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FORWARD BIAS, THE FAILURE OF UNCOVERED INTEREST PARITY AND RELATED **PUZZLES**<sup>[]</sup>

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Abstract

Uncovered interest parity is widely used in open economy macroeconomics. But the

evidence rejects UIP and implies forward bias. There are many suggested explanations for the

failure of UIP and forward bias, but none are widely accepted, at least partially because none

explain the related puzzles discussed below. This paper shows how the liquidity effects of open

market operations and sterilized "leaning against the wind" can explain the failure of UIP and

forward bias even when expectations are rational. They also appear to be able to explain the

related puzzles better than any of the alternatives.

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Key words: risk premiums; rational expectations; uncovered interest parity; intervention, covered

interest parity; forward bias; liquidity effects.

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Author's Note: Unlike most economists, I am a Logical Positivist. As such I assume that all theories are false, but that some are less false than others. I offer this explanation for forward bias and the failure of UIP not as something that is or could be "true", but as something that I believe could be less false than any of the current alternatives.

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#### 1. Introduction.

As Lothian and Wu (2011) point out, uncovered interest parity (UIP) is one of three key international financial relations that are used repeatedly in open-economy macroeconomics. But the evidence rejects UIP and implies forward bias. See Sarno (2005) for a discussion of that evidence.<sup>1</sup>

The importance of UIP has prompted many attempts to explain the apparent failure of UIP and forward bias. Two of the most widely cited explanations are risk premiums and the failure of Rational Expectations. For a discussion of those two alternatives see Sarno (2005).

Less frequently cited alternatives include the following: using interest rates to stabilize exchange rates [McCallum (1994)], a tradeoff between interest rate and exchange rate stability [Anker (1999)], uncertain monetary policy [Lafuente and Ruiz (2006)], nonlinearities [Sarno et al (2006)], fads and fashion [Sercu and Vinaimont (2006)], perpetual learning [Chakraborty and Evans (2008)], adverse selection [Burnside et al (2009)], career risk [Liu and Sercu (2009)], deep habits [Moore and Roche (2010)], infrequent portfolio decisions [Bacchetta and van Wincoop (2010)], carry and momentum trading [Baillie and Chang (2011)], and over confidence [Burnside et al (2011)]. A recent book, Miller (2014), uses an inter-temporal version of UIP with systematic errors in expectations to explain the apparent failure of UIP. But the puzzles remain.

There are several closely related puzzles that these explanations do not seem to be able to resolve.<sup>2</sup> They include (1) the Carry Trade Puzzle, (2) the Commodity Puzzle, (3) the Development Puzzle, (4) the Inflation and Outlier Puzzles, (5) the Maturity Puzzle and (6) the Time Dependency Puzzle

Most references to forward bias and the failure of UIP refer to the bias and failure of UIP under flexible exchange rates between developed countries where short-term interest rate differentials appear to miss-predict the direction of change in future exchange rates. This paper takes a broader view of forward bias and the failure UIP. It also considers evidence from the classical gold standard, pegged exchange rates under the ERM and managed floating. As pointed out below, this research provides substantially more support for UIP. For regimes other than flexible rates, the evidence probably would not be considered clear evidence of forward bias and a strong rejection of UIP.

<sup>&</sup>lt;sup>1</sup> Baillie and Bollerslev (2000) and Maynard and Phillips (2001) among others claim that this evidence is a statistical artifact. Aggarwal et al (2009) reject that claim.

<sup>&</sup>lt;sup>2</sup> A few can explain some of the related puzzles. But all of them appear to be inconsistent with at least one of the related puzzles.

As a result of this difference between flexible rates and other regimes, this paper asks the following question: Why is the forward bias so great and the failure of UIP so dramatic under flexible exchange rates, but not under other regimes? It answers that question by showing for the first time how the liquidity effects of open market operations and sterilized "leaning against the wind" produce forward bias and deviations from UIP under flexible exchange rates.

Section 2 reviews the relevant evidence and all the puzzles. Section 3 discusses Covered Interest Parity (CIP) because it plays an important role in UIP and forward bias. Section 4 provides a formal model for flexible exchange rates between developed countries and explores its implications regarding liquidity effects and sterilized intervention. Section 5 discusses how well the model explains the related puzzles as compared to the alternatives. Section 6 is a brief summary.

#### 2. The Puzzles.

This section begins with the two primary puzzles: the failure of UIP and forward bias. After reviewing the evidence regarding those two puzzles, it takes up the related puzzles.

### 2.1. Uncovered Interest Parity.

Discussions of UIP often take the economic theory behind it for granted, but the theory behind UIP is not obvious. There are at least two different approaches.

Eq. (1) describes UIP. The expected change in the exchange rate equals the appropriate interest rate differential.

$$E(\Delta S_{t+k}|I_t) = i_t \square i^*$$
(1)

Where  $s_t$  is the log of the domestic price of foreign exchange,  $E(x_{t+k}|I_t)$  is the conditional expectation of  $x_{t+k}$  based on the information set  $I_t$  available at t.  $i_t$  and  $i^*$  are risk-free domestic and foreign interest rates with the same maturity as  $s_{t+k}$ . Expectations are "rational" in the sense of Muth (1961) when  $I_t$  contains all of the current information in the model.

One approach to UIP begins with CIP and *assumes* that speculation eliminates expected speculative returns by eliminating the difference between  $E(s_{t+k}|I_t)$  and the log of the appropriate current forward rate denoted  $f_t$ . Together they imply eq. (1). This approach ignores transaction costs and risk premiums, and seems to implicitly assume that expectations are rational.

Given the large body of evidence supporting CIP, when UIP fails, it is natural to question the assumption that  $E(s_{t+k}|I_t)$  equals  $f_t$ . That question appears to be the source of the idea that risk

<sup>&</sup>lt;sup>3</sup> This is the approach used in the New Palgrave Dictionary of Economics, Vol. 1 (2008, 451).

premiums "cause" UIP to fail. Less than Rational Expectations provides another potential explanation for the failure of this version of UIP.

A second approach uses the Fisher equation, an expectations version of purchasing power parity and the simplifying assumption that real interest rates are equal.<sup>4</sup> When nominal interest rates equal real rates plus expected rates of inflation,  $i_t \, \Box \, i^*$  equals the appropriate difference in expected rates of inflation plus the difference in real interest rates. With real differentials zero, the Fisher equation and an expectations version of PPP imply eq. (1) because  $E(\Delta s_{t+k}|I_t)$  and  $i_t \, \Box \, i^*$  both depend on expected inflation. Like the first approach this one ignores transaction costs and risk premiums, and seems to implicitly assume rational expectations.

When UIP fails, this approach suggests that it fails because the Fisher equation fails, expected PPP fails, real rates are not equal, there is a risk premium or expectations are not rational.

The second approach provides an economic explanation for the first; expected changes in exchange rates equal interest rate differentials because both equal expected inflation. This link between differences in expected inflation, nominal interest rate differentials and expected changes in exchange rates appears to be the basic idea behind uncovered interest parity.

But this idea ignores the open market operations that drive inflation in most developed countries. When those open market operations have liquidity effects as the LM schedule implies and is assumed by essentially all Keynesian models, the Fisher equation fails. For example, in Dornbusch (1976), starting in equilibrium with no inflation and no interest rate differential, an expansionary open market operation creates a negative differential and expected inflation.<sup>5</sup>

In both approaches, eq. (2) is the standard test equation where expectations are rational,  $e_{t+k}$  has a zero mean, is uncorrelated and is orthogonal to  $i_t \square i^*$ .

$$\Delta s_{t+k} = a + b(i_t \prod i^*) + e_{t+k}$$
 (2)

UIP implies that estimates of b, denoted  $^{\land}$ , should equal 1.0. As is well known, between developed countries with flexible exchange rates,  $^{\land}$  are routinely negative, and often negative and significant. But as the review of the relevant evidence in Section 2.3 shows,  $^{\land}$  are seldom negative, and hardly ever negative and significant, under other regimes.

### 2.2. Forward-Bias.

<sup>&</sup>lt;sup>4</sup> This appears to be the approach used in Engel (2016).

<sup>&</sup>lt;sup>5</sup> Dornbusch (1976) ignores CIP and *assumes* that UIP holds. UIP is not an implication of the model as it is in Table 1 when there is no intervention and there are no liquidity effects.

The modern forward-bias puzzle begins with Fama (1984) who splits  $f_t$  into  $E(s_{t+1}|I_t)$  and a "premium" denoted  $p_t$ .

$$f_{t} = E(s_{t+1}|I_{t}) + p_{t}$$
(3)

Although subsequent literature almost universally calls  $p_t$  a risk premium, Fama (1984) does not. He first mentions  $p_t$  in his Abstract without any mention of a "risk premium". He then points out that eq. (3) is "no more than a particular definition of the premium component of the forward rate. To give this equation economic content, a model that describes the determination of  $p_t$  is required." Section 4 provides such a model.

Although almost all the relevant literature refers to  $p_t$  as a risk premium, from this point on, as in Engel (2016),  $p_t$  is the expected return to speculation. If  $E(s_{t+1}|I_t)$  is less than  $f_t$ , a speculator can expect to make a profit by selling the currency forward at  $f_t$ , and then buying the currency in the future to cover the forward sale at the lower  $E(s_{t+1}|I_t)$ .

As Sarno (2005, 678) points out, it is unlikely that  $p_t$  is a risk premium.

The message that emerges from the empirical analysis of risk-premium models, in general or partial equilibrium, is that it is hard to explain excess returns in forward foreign exchange by an appeal to risk premia alone; either  $\varphi$ , the coefficient of relative risk aversion, must be incredibly large, or else the conditional covariance of consumption must be incredibly high.

Rather than concentrating on risk premiums in the equilibrium condition that risk premiums equal expected speculative returns, Section 4 concentrates on showing how sterilized intervention and liquidity effects produce expected speculative returns. Those expected returns produce forward bias and risk premiums even when expectations are rational.<sup>6</sup>

Whatever the interpretation of  $p_t$ , eq. (3) implies eq. (4).

$$f_t \prod S_t = E(\Delta S_{t+1}|I_t) + p_t \tag{4}$$

Assuming Rational Expectations and rearranging eq. (4) produces eq. (5).

$$\Delta \mathbf{S}_{t+1} = (\mathbf{f}_t \ \Box \ \mathbf{S}_t) \ \Box \ \mathbf{p}_t + \mathbf{\varepsilon}_{t+1} \tag{5}$$

Omitting  $p_t$  produces the "Fama equation".

$$\Delta \mathbf{S}_{t+1} = \alpha + \beta (\mathbf{f}_t \mid \mathbf{S}_t) + \zeta_{t+1} \tag{6}$$

Where  $\land$  is the estimated  $\beta$ .

<sup>&</sup>lt;sup>6</sup> The combination of forward bias and rational expectations raises an interesting question about the meaning of an "efficient" market.

^ between developed countries with flexible rates are routinely negative and often significant. For some recent evidence see Table 2 in Copeland and Lu (2016). As the next subsection shows, when rates are not flexible, that is not the case.

#### 2.3. Relevant Evidence.

Most discussions of forward bias and the failure of UIP refer to the negative and often significant estimates of ^ and ^ between developed countries under the current float. But there also are estimates under the gold standard, when rates were flexible during the early 1920s and under pegged exchange rates, all between developed countries. In addition there are estimates between developed and emerging countries under both "managed" and flexible rates.<sup>7</sup>

Coleman (2012) estimates b between the U.S. and U.K. from 1888 to 1905, the classical gold standard. His  $^{\land}$  are about 0.5. Paya et all (2010) estimate  $\beta$  between the U.K. and U.S. from December 1921 to May 1925 when rates were flexible. Their  $^{\land}$  is negative and significant.  $^{8}$ 

Flood and Rose (1996) pool their daily data and use SUR to estimate b under both pegged and flexible exchange rates. Their pegged rates are from the Exchange Rate Mechanism of the European Monetary System from March 1979 to March 1994 where Germany is the home country. Their flexible rates use Australia, Canada, France, Germany, Japan, Switzerland and the U.K. from 1981 to October 1994 where the U.S. is the home country. Using flexible rates, a common intercept and a three month interval their  $^{\land}$  is  $\square 0.04$ . With specific intercepts it is  $\square 0.88$ . Both  $^{\land}$  are significantly less than zero.

Using pegged rates, a common intercept and a three month interval their ^ is 0.54. With specific intercepts it is 0.60. Both ^ are significantly greater than zero and significantly less than one. Using one month rather than three month intervals produces similar results. These estimates are designed to avoid the peso problem. When uncorrected for peso problems estimates are close to, but significantly greater than zero

Lothian and Wu (2011) estimate b using 200 years of annual data between the U.K. and France and the U.K. and U.S. Most of their estimates cover such a wide variety of regimes that they are difficult to interpret. The one interval where the regime does not change is the gold standard from 1800 to 1913. For that interval their  $^{\land}$  are significantly below one but not statistically different from zero.

<sup>&</sup>lt;sup>7</sup> Managed covers a wide range of regimes from currency boards to crawling pegs. Particularly with emerging countries, currencies can be flexible *de Jure* but not necessarily *de facto* because of trade and/or capital controls.

<sup>&</sup>lt;sup>8</sup> This interval is a bit unusual because the U.K. was deliberately deflating in order to restore the pre WWI value of sterling versus the U.S. dollar, which had maintained the pre WWI gold content of the dollar.

Their rolling regressions probably provide the most relevant estimates. Those ^ are generally positive up to the adoption of flexible rates in the 1970s. They then turn negative until the 1990s where they begin to turn positive again. But this turn to positive appears to be only temporary. Using data from 1987 to 2006, Aslan and Korap (2010) find that all four flexible currencies versus the U.S. dollar have negative ^ and that two are significant.

Bansal and Dahlquist (2000) and Frankel and Poonawala (2010) provide ^ between developed countries during the current float with the U.S the home country and between the U.S. and emerging countries with both flexible and managed rates. Whether the country pairs are developed or developed and emerging, when rates are flexible their ^ are usually negative and at times significant. Between developed and emerging when rates are managed, ^ are generally not significantly different from zero. (Note that these estimates are not corrected for peso problems.)

This overview of the relevant evidence suggests the following: systematically negative and occasionally negative and significant ^ and/or ^ are restricted almost exclusively to flexible exchange rates. Whether country pairs are developed or developed versus emerging does not seem to make much difference.

### 2.4. Carry Trade.

The "Carry Trade" refers to borrowing where international interest rates are "low" and lending where they are "high" without cover, which appears to produce profit with little risk. Those profits suggest that, for at least some trades, expected speculative returns exceed risk premiums. For some recent articles on the carry trade, see Burnside et al (2008), Hochradl and Wagner (2010), Baillie and Chang (2011) and Copeland and Lu (2016). All four articles evaluate the carry trade during the current float.

#### 2.5. Commodity Puzzle.

One would expect Fama's premium to be as valid for commodity markets as for foreign exchange markets. His premium can refer to the price of wool as well as the price of foreign exchange. Eq. (5) would appear to be as relevant for wool, or any other commodity with forward markets, as it is for foreign-exchange markets.

Given the importance of the bias in foreign-exchange markets, looking for the same bias in commodity markets would seem an obvious and important thing to do. To the best of my knowledge there have been only two attempts to do so: Fama and French (1987) and Kearns (2007). Using futures indexes, Kearns finds positive ^ for commodities. Using individual

futures prices, Fama and French find mostly positive ^ for commodities. Frequent negative ^ for flexible exchange rates and mostly positive ^ for commodities is the Commodity Puzzle.

### 2.6. Development Puzzle.

Bansal and Dahlquist (2000) were the first to suggest that the forward-bias puzzle is confined largely to developed countries. Later Frankel and Poonawala (2010) estimated  $\beta$  between the U.S. and 21 developed countries and 14 emerging countries. The average  $^$  between developed countries and the U.S. is  $\square$ 4.3. The average  $^$  between emerging countries and the U.S. is 0.003. That difference illustrates the Development Puzzle. Grossman et al (2014) do something similar with the euro and pound sterling as home currencies. They find a Development Puzzle only during "non-crisis" periods.

### 2.7. Inflation and Outlier Puzzles.

The Inflation Puzzle is that ^increases with inflation. The Outlier Puzzle is that there are positive (negative) ^ or ^ with outlier (non-outlier) interest rate differentials and/or forward premiums. See for example Sarno et al (2006) and Baillie and Chang (2011). As far as I am aware, all the evidence for these two puzzles uses developed countries under the current float.

### 2.8. Maturity puzzle.

Using developed countries with flexible exchange rates, Alexius (2001), Chinn and Meredith (2004) and Chinn (2006) find that ^are usually negative for maturities of one year or less but that ^are usually positive for over one year. More recently Lothian and Wu (2011) find a similar pattern using over 200 years of annual data. Negative ^ for short maturities and positive ^ for long maturities is the conventional Maturity Puzzle.<sup>9</sup>

Miller (2014, 11) suggests an alternative maturity puzzle: UIP holds when regressing long and short maturity changes in exchange rates against long-maturity interest rate differentials:<sup>10</sup>

### 2.9. Time Dependency Puzzle.

Between developed countries under the current float,  $^{\wedge}$  and  $^{\wedge}$  vary widely over time. For examples of this time dependency for (, see Han (2004). For examples of this time dependency for  $^{\wedge}$ , see Baillie (2011).

Using rolling regressions Lothian and Wu (2011) estimate ^ using over two hundred years of data. Before the early 1970s, ^ are relatively stable, usually positive and not significantly

<sup>&</sup>lt;sup>9</sup> Using pound sterling and the euro as home currencies, Grossman et al (2014) still find negative ^ at 12 month maturities.

<sup>&</sup>lt;sup>10</sup> See Miller (2014, 11) for the relevant references.

different from zero. After that, ^ become unstable, consistently negative and at times negative and significant.

The Time Dependency Puzzle therefore has two parts. Why is there little time dependency before the current float and why is there so much time dependency during the current float?

#### 3.0. CIP.

Covered Interest Parity is based on effective arbitrage and is one of the few theories in open economy macroeconomics for which there is convincing empirical support.<sup>11</sup> Akram et al (2008) probably provide the best available analysis of Covered Interest Parity.<sup>12</sup> They say that one can safely assume that CIP holds for daily and lower frequency data. Given that strong support, any explanation for the failure of UIP or forward bias that is not consistent with CIP is suspect. The model in Section 4 assumes CIP.

Since, as is typical, Akram et al (2008) use maturities of one year or less from developed countries with flexible rates, strictly speaking their conclusions only hold under those conditions. Whether or not CIP holds under other conditions is an issue that needs to be resolved.<sup>13</sup>

Eq. (7) describes the *theory* of Covered Interest Parity.

$$f_{t} \square s_{t} \square (i_{t} \square i^{*}) = \mathbf{d}_{t}$$
 (7)

Where forward premiums and interest rate differentials have the same maturity.

For good data the deviation  $d_t$  should reflect primarily transaction costs, particularly bid-ask spreads. To simplify the discussion, for now it ignores  $d_t$  and assumes that Covered Interest Parity holds exactly. The discussion returns to  $d_t$  later.

CIP implies that the failure of UIP and forward bias are two sides of the same coin. With  $d_t$  zero, eq. (7) implies (8).

$$f_t = s_t + (i_t \prod i^*) \tag{8}$$

Subtracting an appropriate  $E(s_{t+1}|I_t)$  from both sides of eq. (8) produces eq. (9).

$$f_{t} \square E(s_{t+1}|I_{t}) = (i_{t} \square i^{*}) \square [E(s_{t+1}|I_{t}) \square s_{t}]$$

$$(9)$$

The forward bias  $f_t \square E(s_{t+1}|I_t)$  equals the deviation from UIP  $(i_t \square i^*) \square [E(s_{t+1}|I_t) \square s_t]$ .

### 4. A New Approach.

<sup>&</sup>lt;sup>11</sup> Several papers claim that CIP fails after 2008. For a review of those papers and an evaluation of CIP after the Great Recession see Rime et al (2017).

<sup>&</sup>lt;sup>12</sup> For additional work on CIP see Fong et al (2010) and the work cited there.

<sup>&</sup>lt;sup>13</sup> Fletcher and Taylor (1996) find unexploited deviations from long-term CIP. Skinner and Mason (2011) find violations of long-term CIP, but only for emerging markets. Suh and Kim (2016) seem to suggest substantial deviations from CIP for emerging markets.

The primary contribution of this paper is to show for the first time how the liquidity effects of open market operations can cause forward bias and the failure of UIP. This idea is not model specific. Table 1 is only an example of how it works.

Another important and new contribution is to show how sterilized "leaning against the wind" in foreign exchange markets can cause forward bias and the failure of UIP. This effect depends on two assumptions: First "leaning against the wind" introduces inertia into changes in exchange rates and second that sterilized and unsterilized intervention affect nominal interest rate differentials differently. The first assumption is an implication of dropping the idea that exchange rates are asset prices. The second is the result of the fact that sterilized intervention affects short-term interest rates directly, but not the monetary base, while unsterilized intervention does the opposite.

The model in Table 1 begins with a simple and widely known Keynesian model, Dornbusch (1976). As Faust and Rogers (2003, 1407) point out, Dornbusch (1976) uses three standard building blocks: (1) liquidity effects, (2) UIP and (3) long-run PPP. A fourth less standard building block, perfect foresight, is a proxy for rational expectations. For simplicity, Dornbusch ignores risk premiums, holds income constant and treats foreign variables as though they were exogenous. His article also ignores the use of interest rates as tools of monetary policy and focuses on how open market operations affect interest rate differentials, relative price levels and exchange rates.

Table 1 replaces UIP, which is widely rejected, with CIP, for which there is substantial support. But it retains (1) liquidity effects, (3) long-run PPP and (4) rational expectations. For simplicity it also ignores risk premiums, holds income constant, treats foreign variables as though they were exogenous and concentrates on the effects of open market operations.

The model differs from Dornbusch (1976) in at least two other important ways. First it does not implicitly assume that exchange rates are asset prices. That approach appears to have failed. As far as I am aware, no one claims that we can explain the short-run behavior of exchange rates any better today than we could in the early 1970s before the asset approach.

Second Dornbusch (1976) is a Keynesian model where foreign exchange and asset markets clear rapidly, presumable because they are auctions markets, while commodity markets clear slowly, presumable because they are retail markets.

Here there are retail and auction versions of all markets. The interbank market for foreign exchange clears rapidly because it is an auction market. But there also are retail foreign exchange markets where individuals and small businesses buy foreign exchange at local banks and foreign exchange offices. Markets for assets like T bills clear rapidly because they are auction markets. Other asset markets, like those for stock in closely held corporations, clear slowly because they are not auction markets. Most commodity markets clear slowly because they are retail markets. But there are many auction markets for basic commodities like petroleum products, metals and grains that clear rapidly.

One important way this difference manifests itself is in Purchasing Power Parity. In a Keynesian framework PPP *must* use retail prices because those are the only commodity prices available. Here PPP uses auctions prices.

Like all frameworks, this one is not "realistic". It is a caricature that helps us focus on important issues. The question is whether or not it is more useful than the conventional one? Conventional frameworks like the one in Dornbusch (1976) have dominated open economy macroeconomics for a long time and have produced a long list of anomalies. See Miller (2014, 3) for some examples. Perhaps it is time for a new framework.

### 4.1. Empirical support.

The model in Table 1 implies that the liquidity effects of open market operations cause UIP to fail. A large literature uses vector auto-regression to analyze how shocks to monetary policy affect UIP. It includes Eichenbaum and Evans (1995), Grilli and Roubini (1996), Cushman and Zha (1997), Kim and Roubini (2000), Faust and Rogers (2003), Scholl and Uhlig (2008), Bjørnland (2009), Bouakez and Normandin (2010) and Heinlein and Krolzig (2012). Most articles claim that policy shocks create at least temporary deviations from UIP.

There also is support for the idea that intervention causes forward bias and the failure of UIP. Baillie and Osterberg (1997) and (2000) and Mark and Moh (2007) find that intervening in foreign exchange markets can cause UIP to fail.

Table 1
The Model

$f_t \square s_t = i_t \square i^* = (t$	Covered Interest Parity.	(I)
$S_t = (\lambda \square 1)(t + (t \square ZX\Delta S_t + Z_t)$	Flexible spot rate.	(II)
$f_t = \lambda(_t + (_t \square ZX\Delta S_t + Z_t)$	Flexible forward rate.	(III)

```
Changes in the monetary base.
e_t = Du_t \prod FX\Delta s_t + v_t
                                                                                                                (IV)
(t = E(\Delta(t+1)|I_t) + (t \square \Lambda e_t \square HX \Delta s_t)
                                                 Interest rate differential.
                                                                                                       (V)
E(\Delta(t+1}|I_t) = C(Du_t \sqcap FX\Delta s_t + v_t)
                                                 Expected inflation.
                                                                                                       (VI)
\Delta(t+1) = C(Du_t | T F X \Delta S_t + V_t) + X_{t+1}
                                                           Actual inflation.
                                                                                                               (VII)
Definitions:
                                                           Price level differential in logs.
(t
                                                           Real interest rate differential.
(t
                                                           Nominal interest rate differential.
(t
                                                           Actual minus natural rate of unemployment.
u_t
```

#### Shocks and restrictions:

 $v_t = Vv_{t-1} + v_t$ ,  $u_t = Uu_{t-1} + w_t$ ,  $(t = R(t-1} + y_t, z_t = z_{t-1} + \varepsilon_t, C, D, \Lambda, \alpha)$  and h are all  $\geq 0$  while V, U and R are all  $\geq 0$  but less than 1.0. Random variables  $w_t$ ,  $x_t$ ,  $y_t$ ,  $v_t$  and  $\varepsilon_t$  have zero means, zero initial values, are uncorrelated and orthogonal.  $1 \geq F \geq 0$ ,  $Z = \alpha(1+F)X$  and  $H = h(1 \mid F)$ .

#### 4.2. The Model.

All markets in Table 1, including commodity markets, are auction markets. Eq. (I) describes CIP. Like the rest of the model it ignores transaction costs. Eq. (II) describes the market for spot foreign exchange, *e.g.*, the dollar price of sterling, where exchange rates are flexible. Eq. (III) describes the corresponding forward market. As required by CIP, subtracting (II) from (III) produces the short-term nominal interest rate differential ( $\iota$ ). With the log of the exchange rate s(t) exogenous in Eq. (I), which is often an implicit assumption in the relevant literature,  $\lambda \square 1$  is zero in eq. (II).

The primary role of eq. (III) is to show that the model is consistent with CIP. Assuming that  $\lambda \Box 1$  is zero simplifies the later derivations of  $\wedge$  and makes them more consistent with the relevant literature without distorting the main implications of the model.

With  $\lambda \Box 1$  zero, three factors determine spot exchange rates: central bank intervention captured by  $ZX\Delta s_t$ , exogenous shocks captured by  $z_t$  and relative price levels captured by  $(t_t)$ . For simplicity, the foreign price level is assumed to be constant, which implies that foreign expected inflation is zero.

 $ZX\Delta s_t$  describes the most common form of intervention when exchange rates are flexible, leaning against the wind. Central banks lean against the wind by selling a currency as its price rises and buying as its price falls. Neely (2001, 4-6) reports that almost 90% percent of central banks sometimes or always lean against the wind and that 40% fully sterilize while only 30% never sterilize.

X describes the Fed's response to a given  $\Delta s_t$ . The larger X the more the Fed leans against the wind. Z describes how that intervention affects  $\Delta s_t$ . A positive Z introduces positive autocorrelation in  $\Delta s_t$ .

Both Z and the resulting change in the monetary base depend on sterilization. The Fed sterilizes its intervention by selling (buying) assets like U.S. T bills as it buys (sells) sterling. With complete sterilization, intervention does not affect the monetary base and F in eq. (IV) is zero. With no sterilization, F is one and each dollar's worth of sterling that the Fed sells reduces the monetary base by one dollar. There is a general consensus that sterilization reduces Z. There is a less general consensus that Z is positive even when sterilization is complete. Here Z is positive even when F is zero.

Selling spot sterling reduces the Fed's holdings of sterling deposits at the Bank of England. To restore those deposits to their desired level, the Fed sells U.K. assets like T bills. Those sales prevent the intervention from affecting the monetary base in the U.K. Buying sterling as its price falls does the opposite.

As a result, when the Fed fully sterilizes, its intervention does not affect either the U.S. or the foreign monetary base. But purchases (sales) of U.S. short-term assets tend to lower (raise) U.S. short-term interest rates while sales (purchases) of U.K short-term assets tend to raise (lower) U.K. interest rates. As a result, when leaning against the wind causes the Fed to sell foreign exchange, in Table 1 full sterilization causes (, to fall.

If the Fed does not sterilize, then the same sterling sale reduces the U.S. monetary base, which tends to restrict U.S. short-term credit and increase U.S. short-term interest rates. That sterling sale also increases the U.K. monetary base, which tends to increase U.K. short-term credit and lower U.K. short-term interest rates. As a result, unsterilized intervention tends to increase (<sub>t</sub>.

Intervention and how ( $_t$  responds to the degree of sterilization helps explain the Time Dependency Puzzle.  $^{\land}$  changes over time as both the intensity of intervention and the degree of sterilization change over time.

(t captures Purchasing Power Parity. As Rogoff (1996, 649) points out, arbitrage and the Law of One Price (LOP) are the basic building blocks of PPP. Having made the point, he quickly

<sup>&</sup>lt;sup>14</sup> Phillips and Pippenger (1993) find evidence of a temporary effect for sterilized intervention. Although she attributes it to a signaling effect, in her early review of the literature Edison (1993, 54) refers to a temporary effect. In their review of the later literature, Beine et al (2003, 892) also mention a temporary effect. More recently Fatum (2008) finds evidence of a temporary effect. Pippenger and Phillips (1973) probably provide the best estimate of *Z* because they are able to avoid simultaneity by assuming that, in the absence of intervention, the exchange rate is a random walk; an assumption that Pippenger (2008) supports.

goes on to dismiss the LOP by observing that the LOP holds mainly in the breach. This is a typical attitude toward commodity arbitrage and PPP, but not here.<sup>15</sup>

If effective arbitrage and the LOP are the basic building blocks for PPP, and they hold in auction commodity markets as well as Pippenger and Phillips (2008) and Pippenger (2016) claim, then shouldn't we use auction prices when evaluating PPP? Sarno and Taylor (2002, 52-54) make a similar point.

There are other reasons for using auction rather than retail prices. First, at the retail level the distinction between traded and non-traded goods is an illusion. At the retail level *all* goods are effectively non-traded. No one buys shoes at Marks & Spencer in London and sells them to Macy's in New York.. Second, it does not seem appropriate to use retail prices, which are not driven by expectations, to explain auction exchange rates, which are driven by expectations.

Eq. (II) takes commodity arbitrage and the LOP seriously. ( $_t$  is a ratio of price indexes where all prices are auction prices and the two indexes have identical weights. With effective arbitrage in auction markets, the LOP holds for each commodity and  $s_t$  tends to track ( $_t$ .

If both central banks target their price level, they stabilize ( $_t$  and the exchange rate. In that case exchange rates would be stationary as they were under the true gold standard where gold flows stabilized relative price levels.

If either central bank targets inflation, price ratios tend to have unit roots because at least one central bank allows changes in its price level to accumulate. Since Taylor rules target inflation, not price levels, they would imply that, with auction prices, ( $_t$  would have a unit root. Here exchange rates are flexible and ( $_t$  is roughly a random walk.

 $e_t$  in eq. (IV) describes changes in the domestic monetary base. <sup>16</sup> It assumes developed financial markets where governments finance surpluses or deficits by buying or selling securities in open markets. Those transactions affect interest rates, but they do not directly affect the stock of money or PPP. In emerging markets, governments often finance expenditures by selling securities either directly or indirectly to the central bank. Those sales generate inflation, but they produce smaller liquidity effects than equivalent open market operations because they do not involve central bank purchases of short-term securities in the open market.

<sup>&</sup>lt;sup>15</sup> The belief commodity arbitrage fails is based on research like Engel and Rogers (1996), Asplund and Friberg (2001) and Parsley and Wei (2001), all of which use prices from retail markets where arbitrage is not possible.

<sup>&</sup>lt;sup>16</sup> The precise nature of  $e_t$  is not critical. It could be some other appropriate monetary aggregate.

A positive D implies macro-stabilization. When unemployment is above the natural rate  $u_t$  is positive and the Fed acquires short-term assets. When  $u_t$  is negative, it does the opposite. Flexible rates give central banks the freedom to engage in macro-stabilization.

Responses to  $u_t$  are symmetric, but only for simplicity. I know of no central bank that responds symmetrically to high and low unemployment. Asymmetric responses help explain the Time Dependency Puzzle.

A positive X implies intervention. With full sterilization, intervention does not affect the monetary base. With less than full sterilization, selling sterling reduces the monetary base and buying increases it.  $v_t$  captures random changes in the base due to things like bad weather.

Eq. (V) is the most important equation in the model. It describes the nominal interest rate differential ( $\iota$ ). C describes the inflationary effects of open market operations and  $\Lambda$  describes their liquidity effects. If  $C \square \Lambda$  is negative expansionary open market operations lower ( $\iota$ ) and there is a *net* liquidity effect as in most Keynesian models.

When  $\Lambda$  is zero and there is no intervention, the Fisher equation holds. In that case, the model implies that increasing the monetary base by x% percent per year increases prices by x% per year. With foreign prices constant for simplicity, the exchange rate rises by x% per year. With foreign expected inflation zero for simplicity, ( $_t$  would be positive. With ( $_t$  and  $\Delta s_t$  both positive,  $^{\wedge}$  is positive. If ( $_t$  is constant,  $^{\wedge}$  is one. This link between expansionary open market operations, inflation, interest rate differentials and depreciation seems to be the basic idea behind UIP.

Liquidity effects weaken that link. Even with ( $_t$  constant, a liquidity effect, *i.e.*, a positive  $\Lambda$ , implies that ( $_t$  is less than x%, which implies that UIP fails in the sense that  $^i$  is less than one. If there is a net liquidity effect, then ( $_t$  and  $^i$  are negative. Liquidity effects help explain the negative  $^i$  under flexible rates, but not other regimes, because central banks have more freedom to engage in the kind of open market operations that produce liquidity effects under flexible rates than under other regimes.

Liquidity effects fade with maturity and inflationary effects strengthen. The changes in these two effects as maturity increases helps explain the Maturity Puzzle.

Monetary regimes also affect  $C \square \Lambda$ . Liquidity effects tend to be stronger in stable monetary regimes. Inflationary effects tend to be stronger in unstable monetary regimes. The link between  $C \square \Lambda$  and inflation is important for the Inflation and Outlier Puzzles.

There is also a time dimension to liquidity effects. They disappear over time as banks, households and firms adjust their portfolios and commodity prices change. This time dimension helps explain the temporary deviations from UIP found in the VAR research cited earlier. It also helps explain the Maturity Puzzle.

The Fisher equation plays a key role in UIP. Equation (V) says that the Fisher equation holds only under certain conditions. It holds when  $\Lambda$  and X are zero. There is no liquidity effect and there is no intervention.

Eq. (VI) describes expected inflation. How strongly  $E(\Delta(t+1|I_t))$  responds to changes in the monetary base depends on the inflationary effects of open market operations denoted C. C depends on the time horizon and monetary regime. The shorter the time horizon the smaller is C. A substantial increase in the monetary base today would not cause much of a price increase tomorrow, but it should within a year. In a highly stable monetary regime like most of post WWII Germany, one would expect C to be relatively small. In inflationary regimes like Germany in the early 1920s or the highly unstable monetary conditions in France during the early 1920s, one would expect C to be larger. How C responds to inflation helps explain the Inflation and Outlier Puzzles.

Eq. (VII) describes actual inflation  $\Delta(t_{t+1})$ . It equals expected inflation plus  $x_{t+1}$ , as required by the assumption of Rational Expectations.

#### 4.3. ^.

The Appendix shows the solution for  $^{\wedge}$  using the full model. It is complex. Part of that complexity is the result of three things that most other explanations for the failure of UIP ignore: (1) that  $(\lambda \Box 1)$  might not be zero, (2) that several different kinds of shocks affect  $^{\wedge}$  and (3) the possibility that those "shocks" might not be white noise. But most of the complexity is the result of including intervention.

4.3.1. No Intervention: Eq. (10) shows the solution for  $\land$  in a "stripped down" version of the model where ( $\lambda \Box 1$ ), V, U and R are all zero and there is no intervention. That is when spot rates are "exogenous" in the CIP equation, all shocks except  $z_t$  are white noise and there is no intervention. These are common implicit assumptions in most of the relevant models and simplify comparing this model to others.

$$^{1} = \{C(C \square \Lambda)[D^{2}\sigma 2 + \sigma 2]\}/\{(C \square \Lambda)^{2}[D^{2}\sigma 2 + \sigma 2] + \sigma\}$$

$$(10)$$

Although the literature clearly recognizes that real differentials can cause UIP to fail as in eq. (10), I know of no one who claims that real differentials explain the observed forward bias and failure of UIP.

Dropping  $\sigma$  produces an interesting version of eq. (10) that highlights liquidity effects.

$$^{\wedge}_{2} = C/(C \square \Lambda) \tag{11}$$

If  $\Lambda$  is zero,  $^{\wedge}_2$  is one. If  $\Lambda$  is greater than C,  $^{\wedge}_2$  is negative.

The idea behind eqs. (10) and (11) is simple and does not depend on the specifics of this model. It holds even when exchange rates are not flexible. However flexible rates give central banks more freedom to conduct the open market operations that produce liquidity effects.

Consider an expansionary open market operation. With no liquidity effect, let it produce actual inflation, expected inflation, a positive interest rate differential and depreciation. As long as they are equal,  $^{\land}_2$  is one. If liquidity effects partially offset expected inflation, the nominal differential is smaller than the depreciation and  $^{\land}_2$  is less than one. If there is a net liquidity effect,  $^{\land}_2$  is negative because there is depreciation and the differential is negative.

4.3.2. Intervention: Central banks in developed countries with flexible rates routinely lean against the wind and sterilize their intervention. Countries with managed rates have less incentive to do so because they are usually more concerned with longer run movements in their exchange rate. Even if they did lean against the wind, countries with managed rates would have less incentive to sterilize. Sterilized intervention reinforces liquidity effects and helps explain why ^ are negative when exchange rates are flexible, but not otherwise.

Eq. (12) adds intervention to eq. (11).

$$\wedge_3 = \left\{ \left[ \Pi C(C \square \Lambda) + (\Pi C)^2 \Omega \{\Phi / (1 \square \Phi^2) \} \right] \left[ D^2 \sigma^2 + \sigma^2 \right] + \Pi^2 \Omega \{\Phi / (1 \square \Phi^2) \} \left[ \sigma^2 + \sigma^2 \right] \right\}$$

$$/\{[(C \square \Lambda)^2 + (\Omega \Pi C)^2 \{1/(1 \square \Phi^2)\}][D^2 \sigma^2 + \sigma^2] + (\Omega \Pi)^2 \{1/(1 \square \Phi^2)\}[\sigma^2 + \sigma^2]\}$$

(12) Where  $\Pi$  equals 1/(1+ZX),  $\Omega$  equals  $[(\Lambda \Box C)F\Box H]X$ , and  $\Phi$  equals  $[ZX\Box CFX]/[1+ZX]$ .

 $\Omega$  captures the main effects of leaning against the wind.<sup>18</sup> Earlier  $x_t$  and  $\varepsilon_t$  did not affect  $\wedge$  because  $\Omega$  was zero. Now they do. With full sterilization (F=0),  $\Phi$  is positive, but less than one, and  $\Omega$  is negative. As a result, all of the terms in the numerator of eq. (12) containing  $\Omega$  are negative. Leaning against the wind with full sterilization reduces  $\wedge$ .

<sup>&</sup>lt;sup>17</sup> Note that in eq. (10),  $\wedge_1$  goes to zero as ( $\mathbb{C}[\Lambda]$ ) goes to zero.

 $<sup>^{18}</sup>$   $\Pi$  plays a minor role.

The insights provided by eqs. (11) and (12) help explain why ^ are negative when rates are flexible, but not otherwise. It is difficult to see how market "flaws" like "adverse selection", "career risk", "deep habits" or "infrequent portfolio decisions" could explain this difference in regimes because there is no obvious reason to think that these flaws would be greater under flexible rates than under other regimes. In addition, one would expect "flaws to be greater in emerging than in developed markets, but the forward bias and failure of UIP is greater in developed than in emerging markets.

To see how sterilized intervention reduces  $\land$ , consider how a central bank responds to a positive  $\varepsilon_t$ . It sells sterling to moderate the rise in the exchange rate. To sterilize that sale it acquires domestic short-term assets. To rebuild its holdings of foreign exchange, the central bank sells foreign T bills.

Sterilized leaning against the wind contributes to negative  $^{\land}$  when two things happen: <sup>19</sup> First sterilization produces negative ( $_{t}$ . Second leaning against the wind produces positive  $\Delta s_{t+1}$ .

Sterilized intervention also can help explain the Time Dependency Puzzle. ^ changes over time as the intensity of intervention and the degree of sterilization change over time.

### 4.4. Risk premiums.

Ignoring transaction costs, equilibrium implies that expected speculative returns equal risk premiums. That equilibrium implies that exogenous increases in risk premiums increase equilibrium expected returns by reducing the incentive to speculate. It also implies that exogenous increases in those expected returns increase risk premiums by increasing the incentive to speculate. But that equilibrium condition says nothing about "causation"; it only says that the two must be numerically equal. As far as I am aware, no one has explained how risk premiums create expected speculative returns.

Consider the following mental experiment: Start with both sides of that equilibrium condition equal to zero where no one is speculating in either currency. First consider the effects of an exogenous reduction in perceived risk. There may be indirect effects, but there are no direct effects. No one buys speculative assets because of the reduced perceived risks because there is no expected return.

Now, starting with the same initial equilibrium, consider the liquidity effects of an open market operation. Those liquidity effects create expected speculative returns. Those expected

 $<sup>^{19}</sup>$ The following discussion assumes an initial equilibrium where both  $\Delta s$  and ( are zero.

returns induce speculators to take uncovered positions. Those uncovered positions create risk premiums.

This paper concentrates on explaining expected speculative returns. Risk premiums are important, but for simplicity this paper ignores them. Modeling the dynamic adjustment process between risk premiums and expected speculative returns is far beyond the objectives of this paper, which are to show how sterilized leaning against the wind and liquidity effects can produce the expected speculative returns that cause UIP to fail.

#### 4.5. Forward bias.

CIP implies that ^ and ^ are two sides of the same coin. By explaining the failure of UIP the model also explains forward bias.

For example, eq. (13) describes the forward bias when there is no intervention. As in eq. (11),  $(\lambda \Box 1)$  and  $\sigma$  are zero and "shocks" are white noise.

$$E(\mathbf{s}_{t+1}|\mathbf{I}_t) \prod_{t} \mathbf{f}_t = \Lambda(\mathbf{D}\mathbf{u}_t + \mathbf{v}_t) \tag{13}$$

In spite of Rational Expectations, liquidity effects drive a rational wedge between forward rates and expected future spot rates.

To see the economics behind eq. (13) start with all actual and expected changes zero. Then a positive  $v_t$  creates one percent actual inflation, expected inflation and depreciation. First consider the case where  $\Lambda$  is zero and the Fisher equation holds. With  $\sigma$  zero,  $E(\Delta s_{t+1}|I_t)$ ,  $\Delta s_{t+1}$ , (t and  $f_t \square s_t$  are all one percent. There is no forward bias,  $E(s_{t+1}|I_t)$  equals  $f_t$ .

Now consider the same case where  $\Lambda$  is positive. A positive  $\Lambda$  violates the Fisher equation and reduces both  $(t_t \text{ and } f_t \ | \ s_t)$ , but it does not directly affect  $E(\Delta s_{t+1}|I_t)$  or  $\Delta s_{t+1}$ . As a result, even though expectations are rational,  $f_t$  is less than  $E(s_{t+1}|I_t)$  and there is forward bias.

#### 5. Related puzzles.

The articles explaining forward bias and the failure of UIP listed in the Introduction occasionally point out when their explanation is consistent with one of the related puzzles. For example Burnside et al (2009) point out that their approach is consistent with the Inflation Puzzle. But none of those articles discuss whether or not their explanation is consistent with *all* the related puzzles described above.

This section considers how well the model in Table 1 explains all the related puzzles. By doing so it puts this paper at a disadvantage. It does so in the hope that readers will recognize that the crucial issue is not how well this paper explains the related puzzles, but how well it

explains all related puzzles *relative* to the ability of articles listed in the Introduction to do so. For example, sterilized intervention and liquidity effects provide only a partial explanation for the Development Puzzle. But none of the articles listed in the Introduction apparently provides even a partial explanation, at least no one has made such a claim.

Discussions here are brief because a thorough discussion of each puzzle would be as long as this paper.

## 5.1. Carry Trade Puzzle.

Neither Burnside et al (2008) or Hochradl and Wagner (2010) suggest that some earlier explanation for negative ^ or ^ can explain this puzzle. Hochradl and Wagner (2010) stress the problems associated with trying to explain this puzzle with risk premiums and conclude that the related limits to speculation explanation does not provide a complete explanation.

The limits to speculation motivate Baillie and Chang (2011). They point out how momentum traders responding to carry trade profits could increase deviations from UIP. But Baillie and Chang (2011) do not try to explain what causes the deviations from UIP that motivate the Carry Trade. Their primary finding is an upper regime where UIP appears to hold and a lower regime where there is forward bias.

The model in Table 1 shows how liquidity effects create the expected speculative returns that drive the Carry Trade. Banks and other institutions with relatively low transaction costs take risky positions based on rational expected returns. Those risky positions create risk premiums.

### 5.2. Commodity puzzle.

As far as I am aware, no one claims that any of the articles listed in the Introduction explains why there is little of no forward bias in auction commodity markets. The model in Table 1 provides a simple explanation. Liquidity effects and sterilized intervention cause forward bias in foreign exchange markets. However those liquidity effects and intervention do not directly affect auction commodity markets, and governments do not "lean against the wind" in such markets. As a result, nothing in commodity markets drives a rational wedge between forward prices and expected future prices.

# 5.3 Development Puzzle.

Although Grossman et al (2014) find a bit weaker Development Puzzle using the euro and

<sup>&</sup>lt;sup>20</sup> Even developed countries intervene directly or indirectly in basic commodities. Agricultural price supports and export subsidies are examples. But that is not the same as leaning against the wind in the auction markets where those products are bought and sold.

pound sterling, the primary Development Puzzle is that, on average, when the U.S. is the home country, ^ are negative for developed countries, but close to zero for emerging countries.<sup>21</sup> This subsection concentrates on the primary Development Puzzle. This puzzle is probably as much about flexible versus managed exchange rates as it is about developed versus emerging.

Neither Bansal and Dahlquist (2000) nor Frankel and Poonawala (2010) suggest that any of the articles listed in the Introduction provide a solution for the Development Puzzle. Frankel and Poonawala (2010, 597) go farther and point out that risk premiums do not completely explain the Development Puzzle.

Most of the explanations listed in the Introduction are difficult to reconcile with the Development Puzzle. With just a couple of exceptions, *e.g.*, McCallum (1994), Anker (1999) and Lafuente and Ruiz (2006), those explanations involve market "flaws", *e.g.*, "deep habits" or "infrequent portfolio decisions". The Development Puzzle is difficult to reconcile with such "flaws". One would expect them to be larger in emerging than developed markets, but there is more bias in developed than in emerging markets.

Trying to explain the Development Puzzle with risk premiums and flawed expectations creates the same problem. One would expect risk premiums to be larger and expectations more flawed in emerging markets than in developed markets, but there is more bias in developed than in emerging markets.

A closer look at the ^ in Bansal and Dahlquist (2000) and Frankel and Poonawala (2010) suggests that the Development Puzzle is as much about flexible versus managed exchange rates as it is about developed versus emerging. All of the developed countries in both articles have flexible rates over at least part of the interval and all but a couple have flexible rates over the entire interval. Many ^ are significantly negative and the average ^ for all those countries is clearly negative. Four emerging countries have flexible rates, India, South Africa, the Philippines and Mexico. Most of those ^ are negative and one is negative and significant. Their average is negative.

<sup>&</sup>lt;sup>21</sup> Note that Greece and Portugal are emerging in Bansal and Dahlquist (2000) but developed in Frankel and Poonawala (2010). I treat both countries as developed.

<sup>&</sup>lt;sup>22</sup> Frankel and Poonawala (2010) use premiums for both developed and emerging countries. Bansal and Dahlquist (2000) use premiums for developed countries, but they use interest differentials for emerging. Therefore, strictly speaking, they estimate ^ for developed countries and ^ for emerging countries. Given the lack of support for CIP in emerging markets, their implicit assumption that it holds for emerging markets is suspect.

Combining both articles, almost all the ^s for emerging countries with managed rates are positive and not significantly different from zero. Three are negative but none are significant. Two are positive and significant.

Whether rates are flexible or managed, relatively large transaction costs probably bias ^ in all emerging countries toward zero. Transaction costs introduce a neutral range within which any economic link between changes in exchange rates and premiums disappears. According to Burnside et al (2007), bid-ask spreads are two to four times larger in emerging markets.

In addition to that direct effect, transaction costs appear to have an indirect effect by increasing deviations from CIP. As Paya et al (2010) point out, deviations  $d_t$  from CIP bias  $^{\land}$  toward zero. The combination of these two effects of transaction costs presumably bias  $^{\land}$  for all emerging countries toward zero.

For some emerging countries with managed rates, peso problems help explain why ^ are close to zero. Flood and Rose (1996) estimate that the peso effect during the ERM was []0.5. Adding 0.5 to the ^ for emerging countries with managed rates would put them on a par with the ^ during the classical gold standard and during the ERM after accounting for peso effects.

For some emerging countries with managed rates like Hong Kong, Saudi Arabia and the UAE,  $\sigma$ 2 is so small that there is nothing to explain. As an example, the  $\sigma$ 2 for Switzerland in Frankel and Poonawala (2010) is 8.03 while for Hong Kong, Saudi Arabia and the UAE it is 0.01 or less.

For those emerging countries with flexible rates, Table 1 explains why their ^s are generally negative. Their central banks have more freedom to engage in the kind of open market operations that produce liquidity effects, they are more likely to lean against the wind and they are more likely to sterilize that intervention.

### 5.4: Inflation and Outlier Puzzles.

The Inflation Puzzle is that ^ tends to increase with inflation. The Outlier Puzzle is that ^ or ^ tends to increase with outlier interest rate differentials or forward premiums; both of which are likely to be the result of outlier inflation. Both puzzles are based entirely on evidence from developed countries with flexible exchange rates.

Miller (2014, 13) does not suggest an explanation for the Outlier Puzzle. He concludes his discussion by saying "Therefore, the reason why non-outlier IDs yield negative estimates of  $\beta$  appears to be a key toward solving the UIP puzzle." (Note that his  $\beta$  is my b.) Regarding the

Inflation Puzzle, Miller (2014, 9) says the following: "No one, however, has developed a UIP framework that generates a positive relation between estimates of  $\beta$  and the inflation rate.

Table 1 provides such a framework for inflation and outliers. With low inflation, liquidity effects tend to dominate inflationary effects. The resulting negative C— $\lambda$  produces negative  $\wedge$  and  $\wedge$ . As inflation increases C— $\lambda$  increases, which increases  $\wedge$  and  $\wedge$ . To the extent that forward premiums and interest rate differentials reflect inflation, the same mechanism increases  $\wedge$  and  $\wedge$  as premiums and differentials increase

### 5.5. Maturity puzzle.

Negative ^ for short maturities and positive ^ for long maturities is the conventional Maturity Puzzle. Alexius (2001) does not suggest any solution for this puzzle. Chinn and Meredith (2004, 419) point out that "None of the standard explanations for the UIP puzzle – risk premiums, expectational errors, or peso problems – appear at first glance to offer an explanation for why the result should be so different......" They then go on to develop an extension of McCallum (1994) to explain this puzzle. Chinn (2006) refers to several possible solutions including the extension of McCallum, a "preferred habitat" explanation and differing expectations similar to those in Miller (2014).

Sterilized leaning against the wind and liquidity effects can help explain this puzzle. As the maturity of (t) increases  $C \square \Lambda$  increases as liquidity effects fade and inflationary effects increase. An open market operation or intervention that would produce a positive  $\Delta s_{t+1}$  and negative (t) at short maturities produces a positive  $\Delta s_{t+1}$  and a positive (t) at long maturities.

The model also explains Miller's version of the Maturity Puzzle. Let a positive  $v_t$  or  $u_t$  produce a positive  $\Delta s_{t+1}$  and negative (t at short maturities. As maturity increases,  $C \square \Delta t$  increases until (t is positive. The result is a positive  $\Delta s_{t+1}$  and negative (t at short maturities with a positive (t for long maturities. t is negative for short maturities, but the short-maturity t and long maturity (t are both positive.

# 5.6. Time Dependency Puzzle.

As pointed out earlier, the Time Dependency Puzzle has two parts: (1) Why is there relatively little time dependency before the current float? and (2) Why is there so much time dependency during the current float?

<sup>&</sup>lt;sup>23</sup> Larger deviations from CIP at longer maturities also can help explain the Maturity Puzzles. For some evidence of such deviations see Fletcher and Taylor (1996)

It is difficult to see how explanations for forward bias and/or the failure of UIP like "fads and fashions" or "adverse selection" could explain why there is so much time dependency under the current float and so little under other regimes.<sup>24</sup>

Table 1 provides a simple explanation: First central banks are more likely to lean against the wind and sterilize under flexible rates than under other regimes. Second flexible rates give central banks the freedom to engage in the kind of open market operations that produce liquidity effects.

For intervention and liquidity effects to explain time dependency one needs to drop some of the simplifying assumptions like constant C,  $\Lambda$ ,  $\lambda$ , F, X, F, V, U, R,  $\sigma$ 2,  $\sigma$ 2,  $\sigma$ 2, and  $\sigma$ 2. As pointed out in the discussion of the model, the economics behind the model imply that most of these parameters and shocks change over time. For example C increases as inflation increases. As a result, changes in inflation change  $\wedge$ . Intervention, sterilization and  $\sigma$ 2 vary over time as foreign exchange markets cycle between periods of calm and turbulence. Those changes change  $\wedge$ .

Converting this potential explanation into a convincing explanation requires showing that observed changes in these parameters and shocks explain the observed time dependency of ^ and ^. That research is far beyond the objectives of this paper.

#### 6. Summary.

Under regimes other than flexible exchange rates, there is forward bias and UIP fails. But in those regimes, after accounting for peso problems, the coefficients for the relevant test equations tend to be positive and significant, but significantly less than one. Under flexible exchange rates those same coefficients are systematically negative and often negative and significant. It is difficult to see how most of the explanations for forward bias and/or the failure of UIP listed in the Introduction could explain this difference between flexible rates and other regimes.

This paper shows how the liquidity effects of open market operations and sterilized "leaning against the wind" in foreign exchange markets can explain the negative coefficients when rates are flexible..

In addition, liquidity effects and sterilized intervention appear to explain the closely related puzzles listed in the Introduction better than the other explanations listed there.

<sup>&</sup>lt;sup>24</sup> Risk premiums can explain part of this puzzle. It can explain why there is less time dependency under the gold standard. Given the peso problem under pegged rates and many managed regimes, it is not obvious that risk premiums are smaller under those regimes than under flexible rates.

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### **APPENDIX**

- $\land = \{ \left[ \Pi C(C \square \Lambda) \{ 1/(1 \square \Phi U) \} + (\lambda \square 1) (C \square \Lambda)^2 \Pi \{ U + [(\Phi \square 1)/(1 \square \Phi U)] \} \right]$
- $+\Pi^2C^2\Omega\{(U+\Phi)/[(1[\!]\Phi U)(1[\!]\Phi^2)]\} \ +(\lambda[\!]1)(C[\!]\Lambda)\Pi^2\Omega C\{[U^2(1+\Phi)[\!]U[\!]\Phi]/[(1[\!]\Phi U)(1+\Phi)]\}$
- $+\Omega C\Pi^2(\lambda \square 1)(C \square \Lambda)\{(1 \square U)/[(1 \square \Phi U)(1 + \Phi)]\}$
- $+\Pi^2\Omega(\lambda \square 1)^2(C \square \Lambda)^2\{[(1 \square U)(\Phi \square 1 \square U \square \Phi U)]/[(1 \square \Phi U)(1 + \Phi)]\}\right]D^2\sigma 2$
- $+ \left[ (\lambda \Box 1) \Pi \{ R + [(\Phi \Box 1)/(1 \Box \Phi R)] \} \right. \\ \left. + \Pi^2 \Omega (\lambda \Box 1)^2 \{ [(1 \Box R)(\Phi \Box 1 \Box R \Box R \Phi)]/[(1 \Box \Phi R)(1 + \Phi)] \} \right] \sigma$
- $$\begin{split} + \Big[\Pi C(C \square \Lambda) \{1/(1 \square \Phi V)\} + (\lambda \square 1)(C \square \Lambda)^2 \Pi \{V + [(\Phi \square 1)/(1 \square \Phi V)]\} + (\Pi C)^2 \Omega \{(V + \Phi)/[(1 \square \Phi V)]\} \\ + (\lambda \square 1)(C \square \Lambda) \Pi^2 \Omega C \Big] \{ [V^2 (1 + \Phi) \square V \square \Phi]/[(1 \square \Phi V)/(1 + \Phi)]\} + \Pi^2 C \Omega(\lambda \square 1)(C \square \Lambda) \{(1 \square V)/[(1 \square \Phi V)/(1 + \Phi)]\} \\ + (\lambda \square 1)(C \square \Lambda) \Pi^2 \Omega C \Big] \{ [V^2 (1 + \Phi) \square V \square \Phi]/[(1 \square \Phi V)/(1 + \Phi)]\} + \Pi^2 C \Omega(\lambda \square 1)(C \square \Lambda) \{(1 \square V)/[(1 \square \Phi V)/(1 + \Phi)]\} \\ + (\lambda \square 1)(C \square \Lambda) \Pi^2 \Omega C \Big] \{ [V^2 (1 + \Phi) \square V \square \Phi]/[(1 \square \Phi V)/(1 + \Phi)]\} + (\lambda \square 1)(C \square \Lambda) \{(1 \square V)/[(1 \square \Phi V)/(1 + \Phi)]\} \\ + (\lambda \square 1)(C \square \Lambda) \Pi^2 \Omega C \Big] \{ [V^2 (1 + \Phi) \square V \square \Phi]/[(1 \square \Phi V)/(1 + \Phi)]\} + (\lambda \square 1)(C \square \Lambda) \{(1 \square V)/[(1 \square \Phi V)/(1 + \Phi)]\} + (\lambda \square 1)(C \square \Lambda) \{(1 \square V)/[(1 \square \Phi V)/(1 + \Phi)]\} + (\lambda \square 1)(C \square \Lambda) \{(1 \square V)/[(1 \square \Phi V)/(1 + \Phi)]\} + (\lambda \square 1)(C \square \Lambda) \{(1 \square V)/[(1 \square \Phi V)/(1 + \Phi)]\} + (\lambda \square 1)(C \square \Lambda) \{(1 \square V)/[(1 \square \Phi V)/(1 + \Phi)]\} + (\lambda \square 1)(C \square \Lambda) \{(1 \square V)/[(1 \square \Phi V)/(1 + \Phi)]\} + (\lambda \square 1)(C \square \Lambda) \{(1 \square V)/[(1 \square \Phi V)/(1 + \Phi)]\} + (\lambda \square 1)(C \square \Lambda) \{(1 \square V)/[(1 \square \Phi V)/(1 + \Phi)]\} + (\lambda \square 1)(C \square \Lambda) \{(1 \square V)/[(1 \square \Phi V)/(1 + \Phi)]\} + (\lambda \square 1)(C \square \Lambda) \{(1 \square V)/[(1 \square \Phi V)/(1 + \Phi)]\} + (\lambda \square 1)(C \square \Lambda) \{(1 \square V)/[(1 \square \Phi V)/(1 + \Phi)]\} + (\lambda \square 1)(C \square \Lambda) \{(1 \square V)/[(1 \square \Phi V)/(1 + \Phi)]\} + (\lambda \square 1)(C \square \Lambda) \{(1 \square V)/[(1 \square \Phi V)/(1 + \Phi)]\} + (\lambda \square 1)(C \square \Lambda) \{(1 \square V)/[(1 \square \Phi V)/(1 + \Phi)]\} + (\lambda \square 1)(C \square \Lambda) \{(1 \square V)/[(1 \square \Phi V)/(1 + \Phi)]\} + (\lambda \square 1)(C \square \Lambda) \{(1 \square V)/[(1 \square \Phi V)/(1 + \Phi)]\} + (\lambda \square 1)(C \square \Lambda) \{(1 \square V)/[(1 \square \Phi V)/(1 + \Phi)]\} + (\lambda \square 1)(C \square \Lambda) \{(1 \square V)/[(1 \square \Phi V)/[(1 \square \Phi V)/(1 + \Phi)]\} + (\lambda \square 1)(C \square \Lambda) \{(1 \square V)/[(1 \square \Phi V)/$$
- $+(\lambda \Box 1)^2(C\Box \Lambda)^2\Pi^2\Omega\{[(1\Box V)(\Phi\Box 1\Box V\Box V\Phi)]/[(1\Box \Phi V)/(1+\Phi)]\}\Big]\sigma2+\Pi^2\Omega\{\Phi/(1\Box \Phi^2)\}[\sigma2+\sigma2]\Big\}$

 $/ \big\{ \Big[ 1 + 2\Omega\Pi(\lambda \square 1) \{ (1 \square R) / (1 \square \Phi R) \} + 2[\Omega\Pi(\lambda \square 1)]^2 \{ (1 + \Phi R \square \Phi \square R) / [(1 \square \Phi R) (1 \square \Phi^2)] \} \Big] \sigma$ 

- $+ \Big[ (C \square \Lambda)^2 + 2\Omega \Pi C (C \square \Lambda) \{ U/(1 \square \Phi U) \} + 2\Omega \Pi (\lambda \square 1) (C \square \Lambda)^2 \{ (1 \square U)/(1 \square \Phi U) ] \}$
- $+ (\Omega \Pi C)^2 \{ (1 + \Phi U) / [(1 \square \Phi U) (1 \square \Phi^2)] \} + 2 \Omega^2 \Pi^2 C (\lambda \square 1) (C \square \Lambda) \{ (\Phi + U \square \Phi U \square 1) / [(1 \square \Phi U) (1 \square \Phi^2)] \}$
- $+2(\Omega\Pi)^2(\lambda \square 1)^2(C \square \Lambda)^2\{(1+\Phi U \square \Phi \square U)/[(1 \square \Phi U)(1 \square \Phi^2)]\}\Big]D^2\sigma 2$
- $+ \left[ (C \square \Lambda)^2 + 2\Omega \Pi C (C \square \Lambda) \{ V/(1 \square \Phi V) \} + 2\Omega \Pi (\lambda \square 1) (C \square \Lambda)^2 \{ (1 \square V)/(1 \square \Phi V) \} \right]$
- $+ \ (\Omega\Pi C)^2 \{ (1+\Phi V)/[(1 {\rule[b]{0.8ex}{4.5ex}} \Phi V)(1 {\rule[b]{0.8ex}{4.5ex}} \Phi^2)] \} \ + \ 2\Omega^2\Pi^2 C(\lambda {\rule[b]{0.8ex}{4.5ex}} 1)(C {\rule[b]{0.8ex}{4.5ex}} \Lambda) \{ (\Phi + V {\rule[b]{0.8ex}{4.5ex}} \Phi V {\rule[b]{0.8ex}{4.5ex}} 1)/[(1 {\rule[b]{0.8ex}{4.5ex}} \Phi V)(1 {\rule[b]{0.8ex}{4.5ex}} \Phi^2)] \} \ + \ 2\Omega^2\Pi^2 C(\lambda {\rule[b]{0.8ex}{4.5ex}} 1)(C {\rule[b]{0.8ex}{4.5ex}} \Lambda) \{ (\Phi + V {\rule[b]{0.8ex}{4.5ex}} \Phi V {\rule[b]{0.8ex}{4.5ex}} 1)/[(1 {\rule[b]{0.8ex}{4.5ex}} \Phi V)(1 {\rule[b]{0.8ex}{4.5ex}} \Phi^2)] \}$
- $+ 2\Omega^{2}\Pi^{2}(\lambda \Box 1)^{2}(C \Box \Lambda)^{2}\{(1+\Phi V \Box \Phi \Box V)/[(1\Box \Phi V)(1\Box \Phi^{2})]\}\right]\sigma^{2} + (\Omega \Pi)^{2}\{1/(1\Box \Phi^{2})\}[\sigma^{2} + \sigma^{2}\}$

Where  $\Pi$  equals  $1/(1+ZX \square (\lambda \square 1)\Omega)$ ,  $\Omega$  equals  $[(\Lambda \square C)F \square H]X$ , and  $\Phi$  equals  $[ZX \square CFX \square (\lambda \square 1)\Omega]/[1+ZX \square (\lambda \square 1)\Omega]$ .