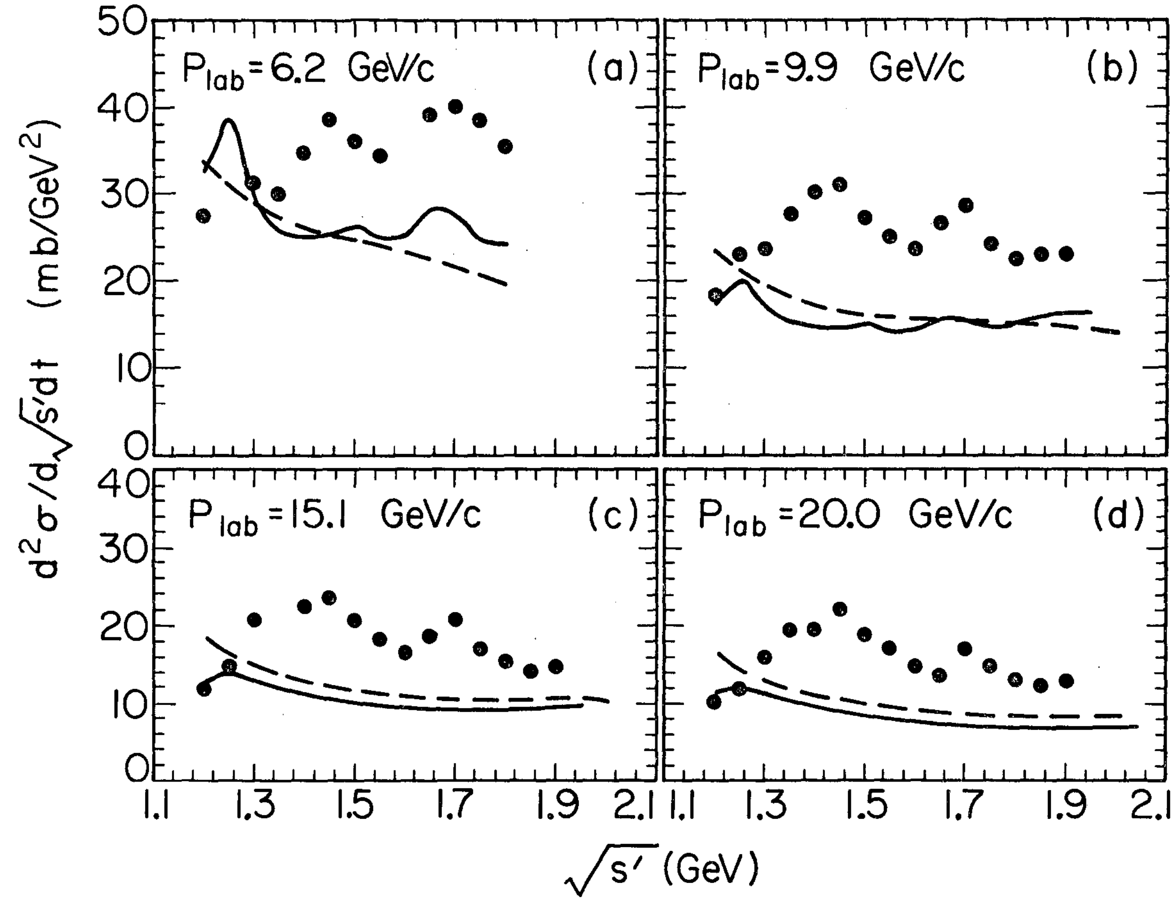


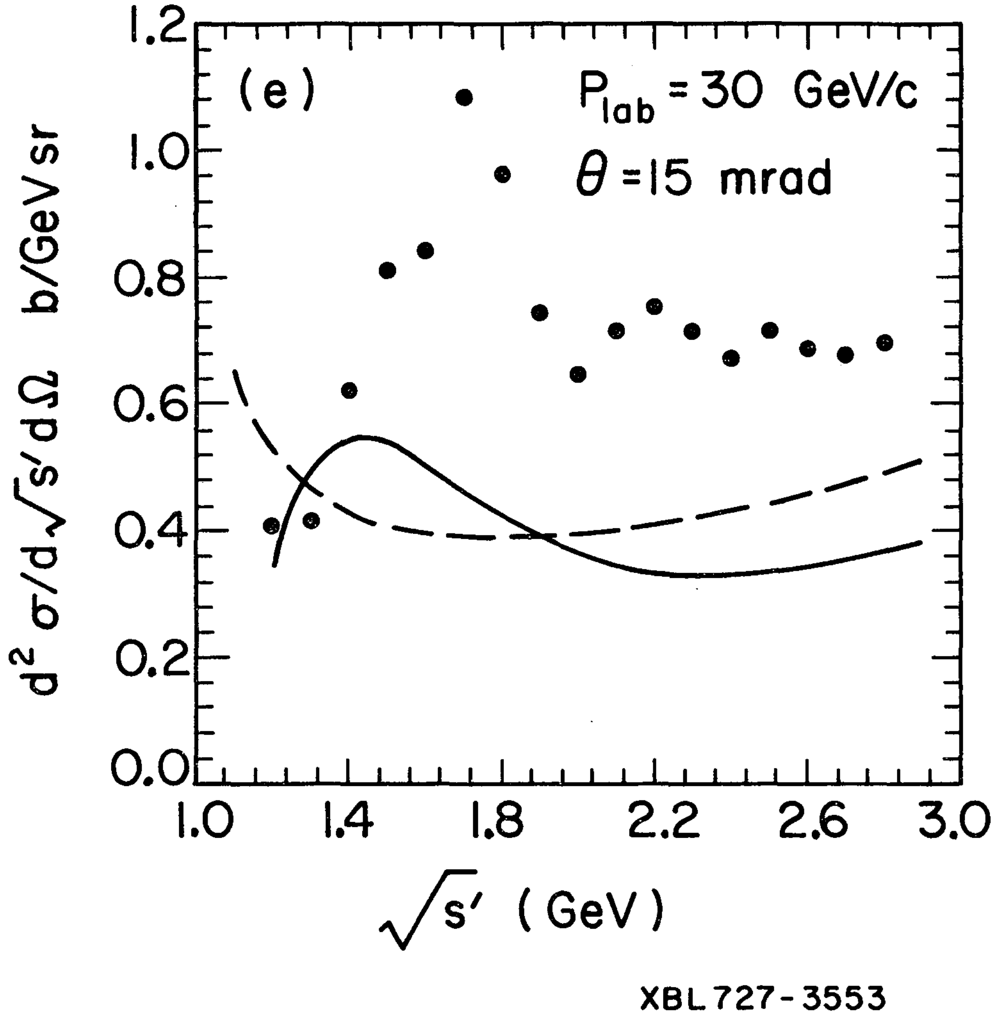
Fig. 3b

XBL724-2836



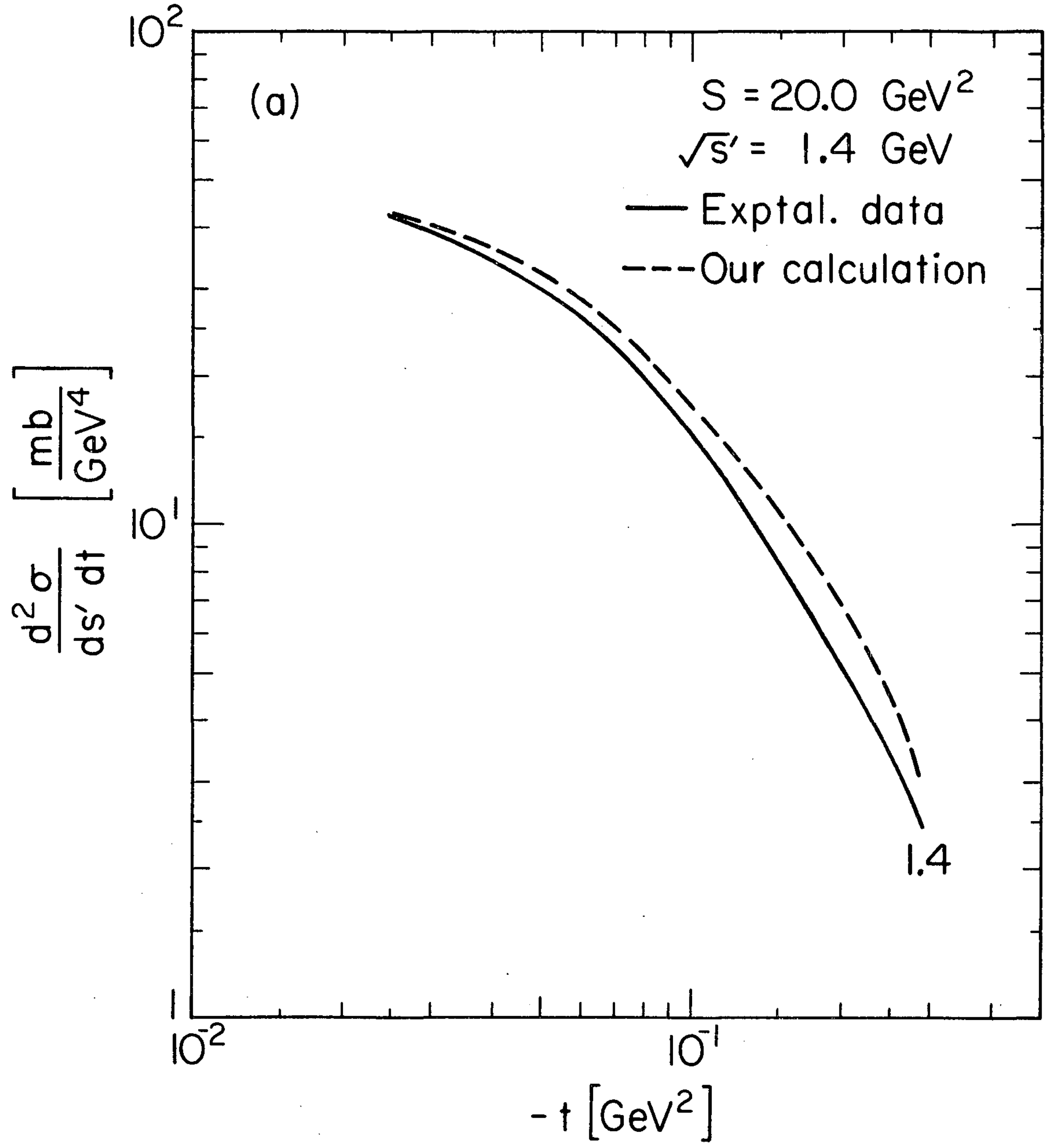
XBL727-3554

Figs. 4a-d.



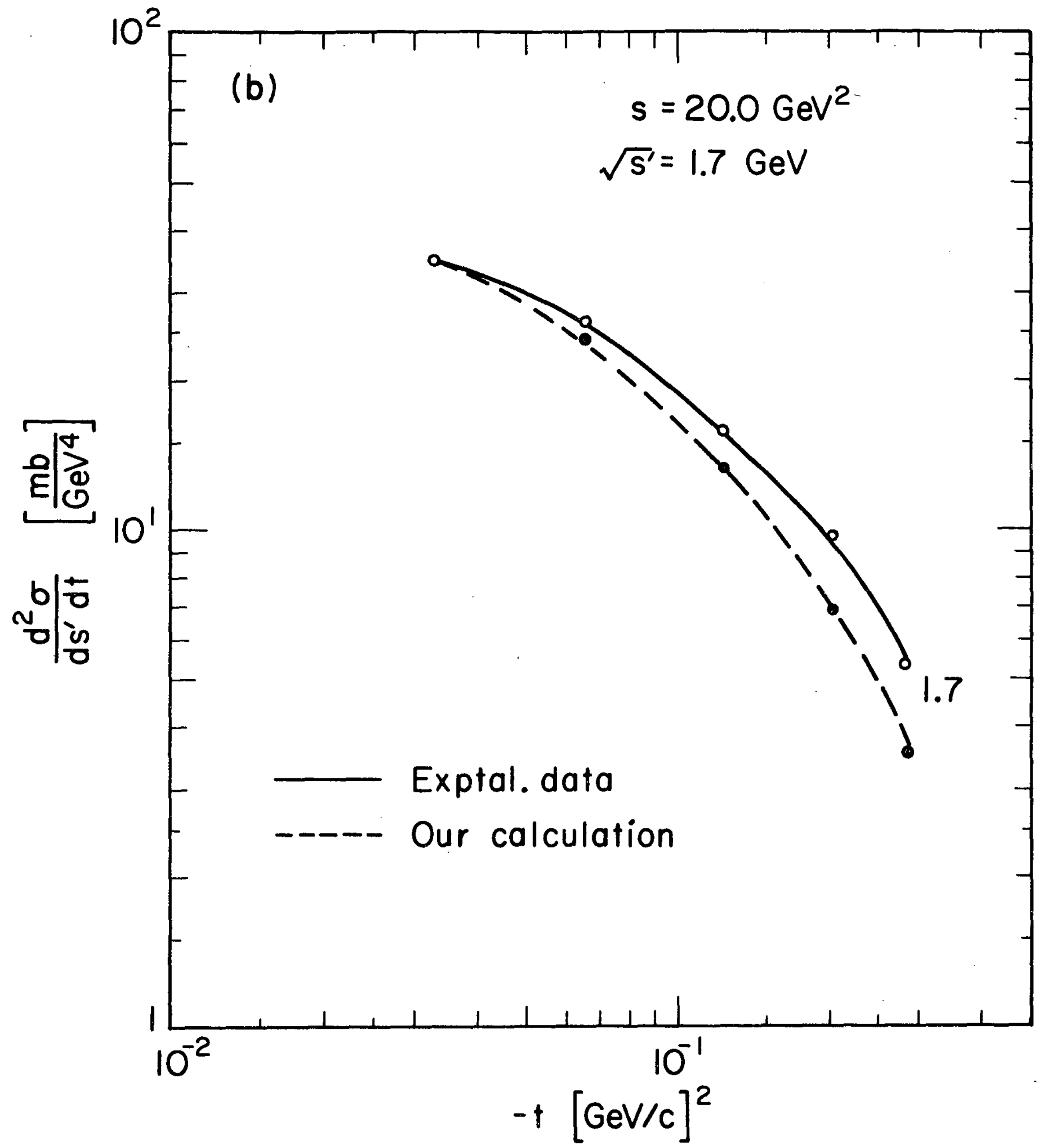
(e)
P_{lab} = 30 GeV/c
θ = 15 mrad
$d^2\sigma/d\sqrt{s'}d\Omega$ b/GeV sr
1.2
1.0
0.8
0.6
0.4
0.2
0.0
1.0
1.4
1.8
2.2
2.6
3.0
$\sqrt{s'}$ (GeV)
XBL 727-3553

Fig. 4e.



XBL724-2840

Fig. 5a



XBL 724-2841

Fig. 5b

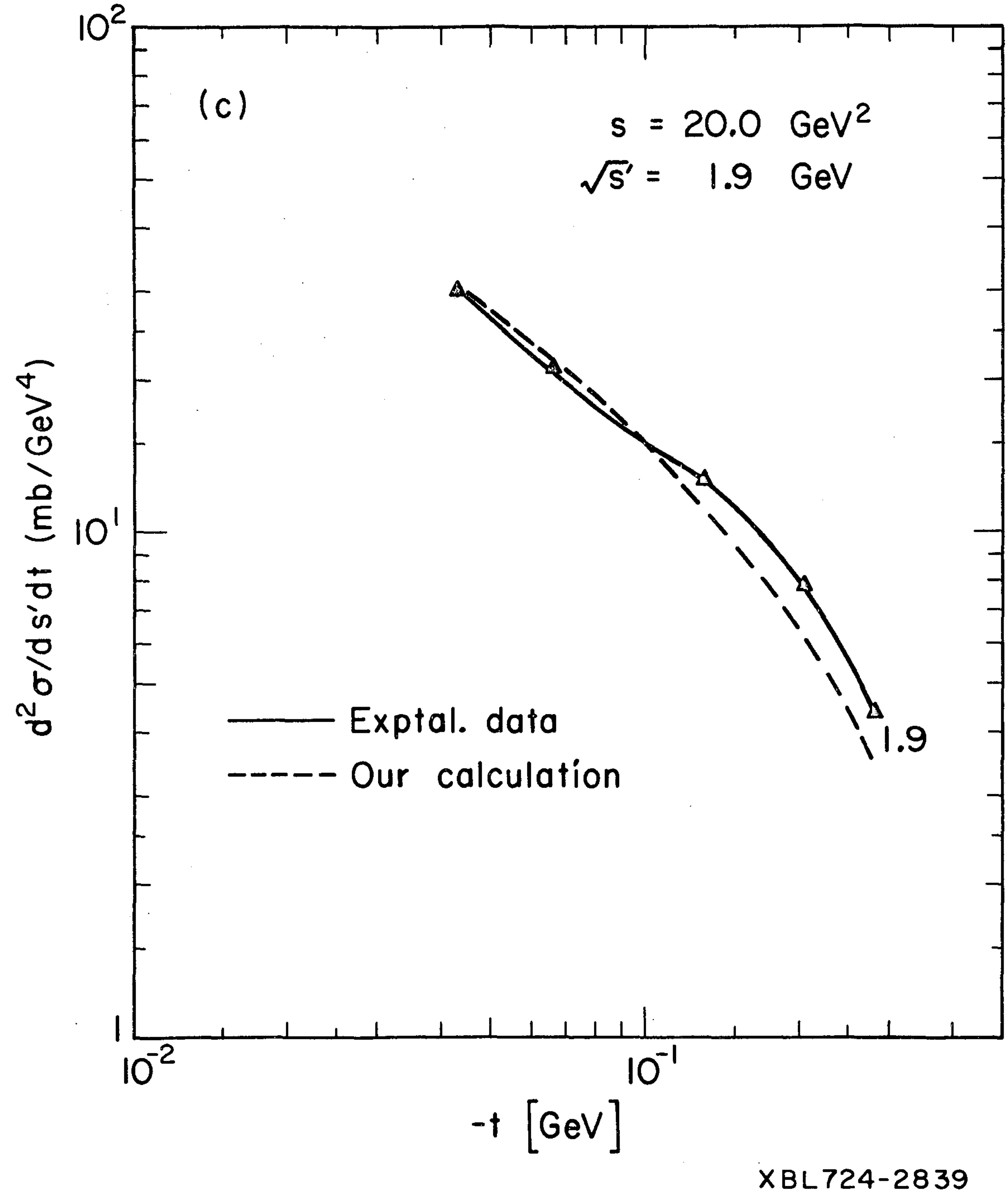
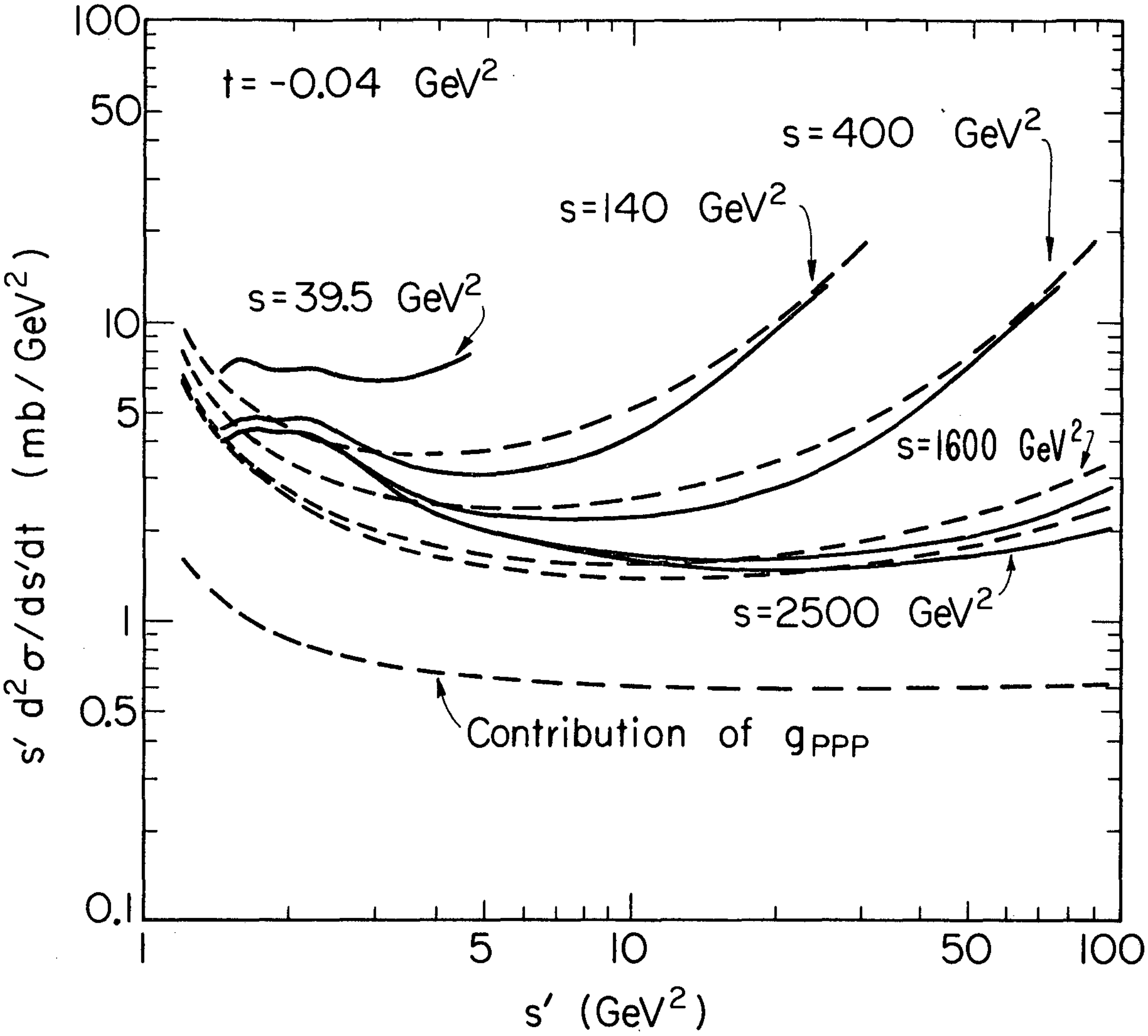
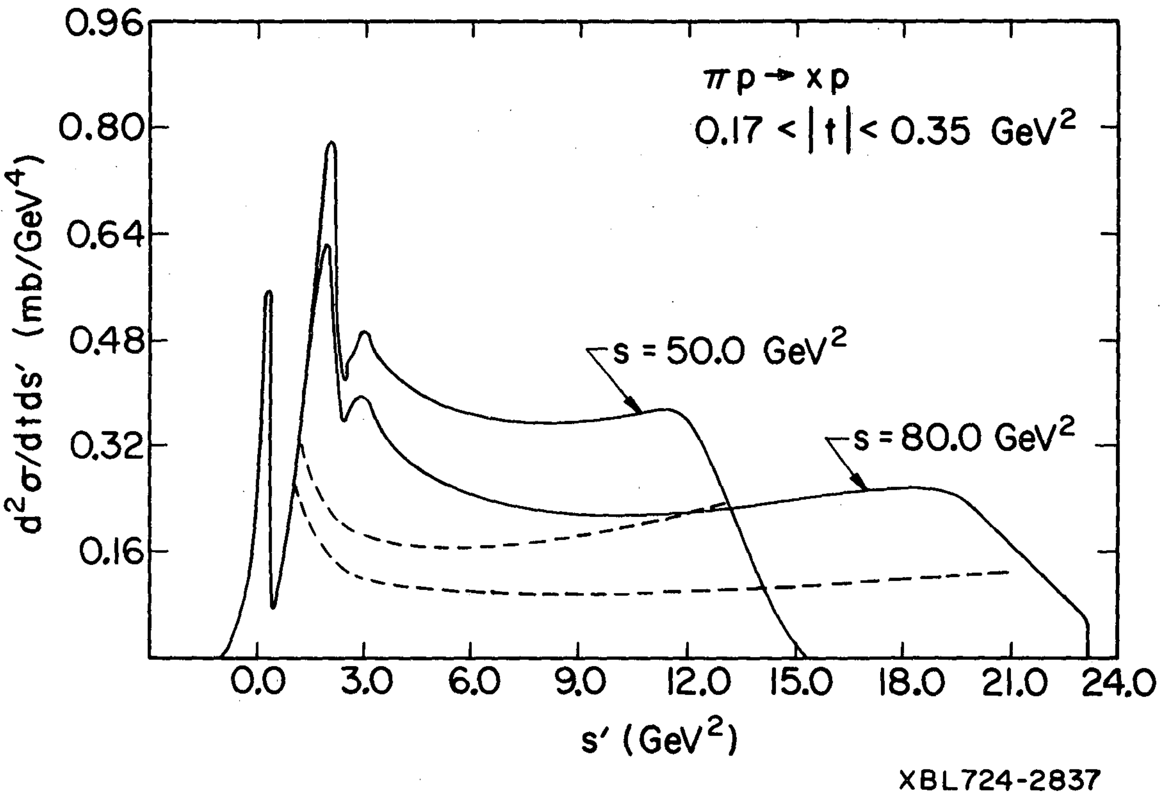

(c)
s = 20.0 GeV²
√s′ = 1.9 GeV
Exptal. data
Our calculation
1.9
d²σ/ds′dt (mb/GeV⁴)
-t [GeV]
10²
10¹
1
10⁻²
10⁻¹
XBL724-2839

Fig. 5c



XBL727-3555

Fig. 6.



0.96
0.80
0.64
0.48
0.32
0.16
d²σ/dtds′ (mb/GeV⁴)
πp → xp
0.17 < |t| < 0.35 GeV²
s = 50.0 GeV²
s = 80.0 GeV²
0.0
3.0
6.0
9.0
12.0
15.0
18.0
21.0
24.0
s′ (GeV²)
XBL724-2837

Fig. 7

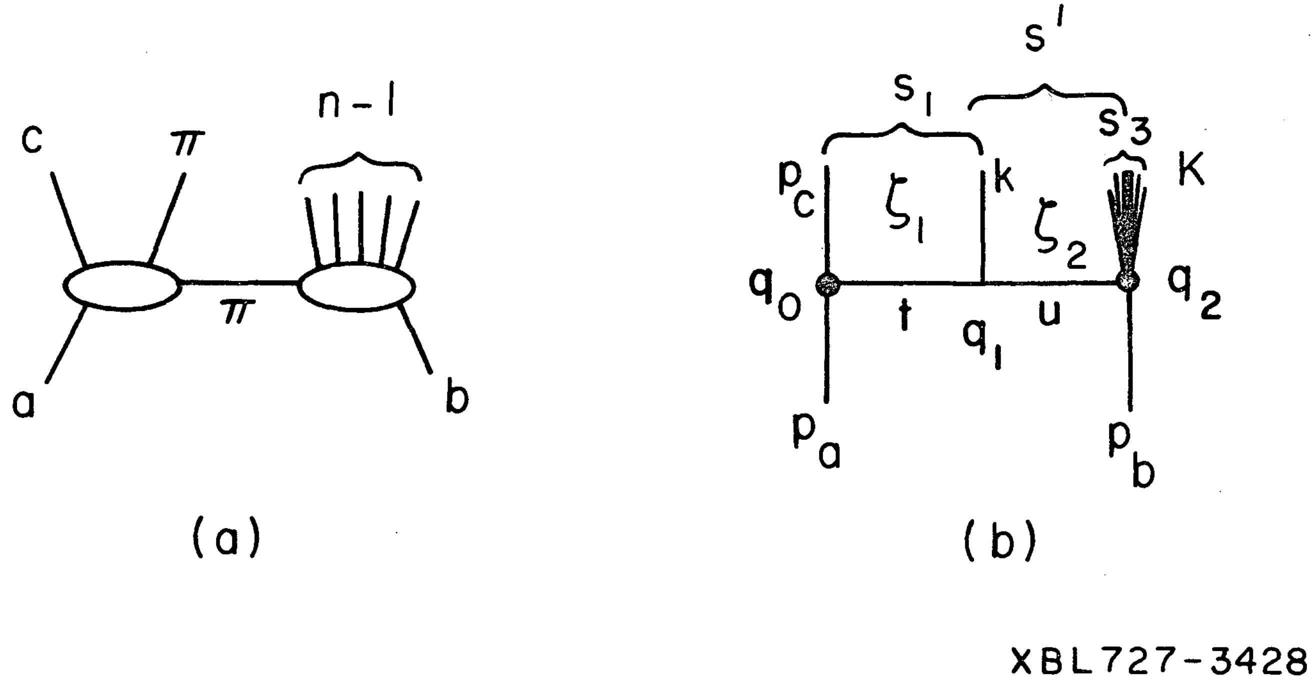
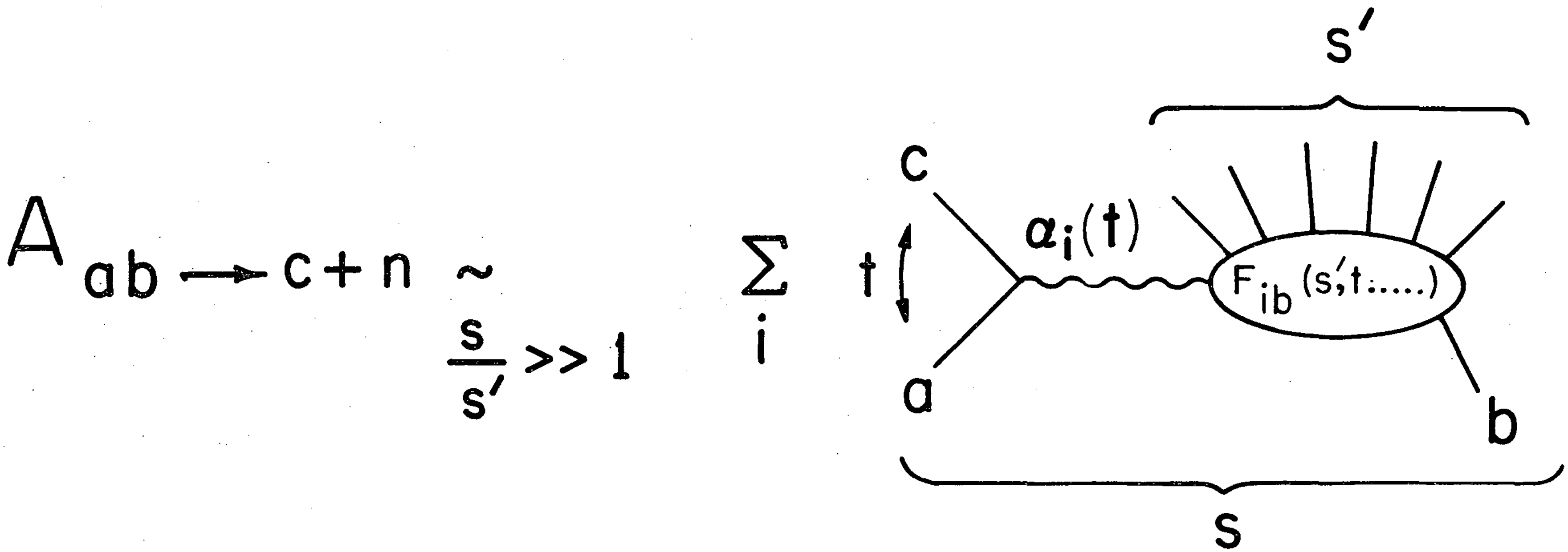


Fig. 8



XBL728-3779

Fig. 9a

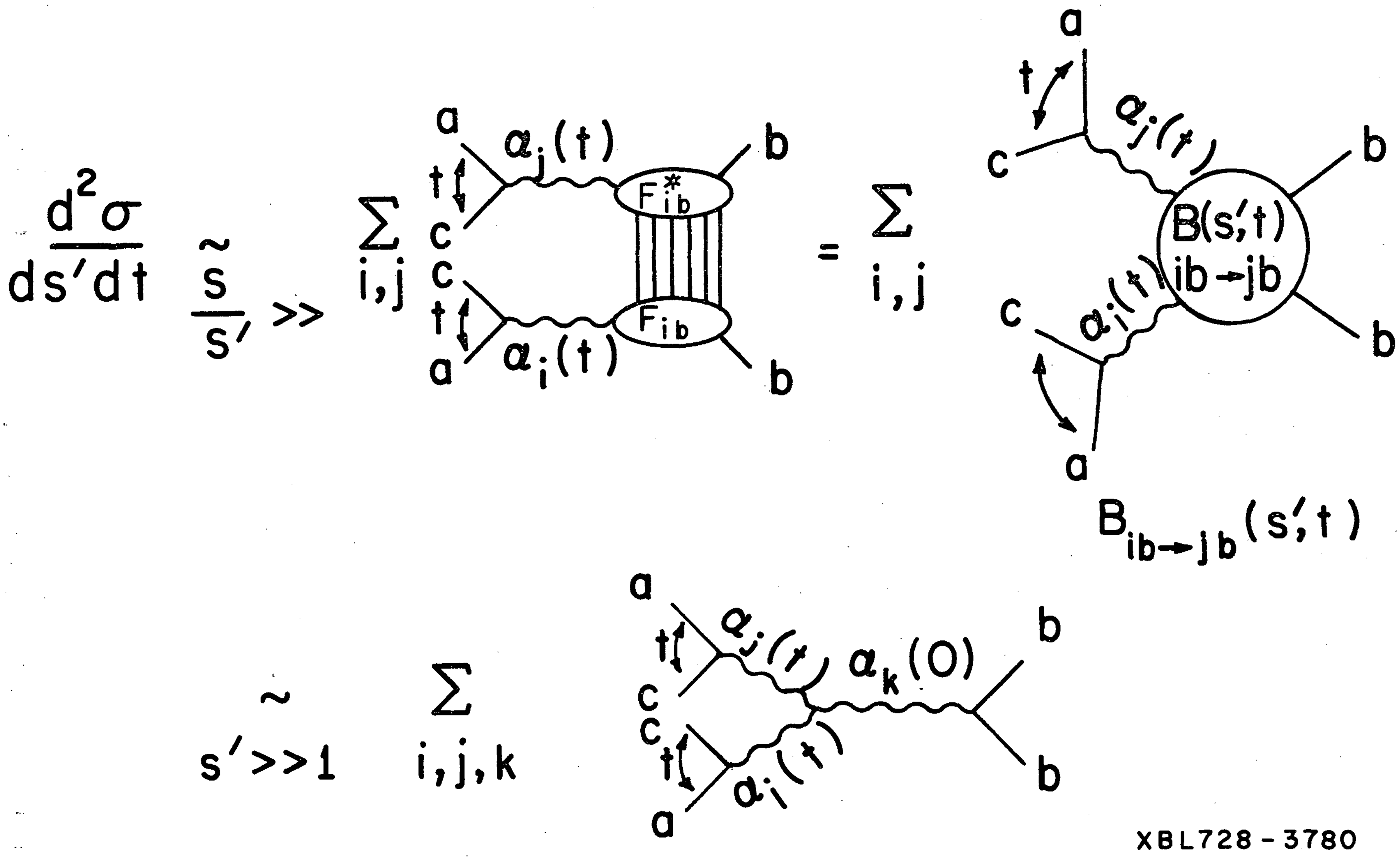
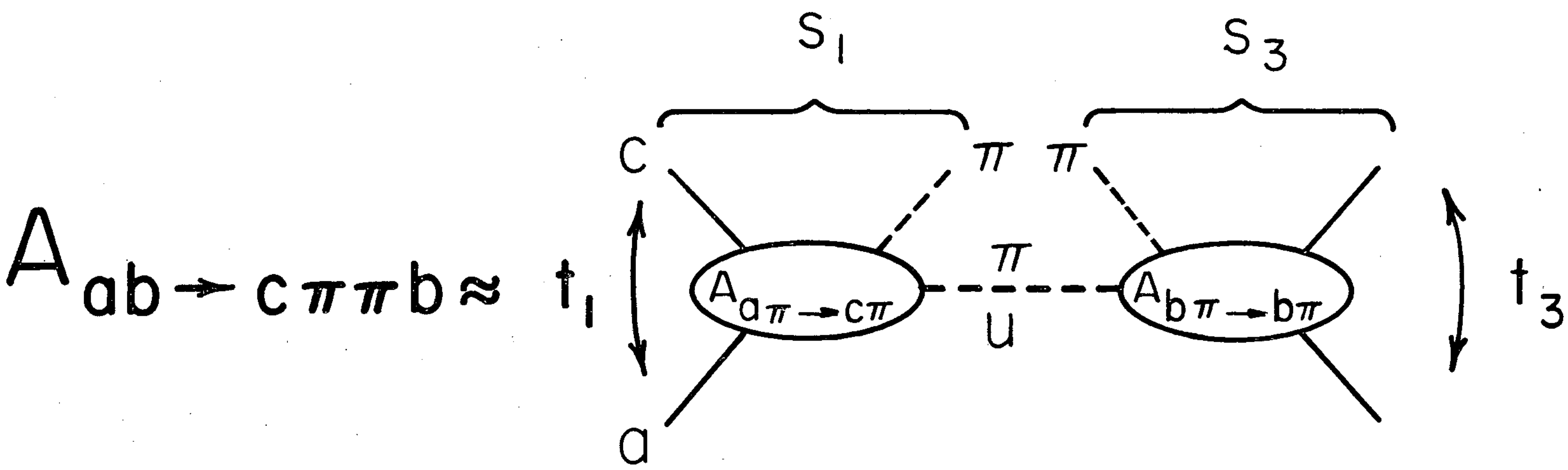
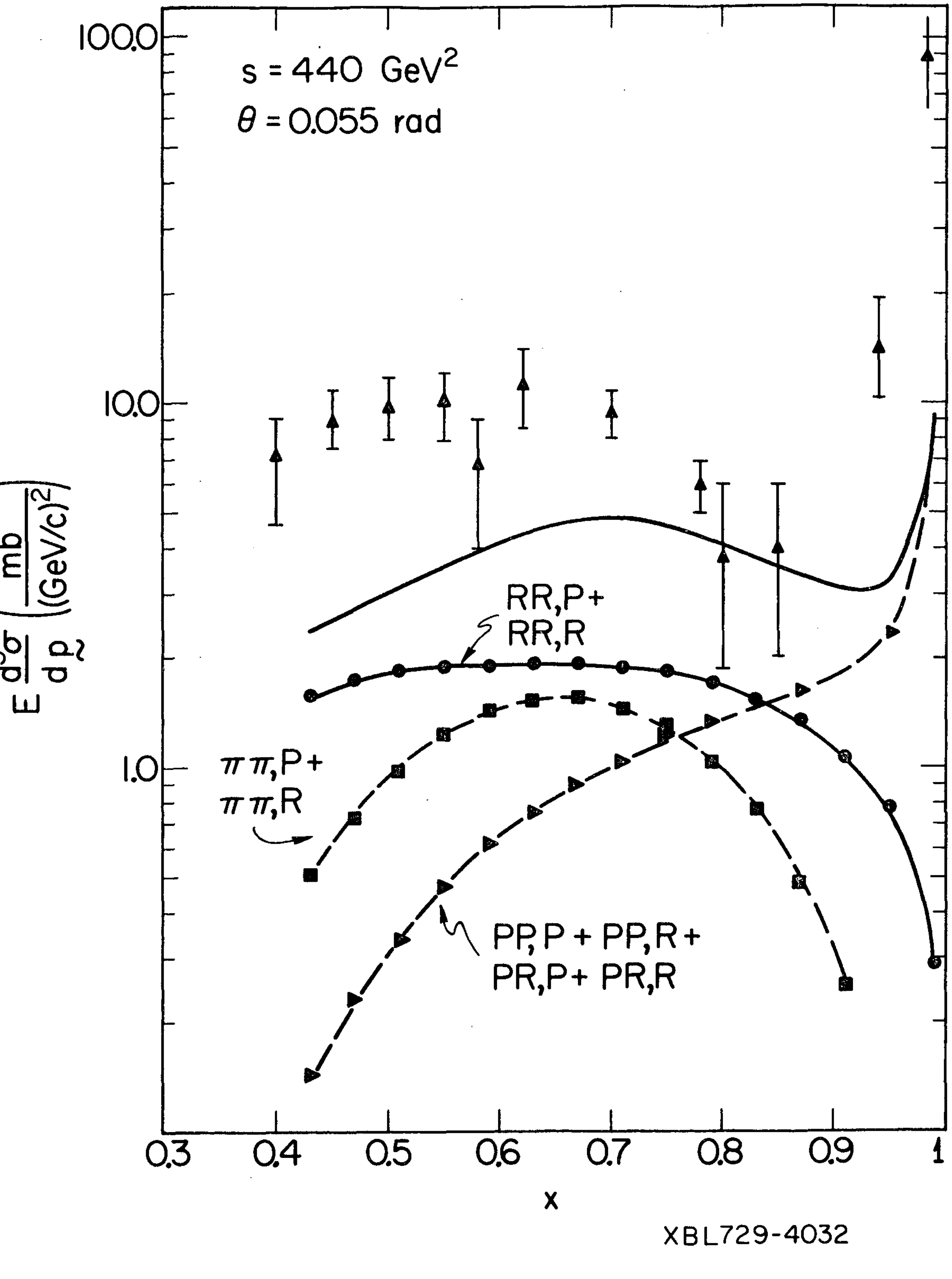

d²σ / ds′dt
~ s/s′ >> ∑ i,j
a c c a b b
αj(t) αi(t)
F*ib Fib
= ∑ i,j
B(s′,t) ib→jb
Bib→jb(s′,t)
~ s′>>1 ∑ i,j,k
αk(0)
XBL728-3780

Fig. 9b



XBL728 -3781

Fig. 10



s = 440 GeV²
θ = 0.055 rad
100.0
10.0
1.0
E d³σ/dp (mb/(GeV/c)²)
RR,P+
RR,R
ππ,P+
ππ,R
PP,P + PP,R +
PR,P+ PR,R
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
x
XBL729-4032

Fig. 11

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