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Physical Preconditioning for Modeling 2D Large Periodic Arrays

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1. Introduction

The number of applications for large array antennas and periodic structures grows increasingly, thus accentuating the need for efficient and accurate means to analyze these structures. A method of moments (MoM) approach provides the required accuracy, although direct MoM analyses require long matrix fill and solve times as well as large amounts of memory to store the MoM matrices. The array's intrinsic translational symmetry along with asymptotic estimates of mutual coupling between array element pairs can be used to reduce the fill time. To reduce the solution time, various algorithms have been developed that use periodicity-induced physical properties. For example, [1],[2] use an a priori estimate of the fields scattered by truncated arrays, which behave as Floquet-modulated-diffracted fields [3], to construct global basis functions. Here, instead, we investigate diffraction effects on truncated periodic structures at the field operator level. A physically-based approximate operator inverse of the electric field integral equation (EFIE) is used either to obtain an accurate first estimate of element currents, or as a preconditioner for a more rigorous, iterative solution. To illustrate, we analyze linear arrays of cylindrical elements constituting antennas or periodic structures with arbitrary element geometry and excitation, though the method can be extended to 3D problems. We show the reduction in the matrix *solution time* that is achieved, and also discuss the physical insight into array truncation effects that the method provides. The MoM matrix and preconditioner, once constructed for a given array, are excitation independent and may be used repeatedly to obtain or analyze element current distributions, element tapering and scanning effects, element input impedances, active input impedances, element-failure impact on the radiating system, or coupling between element pairs in a finite array.

2. Formulation

Consider a finite periodic array with 1D periodicity (see figure) whose elements are invariant in z with an arbitrary cross section in the x-y plane. Consider also the virtual array formed by extending the actual array to form an infinite array. Superscripts a and v denote the actual and virtual array, respectively (see figure below). The standard EFIE for the actual finite array is written as

$$Z^{aa}I^a = V^a \qquad (1)$$

which provides the current I^a on the actual array when solved. The moment matrix Z^{aa}, whose elements represent interactions between basis and testing function pairs, is assumed to be partitioned into blocks representing interactions between pairs of array elements. Consider now the actual and virtual portions comprising the infinite array. For this new problem, after proper partitioning, the EFIE and its inverse can be symbolically written as

$$
\begin{bmatrix} Z^{aa} & Z^{av} \\ Z^{va} & Z^{vv} \end{bmatrix} \begin{bmatrix} I^a \\ I^v \end{bmatrix} = \begin{bmatrix} V^a \\ V^v \end{bmatrix} \qquad \rightarrow \qquad \begin{bmatrix} I^a \\ I^v \end{bmatrix} = \begin{bmatrix} Z^{aa} & Z^{av} \\ Z^{va} & Z^{vv} \end{bmatrix}^{-1} \begin{bmatrix} V^a \\ V^v \end{bmatrix} \equiv \begin{bmatrix} Y^{aa} & Y^{av} \\ Y^{va} & Y^{vv} \end{bmatrix} \begin{bmatrix} V^a \\ V^v \end{bmatrix} \quad (2)
$$

512

which relates the actual and virtual array problems. The EFIE for the actual array (1) can be derived from (2) by requiring that the current I^v on the virtual array vanish. Combining the impedance and admittance matrices for the infinite array we derive the exact identities

$$Y^{aa}Z^{aa} = U - Y^{av}Z^{va}, \qquad \left[Z^{aa}\right]^{-1} = \left[U - Y^{av}Z^{va}\right]^{-1} Y^{aa} \qquad (3a,b)$$

to which we give two interpretations. First, when Y^{aa} is used as a preconditioner for Z^{aa} in (1), it regularizes the hypersingular part of Z^{aa} since $Y^{aa}Z^{aa}$ is equal to the identity operator U plus a diffracted operator $Y^{av}Z^{va}$ that contains no self interactions. Indeed, as shown in the figure below, when $Y^{av}Z^{va}$ operates on a current I of the actual array, it produces voltage excitations $V^v = Z^{va}I$ on the virtual array which in turn produce changes in current $Y^{av}V^v$ back on the actual array. Furthermore, the current $Y^{av}Z^{va}I$ decays away from the array truncation as a diffracted current (ignoring, for simplicity, the possible presence of modes supported by the structure). Second, (3b) relates the admittance (matrix inverse) operator $\left[Z^{aa}\right]^{-1}$ for the *finite* array to the sub-block Y^{aa} of the admittance matrix for an *infinite* array, with truncation effects represented by physical interactions between the actual and virtual array.
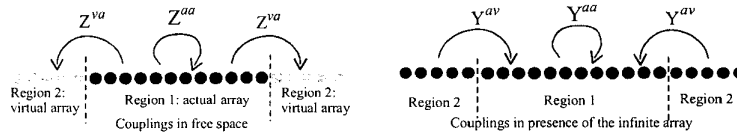
Note that Y^{aa} is the restriction of the *infinite* array admittance matrix to the finite array. Away from the edges of the array, the term $Y^{av}Z^{va}I$ is small since it represents a diffractive term that decays away from the edges. To avoid computing a matrix inverse, we therefore approximate $[U-Y^{av}Z^{va}]^{-1} \approx U + Y^{av}Z^{va}$, which, again, approaches U for currents far from the array edges. The inverse of the MoM matrix thus may be approximated as

$$\left[Z^{aa}\right]^{-1} \approx Y^P, \qquad Y^P = \left[U + Y^{av}Z^{va}\right]Y^{aa} \ . \qquad (4)$$

Further approximations may be made to any of the matrices on the RHS to simplify the preconditioner. Practically, for example, it is likely that the domains and/or ranges of these matrix operators must be windowed. In the following we suppose the array consists of N array elements. Current I^a and known vector V^a are partitioned as $I^a=[[I_1], ...[I_N]]^{\mathrm{T}}$, $V^a=[[V_1],$ $...[V_N]]^{\mathrm{T}}$, and the Z^{aa} matrix is partitioned accordingly. The preconditioner need not include all the interactions since the significant influence of an array element usually extends to just a few nearby elements. Thus only a few interactions need be taken into account in constructing the terms Y^{aa} and $Y^{av}Z^{va}$. In the next section we provide a possible algorithm to construct these terms, though other ways may still be advantageous.

3. Construction of the Preconditioner from the Infinite Array Solution

If the array is formed by N array elements, the matrix Y^{aa} is formed of N sub-blocks, though in practice we use a smaller number $2E+1$ of sub-blocks, accounting only for the interactions between the nearest E array elements to the left and right of an element. The matrix Y^{av} has infinite dimension, however, we approximate the virtual array using only F elements nearest the array edges. In practice, we may choose $F=E$ so that all the sub-blocks of Y^{av} replicate

513

those of Y^{aa}; E and F can be as small as two or three. Analogous arguments hold for the evaluation of Z^{va}, which reuses previously evaluated blocks of Z^{aa}. A key feature of the approach is the fact that matrices Y^{aa} and Y^{av} can be evaluated from the solution of the infinite array problem as follows. We denote by $[Z^{\infty}(\alpha)]$ the $N_e \times N_e$ element MoM matrix for a unit cell of the infinite array problem with progressive interelement phase shift α; $[Y^{\infty}(\alpha)]$ is its inverse. Furthermore, $[Y_n^{\infty}]$ denotes the element-to-element mutual coupling admittance matrices between two array elements of the *infinite* array, n cells apart. Matrices $[Y^{\infty}(\alpha)]$ and $[Y_n^{\infty}]$ are related by the Fourier series transform pair

$$\left[Y^{\infty}(\alpha)\right] = \sum_{n=-\infty}^{\infty}\left[Y_n^{\infty}\right]e^{-jn\alpha} = \left[Z^{\infty}(\alpha)\right]^{-1}, \qquad \left[Y_n^{\infty}\right] = \frac{1}{2\pi}\int_0^{2\pi}\left[Y^{\infty}(\alpha)\right]e^{jn\alpha}d\alpha \qquad (5a,b)$$

which is used to evaluate $[Y_n^{\infty}]$ once $[Y^{\infty}(\alpha)]$ is computed for the infinite array. Note that $[Y^{\infty}(\alpha)]$ is periodic in α with period 2π, and that, unlike the impedance matrices, $[Y_n^{\infty}]\neq[Y_n]$, i.e. the element mutual admittance matrices for infinite and finite arrays are not the same. We summarize the computation of Y^{aa} and Y^{av} as follows: (a) Determine the active impedance matrix $[Z^{\infty}(\alpha)]$ for a unit cell of the infinite array problem for $0 \leq \alpha \leq 2\pi$. This is a small array of size $N_e \times N_e$. (b) Then, compute $[Y^{\infty}(\alpha)]$ by inverting $[Z^{\infty}(\alpha)]$. (c) Compute $[Y_n^{\infty}]$ for all required n (blocks for larger n do not require significantly longer computation times) using (5b). (d) Construct Y^{aa} as the block-Toeplitz matrix having $[Y_n^{\infty}]$ as the nth block from the diagonal. Y^{av} is constructed using the same matrix sub-blocks $[Y_n^{\infty}]$.

4. Numerical Examples

In the following examples, the current on each array element is evaluated by first solving the standard EFIE $Z^{aa}I^a = V^a$ without preconditioning, then using the two simple preconditioners $Y^{aa}Z^{aa}I^a = Y^{aa}V^a$ and $Y^pZ^{aa}I^a = Y^pV^a$, in which the preconditioning matrices Y^{aa} and Y^p are defined in Sec. 2 and Eq. (4), respectively, and computed as outlined above.

Example 1 – Array of 31 Cylinders. The test array is an array of 31 conducting cylinders illuminated by a normally incident TE_z plane wave as shown in the figure. The array period is $d=1.3$m; the radius of the cylinders is 0.5m, and the frequency is 100MHz.
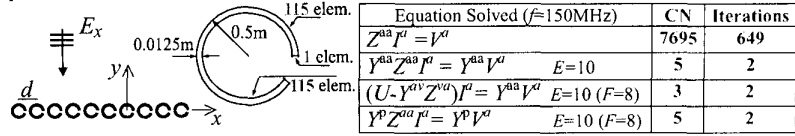

Finite array
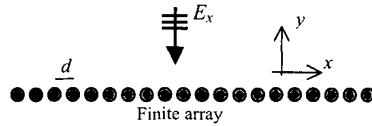
Equation Solved		CN	Iterations
$Z^{aa}I^a = V^a$		45	41
$Y^{aa}Z^{aa}I^a = Y^{aa}V^a$	$E=0$	108	25
	$E=1$	380	20
	$E=2$	78	13
	$E=3$	56	10
	$E=4$	41	6
	$E=5$	30	6
	$E=10$	20	3

In the Table, the spectral condition number (CN) and the number of BiCGstab iterations needed to solve the linear system with a relative error less than 10^{-4} are reported for the EFIE $Z^{aa}I^a = V^a$, and for various forms of its preconditioned counterpart, $Y^{aa}Z^{aa}I^a = Y^{aa}V^a$, in which Y^{aa} is constructed using different numbers E of sub-blocks. Note that the number of iterations decreases with increasing E, but no significant improvement occurs for $E > 4$.

Example 2 – Array of 10 Thick Open Cavities. An array of 10 thick open conducting cavities is illuminated by a perpendicular TE_z plane wave at $f=150$MHz, which is far from cavity internal resonance frequencies. As depicted in the figure below, each array element is finely discretized, with a total of 115 elements on the inner and outer surfaces and 1 element

on each end face, for a total of 232 subelements on each array element. Note the small CN of the preconditioned EFIE, and the dramatic reduction in the number of iterations with Y^{aa} as a preconditioner.

Equation Solved (f=150MHz)		CN	Iterations
$Z^{aa} I^a = V^a$		7695	649
$Y^{aa} Z^{aa} I^a = Y^{aa} V^a$	E=10	5	2
$(U - Y^{av} Z^{va}) I^a = Y^{aa} V^a$	E=10 (F=8)	3	2
$Y^p Z^{aa} I^a = Y^p V^a$	E=10 (F=8)	5	2

Further improvement is obtained using the preconditioned equation $(U - Y^{av} Z^{va}) I^a = Y^{aa} V^a$ (see identity (3a)) since the replacement of $Y^{aa} Z^{aa}$ by $U - Y^{av} Z^{va}$ eliminates the computation of the self interactions; effectively, we analytically extract the identity matrix U from $Y^{aa} Z^{aa}$. In a second test for this array of cavities, plane wave excitation was replaced by *single element excitation* using a voltage generator placed on top of the 4th element. Again, the system was solved in only two iterations using the previously computed matrices Z^{aa} and Y^{aa}.

Example 3 – Inverse, Approximated by Preconditioner. The two preconditioners Y^{aa} and Y^p are used here to approximate the inverse matrix $[Z^{aa}]^{-1}$. Currents directly evaluated as $I^a \approx Y^{aa} V^a$, or as $I^a \approx Y^p V^a$, without solving a linear system, are compared with the "exact" solution provided by the EFIE $Z^{aa} I^a = V^a$ solved numerically. The two previous geometries are investigated for plane wave illumination (with unity magnetic field), and comparison is made with a current sample on each array element (one very sensitive to edge effects). In the first example (array of 31 cylinders), we sample the current on the right-most part of the cylinders. Both approximations $I^a \approx Y^{aa} V^a$ and $I^a \approx Y^p V^a$ provide good results. Note that the current stabilizes away from the edge where edge effects are negligible (in this example no guided modes are excited by the truncation). In the second example (array of 10 cavities) we plot current samples on the top-most point of each element. Again both approximations $I^a \approx Y^{aa} V^a$ and $I^a \approx Y^p V^a$ provide good results. In both geometries, use of the simple preconditioner Y^{aa}, which contains low-order diffraction effects, provides good results, while the more elaborate preconditioner Y^p slightly improves the current estimate. It is expected that arrays that support guided modes (surface waves) would require the more elaborate Y^p to accurately estimate mode reflection at the edges.
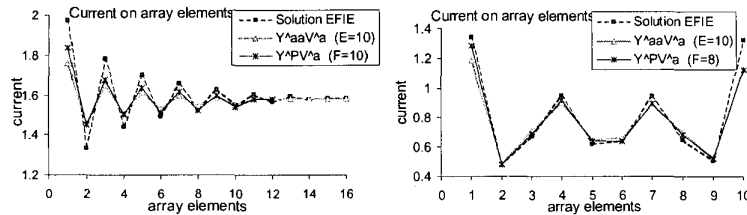
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