Title
NONLINEARITY MINIMUMS AND MAXIMUMS OF A PHASE-SENSITIVE DETECTION SYSTEM

Author
Leskovar, Branko.

Publication Date
1969-06-01

Submitted to IEEE Trans. on Instrumentation
and Measurement

UCRL-19207
Preprint

NONLINEARITY MINIMUMS AND MAXIMUMS OF A PHASE-SENSITIVE DETECTION SYSTEM

Branko Leskovar

June 1969

AEC Contract No. W-7405-eng-48

LAWRENCE RADIATION LABORATORY
UNIVERSITY of CALIFORNIA BERKELEY

NONLINEARITY MINIMUMS AND MAXIMUMS OF A PHASE-SENSITIVE DETECTION SYSTEM*

Branko Leskovar

Lawrence Radiation Laboratory
University of California
Berkeley, California

June 1969

Abstract — Two generalized criteria for minimums and maximums of essential nonlinearity of a phase-sensitive detection system are presented. Minimums and maximums are calculated and plotted by a digital computer over a wide dynamic range of operating conditions, assuming that the input signal is in the narrow-band Gaussian noise.

Recent investigations [1], [2] have shown that in the instrumentation of experimental research the total nonlinearity of a phase-sensitive detection system is of prime importance. In most cases of practical interest, the total system nonlinearity is determined by the essential nonlinearity of the characteristics of the phase-sensitive detector used. The nonlinearity minimums N_{BMIN} and maximums N_{CMAX} of the detector characteristics are particularly important. Both nonlinearities were calculated in previous work [2] for a number of discrete values of the input signal-to-noise ratio $X = V_s/V_\sigma$, and the reference wave-to-noise ratio $\mu = V_c/V_\sigma$; where V_s is the amplitude of the input sine wave,

V_σ is the rms value of the input narrow-band noise, and V_c is the amplitude of the reference wave. Johnson [3] has pointed out a possibility of the existence of additional nonlinearity minimums and maximums which can be larger or smaller in value than those calculated in [2], due to the relatively complicated formulas expressing conditions for N_{BMIN} and N_{CMAX} as well as to the N_{BMIN} and N_{CMAX} calculations made by a relatively small number of discrete values of V_s, V_c, V_σ, and ψ (ψ is the phase angle between the input signal and the reference wave).

Based on reference [2], careful investigations show that a generalized criterion for N_{BMIN} is given by

$$w[f(x_B)]\left\{\gamma^*(x_B,\mu,\psi)\, s[v(x_B)] - \phi^*(x_B,\mu,\psi)\, m[t(x_B)] + y[t(x_B)] - u[v(x_B)]\right\} - \left(\frac{x_B}{2}\right)^2 K[f(x_B)]\left\{y[t(x_B)] - u[v(x_B)]\right\} = 0, \quad (1)$$

where functions $w[f(x_B)]$, $\gamma^*(x_B,\mu,\psi)$, $s[v(x_B)]$, $\phi^*(x_B,\mu,\psi)$, $m[t(x_B)]$, $y[t(x_B)]$, $u[v(x_B)]$, and $K[f(x_B)]$ are given by:

$$w[f(x_B)] = {}_1F_1\left(\frac{1}{2}\,;\,2;\,-\frac{\mu^2+x_B{}^2}{2}\right) \quad (2)$$

$$\gamma^*(x_B,\mu,\psi) = \frac{x_B{}^2}{2} + \frac{\mu\cos\psi}{2}\,x_B \quad (3)$$

$$s[v(x_B)] = {}_1F_1\left(\frac{1}{2}\,;\,2;\,-\frac{\mu^2 + x_B{}^2 + 2\mu x_B\cos\psi}{2}\right) \quad (4)$$

$$\phi^{*}(x_B,\mu,\psi) = \frac{x_B^{\,2}}{2} - \frac{\mu\cos\psi}{2}\,x_B \tag{5}$$

$$m[t(x_B)] = {}_1F_1\left(\frac{1}{2};\ 2;\ -\frac{\mu^2 + x_B^{\,2} - 2\mu x_B\cos\psi}{2}\right) \tag{6}$$

$$y[t(x_B)] = {}_1F_1\left(-\frac{1}{2};\ 1;\ -\frac{\mu^2 + x_B^{\,2} - 2\mu x_B\cos\psi}{2}\right) \tag{7}$$

$$u[v(x_B)] = {}_1F_1\left(-\frac{1}{2};\ 1;\ -\frac{\mu^2 + x_B^{\,2} + 2\mu x_B\cos\psi}{2}\right) \tag{8}$$

$$K[f(x_B)] = {}_1F_1\left(\frac{3}{2};\ 3;\ -\frac{\mu^2 + x_B^{\,2}}{2}\right). \tag{9}$$

where ${}_1F_1$ denotes the confluent hypergeometric function.

By means of computer-aided analysis, using numerical solutions of Eq. (1), and high-density discrete-value calculations, the minimum nonlinearity expressed as

$$N_{BMIN} = f^{*}(x_B)_{\mu,\psi} \tag{10}$$

is calculated and plotted in Fig. 1. From curves it can be seen that N_{BMIN} is a monotonously decreasing function of x_B having a fast rate of decrease of almost a half order of magnitude for $x_B \leqq 10$. N_{BMIN} varies less than 16% for $x_B \geqq 10$ and $\psi \leqq \pi/6$. For $x_B \geqq 10$ and $\psi \geqq \pi/6$, N_{BMIN} has approximately a constant value with variation of x_B. Furthermore, there are N_{BMIN} accumulation points at $x_B = 2.37295$ for $\mu \leqq 0.1$ and for any value of ψ. The N_{BMIN} accumulation points are maximum values of N_{BMIN} for a given value of ψ.

Similarly, a generalized criterion for the maximum nonlinearity N_{CMAX} is given by

$$\left\{\phi[g(x_C)] - z[h(x_C)]\right\}\left\{y^*(x_C,\mu,\psi)\, m[t(x_C)] - w^*(x_C,\mu,\psi)\, s[v(x_C)]\right\} + \left\{s^*(x_C,\mu)\, \rho[g(x_C)] - v^*(x_C,\mu)\, \ell[h(x_C)]\right\}\left\{u[v(x_C)] - y[t(x_C)]\right\} = 0, \tag{11}$$

where functions $m[t(x_C)]$, $s[v(x_C)]$, $u[v(x_C)]$, and $y[t(x_C)]$ are given by relations (6), (4), (8), and (8), respectively. Other functions are defined by:

$$\phi[g(x_C)] = {}_1F_1\left[-\frac{1}{2};\ 1;\ -\frac{(\mu + x_C)^2}{2}\right] \tag{12}$$

$$z[h(x_C)] = {}_1F_1\left[-\frac{1}{2};\ 1;\ -\frac{(\mu - x_C)^2}{2}\right] \tag{13}$$

$$y^*(x_C,\mu,\psi) = \frac{x_C}{2} - \frac{\mu}{2}\cos\psi \tag{14}$$

$$w^*(x_C,\mu,\psi) = \frac{x_C}{2} + \frac{\mu}{2}\cos\psi \tag{15}$$

$$s^*(x_C,\mu) = \frac{x_C + \mu}{2} \tag{16}$$

$$\rho[g(x_C)] = {}_1F_1\left[\frac{1}{2};\ 2;\ -\frac{(\mu + x_C)^2}{2}\right] \tag{17}$$

$$v^*(x_C,\mu) = \frac{x_C - \mu}{2} \tag{18}$$

$$\ell[h(x_C)] = {}_1F_1\left[\frac{1}{2};\ 2;\ -\frac{(\mu - x_C)^2}{2}\right]. \tag{19}$$

The maximum nonlinearity expressed as

$$N_{CMAX} = \varphi^{*}(x_C)_{\mu,\psi} \tag{20}$$

is calculated and plotted in Fig. 2 using a high-density discrete-value calculations approach. From the curves in Fig. 2 we see that N_{CMAX} is a monotonously increasing function of x_C, having a fast rate of increase depending upon ψ value. N_{CMAX} accumulation points are again at $x_C = 2.37295$ for $\mu \leqq 0.1$ and for any value of ψ. Generally N_{CMAX} accumulation points are minimum values of N_{CMAX} for a given value of ψ.

Furthermore, applying the same method as in previous considerations, it is also of interest to calculate over a wide dynamic range of operating conditions the normalized form of the phase-sensitive detector characteristics as a function of ψ, for various values of μ and calculated values of x_B and x_C, considering above given criteria. According to [2], normalized forms of the detector characteristics as a function of ψ, with μ, x_B, and x_C as parameters are given by

$$\left(\frac{V_0}{\eta_d V_\sigma}\right)_B = \left(\frac{\pi}{2}\right)^{1/2} \left\{u[v(x_B)] - y[t(x_B)]\right\} \tag{21a}$$

and

$$\left(\frac{V_0}{\eta_d V_\sigma}\right)_C = \left(\frac{\pi}{2}\right)^{1/2} \left\{u[v(x_C)] - y[t(x_C)]\right\}, \tag{21b}$$

where V_0 and η_d are the detector output signal and detector efficiency, respectively.

Calculations shown that the numerical values of x_B and x_C are very close over a wide range of μ and ψ, although x_B gives the condition for minimum nonlinearity, and x_C for maximum nonlinearity. Consequently, both functions (21a) and (21b) are represented by one curve for a set of value of μ, ψ, and x_B or x_C. Curves show that the normalized output signal is almost independent of the phase angle for a ratio $\psi \leqq 0.2$. For a $\psi \geqq 0.2$ ratio, the normalized output signal considerably decreases its value, achieving $V_0/\eta_d V_\sigma = 0$ for $\psi = \pi/2$.

From conclusions derived from generalized criteria (1), (11), (21a), and (21b), as well as from curves in Figs. 1, 2, and 3 follows a full agreement with results presented in [1] and [2] for N_{BMIN} and N_{CMAX}. Of course, the generalized criteria give more information about behaviour of minimum and maximum nonlinearities than previously published results.

ACKNOWLEDGEMENT

The author wishes to express appreciation to E. Schroeder for writing the computer program.

* This work was done under the auspices of the U.S. Atomic Energy Commission.

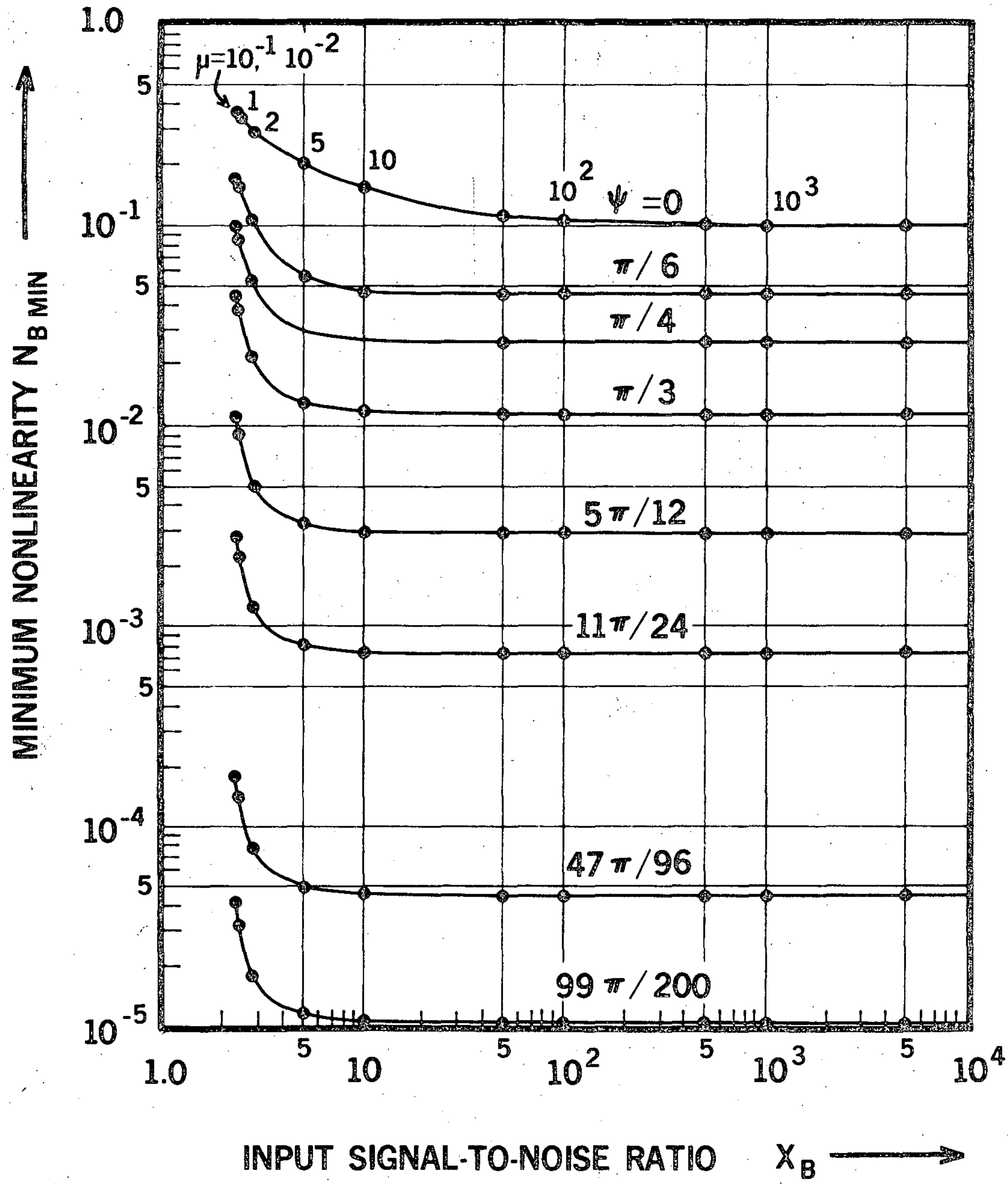
1. B. Leskovar, Phase-Sensitive Detector Nonlinearity at the Signal Detection in the Presence of Noise, IEEE Transactions on Instrumentation and Measurements, Vol. IM-16, No. 4, pp. 285-294, 1967.
2. B. Leskovar, Essential Nonlinearity of Phase-Sensitive Detector Characteristics, in Proceedings of the 6th Allerton Conference on Circuit and System Theory, Urbana, Illinois, 1968.
3. A. R. Johnson, private communication, 1969.

Figure Legends

Fig. 1. Minimum nonlinearity N_{BMIN} as a function of the optimum value of the input signal-to-noise ratio x_B, with the phase angle ψ and the reference wave-to-input noise ratio $\mu = 10^{-2}$, 10^{-1}, 1, 2, 5, 10, 10^2, and 10^3 as parameters.

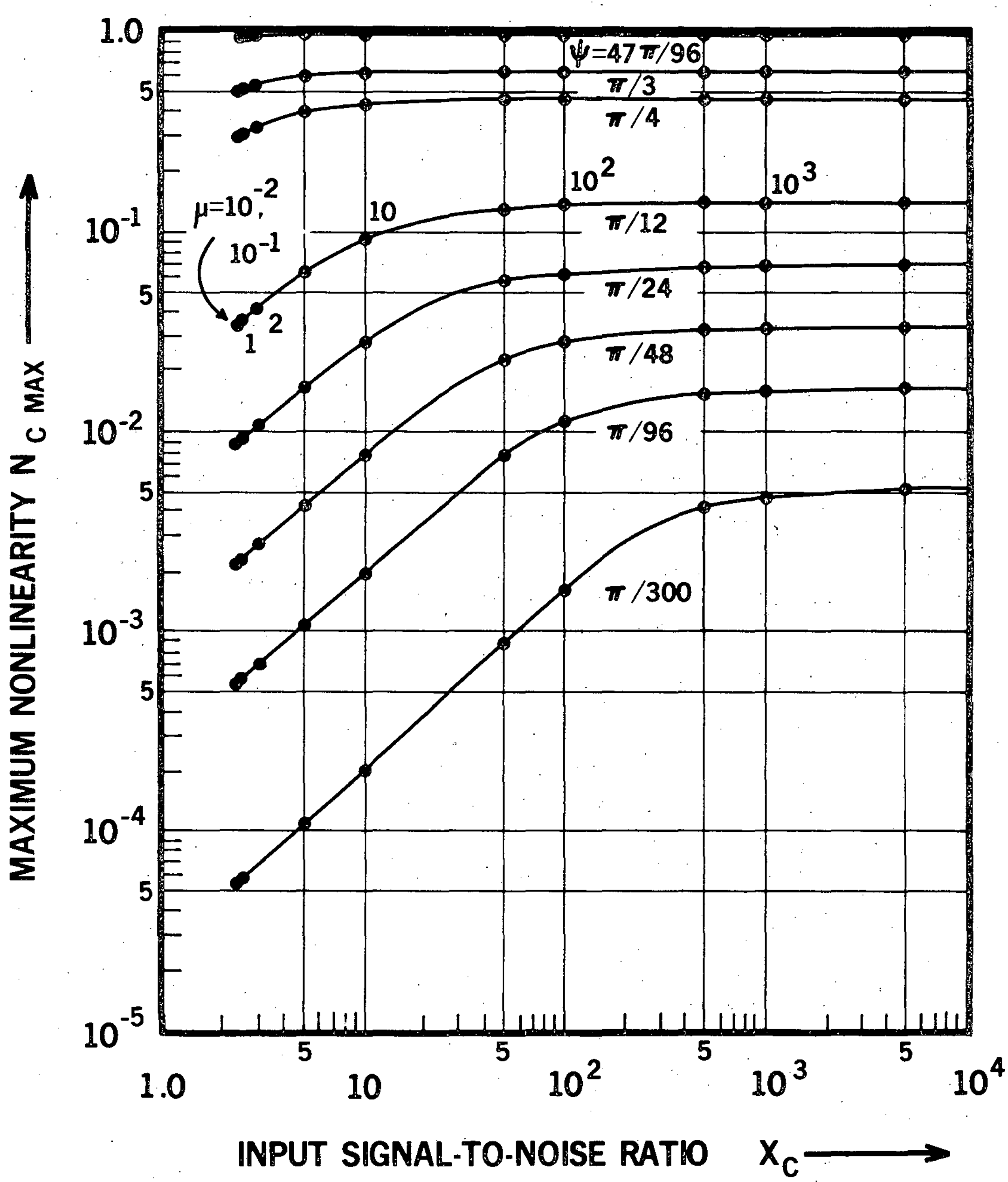
Fig. 2. Maximum nonlinearity N_{CMAX} as a function of the nonoptimum value of the input signal-to-noise ratio x_C, with the phase angle ψ and the reference wave-to-input noise ratio $\mu = 10^{-2}$, 10^{-1}, 1, 5, 10, 10^2, and 10^3 as parameters.

Fig. 3. The normalized phase-sensitive detector characteristics as a function of the phase angle ψ, with the optimum values x_B, the nonoptimum values x_C, and the reference wave-to-input noise ratio $\mu = 10^{-2}$, 10^{-1}, 1.0, 10, 10^2, 10^3 and 10^4 as parameters.



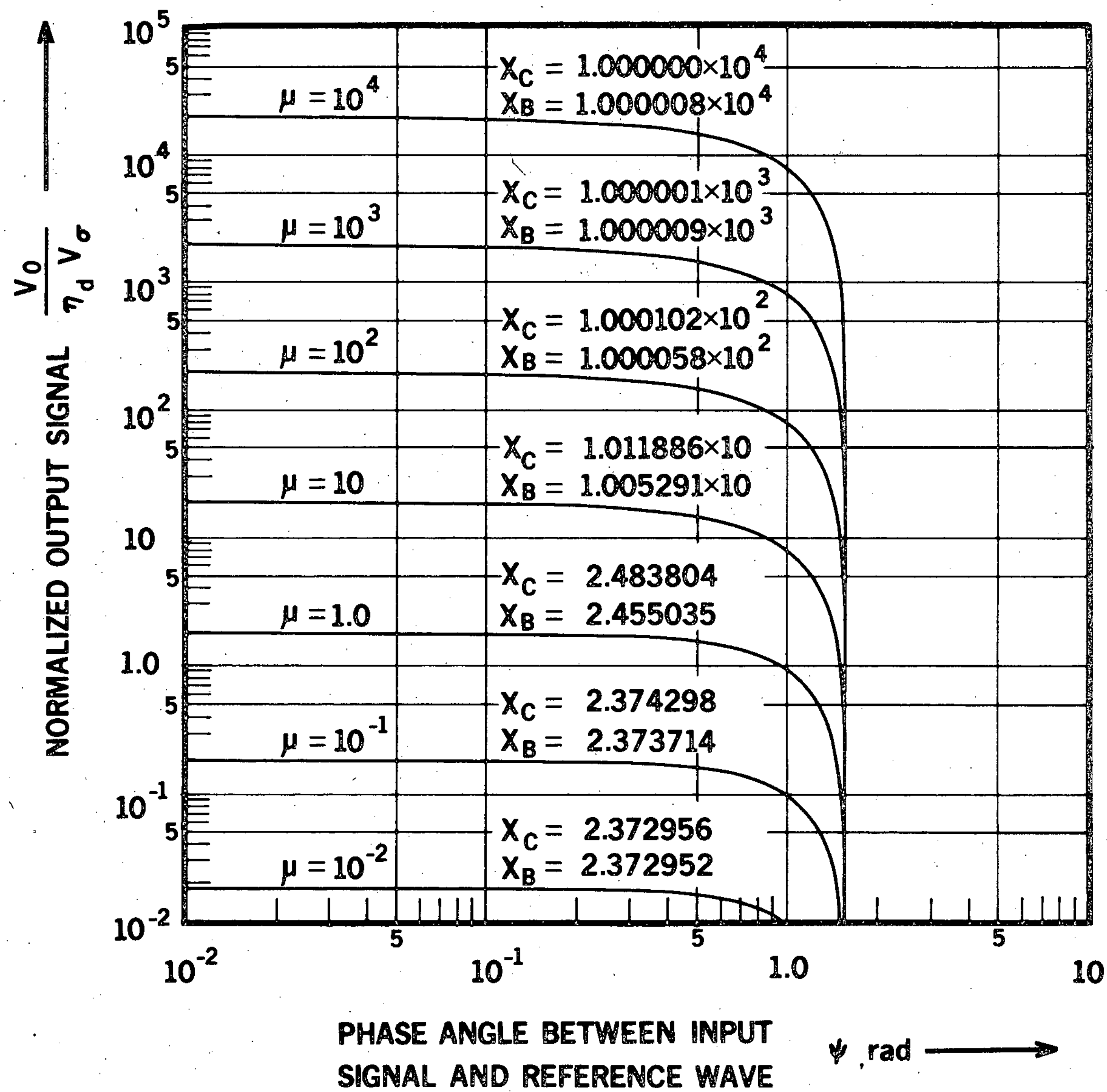
XBL 696-798

Fig. 1



XBL 696-799

Fig. 2



XBL 696-800

Fig. 3

TECHNICAL INFORMATION DIVISION
LAWRENCE RADIATION LABORATORY
UNIVERSITY OF CALIFORNIA
BERKELEY, CALIFORNIA 94720