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Abstract

Non-strategic firms with rational expectations make investment and emissions decisions. The investment rule depends on firms' beliefs about future emissions policies. We compare emissions taxes and quotas when the (strategic) regulator and (nonstrategic) firms have asymmetric information about abatement costs, and all agents use Markov Perfect decision rules. Emissions taxes create a secondary distortion at the investment stage, unless a particular condition holds; emissions quotas do not create a secondary distortion. We solve a linear-quadratic model calibrated to represent the problem of controlling greenhouse gasses. The endogeneity of abatement capital favors taxes, and it increases abatement.

JEL Classification numbers: C61; D8; H21; Q28.

Key Words: Pollution control; Investment; Asymmetric information; Rational expectations; Choice of instruments.

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1 Introduction

The possible relation between carbon stocks and global warming has led to a renewed interest in the problem of controlling emissions when there is asymmetric information about abatement costs. If environmental damages are related to the stock of pollution, as with global warming, the *regulator's* problem is dynamic. Most of the current literature on this dynamic problem assumes that nonstrategic *firms* solve a succession of static problems. If, however, a firm's abatement costs depend on the stock of abatement capital, the firm makes a dynamic investment decision as well as the static emissions decision. We study the regulatory problem with asymmetric information when firms invest in abatement capital. Nonstrategic firms and the regulator solve coupled dynamic problems.

For a variety of pollution problems, capital costs comprise a large part of total abatement costs (Vogan 1991) and investment in abatement capital depends on the regulatory environment. In these cases, the endogeneity of investment is an important aspect of the regulatory problem. Several recent papers, (Buonanno, Carraro, and Galeotti 2001), (Goulder and Schneider 1999), (Goulder and Mathai 2000), (Norhaus 1999), assume that the regulator can choose emissions and also induce firms to provide the first-best level of investment, e.g. by means of an investment tax/subsidy.

We consider the situation where the regulator has a single policy, either an emissions tax or a quota. This assumption is consistent with many regulations and proposals that involve an emissions policy but ignore endogenous investment (e.g., the Kyoto Protocol). In virtually any real-world problem, the regulator is likely to have fewer instruments than targets. Our model is an example of this general disparity between the number of instruments and targets, and therefore is empirically relevant. The restriction enables us to compare our results to those of previous papers that study the asymmetric information, stock pollutant problem in the absence of investment in abatement capital (Hoel and Karp 2001), (Hoel and Karp in press), (Karp and Zhang 2002a) and (Newell and Pizer in press). We identify a previously unrecognized difference between taxes and quantity restrictions, and we provide a simple means of solving the regulatory problem when a certain condition holds. We now describe the problem in more detail.

In each period the representative firm observes a cost shock that is private information. If this cost shock is serially correlated, the regulator learns something about its current value by observing past behavior. The firm knows the current value of the cost shock and therefore is better informed than the regulator. Both types of agents obtain information over time. We assume that the regulator conditions the current emissions policy only on payoff-relevant information: aggregate capital and pollution stocks and (the regulator's) beliefs about the cost shock. The regulator cannot make binding commitments regarding future policies; that is, we restrict policies to be Markov Perfect. Firms have rational expectations; they take the current emissions policy as given and they understand how the regulator chooses future policies. The non-atomic representative firm is not able to affect the economy-wide variables that determine future policies. The firm therefore behaves non-strategically (but not myopically), and uses Markov policies.

The regulator understands that future emissions policies affect the current shadow value of abatement capital and thus affect current investment. Therefore, the regulator might want to commit to future policies as a means of affecting current investment. This incentive is the source of the familiar time-consistency problem. Our setting has the usual ingredients that lead to this problem: the regulator has a second-best instrument (the emissions tax or quota) and he wants to influence forward-looking agents. If the private level of investment is socially optimal, then the regulator has no desire to alter it. In that case, there is no time-consistency problem and we can obtain the equilibrium by solving an optimization problem that contains elements of the regulator's and the firms' problems. If however, the regulator's emissions policy creates a secondary distortion at the investment stage, the time-consistency problem does arise. In that case, the Markov restriction is binding and we need to solve an equilibrium problem (a dynamic game between the regulator and non-strategic firms) rather than a relatively simple dynamic optimization problem. In other words, the type of problem that we need to solve – an equilibrium problem or an optimization problem – depends on whether the Markov Perfect emissions policy (the tax or quota) causes a secondary distortion in investment.

We show that the time-consistency problem does not arise when the regulator uses an emissions quota. It does arise when the regulator uses a tax, unless the primitive functions satisfy a separability condition.

This result is useful for two reasons. First, when the separability condition holds, we can solve the dynamic game by solving a much simpler dynamic optimization problem that combines elements of the regulator's and the firms' optimization problems. The separability condition holds for an important special case that has been used to study the problem of regulating both a flow and a stock pollutant under asymmetric information. We generalize this special case by including endogenous abatement capital. Second, under plausible circumstances the separability condition does not hold. In these cases, an emissions tax creates a secondary investment distortion, whereas the emissions quota does not. Thus, we have identified a difference between taxes and quotas that has previously been unnoticed.[1]

We apply our results to a global warming model in which the separability condition holds. In addition to confirming the robustness of previous results, we show that the endogeneity of abatement capital encourages stricter abatement, and that it favors the use of taxes over quotas. Optimal abatement levels in our model are similar to levels proposed by the Kyoto Protocol.

The next two sections describe the problem. Section 4 establishes the time-consistency results described above, and we show how to solve the game by solving a control problem. Section 5 applies this method to a linear-quadratic model, and Section 6 specializes the model in order to study the problem of greenhouse gasses. We review previous comparisons of taxes and quotas in this linear-quadratic setting, and then discuss how these results change when abatement capital is endogenous.

2 The Basics

The stock of pollutant at the beginning of period t is S_{t-1} and the flow of emissions in period t is x_t. The fraction $0 \leq \Delta \leq 1$ of the pollutant stock lasts into the next period, so the growth equation for S_t is:

$$S_t = \Delta S_{t-1} + x_t. \tag{1}$$

The period t stock-related environmental damage equals $D_t = D\left(S_{t-1}\right)$,with $D' > 0,\ D'' > 0.$

We normalize the number of representative firms to unity. In this section we make no distinction between the firm's and the aggregate levels of emissions and investment. Investment in period t becomes available in the subsequent period. The representative firm's benefit from emissions is a function of emissions, x_t, abatement capital K_{t-1}, and a random cost shock θ_t : $B_t = B\left(K_{t-1}, \theta_t, x_t\right)$.

We can think of the function $B\left(\cdot\right)$ as a restricted profit function in which input and output prices are suppressed. Alternatively, we can interpret $B\left(\cdot\right)$ as the amount of abatement costs

[1] The "more likely" it is that the separability condition holds, the less significant is this potential difference between taxes and quotas, and the more useful is our technique for solving the dynamic game. The converse also holds.

that the firm avoids. For the latter interpretation, define x_t^b as the Business-as-Usual (BAU) level of emissions, i.e. the level of emissions under the status quo. Define $a_t = x_t^b - x_t$ as the level of abatement, i.e. the reduction in emissions due to a new regulatory policy. The additional abatement costs associated with the new regulations are $A_t = A(K_{t-1}, \theta_t, a_t)$. If x_t^b is a function of (K_{t-1}, θ_t), we can rewrite the abatement cost function as $A(K_{t-1}, \theta_t, a_t) = -B(K_{t-1}, \theta_t, x_t)$, with $A_a(\cdot) = B_x(\cdot)$: marginal abatement costs equal the marginal benefit of emissions.

The benefit function is increasing and concave in x and K and increasing in θ ($B_K > 0$, $B_\theta > 0$, $B_x > 0$, $B_{KK} < 0$, $B_{xx} < 0$). More abatement capital decreases the marginal cost of abatement and therefore lowers the marginal benefit of pollution, so $B_{xK} < 0$. A higher cost shock increases the marginal benefits of abatement capital and emissions: $B_{K\theta} \geq 0$, $B_{x\theta} \geq 0$.

At time t only the firm knows the value of the random cost shock θ_t; there is persistent asymmetric information. All agents know the stochastic process for the cost shock, which we assume is $AR(1)$:

$$\theta_t = \rho\theta_{t-1} + \mu_t, \qquad \mu_t \sim iid\left(0, \sigma_\mu^2\right), \qquad \forall t \geq 1, \tag{2}$$

with $-1 < \rho < 1$.[2] The sequence $\{\mu_t\}$ ($t \geq 1$) is generated by an i.i.d. random process with zero mean and common variance σ_μ^2. At time 0 the regulator knows θ_{-1}, so the subjective expectation and variance of θ_0 is $\left(\rho\theta_{-1}, \sigma_\mu^2\right)$. This assumption about the regulator's initial priors makes the problem stationary; it has no bearing on our results, but merely simplifies the notation. At time $t \geq 1$ the regulator's variance for the current shock is σ_μ^2 provided that he has learned the value of the previous shock, θ_{t-1}.

The representative firm invests in abatement capital to reduce future abatement costs, i.e. to increase future benefits from pollution. The flow of investment in period t is I_t. The fraction of abatement capital that lasts into the next period is $0 \leq \delta \leq 1$, so the growth equation for K_t is:

$$K_t = \delta K_{t-1} + I_t. \tag{3}$$

The cost of investment, $C_t = C(I_t, K_{t-1})$, is increasing and convex in I_t. This convexity means that abatement capital does not adjust instantaneously. A greater degree of convexity implies that capital adjusts more slowly.

[2] Throughout the paper we refer to θ as a "cost shock", as an abbreviation for "random cost parameter". In most economically meaningful circumstances, this parameter is positively serially correlated: $\rho > 0$.

The endogeneity of the investment decision means that the marginal abatement cost function, $B_x(\cdot)$, changes endogenously. Slower adjustment of abatement capital means that it is optimal to adjust emissions more slowly.

3 The Game

In this section it is helpful to distinguish between the representative firm's level of capital and the aggregate level of capital. We denote the former by k and the latter by k^A. Where there is no danger of confusion, we denote both using K. Since we normalize the number of representative firms to 1, $k^A = k = K$. The representative firm understands that it controls k, and that this variable affects its payoff directly, via the function $B(\cdot)$. This firm takes the aggregate level of capital k^A as exogenous; k^A has no direct effect on the firm's payoff. However, in a Markov Perfect equilibrium, where the regulator conditions policies on payoff-relevant information, k^A affects the firm's beliefs about future policies.

In order to avoid a proliferation of notation, we do not distinguish between the firm's level of emissions and the aggregate level of emissions. However, it is important to bear in mind that the firm treats aggregate emissions, and therefore the aggregate pollution stock, as exogenous.

The regulator always uses taxes or always uses quotas. The period t policy is the tax p_t or the quota x_t. At time t the regulator knows the aggregate capital stock k^A_{t-1}, the pollution stock S_{t-1} and (as we explain below), the lagged cost shock θ_{t-1}. These are the payoff-relevant variables for the regulator. In a Markov Perfect rational expectations equilibrium, the representative firm takes the current level of the regulatory policy (at time t) as given; it understands that the policy at time $\tau > t$ will be a function of $\left(k^A_{\tau-1}, S_{\tau-1}, \theta_{\tau-1}\right)$. Since the firm takes these conditioning variables to be exogenous, it treats future policies as exogenous. This firm chooses investment I_t under both policies, and it chooses the level of emissions if the regulator uses a tax.

In view of the timing conventions in the model, the regulator's current (tax or quota) policy influences the firm's current emission, but not the current level of investment. Investment depends on the firm's beliefs about *future* policies.

3.1 The Firm's Emission and Investment Responses

The firm wants to maximize the expectation of the present value of the stream of cost saving from polluting (B) minus investment cost (C) minus pollution tax payments (under taxes). The constant discount factor is β, and we use the superscripts T and Q to distinguish functions and variables under taxes and quotas. We assume that emissions are positive under taxes, and that the optimal quota is always binding.[3]

Taxes. The firm's value function under taxes, $V^T\left(k_{t-1},\theta_t,p_t;S_{t-1},k^A_{t-1}\right)$, solves the dynamic programming equation (DPE)

$$V^T\left(k_{t-1},\theta_t,p_t;S_{t-1},k^A_{t-1}\right)=\max_{x_t,I_t}\{B\left(k_{t-1},\theta_t,x_t\right)-p_tx_t-C\left(I_t,k_{t-1}\right)$$
$$+\beta E_t\left[V^T\left(k_t,\theta_{t+1},p_{t+1;}S_t,k^A_t\right)\right]\},$$

subject to the equation of motion for the cost shock (2), the capital stock (3), and the pollution stock (1). The firm's expectation at t of θ_{t+1} and p_{t+1} is conditioned on the payoff-relevant variables $\left(k^A_{t-1},\theta_t,S_{t-1}\right)$.

The optimal level of emissions solves a static problem with the following first-order condition

$$B_x\left(k_{t-1},\theta_t,x_t\right)-p_t=0. \tag{4}$$

Solving for x, we obtain the optimal emission response

$$x^*_t=\chi\left(k_{t-1},\theta_t,p_t\right)\equiv\chi_t. \tag{5}$$

The optimal level of investment equates the marginal cost of investment and the discounted shadow value of abatement capital. Setting $k^A=k=K$, the stochastic Euler equation is[4]

$$\beta E_t\left\{B_K\left(K_t,\theta_{t+1},\chi_{t+1}\right)-C_K\left(I_{t+1},K_t\right)+\delta C_I\left(I_{t+1},K_t\right)\right\}-C_I\left(I_t,K_{t-1}\right)=0. \tag{6}$$

[3]As $t\to\infty$ the support of θ_t covers the real line. Thus, the assumption that emissions are positive under taxes and that the quota is binding with probability 1 (for all t) requires that the marginal effect (on B_x) of θ become small as $\theta\to-\infty$.

[4]For all of the control problems, we merely write the Euler equation since the derivations are standard. The first order condition of the DPE with respect to I_t provides one equation. In this first order condition, the firm's expectation of p_{t+1} is independent of its investment. This independence reflects the fact that the firm is unable to affect aggregate capital or pollution stock, and therefore cannot affect values of the variables that affect future regulation. We differentiate the DPE with respect to k_{t-1}, using the envelope theorem, to obtain a second equation. Combining these two equations gives the stochastic Euler equation.

This second-order difference equation has two boundary conditions, the current abatement capital K_{t-1}, and the transversality condition

$$\lim_{T\to\infty} E_t \left\{ \beta^{T-t} C_I \left(I_T, K_{T-1}\right) K_T \right\} = 0. \tag{7}$$

Quotas. Firms are homogeneous and quotas are not bankable. Thus, under a quota policy, there is no incentive to trade permits.[5] The firm solves the DPE

$$\begin{aligned} V^Q \left(k_{t-1}, \theta_t, x_t; S_{t-1}, k_{t-1}^A\right) = \max\nolimits_{I_t} \{ B\left(k_{t-1}, \theta_t, x_t\right) - C\left(I_t, k_{t-1}\right) \\ + \beta E_t V^Q \left(k_t, \theta_{t+1}, x_{t+1}; S_t, k_t^A\right) \}. \end{aligned}$$

Again, the firm's beliefs about the quota in the next period depend on $\left(k_{t-1}^A, \theta_t, S_{t-1}\right)$.

The optimal level of investment solves the stochastic Euler equation

$$\beta E_t \left\{ B_K \left(K_t, \theta_{t+1}, x_{t+1}\right) - C_K \left(I_{t+1}, K_t\right) + \delta C_I \left(I_{t+1}, K_t\right) \right\} - C_I \left(I_t, K_{t-1}\right) = 0, \tag{8}$$

and the transversality condition (7).

The investment rule Under both taxes and quotas, the current level of investment depends on the firm's beliefs about future policy levels, but it does not depend on the current policy level. The firm has rational expectations about future policies; we discuss this policy rule in the next section. Under either taxes or quotas, the representative firm's equilibrium investment rule at time t is a function of $\left(k_{t-1}, \theta_t; S_{t-1}, k_{t-1}^A\right)$. When there is no danger of confusion, we write the investment rule as $I^j \left(K_{t-1}, \theta_t, S_{t-1}\right)$, $j = T, Q$ (for tax or quota).

3.2 The Regulator's Problem

The regulator's payoff equals the payoff to the representative firm net of taxes, minus environmental damages. The regulator maximizes the expectation of the present discounted value of the flow of the payoff, i.e. the expectation of

$$\sum_{t=0}^{\infty} \beta^t \left(B\left(K_{t-1}, \theta_t, x_t\right) - C\left(I_t, K_{t-1}\right) - D(S_{t-1}) \right).$$

[5]In a model without abatement capital, Karp and Zhang (2002a) show how trade in permits amongst heterogenous firms enables the regulator to learn the value of the cost shock. Without trade in permits (and in the absence of investment decisions), the regulator does not know the previous cost shock when choosing the current quota; in this case, taxes have an informational advantage, relative to quotas. As we point out in the text, when the firm invests in abatement capital, the regulator does not need tradeable quotas in order to learn the cost shock.

His policy (always a tax or always a quota) can be a function of (only) payoff-relevant variables: the current stocks of pollution and capital, and the regulator's current information about the cost shock. Under taxes the regulator knows that equation (5) determines emissions. Under either policy, he knows that investment is given by $I^j\left(K_{t-1}, \theta_t, S_{t-1}\right)$, $j = T, Q$.

The regulator takes as given the investment rule and (under taxes) the emissions rule. At time t the regulator observes the aggregate stocks S_{t-1}, K_{t-1}. If $\rho = 0$, the regulator learns nothing about the current cost shock by observing firms' past behavior. The past cost shock provides information about the current shock if and only if $\rho \neq 0$. Under taxes, the regulator learns the previous cost by observing the response to the previous tax (via equation (5)). Provided that $B_{K\theta} \neq 0$ the regulator who uses quotas can learn the previous cost shock by observing the level of investment in the previous period, i.e. by inverting the investment function $I^Q(\cdot)$. From equation (8), $B_{K\theta} \neq 0$ means that current investment depends on the firm's beliefs about future cost shocks. When $\rho \neq 0$ these beliefs – and therefore current investment – depend on the current cost shock.

If $B_{K\theta} \neq 0$, as we hereafter assume, taxes and quotas give the regulator the same information about the previous cost shock, and thus about the current cost shock. Neither policy has an informational advantage. Of course, using observed emissions (under taxes) to infer the past cost variable requires only that the regulator solve the first order condition of a static problem. Using observed investment (under quotas) to infer the past cost variable requires that the regulator knows the function $I^Q\left(K_{t-1}, \theta_t, S_{t-1}\right)$; that requires the solution of the entire equilibrium. Thus, although both policies have the same informational content (unless $\rho \neq 0$ and $B_{K\theta} = 0$), this information is easier to extract under taxes.

The regulator's decision rule is a function $z^i\left(K_{t-1}, \theta_{t-1}, S_{t-1}\right), j = T, Q$ that determines the current tax ($j = T$) or quota ($j = Q$) as a function of his current information, given his beliefs about the firm's decision rules.

3.3 The Equilibrium

Both the regulator and the representative firm solve stochastic control problems; the exact problem that one agent solves depends on the solution to the other agent's problem. The rational expectations equilibrium investment rule for the firm depends on the regulator's policy rule, and that policy rule depends on the equilibrium investment rule. The investment and the regulatory decision rules generate a random sequence of pollution and capital stocks. Agents have rational expectations about these random variables.

An equilibrium consists of a (possibly non-unique) pair of decision rules $I^{j*}(K_{t-1}, \theta_t, S_{t-1})$ and $z^{j*}(K_{t-1}, \theta_{t-1}, S_{t-1})$ for $j = T, Q$ that are mutually consistent; the superscript "$*$" indicates equilibrium functions. Hereafter we refer to $I^{j*}(K_{t-1}, \theta_t, S_{t-1})$, and $z^{i*}(K_{t-1}, \theta_{t-1}, S_{t-1})$ as Markov Perfect policy rules.

Modern computational methods make it possible to (approximately) solve these kinds of dynamic equilibrium problems, i.e. to find a fixed point in function space (Judd 1998), (Marcet and Marimon 1998), (Miranda and Fackler 2002). These fixed point problems are not trivial, especially when the state space has more than one dimension – it has three in our problem.

4 Finding the Markov Perfect Equilibrium

In many cases, the type of model described in the previous section must be solved as an equilibrium problem rather than as an optimization problem. The next subsection explains why this complication might arise. Using an auxiliary control problem in which the regulator has two policy instruments, we then identify conditions under which the model *can* be solved as a straightforward optimization problem.

4.1 The Time-Consistency Problem

In general, the regulator might want to announce a rule that would determine future levels of the tax or quota. The purpose of such an announcement would be to alter the firm's investment rule – as distinct from altering a stock that appears as an argument of the investment rule. The inability to make binding commitments, and the Markov assumption, exclude this possibility. In a rational expectations equilibrium, current investment depends on beliefs about future policies, and these beliefs and policies depend on the pollution stock. By choice of the current quota or tax level, the regulator affects the future pollution stock, which can affect future investment. Under our assumptions, the only means by which the regulator can influence future levels of investment is by influencing the future level of the pollution stock.

Consider a simpler problem without asymmetric information, where a representative firm with rational expectations makes investment decisions. The firm's optimal decisions depend on its beliefs about future regulations, and the regulator wants to influence the firm's decisions. If the regulator has a first best policy (defined as one that does not cause secondary distortions), he can induce the firm to select exactly the decisions that the regulator would have used, had he been in a position to choose them directly. In that case, the regulatory problem can be solved as standard optimization problem. If, however, the regulator has only a second-best policy, the familiar time-consistency problem arises. (See Xie (1997) for a recent discussion of this problem, and references.) The Markov restriction is binding in this setting, so finding the equilibrium requires solving an equilibrium problem rather than a standard optimization problem.

The presence of asymmetric information in our model leads to the possibility of time-inconsistency of the optimal emissions tax or quota. We know from the literature on principal-agent problems that with asymmetric information, non-linear policies are generally superior to either the linear tax or the quota: neither the linear tax nor the quota is typically the information-constrained first best policy. We showed above that the firm's investment depends on its beliefs about future policies. Since the regulator has two targets, (emissions and investment) and only one instrument (which is typically inferior to some non-linear policy), it appears that the regulator *might* want to use future emissions taxes or quotas to influence the firm's current investment decision. In that case, the information-constrained first best tax or quota would be time-inconsistent: the ability to make commitments about future taxes or quotas would enable the regulator to achieve a higher payoff than under the Markov restriction. If this were the case, we would not be able to obtain a Markov Perfect equilibrium merely by solving a dynamic optimization problem, but would instead have to solve the equilibrium problem described in the previous section.

4.2 An Auxiliary Control Problem

This subsection describes an auxiliary control problem that helps identify conditions under which the Markov Perfect equilibrium can be obtained by solving an optimization problem. In this control problem, in each period the regulator sets an emissions tax or quota using the same information as in the game; later in the same period he observes the current cost shock and then chooses investment directly. (In contrast, in the game the regulator chooses only an emissions policy.) The ability to control current investment directly, knowing the current cost shock, eliminates any incentive to use future emissions policies to control current investment.

In this setting, it does not matter whether the regulator chooses investment directly (e.g. by command and control), or decentralizes this decision by means of an investment tax/subsidy.

In the former case, firms make no investment decision, and in the latter case, firms merely carry out the optimal investment decision induced by the investment tax/subsidy.

As an aid to intuition, it is useful to think of decentralizing the optimal investment decision (from the auxiliary problem) using an investment tax/subsidy. The *optimal* investment tax/subsidy is identically 0 if and only if Markov Perfect rules are equivalent to the optimal policy rules in the auxiliary problem. With an identically zero investment tax, agents have exactly the same optimization problem as in the game. It is optimal to use a non-zero investment tax/subsidy if and only if the Markov Perfect policies do not solve the auxiliary problem.

The Markov Perfect equilibrium investment rule is conditioned on $(K_{t-1}, \theta_t, S_{t-1})$, whereas the emissions tax or quota is conditioned on $(K_{t-1}, \theta_{t-1}, S_{t-1})$. Consequently, in the auxiliary problem we need to consider a two-stage optimization within each period. At the beginning of the period the regulator knows $(K_{t-1}, \theta_{t-1}, S_{t-1})$ and chooses the emissions policy (a tax or quota); the regulator then learns θ_t and chooses the level of investment (equivalently, the investment tax/subsidy).

It does not matter whether this time-line is "plausible". We use this problem only as a means of finding conditions under which the Markov Perfect rules can be obtained by solving a control problem. If the Markov Perfect investment rule is equivalent to the investment rule in the auxiliary problem, then a regulator who had to choose investment (or an investment tax/subsidy) before knowing θ_t would obviously prefer to allow firms to choose investment; i.e., the regulator would use a zero investment tax/subsidy.

We describe the auxiliary control problem when the regulator uses an emissions quota, and then when he uses an emissions tax.

4.2.1 Quotas

The regulator solves the following DPE:

$$\begin{aligned} \mathcal{J}^Q\left(K_{t-1}, S_{t-1}, \theta_{t-1}\right) = \max\nolimits_{x_t} E_{\theta_t|\theta_{t-1}} \{ B\left(K_{t-1}, \theta_t, x_t\right) - D\left(S_{t-1}\right) \\ + \max\nolimits_{I_t} \left[-C\left(I_t, K_{t-1}\right) + \beta \mathcal{J}^Q\left(K_t, S_t, \theta_t\right) \right] \} \end{aligned} \tag{9}$$

subject to equations (1) and (3). The first order condition for the optimal quota is

$$E_{\theta_t|\theta_{t-1}} \{ B_x\left(K_{t-1}, \theta_t, x_t\right) + \beta \mathcal{J}_S^Q\left(K_t, S_t, \theta_t\right) \} = 0 \tag{10}$$

and the Euler equation for investment under quotas is

$$\beta E_{\theta_{t+1}|\theta_t} \left\{ B_K\left(K_t, \theta_{t+1}, x_{t+1}\right) - C_K\left(I_{t+1}, K_t\right) + \delta C_I\left(I_{t+1}, K_t\right) \right\} - C_I\left(I_t, K_{t-1}\right) = 0. \tag{11}$$

The transversality condition is

$$\lim_{T\to\infty} E_{\theta_T|\theta_t}\left\{\beta^{T-t} C_I\left(I_T, K_{T-1}\right) K_T\right\} = 0. \tag{12}$$

4.2.2 Taxes

Using the firm's emission response function (5), the regulator in the auxiliary problem solves the following DPE

$$\begin{aligned}\mathcal{J}^T\left(K_{t-1}, S_{t-1}, \theta_{t-1}\right) = \max\nolimits_{p_t} E_{\theta_t|\theta_{t-1}}\{B\left(K_{t-1}, \theta_t, x_t^*\right) - D\left(S_{t-1}\right) + \\ \max\nolimits_{I_t}\left[-C\left(I_t, K_{t-1}\right) + \beta\mathcal{J}^T\left(K_t, S_t, \theta_t\right)\right]\}\end{aligned} \tag{13}$$

subject to equations (1), (3) and (5). We use the definition

$$H_t \equiv \left[B_x\left(K_{t-1}, \theta_t, x_t^*\right) + \beta\mathcal{J}_S^T\left(K_t, S_t, \theta_t\right)\right],$$

and the abbreviation $\chi_t \equiv \chi\left(K_{t-1}, \theta_t, p_t\right) = x_t^*$. The function H_t is the social benefit of an additional unit of emissions. With this notation, we can write the first-order condition with respect to p_t as

$$E_{\theta_t|\theta_{t-1}}\left\{H_t \frac{\partial \chi_t}{\partial p_t}\right\} = 0, \tag{14}$$

and the stochastic Euler equation for investment as

$$\begin{aligned}\beta E_{\theta_{t+1}|\theta_t}\left\{B_K\left(K_t, \theta_{t+1}, x_{t+1}^*\right) - C_K\left(I_{t+1}, K_t\right) + \delta C_I\left(I_{t+1}, K_t\right) + H_{t+1}\tfrac{\partial \chi_{t+1}}{\partial K_t}\right\} \\ -C_I\left(I_t, K_{t-1}\right) = 0.\end{aligned} \tag{15}$$

The transversality condition is equation (12).

4.3 Social Optimality of the Markov Perfect Rules

Here we find conditions under which the solution to the auxiliary control problem is a Markov Perfect equilibrium to the original game. We refer to the following as the "separability condition" since it requires that $\frac{\partial}{\partial\theta}B_{xx} = \frac{\partial}{\partial\theta}B_{xK} = 0$ when evaluated at the optimal level of emissions:

Condition 1 *(Separability)* B_{xx} *and* B_{xK}*, evaluated at the optimal* x^**, are both independent of the cost variable* θ*.*

We have

Lemma 1 *The separability condition is equivalent to the following two conditions: (a)* $\frac{\partial \chi(K_{t-1},\theta_t,p_t)}{\partial p_t}$ *is independent of* θ_t*. (b)* $\frac{\partial \chi(K_{t-1},\theta_t,p_t)}{\partial K_{t-1}}$ *is independent of* θ_t*, where* p_t *is the time* t *emissions tax .*

Proof. Totally differentiating the first-order condition (4) gives

$$\frac{\partial \chi_t}{\partial p_t} = \frac{1}{B_{xx}\left(K_{t-1}, \theta_t, x_t^*\right)}, \quad \frac{\partial \chi_t}{\partial K_{t-1}} = -\frac{B_{xK}\left(K_{t-1}, \theta_t, x_t^*\right)}{B_{xx}\left(K_{t-1}, \theta_t, x_t^*\right)}.$$

Condition (a) holds if and only if $B_{xx}\left(K_{t-1}, \theta_t, x_t^*\right)$ is independent of θ_t. This independence means that Condition (b) holds if and only if $B_{xK}\left(K_{t-1}, \theta_t, x_t^*\right)$ is independent of θ_t. ■

Our main result is the following

Proposition 1 *(i) When the regulator uses emissions quotas, the solution to the auxiliary problem* (9) *is a Markov Perfect equilibrium to the original game. (ii) When the regulator uses emissions taxes, the solution to the auxiliary problem* (13) *is a Markov Perfect equilibrium to the original game if and only if the separability condition holds.*

The proof, contained in Appendix 1, verifies that the equilibrium conditions in the games and in the auxiliary problems are identical under the conditions stated in the Proposition.

4.3.1 Significance of the proposition

When the regulator uses quotas to control emissions, the Markov Perfect investment rule is (information-constrained) socially optimal. If the regulator uses taxes to control emissions, the Markov Perfect investment rule is socially optimal if and only if the separability condition is satisfied. This condition depends only on the benefit function $B(\cdot)$, not on the damage or the investment cost function. Under the two conditions in Proposition 1, the investment tax that would support the optimal investment (from the auxiliary problem) is identically 0.

Proposition 1 identifies a previously unnoticed difference between taxes and quotas. When the separability condition does not hold, the regulator who uses an emissions tax to control pollution creates a secondary distortion in investment. In these circumstances, private investment is optimal under an emissions quota but not under an emissions tax.

The Proposition also provides a simple way of obtaining the equilibrium for the game when the separability condition holds. This method requires only solving a dynamic optimization problem rather than a dynamic equilibrium problem.

4.3.2 Interpretation of the Separability Condition

We first identify the secondary distortion under emissions taxes, and we explain why it vanishes if the separability condition holds. This discussion also explains why emissions taxes and quotas typically have different effects, as regards the secondary distortion.

In order to identify the secondary distortion, we follow the standard procedure of computing the investment tax/subsidy that supports the information-constrained first best investment policy. Suppose that firms face an investment tax s_t, so their single period payoff is $B(\cdot) - C(\cdot) - s_t I_t - p_t x_t$. We can write the Euler equation for the capital stock corresponding to this problem, and compare it to the optimal investment policy under an emissions tax, equation (15). We omit the details, but the comparison implies that the investment tax supports the socially optimal level of investment if and only if[6]

$$-s_t + \beta\delta E_{\theta_{t+1}|\theta_t} s_{t+1} = \beta E_{\theta_{t+1}|\theta_t} \left\{ H_{t+1} \frac{\partial \chi_{t+1}}{\partial K_t} \right\}. \quad (16)$$

The left side of equation (16) equals the effect of the tax sequence on the marginal incentive to invest in the current period. Under the investment tax, an additional unit of investment costs the firm s_t in the current period, but reduces the cost of tax payments by $\delta E_t s_{t+1}$ in the next period. The right side of equation (16) is the present value of the expectation of the secondary distortion. H_{t+1} is the marginal value to society of an additional unit of emissions in the next period, and $\frac{\partial \chi_{t+1}}{\partial K_t}$ equals the change in emissions in the next period caused by an additional unit of investment in the current period. Thus, the term in brackets in equation (16) is the value to society of the lower future emissions caused by the additional investment. This benefit is external to the firm. The optimal investment tax sequence induces the firm to internalize the present value of the expectation of this additional social benefit of investment – i.e., to internalize the externality.

The optimal emission quota does not create a secondary distortion. Under the quota, the expected social benefit of an additional unit of emissions is zero in each period (equation (10)). The socially optimal rule for determining investment, equation (11), involves only the current and future expected marginal investment and abatement costs. The socially optimal balance of these costs is identical to the balance that firms choose.

The optimal tax, in contrast, requires that an marginal change in the tax has zero expected social value (equation (14)) . This condition is not, in general, equivalent to the requirement that

[6]The right side of equation (16) equals the function τ, used in the proof of Proposition 1.

the expected social marginal benefit of emissions (H_t) is zero. The expected social marginal benefit of an additional unit of emissions is zero if and only if B_{xx} is independent of θ (equivalently, if and only if $\frac{\partial \chi}{\partial p}$ is independent of θ). This independence implies that $E_t H_t = 0$.

Even if this independence holds, H_t is a random variable, a function of θ. If $\frac{\partial \chi}{\partial K}$ is also a function of θ (i.e., if B_{xK} is not independent of θ), then the social marginal benefit of emissions is correlated with $\frac{\partial \chi}{\partial K}$. In that case, the expected marginal value to society of the lower future emissions caused by the additional investment (i.e., the secondary distortion, measured by the right side of equation (16)) is non-zero. Here, the investment externality is non-zero. Consequently, both B_{xx} and B_{xK} must be independent of θ in order for the investment externality to vanish under emissions taxes.

5 The Linear-Quadratic Model

The linear-quadratic model has been widely used both for qualitative and numerical analysis. Here we provide the linear-quadratic model that includes endogenous investment.[7]

The representative firm's benefit function is

$$B\left(K_{t-1}, \theta_t, x_t\right) = f_0 + \left(f_1 + \psi\theta_t\right) K_{t-1} - \frac{f_2}{2} K_{t-1}^2 + \left(a - \phi K_{t-1} + \theta_t\right) x_t - \frac{b}{2} x_t^2$$

with $f_1 > 0$, $f_2 > 0$, $b > 0$, $\psi \geq 0$, $\phi \geq 0$. The function $B\left(\cdot\right)$ (which includes the rental cost of capital) satisfies the separability condition. The cost of changing the level of capital is[8]

$$C\left(I_t\right) = \frac{d}{2}\left(I_t\right)^2, \quad d > 0.$$

Environmental damages are also quadratic:

$$D(S_{t-1}) = \frac{g}{2}\left(S_{t-1} - \bar{S}\right)^2$$

[7] The condition on $B\left(\cdot\right)$ mentioned in footnote 3 does not hold for the linear-quadratic model. Therefore, we can only guarantee that the assumptions of positive emissions under the tax and of a binding quota hold for finite t with high probability. This fact is one of many reasons why the linear-quadratic model can be viewed only as an approximation to the "real world".

[8] We can replace the investment cost function with a quadratic function of net rather than gross investment, so that adjustment costs are zero in the steady state. This slightly more plausible model does not lead to any interesting changes in analysis below. However, it complicates the problem of calibrating the model. Therefore we discuss only the model in which adjustment depends on gross investment.

where $\bar{S}$ is the stock level that minimizes damages.

The following Remark collects a number of useful facts about the comparison of policies. These results will be obvious to readers familiar with the linear-quadratic control problem, so we state them without proof:

Remark 1 *In this linear-quadratic model with additive errors, the Principle of Certainty Equivalence holds. The expected trajectories of all stock and flow variables are the same under taxes and quotas. The higher moments of these trajectories differ under the two policies. Neither the policy ranking nor the magnitude of the payoff difference depends on the information state* $(K_{t-1}, S_{t-1}, \theta_{t-1})$. *The magnitude (but not the sign) of the difference in payoffs depends on the variance of cost,* σ_μ^2.

In the static version of this problem, damages are caused by the flow of pollution, shocks are iid, and there is no abatement capital. The static linear-quadratic model has properties analogous to those listed in Remark 1. In both the static and the dynamic problems, these properties make it possible to compare policies using a minimum of information (e.g., without using information on the magnitude of uncertainty or stocks).

5.1 Regulated Emissions and Investment

For the linear-quadratic model we obtain an explicit equation for the emissions rule (equation (5)) under taxes:

$$x_t^* = e_t - \frac{\phi}{b}K_{t-1} + \frac{\theta_t}{b}; \quad e_t \equiv \frac{a - p_t}{b}.$$

A higher cost variable increases current emissions, and a higher tax or a higher stock of abatement capital decreases emissions.

Using standard methods (e.g. Chapter 14 of Sargent (1987)) we can solve the firm's Euler equation ((6) under taxes and (8) under quotas) to write current investment as a linear function of current capital (K_{t-1}) and the firm's expectations of the future cost variables and policies (taxes or quotas). The optimal investment under emissions taxes is

$$\begin{gathered} I_t^* = \tfrac{\lambda\beta f_1}{d\delta(1-\lambda\beta)} + (\lambda - \delta) K_{t-1} \\ + \tfrac{\lambda\beta}{d\delta} E_t \left[\left(\psi - \tfrac{\phi}{b}\right) \sum_{j=0}^{\infty} (\lambda\beta)^j \theta_{t+1+j} - \phi \sum_{j=0}^{\infty} (\lambda\beta)^j e_{t+1+j} \right] \end{gathered}$$

where $0 < \lambda < 1$ is the smaller root of the quadratic equation $\lambda^2 + \frac{h}{\beta}\lambda + \frac{1}{\beta} = 0$ and $h \equiv -\left[\frac{1}{\delta} + \frac{\beta}{d\delta}\left(f_2 - \frac{\phi^2}{b}\right) + \beta\delta\right]$. A lower expected future tax (i.e., a higher value of e_{t+j}) decreases current investment. A higher expected future cost shock increases (decreases) current investment if $\psi - \frac{\phi}{b}$ is positive (negative). Since $B_{K\theta} = \psi > 0$, a higher expected cost shock increases the expected marginal benefit of capital – and thus increases the marginal shadow value of capital. This effect encourages investment. However, a higher expected cost shock increases expected emissions, reducing the expected marginal benefit of capital ($B_{xK} = -\phi < 0$) and discouraging investment. These offsetting effects are exactly balanced if $\psi = \frac{\phi}{b}$, in which case the cost shock has no effect on investment, under emissions taxes.

The optimal investment under emissions quotas is

$$\begin{array}{c} I_t^* = \frac{\mu\beta f_1}{d\delta(1-\mu\beta)} + (\mu - \delta) K_{t-1} \\ + \frac{\mu\beta}{d\delta} E_t \left[\psi \sum_{j=0}^{\infty} (\mu\beta)^j \theta_{t+1+j} - \phi \sum_{j=0}^{\infty} (\mu\beta)^j x_{t+1+j} \right] \end{array} \quad (17)$$

where $0 < \mu < 1$ is the smaller root of the quadratic equation $\mu^2 + \frac{w}{\beta}\mu + \frac{1}{\beta} = 0$ and $w \equiv -\left(\frac{1}{\delta} + \frac{\beta f_2}{d\delta} + \beta\delta\right)$. Higher expected quotas decrease investment, and higher expected cost shocks increase investment. With quotas, cost shocks have an unambiguous effect, because the firm treats future emissions quotas as exogenous.

5.2 A Limiting Case: Flow Externality

If $\Delta = 0$ the model collapses to the case of a flow externality, in which emissions in the current period cause damages only in the next period: $D(S_{t-1}) = D(x_{t-1})$. By defining $\tilde{D}(x_t) = \beta D(x_t)$ we can write the difference between the benefits and costs of current emissions as $B(K_{t-1}, \theta_t, x_t) - \tilde{D}(x_t)$. This simplification eliminates a state variable (S), making it possible to obtain some analytic results. We can solve the dynamic programming equations under taxes and quotas and compare the payoffs. (Details of the calculations are available on request).

We noted in Section 3.2 that both policies enable the regulator to acquire the same information about the current cost variable if either of these conditions hold: (a) $\rho = 0$; or (b) $\rho \neq 0$ and $\psi \neq 0$. The last inequality implies that the regulator learns the lagged value of θ by observing investment under quotas – see section 3.2 and equation (17). We show that in either of these two cases, the policy ranking does not depend on the parameters associated with abatement capital. If, however, neither of these two conditions hold (and if in addition quotas are not traded) taxes have an informational advantage. In that case, the policy ranking does depend on the parameters associated with abatement costs.

If $\rho = 0$, or if $\rho \neq 0$ and $\psi \neq 0$, the payoff difference under taxes and quotas, is

$$\mathcal{J}^T - \mathcal{J}^Q = \frac{\sigma_\mu^2}{2b(1-\beta)}\left(1 - \frac{\beta g}{b}\right).$$

This expression reproduces a result in Weitzman (1974)'s static model and in two dynamic models ((Hoel and Karp in press) and (Karp and Zhang 2002a)).

If $\rho \neq 0$ and $\psi = 0$, and quotas are not traded, the regulator learns the past cost variable under taxes but not under quotas. Here the payoff difference equals

$$\mathcal{J}^T - \mathcal{J}^Q = \frac{\sigma_u^2}{2b\,(1-\beta)}\left[\Gamma + \left(1 - \frac{\beta g}{b}\right)\right] \tag{18}$$

The function $\Gamma > 0$ (see appendix) embodies the informational advantage of taxes; Γ depends on f_2, d and δ (among other parameters).

We summarize the implications of these expressions in the following:

Remark 2 *For a flow pollutant ($\Delta = 0$): (i) When taxes and quotas have the same informational content (i.e., if (a) $\rho = 0$, or if (b) $\rho \neq 0$ and $\psi \neq 0$), the policy ranking depends only on the relative slopes (appropriately discounted) of the marginal benefit and damage functions. (ii) When taxes have an informational advantage (i.e., when neither conditions (a) or (b) in part (i) hold and quotas are not traded) the policy ranking also depends on the parameters associated with abatement capital.*

The next section considers the problem of a stock-related pollutant. There, even when neither policy has an informational advantage, the policy ranking does depend on the parameters associated with abatement capital – in contrast to Remark 2.i. Here we explain why stock and flow pollutants have this qualitative difference.

As Remark 1 notes, the expected levels of emissions and of investment are the same under taxes and quotas. The first order condition for investment (using equation (9) or (13)) is

$$-C_I\left(I_t, K_{t-1}\right) + \beta \mathcal{J}_K^i\left(K_t, S_t, \theta_t\right) = 0, \quad i = T, Q.$$

The linear-quadratic structure with additive variables implies that $\mathcal{J}_K^T\left(K_t, S_t, \theta_t\right) \equiv \mathcal{J}_K^Q\left(K_t, S_t, \theta_t\right)$: the investment rules under taxes and quotas, conditional on (K_{t-1}, S_t, θ_t), are identical.

For a stock pollutant, $\mathcal{J}_{K,S}^i \neq 0$, so investment at time t depends on the pollution stock at the beginning of the next period, S_t. That pollution stock depends on current emissions; therefore, emissions in period t affect investment in period t. Conditional on the regulator's information at the beginning of a period, the current level of emissions is random under taxes and is a choice variable under quotas. Therefore, conditional on the information at the beginning of a period, the distribution function for the current level of investment differs under the two policies. The expected payoff difference therefore depends on the parameters associated with abatement capital.

In contrast, with a flow pollutant, the current level of emissions has no effect on future payoffs. The shadow value of capital $\mathcal{J}^i_K$ depends only on (K_t, θ_t). With a flow pollutant, the *current* investment and *current* emissions decisions are decoupled. Therefore, the value to the regulator of the difference in emissions under taxes and quotas does not depend on investment costs.

6 An Application to Global Warming

With a stock externality problem such as greenhouse gasses, we have three state variables (greenhouse gasses, the capital stock, and the expected cost shock) and therefore cannot obtain an analytic solution. However, using Proposition 1, it is straightforward to solve the tax and quota problems numerically. The resulting control problem is almost standard, except that new information arrives within a period, so there are two stages of optimization within a period. This fact accounts for the nested maximization in equations (9) and (13). For the linear-quadratic model, we can solve each of these dynamic programming problems by solving a matrix Riccati equation. (Details are available upon request.)

6.1 Model Calibration

Table 1 describes the model. In order to calibrate the general linear quadratic model described in the previous section, we assume that benefits are equal to the value of abatement cost that the firm avoids by increasing emissions. Abatement costs are a quadratic function of abatement, $x^b_t - x_t$ (row 6), where the BAU emissions x^b_t is a decreasing linear function of abatement capital (row 5). A higher level of abatement capital makes it cheaper to reduce emissions, and also decreases the marginal abatement costs. The cost variable $\tilde{\theta}$ (which is proportional to the random variable θ used above) changes the level of BAU emissions and therefore changes marginal abatement costs. Row 7 of Table 1 repeats the general linear quadratic model; the final row gives the parameter restrictions under which this general model reproduces the special model described in the rows 2- 6 of the table.[9] If $m_1 = 0$, capital does not affect abatement

[9]We ignore the effect of $\tilde{\theta}$ on the constant term since the constant has no effect on the regulator's control.

1. Pollutant stock growth	$S_t - \bar{S} = \Delta\left(S_{t-1} - \bar{S}\right) + x_t.$
2. Environmental damage	$D\left(S_{t-1}\right) = \frac{g}{2}\left(S_{t-1} - \bar{S}\right)^2.$
3. Abatement capital growth	$K_t = \delta K_{t-1} + I_t.$
4. Investment cost	$C\left(I_t\right) = \frac{d}{2} I_t^2.$
5. "Business as usual" emissions	$x_t^b = m_0 - m_1 K_{t-1} + \tilde{\theta}_t.$
6. Abatement cost	$A\left(x_t\right) = \frac{b}{2}\left(x_t^b - x_t\right)^2.$
7. "General" benefit function	
$B\left(K_{t-1}, \theta_t, x_t\right) = f_0 + \left(f_1 + \psi\theta_t\right) K_{t-1} - \frac{f_2}{2} K_{t-1}^2 + \left(a - \phi K_{t-1} + \theta_t\right) x_t - \frac{b}{2} x_t^2.$	
Parameter restriction:	
$0 \leq \Delta \leq 1, \quad g > 0, \quad 0 \leq \delta \leq 1, \quad d > 0, \quad m_0 > 0, \quad m_1 \geq 0, \quad b > 0.$	
Relation of parameters:	
$\theta_t = b\tilde{\theta}_t$, $f_0 = -\frac{b}{2} m_0^2$, $f_1 = b m_0 m_1$, $f_2 = b m_1^2$, $a = b m_0$, $\phi = b m_1$, and $\psi = \frac{\phi}{b} = m_1$.	

Table 1: The Model of Global Warming.

costs. This limiting case reproduces previous linear-quadratic models of a stock pollutant (Karp and Zhang 2002a). (When $\rho \neq 0$ taxes have an informational advantage in this limiting case.)

Table 2 lists baseline parameter values. In presenting the simulation results, we use the parameter π, defined as the percentage loss in Gross World Product due to a doubling of greenhouse gasses. This parameter is linearly related to g, the slope of marginal damages. Our baseline parameters assume that $\pi = 1.33$, an estimate that has been widely used (Nordhaus 1994b). For comparison, we also discuss results when $\pi = 3.6$ (the average of expert opinions, reported in Nordhaus (1994a)) and $\pi = 21$ (the maximum of these expert opinions).

Appendix 3 explains our calibration of the abatement costs (rows 3-6 of Table 1). Our companion paper (Karp and Zhang 2002b)[10] describes the calibration of the growth and damage functions (rows 1 and 2 of Table 1) and of the equation for the random shock (equation (2)). Appendix 4 (not intended for publication) reviews this material.

[10]That paper studies the problem in which the regulator learns about the relation between pollution stocks and environmental damages; there we ignore abatement capital.

Parameter	Note	Value
β	a continuous discount rate of 5%	0.9512
Δ	pollutant stock persistence	0.9917
δ	capital stock persistence	0.85
π	the percentage loss in GWP from doubling $\bar{S}$	1.33
g	slope of the marginal damage billion \$/(billion tons of carbon)^2	0.0022
b	slope of the marginal abatement cost, billion \$/(billion tons of carbon)^2	26.992
d	slope of the marginal investment cost, billion \$	703.31
m_0	intercept of the BAU emissions, billion tons of carbon	12.466
m_1	slope of the BAU emissions, (billion tons of carbon)/(billion \$)	0.7266
ρ	cost correlation coefficient	0.90
σ_μ	standard deviation of cost shock, \$/(ton of carbon)	1.7275
x_0^b	current CO_2 emissions into the atmosphere billion tons of carbon	5.20
$\bar{S}$	preindustrial stock, billion tons of carbon	590
S_{-1}	current pollutant stock, billion tons of carbon	781
K_{-1}	initial capital stock, billion \$	10

Table 2: Parameter Values for the Baseline Model.

6.2 Numerical Results

We begin by summarizing results from earlier static and dynamic models that exclude abatement capital. We then discuss new results – those directly related to abatement capital.

6.2.1 Previous results

Previous papers study the relation between the policy ranking and parameters in the linear-quadratic model with additive errors (Hoel and Karp in press), (Karp and Zhang 2002a), (Newell and Pizer in press). Those papers show that the difference in payoffs under optimal taxes and quotas, $\mathcal{J}^T - \mathcal{J}^Q$, is decreasing in $\frac{g}{b}$. The intuition is the same as in Weitzman (1974)'s static model. A larger value of g means that damages are more convex in S. In view of Jensen's inequality, as damages become more convex it becomes more important to control emissions exactly (as under a quota) rather than to choose only the expected value of emissions (as under a tax). A higher value of b makes it more important for the firm to be able to respond to changes in the cost variable by changing emissions. It is able to respond under a tax but not under a quota.

There is a critical value of $\frac{g}{b}$ above which quotas are preferred. This critical value is decreasing in both β and Δ. When more weight is put on future costs and benefits (higher β), or when the stock is more persistent (higher Δ), it is more important to control the exact level of emissions (as under quotas) rather than the first moment of emissions (as under taxes).

The previous papers calibrate models using parameter values that are consistent with published estimates of the abatement costs and environmental damages associated with greenhouse gasses. These studies find that taxes dominate quotas for the control of greenhouse gasses.

These qualitative results also hold for our parameterization of the model with endogenous abatement capital. This robustness is worth noting, but our analysis adds nothing to the intuition for these results, and therefore we do not discuss them further. Instead, we emphasize the comparative statics and dynamics associated with endogenous abatement costs.

6.2.2 The role of abatement capital

There are three important parameters related to abatement capital: δ, d, and m_1. We consider the first two briefly, and then concentrate on the third. In all cases, we perform the obvious experiment of varying one of these parameters, holding all others constant. This experiment has a shortcoming that we discuss later, where we consider a second type of experiment.

We explained why a more durable pollution stock (higher Δ) decreases the preference for taxes. However, a more durable capital stock (higher δ) increases the preference for taxes. Under taxes, the firm responds to a cost shock by changing the level of emissions. Under both taxes and quotas, the firm responds to a cost shock by changing the level of capital, provided that $\rho \neq 0$.[11] The adjustment mechanism via capital provides a partial substitute for the inability

[11] If $\rho = 0$, the current cost shock provides no information about the future cost shocks. Since current investment reduces abatement costs only in future periods, the firm's investment does not depend on the current cost shock if

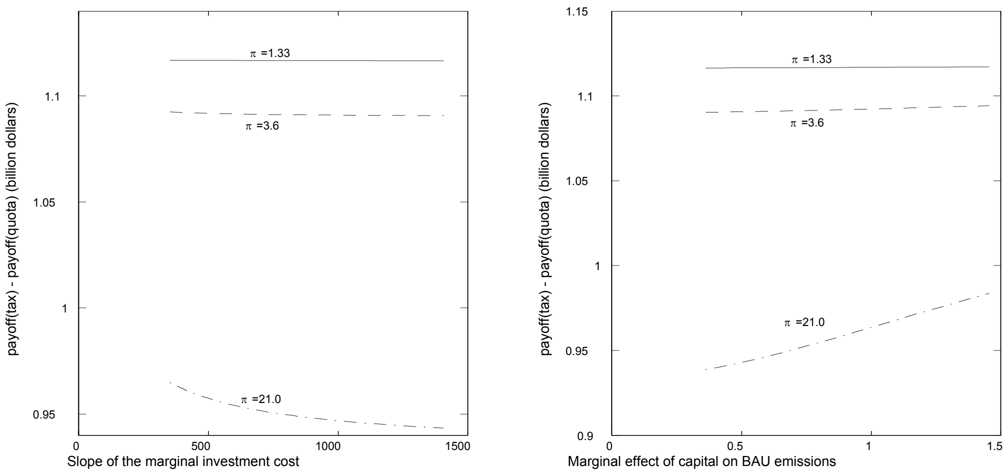

Figure 1: Dependence of expected payoff difference on cost-related parameters

to change emissions under quotas. A large value of δ means that current investment has long-lasting effects, tending to make capital less flexible. The decreased flexibility associated with larger values of δ increases the value of being able to respond to cost variables by changing emissions. A larger value of δ therefore increases the advantage of taxes.

A lower value of m_1 (a decrease in the marginal effect of capital on BAU emissions) or a larger value of d (an increase in the adjustment cost for abatement capital), favors quotas. Figure 1 shows the relation between the difference in payoffs (the value of using taxes minus the value of using quotas) and the parameters d and m_1 for three values of π, holding all other parameters constant. (Recall that π is the percentage loss in global world product due to a doubling of greenhouse gasses.) When environmental damages are moderate ($\pi = 1.33$ or $\pi = 3.6$) the difference in payoffs is insensitive to changes in d and m_1; for large environmental damages ($\pi = 21$) the change in either parameter has a noticeable affect on the payoff difference. Previous linear-quadratic models that do not include investment capital are a special case of the model here, obtained by letting $d \to \infty$ or $m_1 \to 0$. Those models tend to understate the advantage of using taxes.

As d increases, capital increasingly resembles a fixed input; as m_1 decreases, abatement capital has less effect on the marginal benefit of pollution. A larger value of d or a smaller value of m_1 both imply less flexibility of marginal abatement costs. This diminished flexibility

$\rho = 0$.

favors quotas, just as does the diminished flexibility in marginal abatement costs associated with a smaller value of b (the slope of B_x).

In all cases, the present discounted value of the payoff difference under taxes and quotas is approximately 1 billion dollars, implying an annualized cost of about 50 million dollars. Our parameterization of abatement costs assumes that the annualized cost of stabilizing emissions is about 1 percent of income, or 290 billion dollars. Thus, the payoff difference of the two policies is less than 0.02% of the estimated costs of stabilizing emissions.

The small difference in the expected payoffs may be due largely to the Principle of Certainty Equivalence, mentioned in Section 5: the expected stock trajectories are identical under taxes and quotas – only higher moments differ. Uncertainty in our calibrated model (but not in the general formulation) arises only because BAU emissions are uncertain. Given the (small) magnitude of this particular type of uncertainty, the higher moments of stocks simply are not very important. Models that do not satisfy the Principle of Certainty Equivalence find a larger payoff difference under taxes and quotas (Hoel and Karp 2001), (Pizer 1999).

The relations between the equilibrium decision rules and levels of the state variables are as expected. The optimal quota (which equals the expected level of emissions under the optimal tax) decreases with the level of pollution and with the capital stock and increases with the lagged cost shock (for $\rho > 0$, as in our calibration). Equilibrium investment is an increasing function of the stock of pollution and a decreasing function of capital stock. Firms understand that a higher pollution stock will lead to lower future equilibrium emissions, increasing the marginal value of investment. A higher aggregate capital stock encourages the regulator to reduce future emissions, increasing the value of investment. However, the representative firm's level of capital equals the aggregate level. For a given quota or tax, a higher capital stock reduces the marginal value of investment. The net effect of higher capital stocks is to reduce investment.

For our baseline parameters, the optimal quota during the initial period is 4.77 GtC, an 8.3% decrease from the BAU level. It is interesting to compare this level of abatement with the Kyoto Protocol proposals. The Protocol required industrialized countries to reduce their collective emissions of greenhouse gasses by 5.2% of 1990 levels by the year 2012. Targets for individual countries varied; the reduction for the US was to have been 7%. (UNFCC 1997).

In 2001 the US rejected the Kyoto Protocol. A number of prominent economists supported this rejection on the grounds that satisfying the Protocol would be too expensive. Our results

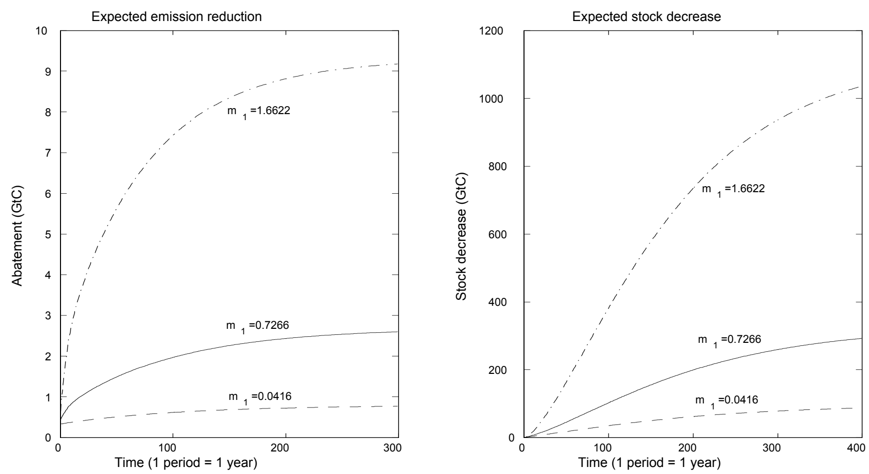

Figure 2: Changes in expected pollution flows and stocks relative to BAU levels

suggest that the proposed levels of abatement might have been close to optimal. Our simulation results do not provide independent evidence supporting the Kyoto Protocol, since our calibration uses the same kinds of assumptions that form the basis for the targets in the Protocol. However, our results suggest that the targets in the Protocol are at least approximately consistent with an optimizing model based on widely held assumptions.

As we mentioned above, the comparative dynamics associated with a change in a single parameter value might be misleading. For example, when we decrease m_1 holding other parameters constant, we change the BAU level of emissions and the abatement costs associated with a particular emissions trajectory, in addition to changing the marginal effect of capital on abatement costs. Here we consider a slightly different experiment: When we vary m_1 we make offsetting changes in m_0 in order to maintain current BAU emissions at 5.2, and we require that the year 2100 BAU emissions are consistent with a particular IPCC scenario.

Our baseline calibration ($m_1 = 0.7266$) makes our model consistent with the *IPCC IS92a* scenario that projects BAU CO_2 stocks of 1500 GtC in the year 2100 – an approximate doubling of stocks relative to pre-industrial levels. For comparison we also choose parameters that are consistent with the *IS92c* scenario of a 35% increase in CO_2 concentration ($m_1 = 0.0416$) and with the *IS92e* scenario of a 170% increase in CO_2 concentration ($m_1 = 1.6622$).

Figure 2 graphs optimal abatement levels, i.e. the difference in the BAU and the optimal levels of emissions (the left panel) and the difference between BAU and the regulated pollution stock (the right panel), as a function of time. The three graphs in each panel correspond to the three values of m_1. In all cases, abatement increases over time. Both the level and the change over time of abatement is greatest when abatement capital has a large effect on marginal abatement costs (m_1 is large). This result is further evidence that the consideration of endogenous investment in abatement capital increases the optimal level of abatement.

7 Conclusions

We studied the problem of optimally controlling a stock externality under asymmetric information about abatement costs. Non-strategic firms have rational expectations about future policies, and they make investment decisions in abatement capital. The regulator is able to use either emissions taxes or quotas, but he is not able to make credible commitments about levels of future emissions policies. We considered a Markov Perfect equilibrium, i.e. one in which agents condition their beliefs on payoff-relevant variables.

When the regulator uses quotas, the inability to make binding commitments (the Markov Perfect restriction) is not binding. In general, this inability to make commitments does reduce the regulator's payoff when he uses emissions taxes. These taxes create a secondary distortion in investment. Thus, we have identified a previously unnoticed difference between taxes and quotas.

If a "separability condition" holds, the firm's emission response to the tax and to the level of capital are both independent of the cost shock. In this case, the secondary distortion associated with taxes vanishes. If the regulator uses quotas, or if he uses taxes and the separability condition holds, we can obtain the Markov Perfect equilibrium by solving an optimization problem rather than an equilibrium problem.

The linear quadratic model satisfies the separability condition. In this model, if damages are caused by a flow, we obtain closed-form expressions for the policy ranking. When neither policy has an informational advantage – the typical case – the policy ranking is independent of parameters associated with abatement capital (for a flow pollutant). This independence does not hold when damages are caused by the stock rather than the flow of pollution.

We calibrated the linear-quadratic model to describe the problem of global warming. We confirmed that previous results hold in the more general setting where abatement capital is endogenous. In addition, we showed that the endogeneity of abatement capital increases both the level of abatement, and the preference for taxes. We also found that our estimates of optimal abatement levels are similar to target levels in the Kyoto Protocol.

8 Appendix

The appendix consists of four parts. Section 1 contains the proof of Proposition 1. Section 2 provides the formulae for Γ used in equation (18). Section 3 contains information on calibrating adjustment costs. Section 3 contains other calibration information similar to that used in Karp and Zhang (2002b). Section 4 is not intended for publication, but is included here to enable the referee to evaluate the calibration.

8.1 Proof of Proposition 1

We use $\mathbb{J}^j(\cdot)$ $(j = T, Q)$ to denote the regulator's value function in the dynamic game (where the regulator chooses only an emissions policy), and $\mathcal{J}^j(\cdot)$ $(j = T, Q)$ to denote the regulator's value function in the corresponding auxiliary problem (where the regulator chooses an emissions policy and then chooses investment after observing the current cost variable). We want to find conditions under which the equilibrium capital and pollution stocks are identical in the Markov Perfect equilibrium to the game and in the auxiliary problem. Equivalently, we want to find conditions under which the optimal investment tax/subsidy is identically 0 in the auxiliary problem.

(i) *Quotas* When the regulator uses an emissions quota, the Euler equations for investment in the Markov perfect equilibrium (equation (8)) and investment in the auxiliary problem (equation (11)) are identical, as are the corresponding transversality conditions. We need to confirm that the Euler equations for the pollution stock are also identical in the two settings.

In the Markov Perfect equilibrium with quotas the regulator solves the following DPE:

$$\mathbb{J}^Q(K_{t-1}, S_{t-1}, \theta_{t-1}) = \max_{x_t} E_{\theta_t|\theta_{t-1}}\{B(K_{t-1}, \theta_t, x_t) - D(S_{t-1}) - C\left(I_t^Q, K_{t-1}\right) + \beta\mathbb{J}^Q\left(\delta K_{t-1} + I_t^Q, \Delta S_{t-1} + x_t, \theta_t\right)\},$$

subject to the private investment rule $I_t^Q \equiv I^Q(K_{t-1}, \theta_t, S_{t-1})$, which is independent of the current quota level x_t. The stochastic Euler equation for pollution stock is:

$$E_{\theta_t|\theta_{t-1}} B_x(K_{t-1}, \theta_t, x_t) - \beta D'(\Delta S_{t-1} + x_t) - \beta\Delta E_{\theta_{t+1}|\theta_{t-1}} B_x(K_t, \theta_{t+1}, x_{t+1}) = 0.$$

The transversality condition is

$$\lim_{T\to\infty} E_{\theta_T|\theta_{t-1}} \left\{\beta^{T-t} B_x\left(K_{T-1}, \theta_T, x_T\right) S_T\right\} = 0.$$

A straightforward calculation confirms that the corresponding Euler equation and transversality condition in the auxiliary problem are identical to the last two equations. (To obtain the Euler equation in the auxiliary problem we differentiate the DPE (9) with respect to S_{t-1}, using the envelope theorem; we combine the resulting equation with the first order condition equation (10).)

(ii) *Taxes* We first consider the equations that determine the evolution of capital stock. Inspection of the Euler equations for capital (equation (6) in the Markov Perfect equilibrium and equation (15) in the auxiliary problem) establishes that these are identical if and only if the function τ, defined as

$$\tau_t \equiv \beta E_{\theta_{t+1}|\theta_t} \left\{H_{t+1} \frac{\partial \chi_{t+1}}{\partial K_t}\right\},$$

is identically 0. We therefore find necessary and sufficient conditions for $\tau_t \equiv 0$. Note that the assumptions that $B_{xK} < 0$ and $B_{KK} < 0$ imply that $\frac{\partial \chi_{t+1}}{\partial K_t} \neq 0$.

By Lemma 1, the separability condition is equivalent to

Condition 2 *(a)* $\frac{\partial \chi(K_{t-1},\theta_t,p_t)}{\partial p_t}$ *is independent of* θ_t. *(b)* $\frac{\partial \chi(K_{t-1},\theta_t,p_t)}{\partial K_{t-1}}$ *is independent of* θ_t.

We therefore need only show that Condition 2 is necessary and sufficient for $\tau_t \equiv 0$. We first consider sufficiency. If Condition (2a) holds, the first-order condition (14) implies

$$E_{\theta_t|\theta_{t-1}} \left\{H_t\right\} = 0, \qquad \forall t. \tag{19}$$

If Condition (2b) also holds, we can write τ_t as

$$\tau_t \equiv \beta \left(\frac{\partial \chi_{t+1}}{\partial K_t}\right) E_{\theta_{t+1}|\theta_t} \left\{H_{t+1}\right\}.$$

Using equation (19), the last equality implies that $\tau_t \equiv 0$. Clearly the transversality conditions in the two problems are the same.

The necessity of the separability condition follows from the previous argument. If either part of Condition 2 does not hold the function τ is not identically 0. (Of course the equality $\tau = 0$ might hold for some values of the information state, but we need the stronger condition that the equality hold identically, i.e., for all possible values of the information state.)

To complete the proof, we need only check that the Euler equations and transversality conditions for the pollution stock are also the same in the two problems. In the Markov Perfect equilibrium with taxes, the regulator solves the following DPE:

$$\begin{aligned}\mathbb{J}^T\left(K_{t-1}, S_{t-1}, \theta_{t-1}\right) = \max\nolimits_{p_t} E_{\theta_t|\theta_{t-1}} \Big\{ B\left(K_{t-1}, \theta_t, \chi_t\right) - D\left(S_{t-1}\right) - C\left(I_t^T, K_{t-1}\right) \\ + \beta \mathbb{J}^T\left(\delta K_{t-1} + I_t^T, \Delta S_{t-1} + \chi_t, \theta_t\right) \Big\},\end{aligned} \tag{20}$$

subject to emissions χ_t given by equation (5), and the private investment rule $I_t^T \equiv I^T\left(K_{t-1}, \theta_t, S_{t-1}\right)$. I_t^T is independent of the current tax level p_t as discussed in Section 3; $\frac{\partial \chi_t}{\partial p_t}$ is independent of θ_t because of Condition 1. Thus the first order condition for the optimal tax is

$$E_{\theta_t|\theta_{t-1}} \Big\{ B_x\left[K_{t-1}, \theta_t, \chi\left(K_{t-1}, \theta_t, p_t\right)\right] + \beta \mathbb{J}_S^T\left[K_t, \Delta S_{t-1} + \chi\left(K_{t-1}, \theta_t, p_t\right), \theta_t\right] \Big\} = 0. \tag{21}$$

Differentiating the DPE (20) with respect to S_{t-1}, using the envelope theorem, and combining the resulting equation with the first order condition (21) gives the stochastic Euler equation for the pollution stock in the dynamic game:

$$\begin{aligned}E_{\theta_t|\theta_{t-1}} \left\{ B_x\left[K_{t-1}, \theta_t, \chi\left(K_{t-1}, \theta_t, p_t\right)\right] - \beta D'\left[\Delta S_{t-1} + \chi\left(K_{t-1}, \theta_t, p_t\right)\right] \right\} \\ - \beta \Delta E_{\theta_{t+1}|\theta_{t-1}} B_x\left[K_t, \theta_{t+1}, \chi\left(K_t, \theta_{t+1}, p_{t+1}\right)\right] = 0\end{aligned} \quad . \tag{22}$$

The transversality condition is

$$\lim_{T\to\infty} E_{\theta_T|\theta_{t-1}} \left\{ \beta^{T-t} B_x\left[K_{T-1}, \theta_T, \chi\left(K_{T-1}, \theta_T, p_T\right)\right] S_T \right\} = 0.$$

Again, it is straightforward to obtain the Euler equation for pollution stocks in the auxiliary problem. We differentiate equation (13) with respect to S_{t-1}, using the envelope theorem. Combining the resulting equation with the first order condition (19) leads to the stochastic Euler equation for the pollution stock in the auxiliary problem. This equation is identical to equations (22). The transversality conditions are also the same. *QED*

8.2 Formulae for Γ

The function Γ used in equation (18) is

$$\Gamma = \frac{\beta^2 \rho^2 \phi^2 \frac{(d-\beta h)}{b\left(1+\frac{\beta g}{b}\right)^2 (d-\beta h - d\beta\rho)^2} + \frac{\beta\rho^2}{1+\frac{\beta g}{b}}}{1-\beta\rho^2} > 0$$

with

$$\Xi \equiv \left(f_2 - \frac{\phi^2}{b+\beta g}\right)\beta - d\left(1-\beta\delta^2\right)$$

$$h=\frac{-\Xi-\sqrt{\Xi^2+4\beta d\left(f_2-\frac{\phi^2}{b+\beta g}\right)}}{2\beta}<0$$

8.3 Calibration of Abatement costs and the shock

We assume that abatement capital depreciates at an annual rate of 16.25%, the mean of capital stock depreciation rates in 14 OECD countries (Cummins, Hassett, and Hubbard 1996). This depreciation rate implies that $\delta=0.85$.

A higher unit of abatement capital decreases the BAU emissions by m_1 units. When $m_1=0$, BAU emissions are constant, and abatement capital has no effect on the marginal benefit of pollution (i.e., on marginal abatement costs). In this special case, the firm's emission decision and investment decision are decoupled, and the firm's capital stock has no effect on the regulator's optimal policy. The restriction $m_1=0$ therefore reproduces the linear-quadratic models of global warming in Karp and Zhang (2002a).

The dependence of adjustment costs on gross rather than net investment leads to a simple method of calibration. In the absence of additional regulation – i.e., under Business as Usual – firms never invest: $I_t^b=0, \forall t\geq 0$. If the initial level of abatement capital is positive, the level monotonically decreases over time, so BAU emissions monotonically increase:

$$K_t^b=\delta^{t+1}K_{-1},\qquad x_t^b=m_0-m_1K_{t-1}^b+\tilde{\theta}_t=m_0-m_1\delta^tK_{-1}+\tilde{\theta}_t,$$

where $K_{-1}>0$ is the abatement capital at the beginning of the initial period ($t=0$). Our assumptions provide a simple way to include endogenous investment, and also to reproduce the stylized fact that BAU emissions will increase. The model is "incomplete", since it does not explain why $K_{-1}>0$. The expected future BAU atmospheric CO_2 stock is:

$$S_t=\Delta^{t+1}S_{-1}-m_1K_{-1}\frac{\delta^t\left[1-\left(\frac{\Delta}{\delta}\right)^{t+1}\right]}{1-\frac{\Delta}{\delta}}+\left[m_0+(1-\Delta)\bar{S}\right]\frac{1-\Delta^{t+1}}{1-\Delta},\tag{23}$$

where S_{-1} is the pollutant stock at the beginning of the initial period.

The current anthropogenic fluxes of CO_2 into the atmosphere is 5.2 GtC[12] so we set $Ex_0^b=$

[12] We use "current" to mean the year 2000. The current total anthropogenic CO_2 emissions are about 8.12 GtC, which equals the sum of 6.518 GtC of global CO_2 emissions from fossil fuel combustion and cement production (Marland, Boden, Andres, Brenkert, and Johnston 1999) and 1.6 GtC annual average net CO_2 emissions from changes in tropical land-use (Intergovernmental Panel on Climate Change 1996). We obtain the current anthropogenic fluxes of CO_2 into the atmosphere 5.20 GtC by multiplying the total anthropogenic emissions by 0.64, the marginal atmospheric retention ratio.

$m_0 - m_1 K_{-1} = 5.2$ to obtain one calibration equation. The *IPCC IS92a* scenario projects BAU CO_2 stocks at 1500 GtC in 2100 (Intergovernmental Panel on Climate Change 1996), page 23. This estimate, equation (23), and the estimate of current atmospheric CO_2 concentration at $S_{-1} = 781$GtC (Keeling and Whorf 1999), gives a second calibration equation. The two equations imply

$$m_0 = 12.466, \qquad m_1 K_{-1} = 7.2661.$$

We have no data on abatement capital, so we choose an arbitrary value for K_{-1}.[13] We set $K_{-1} = 10$.

We choose the baseline values of d (the slope of the marginal investment cost) and b (the slope of the marginal abatement cost) to satisfy a scenario in which firms are required to maintain emissions at the current level in each period. Firms begin with the initial abatement capital and solve an infinite horizon investment problem to minimize the present discounted sum of investment and abatement cost under emission stabilization. In order to determine the two unknown parameters, we assume:

- The annualized discounted present value of firms' total (abatement-related) costs is about 1% of 1998 GWP (Manne and Richels 1992).[14]
- In the steady state the ratio of investment costs to total abatement costs is about 0.5 (Vogan 1991).

These two assumptions lead to the baseline parameter values: $d = 703.31$, and $b = 26.992$.

8.4 Calibration material not intended for publication

Row 1 in Table 1 is pollutant stock growth equation. We measure S_t, the CO_2 atmospheric concentration, in billions of tons of carbon equivalent (GtC). $\bar{S}$ equals 590GtC, the preindustrial CO_2 concentration (Neftel, Friedli, Moor, and Lötscher and H. Oeschger and U. Siegenthaler and B. Stauffer 1999). Let e_t be total anthropogenic CO_2 emissions in period t. The proportion

[13] Even for pollution problems that have been studied in more detail, data on abatement capital is difficult or impossible to obtain. For example, Becker and Henderson (1999) note the absence of estimates of abatement capital stocks associated with U.S. air quality regulation.

[14] Manne and Richels (1992) estimate that the total global costs of stabilizing CO_2 emissions at the 1990 level are about 4,560 billions of 1990 US dollars, or 20.25% of the 1990 GWP. We take the same percentage loss and use the annuanlized value $(1 - \beta) \times 20.25\% = 1\%$.

of emissions contributing to the atmospheric stock is estimated at 0.64 (Goulder and Mathai 2000), (Nordhaus 1994b). This fraction accounts for oceanic uptake, other terrestrial sinks, and the carbon cycle (Intergovernmental Panel on Climate Change 1996). The linear approximation of the evolution of the atmospheric pollutant stock is

$$S_t - 590 = \Delta \left(S_{t-1} - 590\right) + 0.64 e_t.$$

This equation states that 64% of current emissions contribute to atmospheric CO_2, and that CO_2 stocks in excess of the preindustrial level decays naturally at an annual rate of $1 - \Delta$. We take $x_t \equiv 0.64 e_t$, the anthropogenic fluxes of CO_2 into the atmosphere, as the control variable. The stock persistence is $\Delta = 0.9917$ (an annual decay rate of 0.0083 and a half-life of 83 years) (Goulder and Mathai 2000), (Nordhaus 1994b).

We assume that the preindustrial CO_2 concentration has zero environmental damage. Damages from higher CO_2 concentration are $\frac{g}{2}\left(S - \bar{S}\right)^2$. (Row 2 in Table 1). For ease of interpreting the numerical values, we use π to denote the percentage loss in GWP (Gross World Product) from a doubling of the preindustrial CO_2 concentration. With the 1998 GWP of 29,185 billion dollars (International Monetary Fund 1999) we have

$$\pi\% \cdot 29185 = g/2 \cdot 590^2 \implies g = 0.0017\pi.$$

For example, $\pi = 1.33$ which is widely used corresponds to $g = 0.0022$. For the sensitivity analysis we consider two other damage parameters, $\pi = 3.6$ and $\pi = 21.0$, the mean and the maximum of expert opinions.

Using maximum likelihood, we fit the following data generating process for global carbon emissions over the 50 year period 1947-1996 from Marland, Boden, Andres, Brenkert, and Johnston (1999).

$$e_t = e_0 + nt + \varepsilon_t, \quad \varepsilon_t = \rho\varepsilon_{t-1} + \nu_t, \quad \nu_t \sim iid\, N\left(0, \sigma_v^2\right).$$

The estimates are $\rho = 0.9$ and $\sigma_v = 0.1$GtC. We convert the emission uncertainty σ_v into cost uncertainty σ_μ by multiplying it by 0.64 (because $x_t \equiv 0.64 e_t$), and then by the slope of marginal abatement cost $b = 26.992$ (because $\theta_t \equiv b\tilde{\theta}_t$). The result is $\sigma_\mu = 0.1 \times 0.64 \times 26.992 = 1.7275\$/$(ton of carbon).

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