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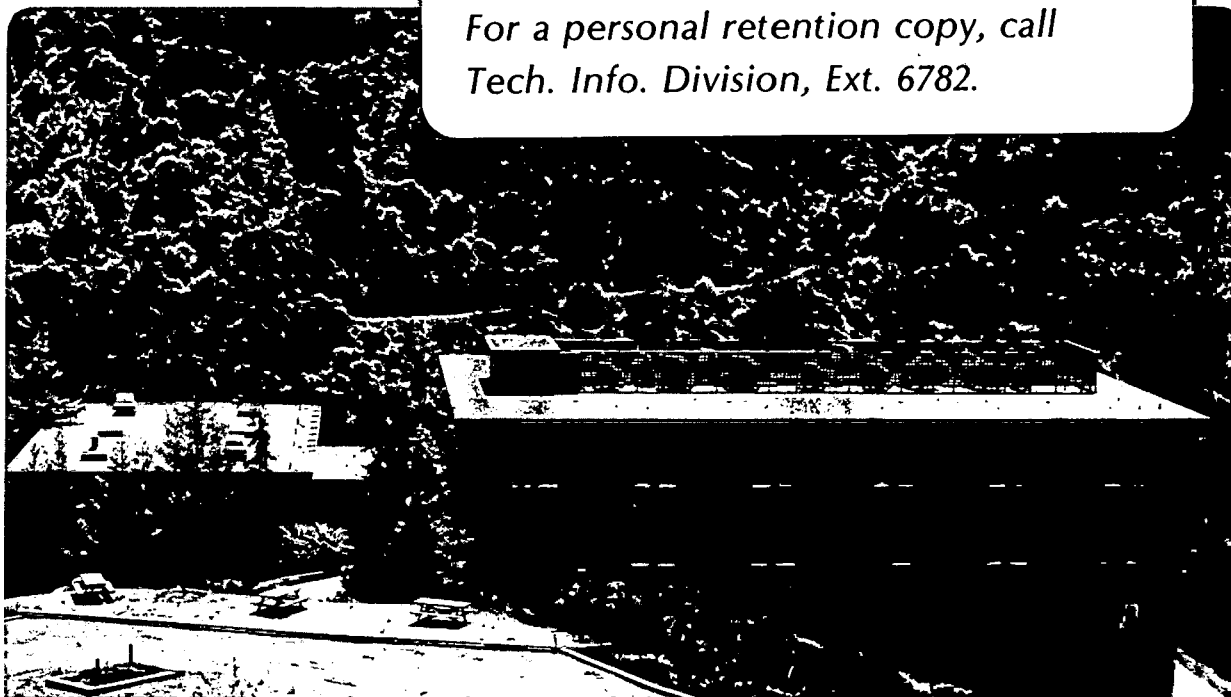
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A MODEL FOR THE CONTRIBUTION TO PLASTICITY-INDUCED  
FATIGUE CRACK CLOSURE FROM RESIDUAL COMPRESSIVE STRESSES

by

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Submitted to Materials Science and Engineering

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FATIGUE CRACK CLOSURE FROM RESIDUAL COMPRESSIVE STRESSES

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SUMMARY

A simple model is developed for the contribution to plasticity-induced crack closure arising from the residual compressive stresses existing within the cyclic plastic zones during fatigue crack propagation. Using assumptions of elastic-perfectly plastic behavior, the model is shown to accurately predict the existence of a critical load ratio, above which such closure is insignificant for a given stress intensity range, and further to be quantitatively in agreement with the load ratio-dependence of fatigue thresholds measured experimentally in dry atmospheres at low load ratios.

1. INTRODUCTION

Plasticity-induced crack closure during fatigue crack growth [1] can be considered to occur from interference, at positive loads in the loading cycle, between mating fracture surfaces in the *wake* of the crack tip, arising from the constraint of surrounding elastic material on the residual stretch in material elements previously plastically-strained at the tip. Since the crack cannot propagate whilst it remains closed, the net effect of this closure is to reduce the nominal stress intensity range ( $\Delta K$ ), computed as  $K_{\max} - K_{\min}$  from applied loads and crack length

measurements, to some lower effective value ( $\Delta K_{\text{eff}}$ ) actually experienced at the crack tip, i.e.,  $\Delta K_{\text{eff}} = K_{\text{max}} - K_{\text{cl}}$ , where  $K_{\text{cl}}$  is the stress intensity at closure ( $\geq K_{\text{min}}$ ).

An important contribution to this phenomenon can be considered to arise from the state of residual compressive stress which initially exists *ahead* of the crack tip within the cyclic plastic zone following unloading from  $K_{\text{max}}$  to  $K_{\text{min}}$  [2]. With crack extension, a proportion of such residual stresses, not relieved by the formation of new crack surface, will act to close the crack within the wake of plastic zones *behind* the crack tip. The objective of the present paper is to model, in simple terms, the contribution to crack closure arising from this phenomenon by considering that the maximum effect from the residual stresses results from the most recent plastic zone.

## 2. THE MODEL

Following Rice's solutions [3] for the plastic superposition of loading and unloading stress distributions in an elastic-perfectly plastic solid during cyclic stressing (Fig. 1), such residual compressive stresses ( $\sigma_r$ ) exist over a distance comparable with the extent of the cyclic plastic zone dimension ( $r_{\Delta}$ ) and are equal in magnitude to the flow stress in compression ( $-\sigma_0$ ). By invoking Irwin's physically-adjusted (equivalent) crack length concept [4], we can envisage the unloaded crack, of actual length  $a$ , as an equivalent elastic crack of length  $(a + r_{\Delta})$ , partially loaded by a uniform stress ( $\sigma_r = -\sigma_0$ ) over a distance  $b \sim r_{\Delta}$  *behind* the notional crack tip (Fig. 2). For small-scale yielding conditions, i.e.  $r_{\Delta}$  small compared with the overall dimensions of

the body, the stress intensity of a partially-loaded elastic crack is given as [5]:

$$K_{Ir} = \frac{2\sqrt{2}}{\sqrt{\pi}} \sigma_r \sqrt{b} , \quad (1)$$

where  $K_{Ir}$  can be considered now as the stress intensity resulting from the residual stresses  $\sigma_r$ . Taking  $\sigma_r = -\sigma_0$ ,  $b \sim r_\Delta$  and recognizing that the maximum extent of the cyclic plastic zone  $r_\Delta$  in plane strain is approximated by [6]:

$$r_\Delta \approx \frac{1}{2\pi} \left( \frac{\Delta K}{2\sigma_0} \right)^2 , \quad (2)$$

this leads to an expression for  $K_{Ir}$  of:

$$|K_{Ir}| = \frac{2\sqrt{2}}{\sqrt{\pi}} \sigma_0 \sqrt{r_\Delta} \approx \frac{\Delta K}{\pi} . \quad (3)$$

Using Eq. (3), the effective stress intensity range ( $\Delta K_{eff}$ ) in the presence of this residual compressive stress field during cyclic crack extension can then be estimated by linear superposition of  $K_{Ir}$  and the maximum and minimum stress intensities (i.e.,  $K_{max}$  and  $K_{min}$ ) due to the applied loading, viz:

$$\begin{aligned} K_{max,eff} &= K_{max} + K_{Ir} , \\ K_{min,eff} &= K_{min} + K_{Ir} , \quad \text{for } |K_{Ir}| \leq |K_{min}| , \\ &= 0 , \quad \text{for } |K_{Ir}| \geq |K_{min}| , \end{aligned} \quad (4a)$$

such that

$$\begin{aligned} \Delta K_{eff} &= K_{max,eff} - K_{min,eff} , \\ &= K_{max} - K_{min} = \Delta K , \quad \text{for } |K_{Ir}| \leq |K_{min}| , \\ &= K_{max} + K_{Ir} , \quad \text{for } |K_{Ir}| \geq |K_{min}| . \end{aligned} \quad (4b)$$

Substituting Eq. (3) into Eq. (4), and noting that the load ratio R is given by

$$R = \frac{K_{\min}}{K_{\max}} = 1 - \frac{\Delta K}{K_{\max}}, \quad (5)$$

yields expressions for  $\Delta K_{\text{eff}}$  as,

$$\begin{aligned} \Delta K_{\text{eff}} &= K_{\max} - K_{\min} = \Delta K, \quad \text{for } |K_{\text{Ir}}| \leq |K_{\min}|, \\ &= K_{\max} \left[ 1 - \left( \frac{1-R}{\pi} \right) \right], \quad \text{for } |K_{\text{Ir}}| \geq |K_{\min}|, \end{aligned} \quad (6)$$

where R is the "nominal" load ratio based on the applied stresses.

Eq. (6) suggests that crack closure arising from residual compressive stresses within the cyclic plastic zones will influence the driving force for cyclic crack extension (i.e.,  $\Delta K_{\text{eff}}$ ) only at low nominal load ratios, where in the limit as  $R \rightarrow 0$ , the effective stress intensity range is given by

$$\Delta K_{\text{eff}} \Big|_{R \rightarrow 0} = K_{\max} \left( 1 - \frac{1}{\pi} \right) = 0.68 K_{\max}. \quad (7)$$

Expressing  $K_{\text{Ir}}$  as the effective stress intensity at closure  $K_{\text{c1}}$  yields an expression for closure induced solely by compressive stress fields of

$$\frac{K_{\text{c1}}}{K_{\max}} = \frac{K_{\text{Ir}}}{K_{\max}} = \frac{\Delta K}{\pi K_{\max}} = \frac{1}{\pi} (1 - R), \quad \text{for } |K_{\text{Ir}}| \geq |K_{\min}|, \quad (8a)$$

which, in the limit of  $R \rightarrow 0$ , gives the maximum extent of closure at positive load ratios from this mechanism as:



$$\left. \frac{K_{c1}}{K_{max}} \right|_{R \rightarrow 0} = \frac{1}{\pi} = 0.32 \quad . \quad (8b)$$

However, at high load ratios, where  $|K_{min}|$  exceeds  $|K_{Ir}|$ , closure via this mechanism will be negligible, such that the effective stress intensity  $\Delta K_{eff}$  will be independent of R and equal to the nominal  $\Delta K$  computed from the applied loads (Eq. 6). This implies that, for a particular alternating stress intensity, there will be a critical load ratio  $R = R_{cr}$  above which residual compressive stresses will not decrease the effective stress intensity range, i.e.,

$$\begin{aligned} \Delta K_{eff} &< \Delta K \quad , \quad \text{for } R < R_{cr} \quad , \\ &= \Delta K \quad , \quad \text{for } R \geq R_{cr} \quad . \end{aligned} \quad (9)$$

### 3. IMPLICATIONS

An example of this concept of a critical load ratio can be appreciated from examining the variation in the threshold stress intensity range for fatigue crack growth as a function of R, following the arguments of Schmidt and Paris [7] shown in Fig. 3. On the assumptions that the *effective* stress intensity range at the threshold  $\Delta K_{th}$  is constant, and that the closure stress intensity  $K_{c1}$  is independent of R, these authors reasoned that the variation in measured  $\Delta K_0$  threshold values, computed from applied stress and crack length measurements, will be one of a decreasing  $\Delta K_0$  with increasing R at low load ratios, where  $\Delta K_0 = \Delta K_{th} + K_{c1} - K_{min}$  (since  $K_{c1} \geq K_{min}$ ), and a constant  $\Delta K_0$  with R at high load ratios, where  $\Delta K_0 = \Delta K_{th}$  (since  $K_{c1} \leq K_{min}$ ). The

transition in this behavior, above which the measured  $\Delta K_0$  remains constant with further increase in R, corresponds to the critical load ratio  $R_{cr}$  where  $K_{min} = K_{c1}$ , since at higher load ratios closure mechanisms will be insignificant. Experimental  $\Delta K_0$  data for lower strength steels [7-9] and aluminum alloys [7] tested in moist air and dry gaseous environments are generally found to conform to this pattern, as shown by the variation in  $\Delta K_0$  with R for two 2½Cr-1Mo pressure vessel steels (SA387 and SA542-3, respectively) in Fig. 4 [8,9]. Focussing attention on the data for dry hydrogen,\* since in moist air additional crack closure mechanisms, principally induced by crack surface oxide debris [8-10], are more active, it is apparent that the critical load ratio  $R_{cr}$  is approximately 0.3 for both materials. In terms of the present analysis, Eqs. (3) and (5) indicate that where  $K_{min} = K_{Ir} = \Delta K/\pi$ , the value of  $R_{cr}$  at threshold levels will be

$$R_{cr} = \frac{1}{1 + \pi} = 0.24 , \quad (10)$$

in good agreement with the experimental results. Furthermore, by applying Eq. (6) at the threshold for  $|K_{Ir}| > |K_{min}|$  with the physically reasonable postulate of a constant effective  $\Delta K_{th}$  at the threshold [7], the current model predicts that the maximum stress intensity at the threshold ( $K_{o,max}$ ), which was formerly assumed to be constant at  $R < R_{cr}$  [7,8], will *decrease* marginally with increasing load ratio below  $R_{cr}$ . Examination of previously published data [8-10], shown in Fig. 4 for dry

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\* At the high frequencies ( $\geq 50$  Hz) associated with near-threshold fatigue measurements in these lower strength steels, behavior in dehumidified hydrogen gas is similar to that in dry inert gases, such as helium and argon [8-10].

gaseous hydrogen, indicates that such a small decrease is discernable. Comparison of predicted and experimentally measured values of  $K_{O,max}$  at low load ratios ( $R < R_{cr}$ ), indicates in fact that agreement is numerically quite close (Fig. 5). The predictions of both  $R_{cr}$  and  $K_{O,max}$  may be expected, however, to be somewhat lower than experimental values (Figs. 4 and 5) since the present analysis is based on crack closure arising solely from plasticity, specifically residual compressive stress, effects. However, as discussed in detail elsewhere [10], even in dehumidified environments such as dry hydrogen gas, at near-threshold levels some crack face oxidation products will form at low load ratios such that a small contribution from oxide-induced crack closure is inevitable.

#### 4. DISCUSSION

The current model represents a simple way to estimate the contribution to plasticity-induced closure arising from the influence of the compressive residual stresses existing within the cyclic plastic zones. It is perhaps somewhat less appealing physically, however, since it does not address specifically the associated effect of excess material inside crack, arising from the residual plastic displacements in crack tip material elements, which cause interference between the crack faces in the *wake* of the tip, as modelled in the detailed plane stress analysis of Budiansky and Hutchinson [11]. Furthermore, as shown by McMeeking and Evans [12] and Budiansky et al. [13] in their treatment of phase transformation-toughening in ceramics, the principal influence of residual compressive stresses in reducing crack tip stress intensities is

generated *behind* the tip, whereas the current model merely treats the situation behind a notional crack tip. In fact for the case of purely dilatant inelasticity examined by the latter authors, the actual shape of the "plastic" zone ahead of the tip was found to be relatively less important. Despite these obvious deficiencies, however, the present model does provide a simple means to quantitatively estimate the extent of plasticity-induced fatigue crack closure, arising from residual compressive stresses, without recourse to complex analytical or numerical analyses. Moreover, contrary to the assumption of Schmidt and Paris [7], the model suggests that the closure stress intensity at threshold is not independent of load ratio, and this is reflected in the predictions of a small load ratio-dependence of  $K_{o,max}$  threshold values at  $R < R_{cr}$ . As shown by the threshold data in Figs. 4 and 5 for dry gaseous environments, such predictions are consistent with experimental observations, provided additional contributions to crack closure, such as arising from the wedging action of crack surface oxidation products [8-10], are minimized.

## 5. CONCLUSIONS

A simple model is presented for the contribution to plasticity-induced crack closure due to the residual compressive stress field, generated within the cyclic plastic zones during fatigue crack propagation in an elastic-perfectly plastic solid. The model is shown to accurately predict the existence of a critical load ratio, above which such closure does not occur (for a given  $\Delta K$ ), and to be quantitatively in agreement with the load ratio-dependence of fatigue thresholds measured in dry atmospheres at low load ratios.

## ACKNOWLEDGMENTS

The work was supported by the Director, Office of Energy Research, Office of Basic Energy Sciences, Materials Science Division of the U.S. Department of Energy under Contract No. DE-AC03-76SF00098.

## NOMENCLATURE

a	crack length
b	distance behind notional crack tip of partial loading (Fig. 2)
K	Mode I stress intensity factor
$K_{cl}$	stress intensity at closure of fatigue crack
$K_{Ir}$	stress intensity due to residual compressive stresses
$K_{max}$	maximum stress intensity
$K_{max,eff}$	effective value of $K_{max}$ experienced at crack tip
$K_{min}$	minimum stress intensity
$K_{min,eff}$	effective value of $K_{min}$ experienced at crack tip
$K_{0,max}$	maximum stress intensity at threshold for no crack growth
$\Delta K$	alternating stress intensity ( $K_{max} - K_{min}$ )
$\Delta K_{eff}$	effective stress intensity range experienced at crack tip
$\Delta K_0$	threshold stress intensity range for no crack growth
$\Delta K_{th}$	effective value of $\Delta K_0$ experienced at crack tip
R	load ratio ( $K_{min}/K_{max}$ )
$R_{cr}$	critical load ratio above which closure is insignificant (for given $\Delta K$ )
$r_{max}$	maximum (monotonic) plastic zone size
$r_{\Delta}$	cyclic plastic zone size
$\sigma_0$	flow stress

$\sigma_r$  residual stress  
 $\sigma_{yy}(x,0)$  maximum local tensile stress directly ahead of crack tip

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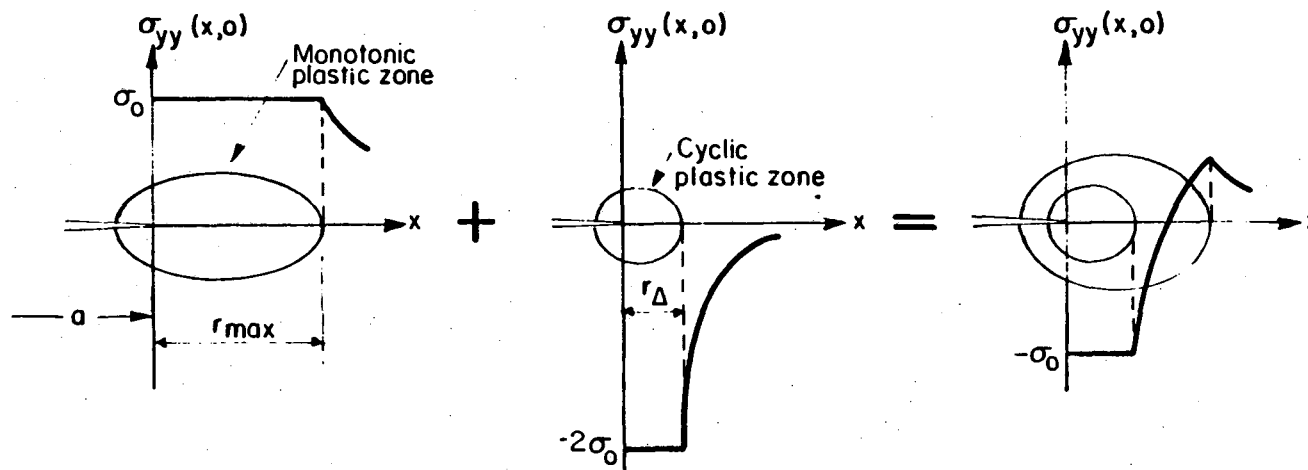
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List of Figure Captions

- Fig. 1: Plastic superposition of loading and unloading stress distributions for an elastic-perfectly plastic solid of flow strength  $\sigma_0$  during fatigue crack propagation.  $\sigma_{yy}(x,0)$  is the maximum principal stress directly ahead of the crack,  $r_{max}$  is the maximum plastic zone and  $r_{\Delta}$  is the cyclic plastic zone. After Rice [3].
- Fig. 2: Idealization of an unloaded fatigue crack of length (a), with cyclic plastic zone  $r_{\Delta}$ , as an equivalent elastic crack of length  $(a + r_{\Delta})$ , partially loaded by residual compressive stresses  $\sigma_r = -\sigma_0$  over a distance behind the notional crack tip of  $b \sim r_{\Delta}$ .
- Fig. 3: Predicted variation of alternating and maximum stress intensities at the threshold ( $\Delta K_0$  and  $K_{0,max}$ , respectively) with load ratio (R), after Paris and Schmidt [7]. Constant effective stress intensity range at threshold ( $\Delta K_{th}$ ), and constant closure stress intensity ( $K_{c1}$ ), both independent of R, are assumed.
- Fig. 4: Experimentally-measured variation of alternating and maximum threshold stress intensities ( $\Delta K_0$  and  $K_{0,max}$ , respectively) with load ratio (R) in two 2½Cr-1Mo pressure vessel steels, SA387 and SA542-3. Tests in ambient temperature environments (at 50 Hz cyclic frequency) of moist air (30 pct relative humidity) and dehumidified hydrogen gas (138 kPa pressure). Sample points shown for dry inert gas atmospheres of argon and helium. Data from refs. [8-10].

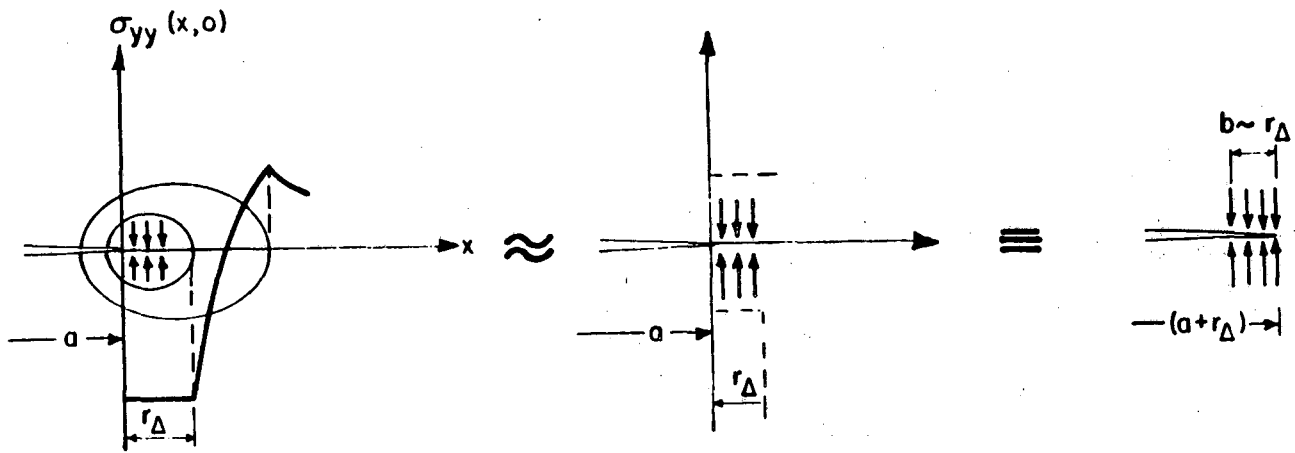
Fig. 5: Variation of maximum threshold stress intensity  $K_{o,max}$  with load ratio (R) in SA387 and SA542-3 steels (yield strengths 290 and 500 MPa, respectively) at low load ratios for tests at 50 Hz in dehumidified gaseous hydrogen, replotted from Fig. 4 [8-10]. Model predictions of  $R_{cr}$  and  $K_{o,max}(R)$  for  $R < R_{cr}$  from Eqs. 9 and 6, respectively.





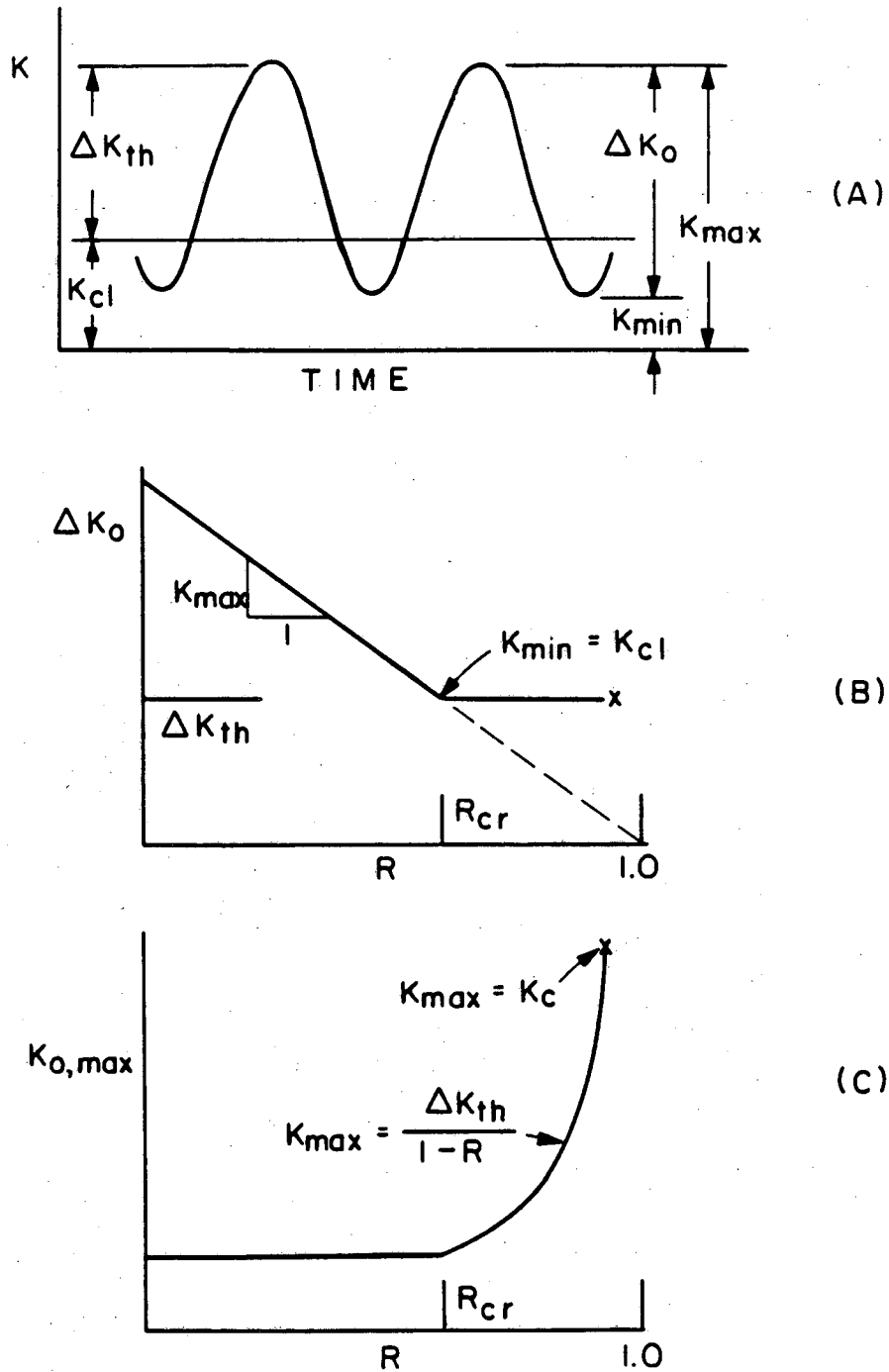
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Fig. 2: Idealization of an unloaded fatigue crack of length ( $a$ ), with cyclic plastic zone  $r_{\Delta}$ , as an equivalent elastic crack of length  $(a + r_{\Delta})$ , partially loaded by residual compressive stresses  $\sigma_r = -\sigma_0$  over a distance behind the notional crack tip of  $b \sim r_{\Delta}$ .



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Fig. 3: Predicted variation of alternating and maximum stress intensities at the threshold ( $\Delta K_0$  and  $K_{0,max}$ , respectively) with load ratio ( $R$ ), after Paris and Schmidt [7]. Constant effective stress intensity range at threshold ( $\Delta K_{th}$ ), and constant closure stress intensity ( $K_{cl}$ ), both independent of  $R$ , are assumed.

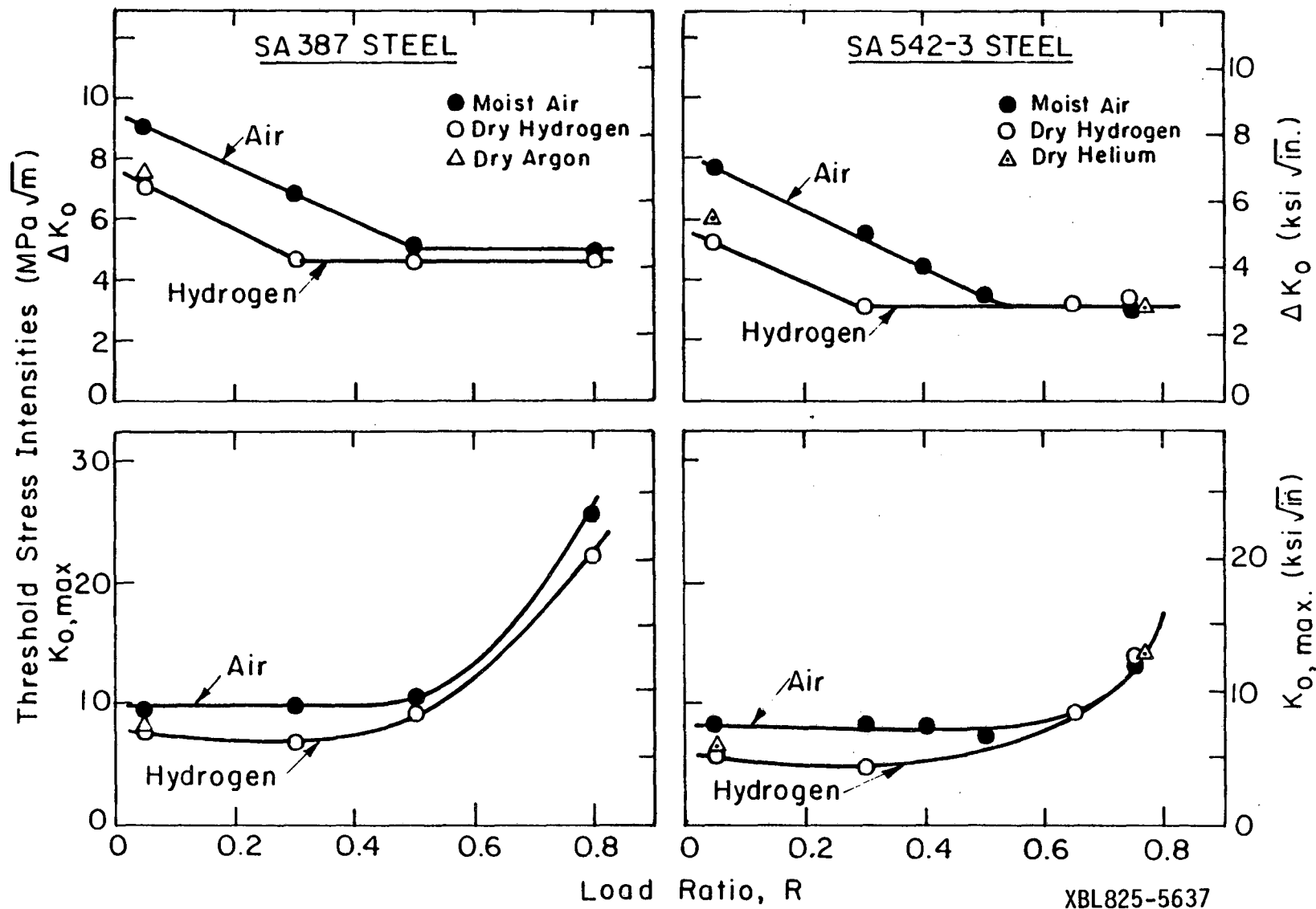
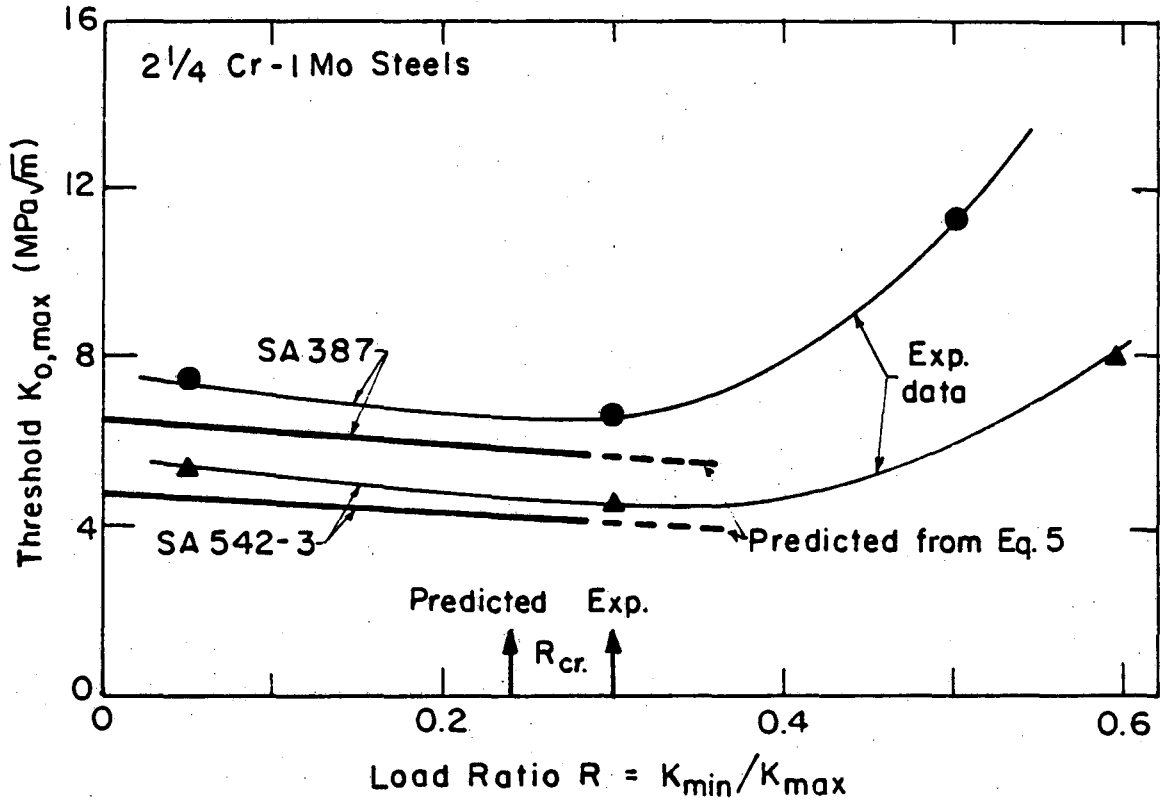


Fig. 4: Experimentally-measured variation of alternating and maximum threshold stress intensities ( $\Delta K_0$  and  $K_{0,max}$ , respectively) with load ratio (R) in two  $2\frac{1}{2}$ Cr-1Mo pressure vessel steels, SA387 and SA542-3. Tests in ambient temperature environments (at 50 Hz cyclic frequency) of moist air (30 pct relative humidity) and dehumidified hydrogen gas (138 kPa pressure). Sample points shown for dry inert gas atmospheres of argon and helium. Data from refs. [8-10].



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