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# SU(3) vs. SU(3) x SU(3) Breaking in Weak Hyperon Decays

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## Abstract

We consider the predictions of chiral perturbation theory for  $SU(3)$  breaking in weak semileptonic and  $s$ -wave nonleptonic hyperon decays. By defining an expansion sensitive only to  $SU(3)$  breaking, we show that the leading corrections give rise to moderate corrections to  $SU(3)$  relations ( $\lesssim 20\%$ ), even though the *chiral* symmetry  $SU(3)_L \times SU(3)_R$  appears to be rather badly broken. This explains why  $SU(3)$  fits to weak hyperon decays work well even though chiral-symmetry breaking corrections are large. Applying these  $SU(3)$ -breaking corrections to the analysis of the EMC data, we find that the predicted value of  $\langle p | \bar{s} \gamma_\mu \gamma_5 s | p \rangle$  is reduced by  $\simeq 35\%$ , suggesting that the “EMC effect” may be less striking than commonly thought.

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## 1. Introduction

In this paper, we consider corrections to the  $SU(3)$  predictions for weak semileptonic and  $s$ -wave nonleptonic hyperon decay rates. The  $SU(3)$  predictions are valid in the limit where  $m_u = m_d = m_s$  (we neglect electromagnetism), and experimentally they work to better than 20%. This remarkable agreement is certainly not due to the fact that the quark masses are nearly equal; if they were, the  $\pi^0$  and  $\eta$  would be nearly degenerate in mass, while we know that  $m_\eta/m_\pi \simeq 4$ . Understanding why some  $SU(3)$  predictions work well while others fail completely has been a theoretical challenge since the discovery of these relations.

To make progress on this question it is clearly necessary to have a systematic framework to study deviations from  $SU(3)$  symmetry. Chiral perturbation theory provides such a framework, giving a rigorous expansion around the *chiral* limit:  $m_u, m_d, m_s \rightarrow 0$ . In the chiral limit, the octet mesons  $\pi$ ,  $K$ , and  $\eta$  are massless Nambu–Goldstone bosons whose couplings are constrained by the low-energy theorems of spontaneous symmetry breaking. These theorems can be encoded in an effective lagrangian with a non-linearly realized  $SU(3)_L \times SU(3)_R$  symmetry. The lowest-order predictions of chiral perturbation theory embody the  $SU(3)$  predictions, and deviations from  $SU(3)$  symmetry relations can be studied by considering corrections to the chiral limit.

Chiral perturbation theory for baryons was recently reformulated by Jenkins and Manohar using an effective lagrangian in which the baryons are treated as heavy fields [1]. These authors computed the  $O(m_s \ln m_s)$  corrections to the hyperon weak decay form factors [1][2] and found that corrections to the lowest-order predictions were  $\sim 100\%$ . The logarithmically-enhanced corrections are not expected to dominate the uncalculable  $O(m_s)$  contributions in the real world. However, the large size of the logarithmically-enhanced corrections does suggest that chiral perturbation theory is breaking down for these processes, and makes the success of the lowest-order predictions difficult to understand. Also puzzling is that the “corrected” predictions still fit the data well, at the price of large shifts in the values of the couplings which define the chiral expansion. For example, in ref. [1], the values to the axial-vector form factors including the corrections were found to give  $D = 0.56$ ,  $F = 0.33$ , while their lowest-order fit gives  $D = 0.80$ ,  $F = 0.50$ .

The authors of ref. [1] propose that the breakdown of chiral perturbation theory for baryons coupled to mesons is due to the presence of the nearby decuplet states [5]. They find that including decuplet intermediate states reduces the size of the logarithmically-enhanced corrections, but they still require large shifts parameters to accommodate the data. We will not consider this point of view here.

In this paper, we propose a well-motivated and well-defined resummation of the chiral expansion which is sensitive only to  $SU(3)$  breaking. We compute the logarithmically-enhanced contributions to the weak decay form factors in this expansion, and find that all corrections are  $\lesssim 20\%$ . We conclude that there is no reason to believe that this  $SU(3)$  expansion is breaking down, even though the chiral expansion does not seem to work well. This is somewhat surprising, since both expansions are controlled by  $m_s$  in the limit  $m_s \gg m_u, m_d$ . Our conclusion is also supported by the fact that predictions for the  $p$ -wave nonleptonic decays, which follow from chiral symmetry but not from  $SU(3)$  alone, do not work well.

We also apply our results to consider the effects of  $SU(3)$  breaking on the interpretation of the EMC effect. We find that  $SU(3)$  breaking reduces the predicted value of  $\langle p | \bar{s} \gamma_\mu \gamma_5 s | p \rangle$  by 35%, reducing the size of the ‘‘EMC effect.’’

The plan of this paper is as follows. In section 2, we briefly review the effective lagrangian formalism we will use to carry out our computations. In section 3, we discuss the computation of the semileptonic decay rates. In section 4, we apply our results to the EMC data. In section 5, we discuss the computation of the nonleptonic decay rates. Section 6 contains our conclusions.

## 2. The Effective Lagrangian

In this section, we briefly review the effective lagrangian we use to carry out the computation. The notation and conventions we use are the same as those of ref. [7]. We briefly review the formalism here for completeness. The reader familiar with this formalism is urged to skip to section 3.

### 2.1. Mesons

The field

$$\xi(x) = e^{i\Pi(x)/f}, \quad (1)$$

is taken to transform under  $SU(3)_L \times SU(3)_R$  as

$$\xi \mapsto L\xi U^\dagger = U\xi R^\dagger, \quad (2)$$

where this equation implicitly defines  $U$  as a function of  $L$ ,  $R$ , and  $\xi$ . The meson fields are

$$\Pi = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & K^0 \\ K^- & \bar{K}^0 & -\frac{2}{\sqrt{6}}\eta \end{pmatrix}. \quad (3)$$

Since we are interested in matrix elements of the vector- and axial-vector Noether currents, we add source terms

$$\delta\mathcal{L} = \mathcal{V}^\mu J_\mu^V + \mathcal{A}^\mu J_\mu^A \quad (4)$$

by defining the covariant derivatives

$$D_\mu\xi \equiv \partial_\mu\xi - i\ell_\mu\xi, \quad D_\mu\xi^\dagger \equiv \partial_\mu\xi^\dagger - ir_\mu\xi^\dagger. \quad (5)$$

(Note that  $(D_\mu\xi)^\dagger \neq D_\mu\xi^\dagger$ .) Here,

$$r_\mu = \mathcal{V}_\mu + \mathcal{A}_\mu, \quad \ell_\mu = \mathcal{V}_\mu - \mathcal{A}_\mu. \quad (6)$$

The effective lagrangian is most conveniently written in terms of

$$V_\mu \equiv \frac{i}{2} (\xi D_\mu\xi^\dagger + \xi^\dagger D_\mu\xi), \quad A_\mu \equiv \frac{i}{2} (\xi D_\mu\xi^\dagger - \xi^\dagger D_\mu\xi), \quad (7)$$

which transform under *local*  $SU(3)_L \times SU(3)_R$  as

$$V_\mu \mapsto UV_\mu U^\dagger + iU\partial_\mu U^\dagger, \quad A_\mu \mapsto UA_\mu U^\dagger. \quad (8)$$

The covariant derivative

$$\nabla_\mu A_\nu \equiv \partial_\mu A_\nu - i[V_\mu, A_\nu], \quad (9)$$

transforms under *local*  $SU(3)_L \times SU(3)_R$  as

$$\nabla_\mu A_\nu \mapsto U\nabla_\mu A_\nu U^\dagger. \quad (10)$$

The chiral symmetry is broken explicitly by the quark masses. (We neglect the effects of electromagnetism in this paper.) We will ignore isospin breaking, so that the quark mass matrix is taken to be

$$M_q = \begin{pmatrix} \hat{m} & & \\ & \hat{m} & \\ & & m_s \end{pmatrix}. \quad (11)$$

It is convenient to define the even- and odd-parity fields

$$M \equiv \frac{1}{2} (\xi^\dagger M_q \xi^\dagger + \text{h.c.}) \mapsto UMU^\dagger, \quad (12)$$

$$P \equiv \frac{1}{2i} (\xi^\dagger M_q \xi^\dagger - \text{h.c.}) \mapsto UPU^\dagger. \quad (13)$$

The simple transformation rules of the fields defined above makes it easy to write down the effective lagrangian. For example, the leading terms can be written

$$\mathcal{L}_0 = f^2 \text{tr}(A^\mu A_\mu) + af^3 \text{tr} M. \quad (14)$$

## 2.2. Baryons

Because we are interested in processes with characteristic energy much smaller than baryon masses, the baryons may be treated as heavy particles [3][1]. The basic idea is to write the baryon momentum as  $P = Mv + k$ , where  $M$  is the common baryon mass in the  $SU(3)$  limit and  $v$  is chosen so that all of the components of the residual momentum  $k$  are small compared to hadronic scales,  $\Lambda$ , for the process of interest. The effective lagrangian is then labelled by  $v$  and is written in terms of fields  $B$  satisfying the positive energy condition  $\not{v}B = B$ , and whose momentum modes are the residual momenta of the baryons. This explicitly removes  $M$  as a kinematic scale in the problem.

The octet baryon fields  $B$  transform under  $SU(3)_L \times SU(3)_R$  as

$$B \mapsto UBU^\dagger. \quad (15)$$

Explicitly, we have

$$B = \begin{pmatrix} \frac{1}{\sqrt{2}}\Sigma^0 + \frac{1}{\sqrt{6}}\Lambda & \Sigma^+ & p \\ \Sigma^- & -\frac{1}{\sqrt{2}}\Sigma^0 + \frac{1}{\sqrt{2}}\Lambda & n \\ \Xi^- & \Xi^0 & -\frac{2}{\sqrt{6}}\Lambda \end{pmatrix}. \quad (16)$$

The lowest order terms in the effective lagrangian involving baryon fields are

$$\begin{aligned} \mathcal{L} = & \text{tr}(\bar{B}i v \cdot \nabla B) + 2D \text{tr}(\bar{B}s^\mu \{A_\mu, B\}) + 2F \text{tr}(\bar{B}s^\mu [A_\mu, B]) \\ & + \sigma \text{tr}(M) \text{tr}(\bar{B}B) + b_D \text{tr}(\bar{B}\{M, B\}) + b_F \text{tr}(\bar{B}[M, B]), \end{aligned} \quad (17)$$

where the spin matrix is given by

$$s^\mu \equiv \frac{1}{2}(\gamma^\mu - \not{v}v^\mu)\gamma_5, \quad (18)$$

and the covariant derivative acts on  $B$  as in eq. (9).

## 3. Semileptonic Decays

In this section, we consider the  $\Delta S = 1$  semileptonic decays of hyperons. These decays are governed by the form factors

$$\langle B_a | J_{\mu c}^V(0) | B_b \rangle = \bar{u}(p_a) \left[ f_1^{abc}(q^2)\gamma_\mu + \frac{if_2^{abc}(q^2)}{M_a + M_b}\sigma_{\mu\nu}q^\nu + \frac{if_3^{abc}(q^2)}{M_a + M_b}q_\mu \right] u(p_b), \quad (19)$$

$$\langle B_a | J_{\mu c}^A(0) | B_b \rangle = \bar{u}(p_a) \left[ g_1^{abc}(q^2)\gamma_\mu\gamma_5 + \frac{ig_2^{abc}(q^2)}{M_a + M_b}\sigma_{\mu\nu}\gamma_5q^\nu + \frac{ig_3^{abc}(q^2)}{M_a + M_b}\gamma_5q_\mu \right] u(p_b), \quad (20)$$

where  $q \equiv p_a - p_b$ . In the  $SU(3)$  limit  $m_u = m_d = m_s$ , the form factors at zero momentum transfer are determined in terms of two parameters,  $D$  and  $F$ :  $f_2(0) = f_3(0) = g_2(0) = g_3(0) = 0$ , the  $f_1(0)$  are  $SU(3)$  Clebsch–Gordan coefficients, and the  $g_1(0)$  are simple linear combinations of  $D$  and  $F$  (see below). We consider the form factors at zero momentum transfer because the masses of the baryon octet become degenerate in the  $SU(3)$  limit, so the  $q^2$  dependence of the form factors is higher order in the  $SU(3)$  expansion.

We will study deviations from the  $SU(3)$  limit using chiral perturbation theory. The contribution of the form factors  $f_3$  and  $g_3$  is suppressed by the electron mass, and can be safely neglected. The corrections to  $f_1$  and the values of  $f_2$  and  $g_2$  are  $O(m_s)$  and are not calculable in chiral perturbation theory. The corrections to  $f_1$  are  $O(m_s)$  and are calculable due to the Ademollo–Gatto theorem; numerically, they are  $\lesssim 5\%$  [6][7]. The corrections to  $g_1$  are  $O(m_s \ln m_s)$ , and are therefore formally the largest corrections in the chiral expansion. We therefore focus on  $g_1$  for the remainder of this section. In ref. [1], these corrections were computed, and were found to be  $\sim 100\%$ .\*

Aside from the distinction between  $SU(3)$  and chiral symmetry breaking, our calculation differs from that of ref. [1] only in that we keep  $m_\pi \neq 0$ . The  $\pi$  corrections are expected to be only  $\sim 20\%$  of the  $K$  and  $\eta$  corrections, but setting  $m_\pi = 0$  systematically *increases* the amount of predicted  $SU(3)$  violation.

We write

$$g_1^{abc}(0) = \alpha_{ab}^c + \frac{1}{16\pi^2 f_2^c} \beta_{ab}^c, \quad (21)$$

where the lowest-order results are

$$\alpha_{ab}^c = D d_{ab}^c + F f_{ab}^c, \quad (22)$$

where  $d_{ab}^c$  and  $f_{ab}^c$  are the symmetric and antisymmetric structure constants of  $SU(3)$ , respectively. Specifically,

$$\begin{aligned} \alpha_{pn}^{1+i2} &= D + F, \\ \alpha_{\Lambda\Sigma^-}^{1+i2} &= \frac{2}{\sqrt{6}} D, \\ \alpha_{p\Lambda}^{4+i5} &= -\frac{1}{\sqrt{6}} (D + 3F), \\ \alpha_{\Lambda\Sigma^-}^{4+i5} &= -\frac{1}{\sqrt{6}} (D - 3F), \\ \alpha_{n\Sigma^-}^{4+i5} &= D - F, \\ \alpha_{\Sigma^0\Sigma^-}^{4+i5} &= \sqrt{2} \alpha_{\Sigma^+\Xi^0}^{4+i5} = \frac{1}{\sqrt{2}} (D + F). \end{aligned} \quad (23)$$

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\* An earlier calculation [4] which found smaller corrections is incorrect.

The leading chiral corrections are

$$\begin{aligned}
\beta_{pn}^{1+i2} &= -(D+F)(2D^2+4DF+2F^2+1)m_\pi^2 \ln \frac{m_\pi^2}{\mu^2} \\
&\quad - \frac{1}{6}(13D^3-D^2F+3D+3DF^2+3F+33F^3)m_K^2 \ln \frac{m_K^2}{\mu^2} \\
&\quad - \frac{1}{3}(D+F)(D-3F)^2 m_\eta^2 \ln \frac{m_\eta^2}{\mu^2},
\end{aligned} \tag{24}$$

$$\begin{aligned}
\beta_{\Lambda\Sigma^-}^{1+i2} &= -\frac{2}{3\sqrt{6}}D(7D^2+3F^2+3)m_\pi^2 \ln \frac{m_\pi^2}{\mu^2} \\
&\quad - \frac{1}{\sqrt{6}}D(3D^2+13F^2+1)m_K^2 \ln \frac{m_K^2}{\mu^2} \\
&\quad - \frac{4}{3\sqrt{6}}D^3 m_\eta^2 \ln \frac{m_\eta^2}{\mu^2},
\end{aligned} \tag{25}$$

$$\begin{aligned}
\beta_{p\Lambda}^{4+i5} &= \frac{3}{8\sqrt{6}}(3D^3+27D^2F+D+25DF^2+3F+9F^3)m_\pi^2 \ln \frac{m_\pi^2}{\mu^2} \\
&\quad + \frac{1}{12\sqrt{6}}(31D^3+15D^2F+9D+9DF^2+27F+297F^3)m_K^2 \ln \frac{m_K^2}{\mu^2} \\
&\quad + \frac{1}{24\sqrt{6}}(D+3F)(19D^2-30DF+27F^2+9)m_\eta^2 \ln \frac{m_\eta^2}{\mu^2},
\end{aligned} \tag{26}$$

$$\begin{aligned}
\beta_{\Lambda\Xi^-}^{4+i5} &= \frac{3}{8\sqrt{6}}(3D^3-27D^2F+D+25DF^2-3F-9F^3)m_\pi^2 \ln \frac{m_\pi^2}{\mu^2} \\
&\quad + \frac{1}{12\sqrt{6}}(31D^3-15D^2F+9D+9DF^2-27F-297F^3)m_K^2 \ln \frac{m_K^2}{\mu^2} \\
&\quad + \frac{1}{24\sqrt{6}}(D-3F)(19D^2+30DF+27F^2+9)m_\eta^2 \ln \frac{m_\eta^2}{\mu^2},
\end{aligned} \tag{27}$$

$$\begin{aligned}
\beta_{n\Sigma^-}^{4+i5} &= -\frac{1}{24}(35D^3+23D^2F+9D+33DF^2-9F-123F^3)m_\pi^2 \ln \frac{m_\pi^2}{\mu^2} \\
&\quad - \frac{1}{12}(31D^3-53D^2F+9D+57DF^2-9F-51F^3)m_K^2 \ln \frac{m_K^2}{\mu^2} \\
&\quad - \frac{1}{24}(D-F)(11D^2-6DF+27F^2+9)m_\eta^2 \ln \frac{m_\eta^2}{\mu^2},
\end{aligned} \tag{28}$$

$$\begin{aligned}
\beta_{\Sigma^0\Xi^-}^{4+i5} &= -\frac{1}{24\sqrt{2}}(35D^3-23D^2F+9D+33DF^2+9F+123F^3)m_\pi^2 \ln \frac{m_\pi^2}{\mu^2} \\
&\quad - \frac{1}{12\sqrt{2}}(31D^3+53D^2F+9D+57DF^2+9F+51F^3)m_K^2 \ln \frac{m_K^2}{\mu^2} \\
&\quad - \frac{1}{24\sqrt{2}}(D+F)(11D^2+6DF+27F^2+9)m_\eta^2 \ln \frac{m_\eta^2}{\mu^2}.
\end{aligned} \tag{29}$$

Here  $\mu$  is an arbitrary renormalization scale. The  $\mu$  dependence of these results is cancelled



by the  $\mu$  dependence of  $O(m_s)$  terms in the effective lagrangian such as

$$\frac{c(\mu)}{\Lambda} \text{tr} (\bar{B} M_s \cdot AB). \quad (30)$$

If we take  $\mu \simeq \Lambda$ , there are no large logarithms in the higher order coefficients, and the correction is dominated by the logarithmically enhanced terms (computed above) near the chiral limit. In the real world these logarithms are not very large, but we expect that the logarithmic terms will give a good indication of the actual size of the corrections.

In the  $SU(3)$  limit, using (24)–(30) we find

$$g_{ab}^c(0) = D' d_{ab}^c + F' f_{ab}^c, \quad (31)$$

where

$$\begin{aligned} D' &= D - \frac{3}{2} D(3D^2 + 5F^2 + 1) \frac{m^2}{16\pi^2 f^2} \ln \frac{m^2}{\mu^2}, \\ F' &= F - \frac{1}{6} F(25D^2 + 63F^2 + 9) \frac{m^2}{16\pi^2 f^2} \ln \frac{m^2}{\mu^2}, \end{aligned} \quad (32)$$

and  $m$  is the common meson mass.

This shows that for purposes of evaluating  $SU(3)$  breaking in semileptonic hyperon decays, it is misleading to present the results in terms of  $D$  and  $F$  defined in the effective lagrangian eq. (17), since large corrections to  $D$  and  $F$  do not necessarily correspond to large  $SU(3)$  breaking. We therefore consider an expansion in  $D'$  and  $F'$ , where  $m$  is chosen to be some appropriate average meson mass (see below) treated as  $O(m_s)$  for purposes of power counting. This expansion can easily be made well-defined to all orders, for example by defining the relations eq. (31) to be exact in the limit where all mesons have a common mass  $m$ .

The parameter  $m$  in eq. (32) is a redundant parameter in this expansion analogous to the renormalization scale  $\mu$  in conventional perturbation theory. In a world where the quark mass differences are small compared to the average quark mass, it is clear that  $m$  should be chosen to be close to the average meson mass. In our world,  $SU(3)$ -breaking quark mass differences are of order  $m_s$ , and it is not clear *a priori* how to choose  $m$ . We simply choose  $m$  in order to minimize the corrections to the lowest-order results. This choice is justified *a fortiori* by the fact that we obtain a reasonable value for  $m$  ( $\simeq 300$  MeV), and by the fact that the corrections expressed in terms of  $D'$  and  $F'$  are small. This is a non-trivial feature of the logarithmically-enhanced corrections, since both  $SU(3)$  and chiral symmetry breaking are controlled by the same parameter, namely  $m_s$ .

The large corrections to the lowest-order results in terms of  $D$  and  $F$  indicate that chiral perturbation theory is breaking down for this process. However, we wish to emphasize

that this breakdown of chiral perturbation theory does not necessarily imply a breakdown of the expansion in terms of  $D'$  and  $F'$ . In (17),  $D$  and  $F$  have an absolute physical significance in terms of the couplings of the light mesons to baryons in chiral perturbation theory. In contrast our parameters  $D'$  and  $F'$  are defined through  $SU(3)$  relations.

In order to determine  $D'$  and  $F'$  we performed a fit to the decay rates and asymmetry data quoted by the Particle Data group [8]. Because we expect that higher-order terms in the chiral expansion give corrections of order

$$\frac{m_K^2}{16\pi^2 f^2} \sim 0.25, \quad (33)$$

we have increased the uncertainties on the measured values of  $g_1$  by 20%. (More information about our fit is presented in appendix A.) Fitting to the lowest-order results gives

$$D = 0.85 \pm 0.06, \quad F = 0.52 \pm 0.04, \quad (34)$$

with  $\chi^2 = 6.1$  for 9 degrees of freedom. (Recall that  $D' = D$ ,  $F' = F$  at lowest order.)

Using  $m = 260$  MeV and  $\mu = m_\rho$ , the corrections to  $g_1(0)$  for all decay modes are less than 20%, and we obtain the best-fit values

$$D' = 0.87 \pm 0.06, \quad F' = 0.53 \pm 0.04, \quad (35)$$

with  $\chi^2 = 6.3$ .

#### 4. $SU(3)$ Breaking and the “EMC Effect”

$SU(3)$  breaking is important for determining the value of various strange-quark matrix elements of nucleons. In this section, we briefly present the predictions of the expansion discussed in section 3 to the extraction of the matrix element

$$\Delta s(Q^2) \equiv \langle p, s | \bar{s} \gamma_\mu \gamma_5 s |_{Q^2} | p, s \rangle, \quad (36)$$

where  $|p, s\rangle$  is a proton state with spin  $s$ . The unexpectedly large value of this matrix element extracted from analysis of EMC data [10] is often called the “EMC effect” and has attracted a good deal of attention in the theoretical literature [11].

Combining a (rigorous, QCD-derived) sum rule with isospin invariance allows us to derive the relation

$$\int_0^1 dx g_1(x, Q^2) = \frac{1}{36} [3g_A + 5(\Delta u + \Delta d - 2\Delta s) + 12\Delta s(Q^2)] \times \left[ 1 - \frac{\alpha_s(Q^2)}{\pi} + O(\alpha_s^2) \right] + O(\Lambda^2/Q^2), \quad (37)$$

where  $g_A \simeq 1.25$  is the nucleon axial coupling. The left-hand side extracted (with extrapolation) from the EMC data is  $0.126 \pm 0.018$  [10], where we have added systematic and statistical errors in quadrature. We have

$$\Delta u + \Delta d - 2\Delta s = (3F - D) \left[ 1 + \frac{1}{16\pi^2 f^2} \gamma \right], \quad (38)$$

where

$$\begin{aligned} \gamma = & 3(D + F)^2 m_\pi^2 \ln \frac{m_\pi^2}{\mu^2} - \frac{1}{6}(9 + 7D^2 - 18DF + 27F^2) m_K^2 \ln \frac{m_K^2}{\mu^2} \\ & - \frac{1}{3}(D - 3F)^2 m_\eta^2 \ln \frac{m_\eta^2}{\mu^2}. \end{aligned} \quad (39)$$

Expressing the results in terms of  $D'$  and  $F'$  and using our best-fit values, we obtain

$$\begin{aligned} \Delta s &= (-0.13 \pm 0.07) M_p, \\ \Delta u + \Delta d + \Delta s &= (0.12 \pm 0.19) M_p, \end{aligned} \quad (40)$$

whereas we obtain  $\Delta s = (-0.20 \pm 0.06) M_p$  and  $\Delta u + \Delta d + \Delta s = (0.06 \pm 0.18) M_p$  if we do not include  $SU(3)$ -breaking corrections.

We may not trust the predicted  $SU(3)$  breaking in eq. (39) quantitatively, since  $O(m_s)$  corrections are not included. However, it is worth noting that the corrections we have computed significantly reduce the value of  $\Delta s$ , suggesting that the ‘‘EMC effect’’ may be less striking than commonly thought.

## 5. Nonleptonic Decays

In this section, we consider nonleptonic decays as another application of the formalism discussed in section 3. We will find that our results tell much the same story as the semileptonic decays: there are large corrections to the lowest-order predictions of chiral symmetry, but corrections to  $SU(3)$  relations are  $\lesssim 10\%$ .

We consider only the predictions for the  $s$ -wave nonleptonic decay amplitudes here, since the chiral perturbation theory predictions for the  $p$ -wave amplitudes do not follow from  $SU(3)$  alone. The effective  $\Delta S = 1$  lagrangian at the weak scale can be written

$$\mathcal{L}_{\Delta S=1} = \frac{4G_F}{\sqrt{2}} V_{ud} V_{us}^* (\bar{q}_L \gamma^\mu S_1 q_L) (\bar{q}_L \gamma_\mu S_2 q_L), \quad (41)$$

where

$$q = \begin{pmatrix} u \\ d \\ s \end{pmatrix}, \quad S_1 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad S_2 = \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}. \quad (42)$$

We follow standard practice and assume the dominance of the  $\Delta I = \frac{1}{2}$  amplitudes. We therefore add to the effective lagrangian the terms

$$\delta\mathcal{L}_{\Delta S=1} = h_D \text{tr}(\bar{B}\{H, B\}) + h_F \text{tr}(\bar{B}[H, B]), \quad (43)$$

where

$$H \equiv \xi^\dagger S_2 S_1 \xi \mapsto U H U^\dagger. \quad (44)$$

Previous authors [2] have also included a term which is higher order in the derivative expansion on the grounds that its coefficient,  $h_\pi$ , as measured in  $\Delta S = 1$  kaon decays is larger than expected by dimensional analysis. We choose to work to a consistent order in the chiral expansion and will neglect this term. The enhancement of  $h_\pi$  is attributed to the  $\Delta I = \frac{1}{2}$  rule which may be violated in these decays (see below), making special treatment of this term somewhat suspect. Also, we have no information about other higher order terms which could also have anomalously large coefficients. In any case, we are interested primarily in the question of the size of  $SU(3)$  violation, and barring accidental cancellations, we expect that the logarithmically-enhanced corrections to give a good indication of the size of the corrections.

The  $s$ -wave decay amplitude for  $B_a \rightarrow B_b \pi$  can be written as

$$\mathcal{M}_s = G_F m_\pi^2 \bar{u}_a \mathcal{A}_{ab} u_b, \quad (45)$$

where  $\mathcal{A}_{ab}$  is the dimensionless  $s$ -wave (parity violating) amplitude as defined in ref. [8].

Assuming the  $\Delta I = \frac{1}{2}$  rule, there are three isospin relations among the seven decay amplitudes that have been measured:

$$\begin{aligned} \sqrt{2}\mathcal{A}(\Sigma^+ \rightarrow p\pi^0) - \mathcal{A}(\Sigma^+ \rightarrow n\pi^+) + \mathcal{A}(\Sigma^- \rightarrow n\pi^-) &= 0, & (5\%) \\ \mathcal{A}(\Lambda \rightarrow p\pi^0) + \sqrt{2}\mathcal{A}(\Lambda \rightarrow n\pi^0) &= 0, & (1\%) \\ \mathcal{A}(\Xi^- \rightarrow \Lambda\pi^-) + \sqrt{2}\mathcal{A}(\Xi^0 \rightarrow \Lambda\pi^0) &= 0. & (8\%) \end{aligned} \quad (46)$$

The experimental deviation from these relations is shown in parentheses. (Details on the data and fits can be found in appendix B.)

These ‘‘isospin’’ relations do not work significantly better than the  $SU(3)$  relations (see below), suggesting that the  $\Delta I = \frac{1}{2}$  rule may not be accurate for these decays. However, since we are interested primarily in the size of  $SU(3)$  violation, it is sufficient to assume the  $\Delta I = \frac{1}{2}$  form eq. (43) for the lagrangian.

The predictions for the remaining independent  $s$ -wave amplitudes are

$$\mathcal{A}_{ab} = \alpha_{ab} \left[ 1 + \frac{1}{16\pi^2 f^2} (\beta_{ab} + \epsilon) \right], \quad (47)$$

where  $\alpha$  is the lowest-order prediction, and  $\beta$  and  $\epsilon$  are the corrections.  $\epsilon$  contains pion wavefunction renormalization and renormalization of  $f_\pi$ . These effects are the same for all decays, and therefore do not affect the  $SU(3)$  predictions. We will not need the explicit expressions for these corrections.

The tree-level results for the four independent amplitudes are

$$\begin{aligned}
\alpha_{n\Sigma^+} &= 0, \\
\alpha_{n\Sigma^-} &= -h_D + h_F, \\
\alpha_{p\Lambda} &= \frac{1}{\sqrt{6}}(h_D + 3h_F), \\
\alpha_{\Lambda\Xi^-} &= \frac{1}{\sqrt{6}}(h_D - 3h_F),
\end{aligned} \tag{48}$$

At lowest order we can eliminate  $h_D$  and  $h_F$  to obtain an  $SU(3)$  relation among the three non-vanishing amplitudes: the Lee–Sugawara relation

$$\Delta_{\text{LS}} \equiv \frac{3}{\sqrt{6}}\mathcal{A}_{n\Sigma^-} + \mathcal{A}_{p\Lambda} + 2\mathcal{A}_{\Lambda\Xi^-} = 0. \tag{49}$$

(This relation is often written including a term proportional to  $\mathcal{A}_{n\Sigma^+}$ .)

The leading chiral corrections are

$$\beta_{n\Sigma^+} = 0, \tag{50}$$

$$\begin{aligned}
\beta_{n\Sigma^-} &= \frac{1}{24} \left[ 7(h_F - h_D) + h_D(51D^2 - 6DF + 27F^2) \right. \\
&\quad \left. - h_F(3D^2 - 54DF + 27F^2) \right] m_\pi^2 \ln \frac{m_\pi^2}{\mu^2} \\
&\quad + \frac{1}{12} \left[ 5(h_D - h_F) + h_D(39D^2 - 30DF + 27F^2) \right. \\
&\quad \left. - h_F(15D^2 - 54DF + 27F^2) \right] m_K^2 \ln \frac{m_K^2}{\mu^2} \\
&\quad + \frac{3}{8} (h_D - h_F)(1 + 3D^2 - 6DF + 3F^2) m_\eta^2 \ln \frac{m_\eta^2}{\mu^2},
\end{aligned} \tag{51}$$

$$\begin{aligned}
\beta_{p\Lambda} &= \frac{1}{24\sqrt{6}} \left[ 7(h_D + 3h_F) - h_D(171D^2 - 162DF + 27F^2) \right. \\
&\quad \left. - h_F(81D^2 + 54DF + 81F^2) \right] m_\pi^2 \ln \frac{m_\pi^2}{\mu^2} \\
&\quad + \frac{1}{12\sqrt{6}} \left[ -5(h_D + 3h_F) + h_D(9D^2 - 90DF - 27F^2) \right. \\
&\quad \left. - h_F(45D^2 + 54DF + 81F^2) \right] m_K^2 \ln \frac{m_K^2}{\mu^2} \\
&\quad - \frac{1}{8\sqrt{6}} (h_D + 3h_F)(3 + D^2 + 6DF + 9F^2) m_\eta^2 \ln \frac{m_\eta^2}{\mu^2},
\end{aligned} \tag{52}$$

$$\begin{aligned}
\beta_{\Lambda\Sigma^-} = & \frac{1}{24\sqrt{6}} \left[ 7(h_D - 3h_F) - h_D(171D^2 - 162DF + 27F^2) \right. \\
& \left. + h_F(81D^2 - 54DF + 81F^2) \right] m_\pi^2 \ln \frac{m_\pi^2}{\mu^2} \\
& + \frac{1}{12\sqrt{6}} \left[ -5(h_D - 3h_F) + h_D(9D^2 + 90DF - 27F^2) \right. \\
& \left. + h_F(45D^2 - 54DF + 81F^2) \right] m_K^2 \ln \frac{m_K^2}{\mu^2} \\
& - \frac{1}{8\sqrt{6}} (h_D - 3h_F)(3 + D^2 - 6DF + 9F^2) m_\eta^2 \ln \frac{m_\eta^2}{\mu^2}. \tag{53}
\end{aligned}$$

Defining  $h'_D$  and  $h'_F$  in analogy to  $D'$  and  $F'$  we obtain from (50)–(53)

$$\begin{aligned}
h'_D &= h_D - \frac{1}{2} \left[ h_D(1 + 13D^2 + 9F^2) + 18h_F DF \right] \frac{m^2}{16\pi^2 f^2} \ln \frac{m^2}{\mu^2}, \\
h'_F &= h_F - \frac{1}{2} \left[ h_F(1 + 5D^2 + 9F^2) + 10h_D DF \right] \frac{m^2}{16\pi^2 f^2} \ln \frac{m^2}{\mu^2}. \tag{54}
\end{aligned}$$

A fit to the data using the lowest-order predictions gives

$$h_D = -0.55 \pm 0.32, \quad h_F = 1.37 \pm 0.17, \tag{55}$$

with  $\chi^2 = 0.06$  for 1 degree of freedom. To account for the theoretical error due to  $O(m_s)$  terms in the expansion we have again added 20% in quadrature to the experimental errors before doing the fit. With only one degree of freedom the errors quoted should be taken as indicative only, but it is clear that the lowest-order predictions fit the data well.

Expressing our results in terms of  $h'_D$  and  $h'_F$  and taking  $\mu = m_\rho$  and  $m = 320$  MeV, we find that all the logarithmically-enhanced  $SU(3)$  corrections are less than 10% and the fit still works well:

$$h'_D = -0.56 \pm 0.40, \quad h'_F = 1.31 \pm 0.18, \tag{56}$$

with  $\chi^2 = 0.30$ . Thus there is every indication that the  $SU(3)$  expansion is well-behaved. This is to be contrasted to the *chiral* expansion, in which corrections to the individual decay amplitudes are  $\sim 50\%$ .

Chiral symmetry also gives a prediction for the  $p$ -wave decay amplitudes which does not follow from  $SU(3)$  alone. These predictions do not work well [2], supporting our conclusion that  $SU(3)$  may be a better symmetry than chiral symmetry.

Including the corrections for the best fit  $h'_D$  and  $h'_F$  the Lee–Sugawara relation becomes

$$\Delta_{LS} = 0.29 \pm 0.13, \tag{57}$$

which is to be compared with the experimental value of  $-0.23 \pm 0.03$ . The expected size of the  $O(m_s)$  contributions is  $\sim 0.4$ , so the fact that the predicted sign of  $\Delta_{LS}$  is wrong does not imply that our expansion is breaking down.

## 6. Conclusions

We have investigated the question of  $SU(3)$  breaking for weak hyperon decays in the context of chiral perturbation theory. One major difference between our work and previous work is that we have emphasized that large explicit chiral symmetry breaking does not necessarily imply large  $SU(3)$  breaking. We have found that  $SU(3)$  breaking is less than 20%, which is what is expected on the basis of dimensional analysis. Although we cannot conclude from our analysis that the expansion is under control, there is no sign that it is breaking down, unlike the usual chiral expansion.

We also used this expansion to analyze the ‘‘EMC effect,’’ and showed that the  $SU(3)$ -breaking corrections reduce the extracted value of the matrix element  $\langle p | \bar{s} \gamma_\mu \gamma_5 s | p \rangle$  by 35%.

### Appendix A. Fit to Semileptonic Decays

In this appendix, we present some details of the fit to semileptonic hyperon decays used in this paper. We use both decay rate and asymmetry data taken from the most recent Particle Data Group (PDG) compilation [8]. For the asymmetry data, we directly use the average values for  $g_A/g_V$  quoted by the PDG. To convert the decay rates into values for  $g_1$ , we keep the full kinematic dependence on the baryon masses, since these effects turn out to be numerically important. The data we use is displayed in table 1.

	lifetime	asymmetry
$n \rightarrow p$	$1.323 \pm 0.003$	$1.257 \pm 0.003$
$\Sigma^- \rightarrow \Lambda$	$0.609 \pm 0.029$	$0.62 \pm 0.44$
$\Lambda \rightarrow p$	$-0.972 \pm 0.018$	$-0.879 \pm 0.021$
$\Sigma^- \rightarrow n$	$0.442 \pm 0.021$	$0.340 \pm 0.017$
$\Xi^- \rightarrow \Sigma^0$	$0.96 \pm 0.19$	—
$\Xi^- \rightarrow \Lambda$	$0.473 \pm 0.026$	$0.306 \pm 0.061$

Table 1: Values for  $g_1(0)$  extracted from 1992 PDG

The decay rate and asymmetry determinations of  $g_1$  are inconsistent if we assume only the errors quoted by the PDG. This is either a symptom of systematic errors in the experiments or an indication that higher-order corrections are important. We expect

that higher order terms in the chiral expansion will give rise to  $\sim 20\%$  corrections, and so we added this amount in quadrature to all the quoted errors to take into account the theoretical uncertainty. When we do this, all the errors on all determinations have a sizable overlap, and reasonable fits are obtained (see the text).

## Appendix B. Fit to Nonleptonic Decays

In this appendix, we give some details about the data used to fit the  $s$ -wave nonleptonic decay amplitudes. The decays have  $s$ - and  $p$ -wave components with a possible relative phase, and so in principle three pieces of information are required to extract the  $s$ -wave amplitudes. We used the total lifetime and the asymmetry parameter  $\alpha$  quoted in the 1992 PDG [8], and neglected final-state phase shifts. This is consistent since final-state phase shifts are higher order in the  $SU(3)$  expansion. Table 2 shows the amplitudes obtained in this way.

Decay	$\mathcal{A}$
$\Lambda \rightarrow p$	$1.43 \pm 0.01$
$\Lambda \rightarrow n$	$1.04 \pm 0.01$
$\Sigma^+ \rightarrow n$	$0.06 \pm 0.01$
$\Sigma^+ \rightarrow p$	$1.44 \pm 0.05$
$\Sigma^- \rightarrow n$	$1.88 \pm 0.01$
$\Xi^0 \rightarrow \Lambda$	$1.51 \pm 0.01$
$\Xi^- \rightarrow \Lambda$	$-1.98 \pm 0.01$

Table 2: Values for  $s$ -wave amplitude  $\mathcal{A}$  from 1992 PDG

Just as for the semileptonic decay amplitudes, we increased the errors on the  $\mathcal{A}$  by 20% to account for the theoretical uncertainty arising from  $O(m_s)$  corrections. When this is done, the data is consistent, and good fits are obtained (see text).

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